CHAPTER II

CONCERNING MORE COMPOSITE MICROSCOPES OF THIS KIND

PROBLEM 1

250. *To construct a microscope of this kind from five lenses, which shall be put in place thus, so that the first real image shall fall between the second and third lenses, truly the latter between the third and fourth lenses.*

SOLUTION

Therefore since the first image must fall in the second interval and the second image in the third interval, the second Q and third R of the letters P, Q, R, S must be negative. Therefore there may be put $Q=-k$ and $R=-k'$, so that there shall be $Pkk'S=\mathfrak{M}$, with there being

$$\mathfrak{M}=\frac{ma}{h}.$$

Thence truly there shall become

$$M=\frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t}}{\mathfrak{M}-1},$$

so that the radius of the area viewed within the object shall become

$$z=Ma\xi$$

Whereby, so that the field of view may emerge a maximum, it is required to be effected , that as many of the letters $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$ may become equal to unity, as the remaining circumstances permit ; concerning which since it may not be able for everything to be put in place, we may put $\mathfrak{s}=1$ and $\mathfrak{t}=1$, for the latter ones, so that at least there shall be

$$M=\frac{\mathfrak{q}+\mathfrak{r}+2}{\mathfrak{M}-1}.$$

Then truly we will have the following equations:

$$1.\quad \mathfrak{B}\mathfrak{q}=(P-1)M,$$
$$2.\quad \mathfrak{C}\mathfrak{r}=-(Pk+1)M-\mathfrak{q},$$
$$3.\quad \mathfrak{D}=-(Pkk'+1)M-\mathfrak{q}-\mathfrak{r},$$

to which the equation may be added, by which the colored margin is removed,

$$0=\frac{\mathfrak{q}}{P}-\frac{\mathfrak{r}}{Pk}+\frac{1}{Pkk'}+\frac{1}{Pkk'S},$$

from which there is deduced

$$k' = \frac{1+\frac{1}{S}}{\mathfrak{r}-k\mathfrak{q}},$$

from which it is clear there must be $\mathfrak{r} > k\mathfrak{q}$. But hence before we may wish to define anything, we must consider the intervals of the lenses, which are

$$\text{first} = Aa\left(1-\tfrac{1}{P}\right),\ \text{second} = -\tfrac{AB}{P}\cdot a\left(1+\tfrac{1}{k}\right),$$
$$\text{third} = -\tfrac{ABC}{Pk}\cdot a\left(1+\tfrac{1}{k'}\right),\ \text{fourth} = -\tfrac{ABCD}{Pkk'}\cdot a\left(1-\tfrac{1}{S}\right),$$

which all must be positive; to which the focal length of the final lens must be added

$$t = \frac{ABCD}{Pkk'S}\cdot a = \frac{ABCD}{\mathfrak{M}}\cdot a,$$

which also must be positive, so that the distance of the eye after that may become positive :

$$O = \frac{t\mathfrak{t}}{M\mathfrak{M}};$$

from which on account of $\mathfrak{t} = 1$ it is evident t must be positive and thus $ABCD > 0$. Therefore, in order that the fourth interval also shall become positive, it is necessary that there shall be $1-\frac{1}{S} < 0$ or $S < 1$, from which it is evident the product Pkk' to become $> \mathfrak{M}$ and thus a very great number; from which that third equation will give, if the value may be substituted in place of M :

$$\mathfrak{D} = \frac{(Pkk'-1)(\mathfrak{q}+\mathfrak{r}+2)}{\mathfrak{M}-1} - \mathfrak{q} - \mathfrak{r};$$

where, since Pkk' and $\mathfrak{M}$ shall be very large numbers and $Pkk' > \mathfrak{M}$, it will become approximately

$$\mathfrak{D} = \frac{Pkk'}{\mathfrak{M}}(\mathfrak{q}+\mathfrak{r}+2) - \mathfrak{q} - \mathfrak{r} = \frac{2Pkk'}{\mathfrak{M}} + (\mathfrak{q}+\mathfrak{r})\left(\frac{Pkk'}{\mathfrak{M}}-1\right)$$

or if on account of $Pkk' = \frac{\mathfrak{M}}{S}$, it will be

$$\mathfrak{D} = \frac{2}{S} + (\mathfrak{q}+\mathfrak{r})\left(\frac{1}{S}-1\right);$$

from which, since $\mathfrak{q}+\mathfrak{r}$ certainly shall be < 2, it is evident to become $\mathfrak{D} > 1$ and thus D to become negative. Therefore there will become $ABC < 0$; hence the third interval at once becomes positive. Whereby, since on account of the second interval there must become $AB < 0$, there will be required to be $C > 0$ and hence $\mathfrak{C} > 0$ and $\mathfrak{C} < 1$. From which, if there were $A > 0$ and thus $P > 1$ on account of the first interval, there must become $B < 0$ and hence either $\mathfrak{B} < 0$ or $\mathfrak{B} > 1$; from which it follows to become for the first case $\mathfrak{q} < 0$, for the other case $\mathfrak{q} > 0$; from which on account of $\mathfrak{q} + \mathfrak{C}\mathfrak{r} < 0$ there will become $\mathfrak{r} < 0$ and thus $k' < 0$, which is absurd. But if $A < 0$ may be assumed and hence $P < 1$, there will have to become $B > 0$ and thus $\mathfrak{B} > 0$ and $\mathfrak{B} < 1$, from which again there will become $\mathfrak{q} < 0$. From which even now is it agreed, whether both these cases may be able to be present. Truly since in each there shall become $\mathfrak{q} < 0$, we may put as before $\mathfrak{q} = -\omega$, so that there shall be

$$\omega = \frac{(1-P)M}{\mathfrak{B}},$$

and on account of the second equation there must become $\omega - \mathfrak{C}\mathfrak{r} > 0$; then truly there will become

$$k' = \frac{1+\frac{1}{S}}{\mathfrak{r}+k\omega};$$

from which because $Pkk' = \frac{\mathfrak{M}}{S}$ there will become

$$Pk = \frac{\mathfrak{M}(\mathfrak{r}+k\omega)}{1+S}.$$

Therefore now we may eliminate the letters ω and $\mathfrak{t}$ from the calculation, and since there shall be $\omega = \frac{1-P}{\mathfrak{B}}M$, we may put for the sake of brevity

$$\frac{1-P}{\mathfrak{B}} = \zeta,$$

so that there shall become

$$\omega = \zeta M;$$

from which also for the sake of brevity there shall become

$$1+Pk = \eta,$$

and there will become

$$\mathfrak{r} = \frac{(\zeta-\eta)M}{\mathfrak{C}}.$$

Again therefore,

$$\mathfrak{r} - \omega = \frac{\left(\zeta(1-\mathfrak{C})-\eta\right)M}{\mathfrak{C}},$$

and

$$2 - \omega + \mathfrak{r} = \frac{2\mathfrak{C}+\left(\zeta\left(1-\mathfrak{C}\right)-\eta\right)M}{\mathfrak{C}};$$

but since there shall be $M = \frac{2+\mathfrak{r}-\omega}{\mathfrak{M}-1}$, there will become

$$2+\mathfrak{r}-\omega = M\left(\mathfrak{M}-1\right),$$

from which we conclude to become

$$M = \frac{2\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta};$$

with which value found, there will become

$$\omega = \frac{2\mathfrak{C}\zeta}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta} \quad \text{and} \quad \mathfrak{r} = \frac{2(\zeta-\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta},$$

from which values again there is produced

$$\mathfrak{r}+k\omega = \frac{2(\zeta-\eta)+2k\mathfrak{C}\zeta}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}$$

and hence

$$Pk = \frac{2\mathfrak{M}(\zeta-\eta+k\mathfrak{C}\zeta)}{(1+S)(\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta)} = \eta-1,$$

from which there is defined

$$k = \frac{(\eta-1)(1+S)\mathfrak{C}(\mathfrak{M}+\zeta-1)-2\mathfrak{M}(\zeta-\eta)-(\eta-1)(1+S)(\zeta-\eta)}{2\mathfrak{M}\mathfrak{C}\zeta},$$

from which again there is found

$$P = \frac{\eta-1}{k}.$$

But since k must be positive, the coefficient of $\mathfrak{M}$ in the numerator of that must be positive, from which there follows:

$$\mathfrak{C} > \frac{2(\zeta-\eta)}{(\eta-1)(1+S)};$$

but truly we have seen $\mathfrak{C} < 1$ and thus it is necessary, that there shall become

$$\frac{2(\zeta-\eta)}{(\eta-1)(1+S)} < 1 \text{ or } 2(\zeta-\eta) < (\eta-1)(1+S);$$

from which condition we conclude

$$\zeta < \eta + \frac{(\eta-1)(1+S)}{2}.$$

Moreover we know there must be $\zeta > \eta$; from which the letter ζ will have to be taken within the bounds

$$\eta \text{ and } \eta + \frac{(\eta-1)(1+S)}{2},$$

where it is evident there must become $\eta > 1$. Therefore our problem will be agreed to be resolved in the following manner.

The letters S and η can be taken as it pleases, provided there is observed to be $S < 1$ and $\eta > 1$, since $Pk = \eta - 1$. Then the letter ζ may be taken within the limits η and $\eta + \frac{(\eta-1)(1+S)}{2}$. But $\mathfrak{C}$ may be taken within the limits

$$\mathfrak{C} < 1 \text{ and likewise } \mathfrak{C} > \frac{2(\zeta-\eta)}{(\eta-1)(1+S)},$$

from which C is defined. Then truly there may be taken

$$k = \frac{(\eta-1)(1+S)\mathfrak{C}(\mathfrak{M}+\zeta-1)-2\mathfrak{M}(\zeta-\eta)-(\eta-1)(1+S)(\zeta-\eta)}{2\mathfrak{M}\mathfrak{C}\zeta},$$

from which there will be had

$$P = \frac{\eta-1}{k} \text{ and } k' = \frac{\mathfrak{M}}{(\eta-1)S}.$$

Finally there may be taken $\mathfrak{B} = \frac{1-P}{\zeta}$, from which B is defined. Finally on account of $Pkk' = \frac{\mathfrak{M}}{S}$ the above formula for $\mathfrak{D}$ will be found

$$\mathfrak{D} = (\tfrac{\mathfrak{M}}{S} - 1)M - (\mathfrak{M} - 1)M + 2 = M\mathfrak{M}(\tfrac{1}{S} - 1) + 2;$$

where with the value of M itself substituted, there will be produced :

$$\mathfrak{D} = \frac{\frac{2\mathfrak{M}\mathfrak{C}}{S} + 2\mathfrak{C}(\zeta-1) - 2(\zeta-\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta},$$

which value, since $\mathfrak{M}$ shall be a very great number, will become approximately $\mathfrak{D} = \frac{2}{S}$ and hence with more care

$$\mathfrak{D} = \frac{2}{S} + \frac{\frac{2}{S}(\zeta-\eta-\mathfrak{C}(\zeta-1)) + 2\mathfrak{C}(\zeta-1) - 2(\zeta-\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}$$

or either,

$$\mathfrak{D} = \frac{2}{S} + \frac{2(\frac{1}{S}-1)(\zeta-\eta-\mathfrak{C}(\zeta-1))}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}\sigma$$

or

$$\mathfrak{D} = \frac{2}{S} - \frac{2(\frac{1}{S}-1)(\mathfrak{C}(\zeta-1)-\zeta+\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}$$

from which anyhow it will certainly be clear to become $\mathfrak{D} > 1$ and thus D to be negative, as now we have observed above . Now just as there were P either > 1 or < 1, there will have to be taken $A>0$ or $A<0$, thus so that now all the elements may be determined; for we will have the focal lengths

$$p=\mathfrak{A}a,\ \ q=-\tfrac{A\mathfrak{B}}{P}\cdot a,\ \ r=-\tfrac{AB\mathfrak{C}}{Pk}\cdot a,\ s=-\tfrac{ABC\mathfrak{C}}{Pkk'}\cdot a\ \text{ and }\ t=\tfrac{ABCD}{\mathfrak{M}}\cdot a,$$

thence the intervals

$$\text{first}=Aa(1-\tfrac{1}{P}),\ \ \text{second}=-\tfrac{ABa}{P}(1+\tfrac{1}{k}),$$

$$\text{third}=-\tfrac{ABCa}{Pk}(1+\tfrac{1}{k'}),\ \text{fourth}=-\tfrac{ABCDa}{Pkk'}(1-\tfrac{1}{S});$$

then truly there will be

$$M=2:\left(\mathfrak{M}+\zeta-1-\tfrac{(\zeta-\eta)}{\mathfrak{C}}\right),$$

and

$$O=\tfrac{\mathfrak{t}}{\mathfrak{M}M};$$

and the aperture of the first lens is defined from the most noteworthy equation $\frac{1}{k^3}=\cdots$.

COROLLARY 1

251. The condition, as we have found here, cannot involve more than $\zeta>\eta$, as ζ may not be taken smaller than η. However nothing prevents, nevertheless, that we may take $\zeta=\eta$; even if this value diminishes the field somewhat, yet this still is produced notable enough. But then there becomes $\mathfrak{r}=0$ and thus the third lens will be required as the minimum aperture, so thus likewise it may perform the duty of the narrowest aperture.

COROLLARY 2

252. But if indeed we may put $\zeta=\eta$ in place, it suffices to take $\mathfrak{C}$ within the limits 0 and 1, from whch at once C becomes positive. Then truly there may be taken

$$k=\frac{(\eta-1)(1+S)(\mathfrak{M}+\eta-1)}{2\mathfrak{M}\eta},$$

from which there will be had

$$P=\frac{2\mathfrak{M}\eta}{(1+S)(\mathfrak{M}+\eta-1)},$$

thus so that for the greater magnifications there shall be approximately

$$k=\frac{(\eta-1)(1+S)}{2\eta}\ \text{ and }\ P=\frac{2\eta}{1+S};$$

from which it is apparent to be $P>1$ and thus A positive. Then truly again there will be

$$\mathfrak{B} = -\frac{(P-1)}{\eta},$$

thus so that there shall be both $\mathfrak{B} < 0$ as well as $B < 0$. Finally in this case there will become

$$\mathfrak{D} = \frac{2\mathfrak{M}+2S(\eta-1)}{S(\mathfrak{M}+\eta-1)}$$

and hence

$$D = -\frac{2\mathfrak{M}+2S(\eta-1)}{\mathfrak{M}(2-S)+S(\eta-1)}$$

and

$$M = \frac{2}{\mathfrak{M}+\eta-1}.$$

SCHOLIUM

253. Besides which, this case $\zeta = \eta$ is especially convenient in practice, this prerogative is included also, as in short the remaining elements shall not depend on the letter $\mathfrak{C}$, thus so that clearly, whatever value we may allow $\mathfrak{C}$ to take within the limits 0 and 1, the remaining elements thence undergo no change. Moreover in this way it will be able to avoid easily, so that the objective lens may not become exceedingly small, which truly in addition performs better by the letter *A*, unless perhaps it may be said for telescopes, where $Aa = p$; therefore it would be superfluous here to set out other cases besides that same one $\zeta = \eta$, and now it will be especially worth the effort to consider some other values for η, so that thence we may be able to understand, which future values shall be going to be the most useful. Truly for the letters *S*, which we have seen must be less than unity, we may always put in place $S = \frac{1}{2}$, since hence a suitable enough interval arises between the two latter lenses. Moreover then our conditions will be expressed in the following manner:

$$1.\ k = \frac{3(\eta-1)(\mathfrak{M}+\eta-1)}{4\mathfrak{M}\eta},$$

$$2.\ P = \frac{4\mathfrak{M}\eta}{3(\mathfrak{M}+\eta-1)},$$

$$3.\ \mathfrak{B} = \frac{-(4\eta-1)\mathfrak{M}+3(\eta-1)}{3\eta(\mathfrak{M}+\eta-1)},$$

4. $\mathfrak{C}$, as we have observed now, is allowed to be chosen by us, provided it shall be between 0 and 1,

$$5.\ \mathfrak{D} = \frac{4\mathfrak{M}+2(\eta-1)}{\mathfrak{M}+\eta-1} \text{ and } D = \frac{-4\mathfrak{M}-2(\eta-1)}{3\mathfrak{M}+\eta-1}.$$

And thus hence the focal lengths will be

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{B}}{P}\cdot a = \frac{(4\eta-3)\mathfrak{M}-3(\eta-1)}{4\mathfrak{M}\eta^2}\cdot Aa,$$
$$r = -\frac{B\mathfrak{C}}{Pk}\cdot Aa = -\frac{B\mathfrak{C}}{\eta-1}\cdot Aa, \quad s = -\frac{BC\mathfrak{D}}{2\mathfrak{M}}\cdot Aa \text{ and } t = \frac{BCD}{\mathfrak{M}}\cdot Aa.$$

Thence the intervals of the lenses :

$$\text{first} = Aa(1-\tfrac{1}{P}) = \frac{(4\eta-3)\mathfrak{M}-3(\eta-1)}{4\mathfrak{M}\eta}\cdot Aa,$$
$$\text{second} = -B\left(\frac{\mathfrak{M}(7\eta-3)+3\eta(\eta-2)+3}{4\mathfrak{M}\eta(\eta-1)}\right)Aa,$$
$$\text{third} = -BC(\tfrac{1}{\eta-1}+\tfrac{1}{2\mathfrak{M}})Aa,$$
$$\text{fourth} = \frac{BCD}{2\mathfrak{M}}\cdot Aa.$$

Thence on account of $M = \frac{2}{\mathfrak{M}+\eta+1}$ there will become

$$O = \tfrac{1}{2}t\left(1+\frac{\eta-1}{\mathfrak{M}}\right).$$

For the first lens the radius of the aperture will be $= x$ required to be found from our best known equation; but for the second on account of

$$\omega = \frac{2\eta}{\mathfrak{M}+\eta-1}$$

the radius of the aperture will be

$$= \frac{\eta}{2(\mathfrak{M}+\eta-1)}q + \frac{x}{P},$$

and on account of $\mathfrak{r} = 0$, the radius of the aperture of the third lens

$$= \frac{x}{Pk} = \frac{x}{\eta-1};$$

but for the two remaining lenses the maximum aperture is attributed.

EXAMPLE 1

254. We may assume $\eta = 2$ and there will become $k = \frac{3(\mathfrak{M}+1)}{8\mathfrak{M}}$, and since so much is concerned with the greater magnifications, it will be able to assume $k = \frac{3}{8}$, hence

$$P = \frac{8\mathfrak{M}}{3(\mathfrak{M}+1)} = \frac{8}{3} \text{ approx.}$$

Again

$$\mathfrak{B} = -\tfrac{5}{6} \text{ and } B = -\tfrac{5}{11}, \quad \mathfrak{D} = 4 \text{ and } D = -\tfrac{4}{3},$$

from which the focal lengths will be

$$p = \mathfrak{A}a, \quad q = \tfrac{5}{16}Aa, \quad r = \tfrac{5}{11}\mathfrak{C}Aa, \quad s = \tfrac{10}{11}\tfrac{C}{\mathfrak{M}} \cdot Aa, \quad t = \tfrac{20}{33} \cdot \tfrac{C}{\mathfrak{M}} \cdot Aa,$$

and the intervals of the lenses

$$\text{first} = \tfrac{5}{8}Aa, \quad \text{second} = \tfrac{5}{8}Aa,$$
$$\text{third} = \tfrac{5}{11}C(1 + \tfrac{1}{2\mathfrak{M}})Aa, \quad \text{fourth} = \tfrac{10}{33} \cdot \tfrac{C}{\mathfrak{M}} \cdot Aa,$$

and

$$M = \tfrac{2}{\mathfrak{M}+1}, \text{ hence } z = \tfrac{1}{2} \cdot \tfrac{a}{\mathfrak{M}+1}$$

and

$$O = \tfrac{1}{2}t\left(1 + \tfrac{1}{\mathfrak{M}}\right);$$

then truly the radius of the aperture of the second lens $= \frac{q}{\mathfrak{M}+1} + \frac{3}{8}x$ and of the third $= x$.

But for the x requiring to be found, it shall satisfy this equation:

$$\frac{1}{k^3} = \frac{\mu \mathfrak{M} x^3}{\mathfrak{A}^3 a^3}\left\{\begin{array}{l} \lambda + \nu\mathfrak{A}(1-\mathfrak{A}) - \frac{(1-\mathfrak{A})^3}{\mathfrak{B}^3 P}\left(\lambda' + \nu\mathfrak{B}(1-\mathfrak{B})\right) \\ -\frac{(1-\mathfrak{A})^3}{B^3\mathfrak{C}^3 Pk}\left(\lambda'' + \nu\mathfrak{C}(1-\mathfrak{C})\right) \\ -\frac{(1-\mathfrak{A})^3}{B^3C^3\mathfrak{D}^3 Pkk'}\left(\lambda''' + \nu\mathfrak{D}(1-\mathfrak{D})\right) + \frac{(1-\mathfrak{A})^3}{B^3C^3D^3} \cdot \frac{\lambda''''}{Pkk'S} \end{array}\right\}$$

which equation thus will be represented more conveniently :

$$\frac{1}{k^3} = \mu \mathfrak{M} x^3 \left\{\begin{array}{l} \frac{\lambda + \nu\mathfrak{A}(1-\mathfrak{A})}{P^3} + \frac{1}{P^4 q^3}\left(\lambda' + \nu\mathfrak{B}(1-\mathfrak{B})\right) \\ + \frac{1}{(Pk)^4 r^3}\left(\lambda'' + \nu\mathfrak{C}(1-\mathfrak{C})\right) \\ + \frac{1}{(Pkk')^4 s^3}\left(\lambda''' + \nu\mathfrak{D}(1-\mathfrak{D})\right) + \frac{\lambda''''}{(Pkk'S)^4 t^3} \end{array}\right\}$$

COROLLARY

255. If these may be transferred to telescopes by putting $\mathfrak{A}a = Aa = p$ and $\mathfrak{M} = m$, the focal lengths will become :

$$q = \tfrac{5}{16}, \quad r = \tfrac{5}{11}\mathfrak{C}p, \quad s = \tfrac{10}{11}\tfrac{C}{m} \cdot p, \text{ and } t = \tfrac{20}{33} \cdot \tfrac{C}{m} \cdot p,$$

the intervals will become

$$\text{first} = \tfrac{5}{8}p, \quad \text{second} = \tfrac{5}{8}p, \;\; \text{third} = \tfrac{5}{11}C(1+\tfrac{1}{2m})p \;\; \text{and the fourth} = \tfrac{10}{33}\cdot\tfrac{C}{m}\cdot p$$

and the radius of the field of view $\Phi = \frac{1718}{\mathfrak{M}+1}$ min.
Therefore the length will become almost $= \frac{110}{88}p + \frac{5}{11}C\cdot p$.
But now p will be able to be defined from the equation given before, where there will become $\mathfrak{A}=0$ on account of $a=\infty$.

EXAMPLE 2

256. Now let there become $\eta = 3$; there will be

$$k = \tfrac{\mathfrak{M}+2}{2\mathfrak{M}},$$

therefore, there will be assumed that

$$k = \tfrac{1}{2}, \;\; P = \tfrac{4\mathfrak{M}+2}{3(\mathfrak{M}+2)} = 4, \;\; \mathfrak{B} = -\tfrac{3\mathfrak{M}+2}{3(\mathfrak{M}+2)} = -1, \text{ therefore } B = -\tfrac{1}{2},$$
$$\mathfrak{D} = \tfrac{4\mathfrak{M}+4}{\mathfrak{M}+2} = 4, \;\; \text{therefore } D = -\tfrac{4}{3}.$$

Hence therefore the focal distances will become

$$p = \mathfrak{A}a, \;\; q = \tfrac{1}{4}Aa, \;\; r = \tfrac{1}{4}\mathfrak{C}Aa, \quad s = \tfrac{C}{\mathfrak{M}}\cdot Aa, \text{ et } t = \tfrac{2}{3}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa.$$

and the intervals of the lenses

$$\text{first} = \tfrac{3}{4}Aa, \quad \text{second} = \tfrac{3}{8}Aa,$$
$$\text{third} = \tfrac{1}{4}C(1+\tfrac{1}{m})Aa, \;\; \text{fourth} = \tfrac{1}{3}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa;$$

hence again there will be

$$M = \tfrac{2}{\mathfrak{M}+2},$$

hence

$$z = \tfrac{1}{2}\cdot\tfrac{a}{\mathfrak{M}+2}$$

and

$$O = \tfrac{1}{2}t\left(1+\tfrac{2}{\mathfrak{M}}\right);$$

then truly the radius of the aperture

$$\text{of the second lens} = \frac{3q}{2(\mathfrak{M}+2)} + \frac{1}{4}x \quad \text{and of the third} = \frac{1}{2}x.$$

The others are had as before.

COROLLARY

257. With the application made to telescopes, where there becomes $Aa = p$, all the elements are determined easily as before; then indeed the length of the instrument, with the parts divided by $\mathfrak{M}$ omitted, will be $= \frac{9}{8}p + \frac{1}{4}Cp$, which is less than in the previous case.

EXAMPLE 3

258. We may put $\eta = 6$, thus so that there shall become $M = \frac{2}{\mathfrak{M}+5}$, and there will become

$$k = \frac{5(\mathfrak{M}+5)}{8\mathfrak{M}} = \frac{5}{8} \text{ approx.,}$$

$$P = 8, \quad \mathfrak{B} = \frac{-7\mathfrak{M}+5}{6(\mathfrak{M}+5)} = -\frac{7}{6} \quad \text{approx.,}$$

from which there shall become

$$B = -\frac{7}{13}, \quad \mathfrak{D} = 4, \quad \text{and} \quad D = -\frac{4}{3};$$

from which the focal lengths will be produced :

$$p = \mathfrak{A}a, \quad q = \frac{7}{48}Aa, \quad r = \frac{7}{65}\mathfrak{C}Aa, \quad s = \frac{14}{13} \cdot \frac{C}{\mathfrak{M}} \cdot Aa, \quad t = \frac{28}{39} \cdot \frac{C}{\mathfrak{M}} \cdot Aa$$

and the intervals of the lenses

$$\text{first} = \frac{7}{8}Aa, \quad \text{second} = \frac{7}{40}Aa,$$
$$\text{third} = \frac{7}{13}C\left(\frac{1}{5} + \frac{1}{2\mathfrak{M}}\right)Aa, \quad \text{fourth} = \frac{14}{39} \cdot \frac{C}{\mathfrak{M}} \cdot Aa.$$

Besides,

$$z = \frac{1}{2} \cdot \frac{a}{\mathfrak{M}+5} \quad \text{and} \quad O = \frac{1}{2}t\left(1 + \frac{5}{\mathfrak{M}}\right);$$

then truly the radius of the aperture of the

$$\text{of the second lens} = \frac{3q}{\mathfrak{M}+5} + \frac{1}{8}x \quad \text{and of the third lens} = \frac{1}{5}x.$$

COROLLARY

259. Therefore in the translation made to telescopes the length of these produced in this case $=\frac{21}{20}p+\frac{7}{65}Cp$, which length is short enough, so that also scarcely a shorter length can be hoped for in the other kinds.

SCHOLIUM

260. Even if this case $\zeta=\eta$ may be seen to be of the greatest use in practice, yet also it will be agreed to be considering certain cases, where $\zeta>\eta$, since in this manner for some field to be produced greater. Moreover with $S=\frac{1}{2}$ the other bound for ζ was

$$\zeta<\eta+\frac{3(\eta-1)}{4} \text{ or } \zeta<\frac{7}{4}\eta-\frac{3}{4}.$$

But it is unable for ζ to be equal to this limit, since otherwise $\mathfrak{C}$ must become $=1$ and hence $C=\infty$. Therefore we may accept

$$\zeta=\eta+\frac{2}{4}(\eta-1)=\frac{3}{2}\eta-\frac{1}{2}$$

and there will be found $\mathfrak{C}>\frac{2}{3}$ and $\mathfrak{C}<1$. Therefore we may accept $\mathfrak{C}=\frac{3}{4}$, so that there may become $C=3$, and hence there will become

$$k=\frac{(\eta-1)(2\mathfrak{M}+15(\eta-1))}{12\mathfrak{M}(3\eta-1)} \quad\text{and}\quad P=\frac{12\mathfrak{M}(3\eta-1)}{2\mathfrak{M}+15(\eta-1)}$$

and hence again

$$\mathfrak{B}=\frac{-4\mathfrak{M}(18\eta-7)+30(\eta-1)}{(3\eta-1)(2\mathfrak{M}+15(\eta-1))} \quad\text{and}\quad \mathfrak{D}=\frac{24\mathfrak{M}+10(\eta-1)}{6\mathfrak{M}+5(\eta-1)} \text{ or } \mathfrak{D}=4 \text{ approx.}$$

Then truly there will be produced

$$M=\frac{12}{6\mathfrak{M}+5(\eta-1)}=\frac{2}{\mathfrak{M}+\frac{5}{6}(\eta-1)},$$

since there was before

$$M=\frac{2}{\mathfrak{M}+\eta-1}.$$

So that if now we may take $\eta=3$ as in the second example, these elements will become

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hence

$$\mathfrak{B} = \frac{-47\mathfrak{M}+15}{4(\mathfrak{M}+15)} \quad \text{and} \quad B = \frac{-47\mathfrak{M}+15}{51\mathfrak{M}+45},$$

then

$$\mathfrak{C} = \tfrac{3}{4}, \quad C = 3, \quad \mathfrak{D} = 4 \quad \text{and} \quad D = -\tfrac{4}{3};$$

from which the individual moments for the construction can be defined. Here indeed since so large numbers are present, which are no longer allowed to be neglected in comparison with $\mathfrak{M}$, in the application to examples it will be appropriate to assume some values for these likewise at once, as well as for $\mathfrak{M}$. Truly here besides we will not progress to become more specific, since at this stage we make do with a simple objective lens, thus so that the confusion may not need to be removed in any other way except by diminishing the aperture of the objective lens ; which remedy, since it may be offered at once, there will be no need to define the letter x by that troublesome calculation; indeed for whatever microscope to have been constructed according to the measures of this kind, will indicate itself the aperture used; moreover in the following chapter we will reduce all the confusion to zero by the magnification of the objective lens, then at last it will be necessary to show the complete determination of the individual moments, we have produced to this stage.

PROBLEM 2

261. *To construct a microscope of this kind from five lenses, which shall be put in place thus, so that the first real image may fall within the second interval, the final truly in the fourth interval.*

SOLUTION

Therefore of the four letters P, Q, R, S the second and the fourth will be negative; from which there may be put $Q = -k$ and $S = -k'$, so that there shall become $PkRk' = \frac{ma}{h} = \mathfrak{M}$; hence the focal length of the final lens will be

$$t = \frac{ABCD}{PkRk'} \cdot a = \frac{ABCD}{\mathfrak{M}} \cdot a,$$

which must be positive, and equal to the interval between the lenses, which are

$$\text{first} = Aa(1-\tfrac{1}{P}), \quad \text{second} = -\tfrac{AB}{P} \cdot a(1+\tfrac{1}{k}),$$
$$\text{third} = -\tfrac{ABC}{Pk} \cdot a(1-\tfrac{1}{R}), \quad \text{fourth} = +\tfrac{ABCD}{PkR} \cdot a(1+\tfrac{1}{k'});$$

therefore so that both the final lens as well as the last interval shall become positive, there must be $ABCD > 0$. Hence as the third also may become positive, there must be

$$-D\left(1-\tfrac{1}{R}\right) > 0$$

and thus D is defined as almost zero. But on account of the second interval there must become $-AB>0$ and on account of the first $Aa(1-\frac{1}{P})>0$. Therefore then there will have to be $CD<0$.

