CHAPTER V

CONCERNED WITH THE FURTHER PERFECTION OF TELESCOPES OF THE FIRST KIND, BY THE ADDITION OF ONE OR MORE LENSES.

PROBLEM 1

152. *If a telescope of the first kind shall be constructed from three lenses separated in turn from each other, to investigate the main features, for which the maximum degree of perfection may be able to be acquired.*

SOLUTION

With all the elements remaining the same, as have been established in the beginning, initially we will consider the equation $m = \frac{\alpha}{b} \cdot \frac{\beta}{c}$, in which both the fractions $\frac{\alpha}{b}$ and $\frac{\beta}{c}$ must be negative and in addition the intervals $\alpha + b$ and $\beta + c$ positive, and now since there must not be $\frac{\alpha}{b} = -1$, lest both the prior lenses may coalesce, we may put $\frac{\alpha}{b} = -k$, so that there may become $m = -k \cdot \frac{\beta}{c}$ and hence $\beta = \frac{-mc}{k}$ or $c = -\frac{\beta k}{m}$ and $\alpha = -bk$, from which on account of $\beta = Bb$, all these distances may be determined in terms of α in the following manner:

$$b = -\frac{\alpha}{k}, \quad \beta = -\frac{B\alpha}{k} \quad \text{and} \quad c = +\frac{B\alpha}{m},$$

with $\gamma = \infty$ being present. Hence therefore there will be required to be $\alpha\left(1 - \frac{1}{k}\right) > 0$, $\alpha B\left(\frac{1}{m} - \frac{1}{k}\right) > 0$ or, since m and k are positive,

$$\alpha\left(k - 1\right) > 0 \quad \text{and} \quad \alpha B\left(k - m\right) > 0$$

and thus also $\frac{B(k-m)}{k-1} > 0$. On account of which two cases will be required to be considered:

The first case, where α is a positive quantity ; then there must be $k > 1$; then truly either $k > m$, if B shall be positive, or $k < m$, if $B < 0$. *In the other case,* where α is negative, there must be $k < 1$; then truly either $k > m$, if B shall be negative, or $k < m$, if B shall be positive, where on account of $m > 1$ that condition $k > m$ falls at once.

Therefore from these premises we may reduce the formula given above (§ 42) for the radius of the circle of confusion to zero :

$$0=\mu\lambda+\frac{\mu' q}{\mathfrak{B}^2 p}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu''\lambda''}{B^3 m}$$

whether on account of $q=-\frac{\alpha\mathfrak{B}}{k}$ and $p=\alpha$

$$0=\mu\lambda-\frac{\mu'}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu''\lambda''}{B^3 m},$$

which reverts either to this form:

$$\text{I.)}\quad 0=\mu\lambda-\frac{\mu'\lambda'}{\mathfrak{B}^3 k}+\frac{\mu''\lambda''}{B^3 m}-\frac{\mu'\nu'}{\mathfrak{B}Bk}.$$

From which so that the colored margin may be removed, on account of $O=0$, this equation is obtained (§ 52) [*i.e.* the eye is placed next to the eyepiece]

$$0=\frac{dn}{n-1}\cdot B\pi'-\frac{dn'}{n'-1}\cdot\frac{1}{k}\left((B+1)\pi'-\pi\right)$$

and so that this confusion may be completely removed, there is had from § 54 [or see E266 on this Contents Page]:

$$0=\frac{dn}{n-1}\cdot\frac{1}{p}+\frac{dn'}{n'-1}\cdot\frac{1}{k^2}\cdot\frac{1}{q}+\frac{dn''}{n''-1}\cdot\frac{1}{m^2}\cdot\frac{1}{r}.$$

For these equations requiring to be resolved in the first place the ratio between π and π' must be defined, that which will be presented easily by the fundamental formulas proposed at the beginning :

$$\frac{\pi}{\Phi}=\frac{\alpha+b}{q}=\frac{1-k}{\mathfrak{B}} \quad\text{and}\quad \frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=\frac{B\alpha}{c}=m,$$

from which there is deduced

$$\pi=\frac{(1-k)\Phi}{\mathfrak{B}}$$

and

$$\pi'=(m-1)\Phi+\pi=\frac{1-k+(m-1)\mathfrak{B}}{\mathfrak{B}}\Phi$$

thus so that there shall become

$$\pi:\pi'=1-k:1-k+(m-1)\mathfrak{B}=1:1+\frac{m-1}{1-k}\mathfrak{B};$$

then for the sake of brevity we may put :

$$\frac{dn}{n-1}=N,\quad \frac{dn'}{n'-1}=N',\quad \frac{dn''}{n''-1}=N'',$$

and hence the second and third equation will be transformed into the following :

II.) $0 = NB(1-k+(m-1)\mathfrak{B}) - N'\frac{1}{k}(B(1-k)+(B+1)(m-1)\mathfrak{B})$
or

$$0=(m-1)N\mathfrak{B}+(1-k)N-\frac{m-k}{k}N',$$

$$\text{III.)}\ \ 0=N-\frac{N'}{k\mathfrak{B}}+\frac{N''}{mB}.$$

The value of $\mathfrak{B}$ can be defined from each of these equations. From the second

$$\mathfrak{B}=\frac{m-k}{(m-1)k}\frac{N'}{N}-\frac{1-k}{m-1}.$$

Truly from the third there follows

$$\mathfrak{B}=\frac{mN'-kN''}{k(mN-N'')}.$$

We may note, or after the value of $\mathfrak{B}=\frac{mN'-kN''}{k(mN-N'')}$ with the condition found before $\frac{B(k-m)}{k-1}>0$ may be able to remain in place. Finally we may find this $1-\mathfrak{B}$ on account of $B=\frac{\mathfrak{B}}{1-\mathfrak{B}}$, and there becomes

$$1-\mathfrak{B}=\frac{m(kN-N')}{k(mN-N'')}$$

and there will be

$$B=\frac{mN'-kN''}{m(kN-N')},$$

from which our condition requires, that there shall become

$$\frac{(mN'-kN'')(k-m)}{m(kN-N')(k-1)}>0$$

which, if there were $N=N'=N''$, will be changed into this form $\frac{-(k-m)^2}{m(k-1)^2}>0$, which is impossible; therefore at this point only this condition can be found, so that the letters N, N', N'' are unequal, which will come about , if the numerator may be made positive, which happens , if both of its factors may be made either positive or negative ; in the first case $mN'-kN''>0$ and thus $k<m\frac{N'}{N''}$ and $k>m$, which can happen, only if there shall be $\frac{N'}{N''}>1$, or if $N'>N''$. Truly for the other case, where each factor is negative, there will be $k<m$ and $k>\frac{N'}{N''}m$, which can happen, only if there shall be $\frac{N'}{N''}<1$ or $N'<N''$; from which it is evident for each case the letters N' and N'' must be unequal, or the second and third lenses must be made from different kinds of glass. But it is apparent in general k cannot differ much from m .

These follow, if a positive numerator may be put in place ; truly if the numerator shall be negative, the denominator also must be negative, for which also we have two cases. For the first case, if there shall be $k > 1$, there must be $kN < N'$ and thus $k < \frac{N'}{N}$; for the latter case, if $k < 1$, likewise there must be $k > \frac{N'}{N}$; for each of which the first and second lens must be formed from different kinds of glass. Truly from these four cases it will be agreed to select that, which may return both the values found for $\mathfrak{B}$ approximately equal; moreover finally, after we will have conveniently defined the values of $\mathfrak{B}$ and k, from the first equation either λ or λ' must be sought, since now λ'' is given thence, so that each side of the eyepiece lens may become equally concave.

COROLLARY 1

153. These four cases for the determination of the letter k are easily reduced to the two following conditions ; for

either 1. k must be taken between the limits 1 and $\frac{N'}{N}$,

or 2. k must be taken between the limits m and $\frac{N'}{N''}m$,

thus so that this same number k must be taken approximately equal to one or to a multiple of m, since the fractions $\frac{N'}{N}$ and $\frac{N'}{N''}$ differ only a little from unity.

COROLLARY 2

154. Therefore there will be a need to investigate the case, for which k itself is put equal to either of the limits.

1. If $k = 1$, the interval between the first and second lens shall become $= 0$ and

$$\mathfrak{B} = \frac{mN'-N''}{mN-N''}, \text{ while } B = \frac{mN'-N''}{m(N-N')}$$

and the distance between the second and third lens $= B\alpha\left(\frac{1-m}{m}\right)$, from which it is deduced, α must be taken either positive or negative.

2. If $k = \frac{N'}{N}$, the interval between the first and second lens $= \frac{N'-N}{N}\alpha$, which since it must be positive, it is clear, α may be required to be either positive or negative ; then truly $\mathfrak{B} = 1$ and $B = \infty$, from which $\beta + c$ or the distance between the second and third lens will become $= \infty$.

3. If $k = m$, the interval of the second and third lens will vanish and there will become

$$\mathfrak{B}=\frac{N'-N''}{mN-N''} \text{ and } B=\frac{N'-N''}{mN-N'}$$

truly the interval between the first and second lens $\frac{\alpha(k-1)}{k}=\frac{\alpha(m-1)}{m}$, where it is evident α must be a positive quantity.

4. If $k=\frac{N'}{N''}m,$ there becomes $\mathfrak{B}=0$ and $B=0$; from which the distance between the second and third lens $=0$.
Therefore since the separations of the lenses may be agreed neither to become zero nor infinite, the number k evidently cannot be taken equal to any limit.

COROLLARY 3

155. Again so that the apparent field of view may be attained, which depends on the formula $\pi'-\pi$, since we have found $\pi'-\pi=1:1+\frac{m-1}{1-k}\mathfrak{B}$, there will be for the four cases mentioned :

1. If $k=1$, there will become $\pi:\pi'=1:\infty$ hence $\pi=0$; thus so that $\Phi=\frac{\pi'}{m-1}$ will be found for the apparent field of view.

2. If $k=\frac{N'}{N}$, there becomes

$$\pi'-\pi=1:1+\frac{mN-N}{N-N'}=1:1+\frac{mN-N'}{N-N'}$$

and hence

$$\pi=\frac{N-N'}{mN-N'}\pi' \quad\text{and}\quad \pi'-\pi=\frac{N(m-1)}{mN-N'}\pi',$$

and thus there becomes $\pi=\frac{N}{mN-N'}\pi'$ for the apparent field of view; and thus Φ will arise greater, if $\frac{N}{mN-N'}>\frac{1}{m-1}$: that is, if $N<N'$, which therefore will eventuate, if the first lens may be prepared from crown glass, the second from crystal.

3. If $k=m$, there will become

$$\pi:\pi'=\frac{mN-N'}{mN-N''} \text{ or } \pi=\frac{mN-N''}{mN-N'}\pi'.$$

From which it is deduced the apparent field of view to become greater than in the preceding chapter, if there were $\pi<0$; since which cannot happen here, in this case a greater apparent field cannot be expected to arise.

4. If $k=\frac{N'}{N''}m$, there will be $\pi:\pi'=1:1$, from which $\pi=\pi'$ and $\pi'-\pi=0$, for which case therefore the apparent field may vanish.

COROLLARY 4

156. Hence therefore we conclude, in order that we may obtain a greater field of view than before, by necessity to be required, so that there shall be $N' > N$, and k must be taken between the limits 1 and $\frac{N'}{N}$, which latter limit shall be greater than unity, also k will be greater than one ; from which it follows the distance α must be taken positive, since $\alpha(k-1) > 0$.

COROLLARY 5

157. Therefore as mainly on account of this case we may use several lenses, so that we may obtain a greater field of view, from these extra cases, just as the letters $N,\ N',\ N''$ are able to be varied among themselves, this single case is relinquished by us, where $N' > N$ and k is taken between the limits 1 and $\frac{N'}{N}$.

SCHOLIUM 1

158. In the corollaries we have used that value of $\mathfrak{B}$ which we have deduced from the third equation. But now above we have observed this equation to be prepared thus, so that concerning that at no time can we be entirely sure; since indeed the values of the letters $N,\ N',\ N''$ etc. may not be able to be defined from any theory, but may be concluded only by experiment, such as have been done by Dollond, [The wave nature of light had not yet been confirmed by Young's experiments, though it was known from Newton's experiments that different colors had slightly differing refractive indices, and Euler elsewhere had considered light as having a wavelength in analogy with sound waves ; here he used the white light or average refractive index for a medium.], however great the care and skill may be used with these, yet at no time may such a level of precision be hoped for, that we need not fear any perceptible error; for this reason also the value of $\mathfrak{B}$ thence deduced cannot be had as the true value, but it is necessary for us to be content, if we will have got to know this value approximately, since that we may confirm regarding the nature of the matter; for since our third equation, the diffusion distance, by which the images of different colors are diffused, is reduced at once to zero, it is easily understood to suffice in practice, while this distance may be rendered small enough, especially after that in which we have excelled, so that the colored fringe may vanish ; therefore in the first place the value of the letter $\mathfrak{B}$ must be determined from the second equation; which if that were prepared thus, so that so far besides the third equation may be satisfied, thence the confusion arising from that will be able to be ignored more, which also cannot be seized upon without harm with telescopes prepared from a single kind of glass. Truly from the second equation the value of $\mathfrak{B}$ may be allowed to be defined for that case also, where all the lenses may be prepared from the same kind of glass, thus so that there shall become $N = N' = N''$; then indeed it may be concluded

$$\mathfrak{B} = \tfrac{m-k}{(m-1)k} - \tfrac{1-k}{m-1} = \tfrac{m-2k+k^2}{(m-1)k},$$

if we may wish to use this value, so that the above prescribed condition $\mathfrak{B}\tfrac{k-m}{k-1} > 0$ may be fulfilled, since thence there shall be $1-\mathfrak{B} = \tfrac{mk-m+k-k^2}{(m-1)k} = \tfrac{(k-1)(m-k)}{(m-1)k}$, there will be

$$B = \tfrac{m-2k+k^2}{(k-1)(m-k)} \text{ and hence the condition } \tfrac{-m+2k-k^2}{(k-1)^2} > 0\ ;$$

in which since the denominator certainly shall be positive, the numerator of such also must be positive and thus :

$$1-m-(k-1)^2 > 0,$$

which cannot occur. From which it is clear in this case the colored fringe plainly cannot be removed.

Therefore we may observe, if we may use in addition a different glass, whether we may escape from this fault. Towards this end we may put for brevity $\tfrac{N'}{N} = \xi$, so that ξ shall be a number either a little greater or less than unity, and since there shall become:

$$\mathfrak{B} = \tfrac{(m-k)\xi + k(k-1)}{(m-1)k}, \quad \text{there will be} \quad B = \tfrac{(m-k)\xi + k(k-1)}{(m-k)(k-\xi)},$$

from which our condition demands, that there shall be $\tfrac{(k-m)\xi - k(k-1)}{(k-1)(k-\xi)} > 0$. Here two cases are required to be considered.

I. If the denominator shall be positive, so that if there becomes, either $k > \xi$ and $k > 1$, or $k < \xi$ and $k < 1$. Then indeed there must be $(k-m)\xi - k(k-1) > 0$ or

$$\tfrac{(1+\xi)^2}{4} - m\xi > \left(k - \tfrac{1}{2}(1+\xi)\right)^2$$

so that, since *m* clearly shall exceed unity, ξ truly may differ little from unity, evidently which cannot happen.

II. If the denominator shall be negative, which happens, if k is held between the limits ξ and 1. Then truly the numerator also must be negative, or

$$(k-m)\xi - k(k-1) < 0 \quad \text{or} \quad \tfrac{(1+\xi)^2}{4} - m\xi < \left(k - \tfrac{1}{2}(1+\xi)\right)^2,$$

which happens at once, since the first part evidently shall be negative. Therefore this case, as we have observed now, is the only one, which deserves attention, since in this way also the third equation at least may be satisfied approximately.

SCHOLIUM 2

159. But if therefore our proposition shall be to removal of the colored fringe, which property chiefly is accustomed to be desired, first it is required to maintain this in no manner can be established from lenses made from a single kind of glass, but at least the first and second lenses must be constructed from different kinds of glass, thus so that there may be put $\frac{N'}{N}=\xi$ or $N=1,\ N'=\xi$ the letter ξ may differ a little from unity, while for N'' either unity or ξ can be accepted as it pleases ; then we have seen the number k must be taken to lie between the limits 1 and ξ, with which done there will be:

$$B=\frac{(m-k)\xi+k(k-1)}{(m-k)(k-\xi)} \quad \text{and} \quad \mathfrak{B}=\frac{(m-k)\xi+k(k-1)}{(m-1)k};$$

from which the determinable distances will be

$$b=-\frac{\alpha}{k},\ \ \beta=\frac{-(m-k)\xi-k(k-1)}{k(m-k)(k-\xi)}\alpha \quad \text{and} \quad c=\frac{(m-k)\xi+k(k-1)}{k(m-k)(k-\xi)}\alpha$$

and hence the distance between the lenses

$$\alpha+b=\alpha\frac{k-1}{k},$$
$$\beta+c=\frac{-(m-k)\xi-k(k-1)}{(m-k)(k-\xi)}\cdot\frac{m-k}{mk}\alpha=\frac{+(m-k)\xi+k(k-1)}{mk(\xi-k)}\alpha,$$

and hence the length of the whole telescope will be

$$=\frac{m-1}{m}\cdot\frac{\xi-k+1}{\xi-k}\alpha$$

Again to find the maximum present in the apparent field of view, which can be done by determining the value $\pi=\frac{\pi'}{1+\frac{m-1}{1-k}\mathfrak{B}}$, which will be changed into the following

$\pi=\frac{k(1-k)}{(m-k)\xi}\pi'$, from which we come upon

$$\Phi=\frac{\pi'-\pi}{m-1}+\frac{\pi'}{m-1}\left(1+\frac{k(k-1)}{(m-k)\xi}\right).$$

Therefore since the maximum field present, however much it can become increased, hence we will obtain that same conclusion: the number k must be greater than unity, from which, since *k* may be held within the limits 1 and ξ, again this rule may be observed, the letter ξ must be greater than one*;* from which it follows the second lens is required to be prepared from crystal glass, the first truly from common glass ; with which agreed the other case, where there will become $\xi < 1$, is excluded completely in practice. Whereby, since there shall be $k > 1$, the distance α, which at this point has been left uncertain, must be positive.

Now finally we will consider the third equation, for which the color confusion is completely removed, and we may see, how much that shall be going to produce. But that third equation now becomes $0 = 1 - \frac{\xi}{k\mathfrak{B}} + \frac{N'}{mB}$, which now may adopt this form:

$$0 = \frac{-m(k-1)(\xi-k)-(m-k)(\xi-k)N''}{m((m-k)\xi+k(k-1))},$$

which quantity certainly will not be equal to zero, but , since *k,* ξ and N'' may differ a little from unity, always definitely will be small; so that which thence may become clearer, since the numerator may have the small factor $\xi - k$, but the denominator always shall be large enough and therefore greater where the magnification were greater. From which it is evident this confusion may never be perceptible. Moreover besides, since this confusion plainly will vanish, if there may be taken $\xi = k$, a certain plan may be observed to take the number *k* closer to the limit ξ than to unity, since it cannot be equal to the limit ξ itself , because the interval between the second and third lens and the length of the telescope to be used may become infinite ; whereby, lest that may be produced exceedingly great, the opposite rather is required to be proposed, so that the letter *k* may be moved away so much from ξ, and may be taken closer to unity. Consequently a single case, which deserves to be set out, rests on this, so that the number *k* may be assigned a number close to unity and just as the excess shall be so small, as the thickness of the lens is accustomed to allow. If indeed *k* may be equated to unity itself, we will have the case of the preceding chapter, where the distance between the lenses plainly is put equal to zero, which inconvenience we set out to avoid here.

PROBLEM 2

160. *If the first lens may be prepared from crown glass, the second lens from crystal glass, and the interval between these may be put as small as the thickness of the lenses allows, to determine the rules, it is required to observe in the construction of this telescope.*

SOLUTION

Therefore here from Dollond's experiments there will have to be put $\xi = \frac{10}{7}$, and since it will be convenient to take k closer to the limit 1 than to the other limit $\frac{10}{7}$, we may assume $k = \frac{8}{7}$,and which have been treated in the preceding scholia, provide us with the following determinations:

I. For the determinable distances :

$$b = -\tfrac{7}{8}\alpha, \quad \beta = \frac{35(7m-8)+28}{8(7m-8)}\alpha \quad \text{or} \quad \beta = \frac{7(35m-36)}{8(7m-8)}\alpha, \quad c = \frac{-35m+36}{m(7m-8)}\alpha.$$

II. For the separation of the lenses

$$\alpha + b = \tfrac{1}{8}\alpha, \quad \beta + c = \frac{35m-36}{8m}\alpha$$

and the length of the telescope $= \frac{9(m-1)}{2m}\alpha.$
Here it is required to be observed, since α shall be the focal length of the first lens and the radius of its aperture must be $x = my = \frac{m}{50}$ dig. [a digit or French inch was approx. 27.07 mm.], that same distance α cannot be less than $5x$ or $\frac{m}{10}$ dig., thus so that there shall be $\alpha > \frac{m}{10}$ dig.; whereby, if for the sake of an example, there may be taken $m = 50$, the length of the telescope may appear greater than $\frac{9 \cdot 49}{20}$ dig., greater than 22 dig., and if there must become $m = 100$, that must be greater than $\frac{9 \cdot 99}{20}$ dig., greater than 44 dig.; which distance since it may be easily allowed, it is clear these telescopes also can be used for higher magnifications; but for lesser magnifications they shall perform extremely well, since, if there may be put $m = 5$, a length will arise $> \frac{36}{20}$ dig., greater than $\frac{9}{5}$ dig., and on taking $m = \frac{5}{2}$ that may be produce $> \frac{27}{40}$ dig.

Moreover, for the apparent field of view we will have radius of that

$$\Phi = \frac{\pi'}{m-1}\left(1 + \frac{4}{5(7m-8)}\right)$$

and thus a little greater than in the preceding case, especially if the magnification were small. Also the focal lengths of these lenses *p, q, r,* may be observed and since there shall be

$$\mathfrak{B} = \frac{35m-36}{28(m-1)} \quad \text{and} \quad B = \frac{35m-36}{-7m+8},$$

there will be

$$p=\alpha,\quad q=\frac{-(35m-36)\alpha}{32(m-1)},\quad r=\frac{-(35m-36)\alpha}{m(7m-8)},$$

and the corrected radius of the aperture of the second lens will be from § 23 [for $\alpha=5x=\frac{m}{10}$]

$$\frac{m(35m-36)\pi'}{400(m-1)(7m-8)}+\frac{7m}{400}.$$

Finally for the construction of these lenses the numbers $\lambda,\ \lambda',$ et λ'' must be taken thus, so that our first equation may be satisfied, which was

$$0=\mu\lambda-\frac{\mu'\lambda'}{\mathfrak{B}^3k}+\frac{\mu''\lambda''}{B^3m}-\frac{\mu'\nu'}{\mathfrak{B}Bk},$$

where it is required to be observed, in order that each side of the eyepiece lens may become equally concave, there must be put in place $\lambda''=1,60006$, if this lens shall be made from crown glass and thus $\mu''=\mu$; but if it shall be made from crystal glass and thus $\mu''=\mu'$, to become $\lambda''=1,67445$ [§ 137]; but this equation cannot be set out except in special cases for a given magnification; where it will help to be remembered

$$\mu=0,9875,\ \ \mu'=0,8724,\ \ \nu'=0,2529,\ \ \mu'\nu'=0,2206.$$

EXAMPLE 1

161. If the magnification shall be $m=\frac{5}{2}$, to describe a telescope of this kind consisting of three lenses.

Since there shall be $m=\frac{5}{2}$, the determined distances will be

$$b=-\tfrac{7}{8}\alpha,\ \ \beta=\tfrac{721}{152}\alpha,\ \ c=-\tfrac{206}{95}\alpha,\ \ B=-\tfrac{103}{19}$$

and the separation of the lenses $\alpha+b=\frac{1}{8}\alpha,\quad \beta+c=\frac{107}{40}\alpha$,

the length of the telescope $=\frac{27}{10}\alpha$ and for the apparent field of view there will become $\Phi=\frac{2\pi'}{3}\left(1+\frac{8}{95}\right)$; on taking $\pi'=\frac{1}{4}$ and on multiplying by 3437 minutes. the angle will be $\Phi=10°\ 21'\ 4'$.

Now since there shall be $B=\frac{-103}{19}$, there will be $\mathfrak{B}=\frac{103}{84}$ and we will have

$$\mathrm{Log.}\left(-B\right)=0,7340836(-)\ \text{ and }\ \mathrm{Log.}\mathfrak{B}=0,0885580;$$

but the equation for the confusion first must be disposed of, if we make the eyepiece lens from crown glass, so that there shall be $\mu''=\mu$ and $\lambda''=1,60006$, there will be

$$\begin{aligned} 0&=0,9815\lambda-0,4140\lambda'-0,00396\\ &\qquad\qquad\qquad\qquad\quad +0,02903\\ 0&=0,98751-0,4140\lambda'\ +0,02507, \end{aligned}$$

from which λ' is sought, and there will be had $\lambda'=2,3852\lambda+0,06055$. If therefore this may be taken $\lambda=1$, there will become $\lambda'=2,4457$, from which there becomes $\lambda'-1=1,4457$ and $\mathrm{log.}\sqrt{(\lambda'-1)}=0,0800391$; from which the construction of the individual lenses itself will be done in the following manner, if indeed the radii of the faces of the first lens shall be F and G, of the second F' and G', and of the third F'' and G'':

I. For the first lens from crown glass :

$$F=\tfrac{\alpha}{\sigma}=0,6024\alpha,\quad G=\tfrac{\alpha}{\rho}=4,4111\alpha.$$

II. For the second lens from crystal glass:

$$\frac{1}{F'}=\frac{\rho\beta+\sigma b\mp\tau(b+\beta)\sqrt{\lambda'-1}}{b\beta}\qquad \frac{1}{G'}=\frac{\sigma\beta+\rho b\pm\tau(b+\beta)\sqrt{\lambda'-1}}{b\beta};$$

since there shall be

$$\mathrm{log.}\left(-\tfrac{b}{\alpha}\right)=\mathrm{log.}\tfrac{7}{8}=9,9420080\left(-\right)\ \text{and}\ \mathrm{log.}\tfrac{\beta}{\alpha}=0,6760917$$
$$\text{and log. }\left(\tfrac{(b+\beta)}{\alpha}\right)=\mathrm{log.}\tfrac{147}{38}=0,5875336,$$
$$\mathrm{log.}\sigma=0,1993986,\ \mathrm{log.}\rho=9,1501422,\ \mathrm{log.}\tau=9,9432471.$$

From which there is found:

$$\frac{1}{F'}=\frac{-0,7174\mp4,0815}{b\beta}\alpha,\qquad \frac{1}{G'}=\frac{+0,7174\pm4,0815}{b\beta}\alpha;$$

so that the greater numbers may be avoided, the lower signs may be taken, and there will become

$$F'=\frac{b\beta}{3,3668\alpha}=-1,2327\alpha,\ G'=\frac{b\beta}{3,3021\alpha}=-1,2568\alpha.$$

For the eyepiece lens being prepared from crown glass, since there each side shall be concave, and its focal length shall be $c=-\frac{206}{95}\alpha$, the radius of concavity for each face will be $=2(n-1)c=-\frac{106\cdot 206}{95}\alpha=-\,2,2978\alpha$.

The first lens allows an aperture, of which the radius $x=0,1506\alpha$. Now truly the distinctness of the image demands, that there shall be $x=\frac{m}{50}\text{dig.}=\frac{1}{20}\text{dig.}$, from which α shall be greater than $\frac{1}{3}$ dig. There may be taken $\alpha=\frac{1}{2}$ dig. and the construction of the telescope thus will be had:

I. For the first lens the radius of the $\begin{cases}\text{anterior face} & =+0,3012 \text{ dig.}\\ \text{posterior face} & =+2,2065 \text{ dig.}\end{cases}$ Crown Glass.

II. For the second lens the radius of the $\begin{cases}\text{anterior face} & =-0,6163 \text{ dig.}\\ \text{posterior face} & =-0,6284 \text{ dig.}\end{cases}$ Flint Glass.

III. For the third lens the radius of each face $=-1,1492$ dig., which may be prepared from Crown Glass.

Then the interval may be put in place between

$$\text{I and II}=\tfrac{1}{16}\text{dig.}=0,0625\text{ dig.}$$
$$\text{II and III}=\tfrac{103}{80}\text{dig.}=1,2875\text{ dig.},$$

thus so that the length of the whole telescope shall become

$$=1,3500\text{ dig.}=1\tfrac{1}{3}\text{dig.},$$

truly the radius of the space seen will be equal to $=10°21'4''$.

