CHAPTER II

CONCERNING THE FURTHER PERFECTION OF THESE MICROSCOPES,

AS A GREATER DEGREE OF CLARITY MAY ARISE

FROM THESE BY SUBSTITUTING TWO OR MORE

CONVEX LENSES IN PLACE OF THE SINGLE OBJECTIVE LENS

PROBLEM 1

157. *To substitute in place of the objective lens two lenses of this kind joined together close to each other, so that a greater degree of clarity may be obtained from the two remaining lenses prescribed with the given constitution in the above chapter.*

SOLUTION

Since here four lenses shall be required to be considered, of which the first two shall be separated by the minimum interval, the third truly may fall before the real image, the letter P will differ minimally from unity, truly Q at this point will be positive, the third R truly negative ; for this reason we may put $R=-k$, from which the focal lengths of these lenses will be

$$p=\mathfrak{A}a,\quad q=-\tfrac{A\mathfrak{B}a}{P},\quad r=\tfrac{AB\mathfrak{C}a}{PQ}\quad\text{and}\quad s=\tfrac{ABC\mathfrak{C}a}{PQk}.$$

Truly since there shall be

$$PQk=\tfrac{ma}{h},$$

there will become

$$s=\tfrac{ABC}{m}\cdot\tfrac{h}{m}.$$

Moreover the intervals between the lenses will be

$$\text{I and II}=Aa\left(1-\tfrac{1}{P}\right),$$
$$\text{II and III}=-\tfrac{AB}{P}\cdot a\left(1-\tfrac{1}{Q}\right),$$
$$\text{III and IV}=\tfrac{ABC}{PQ}\cdot a\left(1+\tfrac{1}{k}\right).$$

But since all the lenses shall be convex, there will become

$$1.\ \mathfrak{A}>0,\quad 2.\ -A\mathfrak{B}>0,\quad 3.\ \mathrm{AB}\mathfrak{C}>0\quad\text{and}\quad 4.\ ABC>0$$

and thus also

$$\tfrac{C}{\mathfrak{C}}>0\ \text{ or } 1+C>0.$$

But the ratio of the intervals is required to be held, because the first must be a minimum, the letter P to differ little from unity, thus so that, if this interval may be put $=\eta a$, there shall be $P=\frac{A}{A-\eta}$, with η being a small enough fraction. Then there must be

$$-AB(Q-1)>0 \text{ and } ABC>0.$$

Now we will consider the area viewed in the object, the radius if which is

$$z=\frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}}{ma+h}\cdot ah\xi,$$

where $\mathfrak{q}$ will be quite small, so that it may be able to be ignored ; then indeed both $\mathfrak{r}$ as will as $\mathfrak{s}$ will be able to be taken equal to unity, if indeed the two latter lenses may be composed of equally convex sides. For in this manner the maximum field of view will be obtained, as has been shown in the preceding chapter. Therefore we may put

$$M=\frac{2h}{ma+\mathrm{h}}$$

and there will become

$$z=Ma\xi;$$

from which there will be had for the position of the eye

$$O=\frac{s}{Ma}\cdot\frac{h}{m},$$

which is positive, as we have assumed now. From which so that for the colored margin requiring to be removes this equation is found :

$$0=\frac{\mathfrak{q}}{P}+\frac{\mathfrak{r}}{PQ}+\frac{\mathfrak{s}}{PQR};$$

but since there shall be

$$\mathfrak{q}=0 \text{ both } \mathfrak{r}=\mathfrak{s}=1 \text{ and } R=-k$$

there will be had

$$0=1-\tfrac{1}{k} \text{ or } k=1$$

as before, thus so that now there shall be $PQ=\frac{ma}{h}$, and since $P=1$ approx., there will become $Q=\frac{ma}{h}$ approx. , since for the greater magnifications Q will become very large magnum. With these observed the fundamental equations will be:

$$1.\ -\mathfrak{B}\mathfrak{q}=(P-1)M,$$
$$2.\ -\mathfrak{C}\mathfrak{r}=(PQ-1)\mathrm{M}-\mathfrak{q},$$

of which the former no further comes into the computation, since both $\mathfrak{q}$ as well as $P-1$ are very small; truly the other equation gives :

$$\mathfrak{C} = -\left(\frac{ma}{h} - 1\right)M = -2\left(\frac{ma-h}{ma+h}\right),$$

from which there is concluded for the greater magnifications :

$$\mathfrak{C} = -2 \text{ and } C = -\tfrac{2}{3},$$

with which values it will be allowed to use without risk; even if indeed either the field of view may be diminished a little or also the colored margin may not be removed completely, that at no time must be disturbed. Whereby, since so far we have found

$$k = 1, \quad PQ = \frac{ma}{h} \quad \text{and} \quad \mathfrak{C} = -\tfrac{2}{3},$$

produce the focal lengths

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{B}a}{P}, \quad r = -2AB\cdot\frac{h}{m} \quad \text{and} \quad s = -\tfrac{2}{3}\cdot AB\cdot\frac{h}{m},$$

thus so that there shall be $s = \frac{1}{3}r$, and now it is apparent that AB must be negative. Moreover, the intervals are expressed thus :

$$\text{The first} = Aa\left(1-\tfrac{1}{P}\right) = na, \text{ with there being } P = \frac{A}{A-\eta},$$

$$\text{and the second} = -\frac{AB}{P}\cdot a\left(1-\tfrac{1}{Q}\right),$$

so that on account of $AB < 0$ the third $= -\frac{4}{3}AB\cdot\frac{h}{m} = 2s$ consequently shall be positive, and the distance of the eye

$$O = \frac{s}{Ma}\cdot\frac{h}{m} = s\left(\frac{ma+h}{2ma}\right) = \tfrac{1}{2}s \text{ approx.}$$

Finally the equation remains to be considered for determining the aperture z, which is

$$\frac{1}{k^3} = \frac{\mu m x^3}{a^2 h}\left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}} - \frac{1}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}}\right) - \frac{h}{A^3B^3ma}\left(\frac{\lambda''}{8} - \frac{3\nu}{4}\right) - \frac{27h\lambda''}{8A^3B^3ma}\right).$$

For the sake of brevity there may be put

$$\mathit{\Lambda} = [A] = \frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}} - \frac{1}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}}\right) - \frac{h}{A^3B^3ma}\left(\frac{\lambda''}{8} - \frac{3\nu}{4}\right) - \frac{27h\lambda''}{8A^3B^3ma}$$

so that there shall become

$$x = \frac{1}{k}\sqrt[3]{\frac{a^2h}{\mu mA}} = \frac{a}{k}\sqrt[3]{\frac{h}{\mu maA}};$$

which expression, in order that it may be produced greater than in the preceding case, is required to insure that the value $\mathit{\Lambda}$, however large it can become, may be expressed less than one, for which the first letters may be taken $\lambda=\lambda'=1$; but for the two latter lenses, since each must be equally convex on both sides, the letters λ'' and λ''' thus now must be defined, so that there shall be

$$\lambda''=1+25\left(\tfrac{\sigma-\rho}{2\tau}\right)^2 \quad\text{and}\quad \lambda'''=1+\left(\tfrac{\sigma-\rho}{2\tau}\right)^2;$$

therefore only the letters A and B remain requiring to be defined, since $P=1$ just about. But concerning the letters A and B now there is to be prescribed 1. $\mathfrak{A}>0$ and 2. $AB<0$ and equally $A\mathfrak{B}<0$, thus so that there shall be $\frac{B}{\mathfrak{B}}>0$ or $1+B>0$. On account of which all these terms for Λ will be positive, thus so that its value may not be reduced to zero, but it shall be required to be reduced to a minimum only. Indeed for the first term that thus may be rendered smaller, so that $\mathfrak{A}$ may be taken greater ; but since then A shall become negative, the letter B will become positive and thus $\mathfrak{B}<1$, from which the following term alone becomes greater than unity. In a similar manner, if a large number $\mathfrak{B}$ may be put in place, B shall become negative and A will have to be taken positive ; from which $\mathfrak{A}$ will become less by one, thus so that now the first term only shall be going to be greater than one. Then truly also in the first place it is required to be cautioned, lest that negative product AB may become exceedingly small, since otherwise the focal lengths r and s will become as if vanishing, from which it is necessary, that the formula $-AB$ may not be expressed below a certain value. Therefore we may put $AB=-\theta$, thus in order that θ may denote the limit requiring to be observed for this product ; which since that quantity may be able to be considered as a constant quantity, while the letters A and B will be had as variables, there will be