Now there may be put

$$M=\frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t}}{\mathfrak{M}-1},$$

and since the maximum field of view is desired, it will be able to take $\mathfrak{s}=1$ and $\mathfrak{t}=1$ at once, so that there may become

$$M=\frac{\mathfrak{q}+\mathfrak{r}+2}{\mathfrak{M}-1}$$

and hence the distance of the eye

$$O=\frac{t}{M\mathfrak{M}};$$

which since it shall be positive, the removal of the margin provides

$$0=\frac{\mathfrak{q}}{P}-\frac{\mathfrak{r}}{Pk}-\frac{1}{PkR}+\frac{1}{PkRk'},$$

from which there is deduced

$$\frac{1}{k'}=-\mathfrak{q}kR+\mathfrak{r}R+1;$$

hence there will become

$$PkR=\mathfrak{M}\left(1+\mathfrak{r}R-\mathfrak{q}kR\right)$$

and thus it is apparent to be $1+\mathfrak{r}R>qkR$. Truly besides we must consider the following equations:

$$1.\ \mathfrak{B}\mathfrak{q}=\left(P-1\right)M,$$
$$2.\ \mathfrak{C}\mathfrak{r}=-\left(1+Pk\right)M-\mathfrak{q},$$
$$3.\ \mathfrak{D}=-\ (1+PkR)M-\mathfrak{q}-\mathfrak{r}.$$

Here for the sake of brevity we may put, as before

$$\frac{1-P}{\mathfrak{B}}=\zeta \text{ and } 1+Pk=\eta$$

and there will become

$$\mathfrak{q}=-\zeta M \text{ and } \mathfrak{r}=\frac{(\zeta-\eta)M}{\mathfrak{C}},$$

from which there is gathered

$$2+\mathfrak{q}+\mathfrak{r}=\frac{2\mathfrak{C}+\left(\zeta\left(1-\mathfrak{C}\right)-\eta\right)M}{\mathfrak{C}}=\left(\mathfrak{M}-1\right)M;$$

from which equation there is deduced

$$M=\frac{2\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}-1)-\zeta(1-\mathfrak{C})+\eta}=\frac{2\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta};$$

from which value in turn there will be

$$\mathfrak{q}=-\frac{2\zeta\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}\quad\text{and}\quad\mathfrak{r}=\frac{2(\zeta-\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}.$$

Now so that we may have an account of the colored margin, there will be at once

$$1+\mathfrak{r}R-\mathfrak{q}kR=\frac{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta+2(\zeta-\eta)R+2\zeta\mathfrak{C}kR}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}.$$

And since on account of $Pk=\eta-1$ there shall become $PkR=(\eta-1)R,$ there will be

$$\begin{aligned}&\mathfrak{C}(\eta-1)R(\mathfrak{M}+\zeta-1)-(\eta-1)(\zeta-\eta)R-\mathfrak{M}\mathfrak{C}(\mathfrak{M}+\zeta-1)\\&+\mathfrak{M}(\zeta-\eta)-2\mathfrak{M}(\zeta-\eta)R-2\mathfrak{M}\zeta\mathfrak{C}kR=0.\end{aligned}$$

But hence before we may determine either k or R , we must consider the ratio of the letter $\mathfrak{D}$ from the third equation above ; therefore since without doubt PkR shall be a very large number involving $\mathfrak{M}$, it is understood readily the letter $\mathfrak{D}$ to be negative; from which also the letter D will be negative and thus we conclude there is going to become $C>0$ and hence $\mathfrak{C}<1$. On account of the same, truly there will have to be the ratio $R>1$, thus so that this letter may be able to be considered to some extent as known; whereby from that equation we deduce :

$$k=\left\{\begin{array}{l}\mathfrak{C}(\eta-1)R(\mathfrak{M}+\zeta-1)-(\eta-1)(\zeta-\eta)R\\-\mathfrak{M}\mathfrak{C}(\mathfrak{M}+\zeta-1)+\mathfrak{M}(\zeta-\eta)-2\mathfrak{M}(\zeta-\eta)R\end{array}\right\}:2\mathfrak{M}\zeta\mathfrak{C}R$$

and hence $P=\frac{(\eta-1)}{k}$, thus so that there shall be $\eta>1$. Whereby so that the value of k may become positive, there must become

$$\begin{aligned}&R\big(\mathfrak{C}(\eta-1)(\mathfrak{M}+\zeta-1)-(\eta-1)(\zeta-\eta)-2\mathfrak{M}(\zeta-\eta)\big)\\&\qquad>\mathfrak{M}\big(\mathfrak{C}\mathfrak{M}+\mathfrak{C}(\zeta-1)-\zeta+\eta\big),\end{aligned}$$

for which in the first place it is required, that the size of the magnifying letter R shall be positive, and since $\mathfrak{M}$ is a very large number, its coefficient before all must be positive, from which it is gathered

$$\mathfrak{C}(\eta-1)>2(\zeta-\eta),$$

from which it is concluded

$$\mathfrak{C}>\tfrac{2(\zeta-\eta)}{(\eta-1)};$$

there since $\mathfrak{C}<1$, there will become

$$2(\zeta-\eta)<\eta-1 \text{ and thus } \zeta<\tfrac{3\eta-1}{2}.$$

From which condition satisfied there must become

$$R>\tfrac{\mathfrak{M}(\mathfrak{C}\mathfrak{M}+\mathfrak{C}(\zeta-1)-\zeta+\eta)}{\mathfrak{C}(\eta-1)(\mathfrak{M}+\zeta-1)-2\mathfrak{M}(\zeta-\eta)-(\eta-1)(\zeta-\eta)},$$

and hence by retracing our steps all the elements will be determined. Truly the rest is set out as in the preceding problem.

COROLLARY 1

262. Here therefore the letter R will denote a number involving a great $\mathfrak{M}$, then the condition $\zeta<\tfrac{3\eta-1}{2}$ maximally agrees with our set up, as the condition of the field demands especially, lest there shall be a need for ζ to be increased further. Whereby, since there shall be $\eta>1$ always, it will be observed to be most convenient to put $\zeta=\eta$ as in the preceding problem, thus so that the aperture of the third lens again may become a minimum, and there may be produced

$$M=\tfrac{2\mathfrak{C}}{\mathfrak{C}\mathfrak{M}+\mathfrak{C}\eta-\mathfrak{C}}=\tfrac{2}{\mathfrak{M}+\eta-1}.$$

COROLLARY 2

263. Moreover by taking $\zeta=\eta$ for $\mathfrak{C}$ the limits will be $\mathfrak{C}<1$ and $\mathfrak{C}>0$. Again there must be taken $R>\tfrac{\mathfrak{M}}{\eta-1}$ and thence there will become

$$k=\tfrac{((\eta-1)R-\mathfrak{M})(\mathfrak{M}+\eta-1)}{2\mathfrak{M}\eta R}$$

and

$$P=\tfrac{2\mathfrak{M}\eta(\eta-1)R}{((\eta-1)R-\mathfrak{M})(\mathfrak{M}+\eta-1)}.$$

Besides truly there will be

$$\mathfrak{B}=\tfrac{-(\eta-1)R((2\eta-1)\mathfrak{M}-\eta+1)-\mathfrak{M}(\mathfrak{M}+\eta-1)}{\eta((\eta-1)R-\mathfrak{M})(\mathfrak{M}+\eta-1)}$$

Finally indeed there will be found

$$\mathfrak{D} = -\frac{2(1+(\eta-1)R-\eta)}{\mathfrak{M}+\eta-1}$$

or, since $\mathfrak{M}$ and *R* shall be exceedingly large numbers, there will be approximately

$$\mathfrak{D} = -\frac{2(\eta-1)R}{\mathfrak{M}},$$

which value certainly is negative, as we have put above.

COROLLARY 3

264. But if also there will be put $\zeta = 0$; so that the limits for $\mathfrak{C}$ will be $\mathfrak{C} < 1$ and $\mathfrak{C} > -\frac{2\eta}{\eta-1}$; which is satisfied, provided $\mathfrak{C}$ may be contained between 1 and 0. Then truly it will be required to take

$$R > \frac{\mathfrak{M}(\mathfrak{C}\mathfrak{M}-\mathfrak{C}+\eta)}{\mathfrak{C}(\eta-1)(\mathfrak{M}-1)+2\mathfrak{M}\eta+\eta(\eta-1)},$$

or on account of the very large number $\mathfrak{M}$,

$$R > \frac{\mathfrak{C}\mathfrak{M}}{\mathfrak{C}(\eta-1)+2\eta}.$$

Hence truly again it follows $k = \infty$ and $P = 0$, thus so that there shall become $Pk = \eta - 1$. Truly besides there will arise

$$\mathfrak{B} = \infty \text{ and } B = -1.$$

Finally indeed on account of

$$\mathfrak{q} = 0 \text{ and } \mathfrak{r} = -\frac{\eta M}{\mathfrak{C}}$$

there will become

$$\mathfrak{D} = -\left(1+(\eta-1)R-\frac{\eta}{\mathfrak{C}}\right)M$$

and thus on account of

$$M = \frac{2\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}-1)+\eta},$$

which value of *M* is a little smaller than in the preceding case, there will become

$$\mathfrak{D} = -\frac{2(\eta-1)R}{\mathfrak{M}}.$$

SCHOLIUM

265. Although this case may appear unexpected as on account of $\mathfrak{B}=\infty$ then truly as well because $P=0$, yet actually it is true and is reduced to the case set out in the previous chapter; for since $\mathfrak{B}=\infty$, the focal length of the second lens is infinite and thus is reduced to the same thing, and if the second lens becomes plane, thus so that no longer will there be a question concerning its place, even if the first interval may produce

$$=Aa\left(1-\tfrac{1}{P}\right)=-\infty$$

and the second

$$=Aa\left(\tfrac{1}{P}+\tfrac{1}{\eta-1}\right)=+\infty,$$

yet the sum of these, which now alone is required to be considered, shall be finite

$$=Aa\left(1+\tfrac{1}{\eta-1}\right)=\tfrac{\eta}{\eta-1}\cdot Aa.$$

Therefore since only four lenses will be had here, this case is required to be referred to the previous chapter. Yet meanwhile this inconvenience will have to arise, when ζ approaches almost to 0, since then also P will be less than unity, thus so that A will have to become a negative number and $B>0$. But since there shall be $\mathfrak{B}=\frac{1-P}{\zeta}$, certainly there shall be $\mathfrak{B}>0$, in addition truly it is necessary, that there shall be $1-P<\zeta$ or $P>1-\zeta$, or P will have to be contained within the limits 1 and $1-\zeta$ or there must become $k<\frac{\eta-1}{1-\zeta}$; where, since $\mathfrak{M}$ and R shall be exceedingly large numbers, there will have to become

$$R\left(\tfrac{\mathfrak{C}(\eta-1)(1-3\zeta)}{1-\zeta}-2(\zeta-\eta)\right)<\mathfrak{C}\mathfrak{M},$$

which happens at once on account of $\zeta<\eta$, if there were

$$\tfrac{\mathfrak{C}(\eta-1)(1-3\zeta)}{(1-\zeta)(\zeta-\eta)}>2 \text{ or } \mathfrak{C}>\tfrac{2(1-\zeta)(\zeta-\eta)}{(\eta-1)(1-3\zeta)}.$$

But if there shall be $\frac{\mathfrak{C}(\eta-1)(1-3\zeta)}{(1-\zeta)(\zeta-\eta)}<2$, there must become

$$R<\tfrac{(1-\zeta)\mathfrak{C}\mathfrak{M}}{\mathfrak{C}(\eta-1)(1-3\zeta)-2(\zeta-\eta)(1-\zeta)},$$

from which observations, we may set out some cases in more detail.

CASE 1, WHERE $\zeta = \eta$

266. Now in this case we have seen the third lens to be left to our choice, provided for that there may be taken $\mathfrak{C} < 1$ and $\mathfrak{C} > 0$, so that C may become a positive number ; from which, if the circumstances demand, that C shall be a large enough number, then $\mathfrak{C}$ will be able to become a little less than unity; then also we have observed there must be taken $R > \frac{\mathfrak{M}}{\eta - 1}$ from which, since there shall be always $\eta > 1$, if indeed it were > 2, then it will be convenient to take $R = \mathfrak{M}$. Moreover it may be observed the letter η not to be taken exceedingly large, since there becomes for the field

$$M = \frac{2}{\mathfrak{M} + \eta - 1}.$$

From this indeed there will be produced

$$k = \frac{((\eta - 1)R - \mathfrak{M})(\mathfrak{M} + \eta - 1)}{2\mathfrak{M}\eta R},$$

whereby, for the greater magnifications, it will be able to take safely

$$k = \frac{(\eta - 1)R - \mathfrak{M}}{2\eta R};$$

from which it is apparent the letter k to become smaller thus, so that with a smaller R it may exceed the prescribed limit $\frac{\mathfrak{M}}{\eta - 1}$; from which there becomes

$$P = \frac{\eta - 1}{k} = \frac{2\eta(\eta - 1)R}{(\eta - 1)R - \mathfrak{M}}.$$

For the remaining elements at first there is produced

$$\mathfrak{B} = \frac{-(\eta - 1)R((2\eta - 1)\mathfrak{M} - \eta + 1) - \mathfrak{M}(\mathfrak{M} + \eta - 1)}{\eta((\eta - 1)R - \mathfrak{M})(\mathfrak{M} + \eta - 1)}$$

and hence approximately,

$$\mathfrak{B} = \frac{-(\eta - 1)(2\eta - 1)R - \mathfrak{M}}{\eta(\eta - 1)R - \eta\mathfrak{M}},$$

which value evidently is negative, from which also B will be negative. Then since there shall be $P > 1$, by the same reasoning the letter A will have to become positive ; from which the product AB on account of

$$B = \frac{-(\eta - 1)(2\eta - 1)R - \mathfrak{M}}{(\eta - 1)(3\eta - 1)R - (\eta - 1)\mathfrak{M}}$$

at once will become negative, to be used precisely as the prescribed conditions demand. Finally indeed there is found

$$\mathfrak{D} = -\frac{2(1+(\eta-1)R-\eta)}{\mathfrak{M}+\eta-1}$$

and thus approximately,

$$\mathfrak{D} = -\frac{2(\eta-1)R}{\mathfrak{M}};$$

from which there becomes

$$D = -\frac{2(\eta-1)R}{\mathfrak{M}+2(\eta-1)R}.$$

From these defined, in the first place the focal lengths will become :

$$p = \mathfrak{A}a,\quad q = -\frac{A\mathfrak{B}}{P}\cdot a = \frac{(\eta-1)(2\eta-1)R+\mathfrak{M}}{2\eta^2(\eta-1)R}\cdot Aa,$$

$$r = -\frac{AB\mathfrak{C}}{\eta-1}\cdot a,\quad s = \frac{ABC\mathfrak{D}}{(\eta-1)R}\cdot a \text{ and } t = \frac{ABCD}{\mathfrak{M}}\cdot a,$$

truly then the intervals become:

$$\text{first} = Aa\left(1-\frac{1}{P}\right),\quad \text{second} = -ABa\left(\frac{1}{P}+\frac{1}{\eta-1}\right),$$

$$\text{third} = -\frac{ABCa}{\eta-1}\left(1-\frac{1}{R}\right),\quad \text{fourth} = ABCDa\left(\frac{1}{(\eta-1)R}+\frac{1}{\mathfrak{M}}\right).$$

The distance of the eye will become

$$O = \frac{t}{M\mathfrak{M}} = \frac{t}{2}\cdot\frac{\mathfrak{M}+\eta-1}{\mathfrak{M}} = \frac{1}{2}t\cdot\left(1+\frac{\eta-1}{\mathfrak{M}}\right) = \frac{1}{2}t \text{ approx.}$$

Truly for the aperture of the first lens indeed the aperture is defined most securely by experimentation; from which the letter x is found and from that the measure of the clarity $= \frac{20x}{\mathfrak{M}}$, if indeed x may be expressed in inches.

Truly for the second lens, since there shall be

$$\mathfrak{q} = -\eta M = -\frac{2\eta}{\mathfrak{M}+\eta-1} = -\frac{2\eta}{\mathfrak{M}},$$

the radius of its aperture will be

$$= \frac{1}{4}\mathfrak{q}q + \frac{x}{P} = \frac{1}{2}\cdot\frac{\eta}{\mathfrak{M}}\cdot q + \frac{x}{P}.$$

Truly for the third lens on account of $\mathfrak{r} = 0$ the radius of the aperture suffices $= \frac{x}{\eta-1}$; indeed the remaining lenses are agreed to be made equally convex on both sides.

COROLLARY

267. If there may be taken $R=\frac{\mathfrak{M}}{\eta-1}$, so that there may become $k=0$ and $P=\infty$ with there being $Pk=\eta-1$, then there becomes $\mathfrak{B}=\frac{1-P}{\eta}=\infty$ and $B=-1$. Therefore then the second lens falls on that first real image on account of the first interval $=Aa=\alpha$, and its focal length will be $q=\frac{Aa}{\eta}$. But truly the second interval in this case arises $=-\frac{ABa}{\eta-1}=\frac{Aa}{\eta-1}$. Indeed the remaining intervals are defined in general, only if there may be observed to become $\mathfrak{D}=-2$ and $D=-\frac{2}{3}$.

EXAMPLE 1

[267*a*]. We may put $\eta=2$ and there will have to be taken $R>\mathfrak{M}$. Truly nothing stands in the way, why we may not assume the second preceding corollary $R=\mathfrak{M}$, thus so that then there may become $k=0$ and $P=\infty$; whereby in the first place the focal lengths may be expressed thus:

$$p=\mathfrak{A}a,\quad q=\frac{Aa}{2},\quad r=A\mathfrak{C}a,\quad s=\frac{2AC\mathfrak{C}}{\mathfrak{M}}\cdot a \text{ and } t=\frac{2}{3}\cdot\frac{C}{\mathfrak{M}}\cdot Aa=\frac{1}{3}s.$$

Then the intervals will be found thus :

$$\text{first}=Aa,\quad \text{second}=Aa,$$
$$\text{third}=C\left(1-\frac{1}{\mathfrak{M}}\right)Aa,\quad \text{fourth}=\frac{4}{3}\cdot\frac{C}{\mathfrak{M}}\cdot Aa;$$

truly the distance of the eye

$$O=\frac{1}{2}t\left(1+\frac{1}{\mathfrak{M}}\right).$$

For the value of x defined either by trial and error or by the know formula, the radius of the aperture of the second lens $=\frac{q}{\mathfrak{M}}=\frac{1}{2}\cdot\frac{Aa}{\mathfrak{M}}$ and of the third lens $=x$; the radius of the area viewed will be $z=\frac{a}{2\mathfrak{M}}$, and the measure of the clarity $=\frac{20x}{\mathfrak{M}}$.

COROLLARY

268. These formulas also will be able to be applied to telescopes by taking $Aa=p$ and $\mathfrak{M}=m$. Then truly this focal length will have to be taken

$$p=m\sqrt[3]{\mu m}\left(1+\ * \ +\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu}{C\mathfrak{C}}\right)$$

with the following terms omitted divisible by $\mathfrak{M}$.

EXAMPLE 2

269. Now there shall be $\eta = 3$, so that there must be $R > \frac{\mathfrak{M}}{2}$, and there may be assumed $R = \mathfrak{M}$, from which there becomes $k = \frac{1}{6}$ and $P = 12$; whereby the remaining elements will become

$$\mathfrak{B} = -\tfrac{11}{12} \text{ and hence } B = -\tfrac{11}{14},\ \mathfrak{D} = -4 \text{ and hence } D = -\tfrac{4}{5},$$

from which the focal distances emerge

$$p = \mathfrak{A}a,\ \ q = \tfrac{11}{36}Aa,\ \ r = \tfrac{11}{28}\mathfrak{C}Aa,\ \ s = \tfrac{11}{7}\tfrac{C}{\mathfrak{M}}\cdot Aa \text{ and } t = \tfrac{22}{35}\tfrac{C}{\mathfrak{M}}\cdot Aa = \tfrac{2}{5}s$$

and the intervals of the lenses

$$\text{first} = \tfrac{11}{12}Aa, \quad \text{second} = \tfrac{11}{24}Aa,$$
$$\text{third} = \tfrac{11}{28}C\left(1 - \tfrac{1}{\mathfrak{M}}\right)Aa,\ \ \text{fourth} = \tfrac{33}{35}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa$$

and the distance of the eye

$$O = \tfrac{1}{2}t\left(1 + \tfrac{2}{\mathfrak{M}}\right);$$

truly the radius of the area viewed in the object will be $z = \frac{a}{2(\mathfrak{M}+2)}$.

The radius of the aperture of the second lens $= \frac{3}{2}\cdot\frac{q}{\mathfrak{M}} + \frac{x}{12}$, and of the third lens $= \frac{x}{2}$ where x will be known either experimentally, or from the know formula, and the order of the clarity $= \frac{20x}{\mathfrak{M}}$.

COROLLARY

270. These formulas also can be adapted for telescopes ; indeed there will become $Aa = p$ and $\mathfrak{M} = m$. Then truly the focal length of the objective lens is defined by this formula:

$$p = m\sqrt[3]{\mu m}\left(1 + \tfrac{1}{12}\left(\tfrac{3^3}{11^3}\lambda' - \tfrac{3\cdot 14\nu}{11^2}\right) + \tfrac{14^3}{11^3\cdot 2}\left(\tfrac{\lambda''}{\mathfrak{C}^3} + \tfrac{\nu}{C\mathfrak{C}}\right)\right).$$

The length of this telescope will be around $= \left(\frac{11}{8} + \frac{11}{28}C\right)p$.

EXAMPLE 3

271. Now there shall be $\eta = 6$, so that there must become $R > \frac{\mathfrak{M}}{5}$ and there may be taken $R = \frac{\mathfrak{M}}{2}$, from which there becomes $k = \frac{1}{4}$ and $P = 20$. Hence again there will become $\mathfrak{B} = -\frac{19}{6}$ and $B = -\frac{19}{25}$. Then truly $\mathfrak{D} = -5$ and $D = -\frac{5}{6}$.
Therefore the focal lengths will themselves be had thus :

$$p = \mathfrak{A}a,\quad q = \tfrac{19}{120}Aa,\quad r = \tfrac{19}{125}\mathfrak{C}Aa,\quad s = \tfrac{38}{25}\tfrac{C}{\mathfrak{M}}\cdot Aa \text{ and } t = \tfrac{19}{30}\tfrac{C}{\mathfrak{M}}\cdot Aa \text{ or } t = \tfrac{5}{12}s$$

and the intervals of the lenses

$$\text{first} = \tfrac{29}{10}Aa,\quad \text{second} = \tfrac{19}{100}Aa,$$

$$\text{third} = \tfrac{19}{125}C\left(1-\tfrac{2}{\mathfrak{M}}\right)Aa, \text{ and fourth} = \tfrac{133}{150}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa,$$

and the distance of the eye

$$O = \tfrac{1}{2}t\left(1+\tfrac{5}{\mathfrak{M}}\right),$$

the radius of the area viewed

$$z = \frac{a}{2(\mathfrak{M}+5)}.$$

Finally with x defined as before, the radius of the aperture of the second lens $= \frac{3}{\mathfrak{M}}\cdot q + \frac{x}{20}$, and of the third lens $= \frac{1}{5}x$; moreover the degree of clarity remains $= \frac{20x}{\mathfrak{M}}$.

EXAMPLE 4

272. There shall be $\eta = 6$ as before, truly there may be taken $R = \mathfrak{M}$ and there will be found $k = \frac{1}{3}$ and $P = 15$. Hence again there will become

$$\mathfrak{B} = -\tfrac{7}{3} \text{ and } B = -\tfrac{7}{10}, \text{ but } \mathfrak{D} = -10 \text{ and } D = -\tfrac{10}{11}.$$

Therefore the focal lengths will be

$$p = \mathfrak{A}a,\quad q = \tfrac{7}{45}Aa,\quad r = \tfrac{7}{50}\mathfrak{C}Aa,$$

$$s = \tfrac{7}{5}\tfrac{C}{\mathfrak{M}}\cdot Aa,\quad t = \tfrac{7}{11}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa \text{ or } t = \tfrac{5}{11}s$$

and the intervals of the lenses :

$$\text{first} = \tfrac{14}{15}Aa,\quad \text{second} = \tfrac{14}{75}Aa,$$

$$\text{third} = \tfrac{7}{50}C\left(1-\tfrac{2}{\mathfrak{M}}\right)Aa,\quad \text{fourth} = \tfrac{42}{55}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa,$$

and the distance of eye

$$O = \tfrac{1}{2} t \left(1 + \tfrac{5}{\mathfrak{M}}\right),$$

and equally the remaining moments will be had as before.

COROLLARY

273. If these may be transferred to telescopes by putting $Aa = p$ and $\mathfrak{M} = m,$ the latter case will be seen here to be preferred before the previous case, since it may produce a slightly smaller length, certainly which shall be $= (1\tfrac{3}{25} + \tfrac{7}{50} C) p$ with the terms divided by $\mathfrak{M}$ ignored, as from the preceding example it would have been $= (1\tfrac{7}{50} + \tfrac{19}{125} C) p$. Therefore in the case of the final example the focal length of the objective lens will be defined thus, so that it shall become

$$p = m\sqrt[3]{\mu m}\left(1 + \tfrac{1}{15}\left(\tfrac{3^3}{7^3}\lambda' - \tfrac{30\nu}{7^2}\right) + \tfrac{200}{343}\left(\tfrac{\lambda''}{\mathfrak{C}^3} + \tfrac{\nu}{C\mathfrak{C}}\right)\right).$$

CASE 2 , WHERE $\zeta = 1$

274. Since there shall be $\zeta = 1$, the bounds for $\mathfrak{C}$ will be $\mathfrak{C} < 1$ and $\mathfrak{C} > -2$, thus so that $\mathfrak{C}$ may be allowed to be chosen by us as before. Then truly there will have to be

$$R > \frac{\mathfrak{M}(\mathfrak{C}\mathfrak{M} + \eta - 1)}{(\eta - 1)(\mathfrak{C}\mathfrak{M} + 2\mathfrak{M} + \eta - 1)}$$

or with the smaller parts ignored,

$$R > \frac{\mathfrak{C}\mathfrak{M}}{(\eta - 1)(2 + \mathfrak{C})}.$$

Therefore there may be put $R = i\mathfrak{M}$ evidently by taking

$$i > \frac{\mathfrak{C}}{(\eta - 1)(2 + \mathfrak{C})}.$$

Therefore then there will become

$$k = \frac{\mathfrak{C}(\eta i - i - 1) + 2i(\eta - 1)}{2i\mathfrak{C}} \quad \text{and} \quad P = \frac{2i(\eta - 1)\mathfrak{C}}{\mathfrak{C}(i(\eta - 1) - 1) + 2i(\eta - 1)}.$$

Hence therefore there will become

$$P - 1 = \frac{\mathfrak{C}(i(\eta - 1) + 1) - 2i(\eta - 1)}{\mathfrak{C}(i(\eta - 1) - 1) + 2i(\eta - 1)}$$

which value will be positive or $P > 1$, if there were

$$i<\frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})}.$$

Therefore in this case there is produced

$$\mathfrak{B}=\frac{-\mathfrak{C}(i(\eta-i)+1)+2i(\eta-1)}{\mathfrak{C}(i(\eta-i)-1)+2i(\eta-1)};$$

which value since it shall be negative, also B will be negative and on account of $P>1$, A must be positive, as the above conditions demand; but if there shall be

$$i>\frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})},$$

then there shall become $P<1$ and A will have to be taken negative, and there will be produced $\mathfrak{B}>0$; from which, so that also B may become positive, in addition it is necessary, that there shall be $\mathfrak{B}<1$, which is evident, since there shall be $\mathfrak{B}=1-P$. Indeed afterwards for finding $\mathfrak{D}$ there will be observed to become

$$M=\frac{2\mathfrak{C}}{\mathfrak{C}\mathfrak{M}+\eta-1}$$

and

$$\mathfrak{q}=-M \quad\text{and}\quad \mathfrak{r}=-\frac{(\eta-1)M}{\mathfrak{C}},$$

from which there arises

$$\mathfrak{D}=-i(\eta-1)M\mathfrak{M}=-2i(\eta-1);$$

truly those values will be changed into these :

$$\mathfrak{q}=-\frac{2}{\mathfrak{M}} \quad\text{and}\quad \mathfrak{r}=-\frac{(\eta-1)}{\mathfrak{C}\mathfrak{M}}.$$

With those values found the focal lengths will be considered, which are

$$p=\mathfrak{A}a,\quad q=-\frac{A\mathfrak{B}}{P}\cdot a,\quad r=-\frac{AB\mathfrak{C}}{\eta-1}\cdot a,\quad s=\frac{ABC\mathfrak{D}}{(\eta-1)\mathfrak{M}}\cdot a \quad\text{and}\quad t=\frac{ABCD}{\mathfrak{M}}\cdot a.$$

Then truly the intervals become:

$$\text{first}=Aa\left(1-\frac{1}{P}\right),\quad \text{second}=-ABa\left(\frac{1}{P}+\frac{1}{\eta-1}\right),$$
$$\text{third}=-\frac{ABCa}{\eta-1}\left(1-\frac{1}{i\mathfrak{M}}\right),\ \text{fourth}=+\frac{ABCDa}{\mathfrak{M}}\left(\frac{1}{i(\eta-1)}+1\right),$$

the distance of the eye

$$O = \frac{t}{\mathfrak{M}M} = \frac{1}{2}t\left(1+\frac{\eta-1}{\mathfrak{C}\mathfrak{M}}\right).$$

Thence in a similar manner, as we have observed now before, the letter x will be required to be defined either by experimentation or from the observed formula. Then truly the radius of the aperture of the second lens will become

$$= \frac{1}{4}\mathfrak{q}q + \frac{x}{P} = \frac{1}{2}\cdot\frac{q}{\mathfrak{M}} + \frac{x}{P},$$

truly the radius of the aperture of the third lens

$$= \frac{1}{4}\mathfrak{r}r + \frac{x}{\eta-1} = \frac{1}{2}\cdot\frac{(\eta-1)r}{\mathfrak{C}\mathfrak{M}} + \frac{x}{\eta-1},$$

the remaining lenses, since they are equally convex, allow the maximum aperture. Finally for the area viewed there will be

$$z = \frac{1}{2}\cdot\frac{\mathfrak{C}a}{\mathfrak{C}\mathfrak{M}+\eta-1} = \frac{a}{2\mathfrak{M}} \text{ approx.}$$

and the measure of the clarity $= \frac{20x}{\mathfrak{M}}$.

COROLLARY 1

275. If the letter i may be contained within these bounds, evidently

$$i > \frac{\mathfrak{C}}{(\eta-1)(2+\mathfrak{C})} \text{ and } i < \frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})},$$

then there will become $P > 1$ and the letters $\mathfrak{B}$ and B become negative, truly the letter A must be taken positive; from which all the elements are of same kind used in the preceding case.

COROLLARY 2

276. But if there were thus $i > \frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})}$, then the letter P shall become smaller than unity and hence both the letter $\mathfrak{B}$ as well as B become positive; but truly the letter A will have to be negative, which can happen in two ways, the one, where $\mathfrak{A} > 1$, the other indeed, where $\mathfrak{A} < 0$; where for the second case the first lens will become concave and the instrument will be liable to become much more inconvenient.

COROLLARY 3

277. But if there were $i=\frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})}$, then there would become $P=1$ and hence $\mathfrak{B}=0$ and $B=0$. Therefore then, lest there may become $q=0$, it will be necessary to take $A=\infty$, yet still, so that there shall become

$$AB=A\mathfrak{B}=-\frac{q}{a}.$$

From which, since there shall become $1-P=\mathfrak{B}$, the first interval will be

$$=Aa\left(1-\frac{1}{P}\right)=-A\mathfrak{B}a=q;$$

whereby on account of $\mathfrak{A}=1$ the focal lengths will become

$$p=a,\quad q=q,\quad r=\frac{\mathfrak{C}q}{\eta-1}\cdot a,\quad s=-\frac{C\mathfrak{D}}{i(\eta-1)\mathfrak{M}}\cdot q \text{ or } s=\frac{2C}{\mathfrak{M}}\cdot q$$

and

$$t=-\frac{CD}{\mathfrak{M}}\cdot q \quad \text{seu} \quad t=\frac{2i(\eta-1)C}{2i(\eta-1)+1}\cdot\frac{q}{\mathfrak{M}}.$$

The intervals of the lenses:

$$\text{first}=q,\quad \text{second}=\frac{\eta}{\eta-1}\cdot q,\quad \text{third}=\frac{C}{\eta-1}\left(1-\frac{1}{i\mathfrak{M}}\right)q,$$

$$\text{fourth}=-\frac{CDa}{\mathfrak{M}}\left(1+\frac{1}{i(\eta-1)}\right)q=\frac{2i(\eta-1)+2}{2i(\eta-1)+1}\cdot\frac{C}{\mathfrak{M}}\cdot q.$$

Nothing requiring to be changed occurs in the remaining moments.

