CHAPTER III

CONCERNING THE GREATEST PERFECTION OF MICROSCOPES OF THIS KIND, WHEREBY WITH THE AID OF CONCAVE LENSES MADE FROM ANOTHER KIND OF GLASS, ALL CONFUSION CLEARLY IS REDUCED TO ZERO

PROBLEM 1

189. *To replace two lenses in the objective lens, the first of which shall be concave, so that with the two latter remaining lenses, all confusion may be removed.*

SOLUTION

Since here four lenses may be had and likewise therefore three intervals, the last of the letters *P, Q, R* must be negative; hence there may be put $R=-k$, and so that the colored margin may be removed, from the above treatment it is evident there must be taken $k=1$, thus so that there shall be $PQ=\frac{ma}{h}$; thence in order that likewise the same field of view may appear as before, there must become $\mathfrak{C}=-2$ and $C=-\frac{2}{3}$; from which the focal lengths of the lenses will be

$$p=\mathfrak{A}a,\ \ q=-\tfrac{A\mathfrak{B}}{P}\cdot a,\ \ r=-2AB\cdot\tfrac{h}{m}\ \text{and}\ s=-\tfrac{2}{3}AB\cdot\tfrac{h}{m},$$

and truly the spacing of the lens:

$$\text{first}=Aa(1-\tfrac{1}{P}),$$

$$\text{second}=-\tfrac{ABa}{P}+AB\cdot\tfrac{h}{m}$$

and

$$\text{third}=-\tfrac{4}{3}AB\cdot\tfrac{h}{m}=2s,$$

and so that the former distance of the eye

$$O=\tfrac{1}{2}s\left(1+\tfrac{h}{ma}\right),$$

since also the radius of the field of view observed remains

$$=\tfrac{1}{2}\cdot\tfrac{ah}{ma+h};$$

but now since the first lens must become concave, it is necessary that $\mathfrak{A}<0$ and thus also $A<0$, whereby there will be required to be $P<1$; then truly on account of $AB<0$ there must become $B>0$ and thus also $\mathfrak{B}$ must become a positive quantity. Now as before, since the first two lenses must be close to each other, we may put the first interval $=-\zeta p$, and there will become

$$\tfrac{1}{P}=1+\tfrac{\mathfrak{A}}{A}\cdot\zeta=(1-\mathfrak{A})\cdot\zeta.$$

Since the first lens shall be concave, that also shall be prepared from crystal glass, while the remainder are assumed to be made from crown glass, thus so that now n may denote 1,58 and $n'=1,53=n''=n'''$, to which the remaining independent letters must agree to become. With which put in place the equation arising from the different refrangibility of the rays will be removing all the confusion
[§ 27 : recall $\frac{dn}{n-1}=N,\ \frac{dn'}{n'-1}=N',\ \frac{dn''}{n''-1}=N''$ etc.;]

$$0=N\cdot\frac{1}{p}+\frac{N'}{P^2}\cdot\frac{1}{q}+\frac{N'}{P^2Q^2}\cdot\frac{1}{r}+\frac{N'}{P^2Q^2R^2}\cdot\frac{1}{s}$$

or

$$0=\frac{N}{N'}\cdot\frac{1}{p}+\frac{1}{P^2q}+\frac{h^2}{m^2a^2r}+\frac{h^2}{m^2a^2s},$$

where the two latter terms clearly are able to be removed, and since there shall be approximately $\frac{N}{N'}=\frac{10}{7}$ and $P=1$, this equation will be had:

$$0=\frac{10}{7}\cdot\frac{1}{\mathfrak{A}}-\frac{1}{A\mathfrak{B}}$$

and thus

$$\mathfrak{B}=\frac{7}{10}\cdot\frac{\mathfrak{A}}{A}=\frac{7}{10}(1-\mathfrak{A})\quad\text{and}\quad B=\frac{7(1-\mathfrak{A})}{3+7\mathfrak{A}};$$

which value in order that it may be made positive with there being $\mathfrak{A}<0$, it is necessary that $3+7\mathfrak{A}$ shall be positive or $-\mathfrak{A}<\frac{3}{7}$; but if there shall not be $P=1$, we will have with more care

$$\mathfrak{B}=\frac{7}{10}(1-\mathfrak{A})\big(1+(1-\mathfrak{A})\zeta\big),$$

where only $\mathfrak{B}<1$ must be observed, so that B also may be produced positive.
Therefore there may be put

$$(1-\mathfrak{A})\big(1+(1-\mathfrak{A})\zeta\big)<\frac{10}{7},\qquad(1-\mathfrak{A})^2+\frac{1-\mathfrak{A}}{\zeta}<\frac{10}{7\zeta}$$

and by adding $\frac{1}{4\zeta^2}$ to each side there will be required to be

$$1-\mathfrak{A}+\frac{1}{2\zeta}<\sqrt{\frac{10}{7\zeta}+\frac{1}{4\zeta^2}}.$$

Now lest the two first lenses may themselves become excessively close, we may put in place $\zeta = \frac{1}{7}$ and there will have to be taken

$$1-\mathfrak{A} < -\tfrac{7}{2} + \sqrt{10+\tfrac{49}{7}} \text{ or } 1-\mathfrak{A} < 1,217.$$

Therefore we may suppose

$$1-\mathfrak{A} = \tfrac{6}{5}$$

and there will become

$$\mathfrak{A} = -\tfrac{1}{5} \text{ and } A = -\tfrac{1}{6}$$

then truly on account of $\zeta = \frac{1}{7}$ there will become $\frac{1}{P} = \frac{41}{35}$ and hence

$$\mathfrak{B} = \tfrac{7}{10} \cdot \tfrac{6}{5} \cdot \tfrac{41}{35} = \tfrac{246}{250} = \tfrac{123}{125} \text{ and } B = \tfrac{123}{2} = 61\tfrac{1}{2};$$

from which there follows $AB = -10\frac{1}{4}$, which value without doubt is exceedingly small, since for large magnifications an exceedingly small value for *r* must be produced; truly it is required to observe, if $1-\mathfrak{A}$ may be taken a little greater than $\frac{3}{5}$, so that the distinction cannot be perceived in practice, then as great a product *AB* as wished can arise; indeed if we may put $1-\mathfrak{A} = \frac{6}{5}+\omega,$ thence there is elicited

$$B = \tfrac{246+235\omega+25\omega^4}{4-235\omega-25m^2};$$

which value thus will arise infinite, only if there may be assumed $\omega = \frac{1}{60}$ approximately, with which agreed on the values A, $\mathfrak{A}$ and $\mathfrak{B}$ will scarcely be changed perceptibly, and thus with these values taken

$$\mathfrak{A} = -\tfrac{1}{5}, \quad A = -\tfrac{1}{6}, \quad \tfrac{1}{P} = \tfrac{41}{35}$$

on account of

$$\zeta = \tfrac{1}{7} \quad \text{and} \quad \mathfrak{B} = \tfrac{123}{125} = 1 - \tfrac{2}{125}$$

according to this the letter *B* to be considered as undefined and without hesitation thus may be defined, so that the letter *r* may obtain a suitable value. On account of which we will have, as follows :

$$p = -\tfrac{1}{5}a, \quad q = 0{,}19212a, \quad r = \theta \cdot \tfrac{h}{m} \text{ and } s = \tfrac{1}{3}r,$$

where θ can be assumed as it pleases; thence truly the intervals will become :

$$\text{First} = -\tfrac{1}{7}p = \tfrac{1}{35}a,\quad \text{second} = \tfrac{41\theta a}{70} - \tfrac{1}{2}\theta\cdot\tfrac{h}{m},\quad \text{third} = 2s.$$

Now finally, so that also the confusion arising from the aperture of the lens may vanish, for this equation to be satisfied there must become [§ 31]:

$$0 = \tfrac{\mu}{\mu'}\left(\lambda + \nu\mathfrak{A}\left(1-\mathfrak{A}\right)\right) - \tfrac{(1-\mathfrak{A})^3}{P}\left(\tfrac{\lambda'}{\mathfrak{B}^3} + \tfrac{\nu'}{B\mathfrak{B}}\right)$$

So that now, here assuming $\lambda' = 1$, on account of

$$\mu = 0,8724,\quad \nu = 0,2529,\quad \mu' = 0,9875,\quad \nu' = 0,2196$$

with the calculation made there will be found

$$\lambda = 2,4137 + 0,0607 = 2,4744,$$

from which there is deduced $\tau\sqrt{(\lambda-1)} = 1,0655$; whereby for the first lens prepared from crystal glass, of which the focal length is $p = -\tfrac{1}{5}a$ and the numbers

$$\mathfrak{A} = -\tfrac{1}{5} \text{ and } \lambda = 2,4744,$$

the radius

$$\begin{cases} \text{of the anterior face } = \dfrac{p}{\sigma - \mathfrak{A}(\sigma-\rho) \mp \tau\sqrt{(\lambda-1)}} = \dfrac{p}{1,8710 - 1,0655} \\ \text{of the posterior face} = \dfrac{p}{\sigma + \mathfrak{A}(\sigma-\rho) \pm \tau\sqrt{(\lambda-1)}} = \dfrac{p}{-0,1469 + 1,0655}, \end{cases}$$

from which the radius

$$\begin{cases} \text{of the anterior face} = \dfrac{p}{0,8055} = -0,2483a \\ \text{of the posterior face} = \dfrac{p}{0,9186} = -0,2177a \end{cases}$$

of which aperture therefore it is the width, the radius of which $x = 0,0544a$, unless perhaps the second lens may not be allowed so great an aperture.

But for the second lens made from crown glass, the focal length of which $q = 0,19212a$ and the numbers

$$\mathfrak{B} = \tfrac{123}{125} \text{ and } \lambda' = 1,$$

the radius

$$\begin{cases} \text{of the anterior face } = \dfrac{q}{\sigma - \mathfrak{B}(\sigma-\rho)} = \dfrac{q}{0,2496} = 0,7697a \\ \text{of the posterior face} = \dfrac{q}{\sigma + \mathfrak{B}(\sigma-\rho)} = \dfrac{q}{1,6372} = 0,1173a, \end{cases}$$

of which therefore the greater aperture cannot be greater than $x = 0,0293a$. Hence moreover it is deduced

$$y = \frac{hx}{ma} = \frac{0,2344}{m} \text{ in.}$$

and hence the measure of the clarity $= \frac{4,688}{m}$, which therefore is almost six times less than in the last case of the preceding chapter.
Hence therefore the following is deduced:

CONSTRUCTION OF A MICROSCOPE OF THIS KIND COMPOSED FROM FOUR LENSES

190. With the distance of the object from the instrument $= a$ and with the magnification given $= m$ there will be had

I. For the first concave lens being prepared from crystal glass, of which the focal length is $p = -\frac{1}{5}a$, the radius

$$\begin{cases} \text{of the anterior face} = -0,2483a \\ \text{of the posterior face} = -0,2177a, \end{cases}$$

the radius of that aperture on account of the reasoning just advanced $x = 0,0293a$ and the distance to the second lens $= -\frac{1}{7}p = 0,0286a.$

II. For the second lens prepared from crown glass, of which the focal length is $q = 0,1921a$, the radius

$$\begin{cases} \text{of the anterior face} = 0,7697a \\ \text{of the posterior face} = 0,1173a; \end{cases}$$

the aperture will remain as before.
Distance to the third lens $= \frac{41}{560}mar - \frac{1}{2}r$,
where r denotes the focal length of the third lens, which is allowed to be assumed arbitrary.

III. But for the third lens, the focal length of which $= r$, if it may be prepared from crown glass,

$$\text{the radius of each face} = 1,06r;$$

moreover likewise, whereas from which kind of glass this lens and also the fourth may be prepared :

the radius of its aperture $=\frac{1}{4}r$

and the distance to the fourth lens $=\frac{2}{3}r.$

IV. For the fourth lens, its focal length is $s=\frac{1}{3}r$,

the radius of each face may be taken $=1,06s$

or rather following the nature of the glass, from which it is prepared.

The radius of the aperture $=\frac{1}{4}r$

and the distance to the eye $=\frac{1}{2}s.$

V. Again the radius of the area viewed in the object will be as follows:

$$z=\frac{1}{2}\cdot\frac{ah}{ma+h}=\frac{4a}{ma+8}\text{ inch.}$$

VI. But the measure of the clarity, by which objects will be observed, will be $=\frac{4,688}{m}$.

COROLLARY 1

191. It is evident here, unless the two first lenses may become exceedingly small, so that the they may be able to be worked on with care by the artificer, by necessity the distance of the object a must be placed much greater than until now. But this distance a may be seen scarcely smaller than the two inches commonly able to be assumed, which indeed is used in practice advantageously, especially since the clarity may not depend on the distance.

COROLLARY 2

192. But with the distance $a=2$ inches assumed, the second interval $\frac{41}{280}mr-\frac{1}{2}r$ will emerge ; whereby if we may assume $r=1$ in., if indeed on account of $s=\frac{1}{3}r$ it cannot conveniently be taken smaller, for the magnification $m=280$ this interval will be $40\frac{1}{2}$ in.; but if a magnification may be desired twice as great, $m=560$, this interval will become $=80\frac{1}{2}$ in. and thus greater for greater magnifications; which enormous length without doubt will displease greatly .

