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THE SHARING OF MOTIONS IN THE COLLISION OF BODIES.

[*i.e.* The conservation of momentum in a simple collision between bodies.
We may note here that Euler measures the speed of bodies in terms of their heights dropped from rest, due one suspects, to the difficulty involved in measuring speeds accurately experimentally at the time. This in turn means both bodies initially are moving in the same direction with positive speeds, and one overtakes the other in the collision.]

1. It is agreed from experiment that the motion of bodies colliding is changed amongst each other ; the question hence arises, what shall be the cause of this change of the motion. Indeed there can be no doubt, why an account of this phenomena in the colliding of bodies may not be investigated ; indeed a body either is at rest or continuing in its state of uniform motion, unless it may be disturbed from its state by some disrupting force. On account of which the question is reduced to this, so that there may be defined, in what way this force shall be present and how great shall it be, by which the change of the motion may prevail to be produced in the collision of the bodies. Besides also it must be determined, how much change in the motion of each body must be endured in the collision.

2. Now in the previous century the rules of sharing the motion have been found by men of the greatest merit, Wren, Wallis, and Huygens, from which it is known, for two bodies running towards each other, how great the speed of each body shall be after the collision. Indeed the rules are confirmed exceedingly well from experiments, so that the truth of these cannot be held in doubt. Yet these rules have been reached by various approaches, and afterwards many diverse demonstrations have been found by others. But none of these, as far as it may appear to me, is genuine, as all have been derived from different principles.

3. According to that, since at this time no cause of the change of the motion has been shown, nor of how the bodies may be able to act on each other, had been explained. On this account I have judged the reward to be worth the effort, to propose this same dissertation, in which the rules of the transfer of the motion are deduced from the most certain principles of mechanics; and likewise it may be shown, how in the same collision the bodies act on each other and the motions may be changed.

4. This I accept beyond doubt as a principle, that every change of motion, be it a diminution or an increase, or a change of direction, be produced by forces, and that these changes are produced in succession, and not by sudden leaps. From this it may be conceded anywhere : for nothing may be brought against it, except in the motion communicated in a collision, in which it is disputed by many, that by its nature it cannot make a sudden leap. Therefore I assert, that in the meeting of two bodies, the speed of each of the bodies to be changed by the hidden forces acting between the bodies.

5. Bodies striking each other experience a similar force that can be shown especially in softer bodies such as those made of wax or clay [*i.e.* following 's Gravesande's

experiments]: for it is easy to observe the attending impressions, to be made not suddenly but gradually ; for which also it can be concluded for harder bodies, to be changed in successive steps. Moreover all bodies by striking each other make impressions, though this is not evident for all. Indeed there are bodies which are called elastic, which do not retain the changed form or impression in striking, but recover their former shape ; truly those which maintain the change in form in the collision are called soft.

6. Between these two general kinds there are innumerable intermediate degrees; evidently of these bodies, which leave not the whole but only part of the impression, to which class without any doubt all the bodies in the world belong. For neither a perfectly elastic nor perfectly soft body can be found, but all of the intermediate kinds are understood to be between these two kinds. Again bodies with a certain degree of elasticity, or lack of softness, as with the elasticity, also differ on account of hardness, following which other remaining bodies are either harder or softer. Moreover a body may be called harder, because it receives a lesser impression from the same force, and from this it is understood, how a body may be completely hard: because without doubt it may take only and indefinitely small impression for any finite force. Therefore exactly as this impression either is restored or otherwise, a perfectly hard body will belong to the class of elastic or to the class of non-elastic bodies.

7. When two elastic bodies collide together, each one presses on the other, and they inflict impressions on each other ; but afterwards they are each restored to their former shape. But while the bodies are pressed together, either one or the other body perhaps may lose some of its motion. Indeed when they are restored, then also the motion lost in the compression is restored, but is distributed otherwise between the bodies. However in the collision of inelastic bodies, the maximum impression that each body receives is retained. Therefore it is required to be determined for the motion of elastic bodies after the collision, so that then the speed of each body may be investigated, certainly when they are restored . Truly with inelastic bodies, the speed of each body ought to be found, which it had in its state of maximum compression.

