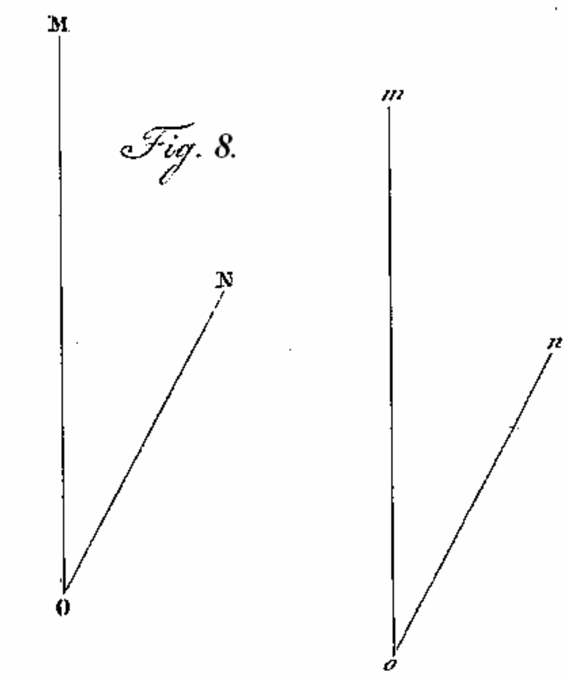
Chapter 10.

On Apparent Movement.

77. The apparent movement is relative to an observer, and is determined by two factors, firstly by the direction in which the body appears to the observer, and secondly by its distance from the observer. This is the apparent location of the body, and from its change one concludes on the apparent movement of the body.

It is assumed that, wherever the observer might be, he has a correct perception of the directions in space. All straight lines that can be drawn from the eye of the observer to anywhere, designate certain directions, and if we consider two observers, one at *O* and the other at *o*, (Fig. 8), then the lines *OM* and *ON* indicate for the observer at *O* certain directions, and the same directions are indicated to the observer at *o* by the lines *om* and *on*, provided *om* is parallel to *OM* and *on* parallel *to On*. Accordingly if the observer *O* sees a body at *M,* but the observer o sees one at *m*, so that the distances *OM* and *om* are equal, then the body *M* appears to the observer *O* to be at the same location at which the body *m* appears to the observer *o*, although the two bodies *M* and *m* are at quite different locations. Similarly will also the apparent location of two bodies *N* and *n*, as viewed by the observers *O* and *o*, be the same. As the apparent location of a body changes with time, the apparent movement is estimated from the change of direction and distance. It is the more necessary to deal with this movement here, as we can not in the world form another concept of apparent movement; for we cannot estimate the locations of bodies other than in relation to our own location, and if we are not always at the same place, there must be a large difference between the true and the apparent movement of a body. This difference can become even bigger, if we do not regard as the same directions that are determined by parallel lines, but by lines that have arbitrary relation to our point of location. For this reason we ascribe movement to the fixed stars, as they change their location as regards direction, which we, on our moving earth, regard as constant. In regard to this apparent movement the point is not which directions according to the above argument are in fact the same, but which we regard erroneously as the same.



78. When the observer always remains motionless at the same location, and if he assesses directions through various lines, then there is no difference between the apparent and the true movement of a body, and the judgment of this observer regarding the movement of all bodies is correct.

If the observer remains immobile at *O* (Fig.8) and views a body at *M*, then he will estimate its location from the direction *OM* and the distance *OM*. Let the body after some time have arrived at *N*, a location estimated by the observer by the direction and distance *ON*; and since he still derives the directions from the same lines, he will still regard the previous location of the body at *M*, and conclude that in this time the body has advanced from *M* to *N,* which is correct. But if the observer had meanwhile changed his concept of direction, he would regard the previous location of the body no longer at *M*, but somewhere else, and consequently judge its movement incorrectly: but if in the imagination of the observer no such change in the concept of direction has occurred, , then all movements of bodies appear to him as in fact they are. Those bodies that appear to him at rest, are in fact at rest, and those that appear to him to move uniformly in a straight line, do in fact follow this type of movement. This observer is therefore not in error, when he decides according to the rules of movement whether or not a movement requires forces; for those bodies that appear to him to remain in their state, actually remain in that state; and those changes that appear to him to occur, actually take place, and for this the forces are required that were determined above. Quite generally all that has so far been said regarding true movement, also applies to apparent movement, provided the observer is at rest and does not change his concept of direction. Not withstanding the fact that this is quite clear, it was nevertheless necessary to be emphasized here in order to make it easier to understand the difference between true and apparent movement in the case where the observer changes his location.