SCHOLIUM

193. Since these microscopes shall be liable to these inconveniences, the cause for this has been put in place, as the focal lengths of the first lenses shall be exceedingly small,

while evidently p and q must be taken equal to as much as the fifth part of a approx., since in the last case of the preceding chapter these focal lengths were thus to be four times greater than the distance a, and hence also it has been established, that the measure of the clarity found here shall be only $=\frac{4{,}688}{m}$, which before would be $\frac{29{,}92}{m}$, that is six times greater, and thus the following true value thirty six times greater. On account of which, even if the artificer may have used all diligence and industry in the construction of these microscopes and the need for that wished may succeed, yet I doubt strongly, whether these microscopes may deserve any prerogative to be used before the preceding, since here also the following confusion arising from the different refrangibility shall be removed completely, which did not happen in the preceding chapter. Indeed here we have taken the first lens concave, the second convex; in truth from the above it is proven well enough that no convenience can be expected, if these lenses may be interchanged with each other; why not rather here the order preferred above now to be observed in practice and thus it would become superfluous, if we might wish that same cause to arise itself. Now on this account we may substitute at once three lenses in place of the objective lens, one of which shall be concave and the remaining two convex, and we may inquire in particular, whether in this case the focal lengths of these lenses may be able to become a little greater than in the case examined here and whether perhaps by increasing the number of lenses further they may be able to provide the greater convenience desired.

PROBLEM 2

194. *To substitute three lenses close to each other in place of the objective lens, the first of which shall be concave and prepared from crystal glass, but the two remaining lenses shall be convex and prepared from crown glass, so that all the confusion may be reduced to zero by the two remaining final lenses.*

SOLUTION

Since here four intervals may be had, the final lens of the letters P, Q, R, S will be negative and the colored margin will be removed, if there were $S=-1$. Truly the two first letters P and Q will be approximately equal to one, thus so that there shall be $PQR=\frac{ma}{h}$. So that it may be extended to the remaining letters, the condition of the field of view demands, that there shall be $\mathfrak{D}=-2$ and $D=-\frac{2}{3}$, and since the first lens shall be concave, $\mathfrak{A}$ will be negative and likewise also A, yet thus, so that there shall be $-A<1$. Thence on account of $q=-\frac{A\mathfrak{B}}{P}\cdot a$, since this lens must be convex, the letter $\mathfrak{B}$ will be positive, and since the third lens, for which $r=\frac{AB\mathfrak{C}}{PQ}\cdot a$ must be convex also, there must become $B\mathfrak{C}<0$, and again since there becomes

$$s=+\frac{2ABC}{PQR}\cdot a=2ABC\cdot\frac{h}{m},$$

lest this lens may not become exceedingly small for the greater magnifications, he product ABC must be equal to a very large positive number, from which it is concluded BC to be a large negative number. But since there shall be also $B\mathfrak{C} < 0$ and this number shall not be very large, it follows that C must be a very large number and hence $\mathfrak{C}$ approximately equal to unity; on account of which B must be a negative number and hence $\mathfrak{B} > 1$, on the other hand truly $\mathfrak{C} < 1$, but with a very small difference present, so that it may produce the very large positive number C.

From these observations we will consider the equation, by which the latter confusion is removed completely, which will be

$$0 = \frac{10}{7} \cdot \frac{1}{p} + \frac{1}{P^2 q} + \frac{1}{P^2 Q^2 r} + \frac{1}{P^2 Q^2 R^2 s} + \frac{1}{P^2 Q^2 R^2 t},$$

where on account of $PQR = \frac{ma}{h}$ the two final terms with care will be allowed to be rejected, and since the letters P and Q shall be equal to unity approximately, we will have this determination:

$$0 = \frac{10}{7} \cdot \frac{1}{p} + \frac{1}{q} + \frac{1}{r},$$

thus so that there shall become

$$\frac{1}{p} = -\frac{7}{10}\left(\frac{1}{q} + \frac{1}{r}\right)$$

of if with the values substituted

$$\frac{A}{\mathfrak{A}} = -\frac{7}{10}\left(\frac{1}{B\mathfrak{C}} - \frac{1}{\mathfrak{B}}\right);$$

and since there is approximately $\mathfrak{C} = 1$, we will obtain

$$1 + A = \frac{7}{10}$$

and thus

$$A = -\frac{3}{10} \quad \text{and} \quad \mathfrak{A} = -\frac{3}{7}.$$

But since it may suffice for the matter to be defined approximately only, we may assume $\mathfrak{A} = -\frac{1}{2}$ and by putting in place

$$= -\frac{1}{7} p = \frac{1}{14} a$$

for both the first intervals, there will become

$$\frac{1}{P} = \frac{17}{14} \quad \text{and} \quad \frac{1}{PQ} = \frac{17}{14} - \frac{3}{14B}$$

and hence

$$p=-\tfrac{1}{2}a,\quad q=+\tfrac{\mathfrak{B}}{3P}\cdot a,\quad r=-\tfrac{B}{3PQ}\cdot a;$$

which values substituted will give

$$0=-\tfrac{20}{7}+\tfrac{3}{\mathfrak{B}P}-\tfrac{3}{BPQ},$$

$$0=-\tfrac{20}{7}+\tfrac{51}{14\mathfrak{B}}-\tfrac{51}{14B}+\tfrac{9}{14B^2}$$

or

$$0=-\tfrac{20}{7}+\tfrac{51}{14}+\tfrac{9}{14B^2}$$

on account of $\tfrac{1}{\mathfrak{B}}-\tfrac{1}{B}=1$, or

$$0=\tfrac{11}{14}+\tfrac{9}{14B^2};$$

hence therefore a very large number will be avoided, whatever may be taken for *B*; but if in addition we may consider the equation for the removal of the other kind of confusion, it will be apparent not to be incongruous to be able to assume $\mathfrak{B}=2$ and thus $B=-2$, thus so that now there shall become

$$\tfrac{1}{PQ}=\tfrac{37}{28},\quad q=\tfrac{17}{21}a\ \text{ and }\ r=\tfrac{37}{42}a\ ;$$

but then the equation requiring to be resolved now will be

$$\lambda=\tfrac{3}{4}v+\tfrac{\mu'}{\mu}\cdot\tfrac{17\cdot27}{14\cdot8^2}\left(\lambda'-2v'\right)+\tfrac{\mu'}{\mu}\cdot\tfrac{27\cdot37\cdot\lambda''}{28\cdot8^2},$$

where there can be put both $\lambda'=1$ as well as $\lambda''=1$, and on account of

$$v=0,2529,\ \ v'=0,2196\ \text{ and }\ \tfrac{\mu'}{\mu}=\tfrac{9875}{8724}$$

there will become

$$\lambda=0,1897+0,3252+0,6322\ \text{ and thus }\ \lambda=1,1471;$$

from where there is deduced

$$\tau\sqrt{(\lambda-1)}\ =0,3365.$$

Whereby for the first crystalline lens, the radius

$$\begin{cases}\text{of the anterior face}=\frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)\mp\tau\sqrt{(\lambda-1)}}=+\frac{p}{1,9668}\ =-0,2542a\\ \text{of the posterior face}\ =\frac{p}{\rho+\mathfrak{A}(\sigma-\rho)\pm\tau\sqrt{(\lambda-1)}}=-\frac{p}{0,2427}=+2,0602a.\end{cases}$$

For the second lens requiring to be prepared from crown glass, the radius

$$\begin{cases}\text{of the anterior face} = \frac{q}{\sigma-\mathfrak{B}(\sigma-\rho)} = +\frac{q}{1,2067} = -0,6708a\\ \text{of the posterior face} = \frac{q}{\rho+\mathfrak{B}(\sigma-\rho)} = +\frac{q}{3,0935} = +0,2617a.\end{cases}$$

In a similar manner for the third lens, the radius

$$\begin{cases}\text{or the anterior face} = \frac{r}{\rho} = \frac{r}{0,2267} = 3,8857a\\ \text{of the posterior face} = \frac{r}{\sigma} = \frac{r}{1,6601} = 0,5306a;\end{cases}$$

for the aperture of which lenses the aperture can be assumed $= 0,0635a$, and hence we will have the following

CONSTRUCTION OF A MICROSCOPE COMPOSED FROM FIVE LENSES,

AND FREED FROM NEARLY ALL CONFUSION

195. Here three things can be assumed as it pleases :

1. distance of the object $= a$,
2. magnification $= m$,
3. focal length of the fourth lens $= s$.

I. For the first crystalline lens, the focal length of which $p = 0,5000a$, the radius

$$\begin{cases}\text{of the anterior face} = -\ 0,2542a\\ \text{of the posterior face} = +\ 2,0602a,\end{cases}$$

the radius of the aperture $x = 0,0635a$, which also is the same in the two following lenses, and the distance to the following lens may be taken $= \frac{1}{14}a$.

II. For the second lens prepared from crown glass , the focal length of which $q = 0,8095a$, the radius

$$\begin{cases}\text{of the anterior face} = -0,6708a\\ \text{of the posterior face} = +0,2617a,\end{cases}$$

and the distance to the third lens may be taken $= \frac{1}{14}a$.

III. For the third lens likewise from crown glass, of which the focal length $r = 0{,}8809a$, the radius

$$\begin{cases}\text{of the anterior face } = 3{,}8857a\\ \text{of the posterior face} = 0{,}5306a,\end{cases}$$

of which the distance to the fourth lens may be taken $= \frac{37mas}{448} - \frac{1}{2}s.$

IV. The fourth lens will be allowed to be constructed from any kind of glass as it pleases, the focal length of which $= s$; then its distance to the eyepiece lens will be $= \frac{2}{3}s$.

V. The focal length of the eyepiece lens itself will be $= \frac{1}{3}s$ and that equally convex on both sides, and the distance as far as to the eye $= \frac{1}{6}s$.

VI. The measure of clarity will be $\frac{10}{m}$ and the radius of the area viewed within the object $z = \frac{4a}{ma+s}$ in.

SCHOLIUM 1

196. Thus if we may change the proposed case, so that the two first lenses may be concave and made from crystalline glass, then they will all be defined in a similar manner to the solution being shown, unless so that now both the letters $\mathfrak{B}$ and B must become negative; and then the removal of the other confusion will give this equation:

$$1 + A = \frac{7}{10} + \frac{3}{10\mathfrak{B}} \text{ or } A = -\frac{3}{10} + \frac{3}{10\mathfrak{B}};$$

so that, if there may be taken $\mathfrak{B} = -2$ and hence $B = -\frac{2}{3}$, there shall be elicited $A = -\frac{9}{20}$ and thus $\mathfrak{A} = -\frac{9}{11}$, which values give rise to the focal lengths

$$p = -\frac{9}{11}\cdot a, \quad q = -\frac{9}{10P}\cdot a, \quad r = \frac{3}{10PQ}\cdot a,$$

where there is

$$\frac{1}{P} = 1 + \frac{20}{11}\zeta \text{ and } \frac{1}{PQ} = 1 + \frac{50}{11}\zeta;$$

moreover we have assumed $\mathfrak{C} = 1$ approx., so that a very large number may be produced for C and on requiring to put $ABC = \theta$ there shall become $s = 2\theta\cdot\frac{h}{m}$ and as before $t = \frac{1}{3}s$ and the final interval $= 2t$. Truly the two first intervals will be $= -\frac{9}{11}\zeta a$ by hypothesis, truly the third interval $= \theta a\left(\frac{1}{PQ} - \frac{h}{ma}\right)$.

