

### **Chapter 3.**

#### **On Mobility as the second General Property of Bodies.**

*16. No material body is tied to its location in such a manner that it should be impossible to move it to any other location. This possibility of occupying another location is called the mobility, which accordingly must be ascribed to bodies as their second general property.*

Whatever concept of space we may have, we cannot imagine a body other than that it occupies a certain part of space, and in a manner of speaking fills it. This part of space, which in extent must be equal to that of the body, is called its location; although a body cannot be simultaneously at more than one location, it is nevertheless not tied to its location in such a manner that it could not at another time be at a different location, for neither in space, nor in the body itself is there anything that would contradict this. The possibility of moving a body from one location to another is so inherent in our concept of bodies, that we cannot imagine a body for which this should not be possible. One can of course imagine a body that is always at the same location; but the question here is not whether or not a body in reality always remains at the same location, but rather whether or not it would be possible to shift it to a different location. It is this mere possibility that represents the mobility, which is justly regarded by all teachers of Natural Science as one of the general properties of bodies. It is therefore possible that a body that presently is here, could at a later time be at any other location; and since this cannot occur without movement, this property is called the mobility or the possibility to move.

*17. As long as a body is at the same place, one says that it is at rest, but if it moves from one place to another, one assigns movement to it. Rest therefore means the remaining of a body at the same location, and movement is the advancement from one place to another.*

As far as its outer circumference is concerned, a body can be at any particular place, although its inner parts execute movement. Thus as far as its outer circumference is concerned, one can say of a clock that it is at rest, although its internal system of gears is in constant motion; therefore one must differentiate between the state of a body as a whole and the state of its parts. However if all parts are at complete rest, then it is certain that the whole must also be at rest, and the whole cannot move without moving the parts as well. In this general investigation we shall in the main deal only with the movement or rest of the whole, and what is found there will subsequently also apply to all the individual parts. That means we shall here consider only the relation of a body's outer circumference to the space, and if this relation remains unaltered, the body is at rest, but if the relation changes, the body is in motion.

18. *Since movement affects only the relation of a body's outer circumference to space, it does not alter the inner state of the body, and consequently movement cannot be regarded either as a property or an accidental characteristic of a body.*

There has been much disputation amongst teachers as to whether or not movement should be counted amongst the properties (*proprietates*) or the accidental characteristics (*accidentiae*) of a body. A property it cannot be, since the properties of a thing are invariant. Moreover, since the accidental characteristics are considered to be such that all changes in a thing occur in the accidental characteristics, such that when these change, the thing itself suffers a change, it is clear that movement can also not be counted amongst the accidental characteristics of a body. One must however admit two types of accidental characteristic, one type concerning the thing itself, whilst the other type is concerned with its relation to other things. If one does this, there remains no doubt that movement must be counted amongst the second type of accidental characteristic. In this way one removes all difficulties that are raised in connection with movement itself, and also with the way it is imparted. But this is only the case if one considers the movement of the body as a whole in terms of the relation of its outer circumference to space. If the inner parts of a body are set in motion amongst themselves, the true state of the body undergoes change. In fact, as will be shown below, this is the only way in which changes in a body can occur. The movement of the whole body can contribute to this in some cases, but this is due not to the movement itself, but rather to particular consequences that flow from it, as will be made clear in the discussion below.

19. *When a body is in motion, it moves continually from one location to an adjacent one, and finds itself at every moment at a different location without remaining anywhere even for the briefest time.*

