Chapter 15.

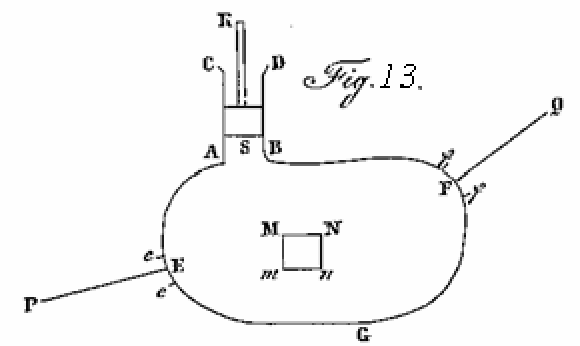
On the liquid state.

112. A liquid matter must first of all have the property that its particles are not attached to each other, so that each particle can without resistance be separated from the others and be set in motion.

One sees this most clearly when one considers the difference between solid and liquid bodies. To tear a part from a solid body and set it in motion more force is necessary than if it were quite unattached; in the latter case even the smallest force would be sufficient, as we have seen at length above; but if a part is to be separated from a solid body, then a force is needed that is able to overcome the attachment; by contrast, in liquid bodies there is no such attachment of the parts to each other, and every part can be separated off without the need for a force for the tearing off itself. But if a force acts only on a part of a solid body, then it can not set this part in motion without at the same time also setting in motion the other parts, on which it is not acting, which has its cause in the attachment of the parts to each other. In this respect a liquid body resembles a heap of sand, from which each grain can be freely removed, because the grains are not attached to each other. It is true that no grain can be taken from the centre of the heap without at the same time setting in motion many others; but it is clearly due to the fact that the other parts are in the way of the movement, and resist it simply because of their persistence. The same must be understood in the case of liquid matter, from the centre of which also no part can be removed, or merely set in motion, without disturbing others. But in the case of the sand heap each grain is a solid body, and it is not possible to remove in the same manner only one half of a grain. Perhaps this is similar in the case of many liquids, since one finds that very small parts of liquid matter frequently display properties of a solid body.

113. The essence of a liquid is that when liquid matter is pressed in one location and can not escape from this force anywhere, then it exerts all round an equal force. For when the liquid matter is enclosed in a vessel, then it presses everywhere on its walls with equal force.

To completely illuminate this property, it is best to consider the liquid matter enclosed in a vessel, the walls of which, because of their solidity, prevent the matter to evade the force that acts on it in one location; let the matter be enclosed in such a vessel *AEGFB* (Fig.13), on which we imagine a tube *ABCD*, through which the matter is pressed by means of a stopper *KS*, and can be distributed throughout the entire space of the vessel; for we attribute to the matter neither weight nor any other force that could act on the particles. Here it is immaterial whether or not the liquid matter is compressible or not; for in the first case the stopper will compress the matter as far as it is able to, and when this has occurred, and the compressing force is in equilibrium, then the state of the liquid matter is as stated above. The latter will then press *everywhere on the walls of the vessel, whatever its shape; at all points such as E and F* the pressure will be the same and be perpendicular, such as *EP* and *FQ*. Thus if we imagine on the inner wall a part *ee*, then the pressure on it will be the greater, the greater we imagine this part to be; therefore if we assume this part *ee* to be the same as the width of the stopper, then the pressure on it must be equal to the force acting on the stopper. Let us put the width or base area of the stopper *S* = *aa*, and the force pressing on it = *p*, then, if the area *ee* = *aa*, the force pressing on it must also be *p*. But if one considers an area *ff*, that may be bigger or smaller, then it would have to withstand a correspondingly bigger or smaller pressure. The force pressing on ff will be $\frac{ff}{aa}p$ and its direction on area *ff* will be perpendicular. This property already includes the previous one, and since all matter that has this property must be called liquid, we justly regard this property as the essence of a liquid.



114. The same pressure is also experienced by all bodies immersed in the liquid matter, as these are compressed with the same force from all directions, assuming them not to have sufficient strength to resist the pressure.