Then again, so that also the first confusion may vanish, the following equation will be required to be satisfied:

$$0=\lambda+\nu\mathfrak{A}\left(1-\mathfrak{A}\right)-\frac{(1-\mathfrak{A})^3}{P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu}{B\mathfrak{B}}\right)+\frac{\mu'}{\mu}\cdot\frac{(1-\mathfrak{A})^3}{B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu'}{C\mathfrak{C}}\right),$$

which becomes with the values substituted

$$0=\lambda-\frac{180\nu}{121}+\frac{8000}{1331P}\left(\frac{\lambda'}{8}-\frac{3\nu}{4}\right)-\frac{\mu'}{\mu}\cdot\frac{30^3}{11^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu'}{C\mathfrak{C}}\right),$$

from which there follows :

$$1331\lambda+\frac{1000\lambda'}{P}=1980\nu+\frac{6000\nu}{P}+\frac{\mu'}{\mu}\cdot\frac{27000}{PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu'}{C\mathfrak{C}}\right).$$

Therefore if here there may be taken $\lambda''=1$ and there may be put in place $\lambda'=\lambda$, so that clearly for each side a minimum value may be found, this equation will be had :

$$\lambda\left(1331+\frac{1000\lambda}{P}\right)=1980\nu+\frac{6000\nu}{P}+\frac{\mu'}{\mu}\cdot\frac{27000}{PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu'}{C\mathfrak{C}}\right),$$

from which it readily apparent the value of λ is going to be produced much greater than 12, from which the construction of these lenses may become extremely difficult. Yet meanwhile we may establish this case with more care by taking $[\nu'=\frac{1}{5},\ \mathrm{C}=12,]\ \zeta=\frac{11}{80}$, so that there may become $\frac{1}{P}=\frac{5}{4}$ and $\frac{1}{PQ}=\frac{13}{8}$, with which in place our equation will be reduced to these numbers:

$$2581\lambda=52888,492,$$

thus so that there shall become

$$\lambda=\frac{52888,5}{2581}=20,492,$$

from which there is deduced

$$\tau\sqrt{(\lambda-1)}=3,8742.$$

Hence there will be for the first lens the radius

$$\begin{cases}\text{of the anterior face}=\frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)\mp\tau\sqrt{(\lambda-1)}}=\frac{p}{2,7620-3,8742}\\ \text{of the posterior face}=\frac{p}{\rho+\mathfrak{A}(\sigma-\rho)\pm\tau\sqrt{(\lambda-1)}}=\frac{p}{-1.0379+3,8742},\end{cases}$$

from which the radius

$$\begin{cases}\text{of the anterioris face} = -\frac{p}{1,1122} = +0,7356a\\ \text{of the posterior face} = \frac{p}{2,8363} = -0,2885a.\end{cases}$$

For the second lens, the radius

$$\begin{cases}\text{of the anterior face} = \frac{p}{\sigma - \mathfrak{B}(\sigma-\rho) \mp \tau\sqrt{(\lambda-1)}} = \frac{p}{0,5911} = -1,9032a\\ \text{of the posterior face} = \frac{p}{\rho + \mathfrak{B}(\sigma-\rho) \pm \tau\sqrt{(\lambda-1)}} = \frac{p}{1.1330} = -0,9930a.\end{cases}$$

But for the third lens made from crown glass, the radius

$$\begin{cases}\text{of the anterior face} = \frac{r}{\rho} = \frac{r}{0,2267} = 2,1504a\\ \text{of the posterior face} = \frac{r}{\sigma} = \frac{r}{1,6601} = 0,2937a;\end{cases}$$

which three lenses shall require a common circular aperture, the radius of which must be taken to be $x = 0,0721a$, from which there becomes $y = \frac{0,5768}{m}$ in. and hence a measure of the clarity $= \frac{11,536}{m}$, which therefore is almost three times greater than in the case of the previous problem.
Hence therefore we deduce the following

CONSTRUCTION OF A MICROSCOPE COMPOSED FROM FIVE LENSES

197. Here evidently at first the distance of the object is given $= a$, then the magnification $= m$ and in the third place the focal length of the fourth lens $= s$; from which there becomes

$$\theta = ABC = \frac{ms}{16}$$

I. For the first lens requiring to be prepared from crystalline glass, of which the focal length is $p = -0,8182a$, the radius

$$\begin{cases}\text{of the anterior face} = +0,7356a\\ \text{of the posterior face} = -0,2885a,\end{cases}$$

the radius of its aperture $a = 0,0721a$, which prevails for the two following lenses, and the distance to the following lens $= 0,1125a$.

II. For the second lens requiring to be prepared from crystalline glass, of which the focal length is $q = -1,125a$, the radius

$$\begin{cases}\text{of the anterior face} = -1,9032a \\ \text{of the posterior face} = -0,9930a,\end{cases}$$

and the distance of which to the third lens $= 0,1125a$.

III. For the third lens requiring to be prepared from crown glass, of which the focal length is $r = 0,4875a$, the radius

$$\begin{cases}\text{of the anterior face} = 2,1504a \\ \text{of the posterior face} = 0,2937a,\end{cases}$$

its distance to the fourth lens

$$= \theta a\left(\tfrac{13}{8} - \tfrac{h}{ma}\right) = \tfrac{13mas}{128} - \tfrac{1}{2}s.$$

IV. The fourth lens will be allowed to be constructed from whatever kind of glass as it pleases, of which the focal length shall be $= s$;then the distance of that to the eyepiece lens $= \frac{2}{3}s$.

V. Moreover the focal length of this eyepiece lens will be $= \frac{1}{3}s$, and its distance to the eye $= \frac{1}{6}s$.

VI. But the measure of the clarity will be $\frac{11,536}{m}$, as we have seen, and the radius of the area viewed as usual, $z = \frac{4a}{ma+8}$ in.

SCHOLIUM 2

198. But although these microscopes may be seen to be preferred to the preceding ones, yet, as we have now indicated, by no means do we dare to recommend these, since the construction of these involves the greatest difficulties, so that it may not be able to be expected to be done, even by the most clever artificer ; evidently the reason for this is contained in the fact that we have found so large a value for the letters λ and λ', evidently rising to twenty. For it is easily understood, if this same value thus were greater or less by one or two, the construction of these lenses would not be changed perceptibly, from which in turn it is deduced, even if these lenses were worked on with the greatest effort, then in all probability the value of the letter λ agreeing with these would be changed not only by one or two, but also to be going to differ by more than 20 ; which if it may eventuate, the confusion thence arising will be much greater, than if a simple objective lens were being used; from which it is evident plainly in no manner can it be hoped for the total removal of the confusion of the latter lenses ; whereby, since at this stage, before the different kinds of glass was understood, we were forced to tolerate this kind of confusion and we had to be content only with the removal of the colored margin,

also now therefore we will be able to remove this condition more easily, since by using crystalline glass perhaps this confusion may be allowed to be diminished in some way, which we may adjoin in some final examples, which are easily adapted for use in practice, since they require values for the letters λ not much greater than unity, yet they may not differ much from the prescribed condition in the problem.

EXAMPLE 1

199. In the formulas found above we may put in place $\mathfrak{A}=-\frac{1}{2}$ and $\mathfrak{B}=2$ and hence $A=-\frac{1}{3}$ and $B=-2$ with the letter $\mathfrak{C}$ remaining a little smaller than one, in order that C may become a very large number. Therefore then there will become from the above formulas

$$\frac{1}{P}=1+\frac{3}{2}\zeta,\quad \frac{1}{PQ}=1+\frac{9}{4}\zeta,$$

from which the focal lengths will become

$$p=-\frac{1}{2}a,\quad q=\frac{2}{3}a+\zeta a,$$

$$r=\frac{2}{3}\mathfrak{C}\left(1+\frac{9}{4}\zeta\right)a=\frac{2}{3}\mathfrak{C}a+\frac{3}{2}\mathfrak{C}\zeta a,$$

$$s=\frac{4}{3}C\cdot\frac{h}{m}\ \text{ and }\ t=\frac{1}{3}s=\frac{4}{9}C\cdot\frac{h}{m}.$$

So that hence there may be produced $s=1$ in. approx. in the case $m=1000$, therefore there must become $C=100$ and thus $\mathfrak{C}=\frac{100}{101}$. The intervals between the lenses will be

$$\text{first and second }=-\zeta p=\frac{1}{2}\zeta a,$$
$$\text{third}=\frac{11mas}{128}-\frac{1}{2}s\quad\text{and}\quad\text{fourth}=\frac{2}{3}s.$$

Moreover here we will be able to assume conveniently $\zeta=\frac{1}{6}$, so that there shall be

$$\frac{1}{P}=\frac{5}{4}\quad\text{and}\quad\frac{1}{PQ}=\frac{11}{8}.$$

Then truly we may put the first concave lens to be prepared from crystalline glass, since in this way the other confusion perhaps may be reduced, truly the second and third from crown glass; and now the first confusion will be reduced to zero, if there shall become

$$8\lambda=6\nu+\frac{\mu'}{\mu}\cdot\frac{5\cdot 27}{4}\left(\frac{\lambda'}{8}-\frac{\nu'}{4}\right)+\frac{\mu'}{\mu}\cdot\frac{11\cdot 27}{8^2}\left(1{,}03\lambda''+\frac{\nu'}{100}\right)$$

or

$$\lambda=\tfrac{3}{4}\nu+\tfrac{\mu'}{\mu}\cdot\tfrac{135}{32}\left(\tfrac{\lambda'}{8}-\tfrac{\nu'}{4}\right)+\tfrac{\mu'}{\mu}\cdot\tfrac{297}{512}\left(1,03\lambda''+\tfrac{\nu'}{100}\right).$$

Now since there shall be

$$\mu=0,8724,\quad \nu=0,2529 \text{ and } \mu'=0,9875,\quad \nu'=0,2196,$$

we may assume $\lambda'=\lambda''=1$ and hence there will become

$$\lambda=0,1897+\tfrac{\mu'}{\mu}\cdot\tfrac{27}{512}\cdot 16,9622$$

or

$$\lambda=1,2022,$$

from which there becomes

$$\tau\sqrt{(\lambda-1)}=0,3946.$$

From which it follows:

For the first lens,

the radius

$$\begin{cases}\text{of the anterior face}=\frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)\mp\tau\sqrt{(\lambda-1)}}=+\frac{p}{1,9087}=-0,2620a\\ \text{of the posterior face}=\frac{p}{\rho+\mathfrak{A}(\sigma-\rho)\pm\tau\sqrt{(\lambda-1)}}=-\frac{p}{0,1846}=+2,7086a.\end{cases}$$

But for the second lens,

the radius

$$\begin{cases}\text{of the anterior face}=\frac{q}{\sigma-\mathfrak{B}(\sigma-\rho)}=-\frac{q}{1,2067}=-0,6906a\\ \text{of the posterior face}=\frac{q}{\rho+\mathfrak{B}(\sigma-\rho)}=+\frac{q}{3,0935}=+0,2694a.\end{cases}$$

For the third lens,

the radius

$$\begin{cases}\text{of the anterior face}=\frac{r}{\sigma-\mathfrak{C}(\sigma-\rho)}=-\frac{r}{0,24103}=3,7656a\\ \text{of the posterior face}=\frac{r}{\rho+\mathfrak{C}(\sigma-\rho)}=+\frac{r}{1,6458}=0,5514a.\end{cases}$$

Therefore the common aperture of the three lenses will be able to be taken

$$x=0,0655a,$$

from which there becomes

$$y=\frac{0,5240}{m}$$

and hence the measure of the clarity will become $=\frac{10,480}{m}$.

CONSTRUCTION OF A MICROSCOPE COMPOSED FROM FIVE LENSES AND MORE ADAPTED TO BE USED IN PRACTICE

200. Here the distance of the object $=a$, the magnification $=m$, and the focal length of the fourth lens $=s$ are given, and hence there will be:

I. For the first lens requiring to be prepared from crystalline glass, the focal length of which is $p=-\frac{1}{2}a$, the radius may be taken

$$\begin{cases}\text{of the anterior face} & =-0,2620a\\ \text{of the posterior face} & =2,7086a,\end{cases}$$

the radius of which aperture $=0,0655a$
and the distance to the second lens $=\frac{1}{12}a$.

II. For the second lens requiring to be prepared from crown glass, of which the focal length is $q=0,8333a$, the radius

$$\begin{cases}\text{of the anterior face} & =-0,6906a\\ \text{of the posterior face} & =+0,2694a,\end{cases}$$

and the distance of which to the third lens $=\frac{1}{12}a$.

III. For the third lens likewise requiring to be prepared from crown glass, the focal length of which $r=0,9075a$, the radius may be taken

$$\begin{cases}\text{of the anterior face} & =3,7656a\\ \text{of the posterior face} & =0,5514a\end{cases}$$

and of which the distance to the third lens $=\frac{11mas}{128}-\frac{1}{2}s$.

IV. Likewise, the fourth lens may be constructed from some kind of glass, and its focal length is allowed to be by our choice, which shall be $=s$, provided this lens shall be equally convex on each side, so that it may allow an aperture, the radius of which $=\frac{1}{4}s$; truly its distance from the fifth lens may be put in place $=\frac{2}{3}s$.

V. Finally the fifth lens or eyepiece shall have a focal length of $=\frac{2}{3}s$ and the radius of the aperture $=\frac{1}{12}s$, if indeed each side is equally convex; then truly the distance of the eye will be $=\frac{1}{5}s$.

VI. The radius of the area viewed in the object will be as usual $=\frac{4a}{ma+8}$.
The measure of the clarity will be indeed $=\frac{10,480}{m}$.