Whilst a body is in motion, one cannot properly say that it is at any stage at a particular location, since this way of speaking seems to suggest a residence time. The expression “passes through” would seem more appropriate, since movement is a continuous passing from one location to another. An incorrect definition of such expressions can easily lead to gross error. Thus some think that movement is nothing other than a succession of many short residences at intermediate locations. Let A be the starting location and Z the location at which the body arrives after some time; one then imagines a certain number of intermediate locations such as B,C,D,E etc. and says that the body in effect hops from A to B, and stays a little while at B, more or less such as to rest. Similarly it hops further from B to C, from there to D and so on. It is possible to imagine such a hopping motion, interrupted by small periods of rest, and this may indeed occur in some cases. However the jump from A to B would still remain a true movement, such as we are discussing here, and would not be interrupted by further small periods of rest. But if we wanted to say that between A and B there were some further locations with small periods of rests, and between these further locations there were additional locations and so forth without end, then it is clear that the body would never leave its original location, since it would have to remain for a little while at each intermediate spot. Alternatively one would have to say that each

period of rest were of infinitely short duration, that is of no duration at all, which would automatically invalidate this strange concept. But if the body goes from A to Z without rest in between, then it passes through all intermediate locations B,C,D etc. and it would be inappropriate to attempt counting all possible intermediate locations. For no two locations can be so close that there could not be between them an additional location, and so on ad infinitum, and these are really all passed through by the body. On this rests the concept of continuity, which applies to extent as well as to movement. It allows one to think of parts, but these are not regarded as separated from each other. In continuity everything hangs together and there is no actual subdivision that would allow parts to be counted.

*20. When a body moves, all points which one can imagine in the body describe certain lines, which one calls the paths described by the points.*

If one deals with a body as a whole, it is difficult to have a proper understanding of its motion, since the different parts which one can distinguish in any body can execute quite different motions. To help our understanding, it is therefore customary to regard separately all points one can imagine in a body. Since each point moves with the body, it is only necessary to consider its paths, i.e. the line in which it moves. Here, as in geometry, one regards a point as having no parts, and since there is no possibility of differentiation within it, its movement is best described by the path it traverses. If one knows the path for every point of the body, one has a sufficient concept of the movement of the body itself. It might appear that, because there is an infinity of points in a body, this way of characterizing the movement is too complicated, and in fact impossible. However apart from the fact that in the case of many bodies it is sufficient to investigate the movement of only a few points, which then automatically determines the movement of all others, there are unfailing methods of solution, which allow all difficulties arising from the infinite number of points under consideration to be overcome.

*21. For a complete description of the movement of a point it is not only necessary to describe the path it traverses, but one must also be able to determine the location on this path at which the point is at any instant of time.*

Here we discuss the movement of a point, not because we regard it as part of a body, but because the movement of a body is most conveniently described in terms of the movement of points within it. But if one can determine the location of the moving point at every instant of time, or rather the location that the point at that time passes through, then one has complete knowledge of its motion, and this includes already the traversed path itself. This is so because, when all points in space are determined through which the point passes at any time, then these describe the whole path. If in addition one has determined in the same fashion the movement of all points in the body, one can claim a complete description of the body, for one cannot conceive of anything concerning the movement of bodies that could not be discussed completely then. Many regard the Science of Movement as most dark and mysterious, which

stems from the fact that they have failed to develop and properly analyse all matters involved. The Science of Movement has however now been clarified to such an extent, that all remaining difficulties are not in the concepts of the Science itself, but solely in the art of solving problems; further development of this art is now the main aim.

*22. Rectilinear and uniform motion takes place when firstly a point moves along a straight line, and when secondly it traverses equal parts of this line in equal times; from this it will also be understood what is meant by curvilinear and non-uniform motion.*

These two conditions, namely that a point firstly moves along a straight line, and that secondly it traverses equal paths in equal times, no doubt describe the type of movement that is easiest understood, and of which it is necessary to have a clear concept. This the more important, because subsequently we also have to deal with curvilinear motion. The straight line and the passing through equal paths in equal times therefore constitute the main example of motion, but it is easily seen that a point could follow a straight line without traversing equal paths in equal time, or a point could traverse equal paths in equal times without following a straight line. But when we speak of equal paths traversed in equal times, then this must apply to all, even the smallest intervals of time. It is not sufficient that equal paths are traversed every hour, but it must also apply to every minute, every second etc. This is best expressed in terms of the Theory of Ratios by saying that the paths must always be proportional to the times. When in this type of movement 60 units of path are traversed in one hour, then in one minute one unit of path, and in one second  $1/60$  of a unit of path will be traversed. On this rests the concept of uniform motion.