$$\frac{dB}{B}=-\frac{dA}{A}.$$

Therefore from these observations the expression for the letter Λ being defined will be:

$$\Lambda=\frac{1}{\mathfrak{A}^3}+\frac{v}{A\mathfrak{A}}-\frac{1}{A^3\mathfrak{B}^3P}-\frac{v}{A^3B\mathfrak{B}P}+\frac{h}{\theta^3ma}\left(\frac{\lambda''}{8}-\frac{3v}{4}\right)+\frac{27h\lambda''}{8\theta^3ma},$$

in which the final terms are constants ; from which for the minimum value of this requiring to be found there will be a need only to differentiate the first terms, where indeed $P=1$. This in the end will be observed to be

$$\frac{1}{\mathfrak{A}}=1+\frac{1}{A} \quad\text{and}\quad \frac{1}{\mathfrak{B}}=1+\frac{1}{B}$$

hence

$$\frac{d\mathfrak{A}}{\mathfrak{A}^2}=\frac{dA}{A^2} \quad\text{and}\quad \frac{d\mathfrak{B}}{\mathfrak{B}^2}=\frac{dB}{B^2}=-\frac{dA}{AB},$$

from which differential equation there will be produced:

$$0=-\frac{3B}{\mathfrak{A}^2}+\frac{3B}{A^2\mathfrak{A}^3}-\frac{3}{A^2\mathfrak{B}^2}-v\left(\frac{B}{A}+\frac{B}{\mathfrak{A}}-\frac{2}{A^2\mathfrak{B}}+\frac{1}{A^2B}\right),$$

which divided by B gives :

$$0=3\left(\frac{1}{A^2\mathfrak{B}^3}-\frac{1}{\mathfrak{A}^2}-\frac{1}{A^2\mathfrak{B}^2B}\right)-v\left(\frac{1}{A}+\frac{1}{\mathfrak{A}}-\frac{2}{A^2\mathfrak{B}B}+\frac{1}{A^2B^2}\right),$$

and hence it will be elicited with the Germanic letters removed

$$0=3\left(\frac{1}{AB}-1\right)\left(1+\frac{2}{A}+\frac{1}{AB}\right)+v\left(\frac{1}{A^2B^2}+\frac{2}{A^2B}-\frac{2}{A}-1\right),$$

which therefore is reduced to these factors:

$$0=(3+v)\left(\frac{1}{AB}-1\right)\left(1+\frac{2}{A}+\frac{1}{AB}\right);$$

from which, since on account of $AB=-\theta$ the second factor shall not be able to vanish, the third factor produces

$$B=-\frac{1}{A+2} \quad \text{and} \quad \mathfrak{A}=-\frac{1}{A+1} \ ;$$

also if both letters will be defined by θ in the following manner:

$$B=\frac{\theta-1}{2} \quad \text{and} \quad \mathfrak{B}=\frac{\theta-1}{\theta+1},$$

then

$$A=-\frac{2\theta}{\theta-1} \quad \text{and} \quad \mathfrak{A}=+\frac{2\theta}{\theta+1},$$

We may conclude to become from these values

$$\Lambda=\frac{(\theta+1)^3}{8\theta^3}\left(1+\frac{1}{P}\right)-\frac{v(\theta^2-1)}{4\theta^2}\left(1-\frac{1}{\theta P}\right)+\frac{h}{\theta^3 ma}\left(\frac{\lambda''}{8}-\frac{3v}{4}\right)+\frac{27h\lambda''}{8\theta^3 ma},$$

where θ generally will be a very large number, so that also for the greater magnifications the focal length s may not become exceedingly small. Hence therefore the equation

$$\Lambda=\tfrac{1}{4}(1-v),$$

will be satisfied well enough and since there shall be almost $v=\frac{1}{5}$, there will be $\Lambda=\frac{1}{5}$, so that without risk it shall be possible to take $\mu\Lambda=\frac{1}{5}$; from which there will be obtained

$$x=\frac{a}{k}\sqrt[3]{\frac{5h}{ma}},$$

whereby that value which we had in the previous chapter, increases in the ratio $\sqrt{5}:1$ or approximately as 17: 10. Whereby here also greater clarity will be obtained increased in the same ratio.

COROLLARY 1

158. Therefore the focal lengths will be expressed in the following manner by the same number θ:

$$p = \mathfrak{A}a = \tfrac{2\theta}{\theta+1}\cdot a, \quad q = \tfrac{2\theta}{(\theta+1)P}\cdot a, \quad r = 2\theta\cdot\tfrac{h}{m} \quad \text{and} \quad s = \tfrac{2}{3}\theta\cdot\tfrac{h}{m},$$

thus so that there shall become $q = p$ approximately and $s = \frac{1}{3}r$ exactly; then truly the separations of the lenses will be

$$\text{in the first place } = -\tfrac{2\theta}{(\theta-1)}\left(1-\tfrac{1}{P}\right)a = \eta a$$

and thus

$$P = \tfrac{2\theta}{2\theta+\eta(\theta-1)} \quad \text{and thus} \quad P<1,$$

$$\text{in the second place } \theta\left(\tfrac{1}{P}-\tfrac{1}{PQ}\right)a = \tfrac{\theta a}{P}-\tfrac{\theta h}{m},$$

$$\text{in the third place} = \tfrac{4}{3}\theta\cdot\tfrac{h}{m} = 2s.$$

COROLLARY 2

159. Since the interval of the first two closest lenses itself is defined most conveniently from the focal lengths of these, we may put to be $\eta a = \zeta p$; hence there will be defined

$$P = \tfrac{(\theta+1)}{(\theta+1)+\zeta(\theta-1)};$$

whereby, since θ shall be a very large number, there will become $P = \frac{1}{1+\zeta}$; whereby if there may be taken $\zeta = \frac{1}{10}$, there will become $P = \frac{10}{11}$, which value will be seen to be most convenient in practice, since this interval may be allowed to change a little at this stage. So that it may be extended to the field of view, the radius of the area viewed in the object will remain the same evidently it is as was used in problem 2 of the preceding chapter and also the distance of the eye likewise is determined here.

SCHOLIUM 1

160. Behold therefore now the conspicuous perfection of these microscopes, which we had set out in the preceding chapter, since the clarity found here shall be notably greater than that found there and to be in the ratio 12:7, and since actually the clarity is perceived to be according to the ratio doubled, this can be considered to be three times greater. On account of which these microscopes with the greater magnification will be able to be extended much further than in the preceding chapter, before the lack of light may become too great. Hence if we wish, so that for the magnification $m = 1000$ the focal length of the eyepiece lens shall not become less than $\frac{1}{4}$ in., it will be required to assume $\theta = 47$, thus so that by taking $\theta = 50$ there shall be no need for concern, that we may have a need for an exceedingly small eyepiece lens. Therefore it will be seen to be worth the effort to have established this case in the following example.