8. Just as there is a need for a generating force for all motion, thus also a force is required for the parts of a body to be compressed, and for the impressions being made; for a body thus resists receiving an impression also with all the force of inertia of the motion used, which must be overcome by the force. So that we may understand more clearly the difficulty of receiving this impression, I consider a spring with the bodies connected in place, by which the impressions are received. Therefore in place of the impression, I put this spring to be compressed ; for the same corresponds to that which is compressed, whither that shall be either a part of the body, or the spring connected to the body.

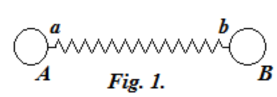


Fig. 1.

9. Therefore I place the spring *ab* between the bodies *A* and *B* running together (Fig. 1), which is compressed, while the bodies go on in turn to approach each other. And this compression of the spring while it endures, by which the force of the spring can overcome the motion by which the bodies approach each other, at which point therefore the spring will be in a state of maximum compression. Then if the bodies are elastic, I

put this spring in between to exert the effect of the restoring force itself; truly if they are not elastic, I consider the spring, when it has been reduced to a state of maximum compression, suddenly to be free from all the force requiring to expand itself.

10. On this account, we will be able to considering the collision from the laws of mechanics, which the forces maintain with the changing motions, to supply the changes of the motions of the bodies in the collision. Indeed it is to be noted, how great a speed is generated by a given force acting on a given body in a given time, but it would be prevailing to destroy the motion only if it were acting contrary to the motion of the body. But the idea of the spring between bodies running together, while it in turn is trying to expand is acted on by the forces, and by which that itself is more compressed, and the more also the motion of the bodies is diminished.

11. So the more the spring itself is compressed, there also a greater force will be necessary for the expansion, but how much that shall be in any degree of the compression, is not needed as we may know; for whatever the law being observed, yet finally after the collision it produces the same division of the motion. Therefore I will designate the magnitude of the elastic force for the general expansion by some letter *P*, and depending on nothing else. Clearly I consider *P* to be a weight, the force of which pressing down is equal to the expansive force of the spring.

12. The body *A* [of weight *A*, here considered interchangeable with mass, and also referring the position of the centre of mass, and likewise for *B*] may encounter (Fig. 2) the spring *AC* requiring the force *P* for its expansion, with the speed just as great, Fig. 2 as a weight would acquire by falling from the height v ; in an instant of time the body *A* may progress through the increment of distance $Aa = dx$, and the speed which it will have at *a*, arising from a body falling through the height $v + dv$. It is evident, if the force $P = A$, the body is going to be retarded from going upwards in the same manner, as it may be retarded by the force of gravity, namely becoming $dv = -dx$; but if there were $P = nA$ then there becomes $dv = -Pdx : A$ [*i.e.* $dv = -ndx$], if the elastic force were opposite to the motion of the body : but if by the motion it should be accelerating forwards, there will be $dv = +Pdx : A$.

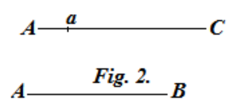
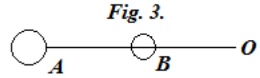


Fig. 2.

13. The body *A* (Fig. 3) may be moving along *AO* with the speed corresponding to the height *a*, while the body *B* may be moving in the same direction with a smaller speed towards *O*, corresponding to the height *b*, these bodies meet and a collision occurs. Then I consider these to begin interacting, if the distance between their centres were $= f$. Therefore with the bodies themselves considered as points (Fig. 4) so that I consider a spring of length f placed between the bodies. That distance shall be *AB*, therefore when the collision begins, the body *A* will be found at *A* , and *B* at

Fig. 3.


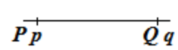


Fig. 4.

B, and the spring, because *A* is moving faster than *B*, will be continually compressed more.

14. Now the spring shall be reduced to the length *PQ*, which I put $= f - x.$ The speed which the body *A* has when it arrives at *P*, shall be derived from the height *v*, and the speed of the body *B* at *Q* from the height *u*, and the force of the spring, which it now has for expanding shall be $= P$. In the element of time that the body *A* may be progressing the smallest distance through the element $Pp = dr$, and the body *B* through $Qq = ds$; the height showing the speed that the body *A* has at *p* will become $= v + dv$, and the corresponding height for the speed of the body B at $q = u + du$. There becomes

$$pq = PQ + Qq - Pp = f - x + ds - dr,$$

but *pq* is equal to *PQ* itself together with their differential, i.e. $pq = f - x - dx.$ Consequently there will be found $dx = dr - ds$.