EXAMPLE 2

162. If the magnification shall be $m = 5$, to describe a telescope of this kind from three agreeing lenses.

Since there shall be $m=5$, there will be $7m-8=27$ and $35m-36=139$, from which the determinable distances will become :

$$b=-\tfrac{7}{8}\alpha=-08750\alpha,\quad \beta=\tfrac{973}{216}\alpha=+4,5046\alpha,\quad c=\tfrac{-139}{5\cdot 27}\alpha=-10296\alpha.$$

From which the intervals become

$$\alpha+b=\tfrac{1}{8}a,\quad \beta+c=3,4750\alpha,$$

from which the length of the telescope $=3,6\alpha$.

Moreover, for the apparent field there will become $\Phi = \frac{\pi'}{4}\left(1+\frac{4}{5\cdot 27}\right)$ and on taking $\pi' = \frac{1}{4}$ and on multiplying by 3437 minutes. there will be $\Phi = 3^\circ\ 41'$.

Now since there shall be $\mathfrak{B} = \frac{139}{112}$ and $B = \frac{-139}{27}$, and with the logarithms taken

$$\text{Log.}\mathfrak{B} = 0,0937968,\ \text{Log.}(-B) = 0,7116510(-),$$

the equation for the first confusion requiring to be removed will be, if we may prepare the eyepiece lens from crown glass, so that there shall be $\mu'' = \mu$ and $\lambda'' = 1,60006$,

$$0 = 0,9875\lambda - 0,39933\lambda' \begin{array}{r} -0,002316 \\ +\ 0,030211 \\ \hline +0,027895, \end{array}$$

from which again there may be sought

$$\lambda' = 2,4729\lambda + 0,06985.$$

Not as before here we may accept $\lambda = 1$, but, so that the first lens may become capable of the maximum aperture and thus the distance α may be able to be taken smaller, there may be taken $\lambda = 1,60006$, so that this lens may have each side equally convex, and there will be had

$$\lambda' = 4,0266,\ \lambda' - 1 = 3,0266,\ \text{and}\ \text{Log.}\sqrt{(\lambda'-1)} = 0,2404775$$

and hence we will obtain :

I. For the first lens made from crown glass the radius of each face

$$= 2(n-1)\alpha\ =\ 1,06\alpha\,;$$

which permits an aperture, of which the radius $x = 0,26\alpha$.

II. For the second lens made from crystal glass, on account of $\text{Log.}\frac{b+\beta}{\alpha} = 0,5598588$ the calculation will be had thus :

$$\frac{1}{F'} = \frac{-0,7479 \pm 5,5409}{b\beta}\alpha \qquad \frac{1}{G'} = \frac{+7,005\ 7 \mp 5,5409}{b\beta}\alpha;$$

and here the upper signs may prevail, and there will be

$$F' = \frac{b\beta}{4,7930\alpha} = -0,8224\alpha, \quad G' = \frac{b\beta}{1,4548\alpha} = -2,6908\alpha.$$

III. For the third lens made from crown glass the radius of each face will be

$$=2(n-1)c=1,06c=-1,0914\alpha .$$

Since now on account of the distinctness there must be $x=\frac{m}{50}$ dig. $=\frac{1}{10}$ dig., α may become greater than $\frac{2}{5}$ dig.

Therefore it will be able to assume $\alpha=\frac{1}{2}$ dig., and the construction of the telescope thus will be had :

I. For the first lens, Crown Glass, the radius of each face $=0,5300$ dig.

II. For the second lens, flint glass, the radius of the $\begin{cases}\text{anterior face } =-0,4111 \text{ dig.}\\ \text{posterior face} =-1,3454 \text{ dig.}\end{cases}$

III. For the third lens, Crown Glass, the radius of each face $=-0,5460$ dig.

Then truly the interval may be established

$$\text{I and II}=\tfrac{1}{16}\text{ dig.,}\quad \text{II and III }=1,7375\text{ dig.,}$$

thus so that the total length shall be $=1,8$ dig. and thus still not equal to two digits.

Finally the radius of the space seen will be equivalent to $3°\ 41'\ 11''$.

COROLLARY

163. Therefore the telescopes constructed in these two examples may be seen to be the most suitable for common use, since these can be easily worn and for these to be especially useful in shows [*i.e.* opera glasses]. But the following examples we will apply to greater magnifications.

EXAMPLE 3

164. The magnification shall be $m=25$, to describe a telescope of this kind consisting of three lenses.

Since there shall be $m=25$, there will become $7m-8=167$ and $35m-36=839$, and the determinable distances will become

$$b=-\tfrac{7}{8}\alpha=-0,875\alpha,\ \beta=\tfrac{7\cdot 839}{8\cdot 167}\alpha=4,3960\alpha,$$
$$\text{Log.}\tfrac{-b}{\alpha}=9,9420081(-),\ \text{Log.}\tfrac{\beta}{\alpha}=0,6430535,$$
$$c=-\tfrac{839}{25\cdot 167}=-\ 0,2009\alpha,$$
$$b+\beta=3,5210\alpha,\quad \text{Log.}(b+\beta)=0,5466660.$$

Hence the separation of the lenses will arise

$$a+b=\tfrac{1}{8}\alpha,\quad \beta+c=4,1951\alpha$$

and the total length $=4,3201\alpha$.

But for the apparent field of view there will become $\Phi=\frac{\pi'}{24}\left(1+\frac{4}{5\cdot167}\right)$ and hence the angle $\Phi=36$ minutes of arc approx.

Again since there shall be $\mathfrak{B}=\frac{839}{28\cdot24}$ and

$$\text{Log.}\mathfrak{B}=0,0963927,\quad \text{Log.}(-B)=\ 0,7010454(-),$$

we may arrive at the following equation

$$0=0,9875\lambda-0,3922\lambda'-0,0004984+0,03077$$

or

$$0=0,9875\lambda-0,3922\lambda'+0,03028.$$

Since we have seen the value $\lambda=1,60006$ hardly to diminish the length of the telescope at all, we may put at once $\lambda=1,60006$ and there will be

$$0=1,6103-0,3922\lambda',$$

from which there becomes

$$\lambda'=4,1057,\ \ \lambda'-1=3,1057 \text{ and } \log.\sqrt{(\lambda'-1)}=0,2460797,$$

from which the construction of the individual lenses thus will be had :

I. For the first lens made from crown glass the radius of each face $=1,06\alpha$, which therefore admits an aperture, of which the radius $x=0,265\alpha$.

II. For the second lens the calculation thus will be had :

$$\frac{1}{F'}=\frac{-0,7688\pm5,4449}{b\beta}\alpha,\qquad \frac{1}{G'}=\frac{6,8338\mp5,4449}{b\beta}\alpha.$$

The upper signs will prevail and there will be

$$F'=\frac{b\beta}{4,6816\alpha}=-0,8216\alpha,\qquad G'=\frac{b\beta}{1,3889\alpha}=-2,7694\alpha\,.$$

III. For the third lens made from crown glass the radius of each face

$$=1,06c=-0,21295\alpha\,.$$

But the distinctness requires $x=\frac{m}{50}$ dig. $=\frac{1}{2}$ dig., from which there is concluded $\alpha>1,89$; therefore $\alpha=2$ may be taken and this will be the construction :

I. For the first lens the radius of each face $=2,12$ dig., Crown Glass.

II. For the second lens the radius of the $\begin{cases}\text{anterior face} & =-1,6432 \text{ dig.}\\ \text{posterior face} & =-5,5388 \text{ dig}\end{cases}$ Flint Glass

III. For the third lens the radius of each face $=-0,42590$ dig., Crown Glass.

Then the interval of the lenses may be put in place I et II $=\frac{1}{4}$ dig., II and III $=8,3902$ dig. and the total length $=8,64$ dig., and the radius of the apparent field of view will be $x=36'$ approximately.

EXAMPLE 4

165. If m should be $=50$, there will be $7m-8=342,\ 35m-36=1714$ and thus the distances

$$b=-\tfrac{7}{8}\alpha=-\ 0,875\alpha,\ \ \beta=4,3852\alpha,\ \ c=-0,10023\alpha.$$

$$\log.\ \tfrac{\beta}{\alpha}=0,6419928, \qquad \log.\ (\tfrac{-b}{\alpha})=9,9420081(-),$$
$$\log.\tfrac{(b+\beta i)}{\alpha}=0,5453319, \quad \log.\ (-\tfrac{b\beta}{\alpha^2})=0,5840009(-).$$

Then truly the separation of the lenses will be

$$a+b=\tfrac{1}{8}\alpha=0,125\alpha,\ \ \beta+c=4,2850\alpha$$

and hence the whole length will be $=4,4100\alpha$.

Again there will be found

$$\text{Log.}(-B)=0,6999847(-), \quad \text{Log.}\mathfrak{B}=0,0966567.$$

There is found for the apparent field

$$\Phi=\tfrac{\pi'}{49}\left(1+\tfrac{4}{5\cdot 842}\right) \text{ or the angle } \Phi=17\tfrac{1}{2} \text{ minutes.}$$

For the confusion required to be removed $\lambda=1,60006$ may be put at once into the equation found and there will be

$$0=1,5801-0,3915\lambda'-0,00025+0,03083$$

or

$$0,39151\lambda'=1,6107, \text{ from which } \lambda'=4,1141;$$

hence

$$\lambda' - 1 = 3{,}1141 \text{ and } \log.\sqrt{(\lambda' - 1)} = 0{,}2466663,$$

from which the construction of the individual lenses thus will be obtained :

I. For the first lens the radius of each face $= 1{,}06\alpha$; which therefore allows an aperture, the radius of which $= 0{,}265\alpha$.

II. For the second lens

$$\tfrac{1}{F'} = \tfrac{-0{,}7653 \pm 5{,}4355}{b\beta}\alpha, \qquad \tfrac{1}{G'} = \tfrac{+6{,}8169 \mp 5{,}4355}{b\beta}\alpha.$$

Therefore the upper signs may prevail, and there will be

$$F' = \tfrac{b\beta}{4{,}6702\alpha} = -0{,}8216\alpha, \quad G' = \tfrac{b\beta}{1{,}3814\alpha} = -2{,}7776\alpha.$$

III. For the third lens the radius of each face

$$= 2(n-1)c = 1{,}06c = -0{,}10624\alpha.$$

Moreover the clarity demands $x = \frac{m}{50} = 1$ dig., from which there follows $\alpha > 3{,}8$; therefore on taking $\alpha = 4$, the following construction of the telescope follows :

I. For the first lens, Crown Glass, radius of each face $= 4{,}24$ dig .

II. For the second lens the radius of the $\left\{\begin{array}{ll}\text{anterior face} & = -\ 3{,}2864 \\ \text{posterior face} & = -11{,}1104\end{array}\right\}$ Flint Glass.

III. For the third lens, Crown Glass, the radius of each face $= -0{,}42496$.

Then truly the separation of the lenses must be put in place :

$$\text{I and II} = 0{,}5 \text{ dig., II and III} = 17{,}1400 \text{ dig.}$$

and thus the length of the telescope $= 17{,}6400$ dig.

Finally the radius of the field of view found is $17{,}6400$ minutes of arc.

SCHOLIUM

166. Since in these solutions the letter λ shall remain undetermined, we have not been able in the three last cases to place unity in place of that, as we have done before, but rather for that we may use that value, where both the faces of the lenses may be restored equal to each other, and in this manner we have obtained a convenient sign, so that the first lens may allow an aperture twice as great and hence the distance α may be reduced almost by half. But so that in general a certain lens, of which the determinable distances are a and α, may have both its faces equal, we have seen above [Lib. I § 56] there must be taken

$$\sqrt{(\lambda-1)}=\frac{(\sigma-\rho)(a-\alpha)}{2\tau(a+\alpha)}=\frac{2(nn-1)}{n\sqrt{(4n-1)}}\cdot\frac{1-A}{1+A}$$

on account of $\alpha=Aa$, from which there becomes

$$\lambda=1+\frac{4(nn-1)^2}{n^2(4n-1)}\cdot\frac{(1-A)^2}{(1+A)^2};$$

whereby, if either a or α were infinite, it can happen both in the objective lens as well as in the eyepiece, there will be had $\lambda=1+\frac{4(nn-1)^2}{n^2(4n-1)}$. But if we may wish, so that some other lens may obtain both its faces equal to each other, then on account of

$$\frac{(1-A)^2}{(1+A)^2}=1-\frac{4A}{(1+A)^2}$$

we must take

$$\lambda=1+\frac{4(nn-1)^2}{n^2(4n-1)}-\frac{16(nn-1)^2A}{n^2(4n-1)(1+A)^2}.$$

But since in our expression for the radius of confusion then such a form may occur $\lambda(A+1)^2+\nu A$, the value of this formula will become

$$(A+1)^2+\frac{4(nn-1)^2(A+1)^2}{n^2(4n-1)}-\frac{16(nn-1)^2A}{nn(4n-1)}+\frac{4(n-1)^2A}{4n-1}.$$

But in this case it will be more convenient to express the value of λ thus to facilitate the calculation :

$$\lambda=1+\frac{(\sigma-\rho)^2(1-A)^2}{4\tau^2(1+A)^2}.$$

EXAMPLE 5

167. If the magnification m must definitely be great or certainly greater than 25, to describe telescopes of this kind consisting of three lenses.

Here I observe at once the first lens taken to be equally convex on each side and its radius of curvature as before $=1,06\alpha$, which allows an aperture, of which the radius $=\frac{1}{4}\alpha = x$; but since on account of the clarity there must be taken $x=\frac{m}{50}$ dig., hence we understand always there can be put in place $\alpha=\frac{2m}{25}=0,08m$ dig. and for the apparent field $\Phi=\frac{\pi'}{m-1}$; and on taking $\pi'=\frac{1}{4}$ there will be $\Phi=\frac{859}{m-1}$.

But now, as before we may consider the remaining parts of the construction, we may consider the case, where $m=\infty$, and there will be

$$b=-\tfrac{7}{8}\alpha,\quad \beta=\tfrac{35}{8}\alpha,\quad c=\tfrac{-5}{m}\alpha=-\tfrac{2}{5}\text{ dig.}$$

[This is not of course physically possible, but assumes at least one of the denominators in the product $m=\frac{\alpha}{b}\cdot\frac{\beta}{c}\ldots$etc. to be essentially zero in comparison with the other values.]
Again the separation of the lenses

$$\alpha+b=\tfrac{1}{8}\alpha \text{ and } \beta+c=\left(\tfrac{35\alpha}{8}-\tfrac{2}{5}\right)\text{ dig. and } \mathfrak{B}=\tfrac{5}{4},\ B=-5.$$

For the following calculation we may assume $\lambda=1,60006$ and the equation will be produced

$$0=1,5801-0,3908\lambda'+0,03088,$$

from which there is found

$$\lambda'=4,1220 \text{ and } \lambda'-1=3,1220 \text{ and } \log.\sqrt{(\lambda'-1)}=0,2472164.$$

Hence on account of

$$\text{Log.}\left(-\tfrac{b}{\alpha}\right)=9,9420081(-),\quad \text{Log.}\tfrac{\beta}{\alpha}=0,6409781$$
$$\text{Log.}\tfrac{b+\beta}{\alpha}=0,5440680,\quad \text{Log.}\left(-\tfrac{b\beta}{\alpha^2}\right)=0,5829862(-).$$

From which we will have for the second lens

$$\frac{1}{F'}=\frac{-0,7662\pm 5,4266}{b\beta}\alpha \qquad \frac{1}{G'}=\frac{+6,8006\mp 5,4266}{b\beta}\alpha$$

or with the upper signs taken

$$F'=\frac{b\beta}{4,6604\alpha}=-\ 0,8214\alpha,\qquad G'=\frac{b\beta}{1,3740\alpha}=-2,7861\alpha,$$

which values have a use for the infinite magnification; but now for any magnification m there may be put

$$F' = -\left(0,8214 + \tfrac{f}{m}\right)\alpha, \quad G' = -\left(2,7861 + \tfrac{g}{m}\right)\alpha,$$

from which the values of the letters f and g must be elicited from the preceding $m = 50$ or also, but with less risk, from the case $m = 25$, and in this manner there is found

$$f = 0,01 \text{ and } g = -0,4250;$$

thus so that in general there shall be

$$F' = -(0,8214 + \tfrac{0,01}{m})\alpha, \quad G' = -(2,7861 - \tfrac{0,4250}{m})\alpha.$$

Then since from above now the focal length of the third lens shall be found $= -\tfrac{2}{5}$ dig. for $m = \infty$, we may establish for some magnification m to be

$$c = -\tfrac{2}{5} - \tfrac{h}{m} \text{ and there will be } c = -\left(\tfrac{2}{5} + \tfrac{1,2480}{m}\right) \text{ dig.;}$$

of which therefore the [corrected] radius of each face will be

$$-\left(0,4240 + \tfrac{0,948}{m}\right) \text{ dig.}$$

Therefore since there shall be $\alpha = 0,08m$ dig., the construction of the telescope will be had in the following manner :

I. For the first lens, Crown Glass, the radius of each face $= 0,0848m$ dig.

II. For the second lens, Flint Glass,

$$\text{the radius of the } \begin{cases} \text{anterior face} & = -(0,0657m + 0,0008) \text{ dig.} \\ \text{posterior face} & = -(0,2228m - 0,0340) \text{ dig.} \end{cases}$$

III. For the third lens, Crown Glass, the radius of each face

$$= -\left(0,4240 + \tfrac{0,948}{m}\right) \text{ dig.}$$

Then truly the intervals between the lenses will be

$$\alpha + b = 0,01m, \quad \beta + c = \left(0,35m - 0,36\right) \text{ dig.},$$

and hence the total length

$$= \left(0,36m - 0,36\right) \text{ dig.}$$

and the radius seen of the field of view $= \frac{859}{m-1}$ minutes of arc,

COROLLARY

168. Therefore if a telescope may be desired, so that it may magnify 100 times, that will be had thus :

I. For the first lens, Crown Glass, the radius of each face $= 8,48$ dig.
II. For the second lens, Flint Glass,

$$\text{the radius of the } \begin{cases} \text{anterior face} & = -\ 6,57 \text{ dig.} \\ \text{posterior face} & = -22,24 \text{ dig.} \end{cases}$$

III. For the third lens, Crown Glass, the radius of each face $= -0,43$ dig.

The interval of the lenses will be :

$$\text{I and II} = 1 \text{dig.}, \ \text{II and III} = 34,64 \text{ dig.}$$

and hence the length of the telescope $= 35,64$ dig. and the radius of the field seen $= 8\frac{1}{2}$ min.

PROBLEM 3

169. *If a telescope of this first kind shall be required to be made from four lenses separated from each other in turn, to determine the matters, from which the maximum degree of perfection may be agreed for that.*

SOLUTION

Therefore here the individuals of these three fractions $\frac{\alpha}{b}$, $\frac{\beta}{c}$ and $\frac{\gamma}{d}$ must be negative and therefore we may put $\frac{\alpha}{b} = -k$ and $\frac{\beta}{c} = -k'$ and, since there shall be $m = -\frac{\alpha}{b} \cdot \frac{\beta}{c} \cdot \frac{\gamma}{d}$, we will have
$b = -\frac{\alpha}{k}$, $\beta = [Bb =] - \frac{B\alpha}{k}$, $c = -\frac{\beta}{k'} = +\frac{B\alpha}{kk'}$, and $\gamma = [Cc =] + \frac{BC\alpha}{kk'}$ and $m = \frac{-kk'\gamma}{d}$,
hence

$$d = -\frac{BC\alpha}{m};$$

from which the spacing of the lenses

$$\alpha+b=\alpha\left(1-\tfrac{1}{k}\right),\quad \beta+c=B\alpha\left(\tfrac{1}{kk'}-\tfrac{1}{k}\right)\quad \text{et}\ \ \gamma+d=BC\alpha\left(\tfrac{1}{kk'}-\tfrac{1}{m}\right);$$

which since they must be positive and equal to the numbers k, k' and $m,$ the two latter divided by the first will give these two conditions :

$$1.\ \frac{B(1-k')}{k'(k-1)}>0,\qquad 2.\ \frac{BC(m-kk')}{mk'(k-1)}>0.$$

Now we will consider the equation, by which the colored margin may be removed, for the case where the distance O is negative[§ 52], on putting as before

$$\tfrac{dn}{n-1}=N,\quad \tfrac{dn'}{n'-1}=N',\quad \tfrac{dn''}{n''-1}=N'',\quad \tfrac{dn'''}{n'''-1}=N'''$$

and there will become

$$0=NBC\pi''\alpha+N'b\left((B+1)C\pi''-\pi\right)+N''c\frac{(C+1)\pi''-\pi'}{B}$$

or

$$0=NBC\pi''-\tfrac{N'}{k}\left((B+1)C\pi''-\pi\right)+\tfrac{N''}{kk'}\left((C+1)C\pi''-\pi'\right);$$

which in the end it will be required to investigate the relations between the letters $\pi,\ \pi',\ \pi''$; which is from chapter I, § 11:

$$\text{I.}\ \ \tfrac{\mathfrak{B}\pi-\Phi}{\Phi}=-k,\text{from which}\ \ \pi=\tfrac{1-k}{\mathfrak{B}}\cdot\Phi.$$

$$\text{II.}\ \ \tfrac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=\tfrac{B\alpha}{c}=kk',\text{from which}\ \ \pi'=\left(\tfrac{1}{B}-\tfrac{k}{\mathfrak{B}}+kk'\right)\tfrac{\Phi}{\mathfrak{C}}.$$

$$\text{III.}\ \ \tfrac{\mathfrak{D}\pi''-\pi'+\pi-\Phi}{\Phi}=\tfrac{BC\alpha}{d}=-m,$$

from which on account of $\mathfrak{D}=1$ there will become

$$\pi''=\left(-m+\tfrac{1}{BC}-\tfrac{k}{\mathfrak{B}C}+\tfrac{kk'}{\mathfrak{C}}\right)\Phi;$$

from which our equation will be

$$0=N\left(-BCm+1-\tfrac{Bk}{\mathfrak{B}}+\tfrac{BCkk'}{\mathfrak{C}}\right)-\tfrac{N'}{k}\left(\tfrac{-BCm}{\mathfrak{B}}-\tfrac{Bk}{\mathfrak{B}}+\tfrac{BCkk'}{\mathfrak{B}\mathfrak{C}}\right)+\tfrac{CN''}{\mathfrak{C}kk'}\left(-m+kk'\right);$$

from which there becomes

$$C=N-N(B+1)k+NBkk'+N'(B+1)-N'(B+1)k'-\tfrac{N''m}{kk'}+N''$$

divided by

$$NBm-NBkk'-\tfrac{N'(B+1)m}{k}+N'(B+1)k'+\tfrac{N''m}{kk'}-N''$$

or more succinctly $C=Nkk'\left(1-k-Bk\left(1-k'\right)\right)+N'kk'\left(B+1\right)\left(1-k'\right)-N''\left(m-kk'\right)$

divided by

$$\left(m-kk'\right)(Nkk'B-N'k'(B+1)+N'')$$

and thus

$$1+C=Nkk'\left(1-k+B\left(m-k\right)\right)-N'(B+1)k'(m-k)$$

divided by

$$\left(m-kk'\right)(Nkk'B-N'k'(B+1)\ \ +N'')$$

and hence

$$\mathfrak{C}=Nkk'\left(1-k-Bk\left(1-k'\right)\right)+N'kk'\left(B+1\right)(1-k')-N''(\ m-kk')$$

divided by

$$Nkk'\left(1-k+B\left(m-k\right)\right)-N'(B+1)k'(m-k)\,;$$

but it is readily apparent to be scarcely possible for us to be allowed to progress further on account of the complexity of these formulas, unless the values for k and k' and for N, N', N'' may be substituted ; yet meanwhile this method also is seen to be going to be successful, if we may wish to apply that to several more lenses; moreover by no means from this difficulty will this matter be able to be tested also in another way.

Clearly from the preceding equations evidently no letters π, π', π'' are to be sought, but rather with these as if given it will be convenient to define the letters $\mathfrak{B}$ and $\mathfrak{C}$; from which we obtain at once

$$\mathfrak{B}=\tfrac{1-k}{\pi}\cdot\varPhi,\quad \mathfrak{C}=\tfrac{(kk'-1)\varPhi+\pi}{\pi'};$$

and finally from the third equation on account of $\mathfrak{D}=1$ there is deduced

$$\varPhi=\tfrac{-\pi+\pi'-\pi''}{m-1},$$

and thus so that $\varPhi$ may be considered as if given. Hence, since there shall be

$$B=\tfrac{\mathfrak{B}}{1-\mathfrak{B}}\ \text{ et }\ C=\tfrac{\mathfrak{C}}{1-\mathfrak{C}},$$

we will have

$$B=\tfrac{(1-k)\varPhi}{\pi-(1-k)\varPhi},\qquad C=\tfrac{(kk'-1)\varPhi+\pi}{\pi'-\pi-(kk'-1)\varPhi}.$$

Now since the first condition shall demand, that there shall be $\frac{B(1-k')}{k'(k-1)}>0$, truly the other divided by this $\frac{C(m-kk')}{1-k'}>0$, with these values substituted these two conditions will be changed into the following :

$$1.\quad \frac{(k'-1)\Phi}{(\pi-(1-k)\Phi)k'}>0 \quad \text{or} \quad \frac{(k'-1)\Phi}{\pi-(1-k)\Phi}>0\,,$$

$$2.\quad \frac{((kk'-1)\Phi+\pi)(m-kk')}{(1-k')(\pi'-\pi-(kk'-1)\Phi)}>0,$$

which multiplied by that gives

$$\frac{-\Phi(m-kk')(\pi+(kk'-1)\Phi)}{(\pi-(1-k)\Phi)(\pi'-\pi-(kk'-1)\Phi)}>0;$$

if here in place of Φ its value may be substituted, which, since it shall always be positive, on account of $m-1$ also being positive, gives in the first place

$$-\pi+\pi'-\pi''>0,$$

then truly these two conditions will give

$$1.\quad \frac{k'-1}{(m-k)\pi+(k-1)\pi'-(k-1)\pi''}>0,$$

$$2.\quad \frac{(m-kk')((m-kk')\pi+(kk'-1)\pi'-(kk'-1)\pi'')}{(k'-1)((m-kk')\pi-(m-kk')\pi'-(kk'-1)\pi'')}>0\;;$$

then truly B and C will be defined thus:

$$B=\frac{(k-1)(\pi-\pi'+\pi'')}{(m-k)\pi+(k-1)\pi'-(k-1)\pi''}$$

$$C=\frac{(m-kk')\pi+(kk'-1)\pi'-(kk'-1)\pi''}{-(m-kk')\pi+(m-kk')\pi'+(kk'-1)\pi''}.$$

Truly if we may wish to substitute these values either into the equation for removing the colored fringe or especially for radius of confusion being reduced to zero, we may be falling upon much greater round about ways, than arise from the first method ; on account of which we must seek another method at this point; but no other is left for us, except that we may investigate the letters k and k' from the above equations ; with which agreed on our investigation will be returned plain enough.

Following this way we will have at once $k=\frac{\Phi-\mathfrak{B}\pi}{\Phi}$ and $kk'=\frac{\Phi-\pi+\mathfrak{C}\pi'}{\Phi}$ with $\Phi=\frac{-\pi+\pi'-\pi''}{m-1}$ present, from which, since k and k' shall be positive numbers and equally the angle Φ, we will have at once these conditions :

$$\Phi-\mathfrak{B}\pi>0,\qquad \Phi-\pi+\mathfrak{C}\pi'>0,\qquad -\pi+\pi'-\pi''>0.$$

Again since from here the separation of the lenses may become

$$1.\quad \alpha+b=\frac{-\mathfrak{B}\pi}{\Phi-\mathfrak{B}\pi}\alpha>0,$$

$$2.\quad \beta+c=\frac{(\mathfrak{B}\pi-B\mathfrak{C}\pi')\alpha\Phi}{(\Phi-\pi+\mathfrak{C}\pi')(\Phi-\mathfrak{B}\pi)}>0,$$

$$3.\quad \gamma+d=\frac{(m-1)\Phi+\pi-\mathfrak{C}\pi'}{m(\Phi-\pi+\mathfrak{C}\pi')}BC\alpha>0,$$

hence we deduce these new conditions:

$$-\mathfrak{B}\pi a>0,$$
$$(\mathfrak{B}\pi-B\mathfrak{C}\pi')\alpha>0,$$
$$((m-1)\Phi+\pi-\mathfrak{C}\pi')BC\alpha>0;$$

and thus also the positive quotients of these must be

$$\frac{\mathfrak{B}\pi-B\mathfrak{C}\pi'}{-\mathfrak{B}\pi}>0,\qquad \frac{((m-1)\Phi+\pi-\mathfrak{C}\pi')BC}{\mathfrak{B}\pi-B\mathfrak{C}\pi'}>0;$$

of which the latter on account of $(m-1)\Phi=-\pi+\pi'-\pi''$ will be changed into this

$$\frac{(\mathfrak{C}\pi'-C\pi'')B}{\mathfrak{B}\pi-B\mathfrak{C}\pi'}>0,$$

and thus five conditions are found for the free α, which it is required to satisfy. But now the equation for the colored fringe being removed thus itself will be found :

$$0=NBC\pi''-\frac{N'\Phi}{\Phi-\mathfrak{B}\pi}((B+1)C\pi''-\pi))+\frac{N''\Phi}{\Phi-\pi+\mathfrak{C}\pi'}((C+1)\pi''-\pi').$$

From these noted the final equation will depend in some way for the removal of the confusion [§ 53]

$$0=N-\frac{N'}{k\mathfrak{B}}+\frac{N''}{kk'B\mathfrak{C}}-\frac{N'''}{mBC}$$

on account of

$$p=\alpha,\quad q=\mathfrak{B}b=-\tfrac{\mathfrak{B}\alpha}{k},\ r=\mathfrak{C}c=\tfrac{\mathfrak{B}\mathfrak{C}\alpha}{kk'}\ \text{ et }\ s=d=-\tfrac{BC\alpha}{m},$$

whether that may be able to be satisfied either completely or perhaps approximately.