SCHOLIUM

278. But here is required to be observed properly the case contained in the latter two corollaries by no means can be transferred to telescopes. Indeed for telescopes, on account of $a=\infty$, it must be assumed by necessity $\mathfrak{A}=0$ and $A=0$, since in these cases A must be either infinite or negative.

EXAMPLE 1

279. We may take $\eta=2$, and since C occurs in these terms only, which are divided by $\mathfrak{M}$, and thus always must indicate an exceptionally large number, for $\mathfrak{C}$ we will be able to assume correctly to be unity; hence therefore for the letter R the first limit will be $i>\frac{1}{3}$; so that our instruments may pertain to the case of the first corollary, there must be

assumed also $i<1$; hence there may be taken therefore $i=\frac{1}{2}$, so that there shall become $R=\frac{1}{2}\mathfrak{M}$; from which we deduce $k=\frac{1}{2}$ and $P=2$, then $\mathfrak{B}=-1$, $B=-\frac{1}{2}$ and $\mathfrak{D}=-1$, hence $D=-\frac{1}{2}$. Hence the focal lengths will be

$$p=\mathfrak{A}a,\quad q=\tfrac{1}{2}Aa,\quad r=\tfrac{\mathfrak{C}}{2}\cdot Aa,\quad s=\tfrac{CAa}{\mathfrak{M}} \text{ and } t=\tfrac{1}{4}\tfrac{C}{\mathfrak{M}}\cdot Aa \text{ or } t=\tfrac{1}{4}s.$$

Truly the intervals of the lenses will be :

$$\text{first}=\tfrac{1}{2}Aa,\quad \text{second}=\tfrac{3}{4}Aa,\quad \text{third}=\tfrac{C}{2}\left(1-\tfrac{2}{\mathfrak{M}}\right)Aa,\quad \text{fourth}=\tfrac{3}{4}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa$$

and the distance of the eye

$$O=\tfrac{1}{2}t\left(1+\tfrac{1}{\mathfrak{M}}\right),$$

finally the radius of the aperture :

$$\text{of the first lens}=x,\ \text{of the second lens}=\tfrac{1}{2}\cdot\tfrac{q}{\mathfrak{M}}+\tfrac{1}{2}x,\ \text{of the third lens}=\tfrac{1}{2}\cdot\tfrac{r}{\mathfrak{M}}+x.$$

EXAMPLE 2

280. There may remain $\eta=2$, truly there may be taken $i=1$ or $R=\mathfrak{M}$ and there will be $k=1$ and $P=1$, then truly $\mathfrak{B}=0=B$; $\mathfrak{D}=-2$ and $D=-\frac{2}{3}$; from which from the third corollary we come upon

$$p=a,\quad q=q,\quad r=q,\quad s=\tfrac{2C}{\mathfrak{M}}\cdot q,\quad t=\tfrac{2}{3}\cdot\tfrac{C}{\mathfrak{M}}\cdot q \text{ or } t=\tfrac{1}{3}s.$$

Truly the intervals of the lenses will be

$$\text{first}=a\quad \text{second}=2q,\quad \text{third}=C\left(1-\tfrac{1}{\mathfrak{M}}\right),\quad \text{fourth}=\tfrac{4}{3}\cdot\tfrac{C}{\mathfrak{M}}\cdot q;$$

truly the remaining moments themselves will be had likewise as in the preceding example.

EXAMPLE 3.

281. There may remain $\eta=2$ and there may be taken $i=-2$, so that there shall be $R=2\mathfrak{M}$; hence therefore $k=\frac{5}{4}$ and $P=\frac{4}{5}$, from which

$$\mathfrak{B}=\tfrac{1}{5},\ \text{hence } B=\tfrac{1}{4};\ \text{then truly } \mathfrak{D}=-4 \text{ and } D=-\tfrac{4}{5}.$$

Hence, since A may be taken negative, there may be put

$$A=-\alpha, \text{ so that there shall become } \mathfrak{A}=-\tfrac{\alpha}{1-\alpha}=\tfrac{\alpha}{\alpha-1};$$

from which the focal lengths will be

$$p=\tfrac{\alpha}{\alpha-1},\ \ q=\tfrac{1}{4}\alpha a,\ \ r=\tfrac{1}{4}\mathfrak{C}\alpha a,\ \ s=\tfrac{1}{2}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a \text{ and } t=\tfrac{1}{5}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a \text{ or } t=\tfrac{1}{5}s.$$

Truly the intervals will be

$$\text{first}=\tfrac{1}{4}\alpha a,\quad \text{second}=\tfrac{9}{16}\alpha a,\quad \text{third}=\tfrac{1}{4}C\left(1-\tfrac{1}{2\mathfrak{M}}\right)\alpha a,\ \ \text{fourth}=\tfrac{3}{10}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a.$$

Truly the remaining moments again are as in the first example. But this is required to be understood properly, if there may be taken $\alpha=1$, $p=\infty$ to arise and thus the first lens clearly can be rejected, thus so that the microscope may be composed from the latter lenses only. But since then the confusion produced will be very great, since $\mathfrak{A}=\frac{1}{0}$ must be taken in the formulas from the preceding chapters, we may reject this instrument deservedly and much more so those, if there shall be $\alpha<1$. But if α may hardly exceed unity, these instruments will be seen to have a use in practice.

EXAMPLE 4

282. Now there shall be $\eta=4$, and since the first limit shall be $i>\frac{1}{9}$, for the case from the first corollary we may take $i<\frac{1}{3}$; Therefore there shall be taken $i=\frac{1}{6}$ with $\mathfrak{C}=1$ and there will become $k=\frac{3}{2}$, $P=2$, hence $\mathfrak{B}=-1$, $B=-\frac{1}{2}$, $\mathfrak{D}=-1$, $D=-\frac{1}{2}$; from which the focal lengths will be

$$p=\mathfrak{A}a,\ \ q=\tfrac{1}{2}Aa,\ \ r=\tfrac{1}{6}Aa,\ \ s=\tfrac{C}{\mathfrak{M}}\cdot Aa \text{ and } t=\tfrac{1}{4}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa \text{ or } t=\tfrac{1}{4}s.$$

Truly the intervals of the lenses :

$$\text{first}=\tfrac{1}{2}Aa,\quad \text{second}=\tfrac{5}{12}Aa,\quad \text{third}=\tfrac{C}{6}\left(1-\tfrac{6}{\mathfrak{M}}\right)Aa,\ \ \text{fourth}=\tfrac{3}{4}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa.$$

The distance of the eye

$$O=\tfrac{1}{2}t\left(1+\tfrac{3}{\mathfrak{M}}\right).$$

Again,

$$z=\frac{a}{2(\mathfrak{M}+3)}.$$

The radius of the aperture

of the first lens $= x$, of the second $= \frac{1}{2} \cdot \frac{q}{\mathfrak{M}} + \frac{1}{2} x$, of the third $= \frac{3}{2} \cdot \frac{r}{\mathfrak{M}} + \frac{1}{3} x$.

EXAMPLE 5

283. There may remain $\eta = 4$, but there shall be taken $i = \frac{1}{3}$, following the third corollary and there will become $k = 3$ and $P = 1$; from which the focal lengths are deduced

$$p = a, \quad q = q, \quad r = \tfrac{1}{3} q, \quad s = \tfrac{2C}{\mathfrak{M}} \cdot q \text{ and } t = \tfrac{2}{3} \cdot \tfrac{C}{\mathfrak{M}} \cdot q = \tfrac{1}{3} s$$

and the intervals of the lenses :

$$\text{first} = q, \quad \text{second} = \tfrac{4}{3} q, \quad \text{third} = \tfrac{C}{3}\left(1 - \tfrac{3}{\mathfrak{M}}\right) q, \quad \text{fourth} = \tfrac{4}{3} \cdot \tfrac{C}{\mathfrak{M}} \cdot q = 2t.$$

The distance of the eye

$$O = \tfrac{1}{2} t \left(1 + \tfrac{3}{\mathfrak{M}}\right),$$

and all the remaining moments are as before.

EXAMPLE 6

284. With $\eta = 4$ remaining there may be taken $i = 1$ and there will be $k = 4$ and $P = \frac{3}{4}$, then truly $\mathfrak{B} = \frac{1}{4}$, $B = \frac{1}{3}$; and $\mathfrak{D} = -6$, $D = -\frac{6}{7}$. Therefore since B shall be positive, the letter A must be negative. Therefore let there be $A = -\alpha$ and there will become $\mathfrak{A} = \frac{\alpha}{\alpha - 1}$; from which the focal lengths will be produced

$$p = \tfrac{\alpha}{\alpha - 1} \cdot a, \quad q = \tfrac{1}{3} \alpha a, \quad r = \tfrac{1}{9} \mathfrak{C} \alpha a, \quad s = \tfrac{2}{3} \cdot \tfrac{C}{\mathfrak{M}} \cdot \alpha a \text{ and } t = \tfrac{2}{7} \cdot \tfrac{C}{\mathfrak{M}} \cdot \alpha a = \tfrac{3}{7} s$$

and the intervals of the lenses

$$\text{first} = \tfrac{1}{3} \alpha a, \quad \text{second} = \tfrac{5}{9} \alpha a, \quad \text{third} = \tfrac{1}{9} C \left(1 - \tfrac{1}{\mathfrak{M}}\right) \alpha a, \quad \text{fourth} = \tfrac{8}{21} \cdot \tfrac{C}{\mathfrak{M}} \cdot \alpha a.$$

The distance of the eye O remains the same as before, as well as the remaining moments.

PROBLEM 3

285. *To construct a microscope from six lenses, which shall be arranged thus, so that the prior real image shall fall in the second interval, truly the posterior shall fall in the fourth interval.*

SOLUTION

Therefore of the five letters P, Q, R, S, T the second and the fourth must be negative; whereby there may be put $Q=-k$ and $S=-k'$, so that there shall become

$$PkRk'T=\mathfrak{M}=\tfrac{ma}{h}.$$

Hence the focal length of the final lens will be

$$u=-\tfrac{ABCDE}{\mathfrak{M}}\cdot a,$$

which must be positive and the intervals of the lenses, which are

$$\text{first}=Aa\left(1-\tfrac{1}{P}\right),\quad \text{second}=-\tfrac{AB}{P}\cdot a\left(1+\tfrac{1}{k}\right),$$
$$\text{third}=-\tfrac{ABC}{Pk}\cdot a\left(1-\tfrac{1}{R}\right),\quad \text{fourth}=\tfrac{ABCD}{PkR}\cdot a\left(1+\tfrac{1}{k'}\right),$$
$$\text{fifth}=\tfrac{ABCDE}{PkRk'}\cdot a\left(1-\tfrac{1}{T}\right).$$

On account of the fifth interval there must become therefore $T<1$, on account of the fourth truly $E<0$ and hence $ABCD>0$. On account of the second truly there must be $AB<0$ and hence also CD negative. Now there may be taken

$$M=\tfrac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t}+\mathfrak{u}}{\mathfrak{M}-1},$$

and since the maximum field may be desired, it will be able to take
$\mathfrak{s}=1,\ \mathfrak{t}=1$ and $\mathfrak{u}=1$,
so that there may become

$$M=\tfrac{\mathfrak{q}+\mathfrak{r}+3}{\mathfrak{M}-1}$$

and hence

$$z=Ma\xi=\tfrac{1}{4}Ma$$

and the distance of the eye

$$O=\tfrac{u}{\mathfrak{M}M};$$

which since it shall be positive, the destruction of the colored margin presents

$$0=\frac{\mathfrak{q}}{P}-\frac{\mathfrak{r}}{Pk}-\frac{1}{PkR}+\frac{1}{PkRk'}+\frac{1}{PkRk'T},$$

from which there is deduced

$$\frac{1}{k'}\left(1+\frac{1}{T}\right)=-\mathfrak{q}kR+\mathfrak{r}R+1,$$

and since there may be agreed to be $T<1$, for the sake of brevity there may be put

$$1+\frac{1}{T}=\theta$$

so that there shall become $\theta>2$, and hence

$$T=\frac{1}{\theta-1},$$

from which there will be had

$$\frac{1}{k'}=\frac{-\mathfrak{q}kR+\mathfrak{r}R+1}{\theta};$$

therefore on account of

$$\frac{PkRk'}{\theta-1}=\mathfrak{M}$$

there will become at once

$$PkR=\frac{(\theta-1)\mathfrak{M}}{k'}=\frac{\mathfrak{M}(\theta-1)(1+\mathfrak{r}R-\mathfrak{q}kR)}{\theta}.$$

Now besides the following equations are required to be considered:

$$1.\ \mathfrak{B}\mathfrak{q}=(P-1)M,$$
$$2.\ \mathfrak{C}\mathfrak{r}=-(1+Pk)M-\mathfrak{q},$$
$$3.\ \mathfrak{D}=-(1+PkR)M-\mathfrak{q}-\mathfrak{r},$$
$$4.\ \mathfrak{E}=(PkRk'-1)M-\mathfrak{q}-\mathfrak{r}-1:$$

for the resolution of which we may put for the sake of brevity

$$\frac{1-P}{\mathfrak{B}}=\zeta \text{ and } 1+Pk=\eta,$$

so that there may become

$$\mathfrak{q}=-\zeta M \text{ and } \mathfrak{t}=\frac{(\zeta-\eta)M}{\mathfrak{C}};$$

therefore

$$3+\mathfrak{q}+\mathfrak{r}=\frac{3\mathfrak{C}+(\zeta(1-\mathfrak{C})-\eta)M}{\mathfrak{C}}=M(\mathfrak{M}-1);$$

from which it is concluded

$$M=\frac{3\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta};$$

from which in turn,

$$\mathfrak{q} = -\frac{3\zeta\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta} \text{ and } \mathfrak{r} = -\frac{3(\zeta-\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}.$$

Now therefore we will have $PkR = (\eta-1)R$ or

$$(\eta-1)R = \mathfrak{M}\cdot\tfrac{\theta-1}{\theta}\cdot\left(1+\frac{3R(\zeta+\zeta\mathfrak{C}k-\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}\right),$$

from which it will be agreed to define the letter *k*, on account of the ratios brought forwards before ; from which finally it will help the letters ζ and η to become known together with $\mathfrak{C}$, always before the magnification $\mathfrak{M}$ may become exceedingly great; for besides otherwise the necessary field may be diminished ; on the other hand truly *R* also may become a very great number; from which that above equation may adopt this form:

$$(\eta-1)R = \tfrac{\theta-1}{\theta}\cdot\left(\frac{\mathfrak{C}\mathfrak{M}+3R(\zeta-\eta)+3R\zeta\mathfrak{C}k}{\mathfrak{C}}\right),$$

from which it follows:

$$k = \frac{(\eta-1)\mathfrak{C}\theta R-(\theta-1)\mathfrak{C}\mathfrak{M}-3(\theta-1)(\zeta-\eta)R}{3(\theta-1)\zeta\mathfrak{C}R};$$

which value, since it must be positive, will have to become

$$R > \frac{(\theta-1)\mathfrak{C}\mathfrak{M}}{(\eta-1)\mathfrak{C}\theta-3(\theta-1)(\zeta-\eta)}$$

with there being

$$(\eta-1)\mathfrak{C}\theta > 3(\theta-1)(\zeta-\eta) \text{ or } \mathfrak{C} > \frac{3(\theta-1)(\zeta-\eta)}{(\eta-1)\theta}.$$

Therefore since there must become $\mathfrak{C} < 1$ in the preceding problem, the numerator of this limit must be less than its denominator, and hence

$$\zeta < \frac{\theta(4\eta-1)-3\eta}{3(\theta-1)}.$$

Therefore with these conditions observed we may put for the sake of brevity again as before $R = i\mathfrak{M}$, thus so that the must become

$$i > \frac{(\theta-1)\mathfrak{C}}{(\eta-1)\mathfrak{C}\theta-3(\theta-1)(\zeta-\eta)};$$

thence we will have:

$$k = \frac{i(\eta-1)\mathfrak{C}\theta-(\theta-1)\mathfrak{C}-3i(\theta-1)(\zeta-\eta)}{3i(\theta-1)\zeta\mathfrak{C}} \text{ and } P = \tfrac{\eta-1}{k};$$

from which value the two cases will be agreed to be considered.

If $P>1$, then there must be $A>0$ and thus $B<0$, which indeed happens at once, since there shall be produced $\mathfrak{B}<0$. Therefore this happens, when $k<\eta-1$; from which it is concluded

$$i<\frac{(\theta-1)\mathfrak{C}}{(\eta-1)\mathfrak{C}\theta-3(\theta-1)((\eta-1)\zeta\mathfrak{C}+\zeta-\eta)};$$

which evidently is the greater limit. Moreover if the letter *i* thus may exceed this limit, then there shall become $P<1$ and thus *A* will have to be taken negative, and since $\mathfrak{B}$ is produced positive, so far only *B* will be positive, as required, just as there shall become $\mathfrak{B}<1$. Truly there shall be always $\mathfrak{B}<1$, unless there were $\zeta<1$; and of also there were $\zeta<1-P$, the case will be impossible. Then since there shall be

$$PkR=(\eta-1)i\mathfrak{M},$$

with the very small terms ignored before $\mathfrak{M}$ on account of $\mathfrak{M}M=3$ there will be approximately

$$\mathfrak{D}=-3i(\eta-1), \text{ hence } D=-\frac{3i(\eta-1)}{3i(\eta-1)+1}.$$

Again since there shall be

$$PkRk'=\frac{\mathfrak{M}}{T}=\mathfrak{M}(\theta-1),$$

in the same way there will become

$$\mathfrak{E}=3\theta-4 \text{ and } E=\frac{3\theta-4}{5-3\theta} \text{ or } E=-\frac{3\theta-4}{3\theta-5}.$$

Hence therefore the focal distances themselves will be had thus :

$$p=\mathfrak{A}a,\quad q=-\frac{A\mathfrak{B}}{P}\cdot a,\quad r=-\frac{AB\mathfrak{C}}{\eta-1}\cdot a,$$

$$s=-\frac{ABC3i(\eta-1)}{(\eta-1)i\mathfrak{M}}\cdot a=-\frac{3ABC}{\mathfrak{M}}\cdot a,$$

$$t=-\frac{3i(\eta-1)(3\theta-4)}{(3i(\eta-1)+1)(\theta-1)}\cdot\frac{ABC}{\mathfrak{M}}\cdot a,$$

$$u=-\frac{3i(\eta-1)(3\theta-4)}{(3i(\eta-1)+1)(3\theta-5)}\cdot\frac{ABC}{\mathfrak{M}}\cdot a;$$

where it will be observed also

$$q=\frac{1}{\zeta}\left(1-\frac{1}{P}\right)Aa,$$

thus so that q shall be to the first interval as $1:\zeta$. Moreover the intervals will be :

$$\text{first} = Aa\left(1-\tfrac{1}{P}\right) = \zeta q,$$

$$\text{second} = -ABa\left(\tfrac{1}{P}+\tfrac{1}{\eta-1}\right),$$

$$\text{third} = -ABCa\left(\tfrac{1}{\eta-1}-\tfrac{1}{i(\eta-1)\mathfrak{M}}\right) = -\tfrac{ABC}{\eta-1}\cdot a\left(1-\tfrac{1}{i\mathfrak{M}}\right),$$

$$\text{fourth} = \tfrac{ABCDa}{\mathfrak{M}}\left(\tfrac{1}{i(\eta-1)}+\tfrac{1}{\theta-1}\right) = -\tfrac{3(i(\eta-1)+\theta-1)}{(3i(\eta-1)+1)(\theta-1)}\cdot\tfrac{ABC}{\mathfrak{M}}\cdot a,$$

$$\text{fifth} = -\tfrac{3i(\eta-1)(3\theta-4)(\theta-2)}{(3i(\eta-1)+1)(\theta-1)(3\theta-5)}\cdot\tfrac{ABC}{\mathfrak{M}}\cdot a.$$

Truly the distance of the eye will be

$$O = \tfrac{u}{\mathfrak{M}M} = \tfrac{1}{3}u \quad \text{approx.}$$

The radius of the area viewed will be

$$z = \tfrac{1}{4}aM = \tfrac{3a}{4\mathfrak{M}}.$$

Then indeed the radius of the aperture of the first lens is $= x$ requiring to be defined either from the known formula or by experimentation, truly

$$\text{the radius of the second lens} = \tfrac{1}{4}\mathfrak{q}q+\tfrac{x}{P} = \tfrac{3}{4}\cdot\tfrac{\zeta}{\mathfrak{M}}\cdot q+\tfrac{x}{P},$$

$$\text{of the third} = \tfrac{1}{4}\mathfrak{r}r+\tfrac{x}{Pk} = \tfrac{3}{4}\cdot\tfrac{\zeta-\eta}{\mathfrak{C}\mathfrak{M}}\cdot r+\tfrac{x}{\eta-1};$$

indeed of the remaining lenses, which must be equally convex on both sides, the radii of the apertures will be respectively $\tfrac{1}{4}s$, $\tfrac{1}{4}t$ et $\tfrac{1}{4}u$. Finally the measure of the clarity will become $=\tfrac{20x}{\mathfrak{M}}$.

COROLLARY 1

286. If there may be taken $\zeta = 0$, for the second lens there will be $q = \infty$, which case returns the same, as if this lens may be absent; but then there will be $k = \infty$ and $P = 0$, $\mathfrak{B} = \infty$ and $B = -1$; from which, even if the first interval may become $=-\infty$, on account of the second lens being missing the interval between the first and third lens nevertheless will become finite $=\tfrac{\eta}{\eta-1}\cdot Aa$.
Thence truly there will have to be taken

$$i > \tfrac{(\theta-1)\mathfrak{C}}{(\eta-1)\mathfrak{C}\theta+3\eta(\theta-1)}.$$

Truly for $\mathfrak{C}$ it suffices, that it may be taken between the limits 1 and 0, since C must be a positive number on account of $D<0$. Truly the remaining determinations remain as before, only if there may be observed to be $B=-1$.

COROLLARY 2

287. Since in this case the second lens may be removed, we will have the solution to the problem in this way, where the microscope is required to be constructed from five lenses, which may be put in place thus, so that the first real image lies in the first interval, the posterior truly in the third; therefore the solution of this problem also shall supply a threefold field of view.

COROLLARY 3

288. Since in general on account of the ratios advanced before the letter C always must designate a large enough number, evidently lest the posterior lenses may become exceedingly small, $\mathfrak{C}=1$ will be satisfied properly and thus it will be allowed to put in place $\mathfrak{C}=1$ with confidence in practice. Then therefore there will have to be taken

$$i>\frac{\theta-1}{(\eta-1)\theta-3(\theta-1)(\zeta-\eta)},$$

then truly

$$k=\frac{i(\eta-1)\theta-3i(\theta-1)(\zeta-\eta)-\theta+1}{3i(\theta-1)\zeta}.$$

CASE 1
WHERE $i=\infty$ AND $\theta=3$

289. In this case there must become

$$\zeta<\frac{3\eta-1}{2};$$

then truly there will be

$$k=\frac{3\eta-2\zeta-1}{2\zeta} \quad \text{and} \quad P=\frac{2(\eta-1)\zeta}{3\eta-2\zeta-1}.$$

So that therefore there may become $P>1$, there will have to be $\zeta>\frac{3\eta-1}{2\eta}$. Whereby there will become $P>1$, if ζ may be taken within the limits $\frac{3\eta-1}{2}$ and $\frac{3\eta-1}{2\eta}$; in which case A will have to be taken positive, and since there is found $\mathfrak{B}<0$, there becomes at once $B<0$. But if there may be taken $\zeta<\frac{3\eta-1}{2\eta}$, then there will be $P<1$ and hence $A<0$, thus so that there must become $B>0$ and hence $\mathfrak{B}<1$;therefore there will be $\mathfrak{B}<1$, if $1-P<\zeta$; or if $P>1-\zeta$, which indeed always happens, unless there shall become $\zeta<1$. Therefore it remains to examine the case $\xi<1$, and since then there must become $P>1-\zeta$, thence this relation arises:

$$\zeta(5\eta-1)-2\zeta^2>3\eta-1;$$

from which it must be apparent

$$\zeta=\tfrac{5\eta-1}{4}-\tfrac{1}{4}\sqrt{25\zeta^2-34\eta+9-\alpha}$$

with α denoting some positive number, or

$$\zeta>\tfrac{5\eta-1}{4}-\tfrac{1}{4}\sqrt{25\zeta^2-34\eta+9};$$

therefore the limit for ζ will depend on η, thus so that on taking $\eta=2$ there shall become

$$\zeta>\tfrac{9-\sqrt{41}}{4} \text{ or } \zeta>\tfrac{5}{8};$$

but if there were $\eta=4$, there must become

$$\zeta>\tfrac{19-\sqrt{273}}{4} \text{ or } \zeta>\tfrac{5}{8} \text{ approx.}$$

But if there shall be $\eta=6$, there is produced

$$\zeta>\tfrac{29-\sqrt{705}}{4} \text{ or } \zeta>\tfrac{5}{8} \text{ approx.}$$

as before and thus it is apparent ζ can never be taken below $\tfrac{5}{8}$. Therefore now we will be able to announce the limits, within which ζ must be taken, to be $\tfrac{5}{8}$ and $\tfrac{3\eta-1}{2}$.
With these observed the focal lengths will be :

$$p=\mathfrak{A}a,\quad q=-\tfrac{A\mathfrak{B}}{P}\cdot a=\tfrac{1}{\zeta}\left(1-\tfrac{1}{p}\right),\quad r=-\tfrac{AB\mathfrak{C}}{\eta-1}\cdot a,$$
$$s=-3\cdot\tfrac{ABC}{\mathfrak{M}}\cdot a,\quad t=-\tfrac{5}{2}\cdot\tfrac{ABC}{\mathfrak{M}}\cdot a,\quad u=-\tfrac{5}{4}\cdot\tfrac{ABC}{\mathfrak{M}}\cdot a$$

and the intervals of the lenses :

$$\text{first}=Aa\left(1-\tfrac{1}{P}\right),\quad \text{second}=-AB\left(\tfrac{1}{P}+\tfrac{1}{\eta-1}\right)a,$$
$$\text{third}=-\tfrac{ABC}{\eta-1}\cdot a,\quad \text{fourth}=-\tfrac{1}{2}\cdot\tfrac{ABC}{\mathfrak{M}}\cdot a,\quad \text{fifth}=-\tfrac{5}{8}\cdot\tfrac{ABC}{\mathfrak{M}}\cdot a.$$

The remaining moments themselves are had to be used in the problem, certainly which will not depend on *u*.

SCHOLIUM

290. Here something amazing will be seen, because in this case *P* shall be able to be both greater and less than unity, since in the solution of the problem we will have shown then *P* only to become >1, when the letter *i* may be contained within the limits

$$\frac{(\theta-1)\mathfrak{C}}{(\eta-1)\mathfrak{C}\theta-3(\theta-1)(\zeta-\eta)} \text{ and } \frac{(\theta-1)\mathfrak{C}}{(\eta-1)\mathfrak{C}\theta-3(\theta-1)(\zeta-\eta+(\eta-1)\zeta\mathfrak{C})},$$

truly also, when *i* will have exceeded the latter limit, then also there will become $P<1$. Whereby, since thus here we have supposed $i=\infty$ hence certainly it is seen to follow always there must be $P<1$, yet which , as we have seen, happens otherwise. Towards resolving which doubt the nature of the latter limit must be investigated more accurately ; indeed if this itself now may become infinite, then certainly we will cease to be amazed, if also on taking $i=\infty$ there may be found $P>1$. But if in this latter limit the denominator not only may vanish, but thus also may emerge negative, then the limit itself will have to be seen to be not only negative but as being infinitely greater as well, thus so that on putting $i=\infty$ at this point it may be agreed to be contained between these limits. Therefore now it is evident the latter limit to become $=\infty$, as if on taking $\mathfrak{C}=1$, there were to become

$$\zeta=\frac{(4\eta-1)\theta-3\eta}{3\eta(\theta-1)},$$

and on assuming $\theta=3$, as we have done , this would become $\zeta=\frac{3\eta-1}{2\eta}$. But since there shall be $\zeta>\frac{3\eta-1}{2\eta}$ (as there must be $\zeta<\frac{3\eta-1}{2\eta}$ always), then this limit shall become as if infinitely greater and hence for $i=\infty$ itself to become smaller; from which be necessity there will have to become $P>1$. But if there shall be $\zeta<\frac{3\eta-1}{2\eta}$, then this limit at this point will be finite and thus the value $i=\infty$ without doubt will be greater than that; from which also only from these cases will there become $P<1$. With this observed we will illustrate this case by some examples.

EXAMPLE 1

291. We may assume $\eta=2$, and since for ζ the prior limit shall be $\zeta<\frac{5}{2}$, truly the posterior limit $\frac{5}{4}$, we may take $\zeta=2$, so that it falls within these limits; from which there will become $k=\frac{1}{4}$ and $P=4$, hence $\mathfrak{B}=-\frac{3}{2}$ and $B=-\frac{3}{5}$; from which the focal lengths will be

$$p=\mathfrak{A}a,\quad q=-\tfrac{1}{4}\mathfrak{B}Aa=\tfrac{3}{8}Aa,\quad r=\tfrac{3}{5}Aa,$$

$$s=\tfrac{9}{5}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa,\quad t=\tfrac{3}{2}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa,\quad u=\tfrac{3}{4}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa$$

and the intervals of the lenses :

$$\text{first} = \tfrac{3}{4}Aa, \quad \text{second} = \tfrac{3}{4}Aa,$$
$$\text{third} = \tfrac{3}{5}CAa, \quad \text{fourth} = \tfrac{3}{10}\tfrac{C}{\mathfrak{M}}\cdot Aa, \quad \text{fifth} = \tfrac{3}{8}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa.$$

Distance of the eye $O = \frac{1}{3}u$ approx., $z = \frac{3a}{4\mathfrak{M}}$,

the radius of the aperture :

$$\text{of the first lens } = x, \text{ of the second} = \tfrac{3}{2\mathfrak{M}}\cdot q + \tfrac{x}{4}, \quad \text{of the third} = x$$

and finally the measure of the clarity $= \frac{20x}{\mathfrak{M}}$ always.