EXAMPLE 2

201. Here there may be put $\mathfrak{A}=-1$ and $\mathfrak{B}=2$ and hence $A=-2$ and $B=2$ and there may be taken $\zeta=\frac{1}{6}$ and there will become

$$\frac{1}{P}=\frac{4}{3},\quad \frac{1}{PQ}=\frac{3}{2}.$$

Whereby the focal lengths will become

$$p=-a,\quad q=\frac{4}{3}a,\quad r=\frac{3}{2}\mathfrak{C}a,\quad s=2C\cdot\frac{h}{m}$$

and thus in turn,

$$C=\frac{ms}{2h}\ \text{ and }\ t=\frac{1}{3}s,$$

truly the intervals

$$\text{first and second}=\frac{1}{6}a,\quad \text{third}=\frac{3mas}{32}-\frac{1}{2}s,\quad \text{fourth}=\frac{2}{3}s.$$

Now in order that the first confusion may be reduced to zero, it will be required for this equation to be satisfied:

$$\lambda=2\nu+\frac{\mu'}{\mu}\cdot\frac{4}{3}\left(\lambda'-2\nu'\right)+\frac{\mu'}{\mu}\cdot\frac{3}{2}\left(1,03\lambda''+\frac{\nu'}{100}\right).$$

Again there may be put in place $\lambda'=\lambda''=1$ and from the calculation to be used as in the previous example there will be found

$$\lambda=0,5058+\frac{\mu'}{\mu}\cdot 2,2960$$

or $\lambda=3,1047$; hence therefore there will become

$$\tau\sqrt{(\lambda-1)}=1,2731.$$

From which there will become,

For the first lens, the radius

$$\begin{cases}\text{of the anterior face} = \frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)\mp\tau\sqrt{(\lambda-1)}} = +\frac{p}{1{,}7509} = -\ 0{,}5711a\\ \text{of the posterior face} = \frac{p}{\rho+\mathfrak{A}(\sigma-\rho)\pm\tau\sqrt{(\lambda-1)}} = -\frac{p}{0{,}0268} = +37{,}3134a.\end{cases}$$

For the second lens

the radius will be as in the previous example

$$\begin{cases}\text{of the anterior face } = -\frac{q}{1{,}2067},\\ \text{of the posterior face} = +\frac{q}{3{,}0935};\end{cases}$$

whereby, since here there shall be $q = 1{,}3333a$, the radius

$$\begin{cases}\text{of the anterior face } = -1{,}1049a\\ \text{of the posterior face} = +0{,}4310a.\end{cases}$$

In a similar manner, as before for the third lens

$$\text{the radius }\begin{cases}\text{of the anterior face } = \frac{r}{0{,}2410},\\ \text{of the posterior face} = \frac{r}{1{,}6458}.\end{cases}$$

Therefore since here $r = \frac{3}{2}\mathfrak{C}a = 1{,}4850a$,

$$\text{the radius}\begin{cases}\text{of the anterior face } = 6{,}1618a,\\ \text{of the posterior f ace} = 0{,}9023a.\end{cases}$$

Therefore for all these lenses the aperture will be able to be taken

$$x = 0{,}1077a,$$

from which there becomes

$$y = \frac{0{,}8616}{m}$$

and the measure of the clarity $= \frac{17{,}232}{m}$.

From which the following arises :

CONSTRUCTION OF A MICROSCOPE COMPOSED FROM FIVE LENSES

202. Here therefore the distance of the object is given $= a$, with the second the magnification given $= m$ and the focal length of the fourth lens $= s$, and there will be:

I. For the first lens requiring to be prepared from crystalline glass, of which the focal length is $p = -a$,

$$\text{the radius}\begin{cases}\text{of the anterior face} & = -\,0,5711a,\\ \text{of the posterior face} & = +37,3134a.\end{cases}$$

the radius of its aperture $= 0,1077a$,
the distance to the second lens $= \frac{1}{6}a$.

II. For the second lens requiring to be prepared from crown glass, the focal length of which $q = 1,3333a$, the radius may be taken

$$\begin{cases}\text{of the anterior face} & = -1,1049a\\ \text{of the posterior face} & = +0,4310a,\end{cases}$$

and its distance to the third lens $= \frac{1}{6}a.$

III. For the third lens likewise, its focal length $r = 1,4850a$, the radius may be taken

$$\begin{cases}\text{of the anterior face} & = 6,1618a\\ \text{of the posterior face} & = 0,9023a,\end{cases}$$

and its distance to the fourth lens $= \frac{3mas}{32} - \frac{1}{2}s.$

IV. Likewise it is the case, from whatever kind of glass the fourth lens may be prepared, its focal length is left to our choice, which shall be $= s$, but only each side shall be equally convex; from which it will admit an aperture, the radius of which $= \frac{1}{4}s$; truly its distance from the fifth lens shall be $= \frac{2}{3}s$.

V. Finally the fifth lens shall have a focal length $=\frac{1}{3}s$ and an aperture, the radius of which shall be $=\frac{1}{12}s$, if indeed each side is equally convex, and the distance to the eye $O=\frac{1}{6}s$.

VI. The radius of the area viewed in the object $=\frac{4a}{ma+8}$, and the measure of the clarity $=\frac{17,232}{m}$.

COROLLARY

203. This microscope may be seen to be preferred to the earlier ones on two accounts:

1. Because the focal lengths of the three first lenses here are greater than before with respect to the object *a;* from which this arises conveniently, so that, even if the distance of the object *a* here may be taken twice as small, yet these lenses will not arise exceedingly small; from which the length of the instrument can be reduced by almost half.

2. Here the measure of the clarity to become almost twice as great than in the preceding case.

PROBLEM 3

204. *If in place of the objective lens four lenses close to each other may be substituted, of which the prior two shall be made from crystalline glass, truly the latter two from crown glass, with the final two lenses remaining so that at this stage they may equip the microscope, in order that each source of confusion may be removed completely.*

SOLUTION

Since here five intervals occur, the first three of which shall be minimal, the letters *P, Q, R* will depart little from unity, truly the letter *T* will be $=-1$, thus so that there shall become $PQRS=\frac{ma}{h}$. Truly of the letters *A, B, C, D, E* this final letter *E* will be $=-\frac{2}{3}$ on account of $\mathfrak{E}=-2$, so that evidently the field of view may become as now $=\frac{4a}{ma+8}$. Now the focal length of the fifth lens will be observed

$$t=ABCD\mathfrak{E}\cdot\frac{h}{m}=-2.ABCD\cdot\frac{h}{m};$$

which lest it may become exceedingly small, on putting $ABCD=-\theta$, thus so that $t=2\theta\cdot\frac{h}{m}$, the number θ must become very large. But now we may establish the solution thus, so that the letters *A, B, C, D* may be removed from the calculation, and finally we may put these equations in place for the sake of brevity:

$$\frac{1}{P}=\alpha,\ \frac{1}{PQ}=\beta,\ \frac{1}{PQR}=\gamma;$$

which letters $\alpha,\ \beta,\ \gamma$ will not differ much from unity, where it needs to be observed properly that these letters which have been used previously [*i.e.* the image distances from each successive lens], must not to be confused with these letters.
Now since the focal lengths of the first four lenses shall be

$$p=\mathfrak{A}a,\quad q=-\alpha A\mathfrak{B}a,\ \ r=\beta AB\mathfrak{C}a,\ s=-\gamma ABC\mathfrak{D}a,$$

from which there is deduced

$$\frac{a}{p}=\frac{1}{\mathfrak{A}}=1+\frac{1}{A},\quad \frac{\alpha a}{q}=-\frac{1}{A\mathfrak{B}}=-\frac{1}{A}-\frac{1}{AB},$$
$$\frac{\beta a}{r}=\frac{1}{AB\mathfrak{C}}=\frac{1}{AB}+\frac{1}{ABC},\quad \frac{\gamma a}{s}=-\frac{1}{ABC\mathfrak{D}}=-\frac{1}{ABC}-\frac{1}{ABCD},$$

therefore evidently to become

$$\frac{1}{p}\cdot a+\frac{\alpha}{q}\cdot a+\frac{\beta}{r}\cdot a+\frac{\gamma}{s}\cdot a=1-\frac{1}{ABCD}=1+\frac{1}{\theta}.$$

Therefore since θ shall be a very large number, there will be required to be approximately

$$\frac{1}{p}+\frac{\alpha}{q}+\frac{\beta}{r}+\frac{\gamma}{s}=\frac{1}{a},$$

which is the first proper equation to be observed. The following equation will provide for the removal of the latter confusion[§ 27], which, for the sake of brevity in place of the fraction $\frac{10}{7}$ or some other proposed, if it were agreed there to be written ζ, may be expressed in this manner:

$$0=\frac{\zeta}{p}+\frac{\zeta\alpha^2}{q}+\frac{\beta^2}{r}+\frac{\gamma^2}{s}.$$

Truly the third equation arising from the destruction of the first confusion is required to be considered [§ 31] ; where since it may be arranged, so that the letters $\lambda,\ \lambda',\ \lambda'',\ \lambda'''$ may not exceed one by much and the values of these on account of the letters *v, v'* etc. may be little affected likewise, as we have seen above, the letters μ and μ' differ little from each other, with the terms depending on *v* ignored, we may put $\lambda=\lambda'=\lambda''=\lambda'''=1$ and our third equation adopts the following form:

$$\frac{1}{p^3}+\frac{\alpha^4}{q^3}+\frac{\beta^4}{r^3}+\frac{\gamma^4}{s^3}=0;$$

and now the whole problem has been reduced thus, so that it may be satisfied by these three equations, where certainly it is required to be observed for the first equation that it must be satisfied with care, but for the two latter equations to suffice, for these to be almost satisfied; so that which resolution may be put in place more easily, again we may put

$$\frac{1}{p}=\frac{z}{a},\quad \frac{\alpha}{q}=\frac{y}{a},\quad \frac{\beta}{r}=\frac{x}{a}\ \text{ and }\ \frac{\gamma}{s}=\frac{v}{a},$$

so that our three equations may be produced :

$$\text{I.}\quad z+y+x+v=1,$$
$$\text{II.}\quad \zeta z+\zeta\alpha y+\beta x+\gamma v=0,$$
$$\text{III.}\quad z^3+\alpha y^3+\beta x^3+\gamma v^3=0,$$

in which for the two latter equations the letters α, β and γ may be able to be taken as unity without notable error. Now we may put in place, so that the resolution may be rendered more clearly,

$$z=f+g,\quad y=f-g,\quad x=h+k,\quad v=h-k,$$

[The letters *f, g, h* and *k* presumably are to be taken as independent variables to expedite a solution of the above equations, not related to the usual uses of these letters in terms of radii of faces, etc.]

and our three equations will be become these:

$$\text{I.}\quad f+h=\tfrac{1}{2},$$
$$\text{II.}\quad \zeta f+h=0,$$
$$\text{III.}\quad f\left(f^2+3g^2\right)+h\left(h^2+3k^2\right)=0.$$

From the first two we deduce

$$f=\frac{1}{2(1-\zeta)},\quad h=\frac{\zeta}{2(\zeta-1)},$$

and since $\zeta=\frac{3}{2}$ approx., we will have now these two values:

$$f=-1\ \text{ and }\ h=\tfrac{3}{2};$$

which substituted into the third will give

$$-1-3g^2+\tfrac{27}{8}+\tfrac{9}{2}k^2=0;$$

from which there is concluded,

$$g=\sqrt{\tfrac{3}{2}k^2+\tfrac{19}{24}}$$

where nothing stands in the way, why k may not be put $=0$; yet meanwhile, since the latter part $h\left(h^2+3k^2\right)$ is increased somewhat on account of the letters β and γ, and that also on account of the terms affected by the letter v, therefore some increment is taken, as this part must be multiplied by $\frac{\mu'}{\mu}$ above, which fraction is greater than unity, it is evident there must be taken $g>\sqrt{\tfrac{19}{24}}$. Therefore there will be taken most conveniently $g=1$; then truly there will become

$$z=0,\ \ y=-2,\ \ x=v=h$$

and hence

$$p=\infty,\ \ q=-\tfrac{\alpha a}{2},\ \ r=\tfrac{2}{3}\beta a\ \text{ and }\ s=\tfrac{2}{3}\gamma a.$$

Therefore since here the focal length of the first lens may become infinite, and that is the same if this first lens were to be removed completely and in place of the objective as many as three lenses will be substituted, of which the first only shall be required to be prepared from crystalline glass; and since here there becomes $\alpha=1$ and $\mathfrak{A}=-\tfrac{1}{2}$, plainly the same case will be had here, which now we have set out above in problem 2, thus so that it would be superfluous to pursue this problem further.

SCHOLIUM

205. Therefore this problem thus is noteworthy chiefly, since here by a single straight forwards method we have used its solution which is requiring to be investigated, which in all occasions is able to bring forth a conspicuous use, from which also it is evident there is no need indeed for anything further to be added to this chapter.