As we have seen, the magnitude of the pressure on the inner walls of the vessel depends firstly on the force *p* that presses on the stopper, and secondly on the base area *S* = *aa* of the stopper, through which the pressure is exerted on the liquid matter. If we then consider on one wall an area ff , then this is subjected to a force $\frac{ff}{aa}p$, but the same pressure is also experienced by all the inner parts of the liquid matter, and if these allowed themselves to be further compressed, and if the force were sufficient for this, then they would really be brought into a smaller space; but here we assume that the liquid matter either does not allow itself to be compressed at all, or that, through the action of the stopper it has already been compressed to the furthest possible extent. If we now imagine a body *MNmn* under this liquid matter, then every side of it will be pressed by the same force, and the direction of the pressure will be perpendicular to it; therefore if one side of this body such as *MN*, = *ff*, then the pressure on it will also be $\frac{ff}{aa}p$. This is the entire pressure that is exerted, for in fact all particles on the same side experience a pressure corresponding to their size, which overall represents the total pressure $\frac{ff}{aa}p$. Because of this, if the area is either convex or concave, then one must imagine it divided into infinitely many small parts, and from the infinitely small forces that press on these, taking into account their directions, determine the total force that press on each, for which the Theory of Equilibrium gives the necessary rules. Accordingly the nature of a liquid consists of the fact that any pressure propagates at once through all parts of the liquid matter with the same force; and in this a sand heap differs significantly from liquid matter; for if the vessel *AEGFB* were filled with sand and is pressed by the stopper *S*, then this pressure will never propagate with equal force through the entire vessel; but the sideways pressure of ee and ff will be smaller than when the matter were liquid.

115. Because the pressure depends on the magnitude of the area on which it acts, it is best if one represents it by a magnitude which, when multiplied by an area, expresses the magnitude of the force that acts on the area, and the constancy of the pressure rests on the fact that this magnitude is constant.

If we express by *aak* the force *p*, which acts on the base area of the stopper $S = aa$, then the pressure on any area *ff* is

$$\frac{ffp}{aa} = ffk.$$

Therefore when $\frac{p}{aa}$, i.e. *k*, remains the same, then the pressure on the liquid matter is also the same. For if one considers different stoppers such that the forces acting on them are proportional to their base areas, then they exert the same pressure on the liquid matter; therefore the smallest force can produce the highest pressure, if only the base area of the stopper is made very small, for it is clear that if the force were a thousand times smaller, but at the same time the base area of the stopper were mad a thousand time smaller, then the pressure on a given area *ff* would have to remain the same. We thus put $p = aak$ or $\frac{p}{aa} = k$, because the magnitude of the pressure depends solely on the magnitude of *k*, and the pressure exerted on area *ff* is given by *ffk*. To picture this completely one regards *k* as a height and expresses the force acting on any area ff by the volume of a cylinder of base area *ff* and height *k*, since the volume of such a cylinder is *ffk*. This picture is also very convenient because the forces are best represented by a weight; for this one selects a uniform matter and says that the force, by which the area ff is pressed, is just as large as the weight of such matter that fills the volume of the cylinder *ffk*. For one comprehends clearly how strongly such a weight would press on an area on which it lies; and just as great is the pressure that the area *ff* has to sustain from the liquid matter. Therefore such a height k gives us a clear understanding of the force that presses on the inner wall of the vessel filled with liquid matter, and that at the same time presses on all parts within the latter. For the larger or smaller this height is, the larger or smaller, in the same proportion, will be the force of the pressure.

116. Liquid matter can not remain at rest if the height, that is determined in the manner just explained, is not everywhere the same .But this is to be understood in relation to such liquid matter ,parts of which are not acted on by weight or any other particular force.