15. Because the elements *Pp* and *Qq* are considered to be traversed in the same time, they will be proportional to the speeds themselves, by which the distances are traversed. Therefore there is:

$$dr : ds = \sqrt{v} : \sqrt{u} \, ,$$

for these speed themselves are as the square roots generated from the heights.
Or if there may be had :

$$\frac{dr}{\sqrt{v}} = \frac{ds}{\sqrt{u}} = \frac{dr - dx}{\sqrt{u}};$$

from this equation there is found :

$$dr = \frac{dx\sqrt{v}}{\sqrt{v} - \sqrt{u}} \text{ and } ds = \frac{dx\sqrt{u}}{\sqrt{v} - \sqrt{u}}.$$

16. Truly the body *A*, while it is progressing along $Pp = dr$, experiences a contrary expansive force *P* to the spring, and on that account from paragraph 12 there will be $dv = -Pdr : A.$ In a similar manner the body *B* passing through $Qq = ds$ is accelerated by the force of the spring *P* and there will be $du = Pds : B.$ From these equations taking together there is found :

$$-Adv - Bdu = Pdr - Pds = Pdx.$$

Taking the integrals there will be

$$\text{const.} - Av - Bu = \int Pdx;$$

but there becomes $\int Pdx = 0$, if there may be put $x = 0$. Towards determining the constant, there may be put $x = 0$, and since there will be $v = a$ and $u = b$, and therefore the const. $= Aa + Bb$. Therefore we will have this equation :

$$A(a-v)+B(b-u)=\int Pdx.$$

17. We may restore the equations $Adv=-Pdr$ and $Bdu=Pds$, and we may substitute the values found for dr and ds, there will be :

$$Adv=\frac{Pdx\sqrt{v}}{\sqrt{u}-\sqrt{v}} \text{ and } Bdu=\frac{Pdx\sqrt{u}}{\sqrt{v}-\sqrt{u}}.$$

From that there will be had:

$$Pdx=\frac{-Adv\sqrt{v}+Adv\sqrt{u}}{\sqrt{v}}.$$

But before there was:

$$Pdx=-Adv-Bdu\ .$$

And from these there arises:

$$Adv\sqrt{u}=-Bdu\sqrt{v} \text{ or } \frac{Adv}{\sqrt{v}}=-\frac{Bdu}{\sqrt{u}}.$$

From which integrated there is found:

$$A\sqrt{v}+B\sqrt{u}=\text{const. }=A\sqrt{a}+B\sqrt{b}.$$

Indeed such must be constant, as also the equation before the collision truly proves to be the case.

18. Therefore we have come upon these two equations,

$$A(a-v)+B(b-u)=\int Pdx \text{ and } A\left(\sqrt{a}-\sqrt{v}\right)+B\left(\sqrt{b}-\sqrt{u}\right)=0.$$

From which the speeds of the bodies are able to be found in some state of compression endured in some compression in the collision itself. For this truly it is required, that P shall be a known function of x, from which it shall be able to take the integral, and in that for x the assumed state of compression to be substituted.

19. Moreover here the particular speeds of each body are desired after the collision. We seek these initially for elastic bodies, and in this case the collision is finite, when again there shall be $x=0$ and thus $\int Pdx=0$. From which there will be :

$$A(a-v)=-B(b-u).$$

This equation divided by the other produces $\sqrt{a}+\sqrt{v}=\sqrt{b}+\sqrt{u}$. And from these two equations there can be elicited readily :

$$\sqrt{v}=\sqrt{a}+\frac{2B\left(\sqrt{b}-\sqrt{a}\right)}{A+B} \text{ and } \sqrt{u}=\sqrt{b}+\frac{2A\left(\sqrt{a}-\sqrt{b}\right)}{A+B}.$$

Here $\sqrt{v}$ and $\sqrt{u}$ themselves denote the speeds of the bodies A and B, which they will have after the collision, while $\sqrt{a}$ and $\sqrt{b}$ indicate the speeds of these before the impact.