EXAMPLE

161. Therefore we may set $\theta = 50$ and to accept $h = 8$ in. and, as in the manner we have observed , $P = \frac{10}{11}$ for the given object distance $= a$, we obtain the following focal lengths:

$$p = \tfrac{100}{51}a,\quad q = \tfrac{110}{51}a,\quad r = \tfrac{800}{m}\text{ in. et } s = \tfrac{800}{3m}\text{ in.}$$

and the separation of the lenses

$$\text{the first} = \tfrac{10}{49}a,\quad \text{second} = 55a - \tfrac{400}{m}\text{in. and the third} = \tfrac{1600}{3m}\text{ in.}$$

and the distance of the eye

$$= \tfrac{400}{3m}\left(1 + \tfrac{8}{ma}\right)\text{ in.}$$

Again since there is $\mathfrak{A} = \frac{100}{51}$ and $\mathfrak{B} = \frac{49}{51}$, on account of $\lambda = 1$ and $\lambda' = 1$ the construction of the two first lenses, certainly if they may be made from common glass, thus will be had:

I. For the first lens

the radius

$$\begin{cases}\text{of the anterior face} = \frac{p}{\sigma - \mathfrak{A}(\sigma-\rho)} = -\frac{p}{1{,}1896} = -0{,}84062p,\\ \text{of the posterior face} = \frac{p}{\rho + \mathfrak{A}(\sigma-\rho)} = \frac{p}{3{,}0077} = 0{,}33248p.\end{cases}$$

II. For the second lens

the radius

$$\begin{cases}\text{of the anterior face} = \frac{q}{\sigma - \mathfrak{B}(\sigma-\rho)} = \frac{q}{0,2470} = 4,0486q, \\ \text{of the posterior face} = \frac{q}{\rho + \mathfrak{B}(\sigma-\rho)} = \frac{q}{1,5711} = 0,6365q.\end{cases}$$

With these noted we may establish the value of the letter Λ, which will be $\Lambda = 0,221$, which multiplied by $\mu = 0,9381$ will give $\mu\Lambda = 0,2073$; which therefore scarcely differs from the above assumed value $\frac{1}{5}$; hence therefore for the aperture of the objective lens we deduce

$$x = \frac{a}{k}\sqrt[3]{\frac{40}{ma}} = \frac{0,17099}{\sqrt[3]{ma}},$$

from which value the measure of the clarity produced

$$= \frac{160}{ma} \cdot x = \frac{27,3584}{m\sqrt[3]{ma}};$$

finally for the field of view there will be

$$z = \frac{4a}{ma+8} \text{ in.},$$

and if at last at the position of the real image we may wish to put a diaphragm in place, the radius of this opening must become

$$= ABCz = \frac{133a}{ma+8} \text{ in.};$$

from which there is made the following

CONSTRUCTION OF MICROSCOPES OF THIS KIND COMPOSED FROM FOUR LENSES FOR SOME MAGNIFICATION

162. These individual lenses will be prepared from common glass, of which the refraction is $n = 1,55$, and the position of the object $= a$, as it will be allowed again to be assumed $= \frac{1}{2}$ in., there will be :

I. For the first lens, the focal length of which is $p = \frac{100}{51}a$, the radius

$$\begin{cases}\text{of the anterior face} = -1,6482a \\ \text{of the posterior face} = +0,6520a,\end{cases}$$

the radius of which aperture

$$x = \frac{0,17099}{\sqrt[3]{ma}} \cdot a$$

and the distance to the second lens

$$=\tfrac{10}{49}a=0,2040a.$$

II. For the second lens, the focal length of which $q=\tfrac{110}{51}a$, the radius

$$\begin{cases}\text{of the anterior face} = \ 8,7323a\\ \text{of the posterior face} = 1,3730a,\end{cases}$$

the radius of which aperture will be a little greater than of the preceding, and the distance to the third lens $=55a-\tfrac{400}{m}$ in.

III. For the third lens, the focal length of which is $r=\tfrac{800}{m}$ in., the

radius of each face will be $r=\tfrac{800}{m}$ in.,

the radius of the aperture $=\tfrac{200}{m}$ in.

and the distance to the fourth lens $=\tfrac{1600}{3m}$ in. $=\tfrac{533}{m}$ in.

IV. For the fourth lens, the focal length of which $r=\tfrac{800}{m}$ in.,

the radius of each face will be

$$r=\tfrac{293}{m}\text{ in.},$$

of which the radius of the aperture $=\tfrac{67}{m}$ in.

and the distance to the eye $=\tfrac{133}{m}$ in.

V. The radius of the area viewed will be

$$z=\tfrac{4a}{ma+8}\text{ in.},$$

the length of the instrument $=55,2040a+\tfrac{267}{m}$ in.,

and the measure of the clarity $=\frac{27,358}{m\sqrt[3]{ma}}$,

where it is required to note as above § 148, IV the two first lenses can be retained for any magnification, indeed the two latter lenses for any object distances, for which the same table will serve, which we have added there.

SCHOLIUM 2

163. The same formulas, which we have found here, also can be transferred to telescopes; where since there shall be $a=\infty$ and $h=a$, lest the lenses may increase indefinitely, there must be $\theta=0$, thus still, so that θa may remain a finite quantity ; clearly since there shall be $p=\frac{2\theta}{\theta+1}\cdot a$, there will be $\theta a=\frac{p}{2}$ and thus the remaining focal distances will be

$$q=\frac{p}{P},\quad r=\frac{p}{m}\ \text{ and }\ s=\frac{p}{3m},$$

thence the intervals of the lenses

$$\text{first } =\left(1-\frac{1}{P}\right)p,\quad \text{second } =\frac{p}{2P}-\frac{p}{2m},\quad \text{third } =\frac{2p}{3m}.$$

Now since it pertains to the letter *P*, the formula given above will have this position

$$P=\frac{\theta+1}{\theta+1+\zeta(\theta-1)},$$

which gives here

$$P=\frac{1}{1-\zeta};$$

but since here it is concerned with telescopes, it will be able to assume $\zeta=\frac{1}{25}$, thus so that there will become $P=\frac{25}{24}$; then truly the distance of the eye will become

$$O=\frac{s(m+1)}{2m}=\frac{1}{2}s\left(1+\frac{1}{m}\right),$$

thus so that the total length may become

$$=p\left(1-\frac{1}{2P}+\frac{1}{3m}+\frac{1}{6m^2}\right)$$

and again the radius of the apparent field

$$\frac{z}{a}=\Phi=\frac{1}{2}\cdot\frac{1}{m+1}=\frac{1718}{m+1}.$$

Now also we may consider the latter equation deduced from the radius of confusion, in which the term enclosed by the brackets will be multiplied by $8\theta^3$, truly the common factor may be divided by the same , and there will be obtained

$$\frac{1}{k^3}=\frac{\mu m x^3}{p^3}\left(1+\frac{1}{P}-\frac{2\nu}{P}+\frac{1}{m}(\lambda''-6\nu)-\frac{27\lambda''}{m}\right);$$

where, since for common glass there shall be $\nu=0,2326$, if there may be put $P=\frac{25}{24}$ and the terms divided by *m* shall be ignored, on account of $\mu=1$ there will become approx.

$$\frac{1}{k^3}=\frac{mx^3}{p^3}\cdot\frac{3}{2},$$

from which there is deduced

$$p=kx\sqrt[3]{\tfrac{3}{2}\cdot m},$$

from which, since the degree of the clarity y shall be accustomed to be given, so that there shall be $x=my$, then truly there may be assumed $ky=1$, indeed if there may be put $y=\frac{1}{50}$ in. and $k=50$, as has been done above, there will be had

$$p=m\sqrt[3]{\tfrac{3}{2}m},\ \text{ or }\ p=\tfrac{8}{7}m\sqrt[3]{m}.$$

But with P known there will be $q=\frac{25}{24}p.$ Moreover here we may assume $\lambda=1$ and $\lambda'=1$, and since there shall be $\mathfrak{A}=0$ and $\mathfrak{B}=-1$, the construction of these lenses for common glass, where $n=1,55$, will be :

I. For the first lens, the radius

$$\begin{cases}\text{of the anterior face}=\frac{p}{\sigma}=0,6145p\\ \text{of the posterior face}=\frac{p}{\rho}=5,2438p.\end{cases}$$

II. But for the second lens the radius

$$\begin{cases}\text{of the anterior face }=\frac{q}{\sigma+(\sigma-\rho)}=+\frac{q}{3,0641}=+0,3264q\\ \text{of the posterior face}=\frac{q}{\rho-(\sigma-\rho)}=-\frac{q}{1,2460}=-0,8026q.\end{cases}$$

hence there will be obtained the following

CONSTRUCTION OF ASTRONOMICAL TELESCOPES MADE FROM FOUR LENSES, FOR COMMON GLASS $n=1,55$

164. The individual moments for the customary construction thus may rendered in order, evidently the proposed magnification m may be defined thence

$$p=\tfrac{8}{7}m\sqrt[3]{m}.$$

I. For the first lens, the focal length of which $=p$, the radius

$$\begin{cases}\text{of the anterior face} = 0,6145p\\ \text{of the posterior face} = 5,2438p.\end{cases}$$

the radius of which aperture $=\frac{m}{50}$ in.
and the distance to the following lens will be $=\frac{1}{25}p = 0,04p$.

