# CHAPTER II

## CONCERNED WITH THE PROPAGATION OF PULSES OF AIR SHOWING THE GENERATION AND PROPAGATION OF SOUND IN EQUALLY WIDE TUBES

### PROBLEM 73

31. *In a tube extended indefinitely in each direction, if somewhere a pulse may be excited, by which the air may be disturbed from equilibrium in some manner, to define the propagation of this disturbance at any time.*

### SOLUTION

*AB* shall be the directrix extended along a right line (Fig. 77) and a disturbance of this kind shall be induced in a small interval $GH = h$ of the air contained in the tube, in order that the line *GM'mH* may show the scale of the density, truly the line *GN'nH* shall be the scale of the speed [*i.e.* of the double pulse, part rarified and part compressed, passing through the air], the applied line of which is *tn*, moving toward *B*, to what extent they may lie above the axis *AB* to be indicated in the figure :

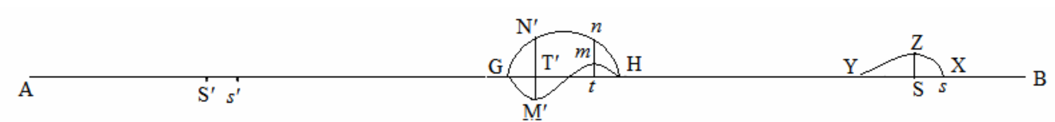

Fig. 77

truly the applied line *tm* falling above the axis shall indicate a density greater than usual, on the other hand truly *T'M'* shall indicate a density smaller than usual, clearly as the state of equilibrium demands, moreover in both directions beyond this interval *GH*, the air remains now in a state of rest also; thus so that there both scales lie on the same axis and the applied lines of these vanish, for this reason I have made each scale to agree with the axis at the ends *G* and *H*. With this in place we consider initially the position of some point S placed towards *B*, and since for the air held in this position the disturbance is required to be found for some time *t* from the start of the elapsed time from the point *S*, from the cut in the axis on both sides, I observe before all else the interval will require to become $= t\sqrt{\frac{2ga}{b}}$, provided that $t\sqrt{\frac{2ga}{b}} < HS$, then no disturbance to arrive at the point *S* and thus the air is going to be in equilibrium there, until the time elapsed from the beginning may arise $= HS\sqrt{\frac{b}{2ga}}$, and finally the air at *S* to respond to the disturbance.

Therefore the elapsed time now shall be greater than the time *t* and the interval may be taken $St = t\sqrt{\frac{2ga}{b}}$, indeed for the remaining part there is no need for a part equal to $ST'$ to be marked off, since here the scale coincides with the axis. Therefore now the element of the air translated from *S* will be at *s*, so that the small interval may become

$$Ss = \tfrac{1}{2}Htn + \tfrac{1}{2}Htm\,,$$

then truly the density of this element will be

$$q = B(1 + \tfrac{1}{2}tm + \tfrac{1}{2}tn)$$

and the speed tending towards *B*

$$\mathfrak{T} = \tfrac{1}{2}(tm + tn)\sqrt{\tfrac{2ga}{b}}$$

[Recall from Ch. 1. §17, in another situation :

$$Ss = \tfrac{1}{2}TNtn,\ q = Q\left(1 - \tfrac{1}{2}TN + \tfrac{1}{2}tn\right) \text{ and } \mathfrak{T} = \tfrac{1}{2}\left(TN + tn\right)\sqrt{\tfrac{2ga}{b}}.]$$

but after the elapse of the time $t = SG\sqrt{\frac{b}{2ga}}$, on account of $tm = 0$ and $tn = 0$, its motion will become zero and the natural density *B* is restored, in which state henceforth it will continue indefinitely.

In the other part of the tube the matter will be resolved in a similar manner in and the point *S'* will remain at rest, until the lapse of the time were $t = S'G'\sqrt{\frac{b}{2ga}}$, but then in the elapsed time $t = S'T'\sqrt{\frac{b}{2ga}}$ a translation will be made through the small distance $S's' = \frac{1}{2}GT'N' + \frac{1}{2}GT'M'$, since in the figure the applied line *T'M'* falls below the axis ; but then the density will become

$$q = B\left(1 - \tfrac{1}{2}T'N' - \tfrac{1}{2}T'M'\right)$$

and the speed directed towards *B*

$$\mathfrak{T} = \tfrac{1}{2}(T'N' + T'M')\sqrt{\tfrac{2ga}{b}}.$$

Moreover once the time $t = S'H\sqrt{\frac{b}{2ga}}$ will have passed, the air at $S'$ will be restored to equilibrium and all the elements of the air will not be subjected to disturbances for a long while, as during the time

$$= GH\sqrt{\tfrac{b}{2ga}} = h\sqrt{\tfrac{b}{2ga}} \text{ sec.}$$

Therefore it is evident, how a pulse excited initially in the interval *GH* will be propagated in each direction in the tube with the passage of the time.
[There seems to be no physical reason why a complex pulse of sound should change its form in transmission along the tube, and this does not appear to happen in reality; in which case we have to conclude that Euler's attempt to marry as it were, sound waves to the transport of air by laminar flow along the tube, to be in vain.]

## COROLLARY 1

32. Therefore putting the distance $HS = s$, a pulse initially excited in the space *GH* if propagated as far as to *S* in a time equal to $= s\sqrt{\frac{b}{2ga}}$ sec., from which it is apparent the propagation to be uniform and in a time of one second for the distance to become $= \sqrt{\frac{2ga}{b}}$ : which, if there may assume one for the density of mercury, so that the height *a* may be had as $2\frac{1}{2}$ English ft., the density of the air will be $b = \frac{1}{14.750} = \frac{1}{10500}$, on account of $g = 16$ English ft., the height of the atmosphere will become around 916 ft.

## COROLLARY 2

33. Also it is understood from the solution, how the particle of the air *S* may originate from the disturbance, certainly at first it may be propelled through the small interval $Ss = \frac{1}{2}Htn + \frac{1}{2}Htm$, then the density will be obtained $q = B\left(1+\frac{1}{2}tm+\frac{1}{2}tn\right)$ and in the third place it will acquire a speed towards *B*, $\mathfrak{T} = \frac{1}{2}(tn+tm)\sqrt{\frac{2ga}{b}}$, from which it is apparent this excess of the density shall be proportional to the normal density, since there shall be

$$\frac{q-B}{B} = \mathfrak{T}\sqrt{\frac{b}{2ga}}$$

## COROLLARY 3

34. But initially this deserves to be observed, the propagation of a pulse excited in the space *GH* is not necessarily to be equally strong in the region *A* and in the region *B*. For if in the initial pulse the scale of the density may agree with the scale of the speed, so that there may become $T'M' = -T'N'$ and $tm = tn$, then the propagation towards *A* clearly will vanish, truly the other towards *A* will be especially vigorous.

## SCHOLIUM 1

35. This propagation of sound by the movement of pulses is shown to be very pretty ; for in whatever way the sound may be produced, always a certain supply of air to be contained in the space *GH* disturbed from the state of equilibrium, induced either only in the density or in the speed, or with each to be induced jointly. But in whatever way this may happen, the propagation of this pulse in each direction in the tube is resolved with equal speed, even if perhaps in the one region of the tube it may be much stronger than in the other. But it can still here be objected, because the measurement of the interval, through which the sound may be propagated in one second, may be shown to become much greater, as it were 1040 English ft., greater than our theory has shown. The cause of this phenomenon either is present in this, as here in the calculation we allow only the smallest pulses, but these sounds, the propagation of which is to be defined by

experiment, as if they were more powerful, as our calculation ought not to be applied to these ; and thus at this point it may be left in doubt, whether they may actually be progressing with that speed which we have found for especially weak sounds. Or if here indeed the experiment may be in doubt, as then it would be allowed to suspect it might be attributed to the huge supply of solid particles of the solid themselves moving around the air, indeed while the disturbance reaches a particle of this kind at one end, also at the same instant the opposite end is struck nor will there be a need for the sound requiring to be propelled by the substance of these particles to be heard. Surely the circumstances for this, as we include this sound in the tube, cannot be attributed to disagreement with the experiment, since below we will see also the same speed for the propagation of the sound to be found in air. Yet meanwhile this dissent does not hinder any the less, hence both the production as well as the propagation of sound shall be considered to be explained correctly. Perhaps also the reason for this dissent may be allowed to be sought there, as we may consider the density of air as 750 times rarer than water [true value is approx. 800 ]; if indeed for that in turn we may attribute a greater rarity 966 , the calculation will agree nicely with experiment.
[The speed of sound in tubes is greater than that in open air, as experiments have shown using electric sparks to generate the sound pulses.]

## SCHOLIUM 2

36. Since in the elapsed time $t = St\sqrt{\frac{b}{2ga}}$ the disturbance of the point *S* thus is defined there by both the applied lines of the scales *tm* and *tn* , thus so that here the density $q = B(1+\frac{1}{2}tm+\frac{1}{2}tn)$ and the speed $\mathfrak{T} = \frac{1}{2}(tm+tn)\sqrt{\frac{2ga}{b}}$ , it is evident the whole initial pulse *GH* to be transferred in this time to the interval $YX = GH$ , so that there shall become $HX = t\sqrt{\frac{2ga}{b}}$ , with the remaining air present in equilibrium, just as in the similar interval taken for the other side of the tube. But in this disturbed interval *YX* the disturbance thus formed nevertheless will be formed by the single scale *YZX* , so that the applied line *SZ* shall be equal to half the sum of these $tm+tn$, certainly which in the same manner, where in the initial pulse, both the density and the speed will be shown at *S*, since now for the point *S* there will become :

$$q = B\left(1+SZ\right) \text{ or } l\,\tfrac{q}{B} = \text{SZ} \text{ and } \mathfrak{T} = SZ\sqrt{\tfrac{2ga}{b}} ,$$

and hence the translation of the element *S* may be determined thus, so that there shall become $Ss =$ area *XSZ* . Therefore this simple new scale *YZX* formed from the two initial scales indicates the nature of the pulse propagating towards *B*, from which also the diverse qualities of the sounds must be explained. But first here the width of the pulses $YX = HG$ is to be distinguished , which provided it were greater or smaller, the sound thence will have a certain nature. Then from the figure of the curve *YZX*, whereas if it had a greater or smaller amplitude *SZ,* either the whole will be situated at the same part of the axis, or partially above and below that, or some other account were desired, the sound

also will be heard in a different way. But indeed it may be observed the strength or intensity of the sound to depend on the amplitude *SZ*, but how such qualities may correspond to the remaining properties of the figure *YZX*, is not at all clear enough
[We may assume the pulse to consist of 'white' sound, the lowest wavelength governed by twice the width of the pulse; thus, the wider the pulse, the deeper the sound];
that perhaps is evident that ought to be explained from the almost infinite variation of the sounds ; of which kind are sounds expressing the different vowels *a*, *e*, *i*, *o*, *u*, and other innumerable differences. Thence also the account of the individual phenomena hence is understood not to be explained, in whatever manner it may be done, so that, if the disturbance *YZX* may be considered at first, that will be propagated further to the single region *B* only, nor will any new pulse be excited backwards towards *A*. Indeed we see, if the initial pulse *GH* must be prepared thus, so that the two scales may arise together, as may happen in the propagation of pulses, then also no propagation is going to follow into the region *A*. Also here in the first place it is required to be observed these same pulses are able to arise initially from an infinitude of pulses, since the propagated pulse *YX* can be produced from the initial pulse *GH* with the two different scales in an endless number of ways , from which it is no wonder, if often different causes can make similar sounds.