20. If the bodies were without any elasticity, the spring will be finished with the collision, when it is compressed maximally, this happens when $dx = 0$ or $dr = ds$, i.e. when $v = u$. Therefore non-elastic bodies have equal speeds after the collision, and thus will remain connoted together. Also their common speed will be

$$\sqrt{v} \text{ or } \sqrt{u} = \frac{A\sqrt{a}+B\sqrt{b}}{A+B}.$$

21. In these I have considered each body to be moving along in the same direction, this indeed does not impede the rules, by which they shall be less universal. For it is to be observed the direction is to be changed, with the speed changed into the negative. Thus there may be put $-\sqrt{b}$ in place of $\sqrt{b}$,the rules may be had for bodies with motions in opposite directions.

22. In a similar way the rules can be found of conserving the motion of bodies not perfectly elastic: indeed for this it is required, that the law governing the elasticity of the spring may be observed, and how it may try to restore itself. Also with these defined, the motion of each body after the collision will be easy to determine.

23. If bodies may strike each other obliquely, or if a number of bodies may collide simultaneously, what the motion they have is going to be after the collision, shall be superfluous to investigate here. For this proposition was only to give a demonstration of the rules of a natural collision : however the more composite cases are resolved from these rules, and which nevertheless to be considered without doubt, since they will depend from these simple rules.

DE COMMUNICATIONE MOTUS IN COLLISIONE CORPORUM

Commentatio 22 indicis ENESTROEMIANI
Commentarii academiae scientiarum Petropolitanae & (1730/1), 1738, p. 159-168

1. Experientia constat corporum in se mutuo incurrentium motus immutari; quaestio igitur hinc nata est, quae sit huius alterationis motus causa Dubitari quidem non potest, quin in ipso corporum conflictu ratio huius phaenomeni investigari debeat; corpus enim omne sive quiescens sive motum perseverat in suo statu, nisi a vi quapiam cieatur et ex statu suo deturbetur. Quamobrem quaestio huc est reducta, ut definiatur, qua in re insit haec vis et quanta sit, quae motus mutationem in conflictu corporum producere valet. Praeterea etiam determinari debet, quantam in collisione utriusque corporis motus mutationem subeat.

2. Inventae sunt iam superiore seculo a viris maxime meritis Wrenno, Wallisio et Hugenio regulae communicationis motus, ex quibus cognoscitur, duobus corporibus in se invicem incurrentibus, quanta futura sit utriusque corporis post conflictum celeritas. Regulae etiam istae experimentis egregie confirmantur, ut de earum veritate nefas esset dubitare. Variis tamen incedentes viis illi ad has regulas pervenerunt, et postmodum ab aliis plures ac diversae inventae sunt demonstrationes. Harum autem nulla, quantum mihi videtur, est genuina, sed derivatae sunt omnes ex alienis principiis.

3. Accedit ad hoc, quod nullus adhuc ipsam alterationis motus causam monstraverit, neque quomodo corpora in se mutuo agere possint, explicuerit. Hanc ob rem operae pretium fore existimavi, istam dissertationem proponere, in qua regulae communicationis motus ex certissimis mechanicae principiis deducantur; simulque ostendatur, quomodo in ipsa collisione corpora in se mutuo agant motusque immutent.

4. Accipio hic tanquam indubitatum principium, omnem motus vel diminutionem vel augmentationem vel directionis mutationem produci a potentiis, idque successive, non saltu. Facile hoc a quoquam concedetur: nihil enim contra id afferri possit, nisi ipsa motus in collisione communicatio, in qua an natura non faciat saltum a multis est disputatum. Statuo igitur in concursu duorum corporum utriusque corporis celeritatem a potentia inter corpora illa delitente immutari.

