

A list of the Definitions and Propositions in Book I of Euler's *Mechanica.*

This list should enable you to locate the sections of interest in the various chapter files.

CH. I: Concerning Motion In General.

DEFINITION 1.

1. Motion is the translation of a body from one place that it occupies to another place. Truly a body remaining in the same place is at rest.

DEFINITION 2.

4. The place is a part of the immense or boundless space which constitutes the whole world. This place is usually called the absolute position which is apparent to the senses, in order that it may be distinguished from relative places, of which mention will soon be made.

DEFINITION 3.

9. Relative motion is the change in the position of a body that it is agreeable to assume within a space. And relative rest means that the body stays in the same place with respect to the same space. Thus taking the earth for this space, we say that those bodies are at rest that maintain their own unchanging places on the earth; and truly we are accustomed to say only those to be moving that proceed from one place to another with respect to the earth. In a similar manner for a ship's movement, those bodies that remain relatively at rest stay in the same place on the ship, while these are relatively moving that change their position on the ship.

PROPOSITION 1. Theorem.

13. Every body, that is carried from one place to another place by relative or absolute motion, passes through all the intermediate positions, and it is not able to suddenly arrive at the final place.

DEFINITION 4.

17. A body is said to be moving equably or uniformly, that in equal intervals of time traverses equal distances. A motion is truly not equable that in equal times travels through unequal distances, or in traveling through equal distances are resolved by unequal intervals of time.

DEFINITION 5.

21. Every body that is moving is said to have a speed or velocity, and this is measured by the distance that body traverses in equal intervals of time, with a uniform motion. Clearly when body B travels twice the distance at a uniform speed that body A travels through in the same time, also moving uniformly, then body B is said to have a speed twice as great as body A.

PROPOSITION 2. Theorem.

25. For two bodies progressing with uniform motion, the speeds vary directly with the distances traveled, and inversely with the time with which these distances are traversed.

PROPOSITION 3. Theorem.

33. In motion with any non-uniformity, the smallest elements of the distance are considered to be traversed by uniform motions

PROPOSITION 4. PROBLEM.

37. *A body moves with some kind of motion along the line AM , with the speed of the body given at some point; it is necessary to determine the time in which the arc AM is completed.*

PROPOSITION 5. Theorem.

44. *Two bodies are moving on the right lines AM and am , and the speeds of these bodies are to be expressed by the similar lines attached to the curves AN and an . I say that these bodies travel homologous distances AM and am in the same time.*

DEFINITION 6.

48. *The extent of the speed is represented by a curve, the attached lines of which represent the speeds, which the body has at places corresponding to the distances that the body has traversed.*

DEFINITION 7.

49. *The scale of the times is a curve, of which the applied lines represent the times at which the corresponding parts of the distance have been completed in the motion.*

PROPOSITION 6. PROBLEM.

51. *From the given scale of the times AT to find and construct the scale of the speed AN .*

PROPOSITION 7. THEOREM.

56. *A body remains in a state of absolute rest, unless it is disturbed to move by some external cause.*

PROPOSITION 8. THEOREM.

63. *A body having uniform absolute motion will always be moving, and with the same speed now that it had at an any earlier time, unless an external cause should act on it or have acted on it.*

PROPOSITION 9. THEOREM.

65. *The body with a given absolute motion shall progress in a straight line, or the distance that it describes shall be a straight line.*

DEFINITION 8.

72. *The direction of the motion is to be determined by a straight line, along which the moving body is trying to progress, and according to this the body progresses, unless it is impeded by some external cause.*

DEFINITION 9.

74. *The force of inertia is in all bodies is that in situ faculty of the body to maintain its state of rest or of continuing in its present state of motion in a straight line.*

PROPOSITION 10. THEOREM.

77. *When the space from which the relative motion is determined, is either absolutely at rest or moving uniformly in a direction, then the given laws prevail for a state of relative rest or of relative uniform motion.*

PROPOSITION 11. PROBLEM.

83. *A body can move uniformly with absolute motion along the line AL , and another body likewise moves uniformly too along the line AM . The size of the relative motion of the other body progressing along AM is sought in relation to the body progressing along AL .*

PROPOSITION 12. PROBLEM.

93. *The body A moves in some manner along the line AL with absolute motion, and the body B along the line BM. The relative motion of body A is required with respect to body B.*

CH. II: CONCERNING THE EFFECT OF FORCES ACTING ON A FREE POINT.