II. For the second lens, the focal length of which is $\frac{25}{24}p$, the radius

$$\begin{cases}\text{of the anterior face will be } = +0,3134p\\ \text{of the posterior face will be} = -0,7705p;\end{cases}$$

the aperture is not defined, provided it shall be greater than the preceding one, and the distance to the third lens $=\frac{12}{25}p-\frac{p}{2m}$.

III. For the third lens, the focal length of which is $r=\frac{p}{m}$,
the radius of each face $=1,1\cdot\frac{p}{m}$,
of which the radius of the aperture $=\frac{p}{4m}$,
and the distance to the fourth lens $=\frac{2p}{3m}$.

IV. For the fourth lens, the focal length of which is $=\frac{p}{3m}$,
the radius of each face $=1,1\cdot\frac{p}{m}$,
the radius of its aperture $=\frac{p}{12m}$
and the distance to the eye $O=\frac{p}{6m}\left(1+\frac{1}{m}\right)$.

V. Therefore the total length will be

$$= p\left(\frac{13}{25}+\frac{1}{3m}+\frac{1}{6m^2}\right)$$

and the radius of the apparent field $\Phi = \frac{1718}{m+1}$ min.

VI. If at the place of the real image, which lies in the middle between the two latter lenses, a diaphragm may be required to be put in place, the radius of its opening will be

$$= ABCz = \frac{p}{6(m+1)}.$$

PROBLEM 2

165. *To construct a microscope with the same four lenses retained, which may be adapted to all the magnifications required to be produced.*

SOLUTION

The focal lengths of these lenses *p, q, r* and *s,* which, it is considered to use from the preceding problem, thus must be prepared, so that in the first place $s = \frac{1}{3}r$, then truly $q = \frac{11}{10}p$; then also both the latter lenses are required to be considered to be equally convex on both sides, truly we will see soon from the previous shape. Therefore the above formulas will be required to be considered :

1. $\theta = \frac{mr}{2h}$, from which, since there shall be $p = \frac{2\theta}{\theta+1}\cdot a$, hence we will deduce

$$a = \frac{\theta+1}{2\theta}\cdot p = \frac{(mr+2h)}{2mr}\cdot p,$$

which therefore also depends on the magnification, thus so that for whatever magnification it will be required to vary the distance of the object.

2. The intervals of the lenses thus themselves will be had :

$$\text{The first interval } = \frac{1}{10}p,$$

$$\text{the second interval} = \frac{11mr+18h}{40h}\cdot p - \frac{1}{2}r \quad \text{on account of } P = \frac{10(mr+2h)}{11mr+18h}$$

or

$$\text{the second interval } = \frac{11mrp}{40h} + \frac{9}{20}p - \frac{1}{2}r,$$

$$\text{the third interval } = \frac{4}{3}\cdot\frac{1}{2}r = \frac{2}{3}r$$

and the distance of the eye

$$O = s\left(\frac{1}{2} + \frac{hr}{(mr+2h)p}\right)$$

or $O = \frac{1}{2}s$ approximately ; and thus for the varied magnification only the second interval will become changeable.
Again truly the radius of the area viewed will be

$$z = \frac{1}{2}\cdot\frac{(mr+2h)hp}{m(mr+2h)p+2mhr},$$

from which, if *m* shall be a very large number, there will become $z = \frac{h}{2m}$
Now so that we may define the shape of the two first lenses, for which above we have assumed $\lambda = 1$ and $\lambda' = 1$, it will be required to the letters carefully :

$$\mathfrak{A} = \frac{2mr}{mr+2h} \text{ and } \mathfrak{B} = \frac{mr-2h}{mr+2h}.$$

But since the shape of these cannot be changed for the various magnifications and according to the nature of the enquiry it suffices for the shape to have been defined approximately only, we consider m as a very large number and we may assume $\mathfrak{A} = 2$ and $\mathfrak{B} = 1$. Also we may put in place the above values to be used, where there was $\theta = 50$, which value surely will correspond to a large magnification ; indeed then it is readily understood the same shape may be agreed on to be satisfied exactly either by a greater or lesser magnification; whereby if we may use common glass, we will have the radius for the first lens

$$\begin{cases} \text{of the anterior face} &= -0,8406p \\ \text{of the posterior face} &= +0,3325p \end{cases}$$

and for the second lens, the radius

$$\begin{cases} \text{of the anterior face} &= 4,0486q \\ \text{of the posterior face} &= 0,6365q. \end{cases}$$

Finally we have found for the aperture of the first lens, its radius

$$x = \frac{0,171a}{\sqrt[3]{ma}}$$

and thence we have obtained the order of the clarity present

$$= \frac{27,358}{m\sqrt[3]{ma}}$$

with there being $a = \frac{mr+2h}{2mr} \cdot p = \frac{1}{2}p$ approximately .

EXAMPLE

166. 1. We may assume for the focal distances of these lenses

$$p = 1 \text{ in.}, \quad q = \tfrac{11}{12} \text{ in.}, \quad r = 1 \text{ in.}, \quad \text{and} \quad s = \tfrac{1}{3} \text{in.},$$

evidently which values will be seen to be especially useful in practice; and if these lenses may be prepared from common glass, the shape of these will be determined thus, so that there shall be :

2. I. For the first lens the radius

$$\begin{cases}\text{of the anterior face} & = -0,84 \text{ in.}\\ \text{of the posterior face} & = +0,33 \text{ in.}\end{cases}$$

II. For the second lens the radius

$$\begin{cases}\text{of the anterior face} & = 4,45 \text{ in.}\\ \text{of the posterior face} & = 0,70 \text{ in.}\end{cases}$$

III. For the third lens

$$\text{the radius of each face } = 1,1 \text{ in.}$$

IV. For the fourth lens

$$\text{the radius of each face } = \tfrac{11}{30} \text{ in.}$$

3. With which lenses prepared the first and second may be established according to an interval $= \frac{1}{10}$ in., the third and fourth truly according to the interval $= \frac{2}{3}$ in., thus however, according to the nature of the eye the fourth lens may be able to be changed a little; but both pairs of tubes of this kind may be inserted, which may be able to be led to a greater or lesser interval as it pleases, just as the magnification demands, if indeed the interval between the second and third lens must be $\left(\frac{11m}{320} - \frac{1}{20}\right)$ in.