CAPUT III

DE SUMMA MICROSCOPIORUM HUIUS GENERIS PERFECTIONE DUM OPE LENTIUM CONCAVARUM ET EX ALIA VITRI SPECIE CONFECTARUM OMNIS PLANE CONFUSIO AD NIHILUM REDIGITUR

PROBLEMA 1

189. *Loco lentis obiectivae duas lentes, quarum prior sit concava, substituere, ut manentibus binis lentibus posterioribus confusio omnis tollatur.*

SOLUTIO

Cum hic ergo quatuor habeantur lentes ideoque tria intervalla, litterarum *P, Q, R* ultima debet esse negativa; ponatur ergo $R=-k$, et ut margo coloratus tollatur, ex supra traditis manifestum est capi debere $k=1$, ita ut sit $PQ=\frac{ma}{h}$; deinde ut simul idem campus comparetur qui ante, debet esse $\mathfrak{C}=-2$ et $C=-\frac{2}{3}$; unde distantiae focales lentium erunt

$$p=\mathfrak{A}a,\quad q=-\frac{A\mathfrak{B}}{P}\cdot a,\quad r=-2AB\cdot\frac{h}{m}\ \text{ et }\ s=-\frac{2}{3}AB\cdot\frac{h}{m},$$

intervalla vero lentium

$$\text{primum}=Aa(1-\tfrac{1}{P}),$$

$$\text{secundum}=-\frac{ABa}{P}+AB\cdot\frac{h}{m}$$

et

$$\text{tertium}=-\tfrac{4}{3}AB\cdot\frac{h}{m}=2s,$$

et ut ante distantia oculi

$$O=\tfrac{1}{2}s\left(1+\frac{h}{ma}\right),$$

quemadmodum etiam spatii in obiecto conspicui semidiameter manet

$$=\tfrac{1}{2}\cdot\frac{ah}{ma+h};$$

nunc autem cum prima lens debeat esse concava, necesse est sit $\mathfrak{A}<0$ ideoque et $A<0$, quare oportebit esse $P<1$; tum vero ob $AB<0$ debet esse $B>0$ ideoque etiam $\mathfrak{B}$ quantitas positiva. Ponamus iam ut ante, quoniam duae priores lentes sibi debent esse proximae, intervallum primum $=-\zeta p$ fietque

$$\frac{1}{P}=1+\frac{\mathfrak{A}}{A}\cdot\zeta=(1-\mathfrak{A})\cdot\zeta.$$

Cum prima lens sit concava, sit ea quoque ex vitro crystallino parata, dum reliquae ex vitro coronario factae esse sumuntur, ita ut nunc n denotet 1,58 et $n'=1,53=n''=n'''$, quibus reliquae litterae independentes consentaneae esse debent. Quo posito aequatio omnem confusionem a diversa radiorum refrangibilitate oriundam tollens erit [§ 27]

$$0=N\cdot\frac{1}{p}+\frac{N'}{P^2}\cdot\frac{1}{q}+\frac{N'}{P^2Q^2}\cdot\frac{1}{r}+\frac{N'}{P^2Q^2R^2}\cdot\frac{1}{s}$$

seu

$$0=\frac{N}{N'}\cdot\frac{1}{p}+\frac{1}{P^2q}+\frac{h^2}{m^2a^2r}+\frac{h^2}{m^2a^2s},$$

ubi duo posteriora membra manifesto reiici possunt, et cum sit circiter $\frac{N}{N'}=\frac{10}{7}$ et $P=1$ proxima, haec aequatio dabit

$$0=\frac{10}{7}\cdot\frac{1}{\mathfrak{A}}-\frac{1}{A\mathfrak{B}}$$

adeoque

$$\mathfrak{B}=\frac{7}{10}\cdot\frac{\mathfrak{A}}{A}=\frac{7}{10}(1-\mathfrak{A})\quad\text{et}\quad B=\frac{7(1-\mathfrak{A})}{3+7\mathfrak{A}};$$

qui valor ut fiat positivus existente $\mathfrak{A}<0$, necesse est, ut $3+7\mathfrak{A}$ sit positivum sive $-\mathfrak{A}<\frac{3}{7}$; si autem non sit $P=1$, adcuratius habebimus

$$\mathfrak{B}=\frac{7}{10}(1-\mathfrak{A})\big(1+(1-\mathfrak{A})\zeta\big),$$

ubi tantum notetur esse debere $\mathfrak{B}<1$, ut etiam B prodeat positivum.

Ponatur igitur

$$(1-\mathfrak{A})\big(1+(1-\mathfrak{A})\zeta\big)<\frac{10}{7},\quad (1-\mathfrak{A})^2+\frac{1-\mathfrak{A}}{\zeta}<\frac{10}{7\zeta}$$

et addito utrinque $\frac{1}{4\zeta^2}$ oportebit esse

$$1-\mathfrak{A}+\frac{1}{2\zeta}<\sqrt{\frac{10}{7\zeta}+\frac{1}{4\zeta^2}}.$$

Ne nunc binae priores lentes sibi nimium fiant vicinae, statuamus $\zeta=\frac{1}{7}$ capique debebit

$$1-\mathfrak{A} < -\tfrac{7}{2}+\sqrt{10+\tfrac{49}{7}} \text{ seu } 1-\mathfrak{A} < 1,217.$$

Sumamus igitur

$$1-\mathfrak{A} = \tfrac{6}{5}$$

eritque

$$\mathfrak{A} = -\tfrac{1}{5} \text{ et } A = -\tfrac{1}{6}$$

tum vero ob $\zeta = \frac{1}{7}$ erit $\frac{1}{P} = \frac{41}{35}$ hincque

$$\mathfrak{B} = \tfrac{7}{10}\cdot\tfrac{6}{5}\cdot\tfrac{41}{35} = \tfrac{246}{250} = \tfrac{123}{125} \text{ et } B = \tfrac{123}{2} = 61\tfrac{1}{2};$$

unde sequitur $AB = -10\frac{1}{4}$, qui valor sine dubio nimis est parvus, quia pro magnis multiplicationibus pro r nimis exiguum praeberet valorem; verum notandum est, si $1-\mathfrak{A}$ tantillo maior caperetur quam $\frac{3}{5}$, ut discrimen in praxi sentiri non posset, tum productum AB quantumvis magnum evadere posse; si enim ponamus $1-\mathfrak{A} = \frac{6}{5}+\omega,$ inde elicitur

$$B = \tfrac{246+235\omega+25\omega^4}{4-235\omega-25m^2};$$

qui valor adeo infinitus evaderet, si tantum sumeretur $\omega = \frac{1}{60}$ proxima, quo pacto valores A, $\mathfrak{A}$ et $\mathfrak{B}$ vix sensibiliter mutarentur, ita ut sumtis hisce valoribus

$$\mathfrak{A} = -\tfrac{1}{5}, \quad A = -\tfrac{1}{6}, \quad \tfrac{1}{P} = \tfrac{41}{35}$$

ob

$$\zeta = \tfrac{1}{7} \quad \text{et} \quad \mathfrak{B} = \tfrac{123}{125} = 1-\tfrac{2}{125}$$

littera B adhuc ut indefinita spectari atque sine haesitatione ita definiri possit, ut littera r idoneum valorem nanciscatur. Quamobrem habebimus, ut sequitur:

$$p = -\tfrac{1}{5}a, \quad q = 0,19212a, \quad r = \theta\cdot\tfrac{h}{m} \text{ et } s = \tfrac{1}{3}r,$$

ubi θ pro lubitu assumi potest; deinde vero intervalla erunt

$$\text{Primum} = -\tfrac{1}{7}p = \tfrac{1}{35}a, \quad \text{secundum} = \tfrac{41\theta a}{70} - \tfrac{1}{2}\theta\cdot\tfrac{h}{m}, \quad \text{tertium} = 2s.$$

Nunc denique, ut etiam confusio ab apertura lentium oriunda evanescat, satisfieri debet huic aequationi [§ 31]:

$$0 = \tfrac{\mu}{\mu'}\left(\lambda+\nu\mathfrak{A}\left(1-\mathfrak{A}\right)\right) - \tfrac{\left(1-\mathfrak{A}\right)^3}{P}\left(\tfrac{\lambda'}{\mathfrak{B}^3}+\tfrac{\nu'}{B\mathfrak{B}}\right)$$

Quodsi iam hic sumatur $\lambda' = 1$, ob

$$\mu = 0,8724, \quad \nu = 0,2529, \quad \mu' = 0,9875, \quad \nu' = 0,2196$$

calculo facto reperietur

$$\lambda = 2,4137 + 0,0607 = 2,4744,$$

unde colligitur $\tau\sqrt{(\lambda-1)} = 1,0655$; quare pro lente prima ex vitro crystallino paranda, cuius distantia focalis est $p = -\frac{1}{5}a$ et numeri

$$\mathfrak{A} = -\frac{1}{5} \text{ et } \lambda = 2,4744,$$

erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{p}{\sigma - \mathfrak{A}(\sigma-\rho) \mp \tau\sqrt{(\lambda-1)}} = \frac{p}{1,8710 - 1,0655} \\ \text{posterioris} = \frac{p}{\sigma + \mathfrak{A}(\sigma-\rho) \pm \tau\sqrt{(\lambda-1)}} = \frac{p}{-0,1469 + 1,0655},\end{cases}$$

unde fit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{p}{0,8055} = -0,2483a \\ \text{posterioris} = \frac{p}{0,9186} = -0,2177a\end{cases}$$

quae ergo aperturae capax est, cuius semidiameter $x = 0,0544a$, nisi forte secunda lens tantam aperturam non patiatur.

Pro secunda autem lente ex vitro coronario, cuius distantia focalis $q = 0,19212a$ et numeri

$$\mathfrak{B} = \frac{123}{125} \text{ et } \lambda' = 1,$$

erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{q}{\sigma - \mathfrak{B}(\sigma-\rho)} = \frac{q}{0,2496} = 0,7697a \\ \text{posterioris} = \frac{q}{\sigma + \mathfrak{B}(\sigma-\rho)} = \frac{q}{1,6372} = 0,1173a,\end{cases}$$

cuius ergo apertura maior esse nequit quam $x = 0,0293a$. Hinc autem colligitur

$$y = \frac{hx}{ma} = \frac{0,2344}{m} \text{ dig.}$$

hincque mensura claritatis $= \frac{4,688}{m}$, quae ergo fere sexies minor est quam in ultimo casu capitis praecedentis.
Hinc ergo colligitur sequens

CONSTRUCTIO HUIUSMODI MICROSCOPIORUM EX QUATUOR LENTIBUS COMPOSITORUM

190. Posita distantia obiecti ab instrumento $=a$ et multiplicatione $=m$ habebitur

I. Pro prima lente concava ex vitro crystallino paranda, cuius distantia focalis est $p=-\frac{1}{5}a$,

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -0,2483a\\ \text{posterioris} &= -0,2177a,\end{cases}$$

eius aperturae semidiameter ob rationes modo allegatas $x=0,0293a$ et distantia ad lentem secundam $=-\frac{1}{7}p=0,0286a.$

II. Pro secunda lente ex vitro coronario paranda, cuius distantia focalis est $q=0,1921a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 0,7697a\\ \text{posterioris} &= 0,1173a;\end{cases}$$

apertura manet ut ante.
Distantia ad lentem tertiam $=\frac{41}{560}mar-\frac{1}{2}r$,
ubi r denotat distantiam focalem tertiae lentis, quam pro arbitrio assumere licet.

III. Pro tertia autem lente, cuius distantia focalis $=r$, si ex vitro coronario paretur,

$$\text{radius faciei utriusque} = 1,06\, r;$$

perinde autem est, ex quanam vitri specie haec lens atque etiam quarta parentur;
eius aperturae semidiameter $=\frac{1}{4}r$
et distantia ad lentem quartam $=\frac{2}{3}r.$

IV. Pro quarta lente, cuius distantia focalis est $s=\frac{1}{3}r$, capiatur
radius utriusque faciei $=1,06s$
vel potius secundum indolem vitri, ex, qua paratur.
Aperturae semidiameter $=\frac{1}{4}r$
et distantia ad oculum $=\frac{1}{2}s.$

V. Porro spatii in obiecto conspicui semidiameter erit ut hactenus

$$z = \frac{1}{2} \cdot \frac{ah}{ma+h} = \frac{4a}{ma+8} \text{ dig.}$$

VI. Claritatis autem, qua obiecta conspicientur, mensura erit $= \frac{4,688}{m}$.

COROLLARIUM 1

191. Hic manifestum est, ne duae priores lentes nimis fiant exiguae, quam ut ab artifice adcurate elaborari possint, necessario distantiam obiecti *a* multo maiorem statui debere quam hactenus. Videtur autem haec distantia *a* vix minor duobus digitis commode assumi posse, quod quidem in praxi pro lucro est habendum, praesertim cum claritas ab hac distantla non pendeat.

COROLLARIUM 2

192. Sumta autem distantia $a = 2$ dig. intervallum secundum evadet $\frac{41}{280} mr - \frac{1}{2} r$; quare si sumamus $r = 1$ dig., siquidem ob $s = \frac{1}{3} r$ commode minus accipi nequit, pro multiplicatione $m = 280$ hoc intervallum erit $40\frac{1}{2}$ dig.; sin autem multiplicatio desideretur duplo maior, $m = 560$, hoc intervallum fiet $= 80\frac{1}{2}$ dig. atque ad eo maius pro maioribus multiplicationibus; quae enormis longitudo sine dubio maxime displicebit.