[See JWS Rayleigh : *Theory of Sound*, *Book* II, p.52, § 257, for a heuristic treatment of sound pulses. Essentially a positive condensed pulse arriving at the open end *B* must be adjusted to atmospheric pressure, and thus the reflected pulse is an inverted rarefied or negative pulse, this process happens a little beyond the opening depending on the width of the tube. In addition, this reflected pulse can again be inverted back into a positive pulse at the open end *A* to reappear as a positive pulse at the other end; the distance gone is thus twice the length of the tube. The case of a tube closed at one end can be resolved in a similar manner, but the pulse must travel four times the length of the tube to be restored to its initial condition. There is no mention of Euler's work on sound in Rayleigh's text.]

## SCHOLIUM 3

37. Thus far I have considered only a single pulse, nor therefore have I contemplated the effects of these sounds, which arise from an arrangement of a succession of several pulses, which are sharp and deep; from which also an infinite variety arise from the font of sounds. Truly since here no universal treatment of the theory of sound has been proposed, I observe only, if initially not one but several pulses $\alpha,\ \beta,\ \gamma$ shall be excited in the air, any of these likewise to be propagated, as if the remaining plainly were absent, nor therefore the several sounds excited at the same time to be confused with each other. Which phenomenon shall be seen to be extremely difficult compared with the other solution,

Fig. 78

follows at once from the principles established. Indeed since from the above directrix *AB* (Fig. 78) both the scales everywhere, besides at the locations $\alpha$, $\beta$, $\gamma$, shall agree with the axis itself, if we may consider some location *S*, for that lapse in the time $t = S\gamma\sqrt{\frac{b}{2ga}}$ the single pulse $\gamma$ is propagated with its individual properties, nor do the remaining pulses $\alpha$ et $\beta$ disturb anything ; but in the elapsed time $t = S\beta\sqrt{\frac{b}{2ga}}$, whereby that pulse has been advanced further , the pulse $\beta$ is advanced to the position *S* with its individual properties and then likewise after the time $t = S\alpha\sqrt{\frac{b}{2ga}}$ the pulse $\alpha$. From which it is understood most clearly, in what manner several sounds either simultaneously or successively thus to become excited at the appointed times at some location *S*, thus so that none of these left shall become hindered, but which will excite the air equally at *S*, plainly as if the rest were absent. Moreover, as far as we may consider enclosed in equally wide tubes here, it is required to be attributed to this cause, how the individual pulses propagated always may retain the same strength ; for however great the distance *S* may be taken from the first distance of the pulses, the pulse to be propagated always are represented by the same scale, from which not only the same strength, but also it is necessary that the same individual properties may be retained. But when such pulses are propagated in the open air in any manner, then certainly we will see these to be weakened more with continually greater distances.

## PROBLEM 74

38. *If the aperture of the tube* (Fig. 79) *shall be at B, truly extended an infinite distance from the other part A, and in some other part some pulse GH may be excited, to define its propagation in the tube.*

## SOLUTION

For the pulse excited in the space *GH*, *GmH* shall be the scale of the density and *GnH* the scale of the speed, of which therefore each may be confined with the axis through the whole tube beyond *GH* . Truly since both the end and the aperture of the tube is at *BB*, and the continuation of the scale on both sides, as must be set up as in figure 74 : hence the scale of the density *BHmGA* in the opposite part inverted becomes *BhMga* ; truly the scale of the speed *BHnGA* for the same part of the axis inverted around *B* will give the continuation *BhNga*.

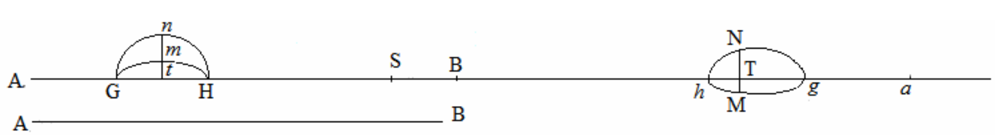

Fig. 79

From which the propagation of the pulse *GH* in the tube likewise will be made, and if outside the tube at an equal distance from the orifice *BB* a similar pulse *gh* may be present, only the scale of the density converted to the other part of the axis, thus so that for the abscissas $BT = Bt$ the applied lines shall become $TN = tn$ and $TM = tm$. Now therefore we may see, when and in what manner the pulse will be going to arrive at whichever location in the tube, and certainly initially the air at the orifice *BB* will be at rest, as long as the time from the start $t = BH\sqrt{\frac{b}{2ga}}$ ; then truly it will begin to be disturbed, as with the lapse of time $t = Bt\sqrt{\frac{b}{2ga}}$ there the density shall become

$$q = B(1 - \tfrac{1}{2}TN - \tfrac{1}{2}TM + \tfrac{1}{2}tn + \tfrac{1}{2}tm) = B$$

and the speed

$$\mathfrak{T} = \tfrac{1}{2}(TN + TM + tn + tm)\sqrt{\tfrac{2ga}{b}} = (tn + tm)\sqrt{\tfrac{2ga}{b}},$$

evidently here the density undergoes no change, indeed the speed towards *a* will be greater there, where the sum of the applied lines *tn* and *tm* were greater, of which *tn* denotes the speed directed towards *B*, and indeed the density *tm* in the initial pulse greater than the natural density. This disturbance of the air will endure in the opening *BB* through the time $= GH\sqrt{\frac{b}{2ga}}$ . But inside the tube two disturbances will generated successively at *S*, clearly the first, when the pulse *GH* arrives there, and then after the time $t = St\sqrt{\frac{b}{2ga}}$ there

$$\text{the density } q = B(1 + \tfrac{1}{2}tn + \tfrac{1}{2}tm) \text{ and the speed } \mathfrak{T} = \tfrac{1}{2}(tn + tm)\sqrt{\tfrac{2ga}{b}}.$$

Then truly it will be disturbed again, when the second pulse *hg* will be transferred there ; indeed in the elapsed time $t = ST\sqrt{\frac{b}{2ga}}$ , there the density will become

$$q = B(1 - \tfrac{1}{2}TN - \tfrac{1}{2}TM)$$

and the speed

$$\mathfrak{T} = \tfrac{1}{2}(tn + tm)\sqrt{\tfrac{2ga}{b}} :$$

clearly in the latter disturbance the density there will be smaller than the natural density, when the former will have been greater, with the speed remaining the same. But at the location *GH* itself, after the first pulse will have ceased, in the elapsed time

$$t = Hh\sqrt{\tfrac{b}{2ga}} = 2BH\sqrt{\tfrac{b}{2ga}} ,$$

it will begin to be disturbed anew and as if there the first echo will be perceived. But at some location *A* of the tube on putting the first pulse *GH*, a twofold disturbance of this kind will be experienced, so that for the first change in the time $= At\sqrt{\frac{b}{2ga}}$ the density there shall be

$$q = B(1 - \tfrac{1}{2}tn + \tfrac{1}{2}tm),$$

truly the speed

$$\mathfrak{T} = \tfrac{1}{2}(tn - tm)\sqrt{\tfrac{2ga}{b}};$$

but for the latter time interval $= AT\sqrt{\tfrac{b}{2ga}}$, the density shall become $q = B(1 + \tfrac{1}{2}tn - \tfrac{1}{2}tm)$ and the speed $\mathfrak{T} = \tfrac{1}{2}(tn - tm)\sqrt{\tfrac{2ga}{b}}$.

But this latter slower pulse will tend towards *A* in the time $= 2BH\sqrt{\tfrac{b}{2ga}}$.

From which it is apparent, if in the principal pulse there shall be $tn = tm$ everywhere, then plainly no disturbances to be going to be excited towards *A*; truly on the other hand, none towards *B*, if there were $tn = -tm$.

## COROLLARY 1

39. Therefore a simple pulse will be excited in the opening *BB* of the tube itself and there a single sound will be heard clearly ; but in the tube on approaching the location of the pulse two successive sounds follow each other, of which the latter will be allowed to considered as the echo of the first; and the interval of those thus may emerge greater, when we may approach more to the principal pulse *GH*.

## COROLLARY 2

40. Moreover from the location of the principal pulse itself *GH* and after that two disturbances in turn to be carried out towards *A* thus will be distant from each other in the time interval $= 2BH\sqrt{\tfrac{b}{2ga}}$ , which if it were notable enough, the latter will be shown as if the *echo* of the first.

## COROLLARY 3

41. If initially there were several pulses excited in the tube, in the same manner as in the previous problem it can be shown the propagation of the individual pulses to be minimally disturbed by the remaining pulses, and thus here also several sounds in turn are not to be confused.

## SCHOLIUM 1

42. The solution of this problem leads us besides beyond expectation to the exposition of two especially interesting phenomena, evidently of the *echo* and the *re-echo.* But first we see the cause of these two phenomena commonly to be ascribed wrongly to some repercussion ; indeed since (Fig. 80) in the tube with the aperture *BB*, truly in the other part for *AA* as if it were extended indefinitely, if the pulse or sound may be excited anywhere in *L*, so that it may be heard clearly at the orifice *BB* ; thence truly by receding to *L* thus the interval will be doubled, so that it may continually be made greater; in the

first place this echo truly arises here and travels to *L* and beyond this place towards *A*, provided that the interval *LB* may be large enough, so that the time, whereby twice its length may be traversed by sound, may be able to be sensed, and the repetition of this sound may be heard clearly, since here no reflection may be discerned, unless perhaps we may wish to say the reflection here at *BB* to arise from the external air,

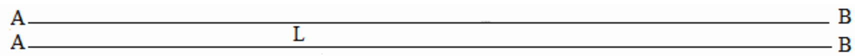

Fig. 80

which yet differs from the common opinion of most people. Thus if the interval *BL* may be 1040 feet, the sound will be heard again clearly produced at *L* after two seconds, and what has been said here concerning the tube, in a certain way would be able to be applied to walkways and narrow streets traversed [by sharp noises], especially if they were covered over above, from which an account can be rendered of the various phenomena in places of this kind.

SCHOLIUM 2

43. Thence also we will be able to put together in some manner an account of sounding tubes ; for if a sound were produced in the tube at *L*, then equally the pulse is almost expelled through the opening *BB* by the force on the free air, and since here it no longer has the nature of propagating pulses, where they may be brought forwards in one region only, also is expanded forwards. Truly since the opening *BB* is not greater than the initial pulse, from a long way off the sound is not stronger, but will be heard together with a certain echo. But if the tube, as it is accustomed to be called generally, may be dilated more around the opening, there is no doubt, why the proposed goal may not be satisfied more : because at *BB* a much greater supply of air may be disturbed and thus generates much stronger pulses in the external air. But now also we will look more closely at the propagation of pulses in the closed part.