DEFINITION 10.

99. *A force is an action on a free body that either leads to the motion of the body at rest, or changes the motion of that body.*

DEFINITION 11.

103. *The direction of the force is the straight line along which the body is trying to move.*

PROPOSITION 13. THEOREM.

107. *When a point is acted on by many forces, the same motion comes about from these, as if the point is acted on by a single force equivalent to all of these forces.*

DEFINITION 12.

111. *The absolute force is the force that acts equally on a body either at rest or moving.*

DEFINITION 13.

113. *A force is relative, which acts in one way on a body at rest, in another on a body in motion.*

PROPOSITION 14. PROBLEM.

118. *For the given effect of an absolute force on a point at rest, to find the effect of the same absolute force on the same point in some kind of motion.*

PROPOSITION 15. PROBLEM.

130. *For a given increment of the speed, that a certain force produces on the point A in the small increment of time dt , to find the increment of the speed, that the same force produces on the same point in the time increment dt .*

PROPOSITION 16. THEOREM.

136. *The force q at the point b has the same effect that the force p has at the point a , if the ratio between the forces and distances is of the form $q : p = b : a$.*

PROPOSITION 17. THEOREM.

142. *The force of inertia of any body is proportional to the quantity of matter, upon which it depends.*

PROPOSITION 18. PROBLEM.

146. *With the effect of one force on some point given, to find the effect of any number of forces acting on the same point.*

PROPOSITION 19. THEOREM.

150. *A point can be moved along the direction AM and it is acted on, while it traverses the small distance Mm , by a force p pulling in the same direction ;the increase in the speed, that the point meanwhile acquires, is as the product of the force by the short time, in which the element of distance Mm is traversed.*

PROPOSITION 20. THEOREM.

154. *The motion of the point in a direction in agreement with the direction of the force, the increment of the speed will be as the force taken with the element of time, and divided by the quantity of matter of the point is composed.*

PROPOSITION 21. PROBLEM.

160. *To determine the effect of any oblique forces acting on a moving point.*

Definition 13.

175. *The force of restitution is that imaginary infinite force, which restores the separate parts of the body again to their previous state.*

PROPOSITION 22. THEOREM.

179. *Let there be two parts of a point separated at b and d ; I say that these are to be joined together by a force of restitution in the point c , at the centre of gravity of the particular b and d .*

PROPOSITION 23. THEOREM.

183. *Let a , b , c , and d be parts of a point mutually separated, which are to be joined together again by the force of restitution, and these are in agreement with a common centre of gravity g .*

GENERAL SCHOLIUM.

188. In which Euler sets out an outline for the great task that lies ahead

CH. IIIa: CONCERNING THE RECTILINEAR MOTION OF A FREE POINT ACTED ON BY ABSOLUTE FORCES.

PROPOSITION 24. THEOREM.

189. *When the directions of the motion and of the force are along the same straight line, the motion will be rectilinear.*

PROPOSITION 25. PROBLEM.

193. *The point A is resting on the line AP , and is to be pulled forwards by a uniform force, or which acts with the same strength everywhere, and the speed of the point is to be determined at any position P .*

DEFINITION 15.

200. *Hereafter we will call the height corresponding to the speed that height, from which a weight falling to the surface of the earth, acquires that same speed.*

PROPOSITION 26. THEOREM.

210. *The heights, by which equal small bodies fall to acquire equal speeds, vary inversely as the forces, under the hypothesis of different uniform [gravitational] forces.*

PROPOSITION 27. PROBLEM.

218. *For the point A is to be moved forwards through the distance AP by a uniform force, and it is required to find the time in which the distance AP is completed.*

PROPOSITION 28. THEOREM.

230. *For a body falling through the distance AP , as we have put in place as hitherto, the speed at P is of such a size, that if it progressed uniformly at this speed for the same time in which the body had fallen through AP , then it would be able to complete a distance twice as great as AP .*

PROPOSITION 29. PROBLEM.

239. *The body now initially at B has a given speed along the line BP , and with a uniform force present acting along the line BP ; the speed of this body is required at any point P of the line BP .*

PROPOSITION 30. PROBLEM.

242. *With everything put in place as in the preceding proposition, to determine the time in which the distance BP is run through.*

PROPOSITION 31. PROBLEM.