Finally, in order also that the former confusion may be removed, for this equation to be satisfied [§ .42]

$$\mu\lambda-\tfrac{\mu'\lambda'}{k\mathfrak{B}^3}+\tfrac{\mu''\lambda''}{kk'\mathfrak{B}^3\mathfrak{C}^3}-\tfrac{\mu'''\lambda'''}{B^3C^3m}-\tfrac{\mu'\nu'}{k\mathfrak{B}B}+\tfrac{\mu''\nu''}{kk'B^3C\mathfrak{C}}=0.$$

SCHOLIUM

170. Since here in general it may be scarcely allowed to progress further, it will be required to descend to particular cases, and since in the preceding chapter the lenses of a perfect triple thus had not excelled the desired use, because the confusion arising from these was not being removed for the eyepiece lens, here we may put a triple objective lens again, so that the first two intervals may vanish, truly we may define its three lenses thus, thus so that the confusion arising may be removed for the eyepiece lens ; hence with this done perhaps the way will be apparent by putting in place small intervals between the first three lenses. Indeed in difficult inquires of this kind always to begin with the easier cases, since then an account may be seen overcoming the difficulties, which when first observed were considered as invincible.

PROBLEM 4

171. *If the three prior lenses may be connected together to each other, so that the objective lens may constitute a triple lens, truly the fourth lens shall be the ocular, to establish the rules for the construction of this telescope.*

SOLUTION

Since here both the intervals $\alpha+b=0$ as well as $\beta+c=0$, there will become at once $k=1$ and $k'=1$, from which the distances follow :

$$b=-\alpha,\ \beta=-B\alpha,\ c=B\alpha,\ \gamma=BC\alpha\ \text{ and }\ d=-\tfrac{BC\alpha}{m}$$

and hence the interval

$$\gamma+d=BC\alpha\left(1-\tfrac{1}{m}\right)=\tfrac{m-1}{m}BC\alpha,$$

which must be positive. But for the letters $\pi,\ \pi',\ \pi''$ we will have

$$1.\ \pi=0,\ \ 2.\ \pi'=0,\ \ 3.\ \pi''=-(m-1)\Phi.$$

And hence the equation for removing the colored margin will be

$$0 = NBC - N'(B+1)C + N''(C+1),$$

from which we deduce

$$C = -\frac{N''}{NB - N'(B+1) + N''},$$

and from which the interval $\gamma + d$ becomes

$$= -\frac{(m-1)}{m} \cdot \frac{N''B\alpha}{NB - N'(B+1) + N''};$$

which since it must be positive, two cases are required to be considered.

The one, where $\alpha > 0$; then there must be

$$\frac{N''B}{NB - N'(B+1) + N''} < 0 \text{ and thus } N - \frac{N''}{B}(B+1) + \frac{N''}{B} < 0$$

or

$$N - N' + \frac{1}{B}(N'' - N') < 0.$$

The other case, if $\alpha < 0$, the contrary must eventuate, evidently

$$N - N' + \frac{1}{B}(N'' - N') > 0.$$

Now the equation will be considered

$$0 = N - \frac{N'}{\mathfrak{B}} + \frac{N''}{B\mathfrak{C}} - \frac{N'''}{mBC};$$

from which multiplied by *BC*, so that there shall be

$$0 = NBC - N'(B+1)C + N''(C+1) - \frac{N'''}{m},$$

since it only differs by the final term $\frac{N'''}{m}$ from the preceding equation, which is very small besides the others, we conclude, if that were satisfied, for this to be satisfied also approximately and that further there, where the multiplier *m* were greater ; which conclusion is based on the fact, that the numbers $N,\ N',\ N'',\ N'''$ differ little from unity.

But for the first for the confusion to be removed it is necessary in addition for this equation to be satisfied

$$0 = \mu\lambda - \frac{\mu'\lambda'}{\mathfrak{B}^3} + \frac{\mu''\lambda''}{\mathfrak{B}^3\mathfrak{C}^3} - \frac{\mu'''\lambda'''}{mB^3C^3} - \frac{\mu'\nu'}{\mathfrak{B}B} + \frac{\mu''\nu''}{B^3C\mathfrak{C}},$$

in which place the value of *C* found above must be substituted

$$C=\frac{-N''}{B(N-N')-N'+N''};$$

that which in general may lead to a great deal of trouble, whereby the solution cannot be found except for particular cases.

COROLLARY 1

172. Even if the conditions for the letter B are given, yet still this letter is left undetermined, while it may be observed,

1. if there were $N-N'+\frac{1}{B}(N''-N')<0$,

then α must be taken positive ;

2. but if there were $N-N'+\frac{1}{B}(N''-N')>0$,

then α must be taken negative.
Since we will have found

$C=\frac{-N''}{B(N-N')-N'+N''}$, there will become $1+C=\frac{B(N-N')-N'}{B(N-N')-N'+N''}$, and hence $\mathfrak{C}=\frac{-N''}{B(N-N')-N'}$.

COROLLARY 2

173. If we may wish to introduce $\mathfrak{B}$ in place of B by putting $B=\frac{\mathfrak{B}}{1-\mathfrak{B}}$, then we may follow with

$$C=\frac{-N''(1-\mathfrak{B})}{(N-N'')\mathfrak{B}-N'+N''} \quad\text{and}\quad \mathfrak{C}=\frac{-N''(1-\mathfrak{B})}{N\mathfrak{B}-N'},$$

the substitution from which noted may resolve the latter more readily ; indeed there becomes for the latter equation :

$$0=\left\{\begin{array}{l}\mu\lambda\mathfrak{B}^2-\mu'\lambda'-\mu'\nu'\mathfrak{B}(1-\mathfrak{B})-\frac{\mu''\lambda''(N\mathfrak{B}-N')^3}{(N'')^3}\\ +\frac{\mu''\nu''(1-\mathfrak{B})((N\mathfrak{B}-N')(N-N'')\mathfrak{B}-N'+N'')}{(N'')^2}\\ \qquad+\frac{\mu'''\lambda'''((N-N'')\mathfrak{B}-N'+N'')^3}{m(N'')^3}\end{array}\right\}.$$

COROLLARY 3

174. With regard to the apparent field, since there shall be $\pi'' = -(m-1)\Phi$, if we may put in place as so far $\pi'' = -\frac{1}{4}$, the radius will be produced $\Phi = \frac{1}{4(m-1)}$ and in minutes of arc $\Phi = \frac{851}{4(m-1)}$, if indeed the eyepiece lens may become equally concave on each side, so that, as we have shown, if $\lambda''' = 1,60006$, evidently as it were with the lens made from crown glass. But if we may wish to make that from crystal glass, there must be put $\lambda''' = 1,67445$.

EXAMPLE

175. If the first and third lens were of crown glass, the middle lens of crystal glass, and from these the objective lens may be constructed, truly the eyepiece lens may be prepared from crown glass, to construct a telescope of some given magnification.

Thus I bring forth this example, since this case has been overlooked in the previous chapter, but which I will treat here in another manner, as it may produce a smaller length.

Therefore since here there shall be $n = 1,53,\ \ n' = 1,58,\ \ n'' = 1,53,\ \ n''' = 1,53,$ there will be, as we have seen, $N = 7,\ \ N' = 10,\ \ N'' = 7,\ \ N''' = 7,$ thus so that there shall be $\mu'' = \mu,\ \nu'' = \nu,\ \mu''' = \mu.$

From these it is gathered

$$C = +\tfrac{7}{3}(1-\mathfrak{B}), \quad \mathfrak{C} = \frac{-7(1-\mathfrak{B})}{7\mathfrak{B}-10}.$$

Therefore as long as this equation remains requiring to be resolved

$$0 = \mu\lambda\mathfrak{B}^3 - \mu'\lambda' - \mu'\nu'\mathfrak{B}(1-\mathfrak{B}) - \frac{\mu\lambda''(7\mathfrak{B}-10)^3}{7^3} - \frac{3\mu\nu(1-\mathfrak{B})(7\mathfrak{B}-10)}{7^2} - \frac{27\mu\lambda'''}{7^3 m};$$

so that therefore we may put in place $\lambda'' = \lambda$ and $\lambda''' = 1,60006$, this equation may adopt this form

$$\mu\lambda\left(\tfrac{30}{7}\mathfrak{B}^2 - \tfrac{300}{49}\mathfrak{B} + \tfrac{1000}{343}\right) - \mu'\nu' + \mu'\nu'\left(\mathfrak{B}^2 - \mathfrak{B}\right)$$
$$+\mu\nu\left(\tfrac{3}{7}\mathfrak{B}^2 - \tfrac{51}{49}\mathfrak{B} + \tfrac{30}{49}\right) - \frac{27\mu\lambda'''}{343m} = 0,$$

which is only a quadratic equation, from which the value of $\mathfrak{B}$ must be elicited; therefore with the terms set out according to the powers of $\mathfrak{B}$ there will be found :

$$\mathfrak{B}^2\left(\mu\lambda+\mu'\nu'+\tfrac{3}{7}\mu\nu\right)+\mathfrak{B}\left(-\tfrac{300}{49}\mu\lambda-\mu'\nu'-\tfrac{51}{49}\mu\nu\right)$$
$$\tfrac{1000}{343}\mu\lambda-\mu'\lambda'+\tfrac{30}{49}\mu\nu-\tfrac{27\mu\lambda'''}{343m}=0.$$

Moreover thus we may put in place the resolution of this equation, so that a greater aperture of the objective lens may be allowed ; which we may not put in the end as before $\lambda=1$, but $\lambda=1,60006$, so that the first lens may emerge with each side similar ; whereby there shall become:

$$\log.\mu=9,9945371 \qquad \log.\mu'=9,9407157$$
$$\log.\mu\nu=9,3360593 \qquad \log.\mu'\nu'=9,3436055,$$
$$\mu\nu=0,2168 \qquad \mu'\nu'=0,2206$$
$$\text{and Log. }\lambda=\text{Log. }\lambda'''=0,2041363;$$

but for the second lens we may not put $\lambda'=1$ as before, but we may leave this letter indeterminate; from which our equation thus will be prepared in numbers:

$$0=7,0852\mathfrak{B}^2-10,1201\mathfrak{B}+4,7393-\mu'\lambda'-\tfrac{0,12437}{m},$$

which is reduced to this

$$\mathfrak{B}^2=\tfrac{10,1201}{7,0852}\mathfrak{B}-\tfrac{4,7393}{7,0852}+\tfrac{\mu'\lambda'}{7,0852}+\tfrac{0,12437}{m\cdot 7,0852}$$
$$\mathfrak{B}^2=1,4283\mathfrak{B}-0,6689+0,12311\lambda'+\tfrac{0,01755}{m}.$$

From which there is found

$$\mathfrak{B}=0,7142\pm\sqrt{\left(-0,1589+0,1231\lambda'+\tfrac{0,01755}{m}\right)}.$$

From which it is apparent λ' must be taken greater than unity. Therefore we may put in place $\lambda'=1\frac{1}{2}$ and there will become

$$\mathfrak{B}=0,7142\pm\sqrt{\left(0,257+\tfrac{0,01755}{m}\right)};$$

but hence further progress may not be allowed unless determined values be attributed to the letter m ; to which in the end we may adjoin the following cases.

CASE 1
$m=10$

176. In this case there will be $\mathfrak{B}=0,7142\pm0,1657$ and either with the lower sign taken $\mathfrak{B}=0,5485$ or with the upper sign taken $\mathfrak{B}=0,8799$. But if we wish, so that the single value $=0,7142$ may be produced for $\mathfrak{B}$, there must be taken

$$\lambda' = \frac{0{,}1572}{0{,}1231} = 1\tfrac{341}{1231},$$

and we may use here in this case.
Therefore since there shall be $\lambda' = 1\tfrac{341}{1231}$, there will be

$$\lambda' - 1 = \tfrac{341}{1231}, \quad \text{Log.}\sqrt{\lambda' - 1} = 9{,}7208976, \quad \text{Log.}(\lambda' - 1) = 9{,}4417952.$$

Now since for all multiplication there shall be $\mathfrak{B} = 0{,}7142$, if indeed we may take

$$\lambda' = 1{,}2908 - \frac{0{,}1425}{m}, \quad \lambda' - 1 = 0{,}2908 - \frac{0{,}1425}{m},$$

hence there will be $1 - \mathfrak{B} = 0{,}2858$, $B = 2{,}4990$, $C = 0{,}6669 = \tfrac{2}{3}$.
From which we will obtain the distances

$$b = -\alpha, \ \beta = -24990\alpha = -2\tfrac{1}{2},$$
$$c = +\ 2{,}4990\alpha, \quad \gamma = +1{,}6667\alpha, \quad d = -0{,}16667\alpha.$$

Now since there shall be $\lambda = \lambda'' = \lambda''' = 1{,}60006$ and $\lambda' = 1{,}2766$, there will be :

I. For the first lens with each face equally convex the radius of each face $= 1{,}06\alpha$.
II. For the second lens from crystal glass :

$$\frac{1}{F'} = \frac{\rho\beta + \sigma b \mp \tau'(b+\beta)\sqrt{\lambda' - 1}}{b\beta}, \quad \frac{1}{G'} = \frac{\sigma\beta + \rho b \pm \tau'(b+\beta)\sqrt{\lambda' - 1}}{b\beta},$$

$$\frac{1}{F'} = \frac{-1{,}9360 \mp 1{,}6152}{b\beta}\alpha, \quad \frac{1}{G'} = \frac{-4{,}0980 \pm 1{,}6152}{b\beta}\alpha,$$

and with the upper signs taken there will become

$$F' = \frac{-b\beta}{3{,}5512} = -0{,}7039\alpha, \quad G' = \frac{-b\beta}{2{,}4828\alpha} = -\ 1{,}0070\alpha.$$

III. For the third lens from crown glass, since here there shall be $\tau\sqrt{\lambda'' - 1} = \frac{\sigma - \rho}{2}$, then truly $c = 2\tfrac{1}{2}\alpha$ and $\gamma = 1\tfrac{2}{3}\alpha$, there will be

$$c + \gamma = 4\tfrac{1}{6}\alpha,$$
$$\frac{1}{F''} = \frac{4{,}5280 \pm 2{,}9870}{c\gamma}\alpha, \quad \frac{1}{G''} = \frac{3{,}3935 \mp 2{,}9870}{c\gamma}\alpha$$

and from the lower signs

$$F'' = \frac{c\gamma}{1{,}5410\alpha} = 2{,}7039\alpha, \ G'' = \frac{c\gamma}{6{,}3205\alpha} = 0{,}6593\alpha.$$

Therefore these three lenses joined to each other will allow an aperture, the radius of which can be estimated to be

$$x = 0,1648\alpha = \tfrac{1}{7}\alpha \quad \text{approximately.}$$

But since on account of the clarity there must be $x = \frac{m}{50}$ dig. $= \frac{1}{5}$ dig., there will be taken approximately $\alpha = \frac{7}{5}$ dig., from which the length of the telescope

$$= 1,5001\alpha = 1\tfrac{1}{2}\alpha = 2,1 \text{ dig.}$$

IV. For the fourth lens with each side equally concave the radius of each face will be

$$= 1,06d = 1,06(-0,1667)\alpha = -0,1767\alpha = -0,2474 \text{ dig.}$$

COROLLARY 1

177. Therefore if in this manner the value of λ' may be defined, the preceding determinations for all the magnifications will prevail with the second lens alone excepted; then moreover for the fourth lens the radius of each face of that must be taken $= -(106)\cdot\frac{5}{3}\cdot\frac{\alpha}{m}$ or $= -1,7666\cdot\frac{\alpha}{m}$.

COROLLARY 2

178. But the construction of the second lens will depend on the magnification, because the value of the letter λ' involves the magnification, since there shall be $\lambda' = 1,2908 - \frac{0,1425}{m}$.

SCHOLlUM

179. Moreover for any magnification the second lens to be defined without any difficulty, after that for the case $m = 10$ has been found; indeed we may put $m = \infty$, there will become $\lambda' = 1,2908$ hence $\lambda' - 1 = 0,2908$ and Log.$\sqrt{(\lambda'-1)} = 9,7317972$, from which the changeable member will be $= 1,6562\alpha$; from which for the second lens there will be

$$\frac{1}{F'} = \frac{-1,9860 \mp 1,6562F}{b\beta}\alpha, \quad \frac{1}{G'} = \frac{-4,0980 \pm 1,6562}{b\beta}\alpha;$$

therefore with the upper signs taken

$$F' = \frac{-b\beta}{3,5922\alpha} = -0,6959\alpha, \; G' = \frac{-b\beta}{2,4418\alpha} = -1,0239\alpha.$$

Therefore now we may put in place for any magnification m to be

$$F' = -(0,6959 + \tfrac{f}{m})\alpha, \quad G' = -\left(1,0239 + \tfrac{g}{m}\right)\alpha,$$

and since on putting $m = 10$

$$0,6959 + \tfrac{f}{10} = 0,7039 \quad \text{and} \quad 1,0239 + \tfrac{g}{10} = 1,0070,$$

there is found $f = 0,0800$, $g = -0,1690$; with which found we arrive at the following construction of the telescope:

I. For the first lens, Crown Glass, radius of each face $= +1,06\alpha$.

II. For the second lens, Flint Glass,

$$\text{the radius of the } \begin{cases} \text{anterior face } = -\left(0,6959 + \frac{0,0800}{m}\right)\alpha \\ \text{posterior face} = -\left(1,0239 - \frac{0,1690}{m}\right)\alpha. \end{cases}$$

III. For the third lens, Crown Glass, the radius of the $\begin{cases} \text{anterior face } = +2,7039\alpha \\ \text{posterior face} = +0,6593\alpha. \end{cases}$

IV. For the fourth lens, Crown Glass, the radius of each face $= -1,7666\frac{\alpha}{m}$; with which lenses prepared the first three may be joined, after which the fourth may be allocated the interval $= \frac{m-1}{m} \cdot \frac{5}{3}\alpha$.

Again since there shall be $x = \frac{m}{50}$ dig. and we will have found $x = 0,1648\alpha$, hence there is deduced to be $\alpha = \frac{m}{8,2400}$, thus so that there may be put in place $\alpha = \frac{4}{33}m$ or $\alpha = \frac{12}{100}m$. Whereby the construction of a telescope of the first kind is obtained:

I. For the first lens, Crown Glass, the radius of each face $= 0,1272m$.

II. For the second lens, Flint Glass,

$$\text{radius of the } \begin{Bmatrix} \text{anterior face } = -0,0835m - 0,0096 \\ \text{posterior face} = -0,1229m + 0,0202. \end{Bmatrix}$$

III. For the third lens, Crown Glass,

$$\text{radius of the } \begin{Bmatrix} \text{anterior face } = 0,3244m \\ \text{posterior face} = 0,0791m; \end{Bmatrix}$$

with which three lenses joined adjacent to each other with the latter interval $=\frac{1}{5}(m-1)$ dig. the ocular lens may be put in place.

IV. For the fourth lens, Crown Glass, the radius of each face will be $=-0,2119$.

CASE 2

180. Since the construction of the telescope shall be applicable for all magnifications, we may set out the solution of the example introduced above in another way. Clearly since here for the value $\mathfrak{B}$ we may have followed with a value less than unity, which had been produced above greater than unity, the case $\mathfrak{B}=1$ is seen noteworthy, which we may set out here. But then there will be $B=\infty$ and since the determinable distances are $\alpha,\ b=-\alpha,\ \beta=-B\alpha,\ c=B\alpha,\ \gamma=BCa,\ d=\frac{-BCa}{m}$, it is necessary, that BC shall be a finite quantity and thus $C=0$ and $\mathfrak{C}=C=0$. Whereby we may put $BC=\theta$, so that there may become $\gamma=\theta\alpha$ and $d=\frac{-\theta\alpha}{m}$ and hence the length of the telescope $=\frac{m-1}{m}\theta\alpha$. With these in place the equation for the colored margin being removed will give on account of $N=7,\ N'=10,\ N''=N'''=7$ and $\pi=0,\ \pi'=0,\ \pi''=-(m-1)\Phi$:

$$0=N\theta-N'\theta+N'',\quad 0=7\theta-10\theta+7 \text{ and hence } \theta=\tfrac{7}{3}.$$

But now the equation for the confusion of the first kind being taken away will become :

$$0=\mu\lambda-\mu'\lambda'+\frac{27\mu\lambda''}{343}-\frac{27\mu\lambda'''}{343m},$$

which divided by μ on account of $\frac{\mu'}{\mu}=0,8834$ will be changed into

$$0=\lambda-0,8884\lambda'+0,0787\lambda''-\frac{0,0787\lambda'''}{m};$$

but with each side of the eyepiece lens made equal there will become $\lambda'''=1,60006$ and

$$0=\lambda-0,8884\lambda'+0,0787\lambda''-\frac{0,1259}{m},$$

from which it will be agreed for the letters λ to be defined thus, so that they may exceed unity; therefore we may put in place $\lambda=1$ and $\lambda''=1$, there will be $0,8884\lambda'=1,0787-\frac{0,1259}{m}$, from which there is found

$$\lambda'=1,2210-\frac{0,1425}{m};$$

therefore only the second lens depends on the magnification m, as henceforth we may set out separately.

Therefore we may put in place the calculation for the first, third and fourth lenses. For the first moreover there is :

$$F=\frac{\alpha}{\sigma}=0,6023\alpha,\quad G=\frac{\alpha}{\rho}=4,4111\alpha.$$

For the third on account of $\lambda''=1$ and $c=-\infty$,

$$F''=\frac{\alpha}{\sigma}=1,4055\alpha,\quad G''=\frac{\gamma}{\rho}=10,2925\alpha.$$

For the fourth lens the radius of each face $=1,06d=-\frac{2,4733\alpha}{m}$.

And the total length of the telescope $=\frac{m-1}{m}\cdot\frac{7}{3}\alpha.$

But for the second lens, sin in general there shall be

$$F'=\frac{b\beta}{\rho'\beta+\sigma'b\pm\tau'(b+\beta)\sqrt{\lambda'-1}},\quad G'=\frac{b\beta}{\sigma'\beta+\rho'b\mp\tau'(b+\beta)\sqrt{\lambda'-1}},$$

on account of $\beta=\infty$ and $b=-\alpha$ there will be

$$F'=\frac{-\alpha}{\rho'\pm\tau'\sqrt{\lambda'-1}},\quad G'=\frac{-\alpha}{\sigma'\mp\tau'\sqrt{\lambda'-1}};$$

for which two cases are required to be established, the one, where $m=10$, and the other, where $m=\infty$.

In the former on account of $m=10$ there will be $\lambda'=1,2068$, $\lambda'-1=0,2068$ and

$$\begin{aligned}\text{Log.}\sqrt{(\lambda'-1)}&=9,6577753\\ \text{Log. }\tau'&=\underline{9,9432471}\\ &9,6010224\end{aligned}$$

to which logarithm there corresponds 0,3990, and thus

$$F'=\frac{-\alpha}{0,1414\pm0,3990},\qquad F'=\frac{-\alpha}{0,5404}=-1,8505\alpha,$$
$$G'=\frac{-\alpha}{1,5827\mp0,3990},\qquad G'=\frac{-\alpha}{1,1837}=-0,8448\alpha.$$

In the other case on account of $m=\infty$ there will be $\lambda'=1,2210$, $\lambda'-1=0,2210$, and

$$\begin{aligned}\text{Log.}\sqrt{(\lambda'-1)}&=9,6721961\\ \text{Log}\,\tau'&=\underline{9,9432471}\\ &9,6154432\end{aligned}$$

hence $\tau'\sqrt{\lambda'-1} = 0,4125$. From which there shall be

$$F' = \frac{-\alpha}{0,1414 \pm 0,4125} = \frac{-\alpha}{0,5589}, \quad G' = \frac{-\alpha}{1,5827 \mp 0,4125} = \frac{-\alpha}{1,1702}$$

and hence

$$F' = -1,8054\alpha, \qquad G' = -0,8545\alpha;$$

whereby we may put in place for the some magnification m :

$$F' = -\left(1,8054 + \tfrac{f}{m}\right)\alpha, \qquad G' = -\left(0,8545 + \tfrac{g}{m}\right)\alpha,$$

and from the case $m = 10$ we elicit

$$f = 0,4510, \ \ g = -0,0970,$$

thus so that there shall become

$$F' = -\left(1,8054 \ + \tfrac{0,4510}{m}\right)\alpha, \quad G' = -\left(0,8545 - \tfrac{0,0970}{m}\right)\alpha;;$$

but for α requiring to be defined the minimum radius $0,6023\alpha$ may be considered occurring in this triple objective lens, of which the fourth part $0,1506\alpha = \frac{m}{50}$; and thus there will be produced $\alpha = \frac{m}{7,5300}$ dig. Therefore there may be taken $\alpha = \frac{2m}{15}$ dig. and the following construction of the telescope of the first kind is found :

I. For the first lens, Crown Glass,

$$\text{radius of the} \begin{cases} \text{anterior face} \ = +0,0803m \ \text{ dig.} \\ \text{posterior face} = +0,5882m \ \text{ dig.} \end{cases}$$

II. For the second lens, Flint Glass,

$$\text{radius of the} \begin{cases} \text{anterior face} \ = (-0,2407m - 0,0601) \ \text{ dig.} \\ \text{posterior face} = (-0,1139m + 0,0124) \ \text{ dig.} \end{cases}$$

III. For the third lens, Crown Glass,

$$\text{radius of the} \begin{cases} \text{anterior face} \ = +0,1874m \ \text{dig.} \\ \text{posterior face} = +1,3723m \ \text{dig.} \end{cases}$$

IV. For the fourth lens, Crown Glass,
radius of each face $= 0{,}3298$ dig.

The distance of the fourth lens from these three lenses joined in turn will be

$$= \tfrac{14}{45}(m-1)\text{ dig.},$$

and the radius of the apparent field will be, as thus far, $\Phi = \frac{859}{m-1}$ min.
[Note that Euler expresses an angle in *radians* as a *radius* in Latin; presumably because there was no equivalent word to be used; the dimensions of course tells us whether he is discussing an angle in radians, or the radius of a circle.]

SCHOLIUM 1

181. Since in this solution we have put $\lambda = 1$ and $\lambda'' = 1$, we have consulted chiefly the skilled artificers, so that in this case errors arising from the construction may not upset the arrangement too much ; but the length of these telescopes has emerged a little greater, therefore so that a small enough radius was occurring in the determination of the objective lens ; but we offer a cure for this inconvenience, if for the first and third lens we may put in place $\lambda' = 1{,}60006$, with which done we will obtain $\lambda' = 1{,}9538 - \frac{0{,}1425}{m}$, which number only has an effect on the second lens, the construction of which we will now investigate next.
Now truly there will be :

For the first lens, Crown Glass, the radius of each face $= 1{,}06\alpha$.
For the third lens, Crown Glass, the radius of each face $= 1{,}06\gamma = 2{,}473\alpha$.
For the fourth lens, Crown Glass, the radius of each face $= -\frac{2{,}4733\alpha}{m}$.