PROBLEM 75

44. *If the tube* (Fig. 81) *shall be equally wide and closed at BB, truly for the other part AA to be extended indefinitely, just as somewhere in there whatever pulse may be excited in a small interval GH, to investigate the propagation of this pulse through the whole tube.*

SOLUTION

If in the interval *GH*, where the pulse is excited, for the point *t* the density were $=Q$, with the natural density being = *B*, and the speed directed towards $B = \Upsilon$, we may put the applied lines

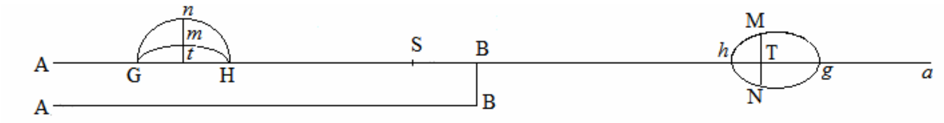

Fig. 81

$$tm = l\tfrac{Q}{B} = \tfrac{Q-B}{B} \text{ and } tn = \Upsilon\sqrt{\tfrac{b}{2ga}},$$

so that the scales of the density and speed *GmH* and *GnH* may be obtained [as lengths on the graphs], which are considered to agree with the remaining extension of the tube with the same axis. Now with the axis *AB* prolonged indefinitely beyond the tube and with the interval taken $BT = Bt$ the applied line $TM = tm$ may be put in place for the same part of the axis, truly for the other $TN = tn$, thus so that scales as if of the secondary pulses *hMg* and *hNg* may be built up. Hence the air meanwhile at the end *BB* will be at rest, as long as each pulse may be propagated there, which likewise will be reached in the time $= BH\sqrt{\tfrac{b}{2ga}}$, moreover in a greater elapsed time $t = Bt\sqrt{\tfrac{b}{2ga}}$; a disturbance of this kind will be excited at *BB* , so that the density there shall be going to become

$$q = B(1+\tfrac{1}{2}TN+\tfrac{1}{2}TM+\tfrac{1}{2}tn+\tfrac{1}{2}tm) = B\left(1+tn+tm\right),$$

truly the speed $\mathfrak{T}$ to be zero, because the air at *BB* cannot be other than at rest. Truly in any other part of the tube at the position *S* within the end *B* and the position of the first pulse *GH* taken for that, this will be called the first pulse, and in the lapsed time $t = St\sqrt{\tfrac{b}{2ga}}$, there the density will become $q = B\left(1+\tfrac{1}{2}tn+\tfrac{1}{2}tm\right)$, truly the speed

$$\mathfrak{T} = \tfrac{1}{2}\left(tn+tm\right)\sqrt{\tfrac{2ga}{b}}.$$

Then truly also the other secondary pulse *gh* will be prepared thus, and in the elapsed time

$$t = ST\sqrt{\tfrac{b}{2ga}} = \left(Bt+BS\right)\sqrt{\tfrac{b}{2ga}}$$

there the density will become $q = B\left(1+\tfrac{1}{2}tn+\tfrac{1}{2}tm\right)$ as before, truly the speed

$$\mathfrak{T} = -\tfrac{1}{2}\left(tn+tm\right)\sqrt{\tfrac{2ga}{b}},$$

opposite to that. But in place of the pulse *GH* and in that one traveling towards *A* both pulses will themselves follow in the elapsed time $2BH\sqrt{\tfrac{b}{2ga}}$; and at *A* for the principal pulse after the time $t = At\sqrt{\tfrac{b}{2ga}}$ the density and the speed will become:

$$q = B(1-\tfrac{1}{2}tn+\tfrac{1}{2}tm) \text{ and } \mathfrak{T} = \tfrac{1}{2}(tn-tm)\sqrt{\tfrac{2ga}{b}},$$

with which disturbance finished for the second pulse in the lapsed time

$$t = AT\sqrt{\tfrac{b}{2ga}} = \left(At+2Bt\right)\sqrt{\tfrac{b}{2ga}}$$

there will be found for *A* the density and speed

$$q = B(1-\tfrac{1}{2}tn+\tfrac{1}{2}tm) \text{ and } \mathfrak{T} = -\tfrac{1}{2}(tn-tm)\sqrt{\tfrac{2ga}{b}},$$

And thus the propagation of the first pulse excited *GH* will become known through the whole length of the tube.

## COROLLARY 1

45. Likewise therefore as in the preceding case, a repetition of the sound will be heard after passing through the interval *BH*, which unless the interval of the time may be able to be perceived, it will be required to have for the echo moreover at that same location *GH* and after that the echo from *A* will be more distinct there, where the interval *BH* were greater.

## COROLLARY 2

46. But on putting the principal pulse put into the interval *GA*, other kinds of pulses will be excited, and the pulse before that produced in the interval *BH*, and thus it can happen, so that in each region no propagation may happen, since in the one case the sum of the applied lines $tn+tm$ is defined, truly in the other the difference of these $tn-tm$.

## SCHOLIUM

47. For the dissimilarity of the pulses propagating in each direction by no means is to be regarded as adverse, if no difference may be noted in the sounds. Indeed even if both scales *GmH* and *GnH* may be prepared thus, so that, just as either the sum or difference of the applied lines *tm* and *tn* must be taken, the greatest difference may arise, yet it is required to be considered, since all the sounds from several successive pulses agree, which arise from some reciprocal motion, these to be prepared thus always, so that these scales may be prepared alternately ordered in the opposite way, and thus if one pulse were more powerful in one region than in another, it will eventuate opposite in the following, and since our sense may not prevail to distinguish the individual pulses, even that distinction may not fall on the senses. But in short if the alternate pulses may not propagate in another region, the sound will be perceived an octave lower, even if the

body may produce twice as many vibrations. But whether a case of this kind may arise at any time, is required to remain in doubt.

PROBLEM 76

48. *If, in an equally wide tube, open at each end AABB* (Fig. 82), *at some point t some pulse may be excited, to determine its propagation through the whole tube.*

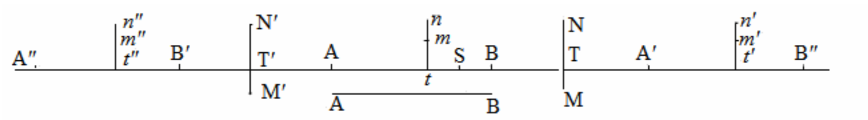

Fig. 82

SOLUTION

Even if the disturbance must be made always in the same interval, as we have assumed up to now, yet now we may consider the pulse with the smallest width, yet from the preceding the effect then arising is understood well enough, and we will consider the disturbance as arising from a single point *t*, where if the density were $=Q$ with the natural density being $=B$, and the speed directed into the region $AB = \Upsilon$ there may be taken

$$tm = l\frac{Q}{B} = \frac{Q-B}{B} \text{ and } tn = \Upsilon\sqrt{\frac{b}{2ga}},$$

[Thus, *tm* can be assumed to be the relative increase in the density of the air, while *tn* is the relative increase in the speed of the pulse over the customary speed of sound.]

but in the remaining points in the tube these two applied lines may. Now with the axis *AB* produced indefinitely in each direction and with the intervals *BA′, A′B″* etc. *AB′, B′A″* taken equal to the length of the tube, from the precepts given above the continuation of both scales thus must be put in place. Initially with the interval taken $BT = Bt$, since the tube is open at $B$, there may be taken $TM = tm$ below, but $TN = tn$ above the axis : then in a similar manner for the other part by taking $AT' = At$, since the tube also is open at *A*, there may be taken $T'M' = tm$ below and $T'N' = tn$ above the axis. Now with this scale *T′M′ N′* collated with the opening *BB*, there may be taken
$Bt' = BT'$, $t'm' = T'M'$ and $t'n' = T'N'$, and each above the axis, and thus all the applied lines *TN* and *tn* above the axis and thus by progressing indefinitely on each side, truly the others *TM* and *tm* will be put in place alternately above and below the axis, and in whatever region the intervals *AT* and *At* in the first place *At*, just as the intervals *BT, Bt* also will be equal, initially *Bt* . With these in place for some point *S* of the tube successively more and thus and infinitude of pulses will be carried through, certainly the principal pulse *tmn* after the time $= St\sqrt{\frac{b}{2ga}}$, then the pulse *TMN* after a time

$$= ST\sqrt{\tfrac{b}{2ga}} = (Bt+BS)\sqrt{\tfrac{b}{2ga}},$$

the third pulse $T'M'N'$ after a time

$$= ST'\sqrt{\tfrac{b}{2ga}} = (At+AS)\sqrt{\tfrac{b}{2ga}}$$

and thus again, the natures of the successive pulses will be shown in the following table:

| Elapsed time from the beginning: | In the tube at S density | speed along $AB$ |
|---|---|---|
| $St\sqrt{\tfrac{b}{2ga}}$ | $B\left(1+\tfrac{1}{2}tn+\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn+\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(2Bt-St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1-\tfrac{1}{2}tn-\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn+\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(2At+St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1+\tfrac{1}{2}tn-\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn-\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(2At+2Bt-St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1-\tfrac{1}{2}tn+\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn-\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(2At+2Bt+St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1+\tfrac{1}{2}tn+\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn+\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(2At+4Bt+St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1-\tfrac{1}{2}tn-\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn+\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(4At+2Bt+St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1+\tfrac{1}{2}tn-\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn-\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(4At+4Bt-St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1-\tfrac{1}{2}tn+\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn-\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| etc. | etc. | etc. |

## COROLLARY 1

49. Therefore the pulses are of a quadruple nature successively disturbing the air at $S$, for each fourth is endowed with the same nature, moreover the times $=2AB\sqrt{\tfrac{b}{2ga}}$ of the fourth pulse arrive at the same location in the tube, and these will follow each other with equal intervals, if there were $Bt = At$ and $St = \tfrac{1}{2}Bt$.

## COROLLARY 2

50. Since four pulses shall be produced in the time $=2AB\sqrt{\tfrac{b}{2ga}}$, $\tfrac{2}{AB}\sqrt{\tfrac{2ga}{b}}$ individual pulses will arise per second, which number, if $AB$ may be expressed in English ft., becomes $\tfrac{1832}{AB}$. If these pulses may be strong enough, they carry a sound which therefore is agreed to arise from a single pulse of air.

## COROLLARY 3

51. If the first pulse may be excited in the opening *BB* itself and as if the ear may be held there, on account of $St = 0,\ Bt = 0$ and $At = AB$ all the pulses will be double and will follow each other in the times

$$2AB\sqrt{\tfrac{b}{2ga}} = \tfrac{AB}{458}\ \text{sec.},$$

if indeed *AB* may be expressed in feet. Hence if the length *AB* shall be as 500 ft., more of the same repeated sounds will themselves be heard repeated clearly following the same number of seconds.

## SCHOLIUM

52. Hence it may be understood, how the echo of several repetitions may be generated, with a long enough tube such a phenomenon may be able to be shown, nor yet may any repercussion arise. Indeed following the calculation, the individual repetitions must be equally strong, yet truly is easily understood several to be the case, by which the following repetitions will be continually weakened, since with the conditions assumed in the calculation, in practice by no means can be satisfied. Yet since examples may not be lacking, for which several repetitions of the same tone are produced distinctly enough, there is no doubt, why the case of these in similar disturbances of the air, such as we have considered here, should not be required to be sought. Then also the account of the echoes hence is considered to be explained chiefly, just as by the nature of the walls of the tube, in as much as likewise they receive the similar motion of the vibrations from the internal air, may act together there. Besides, from the preceding, it is evident to have arisen from these similar phenomena, if the tube may be taken as closed at each end; indeed pulses will be disposed to be derived plainly in the same manner, yet with this difference, so that here the applied lines *tn* arising from the speeds must be located alternately above and below the axis nor do I observe to be a need, that I may set them out separately in this case. But then it is not to be held in contempt for the elucidation of the motions of small drums will be able to be derived; even if the membranes, by which the drums are covered, and the sides of these chiefly regulate the sound, yet some account of the repetition of the pulses is seen to be required for the enclosed air. Therefore it remains finally in this chapter, that we may investigate the agitation of the air in the tube from the one closed part, namely from the other open part.