EXAMPLE 2

292. With $\eta = 2$ remaining, ζ may be equal to the other limit, evidently $\zeta = \frac{5}{4}$, and there will become $k = 1$ and $P = 1$; hence therefore $\mathfrak{B} = 0$ and $B = 0$ arise. Whereby, lest both the second lens as well as the interval may vanish, there must be taken $A = \infty$ and thus $\mathfrak{A} = 1$, thus so that there shall become $A\mathfrak{B}$ or $AB = -\frac{q}{a}$, and since there shall be $\mathfrak{B} = \frac{4}{5}(1-P)$, there will be actually $1-P = \frac{5}{4}\mathfrak{B}$ and hence

$$Aa\left(1-\tfrac{1}{P}\right) = \tfrac{5}{4}q.$$

Whereby the focal lengths will become

$$p = a, \quad q = q, \quad r = \mathfrak{C}q, \quad s = 3\cdot\tfrac{C}{\mathfrak{M}}\cdot q, \quad t = \tfrac{5}{2}\cdot\tfrac{C}{\mathfrak{M}}\cdot q, \quad u = \tfrac{5}{4}\cdot\tfrac{C}{\mathfrak{M}}\cdot q,$$

where the second q is left to our choice. Then truly the intervals of the lenses :

$$\text{first} = \tfrac{5}{4}q, \quad \text{second} = 2q, \ \text{third} = Cq,$$
$$\text{fourth} = \tfrac{1}{2}\cdot\tfrac{C}{\mathfrak{M}}\cdot q, \ \text{fifth} = \tfrac{5}{8}\cdot\tfrac{C}{\mathfrak{M}}\cdot q.$$

The values O and z will be as before, but the radius of the aperture

$$\text{of the second lens} = \tfrac{15}{16\mathfrak{M}}\cdot q + x, \quad \text{and of the third lens} = \tfrac{9}{16\mathfrak{M}}\cdot r + x.$$

EXAMPLE 3

293. With $\eta = 2$ remaining there may be taken $\zeta < \frac{5}{4}$, and since there must be $\zeta > \frac{5}{8}$, as we have shown, there may be taken $\zeta = \frac{3}{4}$ and there will be $k = \frac{7}{3}$ and $P = \frac{3}{7}$; from which there shall become $\mathfrak{B} = \frac{16}{21}$ and hence $B = \frac{16}{5}$; which value since it shall be positive, the letter A must be taken negative, as also the first interval demands on account of $P < 1$. Therefore there shall become $A = -\alpha$ and thus

$$\mathfrak{A} = \frac{\alpha}{\alpha - 1},$$

and the focal lengths will be :

$$p = \frac{\alpha a}{\alpha - 1}, \quad q = \frac{16}{9}\alpha a, \quad r = \frac{15}{5}\mathfrak{C}\alpha a, \quad s = \frac{48}{5}\frac{C}{\mathfrak{M}}\cdot\alpha a, \quad t = 8\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a, \quad u = 4\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a.$$

and the intervals of the lenses :

$$\text{first} = \tfrac{4}{3}\alpha a, \quad \text{second} = \tfrac{32}{3}\alpha a, \ \text{third} = \tfrac{16}{5}C\mathfrak{A}\alpha a,$$
$$\text{fourth} = \tfrac{8}{5}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a, \quad \text{fifth} = 2\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a.$$

The rest will be had as before, and if for which the difference in the apertures is taken, that in practice does not merit attention; yet meanwhile the radius

$$\text{of the second lens} = \tfrac{9}{16}\cdot\tfrac{q}{\mathfrak{M}} + \tfrac{7}{3}x, \ \text{and of the third} = \tfrac{15}{16}\cdot\tfrac{r}{\mathfrak{M}} + x.$$

EXAMPLE 4

294. Now there may be put $\eta = 4$, and since there must become $\zeta < \frac{11}{2}$, truly the posterior limit shall be $\frac{11}{8}$, clearly so that the cases $P > 1$ and $P < 1$ are to be distinguished, there may be taken $\zeta = 3$, $k = \frac{5}{6}$ and $P = \frac{18}{5}$. From which there will become $\mathfrak{B} = -\frac{13}{15}$ and $B = -\frac{13}{28}$. Therefore the focal lengths will be

$$p = \mathfrak{A}a, \quad q = \tfrac{13}{54}Aa, \quad r = \tfrac{13}{84}\mathfrak{C}Aa, \quad s = \tfrac{39}{28}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa,$$
$$t = \tfrac{65}{56}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa, \quad u = \tfrac{65}{112}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa = \tfrac{1}{2}t.$$

The intervals of the lenses :

$$\text{first} = \tfrac{13}{18}Aa, \quad \text{second} = \tfrac{143}{504}Aa, \quad \text{third} = \tfrac{13}{84}CAa,$$
$$\text{fourth} = \tfrac{13}{56}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa, \quad \text{fifth} = \tfrac{65}{224}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa.$$

Finally the radius of the aperture

$$\text{of the second lens} = \frac{9}{4\mathfrak{M}}\cdot q + \frac{5}{18}x, \quad \text{and of the third lens} = \frac{3}{4\mathfrak{M}}\cdot r + \frac{1}{3}x.$$

EXAMPLE 5

295. With $\eta = 4$ remaining, there shall be $\zeta = \frac{11}{8}$; and there will be $k = 3$ and $P = 1$, from which,

$$\mathfrak{B} = \frac{1-P}{\zeta} = \frac{8}{11}(1-P) = 0 \text{ and } B = 0.$$

With which assumed there must become $A = \infty$ thus so that there may become $\mathfrak{A} = 1$; then therefore on introducing q into the calculation there will become

$$A\mathfrak{B} = AB = -\frac{q}{a};$$

from which there becomes

$$Aa\cdot\frac{(1-P)}{P} = -\frac{11}{8}q$$

and thus the focal lengths will be :

$$p = a, \quad q = q, \quad r = \frac{1}{3}\mathfrak{C}q, \quad s = 3\cdot\frac{C}{\mathfrak{M}}\cdot q, \quad t = \frac{5}{2}\cdot\frac{C}{\mathfrak{M}}\cdot q, \quad \text{and} \quad u = \frac{5}{4}\cdot\frac{C}{\mathfrak{M}}\cdot q$$

and the intervals of the lenses :

$$\text{first} = \frac{11}{8}q, \quad \text{second} = \frac{4}{3}q, \ \text{third} = \frac{1}{3}Cq,$$
$$\text{fourth} = \frac{1}{2}\cdot\frac{C}{\mathfrak{M}}\cdot q, \ \text{and fifth} = \frac{5}{8}\cdot\frac{C}{\mathfrak{M}}\cdot q.$$

The radius of the aperture

$$\text{of the second lens} = \frac{33}{32}\cdot\frac{q}{\mathfrak{M}} + x, \quad \text{of the third lens} = \frac{63}{32}\cdot\frac{r}{\mathfrak{M}} + \frac{1}{3}x.$$

EXAMPLE 6

296. At this stage with $\eta = 4$ remaining, there shall be $\zeta = \frac{2}{3}$ and there is found $k = \frac{29}{4}$ and $P = \frac{12}{29}$; from which there shall become $\mathfrak{B} = \frac{51}{58}$ and $B = \frac{51}{7}$, from which with so great a value it is now understood to be conceded microscopes of this kind cannot find a place in practice.

THE CASE WHERE $\eta = 4$ AND $\theta = 3$

297. Since there must be $\eta > 1$ and an exceedingly small value of that is liable to certain inconveniences, on the other hand an exceedingly great value harms the field of view, it will be agreed to use a middle value, and $\eta = 4$ is of this kind; then truly the value $\theta = 3$ or $T = \frac{1}{2}$ gives rise to a suitable interval between the final lenses; but the letter $\mathfrak{C}$ must differ so very little from unity, that it may be allowed to assume $\mathfrak{C} = 1$ in our formulas . With these premises for ζ the limit will be $\zeta < \frac{11}{12}$. For $\mathfrak{C}$ we will have truly

$$\mathfrak{C} < 1 \text{ and } \mathfrak{C} > \tfrac{2}{3}(\zeta - 4);$$

from which it is apparent, even if there shall be $\zeta = \frac{11}{2}$, yet there shall be $\mathfrak{C} = 1$, thus so that certainly it shall be able to assume $\mathfrak{C} = 1$. Again there will become for the letter i

$$i > \frac{2}{33-6\zeta}.$$

Thence we will obtain

$$k = \frac{(33-6\zeta)i-2}{6\zeta i},$$

from which there becomes

$$P = \frac{18\zeta i}{(33-6\zeta)i-2}.$$

Hence there arises

$$\mathfrak{B} = \frac{(33-24\zeta)i-2}{((33-6\zeta)i-2)\zeta}.$$

Here it will be required to distinguish two cases. The first is, where there becomes $P > 1$; and hence A must have a positive value, which happens, if $\mathfrak{B}$ may become < 0 and thus

$$i < \frac{2}{33-24\zeta},$$

or if when i may be contained within the limits $\frac{2}{33-6\zeta}$ and $\frac{2}{33-24\zeta}$, and in this case also B becomes negative, thus so that there shall be $AB < 0$. The other case, where $P < 1$, has a place, if there were $i > \frac{2}{33-24\zeta}$, in which case A will have a negative value; therefore so that B may obtain a positive value, there must be $\mathfrak{B} < 1$ and hence

$$i < \frac{2(1-\zeta)}{6\zeta^2-57\zeta+33}.$$

Therefore since there shall be $i > \frac{2}{33-24\zeta}$, that happens, if there were

$$\frac{1-\zeta}{6\zeta^2-57\zeta+33} > \frac{1}{33-24\zeta},$$

from which it follows $\zeta > 0$. Therefore with these observed the focal lengths will be

$$p = \mathfrak{A}a,\ \ q = -\frac{A\mathfrak{B}}{P}\cdot a,\ \ r = -\frac{1}{3}AB\mathfrak{C}a,\ \ s = -3\cdot\frac{ABC}{\mathfrak{M}}\cdot a,$$
$$t = -\frac{45i}{18i+2}\cdot\frac{ABC}{\mathfrak{M}}\cdot a \ \text{ and } \ u = -\frac{45i}{45i+4}\cdot\frac{ABC}{\mathfrak{M}}\cdot a$$

as also there will become

$$q = +\frac{1}{\zeta}\left(1-\frac{1}{P}\right)Aa$$

and the intervals of the lenses are

$$\text{first} = Aa\left(1-\frac{1}{P}\right),\ \ \text{second} = -ABa\left(\frac{1}{P}+\frac{1}{3}\right),$$
$$\text{third} = -\frac{ABC}{3}\cdot a\left(1-\frac{1}{i\mathfrak{M}}\right),\ \ \text{fourth} = -\frac{3(3i+2)}{2(9i+1)}\cdot\frac{ABC}{\mathfrak{M}}\cdot a,\ \ \text{fifth} = -\frac{45i}{8(9i+1)}\cdot\frac{ABC}{\mathfrak{M}}\cdot a.$$

Thence since there shall be

$$M = \frac{3}{\mathfrak{M}+3},$$

there shall become

$$z = \frac{1}{4}\cdot\frac{3a}{\mathfrak{M}+3}$$

and the distance of the eye

$$O = \frac{1}{3}u\left(1+\frac{3}{\mathfrak{M}}\right).$$

Again the radius of the aperture

of the second lens will be $= \frac{3}{4}\cdot\frac{\zeta}{\mathfrak{M}}\cdot q + \frac{x}{P}$ and of the third lens $= \frac{3}{4}\cdot\frac{(\zeta-4)r}{\mathfrak{M}} + \frac{1}{3}x.$

Finally from the definition x the measure of the clarity will be $= \frac{20x}{\mathfrak{M}}$.

EXAMPLE 1

298. There shall be $\zeta = 0$, and since there must become $i > \frac{2}{33}$, truly besides according to $P > 1$ there shall be $i < \frac{2}{33}$, we may put $i = \frac{2}{33}$ and there will become $P = \frac{0}{0}$ or P is not determined, only it shall not be less than unity, and thus $\mathfrak{B} = -\frac{(P-1)}{\zeta}$, thus so that $\mathfrak{B}$ shall be always ∞ unless there may be taken $P = 1$. Therefore in the first place there shall not

be $P = 1$; there will become $\mathfrak{B} = \infty$ and $B = -1$, thus so that A shall be greater than zero; hence therefore the focal lengths will be

$$p = \mathfrak{A}a, \quad q = \infty, \quad \left(\text{or, which is the same, the second lens is removed}\right),$$
$$r = \tfrac{1}{3} AB\mathfrak{C}a, \quad s = \tfrac{3ABC}{\mathfrak{M}} \cdot a, \quad t = \tfrac{15}{17} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a \text{ and } u = \tfrac{15}{34} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a$$

and the intervals of the lenses:

$$\text{first + second} = \tfrac{4}{3} Aa, \text{ third} = \tfrac{AC}{3}\left(1 - \tfrac{33}{2\mathfrak{M}}\right),$$
$$\text{fourth} = \tfrac{72}{34} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a, \quad \text{fifth} = \tfrac{15}{68} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a.$$

The rest remain the same, except that the radius of the aperture of the third lens shall be

$$= 3 \cdot \tfrac{r}{\mathfrak{M}} + \tfrac{1}{3} x.$$

But if there may be taken $P = 1$, whatever calculation may be put in place, the first interval will disappear always; truly it is superfluous to attend to this case, since in the prior formulas plainly the letter P will have been excluded from the calculation, thus so that these formulas remain in place, whatever value may be attributed to P itself and thus not only if there may be put $P = 1$, but also if P may be taken smaller by one; so that even if it may disagree with our hypothesis, yet on account of the second lens being missing completely this anomaly must be allowed.

EXAMPLE 2

299. There may remain $\zeta = 0$, but there may be taken $i > \tfrac{2}{33}$, if that may be able to be done, in which case there will become $P < 1$; but since in this case again there must be $i < \tfrac{2}{33}$, this case is reduced to the preceding case, which now indeed we have observed to apply both to the values $P < 1$ as well as $P > 1$. Yet meanwhile since the second lens plainly may be missing, the latter limit $i < \tfrac{2}{33}$ shall cease at once, thus so that now it may be allowed to assume $i > \tfrac{2}{33}$, as we have observed in corollary 1 to the next problem. Then therefore the focal lengths will be

$$p = \mathfrak{A}a, \quad q = \infty, \quad \left(\text{or, with the second lens absent}\right),$$
$$r = \tfrac{A\mathfrak{C}}{3} \cdot a, \quad s = \tfrac{3AC}{\mathfrak{M}} \cdot a, \quad t = \tfrac{45i}{18i+2} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a, \quad u = \tfrac{45i}{36i+4} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a.$$

Truly the intervals of the lenses

$$\text{first + second} = \tfrac{4}{3}Aa,\ \text{third} = \tfrac{1}{3}ACa\left(1-\tfrac{1}{i\mathfrak{M}}\right),$$

$$\text{fourth} = \frac{3(3i+2)}{2(9i+2)}\cdot\frac{AC}{\mathfrak{M}}\cdot a,\quad \text{fifth} = \frac{45i}{8(9i+1)}\cdot\frac{AC}{\mathfrak{M}}\cdot a.$$

But so that it may extend to the letter i , since k no further enters into the calculation, from the equation, from which we have defined k , now i may be defined and there will become

$$i = \frac{2}{33-6\zeta} = \frac{2}{33}.$$

But these formulas contain properly the solution of the problem, where only five lenses may be postulated thus requiring to be put in place, so that both the real images may fall in the first and third interval.

EXAMPLE 3

300. There shall be $\zeta = \frac{1}{2}$ and $i > \frac{1}{15}$ to be accepted, and there will become $P > 1$, if there were $i < \frac{2}{21}$; but if there shall be $i > \frac{2}{21}$, likewise there will become $P < 1$. Truly here we may assume $i = \frac{1}{12}$ and there will become $P = \frac{3}{2}$, hence $\mathfrak{B} = -1$ and $B = -\frac{1}{2}$; from which the focal lengths will become

$$p = \mathfrak{A}a,\quad q = \tfrac{2}{3}Aa,\ r = \tfrac{1}{6}A\mathfrak{C}a,$$

$$s = \frac{3}{2}\cdot\frac{AC}{\mathfrak{M}}\cdot a,\quad t = \frac{45}{84}\cdot\frac{AC}{\mathfrak{M}}\cdot a,\quad u = \frac{15}{56}\cdot\frac{AC}{\mathfrak{M}}\cdot a.$$

and the intervals of the lenses :

$$\text{first} = \tfrac{1}{3}Aa,\quad \text{second} = \tfrac{1}{2}Aa,\ \text{third} = \tfrac{1}{6}ACa\left(1-\tfrac{12}{\mathfrak{M}}\right),$$

$$\text{fourth} = \frac{27}{28}\cdot\frac{AC}{\mathfrak{M}}\cdot a,\quad \text{fifth} = \frac{15}{112}\cdot\frac{AC}{\mathfrak{M}}\cdot a.$$

The remaining moments are as before, except that the radius of the aperture

$$\text{of the second lens} = \frac{3}{8}\cdot\frac{q}{\mathfrak{M}} + \frac{2}{3}x,\quad \text{and of the third lens} = \frac{21}{8}\cdot\frac{r}{\mathfrak{M}} + \frac{1}{3}x.$$

These formulas conveniently will be allowed to be applied to telescopes, since by putting $Aa = p$ the length finally becomes

$$\tfrac{5}{6}p + \tfrac{1}{6}Cp,$$

thus so that it may not exceed p by much, even if a large enough number may be taken for C.

EXAMPLE 4

301. There may remain $\zeta = \frac{1}{2}$, but it may be assumed that $i > \frac{2}{21}$, and since hence there shall become $P = \frac{9i}{30i-2}$ and thus $\mathfrak{B} = \frac{21i-2}{15i-1}$, so that there may become $\mathfrak{B} < 1$, it is required for $21i - 2 < 15i - 1$ or $i < \frac{1}{6}$. Therefore there may be taken $i = \frac{1}{8}$ and there will become $P = \frac{9}{14}$ and $\mathfrak{B} = \frac{5}{7}$, hence $B = \frac{5}{2}$; therefore A must be negative. Therefore there may be put $A = -\alpha$, and the focal lengths will become

$$p = \frac{\alpha}{\alpha-1}a,\ \ q = \frac{10}{9}\alpha a,\ r = \frac{5}{6}\mathfrak{C}\alpha a,$$
$$s = \frac{15}{2}\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a,\quad t = \frac{225}{68}\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a,\quad u = \frac{225}{136}\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a.$$

and the intervals of the lenses

$$\text{first} = \frac{5}{9}\alpha a,\quad \text{second} = \frac{85}{18}\alpha a,\ \text{third} = \frac{5}{6}C\alpha a\left(1-\frac{8}{\mathfrak{M}}\right),$$
$$\text{fourth} = \frac{285}{68}\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a,\ \ \text{fifth} = \frac{225}{272}\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a.$$

Then truly the radius of the aperture

$$\text{of the second lens} = \frac{3}{8}\cdot\frac{q}{\mathfrak{M}} + \frac{14}{9}x,\ \ \text{and of the third lens}\ \frac{21}{8}\cdot r + \frac{1}{3}x.$$

But these formulas cannot be applied to telescopes, because A was negative.

EXAMPLE 5

302. There shall be $\zeta = 1$, and since there may be taken $i > \frac{2}{27}$ and, so that there may be produced $P > 1$, $i < \frac{2}{9}$, there may be taken $i = \frac{1}{10}$ and there will become $P = \frac{18}{7}$ and $\mathfrak{B} = -\frac{11}{7}$, and $B = -\frac{11}{18}$; from which the focal lengths will be

$$p = \mathfrak{A}a,\ \ q = \frac{11}{18}Aa,\ r = \frac{11}{54}\mathfrak{C}Aa,$$
$$s = \frac{11}{6}\cdot\frac{AC}{\mathfrak{M}}\cdot a,\quad t = \frac{55}{76}\cdot\frac{C}{\mathfrak{M}}\cdot Aa,\quad u = \frac{55}{152}\cdot\frac{C}{\mathfrak{M}}\cdot Aa.$$

and the intervals of the lenses :

$$\text{first} = \frac{11}{18}Aa,\quad \text{second} = \frac{143}{324}Aa,\ \text{third} = \frac{11}{54}ACa\left(1-\frac{10}{\mathfrak{M}}\right),$$
$$\text{fourth} = \frac{253}{228}\cdot\frac{AC}{\mathfrak{M}}\cdot a,\ \ \text{fifth} = \frac{55}{304}\cdot\frac{AC}{\mathfrak{M}}\cdot a$$

and the radius of the aperture of the
second lens $=\frac{3}{4}\cdot\frac{q}{\mathfrak{M}}+\frac{7}{18}x$, & of the third lens $=\frac{9}{4}\cdot\frac{r}{\mathfrak{M}}+\frac{1}{3}x$.

which formulas also conveniently will be allowed to be transferred to telescopes.

EXAMPLE 6

803. There may remain $\zeta=1$, but there may be taken $i=\frac{2}{9}$ and there will become $P=1$ and $\mathfrak{B}=0$, hence $B=0$; hence A must be taken $=\infty$ and thus $\mathfrak{A}=1$; but then there must be

$$A\mathfrak{B}=AB=-\frac{q}{a},$$

from which there becomes

$$Aa\left(1-\frac{1}{P}\right)=Aa\left(P-1\right)=-A\mathfrak{B}a=q;$$

hence the focal lengths will be

$$p=a,\quad q=q,\ r=\frac{1}{3}\mathfrak{C}q,$$
$$s=3\cdot\frac{C}{\mathfrak{M}}\cdot q,\quad t=\frac{5}{3}\cdot\frac{C}{\mathfrak{M}}\cdot q,\quad u=\frac{5}{6}\cdot\frac{C}{\mathfrak{M}}\cdot q$$

and the intervals of the lenses :

$$\text{first}=q,\quad \text{second}=\frac{4}{3}q,\ \text{third}=\frac{1}{3}Cq\left(1-\frac{9}{2\mathfrak{M}}\right),$$
$$\text{fourth}=\frac{4}{3}\cdot\frac{C}{\mathfrak{M}}\cdot q,\ \ \text{fifth}=\frac{5}{12}\cdot\frac{C}{\mathfrak{M}}\cdot q$$

and the radius of the aperture :

$$\text{of the second lens}=\frac{3}{4}\cdot\frac{q}{\mathfrak{M}}+x,\ \ \text{of the third lens}=\frac{9}{4}\cdot\frac{r}{\mathfrak{M}}+\frac{1}{3}x.$$

EXAMPLE 7

804. There may remain $\zeta=1$, but there may be taken $i>\frac{2}{9}$, and since there shall become

$$P=\frac{18i}{27i-2}\ \text{ and hence }\ \mathfrak{B}=\frac{9i-2}{27i-2},$$

which fraction now at once is less than unity, therefore we may assume $i=1$; there will become $P=\frac{18}{25}$ and $\mathfrak{B}=\frac{7}{25}$, $B=\frac{7}{18}$, from which A must be negative; therefore it may be put in place $=-\alpha$ and the focal lengths will become :

$$p=\frac{\alpha}{\alpha-1}\cdot a,\ \ q=\frac{7}{18}\alpha a,\ r=\frac{7}{54}\mathfrak{C}\alpha a,$$
$$s=\frac{7}{6}\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a,\quad t=\frac{35}{40}\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a,\quad u=\frac{35}{80}\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a$$

and the intervals :

$$\text{first}=\frac{7}{18}\alpha a,\quad \text{second}=\frac{217}{324}\alpha a,\ \text{third}=\frac{7}{54}C\alpha a\left(1-\frac{9}{2\mathfrak{M}}\right),$$
$$\text{fourth}=\frac{7}{24}\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a,\ \ \text{fifth}=\frac{7}{32}\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a$$

and the radius of the aperture

$$\text{of the second lens}=\frac{3}{4}\cdot\frac{q}{\mathfrak{M}}+\frac{25}{18}x,\ \ \text{and of the third lens }=\frac{9}{4}\cdot\frac{r}{\mathfrak{M}}+\frac{1}{3}x.$$

EXAMPLE 8

305. Now there shall become $\zeta=4=\eta$, and since there must be $i>\frac{2}{9}$, so that moreover the may become $P>1$, $i<-\frac{2}{63}$ (which limits will have to be seen as infinitely greater; if indeed we may assume ζ smaller, evidently $\zeta=\frac{11}{8}$, then there would be produced $i<\frac{2}{0}$, which would have indicated *i* some very large number can be taken, and always to become $P>1$; but which prevails from the value $\zeta=\frac{11}{8}$, and prevails much more from greater values), therefore there will become $i=\frac{1}{3}$, and there will become

$$P=-24\ \text{ and }\ \mathfrak{B}=-\frac{23}{96},\text{ and }\ B-=-\frac{23}{27}$$

from which the focal lengths are deduced:

$$p=\mathfrak{A}a,\ \ q=\frac{23}{96}Aa,\ r=\frac{23}{81}A\mathfrak{C}a,$$
$$s=\frac{23}{9}\cdot\frac{AC}{\mathfrak{M}}\cdot a,\quad t=\frac{115}{72}\cdot\frac{AC}{\mathfrak{M}}\cdot a,\quad u=\frac{115}{144}\cdot\frac{AC}{\mathfrak{M}}\cdot a$$

and the intervals of the lenses :

$$\text{first}=\frac{23}{24}Aa,\ \ \text{second}=\frac{23}{72}Aa,\ \text{third}=\frac{23}{81}CAa\left(1-\frac{3}{\mathfrak{M}}\right),$$
$$\text{fourth}=\frac{23}{24}\cdot\frac{AC}{\mathfrak{M}}\cdot a,\ \ \text{fifth}=\frac{115}{288}\cdot\frac{AC}{\mathfrak{M}}\cdot a$$

and the radius of the aperture

$$\text{of the second lens}=\frac{3q}{\mathfrak{M}}+\frac{1}{24}x,\ \ \text{and of the third lens}=\frac{1}{3}x.$$

EXAMPLE 9

306. There may remain $\zeta = 4$ and there shall be $i = 1$; there will become

$$P = \tfrac{72}{7}, \text{ therefore } \mathfrak{B} = -\tfrac{65}{28}, \text{ hence } B = -\tfrac{65}{93};$$

from which the focal lengths are deduced :

$$p = \mathfrak{A}a, \quad q = \tfrac{65}{288} Aa, \; r = \tfrac{65}{279} A\mathfrak{C}a,$$
$$s = \tfrac{65}{31} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a, \quad t = \tfrac{195}{124} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a, \quad u = \tfrac{195}{248} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a = \tfrac{1}{2} t,$$

and the intervals of the lenses :

$$\text{first} = \tfrac{65}{72} Aa, \quad \text{second} = \tfrac{65}{216} Aa, \; \text{third} = \tfrac{65}{279} ACa\left(1 - \tfrac{1}{\mathfrak{M}}\right),$$
$$\text{fourth} = \tfrac{65}{124} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a, \quad \text{fifth} = \tfrac{195}{496} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a,$$

and the radius of the aperture

$$\text{of the second lens} = \tfrac{3q}{\mathfrak{M}} + \tfrac{7}{72} x, \quad \text{and of the third lens} = \tfrac{1}{3} x.$$

SCHOLIUM

307. Of those examples these in the first place are noteworthy, in which there will become $P = 1$, since then the letter A has gone off to infinity and there was $\mathfrak{A} = 1$. Therefore in the second case, which we have just considered above, in general we may put $P = 1$ and there will have to be taken $i = \frac{2}{33 - 24\zeta}$. Then truly there will be $\mathfrak{B} = 0$ and likewise $B = 0$; from which with $\mathfrak{A} = 1$ and $A = \infty$ assumed, there will become $q = -A\mathfrak{B}a$, hence in turn $A\mathfrak{B} = AB = -\frac{q}{a}$, so that now the focal length of the second lens q may be left to our choice entirely; but then for the first interval requiring to be found on account of $\mathfrak{B} = \frac{1-P}{\zeta}$ there will be

$$Aa \cdot \tfrac{P-1}{P} = Aa(P-1) = -A\mathfrak{B}\zeta a = \zeta q,$$

and hence in general the focal lengths will themselves be had:

$$p = a,\quad q = q,\ r = \tfrac{1}{3}\mathfrak{C}q,\ s = \tfrac{3C}{\mathfrak{M}}\cdot q,$$

$$t = \tfrac{15}{17-8\zeta}\cdot\tfrac{C}{\mathfrak{M}}\cdot q,\quad u = \tfrac{15}{34-16\zeta}\cdot\tfrac{C}{\mathfrak{M}}\cdot q \ \text{ or } \ u = \tfrac{1}{2}t.$$

Truly the intervals of the lenses will be :

$$\text{first} = \zeta q,\quad \text{second} = \tfrac{4}{3}q,$$

$$\text{third} = \tfrac{1}{3}q\left(1-\tfrac{33-24\zeta}{2\mathfrak{M}}\right) = \tfrac{1}{3}q\left(1-\tfrac{3(11-8\zeta)}{2\mathfrak{M}}\right),$$

$$\text{fourth} = \tfrac{12(3-2\zeta)}{17-8\zeta}\cdot\tfrac{C}{\mathfrak{M}}\cdot q,\quad \text{fifth} = \tfrac{15}{4(17-8\zeta)}\cdot\tfrac{C}{\mathfrak{M}}\cdot q;$$

where therefore it is evident necessarily there must be taken $\zeta < \tfrac{3}{2}$. Then truly the radius of the aperture

$$\text{of the second lens} = \tfrac{3}{4}\cdot\tfrac{\zeta}{\mathfrak{M}}\cdot q + x,\quad \text{and of the third lens} = \tfrac{3}{4}\cdot\tfrac{(\zeta-4)}{\mathfrak{M}}\cdot r + \tfrac{1}{3}x.$$

Furthermore here it is evident these same formulas in no manner can be applied to telescopes. Therefore so that we can have all the cases, which certainly have a place in practice, thus we will establish the case for six lenses and where we will be concerned only with the field, from which a greater scarcely may be desired, and we will impose the end of this chapter and to be going to move on to the final chapter, in which we have shown, since in place of the objective lens two or more lenses are required to be substituted made either from the same or from different kinds of glass clearly all the confusion will be able to be removed, so that in this way from these numbers we may find absolutely all the microscopes .