Therefore there remains, that we may establish the second lens as before.
Evidently we will consider the two cases, the one, where $m = 10$, the other, where $m = \infty$.
Therefore in the first case there shall be $m = 10$ and there will be

$$\lambda = 1{,}93956,\ \lambda' - 1 = 0{,}93956, \text{ and } \tau'\sqrt{(\lambda'-1)} = 0{,}85056.$$

Whereby :

$$F' = \frac{-\alpha}{0{,}1414 \pm 0{,}85056}, \quad G' = \frac{-\alpha}{1{,}5827 \mp 0{,}85056}$$

or

$$F' = \frac{-\alpha}{0{,}9919} = -1{,}0081\alpha, \quad G' = \frac{-\alpha}{0{,}7322} = -1{,}3659\alpha.$$

Now there may become $m = \infty$, there will be

$$\lambda' = 1,9538, \quad \tau'\sqrt{(\lambda'-1)} = -0,8569$$

and hence

$$F' = \frac{-\alpha}{0,1414 \pm 0,8569}, \quad F' = \frac{-\alpha}{0.9983} = -1,0017\alpha,$$

$$G' = \frac{-\alpha}{1,5827 \mp 0,8569}, \quad G' = \frac{-\alpha}{0,7258} = -1,3778\alpha.$$

Now for any magnification m there may be put

$$F' = -\left(1,0017 + \frac{f}{m}\right)\alpha, \quad G' = -\left(1,3778 + \frac{g}{m}\right)\alpha,$$

and from the first case $m = 10$ there is found :

$$f = 0,065, \quad g = -0,121,$$

thus so that for the second lens

$$\text{the radius of the} \begin{cases} \text{anterior face} & = -\left(1,0017 + \frac{0,65}{m}\right)\alpha \\ \text{posterior face} & = -\left(1,3778 - \frac{0,121}{m}\right)\alpha. \end{cases}$$

Now here there can be taken with care $x = \frac{1}{4}\alpha = \frac{m}{50}$; hence there will be found $\alpha = \frac{8}{100}m.$

Hence therefore the following construction of the first kind of telescope will arise :

I. For the first lens, Crown Glass,

$$\text{the radius of each face } = 0,0848m \text{ dig.}$$

II. For the second lens, Flint Glass,

$$\text{radius of the} \begin{cases} \text{anterior face} = & (-0,08014m - 0,0052) \text{ dig.} \\ \text{posterior face} = & (-0,110224m + 0,0097) \text{ dig.} \end{cases}$$

III. For the third lens, Crown Glass,

$$\text{radius of each face } = 0,19784m \text{ dig.}$$

IV. For the fourth lens, Crown Glass,

$$\text{radius of each face } = -0,19786 \text{ dig.}$$

With the three first lenses in immediate contact with each other for an interval $=0{,}187(m-1)$ dig. the fourth lens may be attached, next to which the eye applied will discern the field of view, the radius of which will be $\frac{859}{m-1}$ minutes of arc.

SCHOLIUM 2

182. In the first place the case is worthy of all attention, since for any magnification a telescope of this kind shall be the shortest to be supplied: thus if we may wish for a magnification of one hundred times, the length will scarcely exceed $18\frac{1}{2}$ digits. Therefore this method, where we have put $\mathfrak{B}=1$, certainly deserves merit, so that also for other kinds of glass or, where for the lenses another combination of crown and flint glass may be put in place connected separately. But since that also can be called into use for solving our problem generally with equal success and by its benefit the conspicuous difficulties mentioned above may vanish, it will be expedient to treat the following more general problem.

PROBLEM 5

183. *If a telescope shall be required to be constructed from four lenses, to describe the rules for the construction, where the two first lenses must be prepared thus, so that the rays transmitted by these may become parallel to each other again.*

SOLUTION

Therefore since the rays refracted again by the second lens may become parallel to the axis, there will become $\beta=\infty$ and thus $\frac{\beta}{b}=B=\infty$ and $\mathfrak{B}=1$, the determinable distances will be

$$b=\frac{\alpha}{k},\ \ \beta=-\frac{B\alpha}{k}=\infty,\ \ c=\frac{B\alpha}{kk'},\ \ \gamma=\frac{BC\alpha}{kk'},\ \ d=-\frac{BC\alpha}{m};$$

and now this will be required to be observed, so that the distance between the second and third lens $\beta+c$ may become finite, there must on account of $\beta=\infty$ be $c=-\infty$, from which there becomes $k'=1$.

But so that we may explain the circumstances more clearly, we may put this distance $=\eta\alpha$, so that there shall be

$$B\alpha\left(\frac{1}{kk'}-\frac{1}{k}\right)=\eta\alpha,$$

from which there becomes

$$k'=\frac{B}{B+\eta k},$$

which on account of $B=\infty$ becomes $k'=1$; yet meanwhile it may be agreed that expression $k'=\frac{B}{B+\eta k}$, to be observed in the following use.

Thence, since $c=\infty$, γ truly a finite quantity, there will be $\frac{\gamma}{c}=C=0$ and hence also $\mathfrak{C}=\frac{C}{1+C}=C=0$; yet meanwhile the product BC must be finite. Therefore there shall be $BC=\theta$, so that there may become $\gamma=\frac{\theta\alpha}{k}$ and $d=\frac{-\theta\alpha}{m}$; moreover since that equation must be connected with this same one, where with the greatest rigor assumed, there is $C=\frac{\gamma}{c}=\frac{\theta}{B}$ and hence $\mathfrak{C}=\frac{\theta}{B+\theta}$. With these observed the separations of the lenses will become :

$$\alpha+b=\alpha\frac{k-1}{k},\quad \beta+c=\eta\alpha,\quad \gamma+d=\left(\frac{1}{k}-\frac{1}{m}\right)\theta\alpha=\frac{m-k}{km}\theta\alpha.$$

From which these fractions $\frac{\eta k}{k-1}$ and $\frac{m-k}{m(k-1)}\theta$ must be positive or

$\frac{\eta}{k-1}>0,\quad \frac{m-k}{k-1}\theta>0,$ or $\frac{m-k}{\eta}>0.$

Now we may inquire into the values of the letters π, π' and π'' from the three following equations requiring to be defined:

I. $\mathfrak{B}\pi-\Phi=-k\Phi,$
II. $\mathfrak{C}\pi'-\pi+\Phi=kk'\Phi,$
III. $(m-1)\Phi=-\pi+\pi'-\pi'';$
the first of which gives at once on account of $\mathfrak{B}=1$

$$\pi=(1-k)\Phi=-(k-1)\Phi,$$

from which, since this value may serve with the field of view increased, π must be negative and thus $k>1$.

But the second equation on account of $\mathfrak{C}=0$ and $k'=1$ will give $-\pi+\Phi=k\Phi,$ from which it will be permitted to conclude zero for π', whereby for $\mathfrak{C}$ and k' it will be required to be more exact for these valued to be written and will become

$$\frac{\theta}{B+\theta}\pi'-\pi+\Phi=\frac{Bk}{B+\eta k}\Phi,$$

which on account of $\pi=-(k-1)\Phi$ will be changed into this:

$$\frac{\theta}{B+\theta}\pi'+k\Phi=\frac{Bk}{B+\eta k}\Phi\quad\text{or}\quad\frac{\theta}{B+\theta}\pi'=\frac{-\eta k^2}{B+\eta k}\Phi,$$

which therefore on account of $B=\infty$ gives

$$\pi'=\frac{-\eta k^2}{\theta}\Phi\ ;$$

but since it may be agreed to take $k>1$, there must be $\alpha>0$ and thus also $\eta>0$; here the value π' will be negative, if there were $\theta>0$; but if $\theta<0$, this will be positive, but where it must be required to remember $(m-k)\theta>0$.

Finally the third equation will be changed into this form:

$$(m-1)\Phi=+(k-1)\Phi-\tfrac{\eta k^2}{\theta}\Phi-\pi''$$

and hence

$$\pi''=(k-m-\tfrac{\eta k^2}{\theta})\Phi \text{ or } \pi''=-(m-k+\tfrac{\eta k^2}{\theta})\Phi;$$

which formula since it may serve also by defining the field of view, if there may be taken $\pi''=-\frac{1}{4}$, there is found:

$$\Phi=\frac{859}{m-k+\frac{\eta k^2}{\theta}} \text{ minutes}$$
$$\Phi=\frac{859\theta}{(m-k)+\eta k^2} \text{ minutes;}$$

whereby care is required, so that $\frac{\eta k^2}{\theta}$ may be returned as a minimum, which may be performed easily by making the interval of the second and third lens as a minimum and thus vanishing, in which case there will be $\Phi=\frac{859}{m-k}$; which therefore becomes greater, where the greater k is taken.

Now therefore we will consider the equation for the removal of the colored margin, which will be

$$0=N\theta\pi''-\tfrac{N'}{k}(\theta\pi''-\pi)+\tfrac{N''}{k}(\pi''-\pi),$$

which with the values substituted gives

$$0=N\Big((m-k)\theta+\eta k^2\Big)+\tfrac{N'}{k}\Big((m-k)\theta+\eta k^2-k+1\Big)-\tfrac{N''}{k}(m-k),$$

from which equation θ can be conveniently defined and there will be found

$$\theta=\frac{-N\eta k^3+N'\eta k^2-N'(k-1)-N''(m-k)}{(m-k)(Nk-N')};$$

moreover since it is agreed to assume η as a minimum and besides it is not necessary, so that this equation may be satisfied with the greatest rigor, with these terms omitted we will have

$$\theta=\frac{-N'(k-1)-N''(m-k)}{(m-k)(Nk-N')},$$

from which there becomes

$$(m-k)\theta = \frac{-N'(k-1)-N''(m-k)}{Nk-N'};$$

which quantity since it must be positive, the numerator clearly shall be negative, also the denominator will be required to be negative and thus $N' > Nk$. So that therefore if N' may have a maximum value evidently from the crystal glass, *N* truly a minimum from the crown glass, so that there shall be $N = 7$ and $N' = 10$, the number *k* is no longer left to be chosen by us, but thus must be taken, so that there may become $7k < 10$ and $k < \frac{10}{7}$, or must be contained within the limits 1 and $\frac{10}{7}$. Here it may be observed, if there may be taken $k = 1$, the preceding case going to arise and hence neither the field of view going to increase ; but if there may be taken $k = \frac{10}{7}$, there may become $\theta = \infty$ and the length of the telescope may become infinite ; from which it will be agreed to assume *k* to be closer to one rather than to the other limit. With these properly considered we may place

$$k = \tfrac{8}{7}, \quad N = 7, \quad N' = 10, \quad N'' = 7,$$

so that θ may obtain a smaller value. From which there will become

$$\theta = \frac{49m-46}{2(7m-8)}$$

and hence $\frac{\eta k^2}{\theta}$ will have this value $\frac{2\cdot 8^2(7m-8)}{7^2(49m-46)}\eta$, which on taking $m = \infty$ becomes $= \frac{128}{343}\eta$, from which it is deduced, only if η may not exceed $\frac{1}{10}$, the diminution of the field of view not to become perceptible.

Finally for the radius of the confusion to be reduced to zero requires this equation to be satisfied

$$0 = \mu\lambda - \frac{\mu'\lambda'}{k} + \frac{\mu''\lambda''}{k\theta^3} - \frac{\mu'''\lambda'''}{m\theta^3},$$

from which we may define λ' most conveniently, which will be on account of $\mu = \mu'' = \mu'''$:

$$\lambda' = \frac{\mu}{\mu'}\left(k\lambda + \frac{\lambda''}{\theta^3} - \frac{k\lambda'''}{m\theta^3}\right);$$

ant this happily is the solution to this problem

COROLLARY 1

184. Therefore the determinable distances of the individual lenses will be :

For the first: ∞ and α with λ, for the second: $b = \frac{-\alpha}{k}$ and $\beta = \infty$ with λ',

for the third : $c = -\infty$ and $\gamma = \frac{\theta\alpha}{k}$ with λ'',

for the fourth: $d = \frac{-\theta\alpha}{k}$ and $\delta = \infty$ with λ''';

where it is required to be noted the first, third and fourth to be prepared from crown glass, the second from crystal glass ; then truly the distances between the lenses to become

$$\alpha + b = \alpha \tfrac{k-1}{k} = \tfrac{1}{8}\alpha, \quad \beta + c = \eta\alpha,$$

from which the distance may be noted that must be put in place as a minimum; and finally

$$\gamma + d = \tfrac{m-k}{km}\theta\alpha,$$

from which the whole length is produced

$$= \alpha\left(\tfrac{k-1}{k} + \eta + \tfrac{m-k}{km}\theta\right).$$

COROLLARY 2

185. But we have substituted these values for k and θ

$$k = \tfrac{8}{7}, \qquad \theta = \tfrac{49m-46}{2(7m-8)};$$

which expression since at this stage it may involve m , the calculation will not be allowed to be resolved as before for any magnification ; yet meanwhile in a similar manner, as we have made use of before, after we have resolved the calculation for two or three multiplications, we will be able to include all magnifications by interpolating the more general formulas .

SCHOLIUM 1

186. These telescopes, which we have described just now before these, thus will be required to be preferred, since with these no lenses are assumed to be joined together, certainly since in practice it cannot happen, then truly also, so that fields of view are granted an increase. Moreover these telescopes may become a little longer, just as on account of the distance between the first and second lenses, as well as mainly on account of greater value of θ, on which the separation of the third and fourth lenses may depend mainly. But here deservedly we have ignored the middle interval $\eta\alpha$. Where still considering the shortness of the instrument, we will consider it as great as may be

allowed to happen, without doubt it may be established, so that only as we have done before, both the first and third lens, ad the fourth to be formed with equal sides, thus so that there shall be $\lambda = \lambda'' = \lambda''' = 1,60006$; then truly there will be $\mu = 0,9875, \quad \mu' = 0,8724$, from which the construction of these lenses follows at once.
Evidently the radius of each face will be :

I. For the first lens $= 1,06\alpha$.

II. For the third lens $= 1,06\frac{\theta\alpha}{k}$.

III. For the fourth lens $= -1,06\frac{\theta\alpha}{m}$.

Therefore nothing other remains, except that for certain magnifications we may establish a calculation; and indeed at first it will be agreed for some small magnification $m = 5$ to be established, so that it may be apparent, to what extent this investigation into the smallest telescopes of this kind may be able to perform ; then truly we may set out a certain greater magnification such as $m = 10$ and finally by substituting $m = \infty$, so that from a comparison of these cases we may be able to form a conclusion for any greater magnification.

EXAMPLE 1

$m = 5$

187. To describe the telescope for the magnification $m = 5$.
In this case there will be

$$\theta = \tfrac{199}{54} = 3,6852, \text{ Log.}\theta = -0,5664611 \text{ and Log.}k = 0,0579920$$

and hence

$$b = -\tfrac{7}{8}\alpha, \quad \beta = \infty, \quad c = -\infty, \quad \gamma = 3,2246\alpha, \quad d = -0,7370\alpha$$

and hence

$$\alpha + b = \tfrac{1}{8}\alpha, \ \beta + c = \eta\alpha = \text{minimum}, \ \gamma + d = 2,4876\alpha$$

and thus the total length of the telescope will be $= 2,6126\alpha + \eta\alpha$.

From which three lenses prepared from crown glass will be found thus so that these will :

I. For the first lens

radius of each face $= 1,06\alpha$.

II. For the third lens

radius of each face $=3,4181\alpha$.

III. For the fourth lens

radius of each face $=-0,7812\alpha$.

IV. For the second lens, Flint Glass, before everything the number λ' must be sought from the formula

$$\lambda' = \tfrac{1,60006\mu}{\mu'}\left(k+\tfrac{1}{\theta^3}-\tfrac{k}{m\theta^3}\right),$$

from which

$$\lambda' = 2,0977, \text{ therefore } \lambda'-1 = 1,0977 \text{ and hence } \tau'\sqrt{(\lambda'-1)} = 0,91936.$$

Whereby for this lens there will be:

$$F' = \tfrac{b}{0,1414\ \pm 0,91936} = \tfrac{b}{1,0608}, \quad G' = \tfrac{b}{1,5827 \mp 0,91986} = \tfrac{b}{0,6633},$$

$$F' = -0,8248\alpha, \quad G' = -1,3192\alpha.$$

From which the following construction of the telescope follows :

I. For the first lens, Crown Glass,
radius of each face $=+1,06\alpha$. Interval $=0,125\alpha$.

II. For the second lens, Flint Glass,

$$\text{radius of the}\begin{cases}\text{anterior face} & = -\ 0,8248\alpha \\ \text{posterior face} & = -1,3192\alpha.\end{cases} \text{Minimum interval.}$$

III. For the third lens, Crown Glass,
radius of each face $=3,4181\alpha$. Intervall $=2,4876\alpha$.

IV. For the fourth lens, Crown Glass,
radius of each face $=-0,7812\alpha$.

An aperture must be attributed to the objective lens, of which the radius shall be $x=\tfrac{1}{4}\alpha$. But since on account of the clarity there must be put in place $x=\tfrac{m}{50}$ dig. $=\tfrac{1}{10}$ dig., from which $\alpha=\tfrac{2}{5}$ dig., the length of the telescope will be $=2,6126\alpha+\eta\alpha=1,0450$ dig. $+\,0,4\eta$ dig. and the radius of the field of view will correspond to $\Phi=223$ min. $=3°43'$.

EXAMPLE 2

188. If the magnification $m=10$ may be desired, to describe a telescope of this kind. On account of $m=10$ there will be

$$\theta=\tfrac{444}{124}=3,5806,\quad \text{Log.}\theta=0,5539613,\ \text{Log.}\ \tfrac{1}{\theta}=-9,4460386,$$

from which

$$b=-\tfrac{7}{8}\alpha=-0,875\alpha,\quad \beta=\infty=-c,\quad \gamma=3,1331\alpha,\ \ d=-0,35806\alpha.$$

Now the number λ' may be established, where there is found

$$\lambda'=2,1049,\ \lambda'-1=1,1049,\ \text{hence}\ \tau'\sqrt{(\lambda'-1)}=0,92238.$$

From which the radii of the faces

$$F'=\frac{b}{0,1414\pm0,9213}=\frac{b}{1,0627},\qquad G'=\frac{b}{1,5827\mp0,9213}=\frac{b}{0,6614}$$

or

$$F'=-0,8234\alpha,\quad G'=-1,3230\alpha.$$

From which the following construction of the telescope is deduced:

I. For the first lens, Crown Glass,
radius of each face $=1,06\ \alpha$. Interval $=0,125\alpha$.

II. For the second lens, Flint Glass,

$$\text{radius if the}\begin{Bmatrix}\text{anterior face}\ =-0,8234\alpha\\ \text{posterior face}=-1,3230\alpha\end{Bmatrix}.\ \text{Minimum interval.}$$

III. For the third lens, Crown Glass,
radius of each face $=3,3211\alpha$. Interval $=2,7751\alpha$.

IV. For the fourth lens, Crown Glass,
radius of each face $=-\ 0,3796\alpha$.

From which the whole length becomes $=2,9001\alpha$. But the aperture attributed to the first lens must be given, of which the radius $x=\frac{m}{50}=\frac{1}{4}\alpha=\frac{1}{5}$ dig. From which therefore if follows $\alpha=\frac{4}{5}$ dig. or greater. Moreover the radius of the field of view will correspond to $\Phi=97$ minut. $=1°37'$.

EXAMPLE 3

189. To describe a telescope of this kind, if the magnification m were infinite.
On account of $m=\infty$ there will be

$$\theta = 3,5 \text{ and Log.}\theta = 0,5440680, \quad \text{Log.}\tfrac{1}{\theta} = 9,4559319$$

and hence

$$b = -0,875\alpha, \ \beta = \infty = -c, \ \gamma = 3,0625\alpha, \ d = -3,5\tfrac{\alpha}{m}.$$

But we have found for the second lens

$$\lambda' = 2,1120, \quad \lambda' - 1 = 1,1120 \text{ and } \tau'\sqrt{(\lambda'-1)} = 0,9253.$$

From which there is deduced

$$F' = \frac{b}{0,1414 \pm 0,9253} = \frac{b}{1,0667}, \quad G' = \frac{b}{1,5827 \mp 0,9253} = \frac{b}{0,6574},$$

$$F' = -\ 0,8203\alpha, \ G' = -\ 1,3309\alpha.$$

From which the following construction of the telescope follows :

I. For the first lens, Crown Glass,

radius of each face $=1,06\alpha$. Interval $=0,125\alpha$.

II. For the second lens, Flint Glass,

$$\text{radius of the } \begin{cases} \text{anterior face} & = -0,8205\alpha \\ \text{posterior face} & = -1,3309\alpha \end{cases}. \text{ Minimum interval.}$$

III. For the third lens, Crown Glass

radius of each face $= 3,2462\alpha$. Interval $= (3,0625 - \frac{3,5}{m})\alpha$.

IV. For the fourth lens, Crown Glass,

radius of each face $=-3,710\frac{\alpha}{m}$.

And hence the length of the telescope will be $=\left(3,1875-\frac{3,5}{m}\right)\alpha$. Therefore the aperture may be attributed to the objective lens, of which the radius $=\frac{1}{4}\alpha=\frac{m}{50}$, thus so that it may be able to take $\alpha=\frac{2}{25}$ dig. $=\infty$.

EXAMPLE 4 IN GENERAL

190. If the magnification were some *m*, perhaps ten times greater, to describe a telescope of this kind.

Since for the case $m=\infty$ we have found $\theta=3,5$, now in general we may put $\theta=3,5+\frac{e}{m}$, and since for $m=10$ there was $\theta=3,5806$, there will be $e=0,806$, thus so that $\theta=3,5+\frac{0,806}{m}$; from which the distances will be had thus :

$$b=-0,875\alpha,\quad \beta=\infty=-c;\quad \gamma=\left(3,0625+\frac{0,7060}{m}\right)\alpha,\quad d=-\left(3,5+\frac{0,8060}{m}\right)\frac{\alpha}{m}.$$

But for the second lens there may be put

$$F'=-\left(0,8205+\frac{f}{m}\right)\alpha,\quad G'=-\left(1,3309+\frac{g}{m}\right)\alpha.$$

Therefore with the case established compared with the case $m=10$ there will be

$$f=0,0290,\quad g=-\,0,0790.$$

Construction of this telescope

I. For the first lens, Crown Glass,

radius of each face $=1,06\alpha$. Interval $=0,125\alpha$.

II. For the second lens, Flint Glass,

$$\text{radius of the}\begin{cases}\text{anterior face } =-\left(\ 0,8205+0,0290\right)\alpha\\ \text{posterior face} =-\left(\ 1,3809-\frac{0,0790}{m}\right)\alpha\end{cases}.\ \text{Minimum interval.}$$

III. For third lens, Crown Glass,

$$\text{radius of each face}=+\left(\ 3,2462+\frac{0,7484}{m}\right)\alpha.$$

$$\text{Interval}=\left(3,0625-\frac{2,7940}{m}-\frac{0,8060}{m^2}\right)\alpha.$$

IV. For the fourth lens, Crown Glass,

$$\text{radius of each face} = -\left(3,710+\frac{0,8544}{m}\right)\frac{\alpha}{m}.$$

And thus the total length will be

$$=\left(3,1875-\frac{2,7940}{m}-\frac{0,8060}{m^2}\right)\alpha;$$

then the radius of the aperture of the objective lens must be $x=\frac{m}{50}$ dig., from which α will have to be taken so that $\alpha=\frac{2}{25}m$ dig. or greater and the radius of the field of view corresponds to the angle

$$\Phi=\frac{859}{m-\frac{8}{7}} \text{ minutes of arc.}$$

SCHOLIUM 2

191. Behold therefore a conspicuous multitude of the various telescopes of the first kind, which at this stage may be able to multiplied indefinitely, if we may wish to grant other values to the letters B and C or also to use more lenses. Truly a search of this kind may be seen to completely superfluous, since a greater degree of perfection cannot be expected and more lenses always stands in the way of clarity nor also may a greater field of view be hoped for. But it is to be observed initially with these telescopes the colored margin may not be able to be destroyed otherwise except by different kinds of glass being used, thus so that we may affirm now, telescopes of this kind cannot be made of the same kind of glass, which do not work with the color margin error, since yet in the following kinds, with lenses made from one kind of glass, such a margin happily may be able to be removed, even if the diffusion space then may not be allowed to be reduced to zero. This restriction also was involved in the cause, whereby notably the apparent field of view was scarcely permitted to increase ; but if we may wish to ignore the colored margin, the field of view may be able to be increased to some extent. For then in the case of the final problem the letters k and θ may remain left to our choice, and since the radius of the field of view shall be $\Phi=-\frac{\pi''}{m-k}$ on putting $\eta=0$, that may be seen able to be increased as it pleases, while k may be assumed only a little less than m, and thus by taking $k=m$ it will depart to infinity; so that it may be testified beyond doubt the trial telescope not to be outstanding. Whereby it will be worth the effort to have resolved this doubt; for which it is required only to have recorded a certain order of the letters π, π' and π'' to be of terms such as $\frac{1}{4}$, which at no time must be transgressed ; whereby, even if in this case the value must be $-\pi''=\frac{1}{4}$ a huge magnitude may be presented for Φ, yet here also it is agreed to consider the value of π ; which since now before was found to be $\pi=-\Phi(k-1)$ and thus $\pi=\frac{\pi''(k-1)}{m-k}$, it is with the greatest

precaution, lest hence there may be produced $\pi > \pi''$; on account of which the letter k will not be allowed to increase as it pleases, but therefore only as far as there may become $k-1=m-k$ or $k=\frac{m+1}{2}$, which put in place the field will be produced twice as large as before, clearly $\Phi=\frac{-\pi''}{m-k}=\frac{-2\pi''}{m-1}$, which therefore we will be able to obtain, only if we may disregard the colored margin. But then for the same case of the final problem the determinable distances may become

$$b=\frac{-2\alpha}{m+1},\ \beta=\infty=-c\ ,\ \gamma=\frac{+2\theta\alpha}{m+1} \text{ and } d=\frac{-\theta\alpha}{m};$$

from which the final interval

$$\gamma+d=+\theta\alpha\left(\frac{2}{m+1}-\frac{1}{m}\right)=\frac{\theta\alpha(m-1)}{m(m+1)},$$

where at this point our arbitrary θ is allowed, provided it may be taken positive ; truly since in this way a very large colored margin may be produced, in no way will telescopes of this kind be recommended; and also in the following we will observe this precept and with no other telescope with the most simple excepted we may mention, unless those which at least shall be devoid of the colored margin, if indeed the whole confusion of the final image may not be able to be avoided.

CAPUT V

DE ULTERIORE TELESCOPIORUM PRIMI GENERIS PERFECTIONE UNA PLURIBUSVE LENTIBUS ADIICIENDIS

PROBLEMA 1

152. *Si huiusmodi telescopium primi generis ex tribus lentibus a se invicem separatis sit conficiendum, investigare momenta, quibus ei maximus perfectionis gradus concillari queat.*

SOLUTIO

Manentibus perpetua omnibus elementis, ut in principio sunt constituta, consideremus primo aequationem $m=\frac{\alpha}{b}\cdot\frac{\beta}{c}$, in qua ambae fractiones $\frac{\alpha}{b}$ et $\frac{\beta}{c}$ debent esse negativae ac praeterea intervalla $\alpha+b$ ac $\beta+c$ positiva, et quoniam nunc non debet esse $\frac{\alpha}{b}=-1$, ne binae priores lentes coalescant, statuamus $\frac{\alpha}{b}=-k$, ut fiat $m=-k\cdot\frac{\beta}{c}$ hincque $\beta=\frac{-mc}{k}$ sive $c=-\frac{\beta k}{m}$ et $\alpha=-bk$, unde ob $\beta=Bb$ omnes hae distantiae per α sequenti modo determinantur:

$$b=-\frac{\alpha}{k},\ \ \beta=-\frac{B\alpha}{k}\ \text{ et }\ c=+\frac{B\alpha}{m},$$

existante $\gamma=\infty$. Hinc igitur esse oportebit $\alpha\left(1-\frac{1}{k}\right)>0,\ \ \alpha B\left(\frac{1}{m}-\frac{1}{k}\right)>0$
seu, quia m et k sunt positiva,

$$\alpha(k-1)>0\ \text{ et }\ \alpha B(k-m)>0$$

ideoque etiam $\frac{B(k-m)}{k-1}>0$. Quocirca duo casus erunt perpendendi:

Prior casus, quo α est quantitas positiva; tum debet esse $k>1$; tum vero vel $k>m$, si B sit positivum, vel $k<m$, si $B<0$. *Altero casu,* quo α est negativum, debet esse $k<1$; tum vero vel $k>m$, si B sit negativum, vel $k<m$, si B sit positivum, ubi ob $m>1$ illa conditio $k>m$ sponte cadit.