4. Also in a similar manner the distance of the object will be a little variable and for any magnification there must be

$$a = \frac{m+16}{2m} \text{ in. } = \left(\frac{1}{2} + \frac{8}{m}\right) \text{ in.}$$

Then truly the position of the eye can be regarded as constant, thus so that its distance shall be

$$O = \frac{1}{6} \text{ dig.}$$

5. The aperture of the third and of the fourth lens is given as large as they are capable.

6. But the maximum aperture of the first lens will depend on the magnification, since its radius shall be

$$x = \frac{0,0855}{\sqrt[3]{\frac{1}{2}m}}$$

from which the measure of the clarity produced will become

$$x = \frac{27,36}{m\sqrt[3]{\frac{1}{2}m}}.$$

7. Apart from the matter concerning this microscope I am going to consider, for any particular magnification , if we may adjoin the following values of the moments of the

variables, which are : 1. the distance of the object $=a$, 2. the separation of the second and third lenses, which we may indicate by the letter *L*, 3. the radius of the aperture of the first lens *x*, and 4. the measure of the clarity $=20y$; which finally is added in the following table

m	*a*	*L*	*x*	*Clarity*
50	0,660	1,669	0,0292	0,1871
100	0,580	3,387	0,0232	0,0743
200	0,540	6,825	0,0184	0,0295
300	0,527	10,262	0,0160	0,0172
400	0,520	13,700	0,0146	0,0117
500	0,516	17,137	0,0136	0,0087
600	0,613	20,576	0,0128	0,0068
700	0,611	24,012	0,0121	0,0065
800	0,510	27,450	0,0116	0,0047
900	0,509	30,887	0,0111	0,0040
1000	0,508	34,325	0,0108	0,0034

PROBLEM 3

167. *To substitute in place of the objective lens three lenses of this kind joined close together, so that with the two remaining lenses established following the precepts given in the above chapter, a greater degree of clarity may be obtained.*

SOLUTION

Since here five lenses shall be required to be considered and a real image may lie in the fourth or final interval, the letters *P, Q, R* will be positive, truly the following *S* may be put $=-k$, thus so that there shall be $PQRk=\frac{ma}{h}$. Hence the focal lengths of the individual lenses will be expressed thus :

$$p=\mathfrak{A}a,\quad q=-\frac{A\mathfrak{B}a}{P},\quad r=\frac{AB\mathfrak{C}a}{PQ},$$

$$s=\frac{ABC\mathfrak{D}a}{PQR}\ \text{ and }\ t=-ABCD\cdot\frac{h}{m}.$$

Truly the intervals of the lenses will be had thus :

$$\text{I and II}=Aa\left(1-\frac{1}{P}\right),\ \text{II and III}=-\frac{AB}{P}\cdot\left(1-\frac{1}{Q}\right),$$

$$\text{III and IV}=\frac{ABC}{PQ}\cdot a\left(1-\frac{1}{R}\right),\ \text{IV and V}=\frac{ABCDa}{PQR}\cdot a\left(1+\frac{1}{k}\right);$$

of which since the two prior ones shall be very small, the letters *P* and *Q* shall differ minimally from unity; on account of which, in the expression of the field the letters

$\mathfrak{q}$ and $\mathfrak{r}$ will be required to be taken as zero, truly the latter $\mathfrak{s}$ and $\mathfrak{t}$ may be taken equal to unity, if indeed both the last lenses may become equally convex. Hence the radius of the area in the object viewed will become

$$z = \frac{2ah}{ma+h} \cdot \xi;$$

but truly with the letter

$$M = \frac{2h}{ma+h},$$

from which the distance of the eye shall become

$$O = \frac{\mathfrak{t}}{Ma} \cdot \frac{h}{m} = \frac{1}{2} t\left(1 + \frac{h}{ma}\right) \text{ or approx.} = \frac{1}{2} t.$$

Again the first and second of the fundamental equations can be omitted, as on account of the letters P and Q being nearly $= 1$ the letters $\mathfrak{q}$ and $\mathfrak{r}$ on their own account become minimal; truly the third lens will give

$$\mathfrak{D}\mathfrak{s} = (PQR - 1)M,$$
$$-\mathfrak{D} = \frac{2(ma - hk)}{k(ma+h)},$$

so that for the greater magnifications there becomes $\mathfrak{D} = -\frac{2}{k}$; but from the equation for the colored fringe, which in this case will be

$$\frac{1}{PQR} - \frac{1}{PQRk} = 0,$$

we may deduce $k = 1$ as before, thus so that there shall be $\mathfrak{D} = -2$ and hence $D = -\frac{2}{3}$; with which found the focal lengths will be

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{B}}{P} \cdot a, \quad r = \frac{AB\mathfrak{C}}{PQ} \cdot a,$$
$$s = 2ABC \cdot \frac{h}{m} \text{ and } t = \frac{2}{3} ABC \cdot \frac{h}{m} = \frac{1}{3} s;$$

from which there follows

$$\mathfrak{A} > 0, \quad A\mathfrak{B} < 0, \quad AB\mathfrak{C} > 0 \text{ and } ABC > 0$$

and the spacing of the lenses :

$$\text{first} = Aa\left(1 - \frac{1}{P}\right), \quad \text{second} = -\frac{AB}{P} \cdot a\left(1 - \frac{1}{Q}\right),$$
$$\text{third} = \frac{ABC}{PQ} \cdot a\left(1 - \frac{1}{R}\right), \text{ fourth} = \frac{4}{3} ABC \cdot \frac{h}{m} = 2t.$$

Finally for the aperture of the first lens or for the letter x requiring to be defined, this equation will be had :

$$\frac{1}{k^3}=\frac{\mu mx^3}{a^2h}\left\{\begin{array}{l}\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}-\frac{1}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu}{B\mathfrak{B}}\right)+\frac{1}{A^3B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu}{C\mathfrak{C}}\right)\\ \frac{1}{8A^3B^3C^3ma}\left(\lambda'''-6\nu\right)+\frac{27h\lambda''''}{8A^3B^3C^3ma}\end{array}\right\},$$

in which expression the numbers λ''' and λ'''' thence are given, since these lenses must be equally convex on both sides ; truly the former λ, λ' and λ'' have positive coefficients, since $[\mathfrak{A}>0,]\ A\mathfrak{B}<0\ [,\ AB\mathfrak{C}>0],$ since from these hypothesis all the lenses shall be convex. Whereby since the whole argument now may be reduced to this, so that the minimum value of this expression may be acquired, in the first place the minimum value may be attributed to these letters λ, λ', λ'' , which is unity; then truly the letters A, B, C must be defined thus, so that the expression may gain the minimum value; which in the end may be agreed to be observed before all else, lest the two final lenses may become extremely small for the greater magnifications, the quantity ABC always must surpass a certain limit ; whereby, since that may be positive, we may put

$$ABC=\theta\,,$$

thus so that θ may be able to be considered as if a given number; from which both the final terms themselves may be determined ; therefore only the three first terms may remain, from which it is required to investigate, how the minimum value may be able to be deduced, where certainly it will be allowed to write unity for the letters P and Q. But since now above we have expedited investigations of this kind more often, thence we may be able to conclude we may be going to depart minimally from the goal, if we may render these three formulas $\mathfrak{A}$, $-A\mathfrak{B}$ and $AB\mathfrak{C}$ equal to each other, thus so that the focal lengths p, q, r may depart only so far from the ratio of equality, as the letters P and Q may differ from unity. Moreover the equality of the first and second of these expressions give

$$\mathfrak{B}=-\tfrac{\mathfrak{A}}{A}=\mathfrak{A}-1 \text{ or } \mathfrak{B}=-\tfrac{1}{A+1};$$

from which there becomes

$$B=\tfrac{\mathfrak{A}-1}{2-\mathfrak{A}}=-\tfrac{1}{2+A}.$$

But the equality of the second and third gives

$$\mathfrak{C}=-\tfrac{\mathfrak{B}}{B}=\mathfrak{B}-1=-\tfrac{1}{B+1};$$

on account of which we will have

$$\mathfrak{C}=\mathfrak{A}-2=-\tfrac{A-2}{A+1}\quad\text{and hence}\quad C=-\tfrac{A+2}{2A+3};$$

but truly there must be $ABC=\theta$, from which all these letters are expressed by θ in the following manner :

$$A=\tfrac{3\theta}{1-2\theta},\quad B=-\tfrac{(1-2\theta)}{2-\theta}\quad \text{and}\quad C=-\tfrac{(2-\theta)}{3}$$

and hence again:

$$\mathfrak{A}=\tfrac{3\theta}{1+\theta},\quad \mathfrak{B}=-\tfrac{(1-2\theta)}{1+\theta}\quad \text{and}\quad \mathfrak{C}=-\tfrac{(2-\theta)}{1+\theta};$$

from which values used our final equation may adopt this form:

$$\frac{1}{k^3}=\frac{\mu m x^3}{a^2h}\left\{\begin{array}{l}\frac{(1+\theta)^3}{27\theta^3}+\frac{\nu(1-2\theta)(1+\theta)}{9\theta^2}+\frac{(1+\theta)^3}{27\theta^3 P}\left(1-\frac{\nu(1-2\theta)(2-\theta)}{(1+\theta)^2}\right)\\ +\frac{(1+\theta)^3}{27\theta^3 PQ}\left(1-\frac{3\nu(2-\theta)}{(1+\theta)^2}\right)+\frac{h}{8\theta^3 ma}\left(\lambda'''-6\nu\right)+\frac{27h\lambda''''}{8\theta^3 ma}\end{array}\right\},$$