*23. In the case of rectilinear and uniform movement the straight line is referred to as the direction of movement; the speed is the ratio of the path to the time in which it is traversed.*

If in the case of this type of movement one knows the path traversed in a certain time, then one can determine from this the path that is traversed in any other time, for however much longer or shorter the time, the longer or shorter will also be the path, i.e. the ratio of path to time will remain constant. One says of this type of movement that it has always the same speed. But if we imagine two uniformly moving points, one of which traverses 2 units of path per second, whilst the other traverses 4 units of path per second, then we say that the speed of the latter is twice that of the former, and if the latter were to traverse 6 or 8 units of path per second,, then its speed is three times or 4 times as great. The greater the ratio of path to the time during which it is traversed, the greater is the speed, and this ratio is obtained by dividing path by time. Therefore if a point traverses path  $S$  in time  $T$ , and another point traverse path  $s$  in time  $t$ , then the speed of the former point to that of the latter is as  $S/T$  to  $s/t$ . If one assumes certain values for time and path, one can say that the speed is equal to the path divided by the time. From this follows that the path equals the speed multiplied by the time, and the time equals the path divided by the speed, relations with which

one should make oneself well acquainted. As regards the direction of movement, in the case of rectilinear movement the straight line indicates whereto the point is moving, and since the line is straight one sees that the movement is always towards the same region. The straight line is therefore referred to as the direction of the movement.

*24. If the movement is curvilinear and non-uniform, one can imagine for every moment in time a rectilinear and uniform movement that in this moment is absolutely the same as the actual movement as regards both its direction and its speed.*

It is customary to say that any movement can, for a single instant, be regarded as rectilinear and uniform, just as in geometry the infinitely small parts of a curve can justly be regarded as straight. But since an incorrect concept of the infinitesimally small could easily lead to difficulties, I have discussed the matter in a different way, which however leads to the same conclusion. It is easy to see that when a point moves along a curve, then the tangents to the curve at any location will indicate the direction of the movement. By means of the differential calculus the speed is found by dividing the differential of the path by the differential of the time, just as if the movement through an infinitesimally short path were uniform. It is therefore possible that a movement is such that both its direction and its speed change at every instant. One sees thus clearly that the entire concept of curvilinear and non-uniform movement rests on the fact that one can assign to the moving point at any instant a direction and a speed, and on this rests the entire science of the movement of bodies.

*25. Mobility distinguishes the bodies from space, to which latter this property can not be ascribed. But the essence of material bodies is not represented merely by their mobility.*

Material bodies have extent just as space has, because the extent of a body is the same as the extent of the location it occupies. But since space has no boundary, and therefore has infinite extent, it follows that extent applies to space in a different manner than to material bodies. A location however, as we imagine it, cannot be regarded as anything other than a part of infinite space that is occupied by a body. Without bodies various locations would have no features that would allow us to distinguish between them, and it would be even less possible to move one location to a different one. Therefore mobility cannot be ascribed either to infinite space, nor to those parts of it which we regard as the extent of material bodies. It is also not possible to say of the empty space between material bodies, as conceived by some scholars, that it has mobility. For example, if in my room there were an empty space, which however at some later time were filled, whilst simultaneously an empty space were created in a different room, then it would not be possible to say that the empty space that had been in my room, had been transferred into the other room. For the latter could have been created without the former being filled. I am speaking here about the commonsense notion of space, without meaning to discuss whether or not space is something real. We must first obtain a complete concept of material bodies, before we can dare to approach that problem. Finally, we can also not say that the essence of material bodies rests entirely in their mobility. This property implies

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certain other properties that appear to characterize the essence of material bodies further, but, as we shall see later, even these other properties are not sufficient to make a thing with extent a material body. A thing can have extent as well as mobility and still not be a material body.