245. *The body has a given speed in the upwards direction and with a uniform force pulling in downwards ; the speed of the body is required at any point in the interval BA that it has travelled through while rising.*

Definition 16.

253. *That fixed point is called the centre of attraction, to which bodies are attracted by a force, which depends on the distance from this point, or which is as some fraction of this distance.*

Definition 17.

260. *The force, which presses upon bodies to the centre of this kind of force, is called the centripetal force. And that, if it is negative, in order that the body is repelled from the centre, is called the centrifugal force.*

PROPOSITION 32. PROBLEM.

264. *Let C be the centre of the forces that attract bodies in some ratio of the distance, and by this force a body at rest at A is drawn forwards; the speed of this body is then sought at any point in the interval AC.*

PROPOSITION 33. PROBLEM.

274. *With the attraction from the centre C to be in some ratio of a multiple of the distances, the body at D now has a given speed ; the point A is required on the line CD produced, from which the descent of the body towards C begins, so that it has acquired this speed when it arrives at D.*

PROPOSITION 34. PROBLEM.

282. *If the centripetal force is in proportion to the distance from the centre C and the body falls from A as far as C, it is required to determine the time in which the body completes any part of this distance.*

CH. IIIb:

PROPOSITION 35. PROBLEM.

286. *If the centripetal force is inversely proportional to the square of the distance from the centre C and the body falls from A as far as C, the time is to be found in which the body traverses any portion of this distance AC.*

PROPOSITION 36. PROBLEM.

290. *To determine the times of descent through the distance AC to the centre of the force C, if the centripetal force is proportional to the reciprocal of the distances considered, and the exponent of this distance is $\frac{2m+1}{2m-1}$, with the number m denoting a whole positive number.*

PROPOSITION 37. PROBLEM.

297. *To determine the time of descent through AC to the centre of force C, if the centripetal force varies inversely with the reciprocal of the distance, the exponent of which is raised to the power $\frac{m-1}{m}$, with m denoting some positive integer.*

PROPOSITION 38. THEOREM.

304. *With the centripetal force present varying inversely as the distance from the centre of force C the time of descent through the whole distance $AC = \frac{a\sqrt{\pi}}{\sqrt{f}}$, with a denoting the distance AC, f the distance at which the centripetal force is equal to the force of gravity, and $\pi : 1$ the ratio of the periphery to the diameter of a circle.*

PROPOSITION 39. THEOREM.

308. *If the centripetal force is as the power of the exponent of the distance n and many bodies are released to fall from different distances, the times of the descents are proportional to the powers of the distances, of which the exponent is $\frac{1-n}{2}$.*

PROPOSITION 40. PROBLEM.

314. *From the centre of force C with the body itself being repelled in the ratio of the n^{th} power of the distances along the line CP; it is required to find the speed of this body at any point P and the time in which the interval CP is traversed.*

PROPOSITION 41. PROBLEM.

321. *If the centripetal force is proportional to some function of the distance from the centre C, and the body is dropped towards C from A, the speed of the body is to be found at any point P and the time in which the interval AP is traversed.*

PROPOSITION 42. PROBLEM.

326. *With the speed of the body traveling on the line AP given at the individual points, the law of the force acting is to be found that brings about this motion.*

PROPOSITION 43. PROBLEM.

332. *With the time given, in which the body progressing on the straight line AC passes through the particular interval AP, it is necessary to define the law of the force which is effected, in order that the body is carried forwards by this motion.*

PROPOSITION 44. PROBLEM.

337. *If the body thus falls along the line AP, so that it has the speed at P in the same time as it has traversed the distance AP, with which it could traverse the distance PM of the adjoining given curve AM with this uniform speed; it is necessary to determine the law of the force acting, by which such motion is generated.*

PROPOSITION 45. PROBLEM.

347. *With the speeds given which a body acquires, falling from any distances towards the centre of force C, to define the law of the centripetal force producing the descents of this kind, with the position in which the body begins its individual descents taken from rest.*

PROPOSITION 46. PROBLEM.

354. *If the times, in which the body reaches the centre of force C from any distances PC, are in the ratio of some multiple of the distances, then the law of the centripetal force can be defined.*

PROPOSITION 47. PROBLEM.

359. *From the given scales of the forces BND, by which the body is acted upon falling through the distance AC, to find innumerable others such as $\beta v \delta$, by which the body acted on by a force at C always acquires the same speed, with the body always starting from rest at A.*

PROPOSITION 48. PROBLEM.