[The fact that sound waves are longitudinal is not set out explicitly here by Euler, though he does group together changes in density with changes in speed by taking $tn + tm$ in his analysis; evidently longitudinal waves have smaller interactions with the walls than transverse waves, as the imposed motions on the air molecules are tangential.]

## PROBLEM 77

53. *If the equally wide tube* (Fig. 83) *AABB were open at the one end AA, truly closed at the other end BB and in that at t some pulse were excited, to investigate its propagation in the tube.*

## SOLUTION

In the air at *t*, from the disturbance, the density shall be $=Q$, with the natural density being $=B$, and the speed along the direction $AA=\Upsilon$, thence there may be taken

$$tm = l\tfrac{Q}{B} \text{ and } tn = \Upsilon\sqrt{\tfrac{b}{2ga}},$$

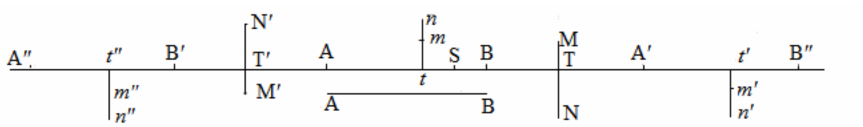

Fig. 83

thus so that the point *m* may refer to the scale of the density, truly the point *n* to the scale of the speed [*i.e.* relative to the quiescent speed], if indeed the remaining points of each lie on the axis itself. Now with that produced on each side and with the interval taken $BT = Bt$, since the tube is closed at *BB*, the applied line $TM = tm$ above, truly the other $TN = tn$ is required to be taken below the axis. Truly for the other part, since the tube is open at *AA*, with the interval taken $AT' = At$, the applied line $T'M' = tm$ below, truly the other $T'N' = tn$ may be placed above the axis, and in a similar manner by taking $At'' = AT$, there will be required to become:

$$t''m'' = -TM = -tm \text{ and } t''n'' = +TN = -tn.$$

Then again by progressing beyond *B* with the interval taken $Bt' = BT'$ there must be taken

$$t'm' = +T'M' = -tm \text{ et } t'n' = -T'N' = -tn,$$

and thus again on each side indefinitely. Now we may see, how all these pulses may arrive successively at some point *S* of the tube, that which may be seen from the adjoined table :

| Elapsed time from the beginning: | At the point S<br>the density will be: | <br>the speed along *AB*: |
|---|---|---|
| $St\sqrt{\frac{b}{2ga}}$ | $B\left(1+\frac{1}{2}tn+\frac{1}{2}tm\right)$ | $+\frac{1}{2}(tn+tm)\sqrt{\frac{2ga}{b}}$ |

| | | |
|---|---|---|
| $(2Bt-St)\sqrt{\frac{b}{2ga}}$ | $B\left(1+\frac{1}{2}tn+\frac{1}{2}tm\right)$ | $-\frac{1}{2}(tn+tm)\sqrt{\frac{2ga}{b}}$ |
| $(2At+St)\sqrt{\frac{b}{2ga}}$ | $B\left(1+\frac{1}{2}tn-\frac{1}{2}tm\right)$ | $+\frac{1}{2}(tn-tm)\sqrt{\frac{2ga}{b}}$ |
| $(2At+2Bt-St)\sqrt{\frac{b}{2ga}}$ | $B\left(1+\frac{1}{2}tn-\frac{1}{2}tm\right)$ | $-\frac{1}{2}(tn-tm)\sqrt{\frac{2ga}{b}}$ |
| $(2At+2Bt+St)\sqrt{\frac{b}{2ga}}$ | $B\left(1-\frac{1}{2}tn-\frac{1}{2}tm\right)$ | $-\frac{1}{2}(tn-tm)\sqrt{\frac{2ga}{b}}$ |
| $(2At+4Bt-St)\sqrt{\frac{b}{2ga}}$ | $B\left(1-\frac{1}{2}tn-\frac{1}{2}tm\right)$ | $+\frac{1}{2}(tn+tm)\sqrt{\frac{2ga}{b}}$ |
| $(4At+2Bt+St)\sqrt{\frac{b}{2ga}}$ | $B\left(1-\frac{1}{2}tn+\frac{1}{2}tm\right)$ | $-\frac{1}{2}(tn-tm)\sqrt{\frac{2ga}{b}}$ |
| $(4At+4Bt-St)\sqrt{\frac{b}{2ga}}$ | $B\left(1-\frac{1}{2}tn+\frac{1}{2}tm\right)$ | $+\frac{1}{2}(tn-tm)\sqrt{\frac{2ga}{b}}$ |
| $(4At+4Bt+St)\sqrt{\frac{b}{2ga}}$ | $B\left(1+\frac{1}{2}tn+\frac{1}{2}tm\right)$ | $+\frac{1}{2}(tn+tm)\sqrt{\frac{2ga}{b}}$ |
| etc. | etc. | etc. |

## COROLLARY 1

54. Therefore in this case a great disparity will be apparent in the successive pulses at the same location, since here after each octave it reverts to the same nature. But the time intervals maintain the same rule as before.

## COROLLARY 2

55. If there shall be $St=0$ and $At=0$, thus so that the sound may be produced at *A* and the ear may be placed there, the order and nature of the pulses carried to the ear successively thus will be had:

| Elapsed time | density | speed |
|---|---|---|
| $2AB\sqrt{\frac{b}{2ga}}$ | $B(1)$ | $-2tn\sqrt{\frac{2ga}{b}}$ |
| $4AB\sqrt{\frac{b}{2ga}}$ | $B(1)$ | $+2tn\sqrt{\frac{2ga}{b}}$ |
| $6AB\sqrt{\frac{b}{2ga}}$ | $B(1)$ | $-2tn\sqrt{\frac{2ga}{b}}$ |

therefore the density here undergoes no change, from which, if the first pulse were to have no speed, all the following mutually cancel each other.

## COROLLARY 3

56. But if a sound may be excited at *BB* and the following pulses may be excepted from the sense of hearing, so that there shall become $St=0$, $Bt=0$ and $At=AB$, then the speed for the named pulses will be zero, and the density alone will alternately become the

$$= B(1-2tm) \text{ et } B(1+2tm)$$

and these pulses will follow each other in the time intervals $2AB\sqrt{\frac{b}{2ga}}$. Moreover in these two cases the four pulses are united into one pulse, for which reason these complex pulses endowed with some other property, also are to be propagated as simple pulses.

SCHOLIUM 1

57. The matters which have been treated here so far concerned with the nature and propagation of pulses, are able to provide the opportunity for both the nature as well as the hearing of sounds requiring to be examined with greater care by natural philosophers. In which business initially there will be required to be considered carefully (Fig. 84), if a pulse excited by some cause may be propagated along the direction *AB* in air and now indeed may occupy an interval *YX* , its nature can be represented always by a certain single curved line *XZY*, which at the same time shall be the scale of the density and of the speed, even if in the principal pulse these two scales were especially diverse; but this is required to be understood to be concerned with simple pulses, truly not with complex ones, which may join together by arriving at the same place from different regions, of which kind are those, which I have described in corollaries 2 and 3. Therefore while that pulse may be situated in the interval *XY*, in the first place its width *XY* must be considered, within which the air has been disturbed from its equilibrium state, outside that truly it remains at rest everywhere: where indeed the greater were this width, there the sound will have been as if fuller. Then that curve *XZY* in this way may be called the disturbance of the air: clearly that particle of the air, which before the pulse had arrived was at *S*, now will be moved to *s*, so that small interval $Ss =$ area $XSZ$ to that right line, by which unity may be designated, for the applied line: then truly its density *q* will be compared with that, so that there shall become $q = B(1+SZ)$, with *B* denoting the natural density, or there shall become $SZ = l\frac{q}{B}$ , but by the motion likewise it will be carried towards [the point] *B* , the speed of which will be $= SZ\sqrt{\frac{2ga}{b}}$. But here the same right line is required to be understood to be taken for unity, which we will have used in the construction of the principal scale of the pulse. Therefore as if the air may be carried to the ear by this disturbance, hence it will be required to be deduced, how the organ of hearing may be affected.

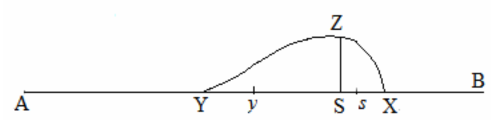

Fig. 84

SCHOLIUM 2

58. In the figure (Fig. 84) I show the whole curve *XZY* described above the axis; since that happens, when the density of the air in the whole interval of the pulse is greater than the natural density and likewise the individual particles of the air shall be provided with a motion directly towards *B* : so if it may eventuate, that the last particle of the pulse will

be translated from *Y* to *y*, in order that the small interval $Yy =$ area $XZY$ , clearly in this case it is required to understand all the air after the pulse arising from the previous agitation has passed through the whole interval successively. But if such a succession may not be happen (Fig. 85), so that the latter part of the pulse *Y* may remain in its natural place, the curve *XZY* showing the disturbance will be prepared thus, so that the part above the lower part shall be put in place, and its negative area *YZ'O* shall be made equal to the positive part *XZO*. Therefore then, if the upper part may precede, the particles of the air of a denser nature will be heard by the ear first and in the region *B* of the motion, truly the rarer particles will follow after, having a backwards motion. From which it may be concluded the essential distinction between the sounds to be put in place in this, just as the upper part *XZO* may either precede or follow. If it may arise, so that this curve may cut the axis *XY* three or five times, then without doubt there may be an abundance of unusual conditions pertaining to the sound. But it is to be agreed always the shape of this curve *XZY* to represent the essential distinction of the sounds; thus without doubt with the distinction removed, which may precede from a succession of several pulses. Moreover though these results have been deduced from the consideration of tubes of equal cross-sections, yet they will extend out much greater, and to be confirmed through those which will be able to be examined concerned with unequally large tubes.