CAPUT II

DE MICROSCOPIIS HUIUS GENERIS MAGIS COMPOSITIS

PROBLEMA 1

250. *Microscopium huius generis ex quinque lentibus construere, quae ita sint dispositae, ut prior imago realis inter lentem secundam et tertiam, posterior vero inter tertiam et quartam cadat.*

SOLUTIO

Cum igitur prior imago in intervallum secundum, posterior vero in tertium cadere debeat, litterarum *P, Q, R, S* secunda et tertia *Q* et *R* debent esse negativae. Statuatur ergo $Q=-k$ et $R=-k'$, ut sit $Pkk'S=\mathfrak{M}$ existente

$$\mathfrak{M}=\frac{ma}{h}.$$

Deinde vero sit

$$M=\frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t}}{\mathfrak{M}-1},$$

ut fiat spatii in obiecto conspicui semidiameter

$$z=Ma\xi$$

Quare, ut campus evadat maximus, efficiendum est, ut litterarum $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$ tot fiant unitati aequales, quam reliquae circumstantiae permittunt; quod cum de omnibus statui nequeat, saltem pro postremis ponamus $\mathfrak{s}=1$ et $\mathfrak{t}=1$, ut sit

$$M=\frac{\mathfrak{q}+\mathfrak{r}+2}{\mathfrak{M}-1}.$$

Tum vero habebuntur sequentes aequationes:

$$1.\quad \mathfrak{B}\mathfrak{q}=(P-1)M,$$
$$2.\quad \mathfrak{C}\mathfrak{r}=-(Pk+1)M-\mathfrak{q},$$
$$3.\quad \mathfrak{D}=-(Pkk'+1)M-\mathfrak{q}-\mathfrak{r},$$

quibus adiungatur aequatio, qua margo coloratus destruitur,

$$0=\frac{\mathfrak{q}}{P}-\frac{\mathfrak{r}}{Pk}+\frac{1}{Pkk'}+\frac{1}{Pkk'S},$$

ex qua deducitur

$$k' = \frac{1+\frac{1}{S}}{\mathfrak{r}-k\mathfrak{q}},$$

unde patet esse debere $\mathfrak{r} > k\mathfrak{q}$. Antequam autem hinc quidquam definire valeamus, lentium intervalla considerare debemus, quae sunt

$$\text{primum} = Aa\left(1-\frac{1}{P}\right),\ \text{secundum} = -\frac{AB}{P}\cdot a\left(1+\frac{1}{k}\right),$$
$$\text{tertium} = -\frac{ABC}{Pk}\cdot a\left(1+\frac{1}{k'}\right),\ \text{quartum} = -\frac{ABCD}{Pkk'}\cdot a\left(1-\frac{1}{S}\right),$$

quae omnia debent esse positiva; quibus adiungatur distantia focalis ultimae lentis

$$t = \frac{ABCD}{Pkk'S}\cdot a = \frac{ABCD}{\mathfrak{M}}\cdot a,$$

quae etiam debet esse positiva, ut prodeat distantia oculi post eam,

$$O = \frac{t\mathfrak{t}}{M\mathfrak{M}},$$

positiva; unde ob $\mathfrak{t} = 1$ evidens est esse debere t positivum ideoque $ABCD > 0$. Quocirca, ut quartum intervallum etiam fiat positivum, necesse est, ut sit $1-\frac{1}{S} < 0$ sive $S < 1$, unde evidens est productum Pkk' fore $> \mathfrak{M}$ ideoque numerum praemagnum; unde tertia illa aequatio, si loco M eius valor substituatur, dabit

$$\mathfrak{D} = \frac{(Pkk'-1)(\mathfrak{q}+\mathfrak{r}+2)}{\mathfrak{M}-1} - \mathfrak{q} - \mathfrak{r}\ ;$$

ubi cum Pkk' et $\mathfrak{M}$ sint numeri praemagni et $Pkk' > \mathfrak{M}$, fiet proxima

$$\mathfrak{D} = \frac{Pkk'}{\mathfrak{M}}(\mathfrak{q}+\mathfrak{r}+2) - \mathfrak{q} - \mathfrak{r} = \frac{2Pkk'}{\mathfrak{M}} + (\mathfrak{q}+\mathfrak{r})\left(\frac{Pkk'}{\mathfrak{M}} - 1\right)$$

sive ob $Pkk' = \frac{\mathfrak{M}}{S}$ erit

$$\mathfrak{D} = \frac{2}{S} + (\mathfrak{q}+\mathfrak{r})\left(\frac{1}{S} - 1\right);$$

unde, cum $\mathfrak{q}+\mathfrak{r}$ certe sit < 2, evidens est fore $\mathfrak{D} > 1$ ideoque D negativum. Erit ergo $ABC < 0$; hinc tertium intervallum sponte fit positivum. Quare, cum ob secundum intervallum esse debeat $AB < 0$, oportet esse $C > 0$ hincque $\mathfrak{C} > 0$ et $\mathfrak{C} < 1$. Unde, si fuerit $A > 0$ ideoque $P > 1$ ob intervallum primum, debebit esse $B < 0$ hincque vel $\mathfrak{B} < 0$ vel $\mathfrak{B} > 1$; unde sequitur fore priori casu $\mathfrak{q} < 0$, altero casu $\mathfrak{q} > 0$; unde ob $\mathfrak{q} + \mathfrak{C}\mathfrak{r} < 0$ fieret $\mathfrak{r} < 0$ ideoque $k' < 0$, quod est absurdum. Sin autem asset $A < 0$ hincque $P < 1$, deberent esse $B > 0$ ideoque $\mathfrak{B} > 0$ et $\mathfrak{B} < 1$, unde iterum fieret $\mathfrak{q} < 0$. Neque igitur etiamnum constat, utrum ambo isti casus consistere queant. Quia vero in utroque fit $\mathfrak{q} < 0$, statuamus ut ante $\mathfrak{q} = -\omega$, ut sit

$$\omega = \frac{(1-P)M}{\mathfrak{B}},$$

et ob secundam. aequationem esse debet $\omega - \mathfrak{C}\mathfrak{r} > 0$; tum vero erit

$$k' = \frac{1+\frac{1}{S}}{\mathfrak{r}+k\omega};$$

unde ob $Pkk' = \frac{\mathfrak{M}}{S}$ fiet

$$Pk = \frac{\mathfrak{M}(\mathfrak{r}+k\omega)}{1+S}.$$

Nunc igitur litteras ω et $\mathfrak{t}$ ex calculo eliminemus, et cum sit $\omega = \frac{1-P}{\mathfrak{B}} M$, ponamus brevitatis gratia

$$\frac{1-P}{\mathfrak{B}} = \zeta,$$

ut sit

$$\omega = \zeta M;$$

deinde sit etiam brevitatis gratia

$$1+Pk = \eta$$

fietque

$$\mathfrak{r} = \frac{(\zeta-\eta)M}{\mathfrak{C}}.$$

Ergo porro

$$\mathfrak{r} - \omega = \frac{\left(\zeta(1-\mathfrak{C})-\eta\right)M}{\mathfrak{C}}$$

et

$$2 - \omega + \mathfrak{r} = \frac{2\mathfrak{C} + \left(\zeta\left(1-\mathfrak{C}\right)-\eta\right)M}{\mathfrak{C}};$$

at cum sit $M = \frac{2+\mathfrak{r}-\omega}{\mathfrak{M}-1}$, erit

$$2+\mathfrak{r}-\omega = M\left(\mathfrak{M}-1\right),$$

unde concludimus fore

$$M = \frac{2\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta};$$

quo valore invento erit

$$\omega = \frac{2\mathfrak{C}\zeta}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta} \quad \text{et} \quad \mathfrak{r} = \frac{2(\zeta-\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta},$$

ex quibus valoribus porro conficitur

$$\mathfrak{r}+k\omega = \frac{2(\zeta-\eta)+2k\mathfrak{C}\zeta}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}$$

hincque

$$Pk = \frac{2\mathfrak{M}(\zeta-\eta+k\mathfrak{C}\zeta)}{(1+S)(\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta)} = \eta-1,$$

ex quo definitur

$$k = \frac{(\eta-1)(1+S)\mathfrak{C}(\mathfrak{M}+\zeta-1)-2\mathfrak{M}(\zeta-\eta)-(\eta-1)(1+S)(\zeta-\eta)}{2\mathfrak{M}\mathfrak{C}\zeta},$$

unde porro invenitur

$$P = \frac{\eta-1}{k}.$$

Quia autem k debet esse positivum, in eius numeratore coefficiens ipsius $\mathfrak{M}$ debet esse positivus, unde sequitur

$$\mathfrak{C} > \frac{2(\zeta-\eta)}{(\eta-1)(1+S)};$$

at vero vidimus esse $\mathfrak{C} < 1$ sicque necesse est, ut sit

$$\frac{2(\zeta-\eta)}{(\eta-1)(1+S)} < 1 \text{ seu } 2(\zeta-\eta) < (\eta-1)(1+S);$$

ex qua conditione concludimus

$$\zeta < \eta + \frac{(\eta-1)(1+S)}{2}.$$

Novimus autem esse debere $\zeta > \eta$; unde littera ζ capi debebit intra limites

$$\eta \text{ et } \eta + \frac{(\eta-1)(1+S)}{2},$$

ubi manifestum est esse debere $\eta > 1$. Nostrum igitur problema sequenti modo resolvi conveniet.

Pro lubitu capi possunt litterae S et η, dummodo observetur esse debere $S < 1$ et $\eta > 1$, quia $Pk = \eta - 1$. Deinde littera ζ capiatur intra limites η et $\eta + \frac{(\eta-1)(1+S)}{2}$.
At $\mathfrak{C}$ capiatur intra hos limites

$$\mathfrak{C} < 1 \text{ simulque } \mathfrak{C} > \frac{2(\zeta-\eta)}{(\eta-1)(1+S)},$$

unde simul C definitur. Tum vero capiatur

$$k = \frac{(\eta-1)(1+S)\mathfrak{C}(\mathfrak{M}+\zeta-1)-2\mathfrak{M}(\zeta-\eta)-(\eta-1)(1+S)(\zeta-\eta)}{2\mathfrak{M}\mathfrak{C}\zeta},$$

ex quo habebitur

$$P = \frac{\eta-1}{k} \text{ et } k' = \frac{\mathfrak{M}}{(\eta-1)S}.$$

Postea capiatur $\mathfrak{B}=\frac{1-P}{\zeta}$, unde definitur *B*. Denique ob $Pkk'=\frac{\mathfrak{M}}{S}$ formula supra pro $\mathfrak{D}$ inventa dabit

$$\mathfrak{D}=(\tfrac{\mathfrak{M}}{S}-1)M-(\mathfrak{M}-1)M+2=M\mathfrak{M}(\tfrac{1}{S}-1)+2;$$

ubi substituto valore ipsius *M* prodit

$$\mathfrak{D}=\frac{\frac{2\mathfrak{M}\mathfrak{C}}{S}+2\mathfrak{C}(\zeta-1)-2(\zeta-\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta},$$

qui valor cum sit $\mathfrak{M}$ numerus praemagnus, erit proxima $\mathfrak{D}=\frac{2}{S}$ hincque adcuratius

$$\mathfrak{D}=\frac{2}{S}+\frac{\frac{2}{S}(\zeta-\eta-\mathfrak{C}(\zeta-1))+2\mathfrak{C}(\zeta-1)-2(\zeta-\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}$$

sive

$$\mathfrak{D}=\frac{2}{S}+\frac{2(\frac{1}{S}-1)(\zeta-\eta-\mathfrak{C}(\zeta-1))}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}$$

vel

$$\mathfrak{D}=\frac{2}{S}-\frac{2(\frac{1}{S}-1)(\mathfrak{C}(\zeta-1)-\zeta+\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}$$

unde saltem patet certe fore $\mathfrak{D}>1$ ideoque *D* negativum, ut supra iam notavimus. Iam prouti fuerit *P* sive >1 sive <1, capi debebit vel $A>0$ vel $A<0$, ita ut nunc omnia elementa sint determinata; habebuntur enim distantiae focales

$$p=\mathfrak{A}a,\quad q=-\frac{A\mathfrak{B}}{P}\cdot a,\quad r=-\frac{AB\mathfrak{C}}{Pk}\cdot a,\quad s=-\frac{ABC\mathfrak{C}}{Pkk'}\cdot a\quad \text{et}\quad t=\frac{ABCD}{\mathfrak{M}}\cdot a,$$

deinde intervalla

$$\text{primum}=Aa(1-\tfrac{1}{P}),\quad \text{secundum}=-\tfrac{ABa}{P}(1+\tfrac{1}{k}),$$

$$\text{tertium}=-\tfrac{ABCa}{Pk}(1+\tfrac{1}{k'}),\quad \text{quartum}=-\tfrac{ABCDa}{Pkk'}(1-\tfrac{1}{S});$$

tum vero erit

$$M=2:\left(\mathfrak{M}+\zeta-1-\frac{(\zeta-\eta)}{\mathfrak{C}}\right)$$

et

$$O=\frac{\mathfrak{k}}{\mathfrak{M}M}$$

et apertura primae lentis ex aequatione notissima $\frac{1}{k^3}=\cdots$ definietur.

COROLLARIUM 1

251. Conditio, quam invenimus, $\zeta > \eta$ hic plus non involvit, quam ne ζ minus quam η accipiatur. Nihil ergo impedit, quominus statuamus $\zeta = \eta$; etsi enim hic valor campum aliquantillum diminuit, tamen is adhuc satis prodit notabilis. Tum autem fiat $\mathfrak{r} = 0$ sicque tertia lens quam minimam requiret aperturam, ita ut simul officium diaphragmatis angustissimi praestet.

COROLLARIUM 2

252. Quodsi vero statuamus $\zeta = \eta$, sufficit capere $\mathfrak{C}$ intra limites 0 et 1, unde simul C fit positivum. Tum vero capiatur

$$k = \frac{(\eta-1)(1+S)(\mathfrak{M}+\eta-1)}{2\mathfrak{M}\eta},$$

ex quo habebitur

$$P = \frac{2\mathfrak{M}\eta}{(1+S)(\mathfrak{M}+\eta-1)},$$

ita ut pro maioribus multiplicationibus sit proxima

$$k = \frac{(\eta-1)(1+S)}{2\eta} \quad \text{et} \quad P = \frac{2\eta}{1+S};$$

unde patet esse $P > 1$ ideoque A positivum. Tum vero erit porro

$$\mathfrak{B} = -\frac{(P-1)}{\eta},$$

ita ut sit tam $\mathfrak{B} < 0$ quam $B < 0$. Denique hoc casu fiet

$$\mathfrak{D} = \frac{2\mathfrak{M}+2S(\eta-1)}{S(\mathfrak{M}+\eta-1)}$$

hincque

$$D = -\frac{2\mathfrak{M}+2S(\eta-1)}{\mathfrak{M}(2-S)+S(\eta-1)}$$

atque

$$M = \frac{2}{\mathfrak{M}+\eta-1}.$$

SCHOLION

253. Praeterquam quod hic casus $\zeta = \eta$ ad praxin inprimis est accommodatus, etiam hanc praerogativam complectitur, ut a littera $\mathfrak{C}$ reliqua elementa prorsus non pendeant, ita ut, quomodocunque $\mathfrak{C}$ accipiamus intra limites scilicet 0 et 1, reliqua elementa nullam inde mutationem patiantur. Hoc autem modo facillime evitari poterit, ne lens obiectiva nimis fiat exigua, quod vero insuper commodius per litteram A praestatur, nisi forte de telescopiis sit sermo, ubi $Aa = p$; superfluum igitur foret hic alios casus praeter istum

$\zeta = \eta$ evolvere, atque nunc inprimis operae pretium erit aliquot valores pro η considerare, ut inde intelligere queamus, quinam inde ad praxin maxima futuri sint idonei. Pro littera vero *S*, quam unitate minorem esse debere vidimus, statuamus semper $S = \frac{1}{2}$, quia hinc satis idoneum intervallum inter binas lentes postremas oritur. Tum autem nostrae conditiones sequenti modo exprimentur:

$$1.\ k = \frac{3(\eta-1)(\mathfrak{M}+\eta-1)}{4\mathfrak{M}\eta},$$

$$2.\ P = \frac{4\mathfrak{M}\eta}{3(\mathfrak{M}+\eta-1)},$$

$$3.\ \mathfrak{B} = \frac{-(4\eta-1)\mathfrak{M}+3(\eta-1)}{3\eta(\mathfrak{M}+\eta-1)},$$

4. $\mathfrak{C}$, ut iam notavimus, arbitrio nostro permittitur, dummodo sit intra 0 et 1,

$$5.\ \mathfrak{D} = \frac{4\mathfrak{M}+2(\eta-1)}{\mathfrak{M}+\eta-1} \quad \text{et} \quad D = \frac{-4\mathfrak{M}-2(\eta-1)}{3\mathfrak{M}+\eta-1}.$$

Hinc itaque distantiae focales erunt

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{B}}{P}\cdot a = \frac{(4\eta-3)\mathfrak{M}-3(\eta-1)}{4\mathfrak{M}\eta^2}\cdot Aa,$$

$$r = -\frac{B\mathfrak{C}}{Pk}\cdot Aa = -\frac{B\mathfrak{C}}{\eta-1}\cdot Aa, \quad s = -\frac{BC\mathfrak{D}}{2\mathfrak{M}}\cdot Aa \ \text{ and } \ t = \frac{BCD}{\mathfrak{M}}\cdot Aa.$$

Deinde lentium intervalla

$$\text{primum} = Aa(1-\tfrac{1}{P}) = \frac{(4\eta-3)\mathfrak{M}-3(\eta-1)}{4\mathfrak{M}\eta}\cdot Aa,$$

$$\text{secundum} = -B\left(\frac{\mathfrak{M}(7\eta-3)+3\eta(\eta-2)+3}{4\mathfrak{M}\eta(\eta-1)}\right)Aa,$$

$$\text{tertium} = -BC(\tfrac{1}{\eta-1}+\tfrac{1}{2\mathfrak{M}})Aa,$$

$$\text{quartum} = \frac{BCD}{2\mathfrak{M}}\cdot Aa.$$

Deinde ob $M = \frac{2}{\mathfrak{M}+\eta-1}$ erit

$$O = \tfrac{1}{2}t\left(1+\frac{\eta-1}{\mathfrak{M}}\right).$$

Pro prima lente semidiameter aperturae erit $= x$ ex aequatione notissima definienda; pro secunda autem ob

$$\omega = \frac{2\eta}{\mathfrak{M}+\eta-1}$$

erit semidiameter aperturae

$$= \frac{\eta}{2(\mathfrak{M}+\eta-1)} q + \frac{x}{P}$$

et ob $\mathfrak{r} = 0$ semidiameter aperturae tertiae lentis

$$= \frac{x}{Pk} = \frac{x}{\eta-1};$$

duabus autem reliquis lentibus apertura maxima tribuitur.

EXEMPLUM 1

254. Sumamus $\eta = 2$ eritque $k = \frac{3(\mathfrak{M}+1)}{8\mathfrak{M}}$, et quia tantum de maioribus multiplicationibus agitur, sumi poterit $k = \frac{3}{8}$, hinc

$$P = \frac{8\mathfrak{M}}{3(\mathfrak{M}+1)} = \frac{8}{3} \text{ approx.}$$

Porro

$$\mathfrak{B} = -\tfrac{5}{6} \text{ et } B = -\tfrac{5}{11}, \ \mathfrak{D} = 4 \text{ et } D = -\tfrac{4}{3},$$

unde distantiae focales erunt

$$p = \mathfrak{A}a, \ q = \tfrac{5}{16}Aa, \ r = \tfrac{5}{11}\mathfrak{C}Aa, \ s = \tfrac{10}{11}\tfrac{C}{\mathfrak{M}} \cdot Aa, \ t = \tfrac{20}{33} \cdot \tfrac{C}{\mathfrak{M}} \cdot Aa.$$

et lentium intervalla

$$\text{primum} = \tfrac{5}{8}Aa, \quad \text{secundum} = \tfrac{5}{8}Aa,$$
$$\text{tertium} = \tfrac{5}{11}C(1+\tfrac{1}{2\mathfrak{M}})Aa, \quad \text{quartum} = \tfrac{10}{33} \cdot \tfrac{C}{\mathfrak{M}} \cdot Aa.$$

et

$$M = \tfrac{2}{\mathfrak{M}+1}, \ \text{hinc } z = \tfrac{1}{2} \cdot \tfrac{a}{\mathfrak{M}+1}$$

et

$$O = \tfrac{1}{2}t\left(1+\tfrac{1}{\mathfrak{M}}\right);$$

tum vero semidiameter aperturae lentis secundae $= \frac{q}{\mathfrak{M}+1} + \frac{3}{8}x.$

Pro x autem inveniendo satisfiat huic aequationi:

$$\frac{1}{k^3} = \frac{\mu\mathfrak{M}x^3}{\mathfrak{A}^3a^3}\left\{\begin{array}{l} \lambda+\nu\mathfrak{A}\left(1-\mathfrak{A}\right)-\frac{(1-\mathfrak{A})^3}{\mathfrak{B}^3P}\left(\lambda'+\nu\mathfrak{B}\left(1-\mathfrak{B}\right)\right) \\ -\frac{(1-\mathfrak{A})^3}{B^3\mathfrak{C}^3Pk}\left(\lambda''+\nu\mathfrak{C}\left(1-\mathfrak{C}\right)\right) \\ -\frac{(1-\mathfrak{A})^3}{B^3C^3\mathfrak{D}^3Pkk'}\left(\lambda'''+\nu\mathfrak{D}\left(1-\mathfrak{D}\right)\right)+\frac{(1-\mathfrak{A})^3}{B^3C^3D^3}\cdot\frac{\lambda''''}{Pkk'S} \end{array}\right\}$$

quae aequatio commodius ita repraesentabitur:

$$\frac{1}{k^3} = \mu\mathfrak{M}x^3 \left\{ \begin{aligned} &\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{P^3} + \frac{1}{P^4q^3}\left(\lambda'+\nu\mathfrak{B}(1-\mathfrak{B})\right) \\ &+\frac{1}{(Pk)^4r^3}\left(\lambda''+\nu\mathfrak{C}(1-\mathfrak{C})\right) \\ &+\frac{1}{(Pkk')^4s^3}\left(\lambda'''+\nu\mathfrak{D}(1-\mathfrak{D})\right)+\frac{\lambda''''}{(Pkk'S)^4t^3} \end{aligned} \right\}$$

COROLLARIUM

255. Si haec ad telescopia transferantur ponendo $\mathfrak{A}a = Aa = p$ et $\mathfrak{M} = m$, fient distantiae focales

$$q = \frac{5}{16}, \quad r = \frac{5}{11}\mathfrak{C}p, \quad s = \frac{10}{11}\frac{C}{m}\cdot p, \quad t = \frac{20}{33}\cdot\frac{C}{m}\cdot p,$$

primum $= \frac{5}{8}p$, secundum $= \frac{5}{8}p$, tertium $= \frac{5}{11}C(1+\frac{1}{2m})p$ et quartum $= \frac{10}{33}\cdot\frac{C}{m}\cdot p$
et semidiameter campi $\Phi = \frac{1718}{\mathfrak{M}+1}$ min.
Longitudo ergo erit propemodum $= \frac{110}{88}p + \frac{5}{11}C\cdot p.$
Nunc autem p ex aequatione ante data definiri poterit, ubi erit $\mathfrak{A} = 0$ ob $a = \infty$.

EXEMPLUM 2

256. Sit nunc $\eta = 3$; erit

$$k = \frac{\mathfrak{M}+2}{2\mathfrak{M}},$$

sumi ergo poterit

$$k = \frac{1}{2}, \quad P = \frac{4\mathfrak{M}+2}{3(\mathfrak{M}+2)} = 4, \quad \mathfrak{B} = -\frac{3\mathfrak{M}+2}{3(\mathfrak{M}+2)} = -1, \text{ ergo } B = -\frac{1}{2},$$

$$\mathfrak{D} = \frac{4\mathfrak{M}+4}{\mathfrak{M}+2} = 4, \text{ ergo } D = -\frac{4}{3}.$$

Hinc ergo fient distantiae focales

$$p = \mathfrak{A}a, \quad q = \frac{1}{4}Aa, \quad r = \frac{1}{4}\mathfrak{C}Aa, \quad s = \frac{C}{\mathfrak{M}}\cdot Aa, \text{ et } t = \frac{2}{3}\cdot\frac{C}{\mathfrak{M}}\cdot Aa.$$

et lentium intervalla

primum $= \frac{3}{4}Aa$, secundum $= \frac{3}{8}Aa$,

tertium $= \frac{1}{4}C(1+\frac{1}{m})Aa$, quartum $= \frac{1}{3}\cdot\frac{C}{\mathfrak{M}}\cdot Aa$;

hinc porro erit

$$M=\frac{2}{\mathfrak{M}+2},$$

hinc

$$z=\frac{1}{2}\cdot\frac{a}{\mathfrak{M}+2}$$

et

$$O=\frac{1}{2}t\left(1+\frac{2}{\mathfrak{M}}\right);$$

tum vero semidiameter aperturae lentis

$$\text{secundae}=\frac{3q}{2(\mathfrak{M}+2)}+\frac{1}{4}x \quad \text{et} \quad \text{tertiae}=\frac{1}{2}x.$$

Cetera se habent ut ante.

COROLLARIUM

257. Facta applicatione ad telescopia, ubi fit $Aa=p$, omnia elementa facile determinantur ut ante; tum vero longitudo instrumenti omissis partibus per $\mathfrak{M}$ divisis erit $=\frac{9}{8}p+\frac{1}{4}Cp$, quae minor est quam casu praecedente.

EXEMPLUM 3

258. Statuamus $\eta=6$, ut fiat $M=\frac{2}{\mathfrak{M}+5}$, eritque

$$k=\frac{5(\mathfrak{M}+5)}{8\mathfrak{M}}=\frac{5}{8}\text{ proxime,}$$

$$P=8,\quad \mathfrak{B}=\frac{-7\mathfrak{M}+5}{6(\mathfrak{M}+5)}=-\frac{7}{6}\quad\text{proxime,}$$

unde fit

$$B=-\frac{7}{13},\quad \mathfrak{D}=4,\quad \text{et}\quad D=-\frac{4}{3};$$

unde distantiae focales prodibunt

$$p=\mathfrak{A}a,\quad q=\frac{7}{48}Aa,\quad r=\frac{7}{65}\mathfrak{C}Aa,\quad s=\frac{14}{13}\cdot\frac{C}{\mathfrak{M}}\cdot Aa,\quad t=\frac{28}{39}\cdot\frac{C}{\mathfrak{M}}\cdot Aa$$

et lentium intervalla

$$\text{primum}=\frac{7}{8}Aa,\quad \text{secundum}=\frac{7}{40}Aa,$$

$$\text{tertium}=\frac{7}{13}C\left(\frac{1}{5}+\frac{1}{2\mathfrak{M}}\right)Aa,\quad \text{quartum}=\frac{14}{39}\cdot\frac{C}{\mathfrak{M}}\cdot Aa.$$

Praeterea

$$z=\frac{1}{2}\cdot\frac{a}{\mathfrak{M}+5}\quad \text{et}\quad O=\frac{1}{2}t\left(1+\frac{5}{\mathfrak{M}}\right);$$

tum vero semidiameter aperturae lentis

$$\text{secundae} = \frac{3q}{\mathfrak{M}+5} + \frac{1}{8}x \quad \text{et tertiae} = \frac{1}{5}x.$$

COROLLARIUM

259. Translatione igitur ad telescopia facta prodiret hoc casu eorum longitudo $= \frac{21}{20}p + \frac{7}{65}Cp$, quae longitudo satis est exigua, ut etiam in aliis generibus vix minor sperari queat.

SCHOLION

260. Etsi iste casus $\zeta = \eta$ in praxi summum usum praestare videtur, tamen etiam considerari conveniet quempiam casum, quo $\zeta > \eta$, quandoquidem hoc modo campo quodpiam augmentum adfertur. Manente autem $S = \frac{1}{2}$ alter limes pro ζ erat

$$\zeta < \eta + \frac{3(\eta-1)}{4} \text{ sive } \zeta < \frac{7}{4}\eta - \frac{3}{4}.$$

Huic autem limiti ipsi aequari nequit, quia alioquin $\mathfrak{C}$ deberet esse $=1$ hincque $C = \infty$. Sumamus igitur

$$\zeta = \eta + \frac{2}{4}(\eta - 1) = \frac{3}{2}\eta - \frac{1}{2}$$

ac reperietur $\mathfrak{C} > \frac{2}{3}$ et $\mathfrak{C} < 1$. Sumatur igitur $\mathfrak{C} = \frac{3}{4}$, ut fiat $C = 3$, hincque fiet

$$k = \frac{(\eta-1)(2\mathfrak{M}+15(\eta-1))}{12\mathfrak{M}(3\eta-1)} \quad \text{et} \quad P = \frac{12\mathfrak{M}(3\eta-1)}{2\mathfrak{M}+15(\eta-1)}$$

hincque porro

$$\mathfrak{B} = \frac{-4\mathfrak{M}(18\eta-7)+30(\eta-1)}{(3\eta-1)(2\mathfrak{M}+15(\eta-1))} \quad \text{et} \quad \mathfrak{D} = \frac{24\mathfrak{M}+10(\eta-1)}{6\mathfrak{M}+5(\eta-1)} \quad \text{seu proxime } \mathfrak{D} = 4.$$

Tum vero prodibit

$$M = \frac{12}{6\mathfrak{M}+5(\eta-1)} = \frac{2}{\mathfrak{M}+\frac{5}{6}(\eta-1)},$$

cum antea fuisset

$$M = \frac{2}{\mathfrak{M}+\eta-1}.$$

Quodsi iam sumamus ut in exemplo secundo $\eta = 3$, fient haec elementa

$$k = \frac{\mathfrak{M}+15}{12\mathfrak{M}(3\eta-1)} \quad \text{et} \quad P = \frac{24\mathfrak{M}}{\mathfrak{M}+15}$$

hinc

$$\mathfrak{B} = \frac{-47\mathfrak{M}+15}{4(\mathfrak{M}+15)} \quad \text{et} \quad B = \frac{-47\mathfrak{M}+15}{51\mathfrak{M}+45},$$

tum

$$\mathfrak{C} = \tfrac{3}{4}, \quad C = 3, \quad \mathfrak{D} = 4 \text{ et } D = -\tfrac{4}{3};$$

unde singula momenta pro constructione definiri possunt. Quia vero hic tanti occurrunt numeri, quos prae $\mathfrak{M}$ negligere non amplius licet, in adplicatione ad exempla statim quoque pro $\mathfrak{M}$ determinatum valorem assumi conveniet. Praeterea vero hic ad specialiora non progredimur, quia adhuc lente obiectiva simplice utimur, ita ut confusio aliter tolli nequeat nisi aperturam lentis obiectivae diminuendo; quod remedium cum praxis sponte offerat, non opus erit litteram x molesto illo calculo definire; siquis enim microscopium secundum huiusmodi mensuras construxerit, ipse usus aperturam declarabit; quando autem in sequente capite per multiplicationem lentis obiectivae omnem confusionem ad nihilum redigemus, tum demum necesse erit completas determinationes pro singulis momentis, uti hactenus fecimus, exhibere.

PROBLEMA 2

261. *Microscopium huius generis ex quinque lentibus construere, quae ita sint dispositae, ut prior imago realis in intervallum secundum, posterior vero in intervallum quartum incidat.*

SOLUTIO

Quatuor ergo litterarum P, Q, R, S secunda et quarta erunt negativae; unde ponatur $Q = -k$ et $S = -k'$, ut sit $PkRk' = \frac{ma}{h} = \mathfrak{M}$; hinc erit ultimae lentis distantia focalis

$$t = \frac{ABCD}{PkRk'} \cdot a = \frac{ABCD}{\mathfrak{M}} \cdot a,$$

quae debet esse positiva aeque ac lentium intervalla, quae sunt

$$\text{primum} = Aa(1 - \tfrac{1}{P}), \quad \text{secundum} = -\tfrac{AB}{P} \cdot a(1 + \tfrac{1}{k}),$$
$$\text{tertium} = -\tfrac{ABC}{Pk} \cdot a(1 - \tfrac{1}{R}), \quad \text{quartum} = +\tfrac{ABCD}{PkR} \cdot a(1 + \tfrac{1}{k'});$$

ergo ut tam ultima lens quam ultimum intervallum fiant positiva, debet esse $ABCD > 0$. Hinc ut tertium quoque fiat positivum, debet esse

$$-D\left(1 - \tfrac{1}{R}\right) > 0$$

sicque circa D nihil definitur. Ob secundum autem intervallum debet esse $-AB > 0$ et ob primum $Aa(1 - \frac{1}{P}) > 0$. Tum ergo debebit esse $CD < 0$.