5. Corpora in se mutuo impingentia vim actionis potentiae similem sentire elucescit praecipue in mollioribus corporibus ut cera vel argilla: facile enim perspicitur impressiones, quas sunt consequuta, non subito sed pedetentim esse factas; ex quo etiam ad duriora corpora concludere licet, successive et per gradus mutationem fieri. Omnia autem corpora conflictu sibi mutuo impressiones imprimunt, quanquam hoc non de omnibus apparet. Sunt enim corpora, quae impressionem seu mutatam in conflictu

formam non retinent, sed pristinam formam recuperant, quae elastica vocantur; illa vero quae mutatam in conflictu formam servant, mollia

6. Inter haec duo genera innumerabiles continentur gradus intermedii; eorum scilicet corporum, quae impressiones acceptas ex parte tantum, non penitus, exuunt, ad quorum classem sine dubio omnia, quae in mundo sunt corpora, pertinent. Neque enim perfecte reperietur corpus elasticum neque perfecte molle, sed omnia medii inter haec generis deprehenduntur. Corpora porro tam elasticitate destituta seu mollia, quam elastica etiam differunt ratione duritiei, secundum quam alia aliis magis vel minus sunt dura Durius autem vocatur corpus, quod ab eadem vi minorem impressionem accipit, ex hocque intelligitur, quid sit corpus perfecte durum, quod nimirum a quaque vi finita infinite parvam tantum impressionem accipiat. Prout ergo haec impressio vel restituitur vel secus, corpus perfecte durum vel ad elasticorum vel non elasticorum classem pertinebit.

7. Quando duo corpora elastica inter se collidunt, alterum comprimit alterum, et sibi impressiones infligunt; postmodum vero se rursus in pristinam formam restituunt. Quamdiu corpora comprimuntur, vel utrumque corpus vel alterutrum saltem de motu suo amittit. Quando vero se restituunt, tum etiam motus in compressione amissus restituitur, sed aliter inter corpora distribuitur. In non elasticorum conflictu autem impressionem, quam utrumque accepit, maximam retinet. Ad motum igitur corporum elasticorum post conflictum determinandum requiritur, ut utriusque corporis investigetur celeritas tum, quando sunt prorsus restituta Pro corporibus vero non elasticis, inveniri debet utriusque corporis celeritas, quam habet in statu maximae impressionis.

8. Quemadmodum ad omnem motum generandum opus est potentia, ita etiam ad partes corporis comprimendas et impressiones faciendas potentia requiritur; corpus enim omne vi inertiae uti motui ita quoque impressioni accipiendae resistit, quae a potentia superari debet. Hanc impressionis accipiendae difficultatem ut clarius percipiamus, corporibus annexa concipio elastra in loco, quo impressiones recipiunt. Loco igitur impressionum elastra haec comprimi pono; eodem enim redit sive id, quod comprimitur, sit ipsius corporis pars, sive elastrum corpori adiunctum.

9. Inter corpora igitur (Fig. 1) *A* et *B* concurrentia pono elastrum *ab,* quod, dum pergunt ad se invicem accedere, comprimatur. Haecque compressio elastri tamdiu durat, quo ad motus, quo ad se invicem accedunt, vim elastri potest superare, tunc ergo elastrum erit in statu maximae compressionis. Deinde si corpora sunt elastica, pono elastrum hoc interpositum vi sese restituendi pollere, si vero non sunt elastica, concipio elastrum, cum in statum maximae compressionis est reductum, subito omnem vim sese expandendi amittere.

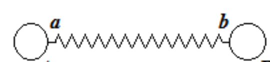


Fig. 1.

10. Hac ratione conflictum considerantes poterimus ex legibus Mechanicis, quas potentiae in alterandis motibus servant, mutationes motuum in collisione corporum supputare. Notum enim est, quantam celeritatem data potentia in datum corpus agens dato tempore generare, nec non si fuerit motus corporis potentiae contrarius, destruere valeat.

Elastrum autem inter corpora concurrentia conceptum, dum se expandere conatur vices potentiae subit, et quo id magis comprimitur, magis etiam corporum motus diminuitur.