363. *From the given scales of the forces BND , by which a body acted upon traverses the interval AC , to find innumerable other curves such as $\beta v\delta$, by which it is effected, that the body completes the interval AC in the same time.*

CH. IVa: CONCERNING THE MOTION OF FREE POINTS IN A MEDIUM WITH RESISTANCE.

DEFINITION 18

367. *The law of the resistance is the force or function of the speed of the body, to which the resistance is in proportion.*

DEFINITION 19

376. *The exponent of the resistance is the height corresponding to its speed, which if the body has this speed, then the resistance experienced is equal to the force of gravity*

DEFINITION 20

380. *Here resistive mediums are called similar that have the same law of resistance. Truly these mediums are dissimilar which have different laws of resistance.*

PROPOSITION 49. PROBLEM.

383. *For a body moving along the line AP in a medium with some kind of resistance, both the law and the exponent of which are known, with a given speed at the point P , to find the decrease in the speed as the element of distance Pp is traversed.*

PROPOSITION 50. PROBLEM.

387. *In a medium with uniform resistance, that offers resistance in some power of the speed, to define the speed of the moving body at particular places.*

PROPOSITION 51. PROBLEM.

397. *For a body moving in a medium with uniform resistance, which makes the resistance to be in proportion to some power of the speeds, to determine the time in which the body travels through some interval AP .*

PROPOSITION 52. PROBLEM.

408. *A body that is moving in some medium with resistance, is acted on by some absolute force; to determine the increase or decrease of the speed, while it runs through any element Pp .*

PROPOSITION 53. PROBLEM.

419. *With a uniform absolute force and medium resistance put in place, to determine the speed of descent of the body at individual points, if the resistance is proportional to the square of the speed.*

PROPOSITION 54. PROBLEM.

426. *To determine the time, in which a body in a medium with a uniform resistance present in proportion to the square of the speed, descends through a given interval AP , acted on by a uniform absolute force.*

PROPOSITION 55. PROBLEM.

434. *If a body in a medium with uniform resistance, which resists in the square ratio, is projected up again from B with a given speed, and is acted upon by a given uniform force g , it is required to determine the speed of the body at individual places.*

PROPOSITION 56. PROBLEM.

441. *To determine the time of ascent through BP of a body in a medium with resistance in the ratio of the square of the speed from B to be projected up with a given speed, and meanwhile to be acted on by an absolute force g pulling downwards.*

CH. IVb:

PROPOSITION 57. PROBLEM.

450. *With the time given, in which a body is projected up from B and has fallen again, in a medium with resistance in the ratio of the square of the speed, and acted on by a constant absolute force g , to determine the height BA to which the body rises, in order that both the initial and the final speed at B after the descent to the same location B are found; and also the ascent time through BA and the descent time through AB.*

PROPOSITION 58. PROBLEM.

461. *If a body after some descent is reflected from O up with the same speed that it acquired in the descent, and again ascends straight up, and these reflections are always repeated when it arrives at O, then the altitudes OA, OB, OC, etc., are sought, which the body in this way successively traverses in a medium with a uniform resistance varying as the square of the speed, and acted on by a constant force g .*

PROPOSITION 59. PROBLEM.

469. *With the resistance of a medium uniform and in proportion to the speed, and with the body acted on by an absolute force pulling downwards, to determine the speed of the body upwards or downwards at any point on a straight line.*

PROPOSITION 60. PROBLEM.

475. *With the resistance of the medium in the simple ratio of the speed and acted on by a uniform absolute force, to determine the time in which the body traverses some interval, either ascending or descending.*

PROPOSITION 61. PROBLEM.

481. *The medium offers resistance in a ratio according to some power of the speed and a uniform force is acting, it is necessary to determine the right motion of the body either ascending or descending.*

PROPOSITION 62. PROBLEM.

485. *With a uniform force present, and the exponent of the resistance to be in proportion to the distances from a fixed point C, and with the law of the resistance in some multiple ratio of the speeds, the speed of the body at some place is required on the line AC, advancing or receding from C.*

PROPOSITION 63. THEOREM.

495. *With the same quantities put in place as in the previous proposition, if many bodies fall towards C from different heights, the times at which they arrive there are in the square root ratio of the distances.*

PROPOSITION 64. PROBLEM.