Now for the sake of brevity we may put the expression enclosed in the brackets

$$\Lambda=\frac{(1+\theta)^3}{27\theta^3}\left(1+\frac{1}{P}+\frac{1}{PQ}\right)-\frac{\nu(1+\theta)}{27\theta^3}\left(3\theta(2\theta-1)+\frac{(2\theta-1)(\theta-2)}{P}+\frac{3(2-\theta)}{PQ}\right)+\frac{h}{8\theta^3 ma}\left(\lambda'''-6\nu\right)+\frac{27h\lambda''''}{8\theta^3 ma};$$

which formula, if θ were a very large number and the letters P and Q considered equal to unity, it will be represented by

$$\Lambda=\tfrac{3-8\nu}{27};$$

which value generally is much smaller, than if the objective lens may be either simple or also double; from which also x may be allocated a greater value, which will be

$$x=\frac{1}{k}\sqrt[3]{\frac{a^2h}{\mu m A}}$$

and the radius of the aperture of the objective lens, provided that were not greater than the figure of the lens allowed. Indeed with x found there will be $y=\frac{h}{ma}\cdot x$ and the measure of the clarity $=\frac{20h}{ma}\cdot x$.

COROLLARY 1

168. These formulae extend equally to telescopes and to microscopes only with this intervening distinction, because for telescopes, where $a=\infty$ and $h=a$, θ shall be infinitely small, but for microscopes θ may become a very great number.

COROLLARY 2

169. Therefore for microscopes there will become approximately

$$\mathfrak{A}=3,\ \ A=-\tfrac{3}{2},\ \ \mathfrak{B}=2,\ \ B=-2,\ \ \mathfrak{C}=1 \text{ and } C=\infty$$

or a very large number; then truly

$$\Lambda=\tfrac{1}{27}\left(1+\tfrac{1}{P}+\tfrac{1}{PQ}\right)-\tfrac{\nu}{27}\left(6+\tfrac{2}{P}\right).$$

COROLLARY 3

170. But if we wish to have the ratio of this very large number θ also, we will have still closer

$$\mathfrak{A}=3-\tfrac{3}{\theta},\quad A=\tfrac{3}{2}-\tfrac{4}{3\theta},$$
$$\mathfrak{B}=2-\tfrac{3}{\theta},\quad B=-2-\tfrac{3}{\theta},$$
$$\mathfrak{C}=1-\tfrac{3}{\theta},\quad \mathrm{C}=\tfrac{\theta}{3}-1;$$

then truly also with greater accuracy there will be

$$\Lambda=\tfrac{1}{27}\left(1+\tfrac{1}{P}+\tfrac{1}{PQ}\right)+\tfrac{1}{9\theta}\left(1+\tfrac{1}{P}+\tfrac{1}{PQ}\right)-\tfrac{\nu}{27}\left(6+\tfrac{2}{P}\right)-\tfrac{\nu}{9\theta}\left(1-\tfrac{1}{P}-\tfrac{1}{PQ}\right).$$

COROLLARY 4

171. Since now it pertains to the intervals of the first lenses, if we may assume each of these ought to be equal to $=\zeta p \ =\zeta\mathfrak{A}a$, with the former values being used approximate to the true values, we will find

$$P=\tfrac{1}{1+2\zeta} \text{ and } Q=\tfrac{1}{1+P\zeta}=\tfrac{1+2\zeta}{1+3\zeta},$$

from which, if we may put in place $\zeta=\tfrac{1}{10}$, there will become

$$P=\tfrac{5}{6} \text{ and } Q=\tfrac{12}{13} \text{ and hence } PQ=\tfrac{10}{13}$$

and

$$\Lambda=\tfrac{7}{54}-\tfrac{14\nu}{45}+\tfrac{7}{18\theta}+\tfrac{\nu}{6\theta}.$$

COROLLARY 5

172. But since the value v may depend on the ratio of the glass, there will be observed for crown glass, where we have $n=1{,}53$, to be approximately $v=\frac{1}{5}$ and for crystal glass, where $n=1{,}58$, to be $v=\frac{1}{4}$; hence therefore there is deduced to become for crown glass:

$$\Lambda=\frac{91}{1350}+\frac{19}{45\theta}.$$

But for crystal glass there will become:

$$\Lambda=\frac{7}{135}+\frac{31}{72\theta};$$

from which it is observed generally to be outstanding, if the lenses may be prepared from crystal glass.

SCHOLIUM 1

173. Since now it may be concerned with the construction of lenses , since for the three first lenses the numbers $\lambda,\ \lambda'$ and λ'' have been put equal to unity, clearly so that the individuals may produce the minimum amount of confusion, it will suffice to have attributed approximate values to the letters $\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C}$, so that without risk it may be allowed to take $\mathfrak{A}=3,\ \mathfrak{B}=2$ and $\mathfrak{C}=1;$ from which it will be able to construct these individual lenses for the given focal distances $p,\ q,\ r$, where it will help to have noted to be

$$q=\frac{p}{P}=\frac{6}{5}p \quad\text{and}\quad r=\frac{p}{PQ}=\frac{13}{10}p;$$

for now it will be allowed to consider the focal length p as known, and from that to define the distance of the object, which will be

$$a=\frac{1+\theta}{3\theta}\cdot p=\frac{1}{3}p\left(1+\frac{1}{\theta}\right);$$

then truly the letter θ is defined most conveniently from the fourth lens, of which the focal length is s, if likewise it may be considered as known, there will be $\theta=\frac{ms}{2h}$, thus so that now there may be had

$$a=\frac{1}{3}p\left(1+\frac{2h}{ms}\right).$$

Then truly there will be $t=\frac{1}{3}s$ and the final interval $=\frac{2}{3}s$, while the two first intervals are by hypothesis $=\frac{1}{10}p.$ Truly the third interval will depend especially on the magnification ; indeed that will become

$$=\theta a\left(\frac{13}{10}-\frac{h}{ma}\right)=\frac{13msa}{20h}-\frac{1}{2}s=\frac{13mps}{60h}+\frac{13}{30}p-\frac{1}{2}s,$$

from which it is apparent, where a greater magnification will be desired, there the instrument must be elongated; then truly also the aperture of the first lens will depend especially on the magnification; just as found from the formula above, since there shall be approximately

$$\mu=1,\quad a=\tfrac{1}{2}p \text{ and } \Lambda=\tfrac{7}{135} \text{ for crystal glass,}$$

if, as we have done above, we may assume $k=20$, $[h=8$ in.], we will obtain

$$x=\tfrac{1}{10}\sqrt[3]{\tfrac{15p^2}{7m}}\text{ in.}=\tfrac{1}{10}\sqrt[3]{\tfrac{135a^2}{7m}}\text{ in.;}$$

whereby, if as above we may put in place the distance of the object to be around half an inch, so that there shall be $p=\frac{3}{2}$ in., there will become

$$x=\tfrac{1}{10}\sqrt[3]{\tfrac{135}{28m}}\text{ in.}=\tfrac{0,1689}{\sqrt[3]{m}}\text{ in.}$$