11. Quo magis elastrum istud comprimitur, eo etiam maiorem habeat oportet vim sese expandendi, sed quanta ea sit in quolibet compressionis gradu, non est opus ut sciamus; quamcunque enim servet legem, eadem tamen denique post conflictum prodit motus distributio. Quantitatem ergo vis elastri expansivae generali litera utcunque variabili P designabo et a nulla alia pendente. Scilicet P mihi erit pondus, cuius nisui deorsum aequalis est vis elastri expansiva

12. Incurrat (Fig. 2) corpus A in elastrum AC vim P se expandendi habens, celeritate tanta, Fig. 2 quanta ex altitudine v grave cadendo acquirit; progrediatur puncto temporis per spatiolum $Aa = dx$, sitque celeritas quam in a habebit, genita ex altitudine $v + dv$. Perspicuum est, si esset $P = A$, corpus eodem modo retardatum iri, quo sursum proiectum a vi gravitatis retardatur, fore nempe $dv = -dx$, si esset $P = nA$ foret $dv = -Pdx : A$, si vis elastri motui corporis est contraria: sed si motum promoveat, erit $dv = +Pdx : A$.

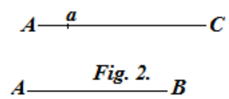
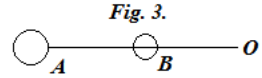


Fig. 2.

13. Moveatur (Fig. 3) corpus A in linea AO celeritate altitudini a debita, corpus vero B minori celeritate in eadem directione versus O ex altitudine b oriunda, occurrent haec corpora sibi invicem fietque conflictus. Pono ea tum in se mutuo agere incipere, cum distantia centrorum fuerit $= f$. Ipsis igitur corporibus (Fig. 4) ut punctis consideratis interpositum concipio elastrum longitudinis f. Sit id AB, quando ergo corpus A reperietur in A, et B in B, conflictus incipiet, elastrumque, quia A celerius movetur quam B, magis continuo comprimetur.

Fig. 3.


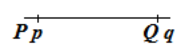


Fig. 4.

14. Reductum iam sit elastrum ad longitudinem PQ, quam pono $= f - x$. Sit celeritas, quam corpus A, cum in P venerit, habet, ex altitudine v orta, celeritasque corporis B in Q ex altitudine u, et vis elastri, quam nunc habet se expandendi sit $= P$. Tempusculo quam minimo progrediatur corpus A per elementum $Pp = dr$ et corpus B per $Qq = ds$; sitque altitudo exhibens celeritatem quam corpus A in p habebit $= v + dv$, et respondens altitudo celeritati corporis B in $q = u + du$. Erit

$$pq = PQ + Qq - Pp = f - x + ds - dr,$$

sed pq aequatur ipsi PQ una cum suo differentiali, i.e. $pq = f - x - dx$. Habebitur consequenter $dx = dr - ds$.

15. Quia elementa Pp et Qq simul ponuntur percursa, erunt ipsis celeritatibus, quibus percurruntur, proportionalia Quocirca est

$$dr : ds = \sqrt{v} : \sqrt{u} \,,$$

sunt enim ipsae celeritates ut radices quadratae ex altitudinibus generantibus.
Sive habetur

$$\frac{dr}{\sqrt{v}} = \frac{ds}{\sqrt{u}} = \frac{dr-dx}{\sqrt{u}};$$

ex hac aequatione reperitur

$$dr = \frac{dx\sqrt{v}}{\sqrt{v}-\sqrt{u}} \text{ atque } ds = \frac{dx\sqrt{u}}{\sqrt{v}-\sqrt{u}}.$$

16. Corpus vero *A*, dum progreditur per $Pp = dr$, contrariam habet vim elastri expansivam *P*, eritque propterea ex paragrapho 12 $dv = -Pdr : A.$ Simili modo corpus *B* per *Qq ds* transiens a vi elastri *P* acceleratur eritque $du = Pds : B.$ Ex his aequationibus coniunctis reperitur

$$-Adv - Bdu = Pdr - Pds = Pdx.$$

Sumantur integralia erit

$$\text{const.} - Av - Bu = \int Pdx;$$

fiat autem $\int Pdx = 0$, si ponatur $x = 0$. Ad constantem determinandam ponatur $x = 0$, eritque tum $v = a$ et $u = b$, est propterea const. $= Aa + Bb$. Habemus igitur istam aequationem

$$A(a-v) + B(b-u) = \int Pdx.$$

17. Resumamus aequationes $Adv = -Pdr$ et $Bdu = Pds$, substituamusque pro *dr* et *ds* valores inventos, erit

$$Adv = \frac{Pdx\sqrt{v}}{\sqrt{u}-\sqrt{v}} \text{ et } Bdu = \frac{Pdx\sqrt{u}}{\sqrt{v}-\sqrt{u}}.$$