SCHOLlON

193. Quod haec microscopia his incommodis sint obnoxia, causa in eo est sita, quod distantiae focales priorum lentium nimis sint exiguae, dum scilicet *p* et *q* tantum parti circiter quintae ipsius *a* aequari debebant, cum in casu postremo capitis praecedentis hae distantiae focales adeo quadruplo essent maiores quam distantia *a,* atque hinc etiam factum est, ut mensura claritatis hic tantum inventa sit $= \frac{4,688}{m}$, cum ante esset $\frac{29,92}{m}$, hoc est sexies maior atque adeo secundum veritatem tricies sexies maior. Quamobrem, etiamsi artifex in constructione horum microscopiorum omnem diligentiam et industriam adhibeat eique opus ex voto succedat, tamen vehementer dubito, an haec microscopia ulIam praerogativam prae antecedentibus mercantur, quamvis hic etiam secunda confusio a diversa refrangibilitate oriunda penitus sit sublata, quod in praecedente capite praestare non licuit. Hic quidem primam lentem sumsimus concavam, secundam vero convexam; verum ex superioribus satis liquet nullum commodum exspectari posse, si hae lentes inter se permutarentur; quin potius hic ordo iam supra

anteferri in praxi debere est observatus ideoque superfluum foret, si istum casum seorsim evolvere vellemus. Quamobrem nunc statim loco lentis obiectivae tres lentes substituamus, quarum una sit concava binaeque reliquae convexae, et inquiramus praecipue, num hoc casu distantia focalis harum lentium aliquanto maior fieri queat quam casu hic tractato et num forte numerum lentium ulterius augendo maiora adhuc commoda sperari queant;

PROBLEMA 2

194. *Loco lentis obiectivae tres lentes sibi proxime iunctas substituere, quarum prima sit concava et ex vitro crystallino parata, binae autem reliquae convexae ex vitro coronario, ut manentibus binis lentibus postremis omnis confusio ad nihilum redigatur.*

SOLUTIO

Cum hic quatuor habeantur intervalla, litterarum P, Q, R, S ultima erit negativa et margo coloratus tolletur, si fuerit $S=-1$. Binae vero primae litterae P et Q unitati proxima erunt aequales, ita ut sit $PQR=\frac{ma}{h}$. Quod ad reliquas litteras attinet, conditio campi postulat, ut sit $\mathfrak{D}=-2$ et $D=-\frac{2}{3}$, et cum prima lens sit concava, erit $\mathfrak{A}$ negativum ideoque etiam A, ita tamen, ut sit $-A<1$. Deinde ob $q=-\frac{A\mathfrak{B}}{P}\cdot a$, quia haec lens debet esse convexa, littera $\mathfrak{B}$ erit positiva, et quia tertia lens, pro qua est $r=\frac{AB\mathfrak{C}}{PQ}\cdot a$ etiam debet esse convexa, esse debet $B\mathfrak{C}<0$, et quia porro fit

$$s=+\frac{2ABC}{PQR}\cdot a=2ABC\cdot\frac{h}{m},$$

ne haec lens pro maioribus multiplicationibus fiat nimis parva, productum ABC aequari debet numero praemagno positivo, unde concluditur BC fore numerum magnum negativum. Cum autem sit etiam $B\mathfrak{C}<0$ hicque numerus non possit esse praemagnus, sequitur C esse debere numerum praemagnum hincque $\mathfrak{C}$ unitati proxima aequale; quamobrem B debet esse numerus negativus hincque $\mathfrak{B}>1$, contra vero $\mathfrak{C}<1$, sed differentia existente valde parva, ut prodeat C numerus praemagnus positivus.

His notatis consideremus aequationem, qua confusio posterior penitus tollitur, quae erit

$$0=\frac{10}{7}\cdot\frac{1}{p}+\frac{1}{P^2q}+\frac{1}{P^2Q^2r}+\frac{1}{P^2Q^2R^2s}+\frac{1}{P^2Q^2R^2t},$$

ubi ob $PQR=\frac{ma}{h}$ bina postrema membra tuto reiicere licet, et cum litterae P et Q proxime unitati aequentur, habebimus hanc determinationem:

$$0=\frac{10}{7}\cdot\frac{1}{p}+\frac{1}{q}+\frac{1}{r},$$

ita ut sit

$$\frac{1}{p}=-\frac{7}{10}\left(\frac{1}{q}+\frac{1}{r}\right)$$

sive substitutis valoribus

$$\frac{A}{\mathfrak{A}}=-\frac{7}{10}\left(\frac{1}{B\mathfrak{C}}-\frac{1}{\mathfrak{B}}\right);$$

et quia proxima est $\mathfrak{C}=1$, obtinebimus

$$1+A=\frac{7}{10}$$

adeoque

$$A=-\frac{3}{10} \text{ et } \mathfrak{A}=-\frac{3}{7}.$$

Cum autem sufficiat rem propemodum tantum definivisse, sumamus $\mathfrak{A}=-\frac{1}{2}$ et statuendo ambo prioro intervalla

$$=-\frac{1}{7}p=\frac{1}{14}a$$

fiet

$$\frac{1}{P}=\frac{17}{14} \text{ et } \frac{1}{PQ}=\frac{17}{14}-\frac{3}{14B}$$

hincque

$$p=-\frac{1}{2}a,\ \ q=+\frac{\mathfrak{B}}{3P}\cdot a,\ \ r=-\frac{B}{3PQ}\cdot a;$$

qui valores substituti dabunt

$$0=-\frac{20}{7}+\frac{3}{\mathfrak{B}P}-\frac{3}{BPQ},$$

$$0=-\frac{20}{7}+\frac{51}{14\mathfrak{B}}-\frac{51}{14B}+\frac{9}{14B^2}$$

seu

$$0=-\frac{20}{7}+\frac{51}{14}+\frac{9}{14B^2}$$

ob $\frac{1}{\mathfrak{B}}-\frac{1}{B}=1$ vel

$$0=\frac{11}{14}+\frac{9}{14B^2};$$

hinc ergo non enormiter aberrabitur, quicquid pro B accipiatur; quodsi autem aequationem ex destructione alterius confusionis consideremus, patebit non incongrue sumi posse $\mathfrak{B}=2$ ideoque $B=-2$, ita ut iam sit

$$\frac{1}{PQ}=\frac{37}{28},\ \ q=\frac{17}{21}a \text{ et } r=\frac{37}{42}a\ ;$$

tum autem aequatio adhuc resolvenda erit

$$\lambda=\frac{3}{4}\nu+\frac{\mu'}{\mu}\cdot\frac{17\cdot 27}{14\cdot 8^2}\left(\lambda'-2\nu'\right)+\frac{\mu'}{\mu}\cdot\frac{27\cdot 37\cdot\lambda''}{28\cdot 8^2},$$

ubi poni potest tam $\lambda' = 1$ quam $\lambda'' = 1$, et ob

$$v = 0,2529 \text{ et } v' = 0,2196 \text{ et } \frac{\mu'}{\mu} = \frac{9875}{8724}$$

fiet

$$\lambda = 0,1897 + 0,3252 + 0,6322 \text{ adeoque } \lambda = 1,1471;$$

unde colligitur

$$\tau\sqrt{(\lambda-1)} = 0,3365.$$

Quare pro prima lente crystallina erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)\mp\tau\sqrt{(\lambda-1)}} = +\frac{p}{1,9668} = -0,2542a\\ \text{posterioris} = \frac{p}{\rho+\mathfrak{A}(\sigma-\rho)\pm\tau\sqrt{(\lambda-1)}} = -\frac{p}{0,2427} = +2,0602a.\end{cases}$$

Pro lente secunda ex vitro coronario paranda erit

$$\text{radius}\begin{cases}\text{anterioris} = \frac{q}{\sigma-\mathfrak{B}(\sigma-\rho)} = +\frac{q}{1,2067} = -0,6708a\\ \text{posterioris} = \frac{q}{\rho+\mathfrak{B}(\sigma-\rho)} = +\frac{q}{3,0935} = +0,2617a.\end{cases}$$

Simili modo pro tertia lente

$$\text{radius}\begin{cases}\text{anterioris} = \frac{r}{\rho} = \frac{r}{0,2267} = 3,8857a\\ \text{posterioris} = \frac{r}{\sigma} = \frac{r}{1,6601} = 0,5306a;\end{cases}$$

pro quarum lentium apertura sumi poterit $= 0,0635a$, hincque sequitur

CONSTRUCTIO MICROSCOPII EX QUINQUE LENTIBUS COMPOSITI ET OMNIS FERE CONFUSIONIS EXPERTIS

195. Hic tres res pro lubitu assumi possunt

1. distantia obiecti $= a$,
2. multiplicatio $= m$,
3. distantia focalis quartae lentis $= s$.

I. Pro prima lente crystallina, cuius distantia focalis $p = 0,5000a$, capiatur

$$\text{radius}\begin{cases}\text{anterioris} &= -\ 0,2542a\\ \text{posterioris} &= +\ 2,0602a,\end{cases}$$

aperturae semidiameter $x = 0,0635a$, qui etiam in duabus sequentibus locum habet, et distantia ad sequentem $= \frac{1}{14}a$.

II. Pro secunda lente ex vitro coronario paranda, cuius distantia focalis $q = 0,8095a$, capiatur

$$\text{radius}\begin{cases}\text{anterioris} &= -0,6708a\\ \text{posterioris} &= +0,2617a,\end{cases}$$

distantia ad lentem tertiam $= \frac{1}{14}a$.

III. Pro tertia lente itidem coronaria, cuius distantia focalis $r = 0,8809a$, capiatur

$$\text{radius}\begin{cases}\text{anterioris} &= 3,8857a\\ \text{posterioris} &= 0,5306a,\end{cases}$$

eius distantia ad lentum quartam $= \frac{37mas}{448} - \frac{1}{2}s$.

IV. Quartam lentem pro lubitu ex quovis vitri genere construere licet, cuius distantia focalis $= s$; tum erit eius distantia ad lentem ocularem $= \frac{2}{3}s$.

V. Ipsius lentis ocularis distantia focalis erit $= \frac{1}{3}s$ eaque pariter utrinque aeque convexa et distantia ad oculum usque $= \frac{1}{6}s$.

VI. Mensura claritatis erit $\frac{10}{m}$ et spatii in obiecto conspicui semidiameter $z = \frac{4a}{ma+s}$ dig.

SCHOLION 1

196. Si casum propositum ita immutemus, ut binae priores lentes sint concavae et ex vitro crystallino factae, tum simili modo solutionem adornando omnia eodem modo definientur, nisi quod nunc ambae litterae $\mathfrak{B}$ et B debeant esse negativae; ac tum destructio alterius confusionis dabit hanc aequationem:

$$1+A=\tfrac{7}{10}+\tfrac{3}{10\mathfrak{B}} \text{ seu } A=-\tfrac{3}{10}+\tfrac{3}{10\mathfrak{B}};$$

unde, si sumatur $\mathfrak{B}=-2$ hincque $B=-\tfrac{2}{3}$, elicietur $A=-\tfrac{9}{20}$ ideoque $\mathfrak{A}=-\tfrac{9}{11}$, qui valores praebent distantias focales

$$p=-\tfrac{9}{11}\cdot a,\quad q=-\tfrac{9}{10P}\cdot a,\quad r=\tfrac{3}{10PQ}\cdot a,$$

ubi est

$$\tfrac{1}{P}=1+\tfrac{20}{11}\zeta \text{ et } \tfrac{1}{PQ}=1+\tfrac{50}{11}\zeta;$$

sumsimus autem $\mathfrak{C}=1$ proxima, ut pro C numerus praemagnus prodeat et ponendo $ABC=\theta$ fiat $s=2\theta\cdot\tfrac{h}{m}$ et ut ante $t=\tfrac{1}{3}s$ ultimumque intervallum $=2t$. Duo priora vero intervalla erunt per hypothesin $=-\tfrac{9}{11}\zeta a$, intervallum vero tertium $=\theta a\left(\tfrac{1}{PQ}-\tfrac{h}{ma}\right)$.