Moreover again for the clarity there will become

$$y=\tfrac{8}{10m}\sqrt[3]{\tfrac{135}{7ma}}$$

and the measure of clarity

$$=\tfrac{16}{m}\sqrt[3]{\tfrac{135}{7ma}}$$

and for the case $a=\frac{1}{2}$ in. that will be

$$=\tfrac{54,060}{m\sqrt[3]{m}}.$$

Therefore from these we will be able to construct microscopes of this kind at once, which with the same lenses retained shall be adapted for all the magnifications required to be produced; moreover in order that we may use common glass at this stage, for which there is $n=1,55$, thus so that the value of x may be agreed to be diminished a little, as it may be desired to use for each; and then for the first lens, of which the focal length $=p$ and $\mathfrak{A}=3$, the radius

$$\begin{cases}\text{of the anterior face } =\frac{p}{\sigma-3(\sigma-\rho)}=-\frac{p}{2,6827}=-0,3728p\\ \text{of the posterior face}=\frac{p}{\rho+3(\sigma-\rho)}=+\frac{p}{4,5008}=+0,2222p,\end{cases}$$

which therefore admits an aperture, the radius of which will be approximately

$$x=0,055p=\tfrac{1}{18}p.$$

Moreover for the second lens, of which the focal length $q=\frac{6}{5}p$, the radius $(\text{on account of } \mathfrak{B}=2)$

$$\begin{cases}\text{of the anterior face} = \frac{q}{\sigma-2(\sigma-\rho)} = -\frac{q}{1{,}2460} = -0{,}8026p\\ \text{of the posterior face} = \frac{q}{\rho+2(\sigma-\rho)} = +\frac{q}{3{,}0641} = +0{,}3274q.\end{cases}$$

However, for the third lens, the focal length of which is $r=\frac{13}{10}p$, the radius

$$\begin{cases}\text{of the anterior face} = \frac{r}{\rho} = 5{,}2438r\\ \text{of the posterior face} = \frac{r}{\sigma} = 0{,}6145r.\end{cases}$$

But we have seen here it can be assumed conveniently $p=1\frac{1}{2}$ in., so that there may become roughly $a=\frac{1}{2}$ in., then truly $s=1$ in., so that there may become $t=\frac{1}{3}$ in.; from which the following will arise :

THE CONSTRUCTION OF MICROSCOPES MADE FROM FIVE LENSES, SUITABLE FOR ALL MAGNIFICATIONS

174. If all the lenses were prepared from common glass, for which $n=1{,}55$, will be had:

I. For the first lens, the focal length of which is $p=1\frac{1}{2}$ in., the radius

$$\begin{cases}\text{of the anterior face} = -0{,}5592 \text{ in.,}\\ \text{and of the posterior face} = +0{,}3333 \text{ in.;}\end{cases}$$

the radius of the aperture of which may become x = 0,0833 in., truly on account of the magnification given $=m$ it will agreed to take

$$x=\frac{0{,}15}{\sqrt[3]{m}} \text{ in.}$$

and the distance to the second lens $=0{,}15$ in.

II. For the second lens, the focal length of which is $q=\frac{18}{10}$ in., the radius may be taken

$$\begin{cases}\text{of the anterior lens} = -1,4447 \text{ in.,}\\ \text{of the posterior lens} = +0,5875 \text{ in.;}\end{cases}$$

only the aperture shall be greater than the preceding, the distance to the third lens $=0,15$ in.

III. For the third lens, the focal length of which is $r=\frac{39}{20}$ in., the radius may be taken

$$\begin{cases}\text{of the anterior face} = 10,2255 \text{ in.}\\ \text{of the posterior face} = 1,1983 \text{ in.;}\end{cases}$$

concerning the aperture the same is required to be maintained as before, and the distance to the fourth lens will become

$$=\left(\tfrac{13m}{320}-\tfrac{1}{2}\right) \text{ in.}$$

IV. For the fourth lens, the focal length of which is $s=1$ in., the radius of each face may be taken $=1,1$ in., the radius of the aperture $=\frac{1}{4}$ in., and the distance to the fifth lens $=\frac{2}{3}$ in. $=0,67$ in.

V. For the fifth lens, the focal length of which is $=\frac{1}{3}$ in., the radius of each face may be taken $=0,37$ in., the radius of that aperture $=\frac{1}{12}$ in., and the distance of the eye $=\frac{1}{6}$ in. $=0,17$ in.

VI. The radius of the area viewed in the object will be $z=\frac{1}{2}\frac{ah}{ma+h}$ with the distance of the object being

$$a=\tfrac{1}{3}p\left(1+\tfrac{2h}{ms}\right)=\tfrac{1}{2}\left(1+\tfrac{16}{m}\right) \text{ in.}$$

VII. Since the radius of the aperture of the objective lens shall be

$$x=\frac{0,15}{\sqrt[3]{m}} \text{ in.},$$

hence there becomes

$$y=\frac{hx}{ma}=\frac{2,40}{m\sqrt[3]{m}}$$

and the measure of the clarity

$$=\frac{48}{m\sqrt[3]{m}},$$

thus so that, if there were $m=512$, the measure of the clarity shall be going to become $=\frac{3}{256}=\frac{1}{85}$, which now is 34 times greater than the clarity of the full moon.

VIII. At this point we may attach the table below, in which m will be shown for particular magnifications

1. distance of the object from the objective lens $a=\frac{1}{2}\left(1+\frac{16}{m}\right)$ in.,
2. interval of the third and fourth lenses, which shall be $l=\left(\frac{13m}{320}-\frac{1}{2}\right)$ in.
3. radius of the aperture of the objective lens $x=\frac{0{,}15}{\sqrt[3]{m}}$ in. and 4. clarity $x=\frac{48}{m\sqrt[3]{m}}$ in.

m	a	l	x	*Clarity*
50	0,660	1,531	0,041	0,261
100	0,580	3,563	0,032	0,103
200	0,540	7,625	0,026	0,041
300	0,527	11,688	0,022	0,024
400	0,520	15,750	0,020	0,016
500	0,516	19,813	0,019	0,012
600	0,513	23,875	0,018	0,009
700	0,511	27,938	0,017	0,008
800	0,510	32,000	0,016	0,006
900	0,509	36,068	0,016	0,006
1000	0,508	40,125	0,015	0,005

SCHOLIUM 2

175. Since our formulae found may be applied equally to telescopes and microscopes, while in the former case there may be put $\theta=0$, truly in this case $\theta=$ a very large number, it will be seen to be worth the effort to investigate more carefully, which kind of instruments are going to be produced and for what use they may be going to be adapted, if the value of the letter θ may be attributed some small value such as 1 or 2 ; this in the end we may assume to be $\theta=2$, so that there shall be

$$s=\frac{4h}{m} \text{ and } t=\frac{4h}{3m} \text{ and hence } m=\frac{4h}{s};$$

then truly the following values will be had :

$$\mathfrak{A}=2,\ \ \mathfrak{B}=1,\ \ \mathfrak{C}=0$$

and thus

$$A=-2,\ \ B=\infty \text{ and } C=0,$$

yet thus, so that there shall be

$$BC=-1 \text{ and } \mathrm{B}\mathfrak{C}=-1;$$

from which values the focal lengths of the first lenses will be

$$p = 2a, \quad q = \tfrac{2a}{P}, \; r = \tfrac{2a}{PQ}$$

and the intervals

$$\text{first} = -2a(1-\tfrac{1}{P}), \quad \text{second} = \infty(1-\tfrac{1}{Q}),$$