79. If the observer continually changes not only his location, but also his concept of direction, then his judgment of the state of bodies, that is of their being at rest or in motion, must be quite different from the facts.

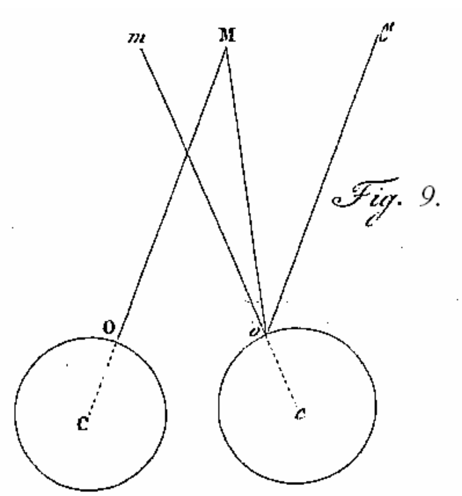


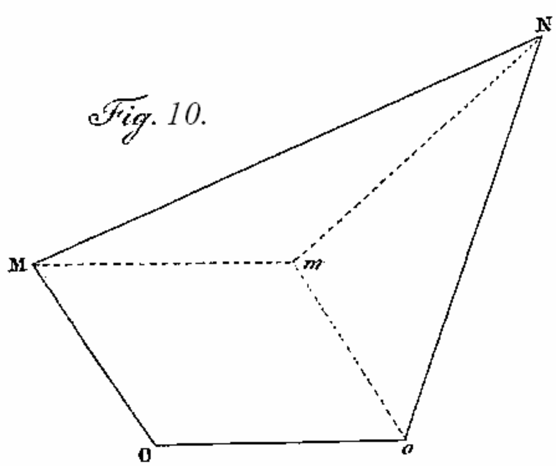
Fig. 9.

We find ourselves in that position when we determine from our perspective the position of heavenly bodies, and regard as directions the lines that we draw from ourselves upwards. For since the earth does not only rotate about the sun, but simultaneously also about its centre, we change not only at every instant our position, but also our directions. To show this clearly, let C be the centre of the earth, and let the observer at *O* (Fig.9) see at *M* a body that we assume at rest, where *OM* is drawn vertically upwards from C.

After some time, let the centre of the earth come to *c*, but the observer, because of the earth's rotation, come to *o*, so that he will see the body, that has remained at its previous location, in the direction *oM*. Since he previously saw the body vertically above himself, he now imagines he had previously seen it at *m*, where the line *com* has been drawn and it is assumed that *om* = *OM*; he therefore thinks that the body has in the meantime moved from *m* to *M*. For he compares the present location not with the one he saw before, but with the one that he believes to be the one he saw before; but he now believes the direction *om* to be the same as the direction *OM*, in which he had previously seen the body *M*. But if he were to judge the directions correctly, and now judge the direction $o\mu$, drawn parallel to line *OM*, to be the same as direction *OM* , that he had seen previously, when he was still at *O,* he would now imagine he had seen the body at μ, and since he now sees it at *M* in direction *oM*, he will believe the body had in the meantime moved from μ to *M*, although it had actually remained at rest. From this it is easy to conclude what his judgment would have been if the body had really moved in the meantime. But here it is necessary to distinguish between two different cases: the first is when the observer changes together with his estimate of the location also that of the direction; the second case is when he judges the directions correctly, and when, in his different positions he considers those directions to be the same, that are determined through parallel lines. It is in this latter way that we describe the movement of heavenly objects, when we determine their location not with reference to the vertical above us, but with reference to the fixed stars; for since the fixed stars are so incredibly far away that all lines drawn to a fixed star from our location are considered parallel, however much that location may change.