Tum vero, ut etiam prior confusio evanescat, sequenti aequationi satisfieri oportebit:

$$0=\lambda+\nu\mathfrak{A}(1-\mathfrak{A})-\tfrac{(1-\mathfrak{A})^3}{P}\left(\tfrac{\lambda'}{\mathfrak{B}^3}+\tfrac{\nu}{B\mathfrak{B}}\right)+\tfrac{\mu'}{\mu}\cdot\tfrac{(1-\mathfrak{A})^3}{B^3PQ}\left(\tfrac{\lambda''}{\mathfrak{C}^3}+\tfrac{\nu'}{C\mathfrak{C}}\right),$$

quae fit substitutis valoribus

$$0=\lambda-\tfrac{180\nu}{121}+\tfrac{8000}{1331P}\left(\tfrac{\lambda'}{8}-\tfrac{3\nu}{4}\right)-\tfrac{\mu'}{\mu}\cdot\tfrac{30^3}{11^3PQ}\left(\tfrac{\lambda''}{\mathfrak{C}^3}+\tfrac{\nu'}{C\mathfrak{C}}\right),$$

unde sequitur

$$1331\lambda+\tfrac{1000\lambda'}{P}=1980\nu+\tfrac{6000\nu}{P}+\tfrac{\mu'}{\mu}\cdot\tfrac{27000}{PQ}\left(\tfrac{\lambda''}{\mathfrak{C}^3}+\tfrac{\nu'}{C\mathfrak{C}}\right).$$

Si igitur hic capiatur $\lambda''=1$ et ponatur $\lambda'=\lambda$, ut scilicet pro utroque valor minimus reperiatur, habebitur ista aequatio:

$$\lambda\left(1331+\tfrac{1000\lambda}{P}\right)=1980\nu+\tfrac{6000\nu}{P}+\tfrac{\mu'}{\mu}\cdot\tfrac{27000}{PQ}\left(\tfrac{\lambda''}{\mathfrak{C}^3}+\tfrac{\nu'}{C\mathfrak{C}}\right),$$

unde facile patet valorem ipsius λ multo maiorem esse proditurum quam 12, unde constructio harum lentium admodum lubrica evaderet. Interim tamen hunc casum diligentius evolvamus sumto [$\nu'=\tfrac{1}{5}$, C $=12$,]$\zeta=\tfrac{11}{80}$, ut fiat $\tfrac{1}{P}=\tfrac{5}{4}$ et $\tfrac{1}{PQ}=\tfrac{13}{8}$, quibus positis aequatio nostra ad hos numeros reducetur:

$$2581\lambda=52888,492,$$

ita ut sit

$$\lambda=\tfrac{52888,5}{2581}=20,492,$$

unde colligitur

$$\tau\sqrt{(\lambda-1)}=3,8742.$$

Hinc pro prima lente erit

$$\text{radius faciei}\begin{cases}\text{anterioris}=\frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)\mp\tau\sqrt{(\lambda-1)}}=\frac{p}{2,7620-3,8742}\\ \text{posterioris}=\frac{p}{\rho+\mathfrak{A}(\sigma-\rho)\pm\tau\sqrt{(\lambda-1)}}=\frac{p}{-1.0379+3,8742},\end{cases}$$

unde fit

$$\text{radius faciei}\begin{cases}\text{anterioris}=-\frac{p}{1,1122}=+0,7356a\\ \text{posterioris}=\frac{p}{2,8363}=-0,2885a.\end{cases}$$

Pro secunda lente

$$\text{radius faciei}\begin{cases}\text{anterioris}=\frac{p}{\sigma-\mathfrak{B}(\sigma-\rho)\mp\tau\sqrt{(\lambda-1)}}=\frac{p}{0,5911}=-1,9032a\\ \text{posterioris}=\frac{p}{\rho+\mathfrak{B}(\sigma-\rho)\pm\tau\sqrt{(\lambda-1)}}=\frac{p}{1.1330}=-0,9930a.\end{cases}$$

Pro lente autem tertia ex vitro coronario erit

$$\text{radius}\begin{cases}\text{anterioris}=\frac{r}{\rho}=\frac{r}{0,2267}=2,1504a\\ \text{posterioris}=\frac{r}{\sigma}=\frac{r}{1,6601}=0,2937a;\end{cases}$$

quae tres lentes cum communem circiter aperturam exigant, eius semidiameter sumi debebit $x=0,0721a$, ex quo fit $y=\frac{0,5768}{m}$ dig. hincque mensura claritatis $=\frac{11,536}{m}$, quae ergo fere triplo maior est quam casu praecedentis problematis.
Hinc ergo deducitur sequens

CONSTRUCTIO MICROSCOPII EX QUINQUE LENTIBUS COMPOSITI

197. Hic scilicet primo datur distantia obiecti $=a$, deinde multiplicatio $=m$ ac tertio distantia focalis quartae lentis $=s$; unde fit

$$\theta=ABC=\frac{ms}{16}$$

I. Pro prima lente ex vitro crystallino paranda, cuius distantia focalis est $p=-0,8182a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} & = +0,7356a\\ \text{posterioris} & = -0,2885a,\end{cases}$$

eius aperturae semidiameter $a = 0,0721a$, quae et pro binis sequentibus valet, et distantia ad secundam lentem $= 0,1125a$.

II. Pro secunda lente ex vitro crystallino paranda, cuius distantia focalis est $q = -1,125a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} & = -1,9032a\\ \text{posterioris} & = -0,9930a,\end{cases}$$

eiusque distantia ad lentem tertiam $= 0,1125a$.

III. Pro lente tertia ex vitro coronario paranda, cuius distantia focalis $r = 0,4875a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} & = 2,1504a\\ \text{posterioris} & = 0,2937a,\end{cases}$$

eius distantia ad lentem quartam

$$= \theta a\left(\tfrac{13}{8} - \tfrac{h}{ma}\right) = \tfrac{13mas}{128} - \tfrac{1}{2}s.$$

IV. Quartam lentem ex quovis vitro pro lubitu construere licet, cuius distantia focalis sit $= s$; tum erit eius distantia ad lentem ocularem $= \tfrac{2}{3}s$.

V. Ipsius autem lentis ocularis erit distantia focalis $= \tfrac{1}{3}s$. eiusque ad oculum distantia $= \tfrac{1}{6}s$.

VI. Mensura claritatis autem erit, ut vidimus, $\frac{11,536}{m}$ spatiique conspicui semidiameter ut hactenus $z = \frac{4a}{ma+8}$ dig.

SCHOLION 2

198. Quanquam autem haec microscopia praecedentibus anteferenda videntur, tamen, uti iam innuimus, ea neutiquam commendare audemus, propterea quod eorum constructio summis difficultatibus est implicata, ut etiam a sollertissimo artifice exspectari nequeat; cuius rei causa manifesto in eo est posita, quod pro litteris λ et λ' tam grandem valorem invenimus, scilicet ad viginti assurgentem. Facile enim intelligitur, si iste valor fuisset unitate vel adeo binario maior vel minor, inde harum lentium constructionem non sensibiliter fuisse mutatam, unde vicissim colligitur, etiamsi hae lentes summo

studio fuerint elaboratae, tum maxime probabile fore valorem litterae λ iis convenientem non solum unitate vel binario, sed etiam magis a 20 esse discrepaturum; quod si eveniat, confusio inde orta adeo multo erit maior, quam si lens obiectiva simplex adhiberetur; ex quo manifestum est perfectam destructionem confusionis posterioris nullo plane modo sperari posse; quare, cum adhuc, ante quam diversa vitri indoles erat comperta, hanc confusionis speciem tolerare sumus coacti et sola destructione marginis colorati contenti esse debuimus, nunc etiam eo facilius huic conditioni renunciare poterimus, cum vitrum crystallinum adhibendo saltem hanc confusionem quodammodo diminuere liceat, quem in finem exempla quaedam subiungamus, quae ad praxin facile accommodari posse videntur, cum pro litteris λ valores unitate non multo maiores requirant neque tamen a praescripta in problemate conditione multum abhorreant.

EXEMPLUM 1

199. In formulis supra inventis statuamus $\mathfrak{A}=-\frac{1}{2}$ et $\mathfrak{B}=2$ hincque $A=-\frac{1}{3}$ et $B=-2$ manente littera $\mathfrak{C}$ aliquantillum minora unitate, ut C fiat numerus praemagnus. Tum igitur erit ex formulis superioribus

$$\frac{1}{P}=1+\frac{3}{2}\zeta,\quad \frac{1}{PQ}=1+\frac{9}{4}\zeta,$$

unde distantiae focales erunt

$$p=-\frac{1}{2}a,\quad q=\frac{2}{3}a+\zeta a,$$

$$r=\frac{2}{3}\mathfrak{C}\left(1+\frac{9}{4}\zeta\right)a=\frac{2}{3}\mathfrak{C}a+\frac{3}{2}\mathfrak{C}\zeta a,$$

$$s=\frac{4}{3}C\cdot\frac{h}{m}\ \text{ et }\ t=\frac{1}{3}s=\frac{4}{9}C\cdot\frac{h}{m}.$$

Ut igitur hinc prodeat $s=1$ dig. circiter casu $m=1000$, debet esse $C=100$ ideoque $\mathfrak{C}=\frac{100}{101}$. Intervalla vero lentium erunt

$$\text{primum et secundum}=-\zeta p=\frac{1}{2}\zeta a,$$

$$\text{tertium}=\frac{11mas}{128}-\frac{1}{2}s\quad\text{et}\quad\text{quartum}=\frac{2}{3}s.$$

Commode autem hic sumere poterimus $\zeta=\frac{1}{6}$, ut sit

$$\frac{1}{P}=\frac{5}{4}\quad\text{et}\quad\frac{1}{PQ}=\frac{11}{8}.$$

Tum vero primam lentem concavam ex vitro crystallino parari ponamus, quandoquidem hoc modo altera confusio saltem diminuetur, secundam vero et tertiam ex vitro coronario; atque nunc prior confusio ad nihilum redigetur, si fiat

$$8\lambda = 6\nu + \frac{\mu'}{\mu}\cdot\frac{5\cdot 27}{4}\left(\frac{\lambda'}{8}-\frac{\nu'}{4}\right)+\frac{\mu'}{\mu}\cdot\frac{11\cdot 27}{8^2}\left(1,03\lambda''+\frac{\nu'}{100}\right)$$

sive

$$\lambda = \frac{3}{4}\nu + \frac{\mu'}{\mu}\cdot\frac{135}{32}\left(\frac{\lambda'}{8}-\frac{\nu'}{4}\right)+\frac{\mu'}{\mu}\cdot\frac{297}{512}\left(1,03\lambda''+\frac{\nu'}{100}\right).$$

Cum nunc sit

$$\mu = 0,8724,\quad \nu = 0,2529 \text{ et } \mu' = 0,9875,\quad \nu' = 0,2196,$$

sumamus $\lambda' = \lambda'' = 1$ hincque fiet

$$\lambda = 0,1897 + \frac{\mu'}{\mu}\cdot\frac{27}{512}\cdot 16,9622$$

seu

$$\lambda = 1,2022,$$

unde fit

$$\tau\sqrt{(\lambda-1)} = 0,3946\,.$$

Ex quo sequitur:

Pro prima lente

$$\text{radius faciei}\begin{cases}\text{anterioris} = \dfrac{p}{\sigma-\mathfrak{A}(\sigma-\rho)\mp\tau\sqrt{(\lambda-1)}} = +\dfrac{p}{1,9087} = -0,2620a\\ \text{posterioris} = \dfrac{p}{\rho+\mathfrak{A}(\sigma-\rho)\pm\tau\sqrt{(\lambda-1)}} = -\dfrac{p}{0,1846} = +2,7086a.\end{cases}$$

Pro secunda autem lente

$$\text{radius faciei}\begin{cases}\text{anterioris} = \dfrac{q}{\sigma-\mathfrak{B}(\sigma-\rho)} = -\dfrac{q}{1,2067} = -0,6906a\\ \text{posterioris} = \dfrac{q}{\rho+\mathfrak{B}(\sigma-\rho)} = +\dfrac{q}{3,0935} = +0,2694a.\end{cases}$$

Pro tertia lente

erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \dfrac{r}{\sigma-\mathfrak{C}(\sigma-\rho)} = -\dfrac{r}{0,24103} = 3,7656a\\ \text{posterioris} = \dfrac{r}{\rho+\mathfrak{C}(\sigma-\rho)} = +\dfrac{r}{1,6458} = 0,5514a.\end{cases}$$

Pro harum igitur trium lentium apertura communi sumi poterit

$$x = 0,0655a,$$

unde fit

$$y = \frac{0,5240}{m}$$

hincque mensura claritatis fiet $= \frac{10,480}{m}$.

CONSTRUCTIO MICROSCOPIl EX QUINQUE LENTIBUS COMPOSITI ET AD PRAXIN MAGIS ACCOMMODATI

200. Dantur hic distantia obiecti $= a$ et multiplicatio $= m$ et quartae lentis distantia focalis $= s$ hincque erit:

I. Pro prima lente ex vitro crystallino paranda, cuius distantia focalis est $p = -\frac{1}{2}a$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} = -0,2620a\\ \text{posterioris} = 2,7086a,\end{cases}$$

eius aperturae semidiameter $= 0,0655a$
et distantia ad lentem secundam $= \frac{1}{12}a$.

II. Pro secunda lente ex vitro coronario paranda, cuius distantia focalis est $q = 0,8333a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} \;\; = -0,6906a\\ \text{posterioris} = +0,2694a,\end{cases}$$

eiusque distantia ad tertiam lentem $= \frac{1}{12}a$.

III. Pro tertia lente itidem ex vitro coronario paranda, cuius distantia focalis $r = 0,9075a$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} \;\; = 3,7656a\\ \text{posterioris} = 0,5514a\end{cases}$$

eiusque distantia ad tertium lentum $= \frac{11mas}{128} - \frac{1}{2}s$.

IV. Perinde est, ex quonam vitri genere lens quarta conficiatur, eiusque distantia focalis arbitrio nostro permittitur, quae sit $= s$, dummodo haec lens sit utrinque aeque convexa, ut aperturam admittat, cuius semidiameter $= \frac{1}{4}s$; eius vero a lente quinta distantia statuatur $= \frac{2}{3}s$.

V. Lens denique quinta seu ocularis habeat distantiam focalem $= \frac{2}{3}s$

et semidiametrum aperturae $=\frac{1}{12}s$, siquidem est utrinque aequaliter convexa; tum vero distantia oculi erit $=\frac{1}{5}s$.

VI. Spatii in obiecto conspicui semidiameter erit ut hactenus $=\frac{4a}{ma+8}$.
Mensura vero claritatis erit $=\frac{10,480}{m}$.