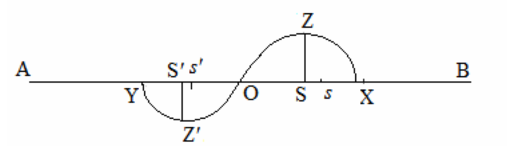

Fig. 85

# CAPUT II

## DE PROPAGATIONE PULSUUM AERIS IN TUBIS AEQUALITER AMPLIS AD SONI GENERATIONEM ET PROPAGATIONEM ILLUSTRANDAM

## PROBLEMA 73

31. *In tubo utrinque in infinitum extenso si alicubi in spatio minimo excitetur pulsus, quo aër utcunque de statu aequilibrii deturbetur, huius agitationis propagationem ad quodvis tempus definire.*

## SOLUTIO

Sit (Fig. 77) *AB* tubi directrix in directum extensa et in spatiolo *G H* = *h* aëri in tubo contento eiusmodi agitatio inducta sit, ut linea *GM' mH* exhibeat scalam densitatum, linea vero *GN'nH* scalam celeritatum, cuius applicatae *tn*, quatenus in figura supra axem *AB* cadunt, motum versus *B* indicent,

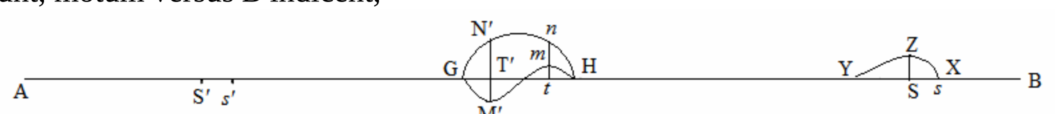

Fig. 77

illius vero curvae applicatae *tm* supra axem cadentes maiorem solito densitatem, contrariae vero *T'M'* minorem solito densitatem ostendant, quam scilicet status aequilibrii postulat utrinque autem ultra hoc intervallum *GH* aër etiam nunc in quiete versetur; ita ut ibi ambae scalae in ipsum axem incidant earumque applicatae evanescant, quam ob causam etiam utramque scalam in terminis *G* et *H* cum axe convenientes feci. Hoc statu initiali constituto consideremus locum tubi quemcunque *S* versus *B* situm, et quia ad aëris hoc loco contenti agitationem inveniendam pro tempore quovis *t* ab initio elapsi utrinque a puncto *s* in axe abscindi oportet intervalla $=t\sqrt{\frac{2ga}{b}}$ ante omnia observo, quamdiu fuerit $t\sqrt{\frac{2ga}{b}}<HS$, nullam agitationem ad punctum *S* pervenire ibique adeo aequilibrium esse futurum, donec tempus ab initio elapsum evadat $=HS\sqrt{\frac{b}{2ga}}$ ac tum demum aërem in *S* agitationem esse sensurum. Elapsum ergo iam sit maius tempus *t* capiaturque intervallum $St=t\sqrt{\frac{2ga}{b}}$, ad alteram enim partem non opus est aequale spatium *ST* abscindi, quia ibi scalae in axem incidunt. Nunc igitur aëris elementum ex *S* translatum erit in *s*, ut sit spatiolum

$$Ss=\tfrac{1}{2}Htn+\tfrac{1}{2}Htm,$$

tum vero densitas huius elementi erit

$$q = B(1 + \tfrac{1}{2}tm + \tfrac{1}{2}tn)$$

et celeritas versus *B* tendens

$$\mathfrak{T} = \tfrac{1}{2}(tm + tn)\sqrt{\tfrac{2ga}{b}}$$

postquam autem elapsum fuerit tempus $t = SG\sqrt{\frac{b}{2ga}}$, ob $tm = 0$ et $tn = 0$ eius motus iterum extinguitur densitasque naturalis *B* restituitur, in quo statu deinceps perpetuo perseverabit.

Simili modo in altera tubi parte res se habebit punctumque $S'$ quiescet, donec elapsum fuerit tempus $t = S'G'\sqrt{\frac{b}{2ga}}$, deinceps autem elapso tempore $t = S'T'\sqrt{\frac{b}{2ga}}$ translatio fiet per spatiolum $S's' = \frac{1}{2}GT'N' + \frac{1}{2}GT'M'$, quia in figura applicata $T'M'$ infra axem cadit; tum autem densitas erit

$$q = B\left(1 - \tfrac{1}{2}T'N' - \tfrac{1}{2}T'M'\right)$$

et celeritas versus *B* directa

$$\mathfrak{T} = \tfrac{1}{2}(T'N' + T'M')\sqrt{\tfrac{2ga}{b}}.$$

Statim autem ac tempus $t = S'H\sqrt{\frac{b}{2ga}}$ effluxerit, aër in S' in aequilibrium restituetur omniaque aëris elementa non diutius agitationi erunt subiecta, quam durante tempore

$$= GH\sqrt{\tfrac{b}{2ga}} = h\sqrt{\tfrac{b}{2ga}} \text{ min. sec.}$$

Manifestum ergo est, quomodo pulsus initio in intervallo *GH* excitatus labente tempore utrinque in tubo propagetur.

## COROLLARIUM 1

32. Posita ergo distantia $HS = s$, pulsus initio in spatio *GH* excitatus ad *S* usque propagatur tempore $= s\sqrt{\frac{b}{2ga}}$ min. sec., unde patet propagationem esse uniformem et tempore unius minuti secundi fieri per spatium $= \sqrt{\frac{2ga}{b}}$ : quod, si densitas mercurii sumatur pro unitate, ut sit *a* altitudo barometri quasi $2\frac{1}{2}$ ped. lond., erit densitas aëris $b = \frac{1}{14.1750} = \frac{1}{10500}$, ob $g = 16$ ped. lond. fiet circiter 916 ped.

## COROLLARIUM 2

33. Ex solutione etiam intelligitur, cuiusmodi agitatione particula aëris *S* concitetur, primo nempe propelletur per spatiolum $Ss = \frac{1}{2}Htn + \frac{1}{2}Htm$, deinde densitatem obtinebit $q = B\left(1+\frac{1}{2}tm+\frac{1}{2}tn\right)$ ac tertio celeritatem versus *B* acquiret $\mathfrak{T} = \frac{1}{2}(tn+tm)\sqrt{\frac{2ga}{b}}$, unde patet celeritatem hanc excessui densitatis supra densitatem naturalem esse proportionalem, quandoquidem est

$$\frac{q-B}{B} = \mathfrak{T}\sqrt{\frac{b}{2ga}}$$

## COROLLARIUM 3

34. Imprimis autem hic notari meretur pulsus in spatio *GH* excitati propagationem in plagam *A* non necessario aeque esse fortem, atque in plagam *B.* Si enim in pulsu initiali scala densitatum congrueret cum scala celeritatum, ut esset $T'M' = -T'N'$ et $tm = tn$, tum propagatio versus *A* prorsus evanesceret, altera vero versus *A* maxime vigeret.

## SCHOLION 1

35. Hac pulsuum promotione soni propagatio pulcerrime illustratur; quocunque enim modo sonus producatur, semper copia quaedam aëris in spatio *GH* contenta de statu aequilibrii deturbatur, sive in sola densitate sive sola celeritate sive utraque coniunctim ipsi mutatio inducatur. Quocunque autem modo hoc eveniat, propagatio huius pulsus utrinque in tubo pari absolvitur celeritate, etiamsi forte in alteram tubi plagam multo sit vehementior quam in alteram. Id autem tantum hic obiici potest, quod experientia spatium, per quod sonus intervallo unius minuti secundi propagatur, multo maius, scilicet 1040 ped.lond., exhibeat, quam nostra Theoria ostendit. Cuius phaenomeni causa vel in eo est posita, quod hic in calculo pulsus tantum minimos admittimus, ii autem soni, quorum propagatio per experimenta est definita, tam fuerint vehementes, ut calculus noster ad eos non debeat accommodari; ideoque adhuc in dubio relinquatur, an non soni maxime debiles ea ipsa celeritate, quam invenimus, revera progrediantur. Vel fietiam hic experientia refragetur, suspicari liceret accelerationem hanc ingenti particularum solidarum in aëre volitantium copiae tribui debere, dum enim agitatio ad unum terminum huiusmodi particulae pertingit, eodem instanti etiam terminus oppositus impellitur neque tempore opus foret ad sonum per substantiam harum particularum propulsandum. Huic certe circumstantiae, quod hic sonum in tubo includimus, hic dissensus ab experientia tribui nequit, quoniam infra videbimus etiam in aëre undique aperto eandem celeritatem pro soni propagatione inveniri. Interim tamen hic dissensus non obstat, quominus hinc tam productio quam propagatio soni recte explicari sit censenda. Fortasse etiam rationem huius dissensus in eo quaeri licebit, quod aërem tantum 750 vicibus rariorem statuimus aqua; si enim ei raritatem 966 vicibus maiorem tribuamus, calculus cum experientia pulcre consentiet.

## SCHOLION 2

36. Quoniam elapso tempore $t = St\sqrt{\frac{b}{2ga}}$ agitatio puncti *S* ita per ambas applicatas *tm* et *tn* scalarum definitur, ut sit ibi densitas $q = B(1+\frac{1}{2}tm+\frac{1}{2}tn)$ et celeritas $\mathfrak{T} = \frac{1}{2}(tm+tn)\sqrt{\frac{2ga}{b}}$, evidens est totum pulsum initialem *GH* hoc tempore transferri in spatium $YX = GH$, ut sit $HX = t\sqrt{\frac{2ga}{b}}$, reliquo aëre in aequilibrio existente, praeterquam in simili spatio ad alteram partem in tubo sumto. In hoc autem spatio *YX* agitatio per unicam scalam *YZX* ita forma tam, ut applicata *SZ* sit semisummae illarum $tm+tn$ aequalis, repraesentabitur, quippe quae
eadem simili modo, quo in pulsu initiali, et densitatem et celeritatem in *S*
exhibebit, cum iam pro puncto *S* sit

$$q = B\left(1+SZ\right) \text{ seu } l\tfrac{q}{B} = \mathrm{SZ} \text{ et } \mathfrak{T} = SZ\sqrt{\frac{2ga}{b}},$$

hincque translatio elementi *S* ita determinetur, ut sit $Ss =$ areae $XSZ$. Haec ergo nova scala simplex *YZX* ex binis initialibus formata indolem pulsuum versus *B* propagatorum declarat, unde etiam diversae sonorum qualitates explicari debebunt. Primum autem hic distinguitur latitudo pulsuum $YX = HG$, quae prout fuerit maior minorve, sonus inde certam indolem habebit. Deinde ex ipsa curvae *Y Z X* figura, prout vel maiorem minoremve habuerit amplitudinem *SZ*, vel tota ad eandem axis partem vel partim supra axem partim infra eum fuerit sita, vel alia quacunque ratione fuerit affecta, sonus quoque diverso modo sensum auditus afficiet. Ab amplitudine quidem *SZ* fortitudo seu vehementia soni pendere videtur, quales autem qualitates reliquis proprietatibus figurae *YZX* respondeant, haud satis liquet; id saltem perspicuum est infinitam fere sonorum varietatem hinc explicari debere; cuiusmodi sunt soni diversas litteras vocales *a, e,* i, *o, u* exprimentes, aliaeque innumerae differentiae. Deinde etiam ratio singularis phaenomeni adhuc non explicati hinc intelligitur, quomodo fiat, ut, si agitatio *YZX* tanquam initialis consideretur, ea tantum in unam plagam *B* ulterius propagetur, neque ullos novos pulsus retro versus *A* excitet. Videmus enim, si ipse pulsus initialis *GH* iam ita esset comparatus, ut binae scalae inter se convenirent, quemadmodum fit in pulsibus propagatis, tum etiam nullam propagationem in plagam *A* esse secuturam. Imprimis etiam hic notandum est eosdem pulsus propagatos ex infinitis pulsibus initialibus oriri posse, quoniam infinitis modis ex binis scalis diversis pulsus initialis *GH* eadem scala pro pulsibus propagatis *YX* produci potest, unde non mirum, si saepe diversae causae similes sonos efficiunt.