Statuatur nunc

$$M = \frac{\mathfrak{q} + \mathfrak{r} + \mathfrak{s} + \mathfrak{t}}{\mathfrak{M} - 1},$$

et quia campus maximus desideratur, statim sumi poterit $\mathfrak{s} = 1$ et $\mathfrak{t} = 1$, ut fiat

$$M=\frac{\mathfrak{q}+\mathfrak{r}+2}{\mathfrak{M}-1}$$

hincque distantia oculi

$$O=\frac{t}{M\mathfrak{M}};$$

quae cum sit positiva, destructio marginis praebet

$$0=\frac{\mathfrak{q}}{P}-\frac{\mathfrak{r}}{Pk}-\frac{1}{PkR}+\frac{1}{PkRk'},$$

unde colligitur

$$\frac{1}{k'}=-\mathfrak{q}kR+\mathfrak{r}R+1;$$

hinc erit

$$PkR=\mathfrak{M}\left(1+\mathfrak{r}R-\mathfrak{q}kR\right)$$

sicque patet esse $1+\mathfrak{r}R>qkR$. Praeterea vero considerare debemus sequentes aequationes:

$$1.\ \mathfrak{B}\mathfrak{q}=\left(P-1\right)M,$$
$$2.\ \mathfrak{C}\mathfrak{r}=-\left(1+Pk\right)M-\mathfrak{q},$$
$$3.\ \mathfrak{D}=-\ (1+PkR)M-\mathfrak{q}-\mathfrak{r}.$$

Ponatur hic ut ante brevitatis gratia

$$\frac{1-P}{\mathfrak{B}}=\zeta \quad \text{et} \quad 1+Pk=\eta$$

fietque

$$\mathfrak{q}=-\zeta M \quad \text{et} \quad \mathfrak{r}=\frac{(\zeta-\eta)M}{\mathfrak{C}},$$

unde colligitur

$$2+\mathfrak{q}+\mathfrak{r}=\frac{2\mathfrak{C}+\left(\zeta\left(1-\mathfrak{C}\right)-\eta\right)M}{\mathfrak{C}}=\left(\mathfrak{M}-1\right)M;$$

ex quo aequatione deducitur

$$M=\frac{2\mathfrak{C}}{\mathfrak{C}\left(\mathfrak{M}-1\right)-\zeta\left(1-\mathfrak{C}\right)+\eta}=\frac{2\mathfrak{C}}{\mathfrak{C}\left(\mathfrak{M}+\zeta-1\right)-\zeta+\eta};$$

ex quo valore vicissim erit

$$\mathfrak{q}=-\frac{2\zeta\mathfrak{C}}{\mathfrak{C}\left(\mathfrak{M}+\zeta-1\right)-\zeta+\eta} \quad \text{et} \quad \mathfrak{r}=\frac{2\left(\zeta-\eta\right)}{\mathfrak{C}\left(\mathfrak{M}+\zeta-1\right)-\zeta+\eta}.$$

Nunc ut marginis colorati rationem habeamus, erit statim

$$1+\mathfrak{r}R-\mathfrak{q}kR=\frac{\mathfrak{C}\left(\mathfrak{M}+\zeta-1\right)-\zeta+\eta+2\left(\zeta-\eta\right)R+2\zeta\mathfrak{C}kR}{\mathfrak{C}\left(\mathfrak{M}+\zeta-1\right)-\zeta+\eta}.$$

Et cum ob $Pk = \eta - 1$ sit $PkR = (\eta - 1)R$, erit

$$\begin{aligned}&\mathfrak{C}(\eta-1)R(\mathfrak{M}+\zeta-1)-(\eta-1)(\zeta-\eta)R-\mathfrak{M}\mathfrak{C}(\mathfrak{M}+\zeta-1)\\&+\mathfrak{M}(\zeta-\eta)-2\mathfrak{M}(\zeta-\eta)R-2\mathfrak{M}\zeta\mathfrak{C}kR=0.\end{aligned}$$

Antequam autem hinc vel k vel R determinemus, considerare debemus rationem litterae $\mathfrak{D}$ ex superiori tertia aequatione; cum igitur PkR sit sine dubio numerus magnus involvens $\mathfrak{M}$, facile intelligitur litteram $\mathfrak{D}$ esse negativam; unde etiam erit D negativum adeoque concludimus fore $C > 0$ hincque $\mathfrak{C} < 1$. Ob eandem vero rationem debebit esse $R > 1$, ita ut haec littera aliquatenus tanquam nota spectari possit; quare ex illa aequatione colligimus

$$k=\left\{\begin{aligned}&\mathfrak{C}(\eta-1)R(\mathfrak{M}+\zeta-1)-(\eta-1)(\zeta-\eta)R\\&-\mathfrak{M}\mathfrak{C}(\mathfrak{M}+\zeta-1)+\mathfrak{M}(\zeta-\eta)-2\mathfrak{M}(\zeta-\eta)R\end{aligned}\right\}:2\mathfrak{M}\zeta\mathfrak{C}R$$

hincque $P = \frac{(\eta-1)}{k}$, ita ut sit $\eta > 1$. Quare ut valor ipsius k fiat positivus, debet esse

$$\begin{aligned}&R\big(\mathfrak{C}(\eta-1)(\mathfrak{M}+\zeta-1)-(\eta-1)(\zeta-\eta)-2\mathfrak{M}(\zeta-\eta)\big)\\&\qquad>\mathfrak{M}\big(\mathfrak{C}\mathfrak{M}+\mathfrak{C}(\zeta-1)-\zeta+\eta\big),\end{aligned}$$

ad quod primo requiritur, ut quantitas litteram R multiplicans sit positiva, et quia $\mathfrak{M}$ est numerus praemagnus, ipsius coefficiens ante omnia debet esse positivus, unde colligimus

$$\mathfrak{C}(\eta-1) > 2(\zeta-\eta),$$

unde concluditur

$$\mathfrak{C} > \frac{2(\zeta-\eta)}{(\eta-1)};$$

quia igitur $\mathfrak{C} < 1$, erit

$$2(\zeta-\eta) < \eta - 1 \text{ sicque } \zeta < \frac{3\eta-1}{2}.$$

Qua conditione impleta debet esse

$$R > \frac{\mathfrak{M}\big(\mathfrak{C}\mathfrak{M}+\mathfrak{C}(\zeta-1)-\zeta+\eta\big)}{\mathfrak{C}(\eta-1)(\mathfrak{M}+\zeta-1)-2\mathfrak{M}(\zeta-\eta)-(\eta-1)(\zeta-\eta)}$$

atque hinc retrogrediendo omnia elementa determinabuntur. Reliqua vero expediuntur ut in praecedente problemate.

COROLLARIUM 1

262. Hic igitur littera R denotabit numerum magnum $\mathfrak{M}$ involventem, deinde conditio $\zeta < \frac{3\eta-1}{2}$ instituto nostro maxime favet, cum campi conditio inprimis postulet, ne ζ ultra necessitatem augeatur. Quare, cum semper sit $\eta > 1$, commodissime videtur statui posse $\zeta = \eta$ uti in praecedente problemate, ita ut tertiae lentis apertura iterum fiat minima prodeatque

$$M = \frac{2\mathfrak{C}}{\mathfrak{C}\mathfrak{M}+\mathfrak{C}\eta-\mathfrak{C}} = \frac{2}{\mathfrak{M}+\eta-1}.$$

COROLLARIUM 2

263. Sumto autem $\zeta = \eta$ pro $\mathfrak{C}$ limites erunt $\mathfrak{C} < 1$ et $\mathfrak{C} > 0$. Porro capi debet $R > \frac{\mathfrak{M}}{\eta-1}$ indeque fiet

$$k = \frac{\left((\eta-1)R-\mathfrak{M}\right)\left(\mathfrak{M}+\eta-1\right)}{2\mathfrak{M}\eta R}$$

et

$$P = \frac{2\mathfrak{M}\eta(\eta-1)R}{\left((\eta-1)R-\mathfrak{M}\right)\left(\mathfrak{M}+\eta-1\right)}.$$

Praeterea vero erit

$$\mathfrak{B} = \frac{-(\eta-1)R\left((2\eta-1)\mathfrak{M}-\eta+1\right)-\mathfrak{M}(\mathfrak{M}+\eta-1)}{\eta\left((\eta-1)R-\mathfrak{M}\right)\left(\mathfrak{M}+\eta-1\right)}$$

Denique vero reperietur

$$\mathfrak{D} = -\frac{2\left(1+(\eta-1)R-\eta\right)}{\mathfrak{M}+\eta-1}$$

sive, cum $\mathfrak{M}$ et R sint numeri praemagni, erit proxima

$$\mathfrak{D} = -\frac{2(\eta-1)R}{\mathfrak{M}},$$

qui valor certo est negativus, ut supra iam posuimus.

COROLLARIUM 3

264. Quin etiam statui poterit $\zeta = 0$; unde pro $\mathfrak{C}$ limites erunt $\mathfrak{C} < 1$ et $\mathfrak{C} > -\frac{2\eta}{\eta-1}$; cui satisfit, dummodo $\mathfrak{C}$ intra limites 1 et 0 contineatur.
Tum vero sumi debebit

$$R > \frac{\mathfrak{M}(\mathfrak{C}\mathfrak{M}-\mathfrak{C}+\eta)}{\mathfrak{C}(\eta-1)(\mathfrak{M}-1)+2\mathfrak{M}\eta+\eta(\eta-1)}$$

sive ob $\mathfrak{M}$ numerum praemagnum

$$R > \frac{\mathfrak{C}\mathfrak{M}}{\mathfrak{C}(\eta-1)+2\eta}.$$

Verum hinc sequitur porro $k=\infty$ et $P=0$, ita ut sit $Pk=\eta-1$. Praeterea vero prodit

$$\mathfrak{B}=\infty \text{ et } B=-1.$$

Denique vero ob

$$\mathfrak{q}=0 \text{ et } \mathfrak{r}=-\frac{\eta M}{\mathfrak{C}}$$

erit

$$\mathfrak{D}=-\left(1+(\eta-1)R-\frac{\eta}{\mathfrak{C}}\right)M$$

ideoque ob

$$M=\frac{2\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}-1)+\eta},$$

qui valor ipsius *M* aliquanto minor est quam casu praecedente, fiet

$$\mathfrak{D}=-\frac{2(\eta-1)R}{\mathfrak{M}}.$$

SCHOLION

265. Quantumvis hic casus paradoxus videatur cum ob $\mathfrak{B}=\infty$ tum vero ob $P=0$, tamen revera est realis et ad casum in praecedente capite expositum reducitur; quia enim , secundae lentis distantia focalis est infinita sicque res eodem redit, ac si secunda lens plane abesset, ita ut non amplius quaestio sit de eius loco; quare, etsi primum intervallum prodeat

$$=Aa\left(1-\frac{1}{P}\right)=-\infty$$

et secundum

$$=Aa\left(\frac{1}{P}+\frac{1}{\eta-1}\right)=+\infty,$$

tamen horum summa, quae sola nunc est spectanda, fit finita

$$=Aa\left(1+\frac{1}{\eta-1}\right)=\frac{\eta}{\eta-1}\cdot Aa.$$

Cum igitur tantum quatuor lentes hic habeantur, hic casus ad praecedens caput est referendus. Interim tamen hinc incommodum nasci debebit, quando ζ prope ad 0 accedit, quia tum *P* etiam erit unitate minus, ita ut *A* debeat esse numerus negativus et $B>0$. Cum autem sit $\mathfrak{B}=\frac{1-P}{\zeta}$, erit quidem $\mathfrak{B}>0$, verum insuper necesse est, ut sit $1-P<\zeta$ vel $P>1-\zeta$ sive *P* contineri debebit intra limites 1 et $1-\zeta$ seu esse debet $k<\frac{\eta-1}{1-\zeta}$; quare, cum $\mathfrak{M}$ et *R* sint numeri praemagni, debebit esse

$$R\left(\frac{\mathfrak{C}(\eta-1)(1-3\zeta)}{1-\zeta}-2(\zeta-\eta)\right)<\mathfrak{CM},$$

quod sponte evenit ob $\zeta<\eta$, si fuerit

$$\frac{\mathfrak{C}(\eta-1)(1-3\zeta)}{(1-\zeta)(\zeta-\eta)}>2 \text{ sive } \mathfrak{C}>\frac{2(1-\zeta)(\zeta-\eta)}{(\eta-1)(1-3\zeta)}.$$

Sin autem sit $\frac{\mathfrak{C}(\eta-1)(1-3\zeta)}{(1-\zeta)(\zeta-\eta)}<2$, debet esse

$$R<\frac{(1-\zeta)\mathfrak{CM}}{\mathfrak{C}(\eta-1)(1-3\zeta)-2(\zeta-\eta)(1-\zeta)},$$

quibus observatis aliquot casus fusius evolvamus.

CASUS 1 QUO $\zeta=\eta$

266. Hoc casu iam vidimus lentem tertiam nostro arbitrio relinqui, dummodo pro ea capiatur $\mathfrak{C}<1$ et $\mathfrak{C}>0$, ut C fiat numents positivus; unde, si circumstantiae postulant, ut C sit numerus satis magnus, tum $\mathfrak{C}$ parum ab unitate deficere debebit; deinde etiam notavimus capi debere $R>\frac{\mathfrak{M}}{\eta-1}$ unde, cum semper sit $\eta>1$, si etiam fuerit >2, tunc commode sumi poterit $R=\mathfrak{M}$. Notetur autem litteram η non nimis magnam sumi convenire, quia pro campo fit

$$M=\frac{2}{\mathfrak{M}+\eta-1}.$$

Deinde vero prodit

$$k=\frac{((\eta-1)R-\mathfrak{M})(\mathfrak{M}+\eta-1)}{2\mathfrak{M}\eta R},$$

quare pro maioribus multiplicationibus tuto sumi poterit

$$k=\frac{(\eta-1)R-\mathfrak{M}}{2\eta R};$$

unde patet litteram k eo fore minorem, quo minus R limitem praescriptum $\frac{\mathfrak{M}}{\eta-1}$ superet; unde fit

$$P=\frac{\eta-1}{k}=\frac{2\eta(\eta-1)R}{(\eta-1)R-\mathfrak{M}}.$$

reliquis elementis primo prodit

$$\mathfrak{B}=\frac{-(\eta-1)R((2\eta-1)\mathfrak{M}-\eta+1)-\mathfrak{M}(\mathfrak{M}+\eta-1)}{\eta((\eta-1)R-\mathfrak{M})(\mathfrak{M}+\eta-1)}$$

hincque proxime

$$\mathfrak{B} = \frac{-(\eta-1)(2\eta-1)R-\mathfrak{M}}{\eta(\eta-1)R-\eta\mathfrak{M}},$$

qui valor manifesto est negativus, ex quo etiam B erit negativum. Deinde cum sit $P>1$, ob eandem rationem littera A debebit esse positiva; ex quo productum AB ob

$$B = \frac{-(\eta-1)(2\eta-1)R-\mathfrak{M}}{(\eta-1)(3\eta-1)R-(\eta-1)\mathfrak{M}}$$

sponte fit negativum, prorsus uti conditiones praescriptae postulant. Denique vero reperitur

$$\mathfrak{D} = -\frac{2(1+(\eta-1)R-\eta)}{\mathfrak{M}+\eta-1}$$

ideoque proxime

$$\mathfrak{D} = -\frac{2(\eta-1)R}{\mathfrak{M}};$$

unde fit

$$D = -\frac{2(\eta-1)R}{\mathfrak{M}+2(\eta-1)R}.$$

His definitis erunt primo distantiae focales

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{B}}{P}\cdot a = \frac{(\eta-1)(2\eta-1)R+\mathfrak{M}}{2\eta^2(\eta-1)R}\cdot Aa,$$
$$r = -\frac{AB\mathfrak{C}}{\eta-1}\cdot a, \quad s = \frac{ABC\mathfrak{D}}{(\eta-1)R}\cdot a \;\text{ et }\; t = \frac{ABCD}{\mathfrak{M}}\cdot a,$$

deinde vero intervalla

$$\text{primum} = Aa\left(1-\tfrac{1}{P}\right), \quad \text{secundum} = -ABa\left(\tfrac{1}{P}+\tfrac{1}{\eta-1}\right),$$
$$\text{tertium} = -\frac{ABCa}{\eta-1}\left(1-\tfrac{1}{R}\right), \quad \text{quartum} = ABCDa\left(\frac{1}{(\eta-1)R}+\frac{1}{\mathfrak{M}}\right).$$

Distantia vero oculi fiet

$$O = \frac{t}{M\mathfrak{M}} = \frac{t}{2}\cdot\frac{\mathfrak{M}+\eta-1}{\mathfrak{M}} = \frac{1}{2}t\cdot\left(1+\frac{\eta-1}{\mathfrak{M}}\right) = \frac{1}{2}t \text{ proxime.}$$

Pro aperturis vero lentium primae quidem apertura tutissime per experientiam definitur; unde reperitur littera x ex eaque mensura claritatis $=\frac{20x}{\mathfrak{M}}$, si scilicet x in digitis exprimatur.

Pro secunda vero lente cum sit

$$\mathfrak{q} = -\eta M = -\frac{2\eta}{\mathfrak{M}+\eta-1} = -\frac{2\eta}{\mathfrak{M}},$$

erit eius aperturae semidiameter

$$= \tfrac{1}{4}\mathfrak{q}q + \tfrac{x}{P} = \tfrac{1}{2}\cdot\tfrac{\eta}{\mathfrak{M}}\cdot q + \tfrac{x}{P}.$$

Pro tertia vero lente ob $\mathfrak{r} = 0$ sufficit aperturae semidiameter $= \frac{x}{\eta-1}$; reliquas vero lentes utrinque aeque convexas confici convenit.

COROLLARIUM

267. Si sumatur $R = \frac{\mathfrak{M}}{\eta-1}$, ut fiat $k = 0$ et $P = \infty$ existente $Pk = \eta - 1$, tum fiet $\mathfrak{B} = \frac{1-P}{\eta} = \infty$ et $B = -1$. Tum igitur secunda lens in ipsam imaginem realem priorem incidet ob primum intervallum $= Aa = \alpha$ eiusque distantia focalis erit $q = \frac{Aa}{\eta}$. At vero secundum intervallum hoc casu evadet $= -\frac{ABa}{\eta-1} = \frac{Aa}{\eta-1}$. Reliqua vero definiuntur ut in genere, si modo notetur fore $\mathfrak{D} = -2$ et $D = -\frac{2}{3}$.

EXEMPLUM 1

[267a]. Ponamus $\eta = 2$ et capi debebit $R > \mathfrak{M}$. Nihil vero impedit, quominus secundum praecedens corollarium sumamus $R = \mathfrak{M}$, ita ut tum fiat $k = 0$ et $P = \infty$; quare primo distantiae focales ita exprimentur:

$$p = \mathfrak{A}a,\quad q = \frac{Aa}{2},\quad r = A\mathfrak{C}a,\quad s = \frac{2AC\mathfrak{C}}{\mathfrak{M}}\cdot a \;\text{ et }\; t = \frac{2}{3}\cdot\frac{C}{\mathfrak{M}}\cdot Aa = \frac{1}{3}s.$$

Deinde intervalla ita se habebunt:

$$\text{primum} = Aa,\quad \text{secundum} = Aa,$$
$$\text{tertium} = C\left(1-\frac{1}{\mathfrak{M}}\right)Aa,\quad \text{quartum} = \frac{4}{3}\cdot\frac{C}{\mathfrak{M}}\cdot Aa;$$

distantia vero oculi

$$O = \tfrac{1}{2}t\left(1+\tfrac{1}{\mathfrak{M}}\right).$$

Pro valore ipsius x sive per experientiam sive per formulam notam definito fiat secundae lentis semidiameter aperturae $= \frac{q}{\mathfrak{M}} = \frac{1}{2}\cdot\frac{Aa}{\mathfrak{M}}$ et tertiae lentis $= x$; semidiameter spatii conspicui erit $z = \frac{a}{2\mathfrak{M}}$ et mensura claritatis $= \frac{20x}{\mathfrak{M}}$.

COROLLARIUM

268. Hae formulae quoque ad telescopia accommodari poterunt sumendo $Aa = p$ et $\mathfrak{M} = m$. Tum vero sumi debebit ipsa distantia focalis

$$p = m\sqrt[3]{\mu m}\left(1 + \ * \ + \frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right)$$

omissis terminis sequentibus per $\mathfrak{M}$ divisis.

EXEMPLUM 2

269. Sit nunc $\eta = 3$, ut esse debeat $R > \frac{\mathfrak{M}}{2}$ sumaturque $R = \mathfrak{M}$, unde fiet $k = \frac{1}{6}$ et $P = 12$; quare reliqua elementa fient

$$\mathfrak{B} = -\frac{11}{12} \text{ hincque } B = -\frac{11}{14} \text{ et } \mathfrak{D} = -4 \text{ et } D = -\frac{4}{5},$$

unde distantiae focales evadent

$$p = \mathfrak{A}a, \quad q = \frac{11}{36}Aa, \quad r = \frac{11}{28}\mathfrak{C}Aa, \quad s = \frac{11}{7}\frac{C}{\mathfrak{M}}\cdot Aa \text{ et } t = \frac{22}{35}\frac{C}{\mathfrak{M}}\cdot Aa = \frac{2}{5}s$$

atque intervalla lentium

$$\text{primum} = \frac{11}{12}Aa, \quad \text{secundum} = \frac{11}{24}Aa,$$

$$\text{tertium} = \frac{11}{28}C\left(1 - \frac{1}{\mathfrak{M}}\right)Aa, \quad \text{quartum} = \frac{33}{35}\cdot\frac{C}{\mathfrak{M}}\cdot Aa$$

oculique distantia

$$O = \frac{1}{2}t\left(1 + \frac{2}{\mathfrak{M}}\right).$$

spatii vero in obiecto conspicui erit semidiameter

$$z = \frac{a}{2(\mathfrak{M}+2)}.$$

Definito x sive per experientiam sive per formulam notam erit semidiameter aperturae

$$\text{secundae lentis} = \frac{3}{2}\cdot\frac{q}{\mathfrak{M}} + \frac{x}{12} \text{ et tertiae} = \frac{x}{2}$$

et gradus claritatis $= \frac{20x}{\mathfrak{M}}$.

COROLLARIUM

270. Hae formulae etiam ad telescopia accommodari possunt; erit enim

$Aa = p$ et $\mathfrak{M} = m$. Tum vero lentis obiectivae distantia focalis definitur per hanc formulam:

$$p = m\sqrt[3]{\mu m}\left(1+\tfrac{1}{12}\left(\tfrac{3^3}{11^3}\lambda' - \tfrac{3\cdot 14\nu}{11^2}\right) + \tfrac{14^3}{11^3\cdot 2}\left(\tfrac{\lambda''}{\mathfrak{C}^3} + \tfrac{\nu}{C\mathfrak{C}}\right)\right).$$

Longitudo huius telescopii erit circiter $= \left(\tfrac{11}{8} + \tfrac{11}{28}C\right)p$.

EXEMPLUM 3

271. Sit nunc $\eta = 6$, ut esse debeat $R > \tfrac{\mathfrak{M}}{5}$ et sumatur $R = \tfrac{\mathfrak{M}}{2}$, unde fiet $k = \tfrac{1}{4}$ et $P = 20$. Hinc porro fiet $\mathfrak{B} = -\tfrac{19}{6}$ et $B = -\tfrac{19}{25}$. Tum vero $\mathfrak{D} = -5$ et $D = -\tfrac{5}{6}$.

Distantiae ergo focales ita se habebunt:

$$p = \mathfrak{A}a,\quad q = \tfrac{19}{120}Aa,\quad r = \tfrac{19}{125}\mathfrak{C}Aa,\quad s = \tfrac{38}{25}\tfrac{C}{\mathfrak{M}}\cdot Aa \text{ et } t = \tfrac{19}{30}\tfrac{C}{\mathfrak{M}}\cdot Aa \text{ seu } t = \tfrac{5}{12}s$$

et intervalla lentium

$$\text{primum} = \tfrac{29}{10}Aa,\quad \text{secundum} = \tfrac{19}{100}Aa,$$
$$\text{tertium} = \tfrac{19}{125}C\left(1-\tfrac{2}{\mathfrak{M}}\right)Aa, \text{ et quartum} = \tfrac{133}{150}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa$$

oculique distantia

$$O = \tfrac{1}{2}t\left(1+\tfrac{5}{\mathfrak{M}}\right),$$

spatii conspicui semidiameter

$$z = \tfrac{a}{2(\mathfrak{M}+5)}.$$

Definito denique x ut ante erit semidiameter aperturae lentis

$$\text{secundae lentis} = \tfrac{3}{\mathfrak{M}}\cdot q + \tfrac{x}{20} \text{ et tertiae} = \tfrac{1}{5}x;$$

gradus autem claritatis manet $= \tfrac{20x}{\mathfrak{M}}$.

EXEMPLUM 4

272. Sit ut ante $\eta = 6$, sumatur vero $R = \mathfrak{M}$ ac reperitur $k = \tfrac{1}{3}$ et $P = 15$. Hinc porro fit $\mathfrak{B} = -\tfrac{7}{3}$ et $B = -\tfrac{7}{10}$, at $\mathfrak{D} = -10$ et $D = -\tfrac{10}{11}$.

Distantiae ergo focales erunt

$$p=\mathfrak{A}a,\quad q=\tfrac{7}{45}Aa,\quad r=\tfrac{7}{50}\mathfrak{C}Aa,$$
$$s=\tfrac{7}{5}\tfrac{C}{\mathfrak{M}}\cdot Aa,\quad t=\tfrac{7}{11}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa \text{ seu } t=\tfrac{5}{11}s$$

et lentium intervalla

$$\text{primum}=\tfrac{14}{15}Aa,\quad \text{secundum}=\tfrac{14}{75}Aa,$$
$$\text{tertium}=\tfrac{7}{50}C\left(1-\tfrac{2}{\mathfrak{M}}\right)Aa,\quad \text{quartum}=\tfrac{42}{55}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa$$

et distantia oculi

$$O=\tfrac{1}{2}t\left(1+\tfrac{5}{\mathfrak{M}}\right),$$

pariter ac reliqua momenta se habet ut ante.

COROLLARIUM

273. Si haec ad telescopia transferantur ponendo $Aa=p$ et $\mathfrak{M}=m,$ casus hic posterior antecedenti praeferendus videtur, siquidem praebet longitudinem parumper minorem, quippe quae neglectis terminis per $\mathfrak{M}$ divisis erit $=(1\tfrac{3}{25}+\tfrac{7}{50}C)p$, cum ex exemplo praecedente fuerit $=(1\tfrac{7}{50}+\tfrac{19}{125}C)p.$ Casu ergo ultimi exempli lentis obiectivae distantia focalis ita definietur, ut sit

$$p=m\sqrt[3]{\mu m}\left(1+\tfrac{1}{15}\left(\tfrac{3^3}{7^3}\lambda'-\tfrac{30\nu}{7^2}\right)+\tfrac{200}{343}\left(\tfrac{\lambda''}{\mathfrak{C}^3}+\tfrac{\nu}{C\mathfrak{C}}\right)\right).$$

CASUS 2
QUO $\zeta=1$

274. Cum sit $\zeta=1$, limites pro $\mathfrak{C}$ erunt $\mathfrak{C}<1$ et $\mathfrak{C}>-2$, ita ut $\mathfrak{C}$ aeque arbitrio nostro permittatur ac ante. Tum vero esse debebit

$$R>\frac{\mathfrak{M}(\mathfrak{C}\mathfrak{M}+\eta-1)}{(\eta-1)(\mathfrak{C}\mathfrak{M}+2\mathfrak{M}+\eta-1)}$$

seu neglectis minoribus partibus

$$R>\frac{\mathfrak{C}\mathfrak{M}}{(\eta-1)(2+\mathfrak{C})}.$$

Statuatur ergo $R=i\mathfrak{M}$ sumendo scilicet

$$i>\frac{\mathfrak{C}}{(\eta-1)(2+\mathfrak{C})}.$$

Tum ergo fiet

$$k=\frac{\mathfrak{C}(\eta i-i-1)+2i(\eta-1)}{2i\mathfrak{C}}\quad\text{et}\quad P=\frac{2i(\eta-1)\mathfrak{C}}{\mathfrak{C}(i(\eta-1)-1)+2i(\eta-1)}$$

Hinc ergo fiet

$$P-1=\frac{\mathfrak{C}\left(i(\eta-1)+1\right)-2i(\eta-1)}{\mathfrak{C}\left(i(\eta-1)-1\right)+2i(\eta-1)}$$

qui valor erit positivus seu $P>1$, si fuerit

$$i<\frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})}.$$

Hoc ergo casu prodit

$$\mathfrak{B}=\frac{-\mathfrak{C}\left(i(\eta-i)+1\right)+2i(\eta-1)}{\mathfrak{C}\left(i(\eta-i)-1\right)+2i(\eta-1)};$$

qui valor cum sit negativus, etiam *B* erit negativum et ob $P>1$ debebit *A* esse positivum, ut superiores conditiones postulant; sin autem esset

$$i>\frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})},$$

tum foret $P<1$ sumique deberet *A* negative ac prodiret $\mathfrak{B}>0$; unde, ut etiam *B* fiat positivum, insuper necesse est, ut sit $\mathfrak{B}<1$, quod manifestum est, cum sit $\mathfrak{B}=1-P$. Postea vero pro $\mathfrak{D}$ inveniendo notetur esse

$$M=\frac{2\mathfrak{C}}{\mathfrak{C}\mathfrak{M}+\eta-1}$$

et

$$\mathfrak{q}=-M \quad \text{et} \quad \mathfrak{r}=-\frac{(\eta-1)M}{\mathfrak{C}},$$

ex quibus prodit

$$\mathfrak{D}=-i\left(\eta-1\right)M\mathfrak{M}=-2i\left(\eta-1\right);$$

illi vero valores abibunt in hos:

$$\mathfrak{q}=-\frac{2}{\mathfrak{M}} \quad \text{et} \quad \mathfrak{r}=-\frac{(\eta-1)}{\mathfrak{C}\mathfrak{M}}.$$

His valoribus inventis considerentur primo distantiae focales, quae sunt

$$p=\mathfrak{A}a, \quad q=-\frac{A\mathfrak{B}}{P}\cdot a, \quad r=-\frac{AB\mathfrak{C}}{\eta-1}\cdot a, \quad s=\frac{ABC\mathfrak{D}}{(\eta-1)\mathfrak{M}}\cdot a \quad \text{et} \quad t=\frac{ABCD}{\mathfrak{M}}\cdot a.$$

Tum vero intervalla

$$\text{primum}=Aa\left(1-\frac{1}{P}\right), \quad \text{secundum}=-ABa\left(\frac{1}{P}+\frac{1}{\eta-1}\right),$$

$$\text{tertium}=-\frac{ABCa}{\eta-1}\left(1-\frac{1}{i\mathfrak{M}}\right), \quad \text{quartum}=+\frac{ABCDa}{\mathfrak{M}}\left(\frac{1}{i(\eta-1)}+1\right),$$

distantia vero oculi

$$O = \frac{t}{\mathfrak{M}M} = \frac{1}{2}t\left(1+\frac{\eta-1}{\mathfrak{C}\mathfrak{M}}\right).$$

Deinde simili modo, ut iam ante notavimus, littera x sive per experientiam sive ex formula nota definiri poterit. Tum vero erit lentis secundae semidiameter aperturae

$$= \frac{1}{4}\mathfrak{q}q + \frac{x}{P} = \frac{1}{2}\cdot\frac{q}{\mathfrak{M}} + \frac{x}{P},$$

tertiae vero lentis

$$= \frac{1}{4}\mathfrak{t}r + \frac{x}{\eta-1} = \frac{1}{2}\cdot\frac{(\eta-1)r}{\mathfrak{C}\mathfrak{M}} + \frac{x}{\eta-1},$$

reliquae vero lentes, quia sunt utrinque aeque convexae, maximam aperturam admittunt. Pro spatio denique conspicuo erit

$$z = \frac{1}{2}\cdot\frac{\mathfrak{C}a}{\mathfrak{C}\mathfrak{M}+\eta-1} = \frac{a}{2\mathfrak{M}} \quad \text{proxime}$$

et mensura claritatis $= \frac{20x}{\mathfrak{M}}$.

COROLLARIUM 1

275. Si littera i contineatur intra hos limites, scilicet

$$i > \frac{\mathfrak{C}}{(\eta-1)(2+\mathfrak{C})} \quad \text{et} \quad i < \frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})},$$

tum fit $P > 1$ et litterae $\mathfrak{B}$ et B negativae, littera vero A sumi debet positive; unde omnia elementa eiusdem sunt naturae uti in casu praecedente.

COROLLARIUM 2

276. Sin autem adeo fuerit $i > \frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})}$, tum littera P unitate fit minor hincque tam littera $\mathfrak{B}$ quam B fiunt positivae; at vero littera A esse debebit negativa, id quod duplici modo evenire potest, altero, quo $\mathfrak{A} > 1$, altero vero, quo $\mathfrak{A} < 0$; quo posteriore casu lens prima evaderet concava et instrumentum multis incommodis foret obnoxium.

COROLLARIUM 3

277. Sin autem fuerit $i = \frac{\mathfrak{C}}{(\eta-1)(2-\mathfrak{C})}$, tum fiet $P = 1$ hincque $\mathfrak{B} = 0$ et $B = 0$. Tum igitur, ne fiat $q = 0$, necesse erit sumi $A = \infty$, ita tamen, ut sit

$$AB = A\mathfrak{B} = -\frac{q}{a}.$$

Unde, cum sit $1 - P = \mathfrak{B}$, erit primum intervallum

$$= Aa\left(1-\tfrac{1}{P}\right) = -A\mathfrak{B}a = q;$$

quare ob $\mathfrak{A}=1$ fient distantiae focales

$$p=a,\quad q=q,\quad r=\tfrac{\mathfrak{C}q}{\eta-1}\cdot a,\quad s=-\tfrac{C\mathfrak{D}}{i(\eta-1)\mathfrak{M}}\cdot q \ \text{ seu } s=\tfrac{2C}{\mathfrak{M}}\cdot q$$

et
$$t=-\tfrac{CD}{\mathfrak{M}}\cdot q \quad \text{seu} \quad t=\tfrac{2i(\eta-1)C}{2i(\eta-1)+1}\cdot\tfrac{q}{\mathfrak{M}}.$$

Intervalla vero lentium

$$\text{primum}=q,\quad \text{secundum}=\tfrac{\eta}{\eta-1}\cdot q,\quad \text{tertium}=\tfrac{C}{\eta-1}\left(1-\tfrac{1}{i\mathfrak{M}}\right)q,$$
$$\text{quartum}=-\tfrac{CDa}{\mathfrak{M}}\left(1+\tfrac{1}{i(\eta-1)}\right)q=\tfrac{2i(\eta-1)+2}{2i(\eta-1)+1}\cdot\tfrac{C}{\mathfrak{M}}\cdot q.$$

In reliquis vero momentis nihil mutandum occurrit.