His igitur praemissis primo ad nihilum redigamus formulam pro semidiametro confusionis supra datam (§ 42):

$$0=\mu\lambda+\frac{\mu' q}{\mathfrak{B}^2 p}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu''\lambda''}{B^3 m}$$

sive ob $q=-\frac{\alpha\mathfrak{B}}{k}$ et $p=\alpha$

$$0=\mu\lambda-\frac{\mu'}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu'}{B}\right)+\frac{\mu''\lambda''}{B^3m},$$

quae redit ad hanc formam

$$\text{I.)}\quad 0=\mu\lambda-\frac{\mu'\lambda'}{\mathfrak{B}^3k}+\frac{\mu''\lambda''}{B^3m}-\frac{\mu'\nu'}{\mathfrak{B}Bk}.$$

Deinde ut margo coloratus tollatur, ob $O=0$ haec habetur aequatio (§ 52)

$$0=\frac{dn}{n-1}\cdot B\pi'-\frac{dn'}{n'-1}\cdot\frac{1}{k}\left((B+1)\pi'-\pi\right)$$

atque ut haec confusio penitus evertatur habetur ex § 54

$$0=\frac{dn}{n-1}\cdot\frac{1}{p}+\frac{dn'}{n'-1}\cdot\frac{1}{k^2}\cdot\frac{1}{q}+\frac{dn''}{n''-1}\cdot\frac{1}{m^2}\cdot\frac{1}{r}.$$

Ad has aequationes resolvendas primo ratio inter π et π' debet definiri, id quod facillime praestabitur per formulas fundamentales in ipso initio propositas:

$$\frac{\pi}{\Phi}=\frac{\alpha+b}{q}=\frac{1-k}{\mathfrak{B}}\quad\text{et}\quad\frac{\mathfrak{C}\pi'-\pi+\Phi}{\Phi}=\frac{B\alpha}{c}=m,$$

ex quibus colligitur

$$\pi=\frac{(1-k)\Phi}{\mathfrak{B}}$$

et

$$\pi'=(m-1)\Phi+\pi=\frac{1-k+(m-1)\mathfrak{B}}{\mathfrak{B}}\Phi$$

ita ut sit

$$\pi:\pi'=1-k:1-k+(m-1)\mathfrak{B}=1:1+\frac{m-1}{1-k}\mathfrak{B};$$

deinde brevitatis gratia statuamus

$$\frac{dn}{n-1}=N,\quad\frac{dn'}{n'-1}=N',\quad\frac{dn''}{n''-1}=N'',$$

atque hinc secunda et tertia aequatio transformabuntur in sequentes:

$$\text{II.)}\ 0=NB(1-k+(m-1)\mathfrak{B})-N'\tfrac{1}{k}(B(1-k)+(B+1)(m-1)\mathfrak{B})$$

sive

$$0=(m-1)N\mathfrak{B}+(1-k)N-\frac{m-k}{k}N',$$

$$\text{III.)}\ 0=N-\frac{N'}{k\mathfrak{B}}+\frac{N''}{mB}.$$

Ex utraque harum aequationum definiri potest valor ipsius $\mathfrak{B}$. Ex secunda

$$\mathfrak{B} = \frac{m-k}{(m-1)k}\frac{N'}{N} - \frac{1-k}{m-1}.$$

Ex tertia vero sequitur

$$\mathfrak{B} = \frac{mN'-kN''}{k(mN-N'')}.$$

Videamus, an posterior valor ipsius $\mathfrak{B} = \frac{mN'-kN''}{k(mN-N'')}$ cum conditone ante inventa $\frac{B(k-m)}{k-1} > 0$ subsistere possit. Hunc in finem ob $B = \frac{\mathfrak{B}}{1-\mathfrak{B}}$ quaeramus $1-\mathfrak{B}$, fietque

$$1-\mathfrak{B} = \frac{m(kN-N')}{k(mN-N'')}$$

eritque

$$B = \frac{mN'-kN''}{m(kN-\mathrm{N}')},$$

unde conditio nostra postulat, ut sit

$$\frac{(mN'-kN'')(k-m)}{m(kN-N')(k-1)} > 0$$

quae, si esset $N = N' = N''$, abiret in hanc $\frac{-(k-m)^2}{m(k-1)^2} > 0$, quod est impossibile; eatenus igitur tantum haec eonditio locum habere poterit, quatenus litterae N, N', N'' sunt inaequales, id quod eveniet, si numerator prodeat positivus, quod fit, si uterque eius factor vel fiat positivus vel uterque negativus; priori casu $mN' - kN'' > 0$ ideoque $k < m\frac{N'}{N''}$ et $k > m$, quod fieri potest, si modo sit $\frac{N'}{N''} > 1$ sive $N' > N''$. Pro altero vero casu, quo uterque factor est negativus, erit $k < m$ et $k > \frac{N'}{N''}m$, quod fieri potest, si modo sit $\frac{N'}{N''} < 1$ seu $N' < N''$; unde patet pro utroque casu litteras N' et N'' inaequales esse debere seu lentem secundam et tertiam ex diversis vitri speciebus confici debere. In genere autem patet k non multum ab m differre posse.

Sequuntur haec, si numerator statuatur positivus; si vero numerator sit negativus, etiam denominatorem oportet esse negativum, pro quo etiam duos casus habemus. Pro priori casu, si sit $k > 1$, debet esse $kN < N'$ ideoque $k < \frac{N'}{N}$; pro posteriori, si $k < 1$, debet simul esse $k > \frac{N'}{N}$; pro quorum utroque prima et secunda lens debent esse ex diverso vitro formatae. Verum ex his quatuor casibus cum eligi convenit, qui ambos valores pro $\mathfrak{B}$ inventos proxima aequales reddat; denique autem, postquam $\mathfrak{B}$ et k convenienter definiverimus, ex prima aequatione sive λ sive λ' quaeri debet, quia λ'' iam inde datur, quod lens ocularis debeat esse utrinque aequaliter concava.

COROLLARIUM 1

153. Quatuor illi casus pro determinatione litterae k facile ad duas sequentes conditiones reducuntur; nam

vel 1. k sumi debet intra limites 1 et $\frac{N'}{N}$

vel 2. k sumi debet intra limites m et $\frac{N'}{N''}m$,

ita ut numerus iste k proxime vel unitati vel multiplicationi m aequalis accipi debeat, quoniam fractiones $\frac{N'}{N}$ et $\frac{N'}{N''}$ parumper tantum ab unitate differunt.

COROLLARIUM 2

154. Operae igitur pretium erit investigare casus, quibus k ipsi alterutri limiti aequalis statuitur.

1. Si $k=1$, foret intervallum inter primam et secundam lentem $=0$ et

$$\mathfrak{B}=\frac{mN'-N''}{mN-N''} \quad \text{et} \quad B=\frac{mN'-N''}{m(N-N')}$$

et inter secundam et tertiam lentem $=B\alpha\left(\frac{1-m}{m}\right)$, unde colligitur, utrum α positivum an negativum sumi debeat.

2. Si $k=\frac{N'}{N}$, fit intervallum inter primam et secundam lentem $=\frac{N'-N}{N}\alpha$, quod cum positivum esse debeat, patet, utrum α positive an negative sumi oporteat; tum vero erit $\mathfrak{B}=1$ et $B=\infty$, unde $\beta+c$ seu distantia inter secundam et tertiam lentem fieret $=\infty$.

3. Si $k=m$, intervallum secundae et tertiae lentis evanescet fietque

$$\mathfrak{B}=\frac{N'-N''}{mN-N''} \quad \text{et} \quad B=\frac{N'-N''}{mN-N'}$$

intervallum vero inter primam et secundam lentem $\frac{\alpha(k-1)}{k}=\frac{\alpha(m-1)}{m}$, ubi manifesta α debet esse quantitas positiva.

4. Si $k=\frac{N'}{N''}m$, fiet $\mathfrak{B}=0$ et $B=0$; unde fieret distantia inter secundam et tertiam lentem $=0$.

Cum igitur neque lentium distantias nullas neque infinitas admitti conveniat, numerus k nulli limitum prorsus aequalis sumi poterit.

COROLLARIUM 3

155. Quod porro ad campum apparentem attinet, qui pendet a formula $\pi' - \pi$, quia invenimus $\pi' - \pi = 1:1 + \frac{m-1}{1-k}\mathfrak{B}$, erit pro memoratis quatuor casibus:

1. Si $k = 1$, erit $\pi : \pi' = 1:\infty$ hinc $\pi = 0$; ita ut pro campo apparente haberetur $\Phi = \frac{\pi'}{m-1}$.

2. Si $k = \frac{N'}{N}$, fiet

$$\pi' - \pi = 1:1 + \frac{mN-N}{N-N'} = 1:1 + \frac{mN-N'}{N-N'}$$

hincque

$$\pi = \frac{N-N'}{mN-N'}\pi' \quad \text{et} \quad \pi' - \pi = \frac{N(m-1)}{mN-N'}\pi',$$

sicque pro campo apparente fiet $\pi = \frac{N}{mN-N'}\pi'$; sicque Φ maius evadet, si $\frac{N}{mN-N'} > \frac{1}{m-1}$: hoc est, si $N < N'$, quod ergo eveniet, si prima lens ex vitro coronario, secunda ex crystallino paretur.

3. Si $k = m$, erit

$$\pi : \pi' = \frac{mN-N'}{mN-N''} \quad \text{seu} \quad \pi = \frac{mN-N''}{mN-N'}\pi'.$$

Unde colligitur campum apparentem maiorem fieri quam in capite praecedente, si fuerit $\pi < 0$; quod cum hic fieri nequeat, in hoc casu campus maior non est exspectandus.

4. Si $k = \frac{N'}{N''}m$, erit $\pi : \pi' = 1:1$, unde $\pi = \pi'$ et $\pi' - \pi = 0$, quo ergo casu campus apparens plane evanesceret.

COROLLARIUM 4

156. Hinc ergo concludimus, ut maiorem campum obtineamus quam ante, necessario requiri, ut sit $N' > N$, atque *k* capi debere intra limites 1 et $\frac{N'}{N}$, qui posterior limes cum sit unitate maior, etiam *k* erit maius unitate; ex quo sequitur distantiam α capi debere positivam, quia $\alpha(k-1) > 0$.

COROLLARIUM 5

157. Cum igitur ob eam causam potissimum plures lentes adhibeamus, ut maiorem campum obtineamus, ex pluribus illis casibus, prout litterae N, N', N'' inter se variare possunt, hic unicus nobis relinquitur, quo $N' > N$ atque *k* inter limites 1 et $\frac{N'}{N}$ sumitur.

SCHOLION 1

158. In his corollariis usi sumus eo valore ipsius $\mathfrak{B}$ quem ex tertia aequatione deduximus. Supra autem iam observavimus hanc aequationem ita esse comparatam, ut de ea nunquam omnino certi esse queamus; cum enim valores litterarum N, N', N'' etc. ex nulla theoria adhuc definiri possint, sed tantum per experimenta, qualia a DOLLONDO sunt instituta, concludantur, quantacunque cura et sollertia in iis adhibeatur, nunquam tamen tantum praecisionis gradum sperare licet, ut non error satis notabills sit pertimescendus; quam ob causam etiam valor ipsius $\mathfrak{B}$ inde deductus pro vero haberi non poterit, sed contentos nos esse oportet, si modo Hunc valorem propemodum cognoverimus, id quod ipsa etiam rei natura confirmatur; quia enim aequatio nostra tertia , spatium diffusionis, per quod imagines diversicolores sunt diffusae, prorsus ad nihilum redigit, facile intelligitur ad praxin sufficere, dummodo hoc spatium reddatur satis exiguum, praecipue postquam id praestiterimus, ut margo coloratus dispareat; inprimis igitur valor litterae $\mathfrak{B}$ ex secunda aequatione determinari debet; qui si ita fuerit comparatus, ut tantum praeterpropter tertiae aequationi satisfaciat, confusio inde oriunda eo magis negligi poterit, quod etiam in telescopiis ex una vitri specie paratis non adeo nocere deprehenditur. Verum ex secunda aequatione valorem ipsius $\mathfrak{B}$ pro eo etiam casu definire licet, quo omnes lentes ex eadem vitri specie, essent confectae, ita ut foret $N = N' = N''$; tum enim concluderetur

$$\mathfrak{B} = \frac{m-k}{(m-1)k} - \frac{1-k}{m-1} = \frac{m-2k+k^2}{(m-1)k}$$

quo valore si velimus uti, ut conditio supra praescripta $\mathfrak{B}\frac{k-m}{k-1} > 0$ adimpleatur, cum inde sit $1-\mathfrak{B} = \frac{mk-m+k-k^2}{(m-1)k} = \frac{(k-1)(m-k)}{(m-1)k}$, erit

$$B = \frac{m-2k+k^2}{(k-1)(m-k)} \text{ hincque conditio } \frac{-m+2k-k^2}{(k-1)^2} > 0 \; ;$$

in qua cum denominator certe sit positivus, etiam numerator talis esse debet adeoque

$$1-m-(k-1)^2 > 0,$$

quod fieri nequit. Ex quo perspicuum est hoc casu marginem coloratum plane tolli non posse.

Videamus igitur, si diverso vitro utamur, num hoc vitium effugere queamus. Hunc in finem ponamus brevitatis gratia $\frac{N'}{N} = \xi$, ut ξ sit numerus unitatem vel tantillum superans vel ab ea deficiens, et cum sit

$$\mathfrak{B}=\frac{(m-k)\xi+k(k-1)}{(m-1)k},\quad \text{erit}\quad B=\frac{(m-k)\xi+k(k-1)}{(m-k)(k-\xi)},$$

unde conditio nostra postulat, ut sit $\frac{(k-m)\xi-k(k-1)}{(k-1)(k-\xi)}>0$. Hic duo casus sunt considerandi.

I. Si denominator sit positivus, quod fit, vel si $k>\xi$ et $k>1$, vel si $k<\xi$ et $k<1$. Tum enim esse debet $(k-m)\xi-k(k-1)>0$ sive

$$\frac{(1+\xi)^2}{4}-m\xi>\left(k-\tfrac{1}{2}(1+\xi)\right)^2$$

quod, cum *m* notabiliter superet unitatem, ξ vero ab unitate parum differat, manifesta fieri nequit.

II. Si denominator sit negativus, quod fit, si *k* continetur intra limites ξ et 1. Tum vero numerator debet etiam esse negativus seu

$$(k-m)\xi-k(k-1)<0\quad \text{sive}\quad \frac{(1+\xi)^2}{4}-m\xi<\left(k-\tfrac{1}{2}(1+\xi)\right)^2,$$

quod sponte evenit, cum pars prior manifesto sit negativa. Hic igitur casus, ut iam notavimus, solus est, qui attentionem meretur, cum hoc modo etiam tertiae aequationi saltim proxime satisfiat.

SCHOLION 2

159. Quodsi ergo nobis propositum sit marginem coloratum tollere, quae proprietas potissimum desiderari solet, primo tenendum est hoc nullo modo lentibus ex una vitri specie factis praestari posse, sed saltem primam et secundam lentem ex diverso vitro constare debere, ita ut posito $\frac{N'}{N}=\xi$ sive $N=1$, $N'=\xi$ littera ξ ab unitate differat, dum pro N'' sive unitas sive ξ pro lubitu accipi potent; deinde vidimus numerum *k* intra limites 1 et ξ sumi debere, quo facto erit

$$B=\frac{(m-k)\xi+k(k-1)}{(m-k)(k-\xi)}\quad \text{et}\quad \mathfrak{B}=\frac{(m-k)\xi+k(k-1)}{(m-1)k};$$

unde distantiae determinatrices erunt

$$b=-\frac{\alpha}{k},\quad \beta=\frac{-(m-k)\xi-k(k-1)}{k(m-k)(k-\xi)}\alpha\quad \text{et}\quad c=\frac{(m-k)\xi+k(k-1)}{k(m-k)(k-\xi)}\alpha$$

hincque lentium intervalla

$$\alpha+b=\alpha\tfrac{k-1}{k},$$
$$\beta+c=\tfrac{-(m-k)\xi-k(k-1)}{(m-k)(k-\xi)}\cdot\tfrac{m-k}{mk}\alpha=\tfrac{+(m-k)\xi+k(k-1)}{mk(\xi-k)}\alpha,$$

hincque tota telescopii longitudo erit

$$=\tfrac{m-1}{m}\cdot\tfrac{\xi-k+1}{\xi-k}\alpha$$

Porro maxime interest in campum apparentem inquirere, quod fit determinando valorem $\pi=\frac{\pi'}{1+\frac{m-1}{1-k}\mathfrak{B}}$, qui abit in sequentem $\pi=\frac{k(1-k)}{(m-k)\xi}\pi'$ unde dipiscimur

$$\Phi=\tfrac{\pi'-\pi}{m-1}+\tfrac{\pi'}{m-1}\left(1+\tfrac{k(k-1)}{(m-k)\xi}\right).$$

Cum igitur maxime intersit campum, quantum fieri potest, augeri, hinc obtinemus istam conclusionem numerum *k* unitate maiorem esse debere, unde, cum *k* contineatur intra limites 1 et ξ, haec porro regula observetur, litteram ξ unitate maiorem esse debere; unde sequitur lentem secundam ex vitro crystallino, primam vero ex communi esse parandam; quo pacto alter casus, quo fieret $\xi<1$, penitus e praxi excluditur. Quare, cum sit $k>1$, distantia α, quae adhuc incerta est relicta, debet esse positiva.

Nunc demum considererons aequationem tertiam, qua confusio colorum penitus tollitur, et videamus, quanta ea nunc sit proditura. Illa autem tertia aequatio nunc fit $0=1-\frac{\xi}{k\mathfrak{B}}+\frac{N'}{mB}$, quae nunc induet hanc formam:

$$0=\tfrac{-m(k-1)(\xi-k)-(m-k)(\xi-k)N''}{m((m-k)\xi+k(k-1))},$$

quae quantitas utique non erit aequalis nihilo, sed, cum *k,* ξ et N'' parum ab unitate differant, semper erit valde parva; id quod clarius inde perspicitur, quod numerator habeat factorem minimum $\xi-k$, denominator autem semper sit satis magnus eoque maior, quo maior fuerit multiplicatio. Ex quo manifestum est hanc confusionem nunquam fore perceptibilem. Praeterea autem, cum haec confusio plane evanesceret, si caperetur $\xi=k$, consultum quidam videtur numerum *k* limiti ξ propiorem capere quam unitati, quandoquidem ipsi limiti ξ aequari nequit, quia intervallum inter secundam et tertiam lentem fieret infinitum uti et longitudo telescopii; quare, ne ea nimis magna prodeat, contrarium potius suadendum est, ut littera *k* a limite ξ, quantum fieri potest, removeatur et unitati propius capiatur. Consequenter unicus casus, qui evolvi meretur, in hoc consistet, ut numero *k* valor unitati proximus assignetur et excessus tam sit exiguus, quam crassities lentium admittere solet. Si enim *k* ipsi unitati aequaretur, haberemus casum praecedentis capitis, quo intervallum lentium plane nullum est positum, quod incommodum hic evitare constituimus.

PROBLEMA 2

160. *Si prima lens ex vitro coronario, secunda vero ex crystallino paretur et inter eas intervallum tam exiguum statuatur, quam crassities lentium admittit, regulas determinare, quas in constructione huius telescopii observare oportet.*

SOLUTIO

Hic ergo ex DOLLONDI experimentis statui debebit $\xi = \frac{10}{7}$, et quia k limiti 1 propius accipi convenit quam alteri limiti $\frac{10}{7}$, sumamus $k = \frac{8}{7}$, et quae in praecedentibus scholiis sunt tradita, sequentes nobis suppeditant determinationes:

I. Pro distantiis determinatricibus

$$b = -\frac{7}{8}\alpha, \quad \beta = \frac{35(7m-8)+28}{8(7m-8)}\alpha \quad \text{vel} \quad \beta = \frac{7(35m-36)}{8(7m-8)}\alpha, \quad c = \frac{-35m+36}{m(7m-8)}\alpha.$$

II. Pro intervallis lentium

$$\alpha + b = \frac{1}{8}\alpha, \quad \beta + c = \frac{35m-36}{8m}\alpha$$

et longitudo telescopii $= \frac{9(m-1)}{2m}\alpha.$

Hic observandum est, cum α sit distantia focalis primae lentis eiusque semidiameter aperturae esse debeat $x = my = \frac{m}{50}$ dig., istam distantiam α minorem esse non posse quam $5x$ seu $\frac{m}{10}$ dig., ita ut sit $\alpha > \frac{m}{10}$ dig.; quare, si capiatur verbi gratia $m = 50$, longitudo telescopii prodiret maior quam $\frac{9\cdot 49}{20}$ dig., maior quam 22 dig., et si fieri debeat $m = 100$, ea maior esse deberet quam $\frac{9\cdot 99}{20}$ dig., maior quam 44 dig.; quae distantia cum facile tolerari queat, manifestum est haec telescopia etiam ad maiores multiplicationes adhiberi posse; pro minoribus autem multiplicationibus eximium certe usum praestant, cum, si statuatur $m = 5$, longitudo prodeat $> \frac{36}{20}$ dig., maior quam $\frac{9}{5}$ dig., sumtoque $m = \frac{5}{2}$ ea prodeat $> \frac{27}{40}$ dig.

Pro campo autem apparente habebimus eius semidiametrum

$$\Phi = \frac{\pi'}{m-1}\left(1 + \frac{4}{5(7m-8)}\right)$$

ideoque aliquantum maior quam casu praecedente, praesertim si multiplicatio fuerit exigua. Notentur etiam distantiae focales harum lentium *p, q, r,* et cum sit

$$\mathfrak{B} = \tfrac{35m-36}{28(m-1)} \quad \text{et} \quad B = \tfrac{35m-36}{-7m+8},$$

erit

$$p = \alpha, \quad q = \frac{-(35m-36)\alpha}{32(m-1)}, \quad r = \frac{-(35m-36)\alpha}{m(7m-8)},$$

et semidiameter aperturae secundae lentis ex § 23 [pro $\alpha = 5x = \frac{m}{10}$]

$$\tfrac{m(35m-36)\pi'}{400(m-1)(7m-8)} + \tfrac{7m}{400}.$$

Denique pro constructione harum lentium numeri λ, λ', et λ'' ita accipi debent, ut satisfiat primae nostrae aequationi, quae erat

$$0 = \mu\lambda - \tfrac{\mu'\lambda'}{\mathfrak{B}^3 k} + \tfrac{\mu''\lambda''}{B^3 m} - \tfrac{\mu'\nu'}{\mathfrak{B}Bk},$$

ubi notandum est, ut lens ocularis utrinque fiat aeque concava, statui debere $\lambda'' = 1,60006$, si haec lens sit ex vitro coronario ideoque $\mu'' = \mu$; sin autem sit ex vitro crystallino ideoque $\mu'' = \mu'$, fore $\lambda'' = 1,67445$ [§ 137]; haec autem aequatio non nisi casibus particularibus pro data multiplicatione evolvi poterit; ubi meminisse iuvabit fore

$$\mu = 0,9875, \quad \mu' = 0,8724, \quad \nu' = 0,2529, \quad \mu'\nu' = 0,2206.$$

EXEMPLUM 1

161. Si multiplicatio sit $m = \frac{5}{2}$, telescopium huius generis ex tribus lentibus constans describere.

Cum sit $m = \frac{5}{2}$, erunt distantiae determinatrices

$$b = -\tfrac{7}{8}\alpha, \quad \beta = \tfrac{721}{152}\alpha, \quad c = -\tfrac{206}{95}\alpha, \quad B = -\tfrac{103}{19}$$

et intervalla lentium

$$\alpha + b = \tfrac{1}{8}\alpha, \qquad \beta + c = \tfrac{107}{40}\alpha$$

et longitudo telescopii $= \frac{27}{10}\alpha$ atque pro campo apparente fiet $\Phi = \frac{2\pi'}{3}\left(1 + \frac{8}{95}\right)$; sumto $\pi' = \frac{1}{4}$ et multiplicando per 3437 minut. erit angulus $\Phi = 10^\circ\ 21'\ 4'..$

Cum nunc sit $B = \frac{-103}{19}$, erit $\mathfrak{B} = \frac{103}{84}$ et habebimus

$$\text{Log.}(-B) = 0,7340836(-) \text{ et Log.}\mathfrak{B} = 0,0885580;$$

aequatio autem pro confusione prima tollenda, si lentem ocularem ex vitro coronario faciamus, ut sit $\mu'' = \mu$ et $\lambda'' = 1,60006$, erit

$$\begin{aligned} 0 &= 0,9815\lambda - 0,4140\lambda' - 0,00396 \\ &\qquad\qquad\qquad\qquad\quad +0,02903 \\ 0 &= 0,98751 - 0,4140\lambda' \quad +0,02507, \end{aligned}$$

unde quaeratur λ', et habebitur $\lambda' = 2,3852\lambda + 0,06055$. Si ergo hic capiatur $\lambda = 1$, fiet $\lambda' = 2,4457$, unde fit $\lambda' - 1 = 1,4457$ et $\log.\sqrt{(\lambda' - 1)} = 0,0800391$;
unde constructio singularum lentium sequenti modo se habebit, siquidem radii facierum primae lentis sint *F* et *G*, secundae *F'* et *G'* et tertiae *F''* et *G''*:

I. Pro prima lente ex vitro coronario

$$F = \tfrac{\alpha}{\sigma} = 0,6024\alpha, \quad G = \tfrac{\alpha}{\rho} = 4,4111\alpha.$$

II. Pro secunda lente ex vitro crystallino

$$\tfrac{1}{F'} = \tfrac{\rho\beta + \sigma b \mp \tau(b+\beta)\sqrt{\lambda' - 1}}{b\beta} \qquad \tfrac{1}{G'} = \tfrac{\sigma\beta + \rho b \pm \tau(b+\beta)\sqrt{\lambda' - 1}}{b\beta};$$

cum nunc sit

$$\log.\left(-\tfrac{b}{\alpha}\right) = \log.\tfrac{7}{8} = 9,9420080(-) \text{ et } \log.\tfrac{\beta}{\alpha} = 0,6760917$$
$$\text{et } \log.\left(\tfrac{(b+\beta)}{\alpha}\right) = \log.\tfrac{147}{38} = 0,5875336,$$
$$\log.\sigma = 0,1993986, \ \log.\rho = 9,1501422, \ \log.\tau = 9,9432471.$$

Unde invenitur

$$\tfrac{1}{F'} = \tfrac{-0,7174 \mp 4,0815}{b\beta}\alpha, \qquad \tfrac{1}{G'} = \tfrac{+0,7174 \pm 4,0815}{b\beta}\alpha;$$

ut maiores numeri evitentur, sumantur signa inferiora, fietque

$$F' = \tfrac{b\beta}{3,3668\alpha} = -1,2327\alpha, \ G' = \tfrac{b\beta}{3,3021\alpha} = -1,2568\alpha.$$

Pro lente oculari ex vitro coronario paranda, cum ea utrinque sit aeque concava eiusque distantia focalis sit $c = -\frac{206}{95}\alpha$, radius concavitatis pro utraque facie erit

$= 2(n-1)c = -\frac{106\cdot 206}{95}\alpha = -2,2978\alpha.$

Prima lens admittit aperturam, cuius semidiameter $x = 0,1506\alpha$. Nunc vero claritas postulat, ut sit $x = \frac{m}{50}\text{dig.} = \frac{1}{20}\text{dig.}$, unde α maius quam $\frac{1}{3}$ dig. Sumatur ergo $\alpha = \frac{1}{2}$ dig. et constructio telescopii ita se habebit:

I. Pro lente prima radius faciei $\begin{cases} \text{anterioris} & = +0,3012 \text{ dig.} \\ \text{posterioris} & = +2,2065 \text{ dig.} \end{cases}$ Crown Glass.

II. Pro lente secunda radius faciei $\begin{cases} \text{anterioris} & = -0,6163 \text{ dig.} \\ \text{posterioris} & = -0,6284 \text{ dig.} \end{cases}$ Flint Glass.

III. Pro lente tertia radius utriusque faciei $= -1,1492$ dig., quae paratur ex Crown Glass.
Tum intervallum statuatur inter

$$\text{I et II} = \tfrac{1}{16}\text{dig.} = 0,0625 \text{ dig.}$$

$$\text{II et III} = \tfrac{103}{80}\text{dig.} = 1,2875 \text{ dig.},$$

ita ut tota telescopii longitudo sit futura

$$= 1,3500 \text{ dig.} = 1\tfrac{1}{3}\text{dig.},$$

spatii vero visi semidiameter erit $= 10°21'4''$.

EXEMPLUM 2

162. Si multiplicatio sit $m = 5$, telescopium huius generis ex tribus lentibus constans describere.

Cum sit $m = 5$, erit $7m - 8 = 27$ et $35m - 36 = 139$, unde distantiae determinatrices fient

$$b = -\tfrac{7}{8}\alpha = -08750\alpha, \quad \beta = \tfrac{973}{216}\alpha = +4,5046\alpha, \quad c = \tfrac{-139}{5\cdot 27}\alpha = -10296\alpha.$$

Ex quibus fiunt intervalla

$$\alpha + b = \tfrac{1}{8}a, \quad \beta + c = 3,4750\alpha,$$

unde telescopii longitudo $= 3,6\alpha$.

Pro campo autem apparente fiet $\Phi = \frac{\pi'}{4}\left(1 + \frac{4}{5\cdot 27}\right)$ sumtoque $\pi' = \frac{1}{4}$ et multiplicando per 3437 minut. erit $\Phi = 3°\ 41'$.