80. If the observer moves at constant speed along a straight line, and estimates directions correctly, then all bodies that are either at rest or are moving at constant speed in a straight line will appear to him to remain at rest in the same state, and this apparent movement could indeed exist without the action of any forces.

Fig. 10.



Let us assume the observer moves at constant speed along the straight line *Oo* (Fig.10), but the body also moves with a constant speed along the straight line *MN*, so that when the observer is at *O*, the body is at *M*, but when the observer is at *o*, the body is at *N*. Therefore when the observer is at *O*, he will see the body in direction *OM* at distance *OM*; but subsequently, when the observer has moved to *o*, he will see the body at *N* in direction and distance *oN*. But now he believes to have seen the body previously in direction and distance *om*, where *om* and *OM* are parallel and of equal length, and he therefore thinks the body had during this time moved fro *m* to *N*, describing the straight line *mN* at constant speed. This arises because, however large one assumes the time elapsed to be, the angle *NMm* remains the same and the ratio of the length *MN* and *Mm* remains constant, and since the angle *MNm* must always be the same, the relation of the line *Nm* to *Mm* or to the time must remain constant.

Accordingly, the apparent movement is likewise uniform in a straight line, and requires no force to maintain it, as does the true movement. Therefore if such an observer that moves uniformly in a straight line, sees a body either at rest or moving uniformly in a straight line, he can conclude that the body remains in its state and is not acted upon by an external force, just as if he were able to observe the true movement of the body, and even if the true movement should be very different from the apparent one. Since we are never able to see the true movement of a body, and our immediate sense impressions always only indicate the apparent movement of bodies, we regard the apparent movement as the true one, and we examine whether or not its maintenance requires forces. If subsequently through other circumstances we are assured as to whether or not forces are acting on the body, and to what extent they agree with the ones we have found, we can conclude as to the extent the apparent and the true movement differ from each other.

81. If the observer moves uniformly in a straight line, and if he judges directions correctly, i.e. according to parallel running lines, then the maintenance of the apparent movement requires the same forces as the true movement, however much the apparent movement may differ from the true movement.

To show this clearly, we will describe the true movement relative to three mutually perpendicular planes *AOB, AOC* and *BOC* as above (Fig.7), and we shall call movement from plane *AOB* u, movement from plane *AOC* v, and movement from plane *BOC* w. The movement of the observer is also expressed in reference to these planes; and since this is uniform in a straight line, his movements relative to the planes will be uniform. If we therefore call his movement from plane *AOB* α, from the plane *AOC* β, from the plane *BOC* γ, then α, β, and γ will be constants. To the observer movement of the body from the plane *AOB* will appear smaller, the faster his own movement from the plane is, and the apparent movement from the plane will be $u - \alpha$. Similarly one sees that the apparent movement from the plane *AOC* will be $v - \beta$, and from the plane *BOC* $w-\gamma$, and these three individually considered apparent movements will together represent the entire apparent movement. From the equations we have given above (74) it follows that for the maintenance of the true movement one requires three forces, $MP = P$, $MQ = Q$ and $MR = R$, so that

$$P = \frac{Mdu}{ndt}, \quad Q = \frac{Mdv}{ndt} \text{ and } R = \frac{Mdw}{ndt}.$$

Now replace u,v,w, by $u-\alpha$, $v-\beta$, $w-\gamma$, then since α, β and γ are constants, the differentials du, dv, dw remain unchanged, so that it is apparent that for the maintenance of the apparent movement the same forces P,Q and R are required as for the true movement. Therefore, provided the observer moves uniformly in a straight line, he will not make an error in estimating the forces needed to maintain the movement of the body, although he derives his estimate from the apparent movement of the body; as we have seen before, if the true movement can occur without any force, the apparent movement also does not require forces.