Habetur ergo ex illa

$$Pdx = \frac{-Adv\sqrt{v}+Adv\sqrt{u}}{\sqrt{v}}.$$

Erat autem ante

$$Pdx = -Adv - Bdu \,.$$

Ex hisque prodit

$$Adv\sqrt{u} = -Bdu\sqrt{v} \text{ seu } \frac{Adv}{\sqrt{v}} = -\frac{Bdu}{\sqrt{u}}.$$

Qua integrata obtinetur

$$A\sqrt{v} + B\sqrt{u} = \text{const.} = A\sqrt{a} + B\sqrt{b}.$$

Talis enim esse debet constans, ut etiam aequatio ante conflictum verum praebeat.

18. Duas ergo invenimus aequationes istas

atque $A(a-v)+B(b-u)=\int Pdx$ atque $A\left(\sqrt{a}-\sqrt{v}\right)+B\left(\sqrt{b}-\sqrt{u}\right)=0.$

Ex quibus celeritates corporum in quovis compressionis statu durante ipso conflictu inveniri possunt. Ad hoc vero requiritur, ut *P* sit cognita functio ipsius *x*, quo possit integrale sumi et in eo pro *x* status compressionis assumtus substitui.

19. Hic autem praecipue celeritates utriusque corporis post conflictum desiderantur. Quaeramus eas primo pro corporibus elasticis, hocque in casu finitus est conflictus, quando fit iterum $x=0$ adeoque $\int Pdx=0.$. Ex quo erit

$$A(a-v)=-B(b-u).$$

Dividatur altera aequatio per hanc, prodibit $\sqrt{a}+\sqrt{v}=\sqrt{b}+\sqrt{u}$. Atque ex postremis his duabus aequationibus facile eruitur

$$\sqrt{v}=\sqrt{a}+\tfrac{2B\left(\sqrt{b}-\sqrt{a}\right)}{A+B} \text{ et } \sqrt{u}=\sqrt{b}+\tfrac{2A\left(\sqrt{a}-\sqrt{b}\right)}{A+B}.$$

Hic $\sqrt{v}$ et $\sqrt{u}$ denotant ipsas corporum *A* et *B* celeritates, quas post conflictum habebunt, at vero $\sqrt{a}$ et $\sqrt{b}$ celeritates eorum ante conflictum.

20. Si corpora fuerint omni elasticitate destituta, conflictu finietur, quando elastrum est maxime compressum, hoc evenit, si est $dx=0$ seu $dr=ds$, i.e. ubi $v=u$. Aequales ergo corpora non elastica habebunt post conflictum celeritates, adeoque coniuncta manebunt. Erit autem eorum communis celeritas

$$\sqrt{v} \text{ vel } \sqrt{u}=\tfrac{A\sqrt{a}+B\sqrt{b}}{A+B}..$$

21. Posui in his utrumque corpus secundum eandem plagam moveri, hoc vero non impedit, quominus hae regulae sint universales. Notum enim est plagam mutari, mutata celeritate in negativam. Ita si poneretur $-\sqrt{b}$ loco $\sqrt{b}$, haberentur regulae communicationis motus pro corporibus in plagas oppositas motis.

22. Simili modo inveniri possunt regulae communicationis motus pro corporibus non perfecte elasticis: ad hoc vero requiritur, ut et nota sit lex vis elasticae elastri, et quousque se restituere valeat. His autem definitis facile erit motum utriusque corporis post conflictum determinare.

23. Si corpora oblique in se impingant, aut si plura corpora simul collidant, quos post conflictum habitura sint motus, hic esset superfluum investigare. Propositum enim tantum erat hic regularum collisionis genuinam dare demonstrationem: magis autem compositi casus ex his regulis resolvuntur, eatenusque sunt extra dubium positi, quatenus ab his simplicibus pendent.