Cum iam sit $\mathfrak{B} = \frac{139}{112}$ et $B = \frac{-139}{27}$ sumtisque logarithmis

$$\text{Log.}\mathfrak{B} = 0,0937968, \ \text{Log.}(-B) = 0,7116510(-),$$

aequatio pro confusione prima tollenda, si lentem ocularem ex vitro coronario paremus, ut sit $\mu'' = \mu$ et $\lambda'' = 1,60006$, erit

$$0=0,9875\lambda-0,39933\lambda' \begin{array}{r} -0,002316 \\ +\ 0,030211 \\ \hline +0,027895, \end{array}$$

ex qua iterum quaeratur

$$\lambda' = \ 2,4729\lambda+0,06985.$$

Hic non ut ante sumamus $\lambda=1$, sed, ut prima lens maximae aperturae fiat capax ideoque distantia α minor accipi possit, capiatur $\lambda=1,60006$, ut haec lens fiat utrinque aequaliter convexa, habebiturque

$$\lambda'=4,0266 \quad \text{et} \quad \lambda'-1 \ = \ 3,0266 \quad \text{et} \quad \text{Log.}\sqrt{(\lambda'-1)}=0,2404775$$

atque hinc obtinebimus:

I. Pro prima lente ex vitro coronario radius utrinsque faciei

$$=2(n-1)\alpha \ = \ 1,06\alpha;$$

quae aperturam admittit, cuius semidiameter $x=0,26\alpha$.

II. Pro secunda lente ex vitro crystallino ob $\text{Log.}\frac{b+\beta}{\alpha}=0,5598588$ calculus ita se habebit:

$$\frac{1}{F'}=\frac{-0,7479\pm5,5409}{b\beta}\alpha \qquad \frac{1}{G'}=\frac{+7,005\ 7\mp5,5409}{b\beta}\alpha;$$

valeant hic signa superiora eritque

$$F'=\frac{b\beta}{4,7930\alpha}=-0,8224\alpha, \quad G'=\frac{b\beta}{1,4548\alpha}=-2,6908\alpha.$$

III. Pro tertia lente ex vitro coronario erit radius utriusque faciei

$$=2(n-1)c=1,06c=-1,0914\alpha.$$

Cum nunc ob claritatem esse debeat $x=\frac{m}{50}$ dig. $=\frac{1}{10}$ dig., fiet α maius quam $\frac{2}{5}$ dig. Sumi igitur poterit $\alpha=\frac{1}{2}$ dig. et constructio telescopii ita se habebit:

I. Pro lente prima, Crown Glass, radius faciei utriusque $=0,5300$ dig.

II. Pro lente secunda, Flint Glass, radius factei $\begin{cases}\text{anterioris} &=-0,4111 \text{ dig.}\\ \text{posterioris} &=-1,3454 \text{ dig.}\end{cases}$

III. Pro lente tertia, Crown Glass, radius utriusque faciei $=-0,5460$ dig.

Tum vero statuatur intervallum

$$\text{I et II} = \tfrac{1}{16}\ \text{dig.},\quad \text{II et III} = 1,7375\ \text{dig.},$$

ita ut tota longitudo sit $= 1,8$ dig. ideoque nondum duos adaequet digitos.
Spatii tandem visi semidiameter erit $= 3°\ 41'\ 11''$.

COROLLARIUM

163. Telescopia igitur in his duobus exemplis constructa aptissima videntur ad usum vulgarem, quoniam ea facile quis secum gerera potest iisque in spectaculis praesertim uti. Sequentia autem exempla ad maiores multiplicationes accommodemus.

EXEMPLUM 3

164. Sit multiplicatio $m = 25$, telescopium huius generis tribus lentibus constans describere.
Cum sit $m = 25$, erit $7m - 8 = 167$ et $35m - 36 = 839$ eruntque distantiae determinatrices

$$b = -\tfrac{7}{8}\alpha = -0,875\alpha,\ \beta = \tfrac{7\cdot 839}{8\cdot 167}\alpha = 4,3960\alpha,$$
$$\text{Log.}\tfrac{-b}{\alpha} = 9,9420081(-),\ \text{Log.}\tfrac{\beta}{\alpha} = 0,6430535,$$
$$c = -\tfrac{839}{25\cdot 167} = -\ 0,2009\alpha,$$
$$b + \beta = 3,5210\alpha,\quad \text{Log.}(b + \beta) = 0,5466660.$$

Hinc prodeunt lentium intervalla

$$a + b = \tfrac{1}{8}\alpha,\ \ \beta + c = 4,1951\alpha$$

et tota longitudo $= 4,3201\alpha$.

Pro campo autem apparente fiet $\Phi = \tfrac{\pi'}{24}\left(1 + \tfrac{4}{5\cdot 167}\right)$ hincque angulus $i\Phi = 36$ minut. prim. circiter.

Cum iam porro sit $\mathfrak{B} = \tfrac{839}{28\cdot 24}$ et

$$\text{Log.}\mathfrak{B} = 0,0963927,\quad \text{Log.}(-B) =\ 0,7010454(-),$$

pervenietur ad sequentem aequationem

$$0 = 0,9875\lambda - 0,3922\lambda' - 0,0004984 + 0,03077$$

seu

$$0 = 0,9875\lambda - 0,3922\lambda' + 0,03028.$$

Quoniam vidimus valorem $\lambda = 1,60006$ longitudinem telescopii haud mediocriter diminuisse, statim ponamus $\lambda = 1,60006$ eritque

$$0 = 1,6103 - 0,3922\lambda',$$

unde prodit

$$\lambda' = 4,1057 \text{ et } \lambda' - 1 = 3,1057 \text{ et } \log.\sqrt{(\lambda' - 1)} = 0,2460797,$$

unde constructio singularum lentium ita se habebit:

I. Pro prima lente ex vitro coronario radius utriusque faciei $= 1,06\alpha$, quae ergo aperturam admittit, cuius semidiameter $x = 0,265\alpha$.

II. Pro secunda lente calculus ita se habebit:

$$\frac{1}{F'} = \frac{-0,7688 \pm 5,4449}{b\beta}\alpha, \quad \frac{1}{G'} = \frac{6,8338 \mp 5,4449}{b\beta}\alpha.$$

Valeant signa superiora eritque

$$F' = \frac{b\beta}{4,6816\alpha} = -0,8216\alpha, \quad G' = \frac{b\beta}{1,3889\alpha} = -2,7694\alpha.$$

III. Pro tertia lente ex vitro coronario radius utriusque faciei
$$= 1,06c = -0,21295\alpha.$$

Claritas autem postulat $x = \frac{m}{50}$ dig. $= \frac{1}{2}$ dig., unde concluditur $\alpha > 1,89$; sumatur ergo $\alpha = 2$ et constructio haec erit:

I. Pro lente prima radius utriusque faciei $= 2,12$ dig., Crown Glass.

II. Pro lente secunda radius faciei $\begin{cases} \text{anterioris} & = -1,6432 \text{ dig.} \\ \text{posterioris} & = -5,5388 \text{ dig} \end{cases}$ Flint Glass

III. Pro lente tertia radius utriusque faciei $= -0,42590$ dig., Crown Glass.
Tum statuatur intervallum lentium
$$\text{I et II} = \tfrac{1}{4} \text{ dig., II et III} = 8,3902 \text{ dig.}$$
et tota longitudo $= 8,64$ dig. campique apparentis semidiameter $x = 36'$ circiter.

EXEMPLUM 4

165. Si m debeat esse $= 50$, erit $7m - 8 = 342$, $35m - 36 = 1714$ adeoque distantiae

$$b=-\tfrac{7}{8}\alpha=-\ 0,875\alpha,\ \ \beta=4,3852\alpha,\ \ c=-0,10023\alpha.$$

$$\log.\ \tfrac{\beta}{\alpha}=0,6419928,\qquad \log.\ (\tfrac{-b}{\alpha})=9,9420081(-),$$
$$\log.\tfrac{(b+\beta i)}{\alpha}=0,5453319,\quad \log.\ (-\tfrac{b\beta}{\alpha^2})=0,5840009(-).$$

Tum vero intervalla lentium erunt

$$a+b=\tfrac{1}{8}\alpha=0,125\alpha,\ \ \beta+c=4,2850\alpha$$

hincque tota longitudo $=4,4100\alpha.$

Porro reperitur

$$\mathrm{Log.}(-B)=0,6999847(-),\quad \mathrm{Log.}\mathfrak{B}=0,0966567.$$

Pro campo apparente reperitur

$\Phi=\frac{\pi'}{49}\left(1+\frac{4}{5\cdot 842}\right)$ seu angulus $\Phi=17\frac{1}{2}$ minut.

Pro confusione tollenda statuatur statim in aequatione inventa $\lambda=1,60006$ eritque

$$0=1,5801-0,3915\lambda'-0,00025+0,03083$$

sive

$$0,39151\lambda'=1,6107,\ \text{unde}\ \lambda'=4,1141\,;$$

hinc

$$\lambda'-1=3,1141\ \text{et}\ \log.\sqrt{(\lambda'-1)}=0,2466663,$$

unde constructio singularum lentium ita se habebit:

I. Pro lente prima radius utriusque faciei $=1,06\alpha$*;* quae ergo aperturam admittit, cuius semidiameter $=0,265\alpha.$

II. Pro lente secunda

$$\tfrac{1}{F'}=\tfrac{-0,7653\pm\ 5,4355}{b\beta}\alpha,\qquad \tfrac{1}{G'}=\tfrac{+6,8169\mp 5,4355}{b\beta}\alpha.$$

Valeant ergo signa superiora eritque

$$F'=\tfrac{b\beta}{4,6702\alpha}=-0,8216\alpha,\quad G'=\tfrac{b\beta}{1,3814\alpha}=-2,7776\alpha.$$

III. Pro lente tertia erit radius utriusque faciei

$$=2(n-1)c=1,06c=-0,10624\alpha.$$

Claritas autem postulat $x=\frac{m}{50}=1$ dig., unde sequitur $\alpha>3,8$; sumto ergo $\alpha=4$ habebitur sequens telescopii constructio:

I. Pro lente prima, Crown Glass, radius utriusque faciei $=4,24$ dig .

II. Pro lente secunda radius faciei $\begin{cases}\text{anterioris} & =-\ 3,2864\\ \text{posterioris} & =-11,1104\end{cases}$ Flint Glass.

III. Pro lenta tertia, Crown Glass., radius utriusque faciei $=-0,42496$.

Tum vero statui debet intervallum lentium

I et II $=0,5$ dig., II et III $=17,1400$ dig.

adeoque telescopii longitudo $=17,6400$ dig.

Campi denique visi semidiameter inventus est $17,6400$ minut.

SCHOLION

166. Cum in his solutionibus littera λ indeterminata sit relicta, in tribus posterioribus exemplis eius loco non unitatem posuimus, ut ante fecimus, sed potius ei tribuimus illum valorem, quo ambae lentis facies inter se aequales redderentur, hocque modo insigne commodum sumus nacti, ut lens prima fere duplo maiorem aperturam admitteret hincque distantia α fere ad dimidium reduci posset. Ut autem in genere quaepiam lens, cuius distantiae determinatrices sunt a et α, ambas suas facies obtineat aequales, supra vidimus [Lib. I § 56] capi debere

$$\sqrt{(\lambda-1)}=\frac{(\sigma-\rho)(a-\alpha)}{2\tau(a+\alpha)}=\frac{2(nn-1)}{n\sqrt{(4n-1)}}\cdot\frac{1-A}{1+A}$$

ob $\alpha=Aa$, unde fit

$$\lambda=1+\frac{4(nn-1)^2}{n^2(4n-1)}\cdot\frac{(1-A)^2}{(1+A)^2};$$

quare, si vel a vel α fuerit infinitum, uti fit tam in lente obiectiva quam in lente oculari, habebitur $\lambda=1+\frac{4(nn-1)^2}{n^2(4n-1)}$ Sin autem velimus, ut alla quaepiam lens obtineat ambas suas facies inter se aequales, tum ob

$$\frac{(1-A)^2}{(1+A)^2}=1-\frac{4A}{(1+A)^2}$$

capere debemus

$$\lambda=1+\frac{4(nn-1)^2}{n^2(4n-1)}-\frac{16(nn-1)^2A}{n^2(4n-1)(1+A)^2}.$$

Cum autem in nosta expressione pro semidiametro confusionis tum occurrat talis forma $\lambda(A+1)^2+\nu A$, valor istius formulae fiet

$$(A+1)^2+\frac{4(nn-1)^2(A+1)^2}{n^2(4n-1)}-\frac{16(nn-1)^2A}{nn(4n-1)}+\frac{4(n-1)^2A}{4n-1}.$$

Commodius autem erit hoc casu valorem ipsius λ pro facilltate calculi ita exprimere

$$\lambda=1+\frac{(\sigma-\rho)^2(1-A)^2}{4\tau^2(1+A)^2}.$$

EXEMPLUM 5

167. Si multiplicatio m debeat esse valde magna vel saltim maior quam 25, huius generis telescopia ex tribus lentibus constantia describere.

Hic statim observo sumta prima lente utrinque aequaliter convexa fore radium utriusque curvaturae ut ante $=1,06\alpha$, quae admittet aperturam, cuius semidiameter $=\frac{1}{4}\alpha=x$; cum autem ob claritatem sumi debeat $x=\frac{m}{50}$ dig., hinc intelligimus semper statui posse $\alpha=\frac{2m}{25}=0,08m$ dig. et pro campo apparent $\Phi=\frac{\pi'}{m-1}$; sumtoque $\pi'=\frac{1}{4}$ erit $\Phi=\frac{859}{m-1}$.

Nunc autem, ante quam reliquas partes constructionis definiamus, contemplemur casum, quo $m=\infty$, eritque

$$b=-\tfrac{7}{8}\alpha,\quad \beta=\tfrac{35}{8}\alpha,\quad c=\tfrac{-5}{m}\alpha=-\tfrac{2}{5}\text{ dig.}$$

Distantiae porro lentium

$$\alpha+b=\tfrac{1}{8}\alpha\ \text{ et }\ \beta+c=\left(\tfrac{35\alpha}{8}-\tfrac{2}{5}\right)\text{ dig. et }\mathfrak{B}=\tfrac{5}{4},\ B=-5.$$

Pro sequente calculo statim sumamus $\lambda=1,60006$ et aequatio prodibit

$$0=1,5801-0,39081\lambda'+0,03088,$$

unde invenitur

$$\lambda'=4,1220\ \text{ et }\lambda'-1=3,1220\ \text{ et }\log.\sqrt{(\lambda'-1)}=0,2472164.$$

Hinc ob

$$\text{Log.}\left(-\tfrac{b}{\alpha}\right)=9,9420081(-),\quad \text{Log.}\tfrac{\beta}{\alpha}=0,6409781$$
$$\text{Log.}\tfrac{b+\beta}{\alpha}=0,5440680,\quad \text{Log.}\left(-\tfrac{b\beta}{\alpha^2}\right)=0,5829862(-).$$

Ex quibus pro secunda lente habebimus

$$\frac{1}{F'}=\frac{-0,7662 \pm 5,4266}{b\beta}\alpha \qquad \frac{1}{G'}=\frac{+6,8006 \mp 5,4266}{b\beta}\alpha$$

seu sumtis signis superioribus

$$F'=\frac{b\beta}{4,6604\alpha}=-\ 0,8214\alpha, \qquad G'=\frac{b\beta}{1,3740\alpha}=-2,7861\alpha,$$

qui valores pro multiplicatione infinita locum habent; at nunc pro multiplicatione quacunque m statuatur

$$F'=-\left(0,8214+\frac{f}{m}\right)\alpha, \quad G'=-\left(2,7861+\frac{g}{m}\right)\alpha,$$

ubi valores litterarum f et g ex casu praecedente $m=50$ vel etiam, sed minus tuto, ex casu $m=25$ erui debent, hocque modo reperitur

$$f=0,01 \text{ et } g=-0,4250;$$

ita ut sit in genere

$$F'=-(0,8214+\tfrac{0,01}{m})\alpha, \quad G'=-(2,7861-\tfrac{0,4250}{m})\alpha.$$

Deinde cum supra iam inventa sit distantia focalis lentis tertiae $=-\frac{2}{5}$ dig. pro $m=\infty$ statuamus pro quavis multiplicatione m esse

$$c=-\frac{2}{5}-\frac{h}{m} \text{ eritque } c=-\left(\frac{2}{5}+\frac{1,2480}{m}\right) \text{ dig.};$$

cuius ergo radius utriusque faciei erit

$$-\left(0,4240+\frac{0,948}{m}\right) \text{ dig.}$$

Cum igitur sit $\alpha=0,08m$ dig., constructio telescopii sequenti modo se habebit:

I. Pro lente prima, Crown Glass, radius faciei utriusque $=0,0848m$ dig.

II. Pro lente secunda, Flint Glass,

$$\text{radius faciei} \begin{cases} \text{anterioris} & =-(0,0657m+0,0008) \text{ dig.} \\ \text{posterioris} & =-(0,2228m-0,0340) \text{ dig.} \end{cases}$$

III. Pro lente tertia, Crown Glass, radius utriusque faciei

$$=-\left(0,4240+\tfrac{0,948}{m}\right)\text{ dig.}$$

Tum vero intervalla erunt

$$\alpha+b=0,01m,\quad \beta+c=\left(0,35m-0,36\right)\text{ dig.},$$

hincque tota longitudo

$$=\left(0,36m-0,36\right)\text{ dig.}$$

campique visi semidiameter $=\frac{859}{m-1}$ minut. prim.

COROLLARIUM

168. Si ergo telescopium. desideretur, quod centies multiplicet, id ita se habebit:

I. Pro lente prima, Crown Glass, radius utriusque faciei $=8,48$ dig.
II. Pro lente secunda, Flint Glass,

$$\text{radius faciei}\begin{cases}\text{anterioris} & =-\ 6,57\text{ dig.}\\ \text{posterioris} & =-22,24\text{ dig.}\end{cases}$$

III. Pro lente tertia, Crown Glass, radius utriusque faciei $=-0,43$ dig.

Intervallum erit lentis

$$\text{I et II}=1\text{dig., II et III}=34,64\text{ dig.}$$

hincque longitudo telescopii $=35,64$ dig. campique visi semidiameter $=8\frac{1}{2}$ min.

PROBLEMA 3

169. *Si huiusmodi telescopium primi generis ex quatuor lentibus α se invicem separatis sit construendum, investigare momenta, quibus ei maximus perfectionis gradus concillatur.*

SOLUTIO

Hic igitur istarum trium fractionum $\frac{\alpha}{b}$, $\frac{\beta}{c}$ et $\frac{\gamma}{d}$ singulae debent esse negativae ponamus ergo $\frac{\alpha}{b}=-k$ et $\frac{\beta}{c}=-k'$ et, cum sit $m=-\frac{\alpha}{b}\cdot\frac{\beta}{c}\cdot\frac{\gamma}{d}$, habebimus
$b=-\frac{\alpha}{k}$, $\beta=-\frac{B\alpha}{k}$, $c=-\frac{\beta}{k'}=+\frac{B\alpha}{kk'}$ et $\gamma=+\frac{BC\alpha}{kk'}$ et $m=\frac{-kk'\gamma}{d}$,

hinc

$$d = -\frac{BC\alpha}{m};$$

unde intervalla lentium

$$\alpha + b = \alpha\left(1 - \frac{1}{k}\right), \quad \beta + c = B\alpha\left(\frac{1}{kk'} - \frac{1}{k}\right) \quad \text{et} \quad \gamma + d = BC\alpha\left(\frac{1}{kk'} - \frac{1}{m}\right);$$

quae cum debeant esse positiva aeque ac numeri k, k' et m, bina posteriora per primum divisa dabunt has duas conditiones

$$1. \ \frac{B(1-k')}{k'(k-1)} > 0, \qquad 2. \ \frac{BC(m-kk')}{mk'(k-1)} > 0.$$

Iam consideremus aequationem, qua margo coloratus tollitur, pro casu, quo distantia O est negativa [§ 52], ponendo ut ante

$$\frac{dn}{n-1} = N, \quad \frac{dn'}{n'-1} = N', \quad \frac{dn''}{n''-1} = N'', \quad \frac{dn'''}{n'''-1} = N'''$$

eritque

$$0 = NBC\pi''\alpha + N'b\left((B+1)C\pi'' - \pi\right) + N''c\frac{(C+1)\pi'' - \pi'}{B}$$

seu

$$0 = NBC\pi'' - \frac{N'}{k}\left((B+1)C\pi'' - \pi\right) + \frac{N''}{kk'}\left((C+1)C\pi'' - \pi'\right);$$

quem in finem investigare oportet relationes inter litteras π, π', π''; est vero ex capite I, § 11:

$$\text{I.} \ \frac{\mathfrak{B}\pi - \Phi}{\Phi} = -k, \text{unde} \ \pi = \frac{1-k}{\mathfrak{B}} \cdot \Phi.$$

$$\text{II.} \ \frac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = \frac{B\alpha}{c} = kk', \text{unde} \ \pi' = \left(\frac{1}{B} - \frac{k}{\mathfrak{B}} + kk'\right)\frac{\Phi}{\mathfrak{C}}.$$

$$\text{III.} \ \frac{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}{\Phi} = \frac{BC\alpha}{d} = -m,$$

unde ob $\mathfrak{D} = 1$ fiet

$$\pi'' = \left(-m + \frac{1}{BC} - \frac{k}{\mathfrak{B}C} + \frac{kk'}{\mathfrak{C}}\right)\Phi;$$

unde aequatio nostra erit

$$0 = N\left(-BCm + 1 - \frac{Bk}{\mathfrak{B}} + \frac{BCkk'}{\mathfrak{C}}\right) - \frac{N'}{k}\left(\frac{-BCm}{\mathfrak{B}} - \frac{Bk}{\mathfrak{B}} + \frac{BCkk'}{\mathfrak{B}\mathfrak{C}}\right) + \frac{CN''}{\mathfrak{C}kk'}\left(-m + kk'\right);$$

unde fit

$$C = N - N(B+1)k + NBkk' + N'(B+1) - N'(B+1)k' - \frac{N''m}{kk'} + N''$$

divisum per

$$NBm - NBkk' - \tfrac{N'(B+1)m}{k} + N'(B+1)k' + \tfrac{N''m}{kk'} - N''$$

vel succinctius $C = Nkk'\left(1-k-Bk\left(1-k'\right)\right) + N'kk'\left(B+1\right)\left(1-k'\right) - N''\left(m-kk'\right)$
divisum per

$$\left(m-kk'\right)(Nkk'B - N'k'(B+1) + N'')$$

adeoque

$$1+C = Nkk'\left(1-k+B\left(m-k\right)\right) - N'(B+1)k'(m-k)$$

divisum per

$$\left(m-kk'\right)(Nkk'B - N'k'(B+1) \ + N'')$$

atque hinc

$$\mathfrak{C} = Nkk'\left(1-k-Bk\left(1-k'\right)\right) + N'kk'\left(B+1\right)(1-k') - N''(\ m-kk')$$

divisum per

$$Nkk'\left(1-k+B\left(m-k\right)\right) - N'(B+1)k'(m-k)\,;$$

sed facile patet hoc modo nobis vix ulterius progredi licere ob harum formularum complicationem, nisi pro k et k' et pro N, N', N'' valores substituantur; interim tamen haec methodus etiam successura videtur, si eam ad plures adhuc lentes applicare vellemus; ceterum haud abs re erit hoc negotium etiam alio modo tentasse.

Ex praecedentibus scilicet aequationibus non litteras π, π', π'' quaeri, sed potius his quasi datis spectatis litteras $\mathfrak{B}$ et $\mathfrak{C}$ definiri conveniet; unde statim obtinemus

$$\mathfrak{B} = \tfrac{1-k}{\pi}\cdot\Phi, \quad \mathfrak{C} = \tfrac{(kk'-1)\Phi+\pi}{\pi'};$$

ex tertia denique aequatione ob $\mathfrak{D} = 1$ colligitur

$$\Phi = \tfrac{-\pi+\pi'-\pi''}{m-1},$$

ita ut et Φ quasi datum spectari queat. Hinc, cum sit

$$B = \tfrac{\mathfrak{B}}{1-\mathfrak{B}} \ \text{ et } \ C = \tfrac{\mathfrak{C}}{1-\mathfrak{C}},$$

habebimus

$$B = \frac{(1-k)\Phi}{\pi-(1-k)\Phi}, \qquad C = \frac{(kk'-1)\Phi+\pi}{\pi'-\pi-(kk'-1)\Phi}.$$

Nunc cum prior conditio postulet, ut sit $\frac{B(1-k')}{k'(k-1)} > 0$, altera vero per hanc divisa $\frac{C(m-kk')}{1-k'} > 0$, valoribus illis substitutis hae duae conditiones abibunt in sequentes:

$$1.\quad \frac{(k'-1)\Phi}{(\pi-(1-k)\Phi)k'}>0 \quad \text{or} \quad \frac{(k'-1)\Phi}{\pi-(1-k)\Phi}>0,$$

$$2.\quad \frac{((kk'-1)\Phi+\pi)(m-kk')}{(1-k')(\pi'-\pi-(kk'-1)\Phi)}>0,$$

quae per illam multiplicata dat

$$\frac{-\Phi(m-kk')(\pi+(kk'-1)\Phi)}{(\pi-(1-k)\Phi)(\pi'-\pi-(kk'-1)\Phi)}>0;$$

si hic loco Φ eius valor substituatur, qui, cum semper sit positivus, ob $m-1$ etiam positivum dat primo

$$-\pi+\pi'-\pi''>0$$

tum vero binae istae conditiones dabunt

$$1.\quad \frac{k'-1}{(m-k)\pi+(k-1)\pi'-(k-1)\pi''}>0,$$

$$2.\quad \frac{(m-kk')((m-kk')\pi+(kk'-1)\pi'-(kk'-1)\pi'')}{(k'-1)((m-kk')\pi-(m-kk')\pi'-(kk'-1)\pi'')}>0;$$

tum vero B et C ita definientur:

$$B=\frac{(k-1)(\pi-\pi'+\pi'')}{(m-k)\pi+(k-1)\pi'-(k-1)\pi''}$$

$$C=\frac{(m-kk')\pi+(kk'-1)\pi'-(kk'-1)\pi''}{-(m-kk')\pi+(m-kk')\pi'+(kk'-1)\pi''}.$$

Verum si hos valores substituere vellemus sive in aequatione pro margine colorato vitando sive inprimis pro semidiametro confusionis ad nihilum redigenda, in multo maiores ambages incideremus, quam priore methodo evenit; quocirca aliam adhuc methodum quaerere debemus; nulla autem alia nobis relinquitur, nisi ut ex superioribus aequationibus litteras k et k' investigemus; quo pacto nostra investigatio satis plana reddetur.

Hanc viam sequentes statim habemus $k=\frac{\Phi-\mathfrak{B}\pi}{\Phi}$ et $kk'=\frac{\Phi-\pi+\mathfrak{C}\pi'}{\Phi}$ existente $\Phi=\frac{-\pi+\pi'-\pi''}{m-1}$, unde, cum k et k' sint numeri positivi pariter atque angulus Φ, habemus statim istas conditiones:

$$\Phi-\mathfrak{B}\pi>0, \qquad \Phi-\pi+\mathfrak{C}\pi'>0, \qquad -\pi+\pi'-\pi''>0.$$

Porro cum hinc intervalla lentium fiant

$$1.\quad \alpha+b=\frac{-\mathfrak{B}\pi}{\Phi-\mathfrak{B}\pi}\alpha>0,$$

$$2.\quad \beta+c=\frac{(\mathfrak{B}\pi-B\mathfrak{C}\pi')\alpha\Phi}{(\Phi-\pi+\mathfrak{C}\pi')(\Phi-\mathfrak{B}\pi)}>0,$$

$$3.\quad \gamma+d=\frac{(m-1)\Phi+\pi-\mathfrak{C}\pi'}{m(\Phi-\pi+\mathfrak{C}\pi')}BC\alpha>0,$$

inde colligimus has novas conditiones:

$$-\mathfrak{B}\pi a>0,$$
$$(\mathfrak{B}\pi-B\mathfrak{C}\pi')\alpha>0,$$
$$((m-1)\Phi+\pi-\mathfrak{C}\pi')BC\alpha>0;$$

ideoque etiam harum quoti positivi esse debent

$$\frac{\mathfrak{B}\pi-B\mathfrak{C}\pi'}{-\mathfrak{B}\pi}>0,\qquad \frac{((m-1)\Phi+\pi-\mathfrak{C}\pi')BC}{\mathfrak{B}\pi-B\mathfrak{C}\pi'}>0;$$

quae posterior ob $(m-1)\Phi=-\pi+\pi'-\pi''$ abit in hanc

$$\frac{(\mathfrak{C}\pi'-C\pi'')B}{\mathfrak{B}\pi-B\mathfrak{C}\pi'}>0,$$

sicque quinque habentur conditiones ab α liberae, quibus satisfieri oportet. Nunc autem aequatio pro destruendo margine colorato ita se habebit:

$$0=NBC\pi''-\frac{N'\Phi}{\Phi-\mathfrak{B}\pi}((B+1)C\pi''-\pi))+\frac{N''\Phi}{\Phi-\pi+\mathfrak{C}\pi'}((C+1)\pi''-\pi').$$

His quomodocunque observatis perpendatur aequatio ultima pro confusione penitus destruenda [§ 53]

$$0=N-\frac{N'}{k\mathfrak{B}}+\frac{N''}{kk'B\mathfrak{C}}-\frac{N'''}{mBC}$$

ob

$$p=\alpha,\quad q=\mathfrak{B}b=-\frac{\mathfrak{B}\alpha}{k},\ \ r=\mathfrak{C}c=\frac{\mathfrak{B}\mathfrak{C}\alpha}{kk'}\ \text{ et }\ s=d=-\frac{BC\alpha}{m},$$

num ei vel absolute vel saltim proxima satisfieri queat.