500. *With a centripetal force to be proportional to some power of the distance from the centre C, and with the uniform resistance of the medium as the square ratio of the speed, to determine the speed of the body at individual points P on the right line AC, either moving either up or down.*

PROPOSITION 65. PROBLEM.

509. *For some centripetal force acting towards C , and with the resistance to the motion following the squares of the velocities changed in some way, the motion of the body is to be determined on the line either accelerating towards, or decelerating from C .*

PROPOSITION 66. PROBLEM.

513. *With the law of the centripetal force pulling towards the centre C given, and with the medium resisting in the ratio of the square of the speed, if the speed of the body is given that the body acquires on being dropped from any height, to determine the density or the exponent of the resistance at individual places.*

PROPOSITION 67. PROBLEM.

520. *With the medium resistance in the square ratio of the speed, and with the density of the medium given or the exponent of this at individual places given, to determine the centripetal force which can be acting, in order that the body released from any height towards the centre C , still always takes the same time to arrive there.*

PROPOSITION 68. PROBLEM.

528. *If the centripetal force is in proportion to the distance from the centre and the medium has a resistance in the simple ratio of the speed, it is required to determine the motion of the body as it approaches towards and as it recedes from the centre C .*

PROPOSITION 69. THEOREM.

537. *If the centripetal force is as the n^{th} power of the distance from the centre C , and the medium resists in the ratio of the $2m$ multiple of the speed, truly the resistance is proportional to the $\frac{mn+m-n}{m}$ power of the distance from the centre C , and the times of all the descents or ascents in the whole distance described are in the ratio of the $\frac{1-n}{2}$ power.*

CH. Va: CONCERNING THE CURVILINEAR MOTION OF FREE POINTS ACTED ON BY ABSOLUTE FORCES OF ANY KIND.

DEFINITION 21.

543. *A body describes the curved line AMB when acted upon by a force. The tangential force on the body is the component of the force along the direction of the tangent TMt to the curve at the point M .*

DEFINITION 22.

547. *The normal force is the force acting on the body describing the line AMB (Fig. 47), the direction of which MR is normal to the element of the curve Mm or the tangent MT .*

PROPOSITION 70. PROBLEM.

552. *If a body, as it traverses the element Mm in a plane, is acted on by two forces, the one normal and the other tangential, to determine the effect of each in altering the motion of the body.*

PROPOSITION 71. PROBLEM.

556. *If the body, while it traverses the element Mm , is acted on by some oblique force in the direction MC , then it is required to determine the effect of this force in changing the motion of the body.*

PROPOSITION 72. PROBLEM.

564. *If there is a constant force the direction of which is normal everywhere to the right line AB and if a body is projected from A with a given speed along the direction AH, then it is required to find the curve AMDB described by the body, and the motion of the body on this curve.*

PROPOSITION 73. PROBLEM.

578. *If the body at A with a given speed and projected in a given direction is always attracted to the line AB in some ratio of some function of the distances from this line, to determine the curve ADB, that the body thus acted upon describes and the whole motion on this curve.*

PROPOSITION 74. PROBLEM.

587. *A body is projected at A and everywhere it is pulled towards the centre of force C by some centripetal force, and it is required to determine the nature of the curve AM, in which the body is moving, and the motion of the body on this curve.*

PROPOSITION 75. PROBLEM.

601. *The nature of the curve AM, that a body acted upon by some centripetal force is to be described, in order the equation between the orthogonal coordinates CP and PM referred to fixed axes AC can be defined.*

PROPOSITION 76. THEOREM.

611. *If the centripetal force is as some function of the distance from the centre C, and a body at A is projected following the direction normal to AC with a speed, of which the corresponding height has a ratio to half AC as the centripetal force at A is to the force of gravity 1, then this body moves uniformly on the circumference of the circle AMBA, the centre of which is C.*

PROPOSITION 77. THEOREM.

619. *If more bodies are moving around the centre of force C and describe the similar curves AM and am about C, the speeds at the similar points M and m are in the square root ratio composed from the ratios of the homologous sides and of the centripetal forces at the homologous points M and m.*

PROPOSITION 78. PROBLEM.

631. *If the centre of force C attracts directly in the ratio of the distances and a body is projected from A following the direction normal to a given radius AC, to determine the curve AMDBH that the body describes, and the speed of the body at particular points.*

CH. Vb:

PROPOSITION 79. PROBLEM.