## SCHOLION 3

37. Hactenus unicum tantum pulsum sum contemplatus neque propterea ad eas sonorum affectiones respexi, quae ex successione et ordine plurium pulsuum nascuntur, quales sunt gravitas et acumen; ex quo fonte soni etiam infinitam varietatem adipiscuntur. Quoniam vero hic non universum sonorum doctrinam tradere est propositum, tantum observo, si initio non unus sed plures pulsus $\alpha,\ \beta,\ \gamma$ in aëre sint excitati, quemlibet eorum perinde propagari, ac si reliqui plane abessent, neque propterea plures sonos simul

excitatos inter se confundi. Quod phaenomenon cum alias solutu perquam difficile sit visum,

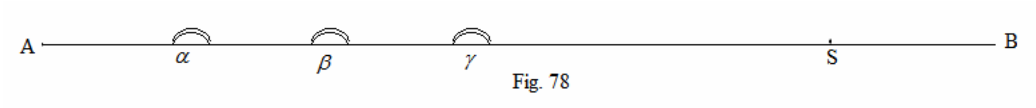

Fig. 78

Fig. 78 ex principiis stabilitis sponte sequitur. Cum enim super directrice *AB* (Fig. 78) ambae scalae ubique, praeterquam in locis $\alpha$, $\beta$, $\gamma$, cum ipso axe congruant, si locum quemcunque *s* consideremus, ad eum elapso tempore $t = S\gamma\sqrt{\frac{b}{2ga}}$ solus pulsus $\gamma$ cum suis affectionibus propagatur neque reliqui pulsus $\alpha$ et $\beta$ quicquam turbant; elapso autem tempore $t = S\beta\sqrt{\frac{b}{2ga}}$, quo ille pulsus iam ultra est promotus, ad locum *S* pulsus $\beta$ cum suis affectionibus perfertur ac deinceps post tempus $t = S\alpha\sqrt{\frac{b}{2ga}}$ pulsus $\alpha$. Ex quo clarissime intelligitur, quemadmodum plures soni vel simul vel successive excitati ita statis temporibus ad quemvis locum *S* proferantur, ut nullus eorum reliquis sit impedimento, sed quilibet aeque aërem in *S* excitet, ac si reliqui plane abessent. Ceterum, quatenus hic pulsus tubo aequaliter amplo inclusos consideramus, huic causae est tribuendum, quod singuli pulsus propagati perpetuo eandem vim retineant; quantumvis enim punctum *S* a pulsibus primitivis distans accipiatur, pulsus propagati semper per similem scalam repraesentantur, unde non solum eandem vim, sed etiam easdem affectiones retineant, necesse est. Quando autem tales pulsus in libero aëre quaquaversus propagantur, tum utique videbimus eos in maioribus distantiis continuo magis debilitari.

## PROBLEMA 74

38. *Si* (Fig. 79) *tubus in B sit apertus, ex altera parte vero A in infinitum extensus, in eoque alicubi GH pulsus quicunque excitetur, eius propagationem in tubo definire.*

## SOLUTIO

Pro pulsu in spatio *GH* excitato sit *GmH* scala densitatum et *GnH* scala celeritatum, quarum ergo utraque extra spatium *GH* per totum tubum cum axe confundatur. Quia vero tubus in *BB* est terminatus et apertus, utriusque scalae continuatio, uti in figura 74 institui debet: hinc scala densitatum *BHmGA* in partem oppositam inversa fiet *BhMga* scala celeritatum vero *BHnGA* ad eandem axis partem circa *B* inversa dabit continuationem

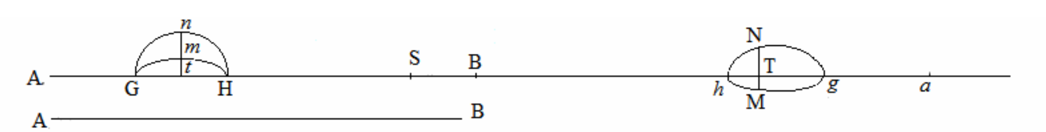

Fig. 79

*BhNga*. Unde in tubo pulsus *GH* propagatio perinde fiet, ac si extra tubum ad parem ab orificio *BB* distantiam similis pulsus *gh* existeret, scala densitatum tantum ad alteram axis partem conversa, ita ut sumta abscissa $BT = Bt$ sint applicatae $TN = tn$ et $TM = tm$. Nunc igitur videamus, quando et quomodo pulsus in quemvis tubi locum sit perventurus, ac primo quidem in orificio *B B* aër quiescet, quoad ab initio effluerit tempus $t = BH\sqrt{\frac{b}{2ga}}$ ; deinceps vero ita agitari incipiet, ut elapso tempore $t = Bt\sqrt{\frac{b}{2ga}}$ ibi futura sit densitas $q = B(1-\frac{1}{2}TN-\frac{1}{2}TM+\frac{1}{2}tn+\frac{1}{2}tm) = B$

et celeritas

$$\mathfrak{T} = \tfrac{1}{2}(TN+TM+tn+tm)\sqrt{\tfrac{2ga}{b}} = (tn+tm)\sqrt{\tfrac{2ga}{b}},$$

densitas scilicet ibi nullam mutationem patietur, celeritas vero versus *a* eo maior erit, quo maior fuerit summa applicatarum *tn* et *tm*, quarum *tn* denotat celeritatem versus *B* directam, *tm* vero densitatem naturali maiorem in pulsu initiali. Agitatio haec aëris in ipso orificio *BB* durabit per tempus $= GH\sqrt{\frac{b}{2ga}}$. Intra tubum autem in *S* successive duae generabuntur agitationes, prior scilicet, quando pulsus *GH* eo appellit, indeque elapso tempore $t = St\sqrt{\frac{b}{2ga}}$ ibi erit

$$\text{densitas } q = B(1+\tfrac{1}{2}tn+\tfrac{1}{2}tm) \text{ et celeritas } \mathfrak{T} = \tfrac{1}{2}(tn+tm)\sqrt{\tfrac{2ga}{b}}.$$

Deinde vero de novo agitabitur, quando pulsus secundarius *hg* eo transferetur; tempore enim elapso $t = ST\sqrt{\frac{b}{2ga}}$, ibi fiet densitas

$$q = B(1-\tfrac{1}{2}TN-\tfrac{1}{2}TM)$$

et celeritas

$$\mathfrak{T} = \tfrac{1}{2}(tn+tm)\sqrt{\tfrac{2ga}{b}} :$$

in agitatione scilicet posteriori densitas eo minor erit naturali, quando priori fuerat maior, celeritate existente eadem. In ipso autem loco *GH*, postquam pulsus primarius cessaverit, elapso tempore

$$t = Hh\sqrt{\tfrac{b}{2ga}} = 2BH\sqrt{\tfrac{b}{2ga}}$$

denuo agitari incipiet ibique quasi echo prioris percipietur. In quovis autem tubi loco *A* pone pulsum primarium *GH* eiusmodi duplex agitatio sentietur, ut pro priori elapso tempore $= At\sqrt{\frac{b}{2ga}}$ ibi sit densitas

$$q = B(1-\tfrac{1}{2}tn+\tfrac{1}{2}tm),$$

celeritas vero

$$\mathfrak{T} = \tfrac{1}{2}(tn-tm)\sqrt{\tfrac{2ga}{b}};$$

pro posteriori vero elapso tempore $= AT\sqrt{\frac{b}{2ga}}$, futura sit densitas $q = B(1+\frac{1}{2}tn-\frac{1}{2}tm)$ et celeritas $\mathfrak{T} = \frac{1}{2}(tn-tm)\sqrt{\frac{2ga}{b}}$.

Tardius autem pulsus hic posterior ad *A* pertinget tempore $=2BH\sqrt{\frac{b}{2ga}}$

Unde patet, si in pulsu principali ubique esset $tn=tm$, tum versus *A* nullas plane agitationes excitatum iri, contra vero nullas versus *B*, si fuerit $tn=-tm$.

COROLLARIUM 1

39. In ipso ergo orificio *BB* tubi pulsus simplex excitabitur ibique unicus sonus exaudietur; in tubo autem ad pulsus locum accedendo duo soni successive se excipient, quorum posteriorem ut resonantiam prioris spectare licebit; eorumque intervallum eo maius evadet, quo magis ad pulsum principalem *GH* appropinquemus.

COROLLARIUM 2

40. In ipso autem pulsus principalis loco *GH* et post eum versus *A* binae agitationes eo perlatae intervallo temporis $=2BH\sqrt{\frac{b}{2ga}}$ a se invicem distabunt, quod si fuerit satis notabile, posterior prioris quasi *echo* exhibebit.

COROLLARIUM 3

41. Si initio plures in tubo excitati fuerint pulsus, eodem modo atque in praecedente problemate ostendi potest singulorum propagationem a reliquis minime perturbari, sicque etiam hic plures sonos invicem non confundi.

SCHOLION 1

42. Solutio huius problematis nos praeter expectationem ad explicationem duorum phaenomenorum imprimis memorabilium manuducit, resonantiae scilicet et *echo.* Primum autem videmus horum phaenomenorum causam vulgo perperam repercussioni cuipiam esse adscriptam; cum enim (Fig.80) in tubo ad *BB* aperto, in altera vero parte ad *AA* quasi in infinitum extenso, si uspiam in *L* pulsus seu sonus excitetur, is nonnisi in orificio *BB* simplex exaudiatur; inde vero ad *L* recedendo ita duplicetur, ut intervallum continuo fiat maius, primo hic resonantia oritur, tum vero in *L* et ultra hunc locum versus *A*, simodo intervallum *LB* satis sit magnum, ut tempus, quo a sono eius duplum percurreretur, sentiri queat, repetitio illius soni exaudietur, cum tamen hic nulla reflexio cernatur, nisi forte dicere velimus repercussionem hic

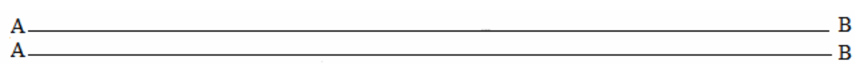

Fig. 80

in *BB* fieri ab aëre externo, quod tamen ab opinione vulgari plurimum abhorret. Ita si intervallum *BL* esset 1040 pedum, sonus in *L* editus post duo minuta secunda ibi iterum audiretur, et quod hic de tubis est dictum, quodammodo etiam ad ambulacra et vicos angustos, praecipue si superne fuerint tecti, transferri licet, unde plurium phaenomenorum in huiusmodi locis observatorum ratio reddi poterit.

## SCHOLION 2

43. Deinde etiam hinc rationem tubarum stentorearum iam quodammodo colligere poterimus; si enim in tubo ad *L* sonus fuerit excitatus, pulsus inde per orificium *BB* pari propemodum vi in liberum aërem expellitur, et quia hic non amplius indolem habet pulsuum propagatorum, qua tantum in unam plagam proferantur, etiam quaqua versus expanditur. Quia vero orificium *BB* non est maius quam pulsus initialis, e longinquo sonus non fortior, sed cum quadam resonantia coniunctus audietur. Sin autem tubus, ut vulgo fieri solet, circa orificium magis dilatetur, nullum est dubium, quin scopo proposito magis satisfiat: quoniam in *BB* multo maior aëris copia agitatur ideoque in aëre externo fortiores pulsus generat. Nunc autem etiam propagationem pulsuum in tubis ex altera parte clausis scrutemur.