Denique, ut etiam confusio prior tollatur, satisfiat huic aequationi [§ .42]

$$\mu\lambda-\frac{\mu'\lambda'}{k\mathfrak{B}^3}+\frac{\mu''\lambda''}{kk'\mathfrak{B}^3\mathfrak{C}^3}-\frac{\mu'''\lambda'''}{B^3C^3m}-\frac{\mu'v'}{k\mathfrak{B}B}+\frac{\mu''v''}{kk'B^3C\mathfrak{C}}=0.$$

SCHOLlON

170. Cum hic in genere vix ulterius progredi liceat, ad casus particulares erit descendendum, et quia in capite praecedente lentes perfectae triplicatae ideo optatum usum non praestiterant, quod confusio a lente oculari oriunda ab iis non destruebatur, hic lentem obiectivam iterum triplicatam statuamus, ut bina priora intervalla evanescant, eius vero tres llentes ita definiamus, ut iis etiam confusio a lente oculari oriunda destruatur; quo facto deinceps forte via patebit inter tres lentes priores exigua intervalla statuendi. Semper enim in huiusmodi disquisitionibus arduis expedit a casibus facilloribus exordiri, quoniam inde ratio perspicitur difficultates superandi, quac primo intuitu invincibiles erant visae.

PROBLEMA 4

171. *Si tres lentes priores inter se immediate iungantur, ut lentem obiectivam triplicatam constituant, quarta vero lens sit ocularis, regulas pro constructione huius modi telescopii exponere.*

SOLUTIO

Cum hic sint intervalla tam $\alpha + b = 0$ quam $\beta + c = 0$, fiet statim $k = 1$ et $k' = 1$, unde sequuntur distantiae

$$b = -\alpha,\ \beta l = -B\alpha,\ c = B\alpha,\ \gamma = BC\alpha \ \text{ et } \ d = -\tfrac{BC\alpha}{m}$$

hincque intervallum

$$\gamma + d = BC\alpha\left(1 - \tfrac{1}{m}\right) = \tfrac{m-1}{m} BC\alpha,$$

quod debet esse positivum. Pro litteris autem $\pi,\ \pi',\ \pi''$ habebimus

$$1.\ \pi = 0,\ \ 2.\ \pi' = 0,\ \ 3.\ \pi'' = -(m-1)\Phi.$$

Atque hinc aequatio pro tollendo margine colorato erit

$$0 = NBC - N'(B+1)C + N''(C+1),$$

unde elicimus

$$C = -\frac{N''}{NB - N'(B+1) + N''},$$

unde intervallum $\gamma + d$ fit

$$=-\frac{(m-1)}{m}\cdot\frac{N''B\alpha}{NB-N'(B+1)+N''};$$

quod cum esse debeat positivum, duo casus sunt perpendendi.

Alter, quo $\alpha>0$; tum esse debet

$$\frac{N''B}{NB-N'(B+1)+N''}<0 \text{ ideoque } N-\frac{N''}{B}(B+1)+\frac{N''}{B}<0$$

sive

$$N-N'+\frac{1}{B}(N''-N')<0.$$

Altero casu, si $\alpha<0$, contrarium evenire debet, scilicet

$$N-N'+\frac{1}{B}(N''-N')>0.$$

Consideretur nunc aequatio, qua ista confusio penitus tollitur, scilicet

$$0=N-\frac{N'}{\mathfrak{B}}+\frac{N''}{B\mathfrak{C}}-\frac{N'''}{mBC};$$

ex qua per BC multiplicata, ut sit

$$0=NBC-N'(B+1)C+N''(C+1)-\frac{N'''}{m},$$

quoniam a praecedente aequatione non differt nisi ultimo termino $\frac{N'''}{m}$, qui prae reliquis est valde parvus, concludimus, si illi fuerit satisfactum, simul quoque huic proxime satisfieri idque eo magis, quo maior fuerit multiplicatio *m;* quae conclusio nititur fundamento, quod numeri N, N', N'', N''' parum ab unitate differunt.

Pro priore autem confusione tollenda insuper satisfieri debet huic aequationi

$$0=\mu\lambda-\frac{\mu'\lambda'}{\mathfrak{B}^3}+\frac{\mu''\lambda''}{\mathfrak{B}^3\mathfrak{C}^3}-\frac{\mu'''\lambda'''}{mB^3C^3}-\frac{\mu'\nu'}{\mathfrak{B}B}+\frac{\mu''\nu''}{B^3C\mathfrak{C}},$$

in qua loco C eius valor supra inventus

$$C=\frac{-N''}{B(N-N')-N'+N''}$$

substitui debet; id quod in genere ad formulam valde molestam deduceret, quare solutio non nisi casibus particularibus absolvi poterit.

COROLLARIUM 1

172. Etsi conditiones pro littera B sunt datae, haec tamen littera prorsus indeterminata relinquitur, dummodo notetur,

1. si fuerit $N-N'+\frac{1}{B}\left(N''-N'\right)<0$,

tum capi debere α positivum;

2. sin autem fuerit $N-N'+\frac{1}{B}\left(N''-N'\right)>0$,

tum capi debere α negativum.
Cum invenerimus

$$C=\frac{-N''}{B(N-N')-N'+N''},\ \text{erit}\ 1+C=\frac{B(N-N')-N'}{B(N-N')-N'+N''},\ \text{hincque}\ \mathfrak{C}=\frac{-N''}{B(N-N')-N'}.$$

COROLLARIUM 2

173. Si velimus loco B introducere $\mathfrak{B}$ ponendo $B=\frac{\mathfrak{B}}{1-\mathfrak{B}}$, tunc consequemur

$$\mathrm{C}=\frac{-N''(1-\mathfrak{B})}{(N-N'')\mathfrak{B}-N'+N''}\quad \text{et}\quad \mathfrak{C}=\frac{-N''(1-\mathfrak{B})}{N\mathfrak{B}-N'},$$

quibus observatis substitutio postrema facillus expedietur; fiet enim postrema aequatio

$$0=\begin{Bmatrix} \mu\lambda\mathfrak{B}^2-\mu'\lambda'-\mu'\nu'\mathfrak{B}\left(1-\mathfrak{B}\right)-\frac{\mu''\lambda''(N\mathfrak{B}-N')^3}{(N'')^3} \\ +\frac{\mu''\nu''(1-\mathfrak{B})\left((N\mathfrak{B}-N')(N-N'')\mathfrak{B}-N'+N''\right)}{(N'')^2} \\ +\frac{\mu'''\lambda'''\left((N-N'')\mathfrak{B}-N'+N''\right)^3}{m(N'')^3} \end{Bmatrix}.$$

COROLLARIUM 3

174. Respectu campi apparentis, cum sit $\pi''=-\left(m-1\right)\Phi$, si statuamus ut hactenus $\pi''=-\frac{1}{4}$, prodibit semidiameter $\Phi=\frac{1}{4(m-1)}$ et in min. primis $\Phi=\frac{851}{4(m-1)}$ min. prim., siquidem lens ocularis fiat utrinque aequaliter concava, quod, uti ostendimus, fiet, si $\lambda'''=1,60006$, hac scilicet lente ex vitro coronario parata. Sin autem eam ex vitro crystallino parare velimus, poni debet $\lambda'''=1,67445$.

EXEMPLUM

175. Si prima et tertia lens fuerit ex vitro coronario, media ex crystallino, ex hisque lens obiectiva constituatur, lens vero ocularis ex vitro coronario paretur, pro quavis data multiplicatione telescopium construere.

Hoc exemplum ideo affero, quod hic casus in capite praecedente est praetermissus, quem autem hic alio modo tractabo, ut longitudo minor prodeat.

Cum igitur hic sit $n = 1,53$, $n' = 1,58$, $n'' = 1,53$, $n''' = 1,53$, erit, uti vidimus, $N = 7$, $N' = 10$, $N'' = 7$, $N''' = 7$, ita ut sit $\mu'' = \mu$, $\nu'' = \nu$, $\mu''' = \mu$.

Ex his colligitur

$$C = +\tfrac{7}{3}(1-\mathfrak{B}), \quad \mathfrak{C} = \frac{-7(1-\mathfrak{B})}{7\mathfrak{B}-10}.$$

Tantum ergo restat haec aequatio resolvenda

$$0 = \mu\lambda\mathfrak{B}^3 - \mu'\lambda' - \mu'\nu'\mathfrak{B}(1-\mathfrak{B}) - \frac{\mu\lambda''(7\mathfrak{B}-10)^3}{7^3} - \frac{3\mu\nu(1-\mathfrak{B})(7\mathfrak{B}-10)}{7^2} - \frac{27\mu\lambda'''}{7^3 m};$$

quodsi ergo statuamus $\lambda'' = \lambda$ et $\lambda''' = 1,60006$, haec aequatio induet hanc formam

$$\begin{aligned}&\mu\lambda\left(\tfrac{30}{7}\mathfrak{B}^2 - \tfrac{300}{49}\mathfrak{B} + \tfrac{1000}{343}\right) - \mu'\nu' + \mu'\nu'\left(\mathfrak{B}^2 - \mathfrak{B}\right)\\ &+\mu\nu\left(\tfrac{3}{7}\mathfrak{B}^2 - \tfrac{51}{49}\mathfrak{B} + \tfrac{30}{49}\right) - \frac{27\mu\lambda'''}{343m} = 0,\end{aligned}$$

quae tantum est aequatio quadratica, ex qua valor ipsius $\mathfrak{B}$ erui debet; terminis igitur secundum potestates ipsius $\mathfrak{B}$ dispositis habebitur:

$$\begin{aligned}&\mathfrak{B}^2\left(\mu\lambda + \mu'\nu' + \tfrac{3}{7}\mu\nu\right) + \mathfrak{B}\left(-\tfrac{300}{49}\mu\lambda - \mu'\nu' - \tfrac{51}{49}\mu\nu\right)\\ &\tfrac{1000}{343}\mu\lambda - \mu'\lambda' + \tfrac{30}{49}\mu\nu - \frac{27\mu\lambda'''}{343m} = 0.\end{aligned}$$

Resolutionem autem huius aequationis ita instituamus, ut lens obiectiva maiorem aperturam admittat; quem in finem non ut ante $\lambda = 1$, sed $\lambda = 1,60006$ statuamus, ut prima lens utrinque sibi similis evadat; quare sit

$$\begin{aligned}&\log.\mu = 9,9945371 &&\log.\mu' = 9,9407157\\ &\log.\mu\nu = 9,3360593 &&\log.\mu'\nu' = 9,3436055,\\ &\mu\nu = 0,2168 &&\mu'\nu' = 0,2206\end{aligned}$$
$$\text{et Log. } \lambda = \text{Log. } \lambda''' = 0,2041363;$$

at pro secunda lente ponatur non ut ante $\lambda' = 1$, sed hanc litteram indeterminatam relinquamus; unde nostra aequatio in numeris ita erit comparata:

$$0 = 7,0852\mathfrak{B}^2 - 10,1201\mathfrak{B} + 4,7393 - \mu'\lambda' - \frac{0,12437}{m},$$

quae reducitur ad hanc

$$\mathfrak{B}^2 = \frac{10,1201}{7,0852}\mathfrak{B} - \frac{4,7393}{7,0852} + \frac{\mu'\lambda'}{7,0852} + \frac{0,12437}{m\cdot 7,0852}$$
$$\mathfrak{B}^2 = 1,4283\mathfrak{B} - 0,6689 + 0,12311\lambda' + \frac{0,01755}{m}.$$

Unde invenitur

$$\mathfrak{B} = 0,7142 \pm \sqrt{\left(-0,1589 + 0,1231\lambda' + \tfrac{0,01755}{m}\right)}.$$

Unde patet λ' capi debere unitate maius. Statuatur ergo $\lambda' = 1\frac{1}{2}$ eritque

$$\mathfrak{B} = 0,7142 \pm \sqrt{\left(0,257 + \tfrac{0,01755}{m}\right)};$$

hinc autem ulterius progredi non licet nisi litterae m valores determinatos tribuendo; quem in finem sequentes casus adiungimus.

CASUS 1
$m = 10$

176. Erit hoc casu $\mathfrak{B} = 0,7142 \pm 0,1657$ sumtoque signo inferiore $\mathfrak{B} = 0,5485$ vel sumto superiore signo $\mathfrak{B} = 0,8799$. Sin autem velimus, ut pro $\mathfrak{B}$ unicus valor $= 0,7142$ prodeat, capi deberet

$$\lambda' = \tfrac{0,1572}{0,1231} = 1\tfrac{341}{1231},$$

hocque casu hic utamur.
Cum igitur si sit $\lambda' = 1\frac{341}{1231}$, erit

$$\lambda' - 1 = \tfrac{341}{1231}, \quad \text{Log.}\sqrt{\lambda' - 1} = 9,7208976, \quad \text{Log.}(\lambda' - 1) = 9,4417952.$$

Cum nunc pro omni multiplicatione sit $\mathfrak{B} = 0,7142$, si quidem capiamus

$$\lambda' = 1,2908 - \tfrac{0,1425}{m}, \quad \lambda' - 1 = 0,2908 - \tfrac{0,1425}{m},$$

hinc erit $1 - \mathfrak{B} = 0,2858, \quad B = 2,4990, \quad C = 0,6669 = \frac{2}{3}.$
Unde obtinemus distantias

$$b = -\alpha, \ \beta = -24990\alpha = -2\tfrac{1}{2},$$
$$c = +\ 2,4990\alpha, \quad \gamma = +1,6667\alpha, \quad d = -0,16667\alpha.$$

Cum nunc sit $\lambda = \lambda'' = \lambda''' = 1,60006$ et $\lambda' = 1,2766$, erit:

I. Pro prima lente utrinque aequaliter convexa radius utriusque faciei $= 1,06\alpha$.
II. Pro secunda lente ex vitro crystallino

$$\frac{1}{F'} = \frac{\rho\beta+\sigma b\mp\tau'(b+\beta)\sqrt{\lambda'-1}}{b\beta}, \qquad \frac{1}{G'} = \frac{\sigma\beta+\rho b\pm\tau'(b+\beta)\sqrt{\lambda'-1}}{b\beta},$$

$$\frac{1}{F'} = \frac{-1{,}9360\mp 1{,}6152}{b\beta}\alpha, \qquad \frac{1}{G'} = \frac{-4{,}0980 \pm 1{,}6152}{b\beta}\alpha,$$

sumtisque signis superioribus erit

$$F' = \frac{-b\beta}{3{,}5512} = -0{,}7039\alpha, \quad G' = \frac{-b\beta}{2{,}4828\alpha} = -1{,}0070\alpha.$$

III. Pro tertia lente ex vitro coronario, cum hic sit $\tau\sqrt{\lambda''-1} = \frac{\sigma-\rho}{2}$, tum vero $c = 2\frac{1}{2}\alpha$ et $\gamma = 1\frac{2}{3}\alpha$ erit

$$c+\gamma = 4\tfrac{1}{6}\alpha,$$

$$\frac{1}{F''} = \frac{4{,}5280\pm 2{,}9870}{c\gamma}\alpha, \qquad \frac{1}{G''} = \frac{3{,}3935\mp 2{,}9870}{c\gamma}\alpha$$

et ex signis inferioribus

$$F'' = \frac{c\gamma}{1{,}5410\alpha} = 2{,}7039\alpha, \; G'' = \frac{c\gamma}{6{,}3205\alpha} = 0{,}6593\alpha.$$

Hae ergo tres lentes sibi iunctae aperturam admittent, cuius semidiameter aestimari potest

$$x = 0{,}1648\alpha = \tfrac{1}{7}\alpha \quad \text{circiter.}$$

Cum autem ob claritatem esse debeat $x = \frac{m}{50}$ dig. $= \frac{1}{5}$ dig., capi debebit circiter $\alpha = \frac{7}{5}$ dig., unde telescopii longitudo

$$= 1{,}5001\alpha = 1\tfrac{1}{2}\alpha = 2{,}1 \text{ dig.}$$

IV. Pro quarta lente aequaliter utrinque concava erit radius utriusque faciei

$$= 1{,}06d = 1{,}06(-0{,}1667)\alpha = -0{,}1767\alpha = -0{,}2474\ dig.$$

COROLLARIUM 1

177. Si ergo hoc modo valor ipsius λ' definiatur, praecedentes determinationes pro omnibus multiplicationibus valebunt excepta sola lente secunda; tum autem pro quarta lente semidiameter utriusque eius faciei capi debet $= -(106)\cdot\frac{5}{3}\cdot\frac{\alpha}{m}$ sive $= -1{,}7666\cdot\frac{\alpha}{m}$.

COROLLARIUM 2

178. Constructio autem secundae lentis a multiplicatione pendebit, quia valor litterae λ' multiplicationem involvit, cum sit $\lambda' = 1,2908 - \frac{0,1425}{m}$.

SCHOLlON

179. Haud difficile autem erit pro quavis multiplicatione secundam lentem definire, postquam ea iam pro casu $m = 10$ est inventa; statuatur enim $m = \infty$, erit $\lambda' = 1,2908$ hinc $\lambda' - 1 = 0,2908$ et Log.$\sqrt{(\lambda' - 1)} = 9,7317972$, unde membrum ambiguum erit $= 1,6562\alpha$; unde pro lente secunda erit

$$\frac{1}{F'} = \frac{-1,9860 \mp 1,6562F}{b\beta}\alpha, \quad \frac{1}{G'} = \frac{-4,0980 \pm 1,6562}{b\beta}\alpha;$$

sumtis ergo signis superioribus

$$F' = \frac{-b\beta}{3,5922\alpha} = -0,6959\alpha, \; G' = \frac{-b\beta}{2,4418\alpha} = -1,0239\alpha.$$

Nunc igitur ponamus pro multiplicatione quacunque m esse

$$F' = -(0,6959 + \tfrac{f}{m})\alpha, \quad G' = -\left(1,0239 + \tfrac{g}{m}\right)\alpha,$$

et quia posito $m = 10$

$$0,6959 + \tfrac{f}{10} = 0,7039 \quad \text{et} \quad 1,0239 + \tfrac{g}{10} = 1,0079,$$

reperitur $f = 0,0800, \;\; g = -0,1690$; quibus inventis adipiscimur sequentem telescopii constructionem:

I. Pro prima lente, Crown Glass, radius utriusque faciei $= +1,06\alpha$.

II. Pro secunda lente, Flint Glass,

$$\text{radius faciei} \begin{cases} \text{anterioris} = -\left(0,6959 + \frac{0,0800}{m}\right)\alpha \\ \text{posterioris} = -\left(1,0239 - \frac{0,1690}{m}\right)\alpha. \end{cases}$$

III. Pro tertia lente, Crown Glass, radius faciei $\begin{cases} \text{anterioris} = +2,7039\alpha \\ \text{posterioris} = +0,6593\alpha. \end{cases}$

IV. Pro quarta lente, Crown Glass, radius utriusque faciei $= -1,7666\frac{\alpha}{m}$; quibus lentibus paratis ternae priores sibi invicem iungantur, post quas quarta collocetur intervallo $= \frac{m-1}{m} \cdot \frac{5}{3}\alpha$.

Cum porro sit $x = \frac{m}{50}$ dig. et invenerimus $x = 0,1648\alpha$, hinc colligitur fore $\alpha = \frac{m}{8,2400}$, ita ut statui possit $\alpha = \frac{4}{33}m$ seu $\alpha = \frac{12}{100}m.$ Quare habetur constructio telescopii primi generis:

I. Pro prima lente, Crown Glass, radius faciei utriusque $= 0,1272m$.

II. Pro secunda lente, Flint Glass,

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -0,0835m - 0,0096\\ \text{posterioris} &= -0,1229m + 0,0202.\end{cases}$$

III. Pro tertia lente, Crown Glass,

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 0,3244m\\ \text{posterioris} &= 0,0791m;\end{cases}$$

quibus tribus lentibus immediate iunctis postea intervallo $= \frac{1}{5}(m-1)$ dig. statuatur lens ocularis.

IV. Pro quarta lente, Crown Glass, radius utriusque faciei $= -0,2119.$

CASUS 2

180. Cum pro omnibus multiplicationibus constructio telescopii sit tradita, solutionem exempli supra allati alio modo expediamus. Scilicet cum hic pro $\mathfrak{B}$ valorem unitate minorem simus consecuti, qui supra unitate maior prodierat, notatu dignus videtur casus $\mathfrak{B} = 1$, quem hic evolvamus. Tum autem erit $B = \infty$ et quia distantiae determinatrices sunt $\alpha,\ b = -\alpha,\ \beta = -B\alpha,\ c = B\alpha,\ \gamma = BCa,\ d = \frac{-BCa}{m}$, necesse est, ut sit BC quantitas finita ideoque $C = 0$ et $\mathfrak{C} = C = 0$. Quare statuamus $BC = \theta$, ut fiat $\gamma = \theta\alpha$ et $d = \frac{-\theta\alpha}{m}$ hincque telescopii longitudo $= \frac{m-1}{m}\theta\alpha$. His positis aequatio pro margine colorato tollendo dabit ob $N = 7,\ N' = 10,\ N'' = N''' = 7$ et $\pi = 0,\ \pi' = 0,\ \pi'' = -(m-1)\Phi$:

$$0 = N\theta - N'\theta + N'',\quad 0 = 7\theta - 10\theta + 7 \text{ hincque } \theta = \tfrac{7}{3}.$$

Nunc autem aequatio pro confusione primae speciei tollenda fiet

$$0 = \mu\lambda - \mu'\lambda' + \frac{27\mu\lambda''}{343} - \frac{27\mu\lambda'''}{343m},$$

quae per μ divisa ob $\frac{\mu'}{\mu} = 0,8834$ abit in hanc

$$0 = \lambda - 0,8884\lambda' + 0,0787\lambda'' - \frac{0,07871\lambda'''}{m};$$

facta autem lente oculari utrinque aequali erit $\lambda''' = 1,60006$ et

$$0=\lambda-0,8884\lambda'+0,0787\lambda''-\tfrac{0,1259}{m},$$

ex qua litteras λ ita definiri convenit, ut unitatem minimum superent; statuamus ergo $\lambda=1$ et $\lambda''=1$, erit $0,8884\lambda'=1,0787-\tfrac{0,1259}{m}$, unde colligitur

$$\lambda'=1,2210-\tfrac{0,1425}{m};$$

sola ergo lens secunda a multiplicatione *m* pendet, quam deinceps seorsim evolvamus.
Calculum ergo pro prima, tertia et quarta instituamus.
Pro prima autem est

$$F=\tfrac{\alpha}{\sigma}=0,6023\alpha,\quad G=\tfrac{\alpha}{\rho}=4,4111\alpha.$$

Pro tertia lente ob $\lambda''=1$ et $c=-\infty$

$$F''=\tfrac{\alpha}{\sigma}=1,4055\alpha,\quad G''=\tfrac{\gamma}{\rho}=10,2925\alpha.$$

Pro quarta lente radius utrisque faciei $=1,06d=-\tfrac{2,4733\alpha}{m}$.

Et tota telescopii longitudo $=\tfrac{m-1}{m}\cdot\tfrac{7}{3}\alpha$.

Pro secunda autem lente, cum in genere sit

$$F'=\frac{b\beta}{\rho'\beta+\sigma'b\mp\tau'(b+\beta)\sqrt{\lambda'-1}},\quad G'=\frac{b\beta}{\sigma'\beta+\rho'b\pm\tau'(b+\beta)\sqrt{\lambda'-1}},$$

ob $\beta=\infty$ et $b=-\alpha$ erit

$$F'=\frac{-\alpha}{\rho'\pm\tau'\sqrt{\lambda'-1}},\quad G'=\frac{-\alpha}{\sigma'\mp\tau'\sqrt{\lambda'-1}};$$

pro quo duo casus sunt evolvendi, alter, quo $m=10$, et alter, quo $m=\infty$.
Priore erit ob $m=10$ $\lambda'=1,2068,$ $\lambda'-1=0,2068$ et

$$\begin{aligned}\text{Log.}\sqrt{(\lambda'-1)}&=9,6577753\\ \text{Log. }\tau'&=\underline{9,9432471}\\ &9,6010224\end{aligned}$$

cui logarithmo respondet 0,3990, ideoque

$$F'=\frac{-\alpha}{0,1414\pm0,3990},\qquad F'=\frac{-\alpha}{0,5404}=-1,8505\alpha,$$
$$G'=\frac{-\alpha}{1,5827\mp0,3990},\qquad G'=\frac{-\alpha}{1,1837}=-0,8448\alpha.$$

Altero casu ob $m=\infty$ erit $\lambda'=1,2210$ et $\lambda'-1=0,2210$ et

$$\begin{aligned}\text{Log.}\sqrt{(\lambda'-1)} &= 9,6721961\\ \text{Log}\,\tau' &= \underline{9,9432471}\\ &\ 9,6154432\end{aligned}$$

hinc $\tau'\sqrt{\lambda'-1} = 0,4125$. Unde fit

$$F' = \frac{-\alpha}{0,1414 \pm 0,4125} = \frac{-\alpha}{0,5589}, \quad G' = \frac{-\alpha}{1,5827 \mp 0,4125} = \frac{-\alpha}{1,1702}$$

hincque

$$F' = -1,8054\alpha, \qquad G' = -0,8545\alpha;$$

quare statuamus pro multiplicatione quacunque m

$$F' = -\left(1,8054 + \tfrac{f}{m}\right)\alpha, \qquad G' = -\left(0,8545 + \tfrac{g}{m}\right)\alpha,$$

et ex casu $m = 10$ elicimus

$$f = 0,4510, \;\; g = -0,0970,$$

ita ut sit

$$F' = -\left(1,8054 + \tfrac{0,4510}{m}\right)\alpha, \quad G' = -\left(0,8545 - \tfrac{0,0970}{m}\right)\alpha;;$$

pro α autem definiendo consideretur radius minimus in lente hae obiectiva triplicata occurrens 0,6023α, cuius pars quarta $0,1506\alpha = \frac{m}{50}$; sicque prodibit $\alpha = \frac{m}{7,5300}$ dig. Sumatur ergo $\alpha = \frac{2m}{15}$ dig. et habetur sequens constructio telescopii primi generis:

I. Pro prima lente, Crown Glass,

$$\text{radius faciei}\begin{cases}\text{anterioris} = +0,0803m \text{ dig.}\\ \text{posterioris} = +0,5882m \text{ dig.}\end{cases}$$

II. Pro secunda lente, Flint Glass,

$$\text{radius faciei}\begin{cases}\text{anterioris} = (-0,2407m - 0,0601) \text{ dig.}\\ \text{posterioris} = (-0,1139m + 0,0124) \text{ dig.}\end{cases}$$

III. Pro tertia lente, Crown Glass,

$$\text{radius faciei}\begin{cases}\text{anterioris} = +0,1874m \text{ dig.}\\ \text{posterioris} = +1,3723m \text{ dig.}\end{cases}$$

IV. Pro quarta lente, Crown Glass,

radius utriusque faciei = 0,3298 dig.

Tribus prioribus lentibus invicem iunctis quartae ab iis intervallum erit

$$=\tfrac{14}{45}(m-1)\text{ dig.},$$

et campi apparentis semidiameter erit, ut hactenus, $\Phi=\frac{859}{m-1}$ min. prim.

SCHOLION 1

181. Quia in hac solutione posuimus $\lambda=1$ et $\lambda''=1$, consuluimus potissimum artifici, quia hoc casu errores in exsecutione commissi non admodum negotium turbant; sed longitudo horum telescopiorum prodiit aliquanto maior, propterea quod radius satis exiguus in determinatione lentis obiectivae occurrebat; huic autem incommodo medelam afferemus, si pro prima et tertia lente statuamus $\lambda'=1,60006$, quo facto obtinebitur $\lambda'=1,9538-\frac{0,1425}{m}$, qui numerus tantum in secundam lentem influit, cuius constructionem deinceps investigemus.