82. But if the observer does not move uniformly in a straight line, but does estimate directions correctly, then to maintain the apparent motion of all bodies, in addition to the forces that are actually acting on the bodies, further forces are required that will at every instant and in every body produce the change that takes place at the location of the observer, but acting in the opposite direction.

However arbitrarily the movement of the observer may change, with reference to the three assumed planes it can always be represented by the three movements α, β and γ, if we take these quantities to be variable. Now if the true movement requires the forces

$$P = \frac{Mdu}{ndt}, \quad Q = \frac{Mdv}{ndt}, \quad R = \frac{Mdw}{ndt}$$

then it is clear that if we replace $u, v, w,$ by $u-\alpha,\ v-\beta,\ w-\gamma,$ maintenance of the apparent movement will require the following three forces:

$$\textit{Force in direction } MP = \frac{Mdu}{ndt} - \frac{Md\alpha}{ndt} = P - \frac{Md\alpha}{ndt}$$

$$\textit{Force in direction } MQ = \frac{Mdv}{ndt} - \frac{Md\beta}{ndt} = Q - \frac{Md\beta}{ndt}$$

$$\textit{Force in direction } MR = \frac{Mdw}{ndt} - \frac{Md\gamma}{ndt} = R - \frac{Md\gamma}{ndt}$$

Therefore, apart from the forces $P, Q, R,$ that actually act on each body, three additional forces are needed that in each body produce the same change that occurs at the location of the observer, but in the opposite direction. If one takes that the mass of the whole body on which the observer is located to be $\mathfrak{M}$, and that the changes occurring there are due to three forces $\mathfrak{P}, \mathfrak{Q},$ and $\mathfrak{R}$, that act in directions *MP, MQ* and *MR*, then we have from the above equations

$$\mathfrak{M} d\alpha = n\mathfrak{P} dt, \quad \mathfrak{M} d\beta = n\mathfrak{Q} dt, \quad \mathfrak{M} d\gamma = n\mathfrak{R} dt$$

Therefore the forces required to maintain the apparent movement will be

$$\text{Force in direction } MP = P - \frac{M\mathfrak{P}}{\mathfrak{M}}$$

$$\text{Force in direction } MQ = Q - \frac{M\mathfrak{Q}}{\mathfrak{M}}$$

$$\text{Force in direction } MR = R - \frac{MR}{\mathfrak{M}}$$

83. For this reason the apparent movement of all heavenly bodies can be determined if one assumes that apart from the forces that are actually acting on any heavenly body, there is acting another force, which relates to the force by which the earth is driven as the mass of the body to the mass of the earth, and that pushes the body in the opposite direction.

It is customary to imagine the observer placed at the centre of the earth, since otherwise his location would be subject to changes that are too large for them to be transferred in the opposite direction to the heavenly bodies. But there are means to vary the apparent movement that was found for this observer, that are also applicable to any observer on he earth's surface. One assumes that all forces acting on any heavenly body are known, and also those from which the earth is driven. One must therefore only apply these latter forces in opposite direction to all heavenly bodies, after having increased or decreased them in the ratio of the mass of the earth to that of the mass of the respective heavenly body, as prescribed above. Since one can know what forces are acting on any heavenly body to produce its apparent movement, the apparent motion itself can be determined with the help of the given equations. This is not the place to make such erudite investigations, but it was nevertheless necessary to point out that the rules, that were derived from the essence of bodies, are applicable in all cases, without exception, and that even the most difficult investigations that have so far been undertaken can be carried out using these rules. To this end have I cast the basic theorems in such equations, which in every case can be easily applied, and whoever is practiced in the art of solving equations, can without further instruction solve the most difficult problems in the Theory of Movement; I therefore hope nobody will take amiss this rather lengthy discussion.