## PROBLEMA 75

44. *Si tubus* (Fig. 81) *aequaliter amplus ad BB sit clausus, ad alteram vero partem AA in infinitum extensus, in eoque alicubi veluti in spatiolo GH, pulsus quicunque excitetur, huius pulsus propagationem per totum tubum investigare.*

## SOLUTIO

Si in spatii *GH*, quo pulsus excitatur, puncto *t* fuerit densitas = *Q*, naturali existente = *B*, et celeritas versus *B* directa $=\Upsilon$, statuantur applicatae

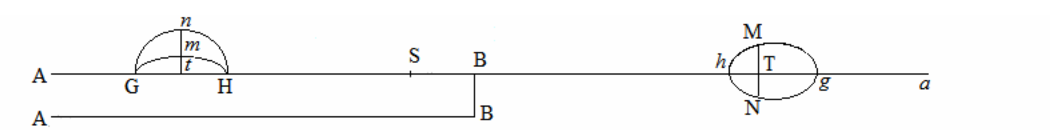

Fig. 81

$$tm = l\tfrac{Q}{B} = \tfrac{Q-B}{B} \quad \text{et} \quad tn = \Upsilon\sqrt{\tfrac{b}{2ga}},$$

ut obtineantur scalae densitatum et celeritatum *GmH* et *GnH*, quae per reliquam tubi extensionem cum ipso axe convenire sunt censendae. Axe iam *AB* extra tubum in infinitum prolongato sumtoque intervallo $BT = Bt$ statuatur applicata $TM = tm$ ad eandem axis partem, ad contrariam vero *TN* = *tn*, ut sic scalae quasi pulsus secundarii *hMg* et *hNg* extruantur. Hinc in ipso termino *BB* aër tamdiu quiescet, quoad uterque pulsus eo propagetur, quod simul continget elapso tempore $= BH\sqrt{\tfrac{b}{2ga}}$, elapso autem tempore maiore $t = Bt\sqrt{\tfrac{b}{2ga}}$, eiusmodi in *BB* agitatio excitabitur, ut futura sit densitas

$$q = B(1+\tfrac{1}{2}TN+\tfrac{1}{2}TM+\tfrac{1}{2}tn+\tfrac{1}{2}tm) = B\left(1+tn+tm\right),$$

celeritas vero $\mathfrak{T}$ nulla, quia aër in *BB* non potest non esse quiescens. In alio vero quovis tubi loco *S* intra terminum *B* et locum primi pulsus *GH* accepto ad eum hic primus pulsus prius appellet, et elapso tempore $t = St\sqrt{\frac{b}{2ga}}$, densitas ibi erit $q = B\left(1+\frac{1}{2}tn+\frac{1}{2}tm\right)$, , celeritas vero

$$\mathfrak{T} = \tfrac{1}{2}\left(tn+tm\right)\sqrt{\tfrac{2ga}{b}}.$$

Deinde vero etiam alter pulsus secundarius *gh* eo perferetur, et elapso tempore

$$t = ST\sqrt{\tfrac{b}{2ga}} = \left(Bt+BS\right)\sqrt{\tfrac{b}{2ga}}$$

ibi erit densitas $q = B\left(1+\frac{1}{2}tn+\frac{1}{2}tm\right)$ ut ante, celeritas vero

$$\mathfrak{T} = -\tfrac{1}{2}\left(tn+tm\right)\sqrt{\tfrac{2ga}{b}},$$

illi contraria. In ipso autem pulsus *GH* loco et pone eum versus *A* ambo pulsus se excipient elapso tempore $2BH\sqrt{\frac{b}{2ga}}$; atque in *A* pro pulsu principali elapso tempore $t = At\sqrt{\frac{b}{2ga}}$ erit

$$\text{densitas } q = B(1-\tfrac{1}{2}tn+\tfrac{1}{2}tm) \text{ et celeritas } \mathfrak{T} = \tfrac{1}{2}(tn-tm)\sqrt{\tfrac{2ga}{b}},$$

qua agitatione finita pro pulsu secundario elapso tempore

$$t = AT\sqrt{\tfrac{b}{2ga}} = \left(At+2Bt\right)\sqrt{\tfrac{b}{2ga}}$$

reperietur ad *A*

$$\text{densitas } q = B(1-\tfrac{1}{2}tn+\tfrac{1}{2}tm) \text{ et celeritas } \mathfrak{T} = -\tfrac{1}{2}(tn-tm)\sqrt{\tfrac{2ga}{b}},$$

Sicque propagatio pulsus primum excitati *GH* per totum tubum innotescit.

## COROLLARIUM 1

45. Perinde ergo atque in casu praecedente per spatium *BH* quaedam soni repetitio percipietur, quae nisi temporis intervallum sentiri queat, pro resonantia erit habenda, in ipso autem loco *GH* et post eum in *A* echo eo magis erit distinctum, quo maius fuerit spatium *BH.*

## COROLLARIUM 2

46. Pulsus autem in spatio *GA* pone pulsum principalem excitati alius erunt indolis, ac pulsus ante eum in spatio *BH* producti, fierique adeo potest, ut in alterutram plagam nulla propagatio contingat, quoniam altero casu agitatio definitur summa applicatarum $tn+tm$, altero vero earum differentia $tn-tm$.

## SCHOLION

47. Dissimilitudini pulsuum in tubo utrinque propagatorum experientia neutiquam adversari est putanda, si in sonis nullum discrimen animadvertitur. Etiamsi enim binae scalae *GmH* et *GnH* ita sint comparatae, ut, prouti vel summa vel differentia applicatarum *tm* et *tn* capi debeat, maximum discrimen oriri debeat, tamen perpendendum est, quoniam omnes soni pluribus pulsibus successive productis constant, qui a motu quodam reciproco nascuntur, eos semper ita esse comparatos, ut alternatim scalas illas contrario modo dispositas praebeant, ideoque si unus pulsus in unam plagam fuerit fortior quam in alteram, contrarium in sequente eveniet, et quoniam sensus nostri singulos discernere non valent, etiam illud discrimen in sensus non cadit. Quodsi alterni pulsus in alteram plagam prorsus non propagentur, sonus percipietur una *octava* gravior, etiamsi corpus sonorum duplo plures edat vibrationes. An autem huiusmodi casus unquam eveniant, in dubio est relinquendum.

## PROBLEMA 76

48. *Si* (Fig. 82) *in tubo aequaliter amplo et utrinque aperto AABB alicubi in t pulsus quicunque excitetur, eius propagationem per totum tubum determinare.*

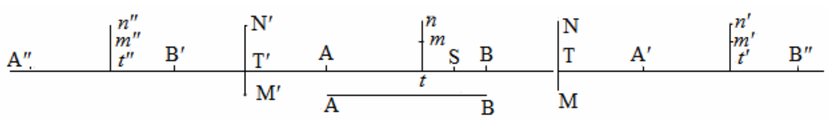

Fig. 82

## SOLUTIO

Etsi agitatio semper in quodam spatio, uti hactenus assumsimus, fieri debet, tamen nunc a latitudine pulsus animum abstrahamus, quandoquidem ex praecedentibus effectus inde oriundus satis intelligitur, et agitationem in unico puncto *t* factam hic contemplemur, ubi si fuerit densitas $=Q$ naturali existente $=B$, et celeritas in plagam *AB* directa $=\Upsilon$ capiatur

$$tm = l\frac{Q}{B} = \frac{Q-B}{B} \quad \text{et} \quad tn = \Upsilon\sqrt{\frac{b}{2ga}},$$

in reliquis autem tubi punctis hae binae applicatae evanescant. Producto iam axe *AB* utrinque in infinitum sumtisque spatiis *BA'*, *A'B''* etc. *AB'*, *B'A''* longitudini tubi aequalibus, ex praeceptis supra datis continuatio ambarum scalarum ita institui debet. Primo sumto intervallo $BT = Bt$, quia tubus in *B* est apertus, capiatur $TM = tm$ infra, at

$TN = tn$ supra axem: tum simili modo ad alteram partem sumto $AT' = At$, quia tubus in *A* etiam est apertus, capiatur $T'M' = tm$ infra et $T'N' = tn$ supra axem. Iam hac scala *T'M' N'* cum orificio *BB* collato, sumatur $Bt' = BT'$, $t'm' = T'M'$ et $t'n' = T'N'$, utrumque supra axem, sicque utrinque in infinitum progrediendo applicatae *TN* et *tn* omnes supra axem, alterae vero *TM* et *tm* alternatim supra et infra axem erunt dispositae, et in quovis spatio intervalla *AT* et *At* primo *At*, uti et intervalla *BT*, *Bt* etiam primo *Bt* erunt aequalia. His positis ad quodvis tubi punctum *S* successive plures adeoque infiniti pulsus perferentur, primo nempe pulsus principalis *tmn* post tempus $= St\sqrt{\tfrac{b}{2ga}}$, tum pulsus *TMN* post tempus

$$= ST\sqrt{\tfrac{b}{2ga}} = (Bt+BS)\sqrt{\tfrac{b}{2ga}},$$

tertio pulsus *T'M' N'* post tempus

$$= ST'\sqrt{\tfrac{b}{2ga}} = (At+AS)\sqrt{\tfrac{b}{2ga}}$$

et ita porro, quorum pulsuum successivorum indoles in sequente tabella exhibetur:

| Elapso tempore ab initio: | In tubi puncto S erit densitas | celeritas sec. *AB* |
|---|---|---|
| $St\sqrt{\tfrac{b}{2ga}}$ | $B\left(1+\tfrac{1}{2}tn+\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn+\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(2Bt-St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1-\tfrac{1}{2}tn-\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn+\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(2At+St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1+\tfrac{1}{2}tn-\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn-\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(2At+2Bt-St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1-\tfrac{1}{2}tn+\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn-\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(2At+2Bt+St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1+\tfrac{1}{2}tn+\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn+\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(2At+4Bt+St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1-\tfrac{1}{2}tn-\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn+\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(4At+2Bt+St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1+\tfrac{1}{2}tn-\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn-\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| $(4At+4Bt-St)\sqrt{\tfrac{b}{2ga}}$ | $B\left(1-\tfrac{1}{2}tn+\tfrac{1}{2}tm\right)$ | $+\tfrac{1}{2}\left(tn-\tfrac{1}{2}tm\right)\sqrt{\tfrac{2ga}{b}}$ |
| etc. | etc. | etc. |

## COROLLARIUM 1

49. Pulsus ergo successive aërem in *S* concitantes ratione indolis sunt quadruplices, quartus enim quisque eadem indole est praeditus, tempore autem $= 2AB\sqrt{\tfrac{b}{2ga}}$ quaterni

pulsus in eundem tubi locum appellunt, hique paribus intervallis se insequentur, si fuerit $Bt = At$ et $St = \frac{1}{2}Bt$.

## COROLLARIUM 2

50. Cum tempore $= 2AB\sqrt{\frac{b}{2ga}}$ quatuor pulsus edantur, singulis minutis secundis evenient $\frac{2}{AB}\sqrt{\frac{2ga}{b}}$ pulsus, qui numerus, si $AB$ in pedibus Lond. exprimatur, fit $\frac{1832}{AB}$. . Si hi pulsus sint satis fortes, sonum referent, qui ergo ab unico aëris pulsu ortus est censendus.

## COROLLARIUM 3

51. Si primus pulsus in ipso orificio $BB$ excitetur ibidemque quasi auris teneatur, ob $St = 0$, $Bt = 0$ et $At = AB$ omnes pulsus erunt geminati seque sequentur temporibus

$$2AB\sqrt{\frac{b}{2ga}} = \frac{AB}{458} \text{ sec.},$$

siquidem $AB$ in pedibus exprimatur. Hinc si longitudo $AB$ sit quasi 500 ped., plures eiusdem soni repetitiones singulis minutis secundis se excipient et exaudientur.