641. *With the attraction from the centre of the forces in the ratio of the distances from C, a body is projected from M with some speed and following some direction MT; it is required to determine the ellipse on which the body is moving.*

PROPOSITION 80. PROBLEM.

644. *If the centripetal force is inversely proportional to the square of the distance and the body is projected with a given speed from A with a given speed in a direction normal to the radius AC, it is required to determine the curve AMDBHA, that the body describes, and the motion itself along this curve.*

PROPOSITION 81. PROBLEM.

656. *With the centripetal force put inversely proportional to the squares of the distances the body is projected from M with any speed and in some nearby direction MT ; from which it is required to determine the ellipse $MDBHAM$, in which the body is moving*

PROPOSITION 82. PROBLEM.

671. *If the centripetal force varies inversely as the cube of the distance from the centre, the curve is required that the body describes projected in any manner, and the motion of the body on that curve.*

PROPOSITION 83. PROBLEM.

685. *With a centripetal force present as some power of the distances, to find the special cases in which a body can be projected in certain ways in order that it moves along a line algebraically.*

PROPOSITION 84. PROBLEM.

695. *If the centripetal force does not disagree much with the ratio of the inverse square with the distances, to determine the motion of the ellipse, and the continual change of the form of the ellipse, and the change of the motion of the body associated with this change of the ellipse.*

PROPOSITION 85. THEOREM.

702. *If a body projected in some way is attracted to several centres of force A, B, C , of which the individual forces are proportional to the distances of the body from these, then the body moves in the same manner, as if it is attracted equally by the common centre of gravity O of the points A, B, C in the simple ratio of the distances.*

CH. Vc:

PROPOSITION 86. PROBLEM.

707. *To find the law of the force continually pulling downwards which can be constructed along the lines MP parallel to each other, in order that the body moves along a given curve AM , and to determine the speed of the body at individual points M .*

PROPOSITION 87. PROBLEM.

714. *For the given curve AMB together with the centre of attraction C , to find the law of the centripetal force, which must act, in order that the body is free to move along that curve, and to find the speed of the body at any position M .*

PROPOSITION 88. THEOREM.

720. *The force pulling towards C , that is put in place in order that the body moves along the given curve AM , has the ratio to the force pulling towards another centre of force c , which is put in place in order that the body can move around the same curve with the same periodic time, as the cube of the line cV from c to the tangent TM drawn parallel to the line CM , is to the volume formed from the line cM multiplied by the square of the line CM .*

PROPOSITION 89. PROBLEM.

729. *If the orbit $(A)(M)(B)$ is revolving around some centre of forces C , it is required to define the centripetal force always pulling towards C , which is put in place, in order that the body is moving in this moveable orbit.*

PROPOSITION 90. PROBLEM.

742. *For the known curve, that a body describes acting under some central force V , to determine the curve, that the body describes acted on the centripetal force $V + \frac{C}{y^3}$, with y*

denoting the distance MC of the body from the centre of forces C .

PROPOSITION 91. PROBLEM.

747. *If the figure that the body describes on being acted on by some central force does differ much from a circle, to determine the motion of the apses.*

CH. Vd:

PROPOSITION 92. PROBLEM.

763. *To find the centripetal forces acting towards two centres C and D , which can be composed, in order that the body moves on a given curve AMB and with the speed given at individual points M .*

PROPOSITION 93. PROBLEM.

767. *A body is moving with a given speed on a curve also given AMB , and it is required to find the centripetal force acting towards the centre C together with a force always acting normally to the line AB pulling the body in the direction MP , which two forces have the effect that the body is free to move on this curve with the prescribed speed.*

PROPOSITION 94. PROBLEM.

771. *If the body is moving on the curve AMB in whatever manner, while the curve itself is revolving around the central fixed point C , it is required to find two forces, one of which is always acting towards the fixed point C , and the other is directed normally to the line in the given position PC , which two forces have the effect that the body is free to move in this orbit.*

PROPOSITION 95. THEOREM.

779. *The body is moving along the curve AM , by some force acting around the point C , and in addition both the body and the point C are acted on by a force in the same direction; there is the relative motion of the body M with respect to the point C or the motion of the body M such as is seen from C , and likewise, if this new force is not to be added.*

PROPOSITION 96. PROBLEMA.

786. *If the body M is revolving around the centre of force L at rest in the curve BM , to determine the force that is effective in order that the body in the same orbit following the same curve AL is moving in a direction always parallel to itself.*

PROPOSITION 97. PROBLEM.