EXEMPLUM 2

201. Statuatur hic $\mathfrak{A}=-1$ et $\mathfrak{B}=2$ hincque $A=-2$ et $B=2$ sumaturque $\zeta=\frac{1}{6}$ fietque

$$\frac{1}{P}=\frac{4}{3},\quad \frac{1}{PQ}=\frac{3}{2}.$$

Quare fient distantiae focales

$$p=-a,\quad q=\frac{4}{3}a,\quad r=\frac{3}{2}\mathfrak{C}a,\quad s=2C\cdot\frac{h}{m}$$

ideoque vicissim

$$C=\frac{ms}{2h}\quad \text{et}\quad t=\frac{1}{3}s,$$

intervalla vero

primum et secundum $=\frac{1}{6}a$, tertium $=\frac{3mas}{32}-\frac{1}{2}s$, quartum $=\frac{2}{3}s$.

Iam ut confusio prior ad nihilum redigatur, satisfieri oportet huic aequationi:

$$\lambda=2\nu+\frac{\mu'}{\mu}\cdot\frac{4}{3}(\lambda'-2\nu')+\frac{\mu'}{\mu}\cdot\frac{3}{2}\left(1,03\lambda''+\frac{\nu'}{100}\right).$$

Statuatur iterum $\lambda'=\lambda''=1$ et uti in praecedente exemplo calculo facto reperietur

$$\lambda=0,5058+\frac{\mu'}{\mu}\cdot 2,2960$$

seu $\lambda=3,1047$; hinc ergo erit

$$\tau\sqrt{(\lambda-1)}=1,2731.$$

Ex quo erit

Pro prima lente

$$\text{radius faciei}\begin{cases}\text{anterioris}=\dfrac{p}{\sigma-\mathfrak{A}(\sigma-\rho)\mp\tau\sqrt{(\lambda-1)}}=+\dfrac{p}{1,7509}=-\,0,5711a\\[2ex]\text{posterioris}=\dfrac{p}{\rho+\mathfrak{A}(\sigma-\rho)\pm\tau\sqrt{(\lambda-1)}}=-\dfrac{p}{0,0268}=+37,3134a.\end{cases}$$

Pro secunda lente

erit uti in praecedente exemplo

$$\text{radius faciei}\begin{cases}\text{anterioris} = -\frac{q}{1,2067}\\ \text{posterioris} = +\frac{q}{3,0935};\end{cases}$$

quare, cum hic sit $q = 1,3333a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = -1,1049a\\ \text{posterioris} = +0,4310a.\end{cases}$$

Simili modo quoque pro tertia lente erit ut ante

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{r}{0,2410}\\ \text{posterioris} = \frac{r}{1,6458}.\end{cases}$$

Cum igitur hic $r = \frac{3}{2}\mathfrak{C}a = 1,4850a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = 6,1618a\\ \text{posterioris} = 0,9023a.\end{cases}$$

Pro communi ergo harum lentium apertura sumi poterit

$$x = 0,1077a,$$

unde fit

$$y = \frac{0,8616}{m}$$

et mensura claritatis $= \frac{17,232}{m}$.

Ex quibus oritur sequens

CONSTRUCTIO MICROSCOPII EX QUINQUE LENTIBUS COMPOSITI

202. Hic igitur dantur distantia obiecti $= a$, secundo multiplicatio $= m$ et tertio distantia focalis quartae lentis $= s$ eritque:

I. Pro prima lente ex vitro crystallino paranda, cuius distantia focalis est $p = -a$,

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -\ 0{,}5711a\\ \text{posterioris} &= +37{,}3134a.\end{cases}$$

eius aperturae semidiameter $=0{,}1077a$,
distantia ad lentem secundam $=\frac{1}{6}a$.

II. Pro secunda lente ex vitro coronario paranda, cuius distantia focalis $q=1{,}3333a$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} &= -1{,}1049a\\ \text{posterioris} &= +\ 0{,}4310a.\end{cases}$$

eiusque ad lentem tertiam distantia $=\frac{1}{6}a.$

III. Pro tertia lente itidem coronaria, cuius distantia focalis $r=1{,}4850a$ capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} &= 6{,}1618a\\ \text{posterioris} &= 0{,}9023a,\end{cases}$$

et distantia ad lentem quartam $=\frac{3mas}{32}-\frac{1}{2}s.$

IV. Perinde est, ex quonam vitri genere lens quarta paretur, eiusque distantia focalis in nostro arbitrio relinquitur, quae sit $=s$, modo sit utrinque aeque convexa; unde aperturam admittet, cuius semidiameter $=\frac{1}{4}s$; eius vero a lente quinta intervallum $=\frac{2}{3}s$.

V. Lens denique quinta habeat distantiam focalem $=\frac{1}{3}s$ et aperturam, cuius semi diameter $=\frac{1}{12}s$, siquidem est utrinque aeque convexa, et distantia oculi $O=\frac{1}{6}s.$

VI. Spatii in obiecto conspicui semidiameter $=\frac{4a}{ma+8}$ et mensura claritatis $=\frac{17{,}232}{m}$.

COROLLARIUM

203. Hoc microscopium ob duplicem causam priori anteferendum videtur:
1. quod distantiae focales trium priorum lentium hic sint maiores quam ante respectu distantiae obiecti *a;* unde hoc commodum nascitur, quod, etiamsi distantia obiecti *a* hic duplo minor capiatur quam ante, tamen istae lentes non evadant nimis exiguae; unde longitudo instrumenti fere ad semissem reduci potest; deinde etiam 2. hic mensura claritatis fere duplo maior est quam casu praecedente.

PROBLEMA 3

204. *Si loco lentis obiectivae quatuor lentes sibi proximae substituantur, quarum binae priores ex vitro crystallino, posteriores vero ex coronario sint factae, manentibus binis ultimis lentibus ut hactenus microscopium ita adornare, ut utraque confusio penitus tollatur.*

SOLUTIO

Cum hic occurrant quinque intervalla, quarum tria prima sint minima, litterae *P, Q, R* parum ab unitate recedent, littera *T* vero erit $=-1$, ita ut sit $PQRS=\frac{ma}{h}$. Litterarum vero *A, B, C, D, E* haec ultima *E* erit $=-\frac{2}{3}$ ob $\mathfrak{E}=-2$, ut scilicet campus fiat ut hactenus $=\frac{4a}{ma+8}$. Iam spectetur distantia focalis quintae lentis

$$t=ABCD\mathfrak{E}\cdot\frac{h}{m}=-2.ABCD\cdot\frac{h}{m};$$

quae ne nimis fiat exigua, posito $ABCD=-\theta$, ut sit $t=2\theta\cdot\frac{h}{m}$, numerus θ debet esse praemagnus. Nunc autem solutionem ita instruamus, ut litterae *A, B, C, D* ex calculo elidantur, huncque in finem statuamus brevitatis gratia

$$\frac{1}{P}=\alpha,\quad \frac{1}{PQ}=\beta,\quad \frac{1}{PQR}=\gamma;$$

quae ergo litterae α, β, γ ab unitate non multum discrepabunt, ubi probe notetur has litteras cum iis, quae supra sunt usurpatae, confundi non debere.
Cum iam distantiae focales quatuor priorum lentium sint

$$p=\mathfrak{A}a,\quad q=-\alpha A\mathfrak{B}a,\quad r=\beta AB\mathfrak{C}a,\quad s=-\gamma ABC\mathfrak{D}a,$$

unde colligitur

$$\frac{a}{p}=\frac{1}{\mathfrak{A}}=1+\frac{1}{A},\quad \frac{\alpha a}{q}=-\frac{1}{A\mathfrak{B}}=-\frac{1}{A}-\frac{1}{AB},$$
$$\frac{\beta a}{r}=\frac{1}{AB\mathfrak{C}}=\frac{1}{AB}+\frac{1}{ABC},\quad \frac{\gamma a}{s}=-\frac{1}{ABC\mathfrak{D}}=-\frac{1}{ABC}-\frac{1}{ABCD},$$

manifestum ergo est fore

$$\frac{1}{p}\cdot a+\frac{\alpha}{q}\cdot a+\frac{\beta}{r}\cdot a+\frac{\gamma}{s}\cdot a=1-\frac{1}{ABCD}=1+\frac{1}{\theta}.$$

Cum ergo θ sit numerus praemagnus, proxime esse oportet

$$\frac{1}{p}+\frac{\alpha}{q}+\frac{\beta}{r}+\frac{\gamma}{s}=\frac{1}{a},$$

quae est prima aequatio probe notanda. Secundam aequationem nobis suppeditabit destructio posterioris confusionis [§ 27], quae, si brevitatis gratia loco fractionis $\frac{10}{7}$ seu quaecunque alia experientiae fuerit consentanea scribatur ζ hoc modo exprimetur:

$$0=\frac{\zeta}{p}+\frac{\zeta\alpha^2}{q}+\frac{\beta^2}{r}+\frac{\gamma^2}{s}.$$

Tertia vero aequatio ex destructione confusionis prioris [§ 31] est petenda; ubi cum expediat, ut litterae λ, λ', λ'', λ''' non multum unitatem superent earumque valores ob litteras v, v' etc. parum adficiantur simulque, ut vidimus, litterae μ et μ' : parum discrepant, neglectis terminis a v pendentibus statuamus $\lambda=\lambda'=\lambda''=\lambda'''=1$ ac tertia nostra aequatio sequentem induet formam:

$$\frac{1}{p^3}+\frac{\alpha^4}{q^3}+\frac{\beta^4}{r^3}+\frac{\gamma^4}{s^3}=0;$$

atque nunc totum negotium eo est reductum, ut his tribus aequationibus satisfiat, ubi quidem est notandum primae aequationi satis adcurate satisfieri debere, pro duabus posterioribus autem sufficere, si iis propemodum fuerit satisfactum; quae resolutio quo facilius instituatur, ponamus porro

$$\frac{1}{p}=\frac{z}{a},\quad \frac{\alpha}{q}=\frac{y}{a},\quad \frac{\beta}{r}=\frac{x}{a}\quad \text{et}\quad \frac{\gamma}{s}=\frac{v}{a},$$

ut tres nostrae aequationes prodeant

$$\text{I.}\quad z+y+x+v=1,$$
$$\text{II.}\quad \zeta z+\zeta\alpha y+\beta x+\gamma v=0,$$
$$\text{III.}\quad z^3+\alpha y^3+\beta x^3+\gamma v^3=0,$$

in quibus duabus posterioribus litterae α, β et γ sine notabili errore pro unitate haberi poterunt. Statuamus nunc, quo resolutio planior reddatur,

$$z=f+g,\quad y=f-g,\quad x=h+k,\quad v=h-k,$$

et tres nostrae aequationes abibunt in has:

$$\text{I.}\quad f+h=\tfrac{1}{2},$$
$$\text{II.}\quad \zeta f+h=0,$$
$$\text{III.}\quad f\left(f^2+3g^2\right)+h\left(h^2+3k^2\right)=0.$$

Ex duabus prioribus colligimus

$$f=\frac{1}{2(1-\zeta)},\quad h=\frac{\zeta}{2(\zeta-1)},$$

et quia proxime $\zeta=\frac{3}{2}$, iam habemus hos duos valores:

$$f=-1 \text{ et } h=\frac{3}{2};$$

qui in tertia substituti dabunt

$$-1-3g^2+\frac{27}{8}+\frac{9}{2}k^2=0;$$

unde concluditur

$$g=\sqrt{\frac{3}{2}k^2+\frac{19}{24}}$$

ubi nihil impedit, quominus k statuatur $=0$; interim tamen, quia ob litteras β et γ posterior pars $h\left(h^2+3k^2\right)$ aliquantum augetur eaque etiam tam ob terminos littera v adfectos aliquod incrementum capit quam ideo, quod haec pars insuper per $\frac{\mu'}{\mu}$ multiplicari debet, quae fractio unitate est maior, manifestum est sumi debere $g>\sqrt{\frac{19}{24}}$. Convenientissime ergo sumetur $g=1$; tum vero erit

$$z=0,\quad y=-2,\quad x=v=h$$

hincque

$$p=\infty,\quad q=-\frac{\alpha a}{2},\quad r=\frac{2}{3}\beta a \text{ et } s=\frac{2}{3}\gamma a.$$

Cum igitur hic primae lentis distantia focalis fiat infinita, idem est ac si haec prima lens penitus tolleretur locoque obiectivae tantum tres lentes substituerentur, quarum sola prima ex vitro crystallino sit paranda; et quia hic fit $\alpha=1$ et $\mathfrak{A}=-\frac{1}{2}$, idem plane hic habetur casus, quem iam supra in problemate 2 evolvimus, ita ut superfluum foret hoc problema ulterius prosequi.

SCHOLION

205. Hoc igitur problema ideo potissimum est notatu dignum, quod hic singulari prorsus methodo sumus usi eius solutionem investigandi, quae in allis occasionibus insignem usum afferre posse videtur, ex quo etiam perspicuum est ne opus quidem esse quicquam insuper ad hoc caput adiicere.