Iam vero erit:

Pro prima lente, Crown Glass, radius utriusque faciei $=1,06\alpha$.
Pro tertia lente, Crown Glass, radius utriusque faciei $=1,06\gamma=2,473\alpha$.
Pro quarta lente, Crown Glass, radius utriusque faciei $=-\frac{2,4733\alpha}{m}$.

Restat igitur, ut secundam lentem evolvamus ut ante.

Scilicet duos casus contemplabimur, alterum, quo $m=10$, alterum, quo $m=\infty$.

Sit igitur primo $m=10$ eritque

$$\lambda=1,93956 \text{ et } \lambda'-1=0,93956 \text{ et } \tau'\sqrt{(\lambda'-1)}=0,85056.$$

Quare:

$$F'=\frac{-\alpha}{0,1414\pm0,85056},\quad G'=\frac{-\alpha}{1,5827\mp0,85056}$$

seu

$$F'=\frac{-\alpha}{0,9919}=-1,0081\alpha,\quad G'=\frac{-\alpha}{0,7322}=-1,3659\alpha.$$

Sit nunc $m=\infty$, erit

$$\lambda'=1,9538,\quad \tau'\sqrt{(\lambda'-1)}=-0,8569$$

hincque

$$F'=\frac{-\alpha}{0,1414\pm0,8569},\quad F'=\frac{-\alpha}{0.9983}=-1,0017\alpha,$$

$$G'=\frac{-\alpha}{1,5827\mp0,8569},\quad G'=\frac{-\alpha}{0,7258}=-1,3778\alpha.$$

Nunc pro multiplicatione quacunque m statuatur

$$F'=-\left(1,0017+\frac{f}{m}\right)\alpha,\quad G'=-\left(1,3778+\frac{g}{m}\right)\alpha,$$

et ex priore casu $m = 10$ colligitur

$$f = 0,065, \quad g = -0,121,$$

ita ut sit pro lente secunda

$$\text{radius faciei} \begin{cases} \text{anterioris} & = -\left(1,0017 + \frac{0,65}{m}\right)\alpha \\ \text{posterioris} & = -\left(1,3778 - \frac{0,121}{m}\right)\alpha. \end{cases}$$

Hic iam tute sumi potest $x = \frac{1}{4}\alpha = \frac{m}{50}$; hinc obtinetur $\alpha = \frac{8}{100}m.$
Hinc ergo orietur sequens telescopii primi generis constructio:

I. Pro lente prima, Crown Glass,

$$\text{radius utriusque faciei } = 0,0848m \text{ dig.}$$

II. Pro lente secunda, Flint Glass,

$$\text{radius faciei} \begin{cases} \text{anterioris} & = \left(-0,08014m - 0,0052\right) \text{ dig.} \\ \text{posterioris} & = \left(-0,110224m + 0,0097\right) \text{ dig.} \end{cases}$$

III. Pro lente tertia, Crown Glass,

$$\text{radius utriusque faciei } = 0,19784m \text{ dig.}$$

IV. Pro lente quarta, Crown Glass,

$$\text{radius utriusque faciei } = -0,19786 \text{ dig.}$$

Tribus lentibus prioribus sibi immediate iunctis ad intervallum $= 0,187(m-1)$ dig.

collocetur lens quarta, cui oculus immediate adplicatus cernet campum, cuius semidiameter erit $\frac{859}{m-1}$ min. prim.

SCHOLION 2

182. Hic casus inprimis est omni attentione dignus, quoniam pro quavis multiplicatione huius generis telescopia brevissima suppeditat: si enim multiplicationem adeo centupiam desideremus, longitudo vix superabit $18\frac{1}{2}$ digitos. Haec igitur methodus, qua posuimus $\mathfrak{B} = 1$, utique mereretur, ut etiam ad alias vitri species seu, ubi pro lentibus alia combinatio vitri coronarii et crystallini statueretur, seorsim adplicaretur. Sed quia ea

etiam ad problema nostrum generale solvendum aeque felici successu in usum vocari potest eiusque beneficio insignes difficultates supra commemoratae evanescunt, expediet sequens problema generalius tractasse.

PROBLEMA 5

183. *Si telescopium ex quatuor lentibus sit construendum, duae priores vero lentes ita debeant esse comparatae, ut radii per eas transmissi iterum inter se fiant paralleli, regulas pro constructione describere.*

SOLUTIO

Cum igitur radii per secundam lentem refracti iterum fiant axi paralleli, erit $\beta=\infty$ ideoque $\frac{\beta}{b}=B=\infty$ et $\mathfrak{B}=1$, erunt distantiae determinatrices

$$b=\frac{\alpha}{k},\ \ \beta=-\frac{B\alpha}{k}=\infty,\ \ c=\frac{B\alpha}{kk'},\ \ \gamma=\frac{BC\alpha}{kk'},\ \ d=-\frac{BC\alpha}{m};$$

hicque iam notari oportet, ut distantia inter secundam et tertiam lentem $\beta+c$ fiat finita, debere ob $\beta=\infty$ esse $c=-\infty$, unde fit $k'=1$.

Quo autem rem clarius explicemus, statuatur haec distantia $=\eta\alpha$, ut sit

$$B\alpha\left(\frac{1}{kk'}-\frac{1}{k}\right)=\eta\alpha,$$

unde fit

$$k'=\frac{B}{B+\eta k},$$

quae ob $B=\infty$ fit $k'=1$; interim tamen conveniet illam expressionem $k'=\frac{B}{B+\eta k}$, in usum sequentem notasse.

Deinde, quia $c=\infty$, γ vero finita quantitas, erit $\frac{\gamma}{c}=C=0$ hincque etiam $\mathfrak{C}=\frac{C}{1+C}=C=0$; interim tamen productum BC debet esse finitum Sit igitur $BC=\theta$, ut fiat $\gamma=\frac{\theta\alpha}{k}$ et $d=\frac{-\theta\alpha}{m}$; cum illa autem aequatione coniungi debet ista, qua summo rigore est $C=\frac{\gamma}{c}=\frac{\theta}{B}$ hincque $\mathfrak{C}=\frac{\theta}{B+\theta}$. His notatis erunt intervalla lentium

$$\alpha+b=\alpha\frac{k-1}{k},\quad \beta+c=\eta\alpha,\quad \gamma+d=\left(\frac{1}{k}-\frac{1}{m}\right)\theta\alpha=\frac{m-k}{km}\theta\alpha.$$

Unde hae fractiones $\frac{\eta k}{k-1}$ et $\frac{m-k}{m(k-1)}\theta$ debent esse positivae seu

$\frac{\eta}{k-1}>0,\quad \frac{m-k}{k-1}\theta>0$ seu $\frac{m-k}{\eta}>0.$

Iam inquiramus in valores litterarum $\pi,\ \pi'$ et π'' ex tribus sequentibus aequationibus definiendos

I. $\mathfrak{B}\pi - \Phi = -k\Phi$,
II. $\mathfrak{C}\pi' - \pi + \Phi = kk'\Phi$,
III. $(m-1)\Phi = -\pi + \pi' - \pi''$;
quarum prima statim dat ob $\mathfrak{B} = 1$

$$\pi = (1-k)\Phi = -(k-1)\Phi,$$

unde, ut hic valor campo augendo inserviat, π numerus negativus esse debet ideoque $k > 1$.

Secunda autem aequatio ob $\mathfrak{C} = 0$ et $k' = 1$ daret $-\pi + \Phi = k\Phi$, unde pro π' nihil concludere liceret, quare pro $\mathfrak{C}$ et k' valores illos exactiores scribi oportebit fietque

$$\tfrac{\theta}{B+\theta}\pi' - \pi + \Phi = \tfrac{Bk}{B+\eta k}\Phi,$$

quae ob $\pi = -(k-1)\Phi$ abit in hanc

$$\tfrac{\theta}{B+\theta}\pi' + k\Phi = \tfrac{Bk}{B+\eta k}\Phi \quad \text{seu} \quad \tfrac{\theta}{B+\theta}\pi' = \tfrac{-\eta k^2}{B+\eta k}\Phi,$$

quae ergo ob $B = \infty$ dat

$$\pi' = \tfrac{-\eta k^2}{\theta}\Phi \;;$$

quia autem convenit sumere $k > 1$, debet esse $\alpha > 0$ ideoque et $\eta > 0$; hic valor π' erit negativus, si fuerit $\theta > 0$; sin autem $\theta < 0$, is erit positivus, ubi autem meminisse oportet esse debere $(m-k)\theta > 0$.

Tertia denique aequatio abit in hanc formam:

$$(m-1)\Phi = +(k-1)\Phi - \tfrac{\eta k^2}{\theta}\Phi - \pi''$$

hincque

$$\pi'' = (k - m - \tfrac{\eta k^2}{\theta})\Phi \;\text{ sive }\; \pi'' = -(m - k + \tfrac{\eta k^2}{\theta})\Phi;$$

quae formula cum etiam inserviat campo definiendo, si capiatur $\pi'' = -\frac{1}{4}$, reperitur

$$\Phi = \frac{859}{m-k+\frac{\eta k^2}{\theta}} \text{ minut.}$$

$$\Phi = \frac{859\theta}{(m-k)+\eta k^2} \text{ minut.};$$

quare curandum est, ut $\frac{\eta k^2}{\theta}$ quam minimum reddatur, quod facile praestatur faciendo intervallum secundae et tertiae lentis quam minimum adeoque evanescens, quo casu erit $\Phi = \frac{859}{m-k}$; qui eo maior fit , quo major sumitur k.

Nunc igitur aequationem pro margine colorato tollendo consideremus, quae erit

$$0 = N\theta\pi'' - \tfrac{N'}{k}(\theta\pi'' - \pi) + \tfrac{N''}{k}(\pi'' - \pi),$$

quae substitutis valoribus dat

$$0 = N\left((m-k)\theta + \eta k^2\right) + \tfrac{N'}{k}\left((m-k)\theta + \eta k^2 - k + 1\right) - \tfrac{N''}{k}(m-k),$$

ex qua aequatione θ commode definiri potest reperieturque

$$\theta = \frac{-N\eta k^3 + N'\eta k^2 - N'(k-1) - N''(m-k)}{(m-k)(Nk-N')};$$

quia autem convenit η quam minimum assumere ac praeterea non necesse est, ut isti aequationi summo rigore satisfiat, his terminis omissis habebimus

$$\theta = \frac{-N'(k-1) - N''(m-k)}{(m-k)(Nk-N')},$$

unde fit

$$(m-k)\theta = \frac{-N'(k-1) - N''(m-k)}{Nk-N'};$$

quae quantitas cum debeat esse positiva, numerator autem manifesto sit negativus, etiam denominatorem negativum esse oportet ideoque $N' > Nk$. Quodsi ergo N' maximum habeat valorem ex vitro scilicet crystallino, N vero minimum ex vitro coronario, ut sit $N = 7$ et $N' = 10$, numerus k non amplius nostro arbitrio relinquitur, sed ita capi debet, ut fiat $7k < 10$ et $k < \frac{10}{7}$, seu contineri debet intra limites 1 et $\frac{10}{7}$. Notetur hic, si caperetur $k = 1$, casum praecedentem esse oriturum neque campum hinc auctum iri; sin autem capiatur $k = \frac{10}{7}$, foret $\theta = \infty$ et longitudo telescopii fieret infinita ; unde conveniet k propius unitati quam alteri limiti assumere. His probe perpensis statuamus

$$k = \tfrac{8}{7},\ \ N = 7,\ \ N' = 10,\ \ N'' = 7,$$

quo θ obtineat valorem minorem. Unde fiet

$$\theta = \frac{49m-46}{2(7m-8)}$$

hincque $\frac{\eta k^2}{\theta}$ habebit hunc valorem $\frac{2\cdot 8^2(7m-8)}{7^2(49m-46)}\eta$, qui sumto $m = \infty$ fit $= \frac{128}{343}\eta$, ex quo colligitur, si modo η non excedat $\frac{1}{10}$, campi diminutionem non fore sensibilem.

Denique pro semidiametro confusionis ad nihilum redigenda satisfiat huic aequationi

$$0 = \mu\lambda - \frac{\mu'\lambda'}{k} + \frac{\mu''\lambda''}{k\theta^3} - \frac{\mu'''\lambda'''}{m\theta^3},$$

ex qua commodissime definiemus λ', qui erit ob $\mu = \mu'' = \mu'''$

$$\lambda' = \frac{\mu}{\mu'}\left(k\lambda + \frac{\lambda''}{\theta^3} - \frac{k\lambda'''}{m\theta^3}\right);$$

sicque hoc problema feliciter est solutum

COROLLARIUM 1

184. Distantiae ergo determinatrices singularum lentium erunt:

Pro prima: ∞ et α cum λ,

pro secunda: $b = \frac{-\alpha}{k}$ et $\beta = \infty$ cum λ',

pro tertia : $c = -\infty$ et $\gamma = \frac{\theta\alpha}{k}$ cum λ'',

pro quarta: $d = \frac{-\theta\alpha}{k}$ et $\delta = \infty$ cum λ''';

ubi notandum primam, tertiam et quartam ex vitro coronario, secundam ex crystallino esse parandam; tum vero fore intervalla lentium

$$\alpha + b = \alpha\frac{k-1}{k} = \frac{1}{8}\alpha, \quad \beta + c = \eta\alpha,$$

de qua distantia notetur eam statui debere quam minimam; ac denique

$$\gamma + d = \frac{m-k}{km}\theta\alpha,$$

unde tota longitudo prodit

$$= \alpha\left(\frac{k-1}{k} + \eta + \frac{m-k}{km}\theta\right).$$

COROLLARIUM 2

185. Pro litteris autem k et θ hos valores statuimus

$$k = \frac{8}{7}, \quad \theta = \frac{49m-46}{2(7m-8)};$$

quae expressio cum adhuc m involvat, calculum non ut ante pro quavis multiplicatione in genere absolvere licebit; interim tamen simili modo, quo ante usi sumus, postquam pro duabus tribusve multiplicationibus calculum absolverimus, interpolando formulas generaliores pro omni multiplicatione concludere poterimus.

SCHOLION 1

186. Haec telescopia iis, quae modo ante descripsimus, ideo erunt praeferenda, quod in his nullae lentes sibi immediate iunctae assumuntur, quippe quod in praxi locum habere nequit, tum vero etiam, quod aliquod campi augmentum largiuntur. Ceterum haec telescopia aliquanto fiunt longiora, tam ob distantiam inter lentes primam et secundam, quam potissimum ob maiorem valorem ipsius θ, a quo intervallum tertiae et quartae lentis potissimum pendet. Intervallum autem medium $\eta\alpha$ hic merito negligimus. Quo tamen brevitati instrumenti, quantum fieri licet, consulamus, expediet sine dubio, ut modo ante fecimus, tam primam et tertiam lentem, quam quartam utrinque aequales formare, ita ut sit $\lambda = \lambda'' = \lambda''' = 1,60006$; tum vero erit $\mu = 0,9875, \quad \mu' = 0,8724$, unde harum lentium constructio statim sequitur.

Erit scilicet radius utriusque faciei:

I. Pro lente prima $= 1,06\alpha.$

II. Pro lente tertia $= 1,06\frac{\theta\alpha}{k}.$

III. Pro lente quarta $= -1,06\frac{\theta\alpha}{m}.$

Nihil igitur aliud restat, nisi ut pro quibusdam multiplicationibus calculum expediamus; ac primo quidem conveniet multiplicationem quandam exiguam $m = 5$ evolvere, ut pateat, quantum haec investigatio in minimis telescopiis huius generis praestare possit; tum vero multiplicationem quandam maiorem veluti $m = 10$ indeque subito $m = \infty$ evolvamus, ut ex horum casuum comparatione conclusionem pro quavis maiore multiplicatione formare queamus.

EXEMPLUM 1

$$m = 5$$

187. Telescopium pro multiplicatione $m = 5$ describere.
Erit hoc casu

$$\theta = \tfrac{199}{54} = 3,6852,\ \text{Log.}\theta = -\ 0,5664611 \text{ et Log.}k = 0,0579920$$

hincque

$$b = -\tfrac{7}{8}\alpha,\ \ \beta = \infty,\ \ c = -\infty,\ \ \gamma = 3,2246\alpha,\ \ d = -0,7370\alpha$$

hincque

$$\alpha + b = \tfrac{1}{8}\alpha,\ \beta + c = \eta\alpha = \text{ minimo},\ \ \gamma + d = 2,4876\alpha$$

sicque longitudo tota telescopii erit $= 2,6126\alpha + \eta\alpha.$

Unde tres lentes ex vitro coronario parandae ita se habebunt:

I. Pro prima lente

$$\text{radius utriusque faciei } = 1,06\alpha .$$

II. Pro tertia lente

$$\text{radius utriusque faciei } = 3,4181\alpha.$$

III. Pro quarta lente

$$\text{radius utriusque faciei } = -\ 0,\ 7812\alpha .$$

IV. Pro secunda lente, Flint Glass, ante omnia quaeri debet numerus λ' ex formula

$$\lambda' = \tfrac{1,60006\mu}{\mu'}\left(k + \tfrac{1}{\theta^3} - \tfrac{k}{m\theta^3}\right),$$

unde

$$\lambda' = 2,0977, \text{ ergo } \lambda' - 1 = 1,0977 \text{ hincque } \tau'\sqrt{(\lambda' - 1)} = 0,91936.$$

Quare pro hac lente erit

$$F' = \tfrac{b}{0,1414\ \pm 0,91936} = \tfrac{b}{1,0608}, \quad G' = \tfrac{b}{1,5827 \mp 0,91986} = \tfrac{b}{0,6633},$$

$$F' = -0,8248\alpha, \quad G' = -1,3192\alpha.$$

Unde fluit sequens constructio telescopii:

I. Pro lente prima, Crown Glass,
radius utriusque faciei $= +1,06\alpha$. Intervallum $= 0,125\alpha$.

II. Pro lente secunda, Flint Glass,

$$\text{radius faciei } \begin{Bmatrix} \text{anterioris} = -\ 0,8248\alpha \\ \text{posterioris } = -1,3192\alpha. \end{Bmatrix} \text{ Intervallum minimum.}$$

III. Pro lente tertia, Crown Glass,
radius faciei utriusque $= 3,4181\alpha.$ Intervallum $= 2,4876\alpha.$

IV. Pro lente quarta, Crown Glass,
radius utriusque faciei $= -0,7812\alpha.$

Lenti obiectivae tribui potest apertura, cuius semidiameter $x = \frac{1}{4}\alpha$. Cum autem ob claritatem statui debeat $x = \frac{m}{50}$ dig. $= \frac{1}{10}$ dig., unde $\alpha = \frac{2}{5}$ dig., erit telescopii longitudo $= 2,6126\alpha + \eta\alpha = 1,0450$ dig. $+ 0,4\eta$ dig. et semidiameter campi $\Phi = 223$ min. $= 3°43'$.

EXEMPLUM 2

188. Si multiplicatio $m=10$ desideretur, telescopium huius generis describere.
Ob $m=10$ erit

$$\theta=\tfrac{444}{124}=3,5806,\ \ \text{Log.}\theta=0,5539613,\ \text{Log.}\ \tfrac{1}{\theta}=-9,4460386,$$

unde

$$b=-\tfrac{7}{8}\alpha=-0,875\alpha,\quad \beta=\infty=-c,\ \ \gamma=3,1331\alpha,\ \ d=-0,35806\alpha.$$

Nunc evolvatur numerus λ', qui reperitur

$$\lambda'=2,1049,\ \lambda'-1=1,1049,\ \text{hinc}\ \tau'\sqrt{(\lambda'-1)}=0,92238.$$

Unde radii facierum.

$$F'=\tfrac{b}{0,1414\pm0,9213}=\tfrac{b}{1,0627},\quad G'=\tfrac{b}{1,5827\mp0,9213}=\tfrac{b}{0,6614}$$

seu

$$F'=-0,8234\alpha,\ \ G'=-1,3230\alpha.$$

Unde colligitur sequens constructio telescopii:

I. Pro prima lente, Crown Glass,
radius utriusque faciei $=1,06\ \alpha$. Intervallum $=0,125\alpha$.

II. Pro secunda lente, Flint Glass,
radius faciei $\begin{cases}\text{anterioris} &= -0,8234\alpha\\ \text{posterioris} &= -1,3230\alpha\end{cases}$. Intervallum minimum.

III. Pro lente tertia, Crown Glass,
radius utriusque faciei $=3,3211\alpha$. Intervallum $=2,7751\alpha$.
IV. Pro lente quarta, Crown Glass,
radius utriusque faciei $=-\ 0,8796\alpha$.

Unde fit tota longitudo $=2,9001\alpha$. Lenti primae autem apertura tribui debet, cuius semidiameter $x=\tfrac{m}{50}=\tfrac{1}{4}\alpha=\tfrac{1}{5}$ dig. Unde sequitur $\alpha=\tfrac{4}{5}$ dig. sive maius. Campi autem visi semidiameter erit $\Phi=97$ minut. $=1°37'$.

EXEMPLUM 3

189. Si multiplicatio m fuerit ∞, telescopium huius generis describere.
Ob $m=\infty$ erit

$$\theta = 3,5 \text{ et Log.}\theta = 0,5440680, \quad \text{Log.}\tfrac{1}{\theta} = 9,4559319$$

hincque

$$b = -0,875\alpha, \ \beta = \infty = -c, \ \ \gamma = 3,0625\alpha, \ \ d = -3,5\tfrac{\alpha}{m}.$$

Pro lente autem secunda invenimus

$$\lambda' = 2,1120, \quad \lambda' - 1 = 1,1120 \ \text{ et } \tau'\sqrt{(\lambda' - 1)} = 0,9253.$$

Ex quibus colligitur

$$F' = \frac{b}{0,1414 \pm 0,9258} = \frac{b}{1,0667}, \quad G' = \frac{b}{1,5827 \mp 0,9253} = \frac{b}{0,6574},$$

$$F' = -\ 0,8203\alpha, \ G' = -\ 1,3309\alpha.$$

Unde colligitur sequens constructio telescopii:

I. Pro prima lente, Crown Glass,

radius utriusque faciei $= 1,06\alpha$. Intervallum $= 0,125\alpha$.

II. Pro secunda lente, Flint Glass,

$$\text{radius faciei}\begin{cases}\text{anterioris} & = -0,8205\alpha \\ \text{posterioris} & = -1,3309\alpha\end{cases}\text{.Intervallum minimum.}$$

III. Pro tertia lente, Crown Glass

radius utriusque faciei $= 3,2462\alpha$. Intervallum $= (3,0625 - \frac{3,5}{m})\alpha$.

IV. Pro quarta lente, Crown Glass,

radius utriusque faciei $= -\ 3,710\frac{\alpha}{m}$.

Hincque longitudo telescopii erit $= \left(3,1875 - \frac{3,5}{m}\right)\alpha$. Lenti vero obiectivae apertura tribuatur, cuius semidiameter $= \frac{1}{4}\alpha = \frac{m}{50}$, ita ut capi possit $\alpha = \frac{2}{25}$ dig. $= \infty$.

EXEMPLUM 4 GENERALE

190: Si multiplicatio fuerit quaecunque m, saltem denario maior, telescopium huius generis describere.

Cum pro casu $m = \infty$ invenerimus $\theta = 3,5$, nunc in genere ponamus

$\theta = 3,5 + \frac{e}{m}$, et quia pro $m = 10$ fuerat $\theta = 3,5806$, erit $e = 0,806$, ita ut sit $\theta = 3,5 + \frac{0,806}{m}$; unde distantiae ita se habebunt:

$$b = -0,875\alpha, \quad \beta = \infty = -c; \quad \gamma = \left(3,0625 + \frac{0,7060}{m}\right)\alpha, \quad d = -\left(3,5 + \frac{0,8060}{m}\right)\frac{\alpha}{m}.$$

Pro lente autem secunda ponatur

$$F' = -\left(0,8205 + \frac{f}{m}\right)\alpha, \quad G' = -\left(1,3309 + \frac{g}{m}\right)\alpha.$$

Comparatione igitur instituta cum casu $m = 10$ erit

$$f = 0,0290, \quad g = -\ 0,0790.$$

Constructio huius telescopii

I. Pro prima lente, Crown Glass,

radius utriusque faciei $= 1,06\alpha$. Intervallum $= 0,125\alpha$.

II. Pro secunda lente, Flint Glass,

$$\text{radius faciei} \begin{cases} \text{anterioris} = -(\ 0,8205 + 0,0290)\alpha \\ \text{posterioris} = -\left(\ 1,3809 - \frac{0,0790}{m}\right)\alpha \end{cases}. \text{ Intervallum minimum.}$$

III. Pro tertia lente, Crown Glass,

$$\text{radius utriusque faciei} = +\left(\ 3,2462 + \frac{0,7484}{m}\right)\alpha.$$

$$\text{Intervallum} = \left(3,0625 - \frac{2,7940}{m} - \frac{0,8060}{m^2}\right)\alpha.$$

IV. Pro quarta lente, Crown Glass,

$$\text{radius faciei utriusque} = -\left(3,710 + \frac{0,8544}{m}\right)\frac{\alpha}{m}.$$

Sicque tota longitudo erit

$$= \left(\ 3,1875 - \frac{2,7940}{m} - \frac{0,8060}{m^2}\right)\alpha;$$

deinde lentis obiectivae semidiameter aperturae debet esse $x = \frac{m}{50}$ dig., unde α capi debebit $\alpha = \frac{2}{25}m$ dig. sive maius campique visi semidiameter

$$\Phi = \frac{859}{m - \frac{3}{7}} \text{ min. prim.}$$

SCHOLION 2

191. En ergo insignem multitudinem variorum primi generis telescopiorum, quae adhuc in infinitum multiplicari possent, si litteris B et C alios valores tribuere vel etiam pluribus lentibus uti vellemus. Verum huiusmodi investigatio prorsus superflua videtur, cum maior perfectionis gradus exspectari nequeat ac plures lentes claritati semper obsint neque etiam maior campus sperari possit. Inprimis autem observandum est in his telescopiis marginem coloratum aliter destrui non potuisse nisi diversis vitri speciebus adhibendis, ita ut iam affirmare possimus, ex eadem vitri specie huiusmodi telescopia confici non posse, quae non vitio marginis colorati laborent, cum tamen in sequentibus generibus, lentibus ex una vitri specie factis, talis margo feliciter tolli possit, etiamsi tunc ipsum diffusionis spatium ad nihilum redigera non liceat. Haec restrictio etiam in causa erat, quod campum apparentem vix notabiliter augere licuerat; sin autem marginem coloratum negligera vellemus, campus haud mediocriter augeri posset. Tum enim in casu ultimi problematis litterae k et θ manerent arbitrio nostro relictae, et cum semidiameter campi esset $\Phi=-\frac{\pi''}{m-k}$ posito $\eta=0$, videtur ea ad lubitum augeri posse, dum tantum k parum ab m deficiens assumatur, atque adeo sumto $k=m$ in infinitum abiret; quod tamen nullo modo praestari posse experientia abunde testatur. Quare hoc dubium solvisse operae erit pretium; ad quod tantum recordari oportet litteris π, π' et π'' certum praescriptum esse terminum veluti $\frac{1}{4}$, quem transgredi nunquam debent; quare, etsi hoc casu valor $-\pi'=\frac{1}{4}$ enormem magnitudinem pro Φ praebet, tamen hic etiam ad valorem ipsius π spectari convenit; qui cum ante iam inventus esset $\pi=-\Phi(k-1)$ ideoque $\pi=\frac{\pi''(k-1)}{m-k}$, maxime cavendum est, ne hinc prodeat $\pi>\pi''$; quamobrem litteram k iam non pro lubitu augere licebit, sed eo usque tantum, quoad fiat $k-1=m-k$ sive $k=\frac{m+1}{2}$, quae positio campum duplo maiorem quam ante produceret, scilicet $\Phi=\frac{-\pi''}{m-k}=\frac{-2\pi''}{m-1}$, quem ergo obtinere possemus, si modo marginem coloratum despiciamus. Tum autem pro eodem casu ultimi problematis forent distantiae determinatrices

$$b=\frac{-2\alpha}{m+1},\ \beta=\infty=-c \text{ et } \gamma=\frac{+2\theta\alpha}{m+1} \text{ et } d=\frac{-\theta\alpha}{m};$$

unde fit postremum intervallum

$$\gamma+d=+\theta\alpha\left(\frac{2}{m+1}-\frac{1}{m}\right)=\frac{\theta\alpha(m-1)}{m(m+1)},$$

ubi adhuc θ nostro arbitrio permittitur, dummodo positive capiatur; verum quia hoc modo margo coloratus praemagnus esset proditurus, huiusmodi telescopia nullo modo commendari poterunt; atque hoc praeceptum etiam in posterum observabimus nullaque alia telescopia exceptis tantum simplicissimis proferemus, nisi quae saltem a margine colorato sint immunia, siquidem tota haec confusio non vitari queat.