795. *With the sun at rest at S and with the earth T moving around it uniformly in the circle TD while the moon L is attracted to the earth T as to the sun S in the inverse square of the distances; with which put in place it is required to determine the motion of the moon, such as can be seen from the earth T .*

CH. Ve:**PROPOSITION 98. THEOREM.**

802. *There are three principal forces which can be put in place, and into which other forces must be resolved, in order that a body can move on a curve that does not exist in a plane; these individual forces are normal to each other; and of these one is the tangential force, and the remaining two are normal to that force, of which one lies in the given plane, and the direction of the other is normal to this plane, and nothing remains of the original forces to change the actions of these forces.*

PROPOSITION 99. PROBLEM.

812. *To determine the inclination of the plane, in which the elements Mm and $m\mu$ described by a body have been placed, relative to the fixed plane APQ and to find the line of intersection of the two planes.*

PROPOSITION 100. PROBLEM.

823. *If a body is acted on by three forces, of which the directions Mf , Mg and MQ are parallel to the three coordinate axes AP , PQ and QM , to determine the motion of the body and the orbit in which it moves.*

PROPOSITION 101. PROBLEM.

830. *If a body is always acted on by a force towards the axis AP along the perpendicular MP dropped to the body, then it is required to find the motion of the body.*

PROPOSITION 102. PROBLEM.

840. *If a body is acted on at individual points M by two forces, the first in the direction MA , and the other, the direction of which is MQ along the normal sent from M to the plane APQ , then it is required to determine the motion of the body M and its orbit.*

PROPOSITION 103. PROBLEM.

851. *If the body is always attracted partially to some fixed point A in the ratio of the distances from the same, and partially normally to the plane APQ in the ratio of the given distances from this plane as well; it is required to determine the nodes or the points in which the body arrives at this plane, and besides also the points, at which the body is at a maximum distance from the plane.*

CH. VIa: CONCERNING THE CURVILINEAR MOTION OF A FREE POINT IN A RESISTIVE MEDIUM.**PROPOSITION 104. THEOREM.**

860. *If a body is moving in a medium with resistance acted on by some number of absolute forces, the resistive force does not disturb the action of the other absolute forces in any way, except that the tangential force arising from that is diminished.*

PROPOSITION 105. PROBLEM.

866. *If a body is moving in a medium with some resistance and is acted upon by some absolute forces, yet thus, so that the motion is completed in the same plane, to define the rules that the body observes in its motion.*

PROPOSITION 106. PROBLEM.

870. *The force acts normally everywhere to the given line in the position AP and the body moves in a medium with some resistance; it is required to determine the curve AMB in which the body moves, and the motion of the body.*

PROPOSITION 107. PROBLEM.

874. *If both the force and the resisting medium is constant and the direction of the force is along the direction MP normal to the given line AP, and the medium has a resistance in the ratio of the square of the speed, to determine the motion of the projected body.*

PROPOSITION 108. PROBLEM.

884. *If the resistance of the medium were as the speed of the body and the direction of the force along MP, and in addition the force is uniform as is the resistance, it is required to determine the curve that the body describes and the speed at individual points.*

PROPOSITION 109. PROBLEM.

884. *If the body is everywhere attracted downwards equally, and it is projected along the horizontal direction at A with a given velocity in a uniform medium that offers resistance in the simple ratio of the speed, then to determine the curve AM that the body describes and to find the motion of the body on this curve.*

PROPOSITION 110. PROBLEM.

903. *With the absolute uniform force put acting along the vertical direction MP and the medium, that is also put as uniform, with the resistance in some ratio of the multiple of the speeds, to determine the curve AM described by the projected body.*

PROPOSITION 111. PROBLEM.

908. *With an absolute uniform force put in place acting downwards, to determine the resistance at the individual places M which can be put in place, in order that the body describes the given curve BAM.*

PROPOSITION 112. PROBLEM.

916. *With the absolute force g constant and always acting downwards, to determine the resistance which must be put in place in order that the body is free to move along the periphery of the circle BAMD.*

CH. VIb:

PROPOSITION 113. PROBLEM.

925. *As before, with the uniform absolute force g put in place and pulling downwards, to find the force of the resistance which is effective in order that the body can move along the hyperbola NAM freely, with the axis CAQ vertical.*

PROPOSITION 114. PROBLEM.