SCHOLION

278. Probe autem hic est notandum casus in his duobus postremis corollariis contentos neutiquam ad telescopia transferri posse. Pro telescopiis enim ob $a=\infty$ necessario sumi debet $\mathfrak{A}=0$ et $A=0$, cum in his casibus debeat esse A vel infinitum vel negativum.

EXEMPLUM 1

279. Sumamus $\eta=2$, et quia C in iis tantum terminis occurrit, qui per $\mathfrak{M}$ sunt divisi, ideoque semper numerum praemagnum significare debet, pro $\mathfrak{C}$ recte unitatem assumere poterimus; hinc ergo pro littera R primus limes erit $i>\tfrac{1}{3}$; ut nostra instrumenta ad casum corollarii primi pertineant, sumi quoque debet $i<1$; hinc ergo capiatur $i=\tfrac{1}{2}$, ut sit $R=\tfrac{1}{2}\mathfrak{M}$; unde colligemus $k=\tfrac{1}{2}$ et $P=2,$ deinde $\mathfrak{B}=-1$ et $B=-\tfrac{1}{2}$ et $\mathfrak{D}=-1$, hinc $D=-\tfrac{1}{2}$. Hinc distantiae focales erunt

$$p=\mathfrak{A}a,\quad q=\tfrac{1}{2}Aa,\quad r=\tfrac{\mathfrak{C}}{2}\cdot Aa,\quad s=\tfrac{CAa}{\mathfrak{M}} \ \text{ et } \ t=\tfrac{1}{4}\tfrac{C}{\mathfrak{M}}\cdot Aa \quad \text{seu} \quad t=\tfrac{1}{4}s.$$

Intervalla vero lentium erunt

$$\text{primum}=\tfrac{1}{2}Aa,\quad \text{secundum}=\tfrac{3}{4}Aa,\quad \text{tertium}=\tfrac{C}{2}\left(1-\tfrac{2}{\mathfrak{M}}\right)Aa,\quad \text{quartum}=\tfrac{3}{4}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa$$

ac distantia oculi

$$O = \tfrac{1}{2}t\left(1+\tfrac{1}{\mathfrak{M}}\right),$$

semidiametri denique aperturarum lentis

$$\text{primae} = x, \quad \text{secundae} = \tfrac{1}{2}\cdot\tfrac{q}{\mathfrak{M}} + \tfrac{1}{2}x, \quad \text{tertiae} = \tfrac{1}{2}\cdot\tfrac{r}{\mathfrak{M}} + x.$$

EXEMPLUM 2

280. Maneat $\eta = 2$, sumatur vero $i = 1$ sive $R = \mathfrak{M}$ atque erit $k = 1$ et $P = 1$, tum vero $\mathfrak{B} = 0 = B$ et $\mathfrak{D} = -2$ et $D = -\tfrac{2}{3}$; ; unde ex corollario tertio nanciscimur

$$p = a, \quad q = q, \quad r = q, \quad s = \tfrac{2C}{\mathfrak{M}}\cdot q, \quad t = \tfrac{2}{3}\cdot\tfrac{C}{\mathfrak{M}}\cdot q \text{ seu } t = \tfrac{1}{3}s.$$

Intervalla vero lentium erunt

$$\text{primum} = a \quad \text{secundum} = 2q, \quad \text{tertium} = C\left(1-\tfrac{1}{\mathfrak{M}}\right), \quad \text{quartum} = \tfrac{4}{3}\cdot\tfrac{C}{\mathfrak{M}}\cdot q;$$

reliqua vero momenta perinde ac in praecedente exemplo se habebunt.

EXEMPLUM 3.

281. Maneat $\eta = 2$ et sumatur $i = -2$, ut sit $R = 2\mathfrak{M}$; erit ergo $k = \tfrac{5}{4}$ et $P = \tfrac{4}{5}$, unde $\mathfrak{B} = \tfrac{1}{5}$, hinc $B = \tfrac{1}{4}$; tum vero $\mathfrak{D} = -4$ et $D = -\tfrac{4}{5}$.
Hinc, cum A debeat esse negativum, statuatur

$$A = -\alpha, \quad \text{ut sit } \mathfrak{A} = -\tfrac{\alpha}{1-\alpha} = \tfrac{\alpha}{\alpha-1};$$

ex quo distantiae focales erunt

$$p = \tfrac{\alpha}{\alpha-1}, \quad q = \tfrac{1}{4}\alpha a, \quad r = \tfrac{1}{4}\mathfrak{C}\alpha a, \quad s = \tfrac{1}{2}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a \text{ et } t = \tfrac{1}{5}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a \text{ sive } t = \tfrac{1}{5}s.$$

Intervalla vero erunt

$$\text{primum} = \tfrac{1}{4}\alpha a, \quad \text{secundum} = \tfrac{9}{16}\alpha a, \quad \text{tertium} = \tfrac{1}{4}C\left(1-\tfrac{1}{2\mathfrak{M}}\right)\alpha a, \quad \text{quartum} = \tfrac{3}{10}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a.$$

Reliqua vero momenta sunt iterum ut in exemplo primo. Hic autem probe notandum est, si capiatur $\alpha = 1$, prodire $p = \infty$ ideoque primam lentem plane reiici posse, ita ut microscopium ex solis lentibus posterioribus componatur. Quia autem tum confusio prodiret enormis, cum in formulis capitis praecedentis sumi deberet $\mathfrak{A} = \tfrac{1}{0}$, hoc instrumentum merito reiicimus multoque magis ea, quae prodirent, si esset $\alpha < 1$. At si α unitatem haud parum superet, haec instrumenta in praxi usum habere posse videntur.

EXEMPLUM 4

282. Sit nunc $\eta = 4$, et cum primus limes sit $i > \frac{1}{9}$, pro casu corollarii primi sumamus $i < \frac{1}{3}$; Sit igitur $i = \frac{1}{6}$ sumto $\mathfrak{C} = 1$ eritque $k = \frac{3}{2}$, $P = 2$, hinc $\mathfrak{B} = -1$, $B = -\frac{1}{2}$, $\mathfrak{D} = -1$, $D = -\frac{1}{2}$; unde distantiae focales erunt

$$p = \mathfrak{A}a, \;\; q = \tfrac{1}{2}Aa, \;\; r = \tfrac{1}{6}Aa, \;\; s = \tfrac{C}{\mathfrak{M}} \cdot Aa \;\text{ et }\; t = \tfrac{1}{4} \cdot \tfrac{C}{\mathfrak{M}} \cdot Aa \;\text{ seu }\; t = \tfrac{1}{4}s.$$

Intervalla vero lentium

$$\text{primum} = \tfrac{1}{2}Aa, \quad \text{secundum} = \tfrac{5}{12}Aa, \quad \text{tertium} = \tfrac{C}{6}\left(1 - \tfrac{6}{\mathfrak{M}}\right)Aa, \;\; \text{quartum} = \tfrac{3}{4} \cdot \tfrac{C}{\mathfrak{M}} \cdot Aa.$$

oculi distantia

$$O = \tfrac{1}{2}t\left(1 + \tfrac{3}{\mathfrak{M}}\right).$$

Porro

$$z = \frac{a}{2(\mathfrak{M}+3)}.$$

Semidiametri porro aperturarum erunt lentis

$$\text{primae} = x, \;\; \text{secundae} = \tfrac{1}{2} \cdot \tfrac{q}{\mathfrak{M}} + \tfrac{1}{2}x, \;\; \text{tertiae} = \tfrac{3}{2} \cdot \tfrac{r}{\mathfrak{M}} + \tfrac{1}{3}x.$$

EXEMPLUM 5

283. Maneat $\eta = 4$, at sumatur $i = \frac{1}{3}$ secundum corollarium tertium eritque $k = 3$ et $P = 1$; unde colliguntur distantiae focales

$$p = a, \;\; q = q, \;\; r = \tfrac{1}{3}q, \;\; s = \tfrac{2C}{\mathfrak{M}} \cdot q \;\text{ et }\; t = \tfrac{2}{3} \cdot \tfrac{C}{\mathfrak{M}} \cdot q = \tfrac{1}{3}s$$

et lentium intervalla

$$\text{primum} = q, \quad \text{secundum} = \tfrac{4}{3}q, \quad \text{tertium} = \tfrac{C}{3}\left(1 - \tfrac{3}{\mathfrak{M}}\right)q, \;\; \text{quartum} = \tfrac{4}{3} \cdot \tfrac{C}{\mathfrak{M}} \cdot q = 2t.$$

Distantia oculi

$$O = \tfrac{1}{2}t\left(1 + \tfrac{3}{\mathfrak{M}}\right),$$

et reliqua momenta omnia sunt ut ante.

EXEMPLUM 6

284. Manente $\eta = 4$ sumatur $i = 1$ eritque $k = 4$ et $P = \frac{3}{4}$, tum vero $\mathfrak{B} = \frac{1}{4}$ et $B = \frac{1}{3}$ et $\mathfrak{D} = -6$, $D = -\frac{6}{7}$. Cum igitur B sit positivum, littera A negativa esse debet. Sit igitur $A = -\alpha$ fietque $\mathfrak{A} = \frac{\alpha}{\alpha - 1}$; unde prodibunt distantiae focales

$$p=\tfrac{\alpha}{\alpha-1}\cdot a,\quad q=\tfrac{1}{3}\alpha a,\quad r=\tfrac{1}{9}\mathfrak{C}\alpha a,\quad s=\tfrac{2}{3}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a \text{ et } t=\tfrac{2}{7}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a=\tfrac{3}{7}s$$

et intervalla lentium

primum $=\tfrac{1}{3}\alpha a$, secundum $=\tfrac{5}{9}\alpha a$, tertium $=\tfrac{1}{9}C\left(1-\tfrac{1}{\mathfrak{M}}\right)\alpha a$, quartum $=\tfrac{8}{21}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a.$

Distantia oculi O cum reliquis momentis eadem ac ante manet.

PROBLEMA 3

285. *Microscopium ex sex lentibus construere, quae ita sint dispositae, ut prior imago realis in intervallum secundum, posterior vero in quartum incidat.*

SOLUTIO

Quinque igitur litterarum P, Q, R, S, T secunda et quarta debent esse negativae; quare ponatur $Q=-k$ et $S=-k'$, ut sit

$$PkRk'T=\mathfrak{M}=\tfrac{ma}{h}.$$

Hinc erit ultimae lentis distantia focalis

$$u=-\tfrac{ABCDE}{\mathfrak{M}}\cdot a,$$

quae debet esse positiva aeque ac lentium intervalla, quae sunt

$$\text{primum}=Aa\left(1-\tfrac{1}{P}\right),\quad \text{secundum}=-\tfrac{AB}{P}\cdot a\left(1+\tfrac{1}{k}\right),$$
$$\text{tertium}=-\tfrac{ABC}{Pk}\cdot a\left(1-\tfrac{1}{R}\right),\quad \text{quartum}=\tfrac{ABCD}{PkR}\cdot a\left(1+\tfrac{1}{k'}\right),$$
$$\text{quintum}=\tfrac{ABCDE}{PkRk'}\cdot a\left(1-\tfrac{1}{T}\right).$$

Ob quintum ergo debet esse $T<1$, ob quartum vero $E<0$ hincque $ABCD>0$.
Ob secundum vero esse debet $AB<0$ hincque etiam CD negativum. Statuatur nunc

$$M=\tfrac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t}+\mathfrak{u}}{\mathfrak{M}-1},$$

et quia campus maximus desideratur, sumi poterit $\mathfrak{s}=1$, $\mathfrak{t}=1$ et $\mathfrak{u}=1$, ut fiat

$$M=\tfrac{\mathfrak{q}+\mathfrak{r}+3}{\mathfrak{M}-1}$$

hincque

$$z=Ma\xi=\tfrac{1}{4}Ma$$

et distantia oculi

$$O=\tfrac{u}{\mathfrak{M}M};$$

quae cum sit positiva, destructio marginis colorati praebet

$$0 = \frac{\mathfrak{q}}{P} - \frac{\mathfrak{r}}{Pk} - \frac{1}{PkR} + \frac{1}{PkRk'} + \frac{1}{PkRk'T},$$

unde colligitur

$$\frac{1}{k'}\left(1+\frac{1}{T}\right) = -\mathfrak{q}kR + \mathfrak{r}R + 1,$$

et quia constat esse $T < 1$, statuatur brevitatis gratia

$$1+\frac{1}{T} = \theta$$

ut sit $\theta > 2$, hincque

$$T = \frac{1}{\theta - 1},$$

ex quo habebitur

$$\frac{1}{k'} = \frac{-\mathfrak{q}kR + \mathfrak{r}R + 1}{\theta};$$

ergo ob

$$\frac{PkRk'}{\theta - 1} = \mathfrak{M}$$

fiet statim

$$PkR = \frac{(\theta-1)\mathfrak{M}}{k'} = \frac{\mathfrak{M}(\theta-1)(1+\mathfrak{r}R-\mathfrak{q}kR)}{\theta}.$$

Praeterea iam considerandae sunt aequationes sequentes:

$$\begin{aligned}
&1.\ \mathfrak{B}\mathfrak{q} = (P-1)M,\\
&2.\ \mathfrak{C}\mathfrak{r} = -(1+Pk)M - \mathfrak{q},\\
&3.\ \mathfrak{D} = -(1+PkR)M - \mathfrak{q} - \mathfrak{r},\\
&4.\ \mathfrak{E} = (PkRk' \text{-} 1)M - \mathfrak{q} - \mathfrak{r} - 1:
\end{aligned}$$

pro quarum resolutione statuamus brevitatis gratia

$$\frac{1-P}{\mathfrak{B}} = \zeta \quad \text{et} \quad 1 + Pk = \eta,$$

ut fiat

$$\mathfrak{q} = -\zeta M \quad \text{et} \quad \mathfrak{t} = \frac{(\zeta-\eta)M}{\mathfrak{C}};$$

ergo

$$3+\mathfrak{q}+\mathfrak{r} = \frac{3\mathfrak{C}+\left(\zeta(1-\mathfrak{C})-\eta\right)M}{\mathfrak{C}} = M\left(\mathfrak{M}-1\right);$$

unde concluditur

$$M = \frac{3\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta};$$

unde vicissim

$$\mathfrak{q} = -\frac{3\zeta\mathfrak{C}}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta} \quad \text{et} \quad \mathfrak{r} = -\frac{3(\zeta-\eta)}{\mathfrak{C}(\mathfrak{M}+\zeta-1)-\zeta+\eta}.$$

Nunc ergo habebimus $PkR = (\eta - 1)R$ seu

$$(\eta - 1)R = \mathfrak{M} \cdot \tfrac{\theta - 1}{\theta} \cdot \left(1 + \tfrac{3R(\zeta + \zeta\mathfrak{C}k - \eta)}{\mathfrak{C}(\mathfrak{M} + \zeta - 1) - \zeta + \eta}\right),$$

unde ob rationes ante allegatas litteram k definire convenit; quem in finem notasse iuvabit litteras ζ et η una cum $\mathfrak{C}$ semper prae multiplicatione $\mathfrak{M}$ fore valde exiguas; alioquin enim campus praeter necessitatem diminueretur; contra vero R etiam fore numerum praemagnum; unde superior illa aequatio induet hanc formam:

$$(\eta - 1)R = \tfrac{\theta - 1}{\theta} \cdot \left(\tfrac{\mathfrak{C}\mathfrak{M} + 3R(\zeta - \eta) + 3R\zeta\mathfrak{C}k}{\mathfrak{C}}\right),$$

ex qua sequitur

$$k = \tfrac{(\eta - 1)\mathfrak{C}\theta R - (\theta - 1)\mathfrak{C}\mathfrak{M} - 3(\theta - 1)(\zeta - \eta)R}{3(\theta - 1)\zeta\mathfrak{C}R};$$

qui valor cum debeat, esse positivus, debebit esse

$$R > \tfrac{(\theta - 1)\mathfrak{C}\mathfrak{M}}{(\eta - 1)\mathfrak{C}\theta - 3(\theta - 1)(\zeta - \eta)}$$

existente

$$(\eta - 1)\mathfrak{C}\theta > 3(\theta - 1)(\zeta - \eta) \text{ seu } \mathfrak{C} > \tfrac{3(\theta - 1)(\zeta - \eta)}{(\eta - 1)\theta}.$$

Cum vero ut in problemate praecedente esse debeat $\mathfrak{C} < 1$, numerator illius limitis minor esse debet suo denominatore hincque

$$\zeta < \tfrac{\theta(4\eta - 1) - 3\eta}{3(\theta - 1)}.$$

His ergo conditionibus observatis ponamus brevitatis gratia iterum ut ante $R = i\mathfrak{M}$, ita ut esse debeat

$$i > \tfrac{(\theta - 1)\mathfrak{C}}{(\eta - 1)\mathfrak{C}\theta - 3(\theta - 1)(\zeta - \eta)};$$

habebimus inde

$$k = \tfrac{i(\eta - 1)\mathfrak{C}\theta - (\theta - 1)\mathfrak{C} - 3i(\theta - 1)(\zeta - \eta)}{3i(\theta - 1)\zeta\mathfrak{C}} \text{ et } P = \tfrac{\eta - 1}{k};$$

pro quo valore duos casus considerari convenit.

Si $P > 1$, tum debet esse $A > 0$ ideoque $B < 0$, quod quidem sponte evenit, cum prodeat $\mathfrak{B} < 0$. Hoc ergo evenit, quando $k < \eta - 1$; ex quo concluditur

$$i < \frac{(\theta-1)\mathfrak{C}}{(\eta-1)\mathfrak{C}\theta-3(\theta-1)\left((\eta-1)\zeta\mathfrak{C}+\zeta-\eta\right)};$$

qui limes manifesto maior est superiore. Sin autem littera i adeo hunc limitem superet, tunc fiat $P< 1$ ideoque A negative sumi debebit, et quia $\mathfrak{B}$ prodit positivum, B eatenus tantum erit positivum, uti requiritur, quatenus fit $\mathfrak{B} < 1$. Fit vero semper $\mathfrak{B} < 1$, nisi fuerit $\zeta < 1$; atque si etiam fuerit $\zeta < 1 - P$, casus erit impossibilis. Deinde cum sit

$$PkR = (\eta-1)i\mathfrak{M},$$

neglectis terminis prae $\mathfrak{M}$ valde parvis ob $\mathfrak{M}M = 3$ proxime erit

$$\mathfrak{D} = -3i(\eta-1), \text{ hinc } D = -\frac{3i(\eta-1)}{3i(\eta-1)+1}.$$

Porro cum sit

$$PkRk' = \frac{\mathfrak{M}}{T} = \mathfrak{M}(\theta-1),$$

fiet eodem modo

$$\mathfrak{E} = 3\theta-4 \text{ et } E = \frac{3\theta-4}{5-3\theta} \text{ seu } E = -\frac{3\theta-4}{3\theta-5}\cdot$$

Hinc ergo distantiae focales ita se habebunt:

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{B}}{P}\cdot a, \quad r = -\frac{AB\mathfrak{C}}{\eta-1}\cdot a,$$
$$s = -\frac{ABC3i(\eta-1)}{(\eta-1)i\mathfrak{M}}\cdot a = -\frac{3ABC}{\mathfrak{M}}\cdot a,$$
$$t = -\frac{3i(\eta-1)(3\theta-4)}{(3i(\eta-1)+1)(\theta-1)}\cdot\frac{ABC}{\mathfrak{M}}\cdot a,$$
$$u = -\frac{3i(\eta-1)(3\theta-4)}{(3i(\eta-1)+1)(3\theta-5)}\cdot\frac{ABC}{\mathfrak{M}}\cdot a;$$

ubi notetur esse quoque

$$q = \frac{1}{\zeta}\left(1-\frac{1}{P}\right)Aa,$$

ita ut sit q ad primum intervallum ut $1:\zeta$. Intervalla autem erunt

$$\text{primum} = Aa\left(1-\tfrac{1}{P}\right) = \zeta q,$$
$$\text{secundum} = -ABa\left(\tfrac{1}{P}+\tfrac{1}{\eta-1}\right),$$
$$\text{tertium} = -ABCa\left(\tfrac{1}{\eta-1}-\tfrac{1}{i(\eta-1)\mathfrak{M}}\right) = -\tfrac{ABC}{\eta-1}\cdot a\left(1-\tfrac{1}{i\mathfrak{M}}\right),$$
$$\text{quartum} = \tfrac{ABCDa}{\mathfrak{M}}\left(\tfrac{1}{i(\eta-1)}+\tfrac{1}{\theta-1}\right) = -\tfrac{3(i(\eta-1)+\theta-1)}{(3i(\eta-1)+1)(\theta-1)}\cdot\tfrac{ABC}{\mathfrak{M}}\cdot a,$$
$$\text{quintum} = -\tfrac{3i(\eta-1)(3\theta-4)(\theta-2)}{(3i(\eta-1)+1)(\theta-1)(3\theta-5)}\cdot\tfrac{ABC}{\mathfrak{M}}\cdot a.$$

Distantia vero oculi erit

$$O = \tfrac{u}{\mathfrak{M}M} = \tfrac{1}{3}u \text{ proxime.}$$

Spatii vero conspicui semidiameter erit

$$z = \tfrac{1}{4}aM = \tfrac{3a}{4\mathfrak{M}}.$$

Tum vero semidiameter aperturae lentis primae est $= x$ sive per formulam notam sive per experientiam definienda, lentis vero

$$\text{secundae} = \tfrac{1}{4}\mathfrak{q}q+\tfrac{x}{P} = \tfrac{3}{4}\cdot\tfrac{\zeta}{\mathfrak{M}}\cdot q+\tfrac{x}{P},$$
$$\text{tertiae} = \tfrac{1}{4}\mathfrak{r}r+\tfrac{x}{Pk} = \tfrac{3}{4}\cdot\tfrac{\zeta-\eta}{\mathfrak{C}\mathfrak{M}}\cdot r+\tfrac{x}{\eta-1};$$

reliquarum vero lentium, quae debent esse utrinque aeque convexae, semidiametri aperturarum erunt respective $\tfrac{1}{4}s$, $\tfrac{1}{4}t$ et $\tfrac{1}{4}u$. Denique autem mensura claritatis fiet $= \tfrac{20x}{\mathfrak{M}}$.

COROLLARIUM 1

286. Si statuatur $\zeta = 0$, pro secunda lente erit $q = \infty$, qui casus eodem redit, ac si haec lens plane abesset; tum autem erit $k = \infty$ et $P = 0$, $\mathfrak{B} = \infty$ et $B = -1$; unde, etsi primum intervallum fit $= -\infty$, ob secundam lentem deficientem intervallum primae et tertiae lentis fiet nihilominus finitum

$$= \tfrac{\eta}{\eta-1}\cdot Aa.$$

Deinde vero capi debebit

$$i > \tfrac{(\theta-1)\mathfrak{C}}{(\eta-1)\mathfrak{C}\theta+3\eta(\theta-1)}.$$

Pro $\mathfrak{C}$ vero sufficit, ut capiatur intra limites 1 et 0, quandoquidem *C* debet esse numerus positivus ob $D<0$. Reliquae vero determinationes manent ut ante, si modo notetur esse $B=-1$.

COROLLARIUM 2

287. Quia hoc casu lens secunda tollitur, hoc modo solutio habebitur problematis, quo microscopium ex quinque lentibus constructum quaeritur, quae ita sint dispositae, ut prima imago realis in primum intervallum, posterior vero in tertium incidat; cuius ergo problematis solutio etiam suppeditat campum triplicatum.

COROLLARIUM 3

288. Quia in genere ob rationes ante allegatas littera *C* semper designare debet numerum satis magnum, ne scilicet lentes posteriores fiant nimis exiguae, satis prope erit $\mathfrak{C}=1$ atque adeo in praxi tuto statuere licebit $\mathfrak{C}=1$. Tum igitur sumi debebit

$$i>\frac{\theta-1}{(\eta-1)\theta-3(\theta-1)(\zeta-\eta)},$$

deinde vero

$$k=\frac{i(\eta-1)\theta-3i(\theta-1)(\zeta-\eta)-\theta+1}{3i(\theta-1)\zeta}.$$

CASUS 1
QUO $i=\infty$ ET $\theta=3$

289. Hoc ergo casu esse debet

$$\zeta<\frac{3\eta-1}{2};$$

tum vero erit

$$k=\frac{3\eta-2\zeta-1}{2\zeta} \quad \text{et} \quad P=\frac{2(\eta-1)\zeta}{3\eta-2\zeta-1}.$$

Ut igitur fiat $P>1$, debebit esse $\zeta>\frac{3\eta-1}{2\eta}$. Quare fiet $P>1$, si capiatur ζ intra limites $\frac{3\eta-1}{2}$ et $\frac{3\eta-1}{2\eta}$; quo ergo casu *A* sumi debet positive, et quia reperitur $\mathfrak{B}<0$, sponte fit $B<0$. Sin autem sit $\zeta<\frac{3\eta-1}{2\eta}$, tunc erit $P<1$ hincque $A<0$, ita ut debeat esse $B>0$ hincque $\mathfrak{B}<1$; erit vero $\mathfrak{B}<1$, si $1-P<\zeta$; sive $P>1-\zeta$, quod quidem semper evenit, nisi sit $\zeta<1$. Superest ergo examinare casum $\xi<1$, et quia tum esse debet $P>1-\zeta$, oritur inde haec relatio:

$$\zeta(5\eta-1)-2\zeta^2>3\eta-1;$$

unde patet esse debere

$$\zeta=\frac{5\eta-1}{4}-\frac{1}{4}\sqrt{25\zeta^2-34\eta+9-\alpha}$$

denotante α numerum quempiam positivum sive

$$\zeta > \frac{5\eta-1}{4} - \frac{1}{4}\sqrt{25\zeta^2 - 34\eta + 9};$$

qui ergo limes pro ζ pendet ab η, ita ut sumto $\eta = 2$ debeat esse

$$\zeta > \frac{9-\sqrt{41}}{4} \text{ seu } \zeta > \frac{5}{8};$$

sin antem fuerit $\eta = 4$, debet esse

$$\zeta > \frac{19-\sqrt{273}}{4} \text{ seu proxime } \zeta > \frac{5}{8}.$$

At si sit $\eta = 6$, prodit

$$\zeta > \frac{29-\sqrt{705}}{4} \text{ seu proxime } \zeta > \frac{5}{8}$$

ut ante sicque patet ζ nunquam infra $\frac{5}{8}$ accipi posse. Nunc igitur pronuntiare poterimus limites, intra quos ζ capi debeat, esse $\frac{5}{8}$ et $\frac{3\eta-1}{2}$.

His notatis distantiae focales erunt

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{B}}{P}\cdot a = \frac{1}{\zeta}\left(1 - \frac{1}{p}\right), \quad r = -\frac{AB\mathfrak{C}}{\eta-1}\cdot a,$$

$$s = -3\cdot\frac{ABC}{\mathfrak{M}}\cdot a, \quad t = -\frac{5}{2}\cdot\frac{ABC}{\mathfrak{M}}\cdot a, \quad u = -\frac{5}{4}\cdot\frac{ABC}{\mathfrak{M}}\cdot a$$

et lentium intervalla

$$\text{primum} = Aa\left(1 - \frac{1}{P}\right), \quad \text{secundum} = -AB\left(\frac{1}{P} + \frac{1}{\eta-1}\right)a,$$

$$\text{tertium} = -\frac{ABC}{\eta-1}\cdot a, \quad \text{quartum} = -\frac{1}{2}\cdot\frac{ABC}{\mathfrak{M}}\cdot a, \quad \text{quintum} = -\frac{5}{8}\cdot\frac{ABC}{\mathfrak{M}}\cdot a.$$

Reliqua momenta se habent uti in problemate, quippe quae ab *u* non pendent.

SCHOLION

290. Mirum hic videbitur, quod hoc casu tam *P* maius unitate quam minus unitate fieri possit, cum in solutione problematis ostenderimus tum solum *P* fieri >1, quando littera *i* contineatur intra limites

$$\frac{(\theta-1)\mathfrak{C}}{(\eta-1)\mathfrak{C}\theta-3(\theta-1)(\zeta-\eta)} \quad \text{et} \quad \frac{(\theta-1)\mathfrak{C}}{(\eta-1)\mathfrak{C}\theta-3(\theta-1)\left(\zeta-\eta+(\eta-1)\zeta\mathfrak{C}\right)},$$

quando vero i etiam posteriorem limitem superaverit, tum semper fore $P<1$. Quare, cum hic adeo sumserimus $i=\infty$ hinc utique sequi videtur semper esse debere $P<1$, quod tamen, ut vidimus, secus evenit. Ad quod dubium solvendum natura posterioris limitis accuratius perpendi debet; si enim is ipse iam fieret infinitus, tum certe mirari desinemus, si etiam sumto $i=\infty$ reperiatur $P>1$. Sin autem in hoc limite posteriore denominator non solum evanescat, sed adeo negativus evadat, tum ipse limes non tam negativus quam infinito maior spectari debebit, ita ut positio $i=\infty$ adhuc inter illos limites contineri sit censenda. Nunc igitur manifestum est limitem posteriorem fieri $=\infty$ si sumto $\mathfrak{C}=1$ fuerit

$$\zeta=\frac{(4\eta-1)\theta-3\eta}{3\eta(\theta-1)},$$

sumtoque $\theta=3$, uti fecimus, si fuerit $\zeta=\frac{3\eta-1}{2\eta}$. Sin autem sit $\zeta>\frac{3\eta-1}{2\eta}$ (semper autem esse debet $\zeta<\frac{3\eta-1}{2\eta}$), tum ille limes fit quasi infinito maior hincque $i=\infty$ ipso minor; unde necessario fieri debebit $P>1$. Sin autem sit $\zeta<\frac{3\eta-1}{2\eta}$, tum ille limes adhuc erit finitus ideoque valor $i=\infty$ illo erit sine dubio maior; unde etiam his tantum casibus fiet $P<1$. Hoc notato istum casum aliquot exemplis illustremus.

EXEMPLUM 1

291. Sumamus $\eta=2$, et cum pro ζ prior limes sit $\zeta<\frac{5}{2}$, posterior vero limes $\frac{5}{4}$, sumamus $\zeta=2$, ut cadat intra hos limites; unde fiet $k=\frac{1}{4}$ et $P=4$, hinc $\mathfrak{B}=-\frac{3}{2}$ et $B=-\frac{3}{5}$; unde distantiae focales erunt

$$p=\mathfrak{A}a, \quad q=-\frac{1}{4}\mathfrak{B}Aa=\frac{3}{8}Aa, \quad r=\frac{3}{5}Aa,$$
$$s=\frac{9}{5}\cdot\frac{C}{\mathfrak{M}}\cdot Aa, \quad t=\frac{3}{2}\cdot\frac{C}{\mathfrak{M}}\cdot Aa, \quad u=\frac{3}{4}\cdot\frac{C}{\mathfrak{M}}\cdot Aa$$

et lentium intervalla

$$\text{primum}=\frac{3}{4}Aa, \quad \text{secundum}=\frac{3}{4}Aa,$$
$$\text{tertium}=\frac{3}{5}CAa, \quad \text{quartum}=\frac{3}{10}\frac{C}{\mathfrak{M}}\cdot Aa, \quad \text{quintum}=\frac{3}{8}\cdot\frac{C}{\mathfrak{M}}\cdot Aa.$$

Distantia oculi $O=\frac{1}{3}u$ proxime, $z=\frac{3a}{4\mathfrak{M}}$,
semidiameter aperturae lentis

$$\text{primae}=x, \quad \text{secundae}=\frac{3}{2\mathfrak{M}}\cdot q+\frac{x}{4}, \quad \text{tertiae}=x$$

ac denique mensura claritatiss $=\frac{20x}{\mathfrak{M}}$ semper.