Here we exclude not only by which all parts of liquid matter are pushed downwards, but all other similar forces that could in particular act on any particle of the liquid matter. Accordingly we consider liquid matter all particles of which, such as *MNmn*, are pressed only by the surrounding liquid matter, where one must carefully distinguish these forces that stem from the surrounding liquid matter from such forces as weight. For although is also caused by the pressure of a surrounding subtle matter, it must be distinguished from the coarser liquid matter that we are considering here, and although all coarser liquid matter, such as for example water, is intimately mixed with the subtle matter of the aether, it will be shown below how the pressure due to the subtle matter is sharply distinguished from that exerted by the coarser matter. Thus if a particle *MNmn* is to remain at rest, then the pressure from all sides must be equal, that means the height k, that determines the pressure, must everywhere be the same. Since this applies to all parts, it is clear that the liquid matter can not remain at rest, unless the height k, that indicates the pressure, is everywhere the same. For if we imagine the particle *MNmn* as a cube, then one sees at once that if the pressure on two opposite sides *MN* and *mn* were not equal, the cube would have to be set in motion by the greater pressure. The truth of this is unaffected by whether or not the matter is compressible, provided is has only once been brought to equilibrium by the pressing force; it is also irrelevant whether or not the density of the liquid matter is everywhere the same.

117. To understand precisely the state of a liquid matter, the main consideration is the pressure that all its parts have to withstand from the surrounding ones, and if this pressure, or the height that determines it, is known, then one is able to judge as to the state of rest or the changes that take place in it.

If one knows the magnitude of the pressure that prevails at all points of liquid matter, and if at the same time the particular forces that act on each particle are given, then one knows all forces that drive every particle of the liquid matter. From this one can judge whether each particle will remain in its state or change it; the former will happen if all forces acting on any particle are in equilibrium with each other; is this not the case, then its state must be changed by the predominating forces. But here one must take into consideration whether or not the particles are compressible, and whether in the former case the forces are able to compress them further, or, if they are to weak, whether the particles will expand into a larger space. Consideration of these circumstances leads to a complete recognition of the state in which a liquid matter finds itself, and this rests mainly on a precise knowledge of the pressure through which the particles act on each other. Whether liquid matter is at rest or in motion, the first question must always be how strongly the particles interact with each other at any point, or what the magnitude of the height is that determines the pressure there in the manner explained above. Here it does not matter whether the pressure is due to an external force, such as we have imagined to be acting on a stopper, or whether it merely arises from a change in the state of the parts, and thus is due to their impenetrability. From this the whole theory of the equilibrium and of the movement of liquid matter must be derived.

118. A liquid matter can not be formed from a number of small particles that are solid and hard, for whatever the shape and arrangement of the particles might be, for it is not possible that a pressure that acts at one location will propagate in all directions with equal force.

If one imagines these particles in the first place to have the shape of a cube, and imagines them to be arranged in an orderly manner on one another, then one sees easily that if the uppermost is pressed down by a force, the lowest one will press on the ground with equal force, but sideways no force will be exerted; therefore if many such rows fill a vessel, and a force presses on them from above, then the bottom of the vessel will experience an equal force, but the sides will experience none at all. If the particles were lying together in a disordered manner, then the pressure could also be propagated sideways, but it would not be the same in all directions. But what can be said of cubic particles is equally valid for all other shapes that involve corners, for which reason most Natural Scientists ascribe to the particles a spherical shape; but it is easy to show that from these, if they are assumed to be solid and hard, this main property of liquids can also not be obtained. It is only necessary to imagine a heap of spheres, arranged as is usual for cannon balls, to see that if they are pressed from above, they exert no force sideways, or that at least such a force would not be the same in all directions. In addition spheres that fill a vessel can not lie together in such a regular manner that a great dissimilarity in their arrangement would not develop, that also would prevent a uniform pressure. If one assumes the spheres to be in uniform motion, then the pressure can through this be changed, but can not be maintained the same in all directions; and one would have to regard as extremely rare a case where the pressure were the same, remembering that this constitutes the essence of a liquid. One need only consider that if three spheres lie in a straight line, the middle one can always be pushed sideways, whatever forces may be acting on the outer ones.