933. *Let the curve NM be a hyperbola of any other kind having a vertical asymptote CP, and it is required to determine the resistance which is effective, in order that the body, always acted on by a downwards force, moves on this hyperbola.*

PROPOSITION 115. PROBLEM.

952. *Let some variable absolute force that acts downwards along MP be given; to determine the required resistance for this, in order that the body moves along the given curve AM.*

PROPOSITION 116. PROBLEM.

961. *If AM is the given curve and the resistance is given by quantities relating to the curve, to find the absolute force P always acting normal to the axis AC, which can be made, so that the body is free to move in this curve,*

PROPOSITION 117. PROBLEM.

968. *If the medium is uniform and the resistance in the ratio of the square of the velocity, to determine the absolute force acting downwards which can be made, in order that the body in the medium with this resistance describes the given curve AM .*

PROPOSITION 118. PROBLEM.

984. *To find the resistance at individual points M for an absolute force acting downwards along MP , which can be put in place in order that a body can move on a given curve AM , and can move with a given speed at the individual points M .*

PROPOSITION 119. PROBLEM.

991. *If the medium is uniform and offers resistance in the ratio of some multiple of the speed, and it is given besides that the body progresses uniformly along the horizontal AP at a constant speed, to find the force acting downwards and the curve that the body describes.*

CH. VIc:

PROPOSITION 120. PROBLEM.

1005. *A body is always attracted by some force in a medium with some kind of resistance towards the fixed point C ; to determine the curve AM that the body describes projected in some manner.*

PROPOSITION 121. PROBLEM.

1016. *If the centripetal force varies as some power of the distances from the centre, and the body moves in a medium with constant resistance, which resists in the square ratio of the speed, to determine the curve AM that the body describes, and the motion of the body on this curve.*

PROPOSITION 122. PROBLEM.

1025. *In a uniform medium, which resists in the simple ratio of the speed, the body moves attracted to the centre C by a force proportional to some power of the distance; to determine the curve AM that the body describes.*

PROPOSITION 123. PROBLEM.

1033. *If the curve AM is given that the body describes, and the resistance is given at the individual points M , to determine the centripetal force always directed towards the centre C , and the speed of the body at individual points.*

PROPOSITION 124. PROBLEM.

1044. *If the resistance is proportional to some power of the speed and the exponent at particular places is given, the centripetal force arising is to be found in order that the body moves on the given curve AM .*

PROPOSITION 125. PROBLEM.

1056. *If the curve AM is given which the curve describes, and the centripetal force acts towards the centre C , to find the requisite resistance at the individual points M and the speed of the body.*

CH. VI d:

PROPOSITION 126. PROBLEM.

1063. *If the curve AM is given, on which the body is moving, and the angular motion about the centre of force C , to find both the centripetal force attracting the body towards C as well as the resistance at individual places.*

PROPOSITION 127. PROBLEM.

1069. *If the resistance is given by some power of the speed and likewise the exponent of the resistance, also in addition the angular motion of the body about the centre C is given, from these to find the curve that the body describes, and the centripetal force pulling towards the centre C .*

PROPOSITION 128. PROBLEM.

1077. *If the body at M is acted on by two forces, of which the one has the direction MP normal to the given line AC , and the other truly has the direction MQ parallel to AC itself or normal to BC , to determine the curve AM which the body describes in any medium with resistance due to action of these forces.*

PROPOSITION 129. PROBLEM.

1087. *If a body moves in a medium that resists as the square ratio of the speed, and if the force P is to the force Q as MP to MQ or, which is the same, if the body is drawn to the centre C by some force, to determine the curve AMB described by the body.*

PROPOSITION 130. PROBLEM.

1093. *In a medium with some resistance acting a body is acted on by three forces, of which one is along the tangent, and the remaining two are normal to the direction of the body, and in two planes with the normals between each normal in turn; to determine the motion of the body and the curve which it describes.*

PROPOSITION 131. PROBLEM.

1103. *If a body M in some resisting medium is drawn by three forces, of which the direction of one is Mf parallel to the AP , the direction of another Mg is parallel to the applied line PQ placed in the plane APQ and the direction of the third is MQ sent normally to the plane APQ from M , to find the motion of the body and the line that it describes.*

PROPOSITION 132. PROBLEM.

1111. *In a uniform medium, which resists in the simple ratio of the speeds, the body is always attracted normally to the line AP ; to define the curve that the body describes projected in any manner.*