## SCHOLION

52. Hinc intelligere licet, quomodo echo plurium repetitionum generetur, cum tubus satis longus tale phaenomenum exhibere possit, neque tamen ulla soni repercussio eveniat. Secundum calculum quidem singulae repetitiones aeque deberent esse fortes, verum tamen facile intelligitur plures esse causas, quibus sequentes repetitiones continuo debilitentur, quoniam conditionibus in calculo assumtis, in praxi neutiquam satisfieri potest. Cum tamen exempla non deficiant, quibus eiusdem vocis plures repetitiones satis distincte eduntur, nullum est dubium, quin earum causa in simili aëris agitatione, qualem hic sumus contemplati, sit quaerenda. Deinde etiam resonantiae ratio hinc potissimum explicari debere videtur, etiamsi laterum tubi natura, quatenus ab aëre interno simul motum vibratorium recipiunt, eo non parum conferat. Praeterea ex praecedentibus satis est manifestum his similia phaenomena prodire debere, si tubus utrinque clausus accipiatur; pulsus enim derivati super axe eodem plane modo erunt dispositi, hoc tantum discrimine, quod hic applicatae *tn* ex celeritatibus natae alternatim infra et supra axem collocari debeant neque ergo opus esse arbitror, ut hunc casum seorsim evolvam. Inde autem non contemnendae dilucidationes pro motu tympanorum derivari poterunt; etsi enim membranae, quibus tympana teguntur, eorumque latera sonum potissimum moderantur, tamen in aëre incluso quoque ratio repetitionis pulsuum quaerenda videtur. Superest igitur in hoc capite tantum, ut agitationem aëris in tubis ex altera parte clausis, ex altera vero apertis investigemus.

## PROBLEMA 77

53. *Si tubus* (Fig. 83) *aequaliter amplus AABB fuerit in altero termino AA apertus, in altero vero BB clausus atque in eo ad t pulsus quicunque fuerit excitatus, eius propagationem in tubo investigare.*

SOLUTIO

In aëre ad $t$ agitato sit densitas $= Q$, naturali existente $= B$, et celeritas secundum directionem $AA = \Upsilon$, inde capiatur

$$tm = l\tfrac{Q}{B} \quad \text{et} \quad tn = \Upsilon\sqrt{\tfrac{b}{2ga}},$$

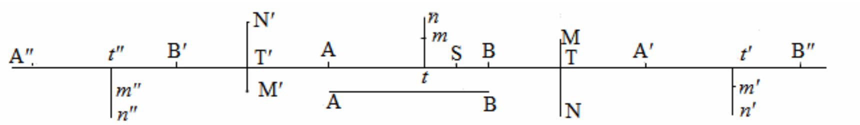

Fig. 83

ita ut punctum *m* referat scalam densitatum, punctum *n* vero scalam celeritatum, siquidem reliqua utriusque scalae puncta in ipsum axem incidunt. Iam eo utrinque producto sumtoque intervallo $BT = Bt$, quia tubus in *BB* est clausus, applicata $TM = tm$ supra, altera vero $TN = tn$ infra axem est ponenda. Ad alteram vero partem, quia tubus in *AA* est apertus, sumto spatio $AT' = At$, applicata $T'M' = tm$ infra, altera vero $T'N' = tn$ supra axem statuatur, similique modo sumto $At'' = AT$, fieri oportet

$$t''m'' = -TM = -tm \quad \text{et} \quad t''n'' = +TN = -tn.$$

Tum iterum ultra *B* progrediendo sumto intervallo $Bt' = BT'$ capi debet

$$t'm' = +T'M' = -tm \quad \text{et} \quad t'n' = -T'N' = -tn,$$

sic porro utrinque in infinitum. Nunc videamus, quomodo omnes isti pulsus successive ad tubi punctum quodcunque *S* perveniant, id quod ex adiuncta tabella perspicietur:

| Elapso tempore ab initio: | In tubi puncto S erit densitas | celeritas sec. *AB* |
|---|---|---|
| $St\sqrt{\frac{b}{2ga}}$ | $B\left(1+\frac{1}{2}tn+\frac{1}{2}tm\right)$ | $+\frac{1}{2}(tn+tm)\sqrt{\frac{2ga}{b}}$ |
| $(2Bt-St)\sqrt{\frac{b}{2ga}}$ | $B\left(1+\frac{1}{2}tn+\frac{1}{2}tm\right)$ | $-\frac{1}{2}(tn+tm)\sqrt{\frac{2ga}{b}}$ |
| $(2At+St)\sqrt{\frac{b}{2ga}}$ | $B\left(1+\frac{1}{2}tn-\frac{1}{2}tm\right)$ | $+\frac{1}{2}(tn-tm)\sqrt{\frac{2ga}{b}}$ |
| $(2At+2Bt-St)\sqrt{\frac{b}{2ga}}$ | $B\left(1+\frac{1}{2}tn-\frac{1}{2}tm\right)$ | $-\frac{1}{2}(tn-tm)\sqrt{\frac{2ga}{b}}$ |
| $(2At+2Bt+St)\sqrt{\frac{b}{2ga}}$ | $B\left(1-\frac{1}{2}tn-\frac{1}{2}tm\right)$ | $-\frac{1}{2}(tn-tm)\sqrt{\frac{2ga}{b}}$ |
| $(2At+4Bt-St)\sqrt{\frac{b}{2ga}}$ | $B\left(1-\frac{1}{2}tn-\frac{1}{2}tm\right)$ | $+\frac{1}{2}(tn+tm)\sqrt{\frac{2ga}{b}}$ |

| | | |
|---|---|---|
| $(4At+2Bt+St)\sqrt{\frac{b}{2ga}}$ | $B\left(1-\frac{1}{2}tn+\frac{1}{2}tm\right)$ | $-\frac{1}{2}(tn-tm)\sqrt{\frac{2ga}{b}}$ |
| $(4At+4Bt-St)\sqrt{\frac{b}{2ga}}$ | $B\left(1-\frac{1}{2}tn+\frac{1}{2}tm\right)$ | $+\frac{1}{2}(tn-tm)\sqrt{\frac{2ga}{b}}$ |
| $(4At+4Bt+St)\sqrt{\frac{b}{2ga}}$ | $B\left(1+\frac{1}{2}tn+\frac{1}{2}tm\right)$ | $+\frac{1}{2}(tn+tm)\sqrt{\frac{2ga}{b}}$ |
| etc. | etc. | etc. |

COROLLARIUM 1

54. Hoc ergo casu maior disparitas in pulsibus successivis ad eundem locum appellentibus cernitur, quoniam hic demum octavus quisque ad eandem indolem revertitur. Temporis autem intervalla eandem legem tenent ut ante.

COROLLARIUM 2

55. Si sit $St=0$ et $At=0$, ita ut sonus in $A$ edatur ibique auris constituatur, ordo et indoles pulsuum ad aurem successive delatorum ita se habebit:

| Elapso tempore | densitas | celeritas |
|---|---|---|
| $2AB\sqrt{\frac{b}{2ga}}$ | $B(1)$ | $-2tn\sqrt{\frac{2ga}{b}}$ |
| $4AB\sqrt{\frac{b}{2ga}}$ | $B(1)$ | $+2tn\sqrt{\frac{2ga}{b}}$ |
| $6AB\sqrt{\frac{b}{2ga}}$ | $B(1)$ | $-2tn\sqrt{\frac{2ga}{b}}$ |

densitas ergo hic nullam subit mutationem, unde, si primus pulsus nullam habuerit celeritatem, sequentes omnes se mutuo destruunt.

COROLLARIUM 3

56. Sin autem sonus in $BB$ excitetur ibique a sensu auditus pulsus sequentes excipiantur, ut sit $St=0$, $Bt=0$ et $At=AB$, , tum celeritas in pulsibus appellentibus erit nulla, densitas sola alternatim erit

$$=B(1-2tm) \text{ et } B(1+2tm)$$

hique pulsus temporis intervallis $2AB\sqrt{\frac{b}{2ga}}$ se insequentur. His autem duobus casibus quaterni pulsus in unum uniuntur, quam ob causam etiam hi pulsus complexi alia proprietate praediti sunt ac pulsus simplices propagati.

SCHOLION 1

57. Quae hactenus hic sunt tradita de pulsuum indole et propagatione, Physicis occasionem

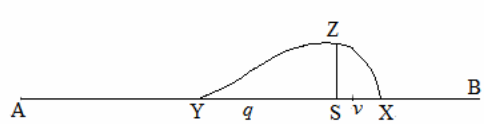

Fig. 84

praebere possunt in naturam tam sonorum quam auditus accuratius inquirendi. In quo negotio imprimis erit perpendendum (Fig. 84), si pulsus a quacunque causa excitatus secundum directionem *AB* in aëre propagetur et nunc quidem spatium *Y X* occupet, eius naturam semper una quadam linea curva *XZY* repraesentari posse, quae simul sit scala densitatum et celeritatum, etiamsi in pulsu principali hae duae scalae maxime fuerint diversae; hoc autem de pulsibus simplicibus est intelligendum, non vero de complexis, qui ex pluribus simplicibus a contrariis plagis in eundem locum venientibus coalescunt, cuiusmodi sunt ii, quos in corollariis 2 et 3 descripsi. Dum igitur pulsus ille simplex in spatio *XY* versatur, primum eius latitudo *XY* considerari debet, intra quam aër de statu aequilibrii est deturbatus, extra eam vero ubique quiescit: quo enim haec latitudo fuerit maior, eo plenior quasi erit sonus. Deinde curva illa *XZY* hoc modo aëris agitationem declarat: ea scilicet aëris particula, quae ante pulsus adventum erat in *S*, nunc erit translata in *s*, ut sit spatiolum $Ss =$ areae $XSZ$ ad lineam illam rectam, qua unitas designatur, applicatae: tum vero eius densitas *q* ita erit comparata, ut sit $q = B(1+SZ)$ denotante *B* densitatem naturalem, seu erit $SZ = l\frac{q}{B}$ , motu autem simul versus *B* feretur, cuius celeritas erit $= SZ\sqrt{\frac{2ga}{b}}$.. Eadem autem hic recta pro unitate assumta est intelligenda, qua in scalis pulsus principalis construendis fuerimus usi. Quodsi ergo aër hac agitatione ad aurem perferatur, hinc erit colligendum, quomodo auditus organum afficiatur.

## SCHOLION 2

58. In figura totam curvam *XZY* supra axem descriptam exhibeo; id quod evenit, cum in toto pulsus intervallo densitas aëris naturali est maior simulque singulae aëris particulae motu versus *B* directo sint praeditae: quod si eveniat, postrema pulsus particula ex *Y* in *y* erit translata, ut sit spatiolum $Yy =$ areae $XZY$ , hoc scilicet casu universus aër post pulsum ab agitatione praecedente per tantum spatiolum successisse est intelligendus. Quodsi (Fig. 85) talis successio non contingat, ut postrema pulsus particula *Y* in loco suo naturali persistat, curva *XZY* agitationem exhibens ita erit comparata, ut partim supra partim infra axem sit disposita, eiusque area negativa *YZ'O* positivae *XZO* fiat aequalis. Tum igitur, si pars superior antecedat, ad aurem prius perferentur aëris particulae naturali densiores et in plagam *B* motae, post vero sequentur particulae rariores, motum retro directum habentes. Ex quo concludendum videtur essentiale sonorum discrimen in hoc esse constituendum, prout pars superior *XZO* vel praecedat vel sequatur. Si eveniret, ut haec curva ter vel quinquies axem *XY* secaret, inde sine dubio peculiares affectiones in sonum redundarent. Semper autem figura huius curvae *X Z Y* maxime essentiale discrimen sonorum repraesentare est censenda; eo nimirum discrimine remoto, quod a successione plurium pulsuum proficiscitur. Quanquam autem haec ex sola consideratione tuborum aeque amplorum sunt deducta, tamen multo latius patent, et per ea, quae circa tubos inaequaliter amplos perscrutari licebit, confirmabuntur.

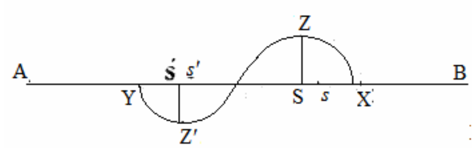

Fig. 85