EXEMPLUM 2

292. Manente $\eta=2$ aequetur ζ ipsi alteri limiti, scilicet $\zeta=\frac{5}{4}$, fietque $k=1$ et $P=1$; hinc ergo prodit $\mathfrak{B}=0$ et $B=0$. Quare, ne tam secunda lens quam intervalla evanescant, sumi debet $A=\infty$ ideoque $\mathfrak{A}=1$, ita ut sit $A\mathfrak{B}$ sive $AB=-\frac{q}{a}$, et cum sit $\mathfrak{B}=\frac{4}{5}(1-P)$, erit revera $1-P=\frac{5}{4}\mathfrak{B}$
hincque

$$Aa\left(1-\frac{1}{P}\right)=\frac{5}{4}q.$$

Quare distantiae focales erunt

$$p=a,\quad q=q,\quad r=\mathfrak{C}q,\quad s=3\cdot\frac{C}{\mathfrak{M}}\cdot q,\quad t=\frac{5}{2}\cdot\frac{C}{\mathfrak{M}}\cdot q,\quad u=\frac{5}{4}\cdot\frac{C}{\mathfrak{M}}\cdot q,$$

ubi secunda q arbitrio nostro relinquitur. Tum vero lentium intervalla

$$\text{primum}=\frac{5}{4}q,\quad \text{secundum}=2q,\ \text{tertium}=Cq,$$
$$\text{quartum}=\frac{1}{2}\cdot\frac{C}{\mathfrak{M}}\cdot q,\quad \text{quintum}=\frac{5}{8}\cdot\frac{C}{\mathfrak{M}}\cdot q.$$

Valores O et z erunt ut ante, at semidiameter aperturae lentis

$$\text{secundae}=\frac{15}{16\mathfrak{M}}\cdot q+x\quad\text{et}\quad\text{tertiae}=\frac{9}{16\mathfrak{M}}\cdot r+x.$$

EXEMPLUM 3

293. Manente $\eta=2$ sumatur $\zeta<\frac{5}{4}$, et cum esse debeat $\zeta>\frac{5}{8}$, uti ostendimus, sumatur $\zeta=\frac{3}{4}$ eritque $k=\frac{7}{3}$ et $P=\frac{3}{7}$; unde fit $\mathfrak{B}=\frac{16}{21}$ hincque $B=\frac{16}{5}$; qui valor cum sit positivus, littera A negative capi debet, uti etiam primum intervallum postulat ob $P<1$. Sit igitur $A=-\alpha$ ideoque

$$\mathfrak{A}=\frac{\alpha}{\alpha-1}$$

eruntque distantiae focales

$$p=\frac{\alpha a}{\alpha-1},\quad q=\frac{16}{9}\alpha a,\quad r=\frac{15}{5}\mathfrak{C}\alpha a,\quad s=\frac{48}{5}\frac{C}{\mathfrak{M}}\cdot\alpha a,\quad t=8\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a,\quad u=4\cdot\frac{C}{\mathfrak{M}}\cdot\alpha a.$$

et intervalla lentium

$$\text{primum} = \tfrac{4}{3}\alpha a, \quad \text{secundum} = \tfrac{32}{3}\alpha a, \ \text{tertium} = \tfrac{16}{5}C\mathfrak{A}\alpha a,$$
$$\text{quartum} = \tfrac{8}{5}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a, \ \text{quintum} = 2\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a.$$

Reliqua se habebunt ut ante, ac si qua differentia in aperturis deprehenditur, ea in praxi attendi non meretur; interim tamen semidiameter

$$\text{secundae lentis} = \tfrac{9}{16}\cdot\tfrac{q}{\mathfrak{M}}+\tfrac{7}{3}x \text{ et tertiae} = \tfrac{15}{16}\cdot\tfrac{r}{\mathfrak{M}}+x.$$

EXEMPLUM 4

294. Statuatur nunc $\eta = 4$, et cum esse debeat $\zeta < \frac{11}{2}$, posterior vero limes sit $\frac{11}{8}$, quo scilicet casus $P > 1$ et $P < 1$ distinguuntur, sumatur $\zeta = 3$ $k = \frac{5}{6}$ et $P = \frac{18}{5}$. Unde fit $\mathfrak{B} = -\frac{13}{15}$ et $B = -\frac{13}{28}$. Distantiae ergo focales lentium erunt

$$p = \mathfrak{A}a, \ q = \tfrac{13}{54}Aa, \ r = \tfrac{13}{84}\mathfrak{C}Aa, \ s = \tfrac{39}{28}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa,$$
$$t = \tfrac{65}{56}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa, \ u = \tfrac{65}{112}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa = \tfrac{1}{2}t.$$

Intervalla vero lentium

$$\text{primum} = \tfrac{13}{18}Aa, \ \text{secundum} = \tfrac{143}{504}Aa, \quad \text{tertium} = \tfrac{13}{84}CAa,$$
$$\text{quartum} = \tfrac{13}{56}\tfrac{C}{\mathfrak{M}}\cdot Aa, \ \text{quintum} = \tfrac{65}{224}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa.$$

Denique semidiameter aperturae lentis

$$\text{secundae} = \tfrac{9}{4\mathfrak{M}}\cdot q+\tfrac{5}{18}x \ \text{ et } \ \text{tertiae} = \tfrac{3}{4\mathfrak{M}}\cdot r+\tfrac{1}{3}x.$$

EXEMPLUM 5

295. Manente $\eta = 4$ sit $\zeta = \frac{11}{8}$; ac erit $k = 3$ et $P = 1$, unde

$$\mathfrak{B} = \tfrac{1-P}{\zeta} = \tfrac{8}{11}(1-P) = 0 \ \text{ et } \ B = 0.$$

Unde assumi debet $A = \infty$ ita ut fiat $\mathfrak{A} = 1$; tum igitur introducto q in calculum fiet

$$A\mathfrak{B} = AB = -\tfrac{q}{a};$$

unde fit

$$Aa\cdot\frac{(1-P)}{P}=-\frac{11}{8}q$$

sicque distantiae focales erunt

$$p=a,\quad q=q,\quad r=\tfrac{1}{3}\mathfrak{C}q,\quad s=3\cdot\tfrac{C}{\mathfrak{M}}\cdot q,\quad t=\tfrac{5}{2}\cdot\tfrac{C}{\mathfrak{M}}\cdot q,\quad \text{et}\quad u=\tfrac{5}{4}\cdot\tfrac{C}{\mathfrak{M}}\cdot q$$

et lentium intervalla

$$\text{primum}=\tfrac{11}{8}q,\quad \text{secundum}=\tfrac{4}{3}q,\quad \text{tertium}=\tfrac{1}{3}Cq,$$
$$\text{quartum}=\tfrac{1}{2}\cdot\tfrac{C}{\mathfrak{M}}\cdot q,\quad \text{et}\quad \text{quintum}=\tfrac{5}{8}\cdot\tfrac{C}{\mathfrak{M}}\cdot q.$$

Semidiameter aperturae lentis

$$\text{secundae}=\tfrac{33}{32}\cdot\tfrac{q}{\mathfrak{M}}+x,\quad \text{tertiae}=\tfrac{63}{32}\cdot\tfrac{r}{\mathfrak{M}}+\tfrac{1}{3}x.$$

EXEMPLUM 6

296. Manente adhuc $\eta=4$ sit $\zeta=\frac{2}{3}$ ac reperitur $k=\frac{29}{4}$ et $P=\frac{12}{29}$; unde fit $\mathfrak{B}=\frac{51}{58}$ et $B=\frac{51}{7}$, ex quo tanto valore iam perspicuum est huiusmodi microscopiis in praxi locum concedi non posse.

CASUS 2
QUO $\eta=4$ ET $\theta=3$

297. Quoniam debet esse $\eta>1$ eiusque valor nimis parvus quibusdam incommodis est obnoxius, nimis magnus vero campo nocet, mediocri semper valore uti conveniet, cuiusmodi est $\eta=4$; tum vero valor $\theta=3$ seu $T=\frac{1}{2}$ idoneum intervallum inter ultimas lentes praebet; littera autem $\mathfrak{C}$ tam parum ab unitate deficere debet, ut in nostris formulis liceat sumere $\mathfrak{C}=1$. His praemissis pro ζ limes erit $\zeta<\frac{11}{12}$. Pro $\mathfrak{C}$ vero habebimus

$$\mathfrak{C}<1\quad\text{et}\quad\mathfrak{C}>\tfrac{2}{3}(\zeta-4);$$

unde patet, etiamsi sit $\zeta=\frac{11}{2}$, tamen fore $\mathfrak{C}=1$, ita ut certe sumi possit $\mathfrak{C}=1$. Porro pro littera *i* fiet

$$i>\frac{2}{33-6\zeta}.$$

Deinde obtinebimus

$$k=\frac{(33-6\zeta)i-2}{6\zeta i},$$

unde fit

$$P=\frac{18\zeta i}{(33-6\zeta)i-2}.$$

Hinc oritur

$$\mathfrak{B} = \frac{(33-24\zeta)i-2}{((33-6\zeta)i-2)\zeta}.$$

Hic duos casus distingui oportet. Prior est, quo fit $P>1$; hincque A valorem positivum habere debet, quod evenit, si $\mathfrak{B}$ fiat < 0 ideoque

$$i<\frac{2}{33-24\zeta},$$

sive quando i continetur intra limites $\frac{2}{33-6\zeta}$ et $\frac{2}{33-24\zeta}$, atque hoc casu etiam B fit negativum, ita ut sit $AB<0$. Alter casus, quo $P < 1$, locum habet, si fuerit $i>\frac{2}{33-24\zeta}$, quo casu A valorem habebit negativum; quo igitur B nanciscatur valorem positivum, debet esse $\mathfrak{B}<1$ hincque

$$i<\frac{2(1-\zeta)}{6\zeta^2-57\zeta+33}.$$

Cum igitur sit $i>\frac{2}{33-24\zeta}$, id evenit, si fuerit

$$\frac{1-\zeta}{6\zeta^2-57\zeta+33}>\frac{1}{33-24\zeta},$$

unde sequeretur $\zeta>0$. His igitur notatis distantiae focales erunt

$$p=\mathfrak{A}a,\quad q=-\frac{A\mathfrak{B}}{P}\cdot a,\quad r=-\frac{1}{3}AB\mathfrak{C}a,\quad s=-3\cdot\frac{ABC}{\mathfrak{M}}\cdot a,$$
$$t=-\frac{45i}{18i+2}\cdot\frac{ABC}{\mathfrak{M}}\cdot a \quad\text{et}\quad u=-\frac{45i}{45i+4}\cdot\frac{ABC}{\mathfrak{M}}\cdot a$$

sive etiam erit

$$q=+\frac{1}{\zeta}\left(1-\frac{1}{P}\right)Aa$$

et intervalla lentium sunt

$$\text{primum}=Aa\left(1-\frac{1}{P}\right),\quad \text{secundum}=-ABa\left(\frac{1}{P}+\frac{1}{3}\right),$$
$$\text{tertium}=-\frac{ABC}{3}\cdot a\left(1-\frac{1}{i\mathfrak{M}}\right),\quad \text{quartum}=-\frac{3(3i+2)}{2(9i+1)}\cdot\frac{ABC}{\mathfrak{M}}\cdot a,\quad \text{quintum}=-\frac{45i}{8(9i+1)}\cdot\frac{ABC}{\mathfrak{M}}\cdot a.$$

Deinde cum sit

$$M=\frac{3}{\mathfrak{M}+3},$$

fiet

$$z=\frac{1}{4}\cdot\frac{3a}{\mathfrak{M}+3}$$

et distantia oculi

$$O=\frac{1}{3}u\left(1+\frac{3}{\mathfrak{M}}\right).$$

Semidiameter porro aperturae

$$\text{lentis secundae erit} = \tfrac{3}{4} \cdot \tfrac{\zeta}{\mathfrak{M}} \cdot q + \tfrac{x}{P} \quad \text{et} \quad \text{tertae} = \tfrac{3}{4} \cdot \tfrac{(\zeta-4)r}{\mathfrak{M}} + \tfrac{1}{3}x.$$

Denique definito x erit mansura claritatis $= \frac{20x}{\mathfrak{M}}$.

EXEMPLUM 1

298. Sit $\zeta = 0$, et cum esse debeat $i > \frac{2}{33}$, praeterea vero pro $P > 1$ sit $i < \frac{2}{33}$, statuamus $i = \frac{2}{33}$ eritque $P = \frac{0}{0}$ sive P non determinatur, modo non sit minus unitate, adeoque $\mathfrak{B} = -\frac{(P-1)}{\zeta}$, ita ut $\mathfrak{B}$ semper sit ∞ nisi capiatur $P = 1$. Primo igitur non sit $P = 1$; erit $\mathfrak{B} = \infty$ et $B = -1$, ita ut A sit maius nihilo; hinc igitur distantiae focales erunt

$$p = \mathfrak{A}a, \quad q = \infty, \quad \big(\text{seu, quod idem est, secunda lens tollitur}\big),$$
$$r = \tfrac{1}{3} AB\mathfrak{C}a, \quad s = \tfrac{3ABC}{\mathfrak{M}} \cdot a, \quad t = \tfrac{15}{17} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a \quad \text{et} \quad u = \tfrac{15}{34} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a$$

et intervalla lentium

$$\text{primum+ secundum} = \tfrac{4}{3} Aa, \ \text{tertium} = \tfrac{AC}{3}\Big(1 - \tfrac{33}{2\mathfrak{M}}\Big),$$
$$\text{quartum} = \tfrac{72}{34} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a, \quad \text{quintum} = \tfrac{15}{68} \cdot \tfrac{AC}{\mathfrak{M}} \cdot a.$$

Reliqua manent, nisi quod sit semidiameter aperturae

$$\text{lentis tertiae} = 3 \cdot \tfrac{r}{\mathfrak{M}} + \tfrac{1}{3}x.$$

Sin autem caperetur $P = 1$, utcunque calculus instituatur, primum intervallum semper evanesceret; verum superfluum est ad hunc casum attendere, cum in prioribus formulis littera P plane ex calculo evanuerit, ita ut illae formulae subsistant, quicunque valor ipsi P tribuatur atque adeo non solum, si ponatur $P = 1$, sed etiam, si P unitate minus sumeretur; quod etsi nostrae hypothesi repugnat, tamen ob lentem secundam prorsus deficientem haec anomalia admitti debet.

EXEMPLUM 2

299. Maneat $\zeta = 0$, sed capiatur $i > \frac{2}{33}$, si fieri queat, quo casu fiet $P < 1$; quia autem hoc ipso casu iterum esse debet $i < \frac{2}{33}$, hic casus ad praecedentem redigitur, quem quidem iam notavimus aeque ad valores $P < 1$ quam ad $P > 1$ patere. Interim tamen cum

secunda lens plane deficiat, posterior limes $i < \frac{2}{33}$ sponte cessat, ita ut nunc liceat assumere $i < \frac{2}{33}$, uti iam observavimus in corollario 1 problemati subnexo. Tum igitur erit

$$p = \mathfrak{A}a, \quad q = \infty, \quad (\text{seu, secunda lens deest}),$$
$$r = \frac{A\mathfrak{C}}{3} \cdot a, \quad s = \frac{3AC}{\mathfrak{M}} \cdot a, \quad t = \frac{45i}{18i+2} \cdot \frac{AC}{\mathfrak{M}} \cdot a, \quad u = \frac{45i}{36i+4} \cdot \frac{AC}{\mathfrak{M}} \cdot a.$$

Intervalla vero lentium

$$\text{primum+ secundum} = \frac{4}{3}Aa, \ \text{tertium} = \frac{1}{3}ACa\left(1 - \frac{1}{i\mathfrak{M}}\right),$$
$$\text{quartum} = \frac{3(3i+2)}{2(9i+2)} \cdot \frac{AC}{\mathfrak{M}} \cdot a, \ \text{quintum} = \frac{45i}{8(9i+1)} \cdot \frac{AC}{\mathfrak{M}} \cdot a.$$

Quod autem ad litteram i attinet, quoniam k non amplius in calculum ingreditur, ex aequatione, unde k definivimus, iam i definiatur eritque

$$i = \frac{2}{33 - 6\zeta} = \frac{2}{33}.$$

Proprie autem hae formulae continent solutionem problematis, quo quinque tantum lentes postulantur ita disponendae, ut ambae imagines reales in primum et tertium intervallum incidant.

EXEMPLUM 3

300. Sit $\zeta = \frac{1}{2}$ sumique debebit $i > \frac{1}{15}$ fietque $P > 1$, si fuerit $i < \frac{2}{21}$; sin autem sit $i > \frac{2}{21}$ simul fiet $P < 1$. Hic vero sumamus $i = \frac{1}{12}$ fietque $P = \frac{3}{2}$, hinc $\mathfrak{B} = -1$ et $B = -\frac{1}{2}$; unde distantiae focales erunt

$$p = \mathfrak{A}a, \quad q = \frac{2}{3}Aa, \ r = \frac{1}{6}A\mathfrak{C}a,$$
$$s = \frac{3}{2} \cdot \frac{AC}{\mathfrak{M}} \cdot a, \quad t = \frac{45}{84} \cdot \frac{AC}{\mathfrak{M}} \cdot a, \quad u = \frac{15}{56} \cdot \frac{AC}{\mathfrak{M}} \cdot a.$$

et lentium intervalla

$$\text{primum} = \frac{1}{3}Aa, \quad \text{secundum} = \frac{1}{2}Aa, \ \text{tertium} = \frac{1}{6}ACa\left(1 - \frac{12}{\mathfrak{M}}\right),$$
$$\text{quartum} = \frac{27}{28} \cdot \frac{AC}{\mathfrak{M}} \cdot a, \ \text{quintum} = \frac{15}{112} \cdot \frac{AC}{\mathfrak{M}} \cdot a.$$

Reliqua momenta sunt ut ante, nisi quod sit semidiameter aperturae

$$\text{secundae lentis} = \frac{3}{8} \cdot \frac{q}{\mathfrak{M}} + \frac{2}{3}x \ \text{ et } \ \text{tertiae} = \frac{21}{8} \cdot \frac{r}{\mathfrak{M}} + \frac{1}{3}x.$$

Has formulas commode ad telescopia adplicare licet, quia posito $Aa = p$ longitudo tantum fit

$$\tfrac{5}{6}p+\tfrac{1}{6}Cp,$$

ita ut ea non multum superet p, etiamsi pro C numerus satis magnus capiatur.

EXEMPLUM 4

301. Maneat $\zeta=\frac{1}{2}$, sed sumatur $i>\frac{2}{21}$, et quia hinc fit $P=\frac{9i}{30i-2}$ ideoque $\mathfrak{B}=\frac{21i-2}{15i-1}$, ut fiat $\mathfrak{B}<1$, debet esse $21i-2<15i-1$ sive $i<\frac{1}{6}$.
Capiatur ergo $i=\frac{1}{8}$ fietque $P=\frac{9}{14}$ et $\mathfrak{B}=\frac{5}{7}$, hinc $B=\frac{5}{2}$; ergo A debet esse negativum. Statuatur ergo $A=-\alpha$ et distantiae focales erunt

$$p=\tfrac{\alpha}{\alpha-1}a,\quad q=\tfrac{10}{9}\alpha a,\ r=\tfrac{5}{6}\mathfrak{C}\alpha a,$$
$$s=\tfrac{15}{2}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a,\quad t=\tfrac{225}{68}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a,\quad u=\tfrac{225}{136}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a.$$

et lentium intervalla

$$\text{primum}=\tfrac{5}{9}\alpha a,\quad \text{secundum}=\tfrac{85}{18}\alpha a,\ \text{tertium}=\tfrac{5}{6}C\alpha a\left(1-\tfrac{8}{\mathfrak{M}}\right),$$
$$\text{quartum}=\tfrac{285}{68}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a,\ \ \text{quintum}=\tfrac{225}{272}\cdot\tfrac{C}{\mathfrak{M}}\cdot\alpha a.$$

Tum vero semidiameter aperturae lentis secundae et tertiae

$$=\tfrac{3}{8}\cdot\tfrac{q}{\mathfrak{M}}+\tfrac{14}{9}x \ \text{ et } \ \tfrac{21}{8}\cdot r+\tfrac{1}{3}x.$$

Has autem formulas ad telescopia adplicare non licet, quia A erat negativum.

EXEMPLUM 5

302. Sit $\zeta=1$, et cum sumi debeat $i>\frac{2}{27}$ atque, ut prodeat $P>1$, $i<\frac{2}{9}$, capiatur $i=\frac{1}{10}$ fietque $P=\frac{18}{7}$ et $\mathfrak{B}=-\frac{11}{7}$ et $B=-\frac{11}{18}$; unde distantiae focales erunt

$$p=\mathfrak{A}a,\ \ q=\tfrac{11}{18}Aa,\ r=\tfrac{11}{54}\mathfrak{C}Aa,$$
$$s=\tfrac{11}{6}\cdot\tfrac{AC}{\mathfrak{M}}\cdot a,\quad t=\tfrac{55}{76}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa,\quad u=\tfrac{55}{152}\cdot\tfrac{C}{\mathfrak{M}}\cdot Aa.$$

et lentium intervalla

$$\text{primum} = \tfrac{11}{18}Aa, \quad \text{secundum} = \tfrac{143}{324}Aa, \ \text{tertium} = \tfrac{11}{54}ACa\left(1-\tfrac{10}{\mathfrak{M}}\right),$$
$$\text{quartum} = \tfrac{253}{228}\cdot\tfrac{AC}{\mathfrak{M}}\cdot a, \ \ \text{quintum} = \tfrac{55}{304}\cdot\tfrac{AC}{\mathfrak{M}}\cdot a$$

et semidiameter aperturae lentis

$$\text{secundae } = \tfrac{3}{4}\cdot\tfrac{q}{\mathfrak{M}}+\tfrac{7}{18}x, \ \ \text{tertiae} = \tfrac{9}{4}\cdot\tfrac{r}{\mathfrak{M}}+\tfrac{1}{3}x.$$

quas formulas etiam commode ad telescopia transferre licet.

EXEMPLUM 6

803. Maneat $\zeta = 1$, sed sumatur $i = \frac{2}{9}$ fietque $P = 1$ et $\mathfrak{B} = 0$, hinc $B = 0$; hinc A capi debet $= \infty$ ideoque $\mathfrak{A} = 1$; tum autem esse debet

$$A\mathfrak{B} = AB = -\tfrac{q}{a},$$

unde fit

$$Aa\left(1-\tfrac{1}{P}\right) = Aa\left(P-1\right) = -A\mathfrak{B}a = q;;$$

hinc distantiae focales erunt

$$p = a, \ \ q = q, \ r = \tfrac{1}{3}\mathfrak{C}q,$$
$$s = 3\cdot\tfrac{C}{\mathfrak{M}}\cdot q, \quad t = \tfrac{5}{3}\cdot\tfrac{C}{\mathfrak{M}}\cdot q, \quad u = \tfrac{5}{6}\cdot\tfrac{C}{\mathfrak{M}}\cdot q$$

et lentium intervalla

$$\text{primum} = q, \quad \text{secundum} = \tfrac{4}{3}q, \ \text{tertium} = \tfrac{1}{3}Cq\left(1-\tfrac{9}{2\mathfrak{M}}\right),$$
$$\text{quartum} = \tfrac{4}{3}\cdot\tfrac{C}{\mathfrak{M}}\cdot q, \ \ \text{quintum} = \tfrac{5}{12}\cdot\tfrac{C}{\mathfrak{M}}\cdot q$$

et semidiametri aperturae lentis

$$\text{secundae } = \tfrac{3}{4}\cdot\tfrac{q}{\mathfrak{M}}+x, \ \ \text{tertiae} = \tfrac{9}{4}\cdot\tfrac{r}{\mathfrak{M}}+\tfrac{1}{3}x.$$

EXEMPLUM 7

804. Maneat $\zeta = 1$, sed capiatur $i > \frac{2}{9}$, et cum sit

$$P = \tfrac{18i}{27i-2} \quad \text{hincque} \quad \mathfrak{B} = \tfrac{9i-2}{27i-2},$$

quae fractio iam sponte unitate est minor, sumamus ergo $i = 1$; erit
$P = \frac{18}{25}$ et $\mathfrak{B} = \frac{7}{25}$, $B = \frac{7}{18}$, unde A debet esse negativum; statuatur ergo $= -\alpha$ eruntque distantiae focales

$$p = \frac{\alpha}{\alpha - 1} \cdot a, \quad q = \frac{7}{18}\alpha a, \; r = \frac{7}{54}\mathfrak{C}\alpha a,$$
$$s = \frac{7}{6} \cdot \frac{C}{\mathfrak{M}} \cdot \alpha a, \quad t = \frac{35}{40} \cdot \frac{C}{\mathfrak{M}} \cdot \alpha a, \quad u = \frac{35}{80} \cdot \frac{C}{\mathfrak{M}} \cdot \alpha a$$

et intervalla

$$\text{primum} = \frac{7}{18}\alpha a, \quad \text{secundum} = \frac{217}{324}\alpha a, \; \text{tertium} = \frac{7}{54}C\alpha a\left(1 - \frac{9}{2\mathfrak{M}}\right),$$
$$\text{quartum} = \frac{7}{24} \cdot \frac{C}{\mathfrak{M}} \cdot \alpha a, \; \text{quintum} = \frac{7}{32} \cdot \frac{C}{\mathfrak{M}} \cdot \alpha a$$

et semidiametri aperturae lentis

$$\text{secundae} = \frac{3}{4} \cdot \frac{q}{\mathfrak{M}} + \frac{25}{18}x, \; \text{tertiae} = \frac{9}{4} \cdot \frac{r}{\mathfrak{M}} + \frac{1}{3}x.$$

EXEMPLUM 8

305. Sit nunc $\zeta = 4 = \eta$, et cum esse debeat $i > \frac{2}{9}$, ut autem fiat $P > 1$, $i < -\frac{2}{63}$ (qui limes ut infinito maior spectari debet; si enim sumsissemus ζ minus, scilicet $\zeta = \frac{11}{8}$, tum prodiisset $i < \frac{2}{0}$, quod indicio fuisset *i* quantumvis magnum accipi posse semperque fore $P > 1$; quod autem de valore $\zeta = \frac{11}{8}$ valet, multo magis de maioribus valet), sit ergo $i = \frac{1}{3}$ eritque

$$P = -24 \;\text{ et }\; \mathfrak{B} = -\frac{23}{96} \;\text{ et }\; B - = -\frac{23}{27}$$

unde colliguntur distantiae focales

$$p = \mathfrak{A}a, \quad q = \frac{23}{96}Aa, \; r = \frac{23}{81}A\mathfrak{C}a,$$
$$s = \frac{23}{9} \cdot \frac{AC}{\mathfrak{M}} \cdot a, \quad t = \frac{115}{72} \cdot \frac{AC}{\mathfrak{M}} \cdot a, \quad u = \frac{115}{144} \cdot \frac{AC}{\mathfrak{M}} \cdot a$$

et intervalla lentium

$$\text{primum} = \frac{23}{24}Aa, \; \text{secundum} = \frac{23}{72}Aa, \; \text{tertium} = \frac{23}{81}CAa\left(1 - \frac{3}{\mathfrak{M}}\right),$$
$$\text{quartum} = \frac{23}{24} \cdot \frac{AC}{\mathfrak{M}} \cdot a, \; \text{quintum} = \frac{115}{288} \cdot \frac{AC}{\mathfrak{M}} \cdot a$$

et semidiameter aperturae lentis

$$\text{secundae} = \frac{3q}{\mathfrak{M}} + \frac{1}{24}x \;\text{ et }\; \text{tertiae} = \frac{1}{3}x.$$

EXEMPLUM 9

306. Maneat $\zeta = 4$ et sit $i = 1$; fiet

$$P = \tfrac{72}{7},\ \text{ergo}\ \mathfrak{B} = -\tfrac{65}{28},\ \text{hinc}\ B = -\tfrac{65}{93};$$

unde colliguntur distantiae focales

$$p = \mathfrak{A}a,\ \ q = \tfrac{65}{288}Aa,\ r = \tfrac{65}{279}A\mathfrak{C}a,$$
$$s = \tfrac{65}{31}\cdot\tfrac{AC}{\mathfrak{M}}\cdot a,\quad t = \tfrac{195}{124}\cdot\tfrac{AC}{\mathfrak{M}}\cdot a,\quad u = \tfrac{195}{248}\cdot\tfrac{AC}{\mathfrak{M}}\cdot a = \tfrac{1}{2}t,$$

et intervalla lentium

$$\text{primum} = \tfrac{65}{72}Aa,\ \ \text{secundum} = \tfrac{65}{216}Aa,\ \text{tertium} = \tfrac{65}{279}ACa\left(1 - \tfrac{1}{\mathfrak{M}}\right),$$
$$\text{quartum} = \tfrac{65}{124}\cdot\tfrac{AC}{\mathfrak{M}}\cdot a,\ \ \text{quintum} = \tfrac{195}{496}\cdot\tfrac{AC}{\mathfrak{M}}\cdot a,$$

et semidiameter aperturae lentis

$$\text{secundae} = \tfrac{3q}{\mathfrak{M}} + \tfrac{7}{72}x\ \ \text{et}\ \ \text{tertiae} = \tfrac{1}{3}x.$$

SCHOLION

307. Horum exemplorum ea inprimis sunt notatu digna, in quibus fiebat $P = 1$, quia tum littera A abibat in infinitum eratque $\mathfrak{A} = 1$. In casu igitur secundo, quem hactenus sumus contemplati, statuamus in genera $P = 1$ ac sumi debebit $i = \frac{2}{33-24\zeta}$. Tum ergo erit $\mathfrak{B} = 0$ simulque $B = 0$; unde sumto $\mathfrak{A} = 1$ et $A = \infty$ fiet $q = -A\mathfrak{B}a$, hinc vicissim $A\mathfrak{B} = AB = -\frac{q}{a}$, ita ut nunc distantia focalis secundae lentis q arbitrio nostro penitus relinquatur; tum autem ad primum intervallum inveniendum ob $\mathfrak{B} = \frac{1-P}{\zeta}$ erit

$$Aa\cdot\tfrac{P-1}{P} = Aa\left(P-1\right) = -A\mathfrak{B}\zeta a = \zeta q$$

atque hinc in genere distantiae focales ita se habebunt:

$$p = a,\ \ q = q,\ r = \tfrac{1}{3}\mathfrak{C}q,\ s = \tfrac{3C}{\mathfrak{M}}\cdot q,$$
$$t = \tfrac{15}{17-8\zeta}\cdot\tfrac{C}{\mathfrak{M}}\cdot q,\quad u = \tfrac{15}{34-16\zeta}\cdot\tfrac{C}{\mathfrak{M}}\cdot q\ \ \text{seu}\ \ u = \tfrac{1}{2}t.$$

Intervalla vero lentium erunt

$$\text{primum} = \zeta q, \quad \text{secundum} = \tfrac{4}{3} q,$$

$$\text{tertium} = \tfrac{1}{3} q\left(1 - \frac{33-24\zeta}{2\mathfrak{M}}\right) = \tfrac{1}{3} q\left(1 - \frac{3(11-8\zeta)}{2\mathfrak{M}}\right),$$

$$\text{quartum} = \frac{12(3-2\zeta)}{17-8\zeta} \cdot \frac{C}{\mathfrak{M}} \cdot q, \quad \text{quintum} = \frac{15}{4(17-8\zeta)} \cdot \frac{C}{\mathfrak{M}} \cdot q;$$

ubi ergo manifestum est necessario sumi debere $\zeta < \tfrac{3}{2}$. Tum vero erit semidiameter aperturae lentis

$$\text{secundae} = \tfrac{3}{4} \cdot \frac{\zeta}{\mathfrak{M}} \cdot q + x \quad \text{et} \quad \text{tertiae} = \tfrac{3}{4} \cdot \frac{(\zeta-4)}{\mathfrak{M}} \cdot r + \tfrac{1}{3} x.$$

Ceterum hic manifestum est istas formulas ad telescopia accommodari neutiquam posse. Cum igitur omnes casus, qui quidem in praxi locum habere possunt, adeo pro sex lentibus evolverimus iisque tantum campum conciliaverimus, quo maior desiderari vix queat, huic capiti finem imponimus ad sequens idque ultimum progressuri, in quo ostendimus, quemadmodum loco lentis obiectivae duas pluresve lentes sive ex eodem sive ex diverso vitri genere factas substituendo omnia plane confusio tolli possit, ut hoc modo microscopia omnibus numeris absoluta nanciscamur.