$$\text{third} = \tfrac{2a}{PQ}\left(1-\tfrac{1}{R}\right) = 2a\left(\tfrac{1}{PQ}-\tfrac{h}{ma}\right) \text{ and fourth} = \tfrac{8h}{3m}$$

with the distance of the eye remaining $O = \frac{1}{2}t.$ So that now the first interval may become $= \frac{1}{10}p$, there will have to be taken $P = \frac{10}{11}$, but always Q must be $=1$, however great the second interval may be taken ; moreover it will be agreed at first to be taken equal, thus so that there shall be $q = \frac{11}{5}a$ and $r = \frac{11}{5}a$ and the third interval $= 2a\left(\frac{11}{10}-\frac{h}{ma}\right)$; thence as there was before

$$z = \tfrac{1}{2}\cdot\tfrac{ah}{ma+h};$$

Indeed now we will obtain

$$\Lambda = \tfrac{2}{5}-\tfrac{\nu}{4}+\tfrac{h}{64ma}\left(\lambda'''-6\nu\right)+\tfrac{27h\lambda''''}{64ma},$$

which value for the case $\nu = \frac{1}{5}$ shall become $\Lambda = \frac{7}{20}$, but for the case $\nu = \frac{1}{4}$ there shall become $\Lambda = \frac{27}{80}$ or in each case approximately $\Lambda = \frac{1}{3}$; hence therefore we deduce:

$$x = \tfrac{1}{k}\sqrt[3]{\tfrac{3a^2h}{\mu m}}$$

or

$$x = \tfrac{1}{20}\sqrt[3]{\tfrac{3a^2h}{m}}$$

and again finally

$$y = \tfrac{h}{20m}\sqrt[3]{\tfrac{3h}{ma}}$$

and the measure of clarity

$$y = \tfrac{h}{m}\sqrt[3]{\tfrac{3h}{ma}}.$$

With these items noted, which pertain to the construction of the instruments, the following will be observed :

1. If there may be taken $s = 1$ in., there will become $m = 4h$ in. or $\frac{h}{m} = \frac{1}{4}$ in., which value will be required to be explained thus, so that the instrument will represent the object to us greater in turn by m times, which if we may consider that to be seen at the distance h ; from which it will be apparent to us for the object to be represented at the same magnitude, as if we may see that with the naked eye at a distance $= \frac{h}{m}$, from which it is

evident the instrument, concerning how it performs, is going to represented the object for us with the same size, as if it were to be discerned by us with the naked eye at a distance $=\frac{1}{4}$ in. , evidently with all the greatest confusion removed, by which objects in the vicinity may affect us so much.

2. If we may assume the focal length s to be greater or less by a single inch, the magnification also will become greater or less; but in practice a smaller value may scarcely be allowed for s, since $t=\frac{1}{3}s$, truly no one will desire a greatly increased magnification ; from which this same value $s=1$ in. will be seen to accommodate our main objective.

3. Moreover an instrument of this kind will be able to be of outstanding use, when thus we choose to observe to observe the object, as if we may look at that at the distance $=\frac{1}{4}$ in., or what amounts to the same, 32 times greater, than if we might look at that at a distance of eight inches, and thus this instrument performs the same as a microscope magnifying 32 times.

4. But since with microscopes the object distance is accustomed to taken exceedingly small, then this instrument will be able to be used chiefly, when it will not be allowed to approach as close as it pleases ; on account of which, if the distance of the object a were to become a little greater, than is accustomed to be allowed with microscopes, we will see, how our instrument then shall be going to be compared; therefore besides we may put $a=1$ ft. $=12$ in. with s remaining $=1$ in., and the focal lengths will be determined in this manner:

$$p=24\text{ in.},\quad q=26,4\text{ in.},\quad r=26,4\text{ dig.},\quad s=1\text{ in. and }\ t=\tfrac{1}{3}\text{ in.}$$

Thence truly the following intervals become:

$$\begin{aligned}&\text{first}=2,4\text{ in.}, &&\text{second}=2,4\text{ in.},\\ &\text{third}=25,9\text{ in.}, &&\text{fourth}=0,67\text{ in.}\end{aligned}$$

Truly the radius of the aperture of the first lens now will be

$$x=\tfrac{1}{20}\sqrt[3]{\tfrac{3\cdot 144}{4}}\text{ in.}=\tfrac{1}{20}\sqrt[3]{108}\text{ in.}=0,238\text{ in.}$$

and hence the measure of the clarity $=0,099$ and thus the clarity itself will be 100 times smaller, than if the same object were viewed by the naked eye, which circumstance alone will exclude instruments of this kind from practical use, unless the length of those may satisfy the inconvenience ; also why not, if it may be allowed to approach the object closer, nothing stands in the way, why use an ordinary microscope may not be, especially if we may wish to be content with such a small magnification ; likewise indeed a simple microscope of focal length $=\frac{1}{4}$ in. might be better.

SCHOLIUM 3

176. Therefore now we may apply our formulas to telescopes also, where $a=\infty$ and there is taken $h=a$; therefore then it will be required to take $\theta=0$, thus however, so that θa may become a finite quantity; therefore since there shall become

$$p=\frac{3\theta}{1+\theta}\cdot a=3\theta a,$$

thus so that there shall be $\theta a=\frac{1}{3}p$, from which again there shall be

$$q=\frac{p}{P} \text{ and } r=\frac{p}{PQ}, \text{ but } s=\frac{2p}{3m} \text{ and } t=\frac{2p}{9m};$$

then truly the intervals of the lenses become:

$$\text{first}=p\left(1-\tfrac{1}{P}\right), \text{ second}=\tfrac{p}{2P}\left(1-\tfrac{1}{Q}\right),$$

$$\text{third }=\tfrac{p}{3}\left(\tfrac{1}{PQ}-\tfrac{1}{m}\right), \text{ fourth }=\tfrac{4p}{9m},$$

for the place of the eye remaining

$$O=\tfrac{1}{2}t\left(1+\tfrac{1}{m}\right).$$

Now we may make the two prior intervals equal to each other, and since the objective lenses now shall be much greater, we may put each equal to $\frac{1}{25}p$ and there will be found

$$P=\tfrac{25}{24} \text{ and } Q=\tfrac{12}{11} \text{ and hence } PQ=\tfrac{25}{22};$$

from which the above values will be

$$q=\tfrac{24}{25}p \text{ and } r=\tfrac{22}{25}p$$

and the interval of the third

$$=\tfrac{p}{3}\left(\tfrac{22}{25}-\tfrac{1}{m}\right)=\tfrac{22}{75}p-\tfrac{p}{3m}.$$

Again for the apparent field of view its radius becomes [expressed as an angle]

$$\Phi=\tfrac{z}{a}=\tfrac{2}{m+1}\cdot\xi=\tfrac{1}{4}\cdot\tfrac{2}{m+1}=\tfrac{1718}{m+1}\text{ min.}$$

Finally the equation for distinct vision will be

$$\frac{\mu m x^3}{p^3}\left(\tfrac{71}{25}-\tfrac{36}{5}\nu+\tfrac{27}{8m}\left(\lambda'''-6\nu\right)+\tfrac{729\lambda''''}{8m}\right)=\tfrac{1}{k^3}.$$

Now here it is usual to propose the order of the clarity, by which the objects shall be represented, which shall be $y=\frac{1}{50}$ in.., and there must be assumed $x=my=\frac{m}{50}$ in. and also there may be taken $k=50$ as in the above book ; with which put in place there will be found

$$p=m\sqrt[3]{\mu m\left(\tfrac{71}{25}-\tfrac{36}{5}\nu+\tfrac{27}{8m}\left(\lambda'''-6\nu\right)+\tfrac{729\lambda''''}{8m}\right)},$$

where, if we may use common glass, there will become $\mu=0,9381$ and $\nu=0,2326$; but now truly we have assumed $\lambda=\lambda'=\lambda''=1$, and since the two latter lenses must be equally convex on each side, there will be required to become

$$\lambda''''=1,6298 \text{ et } \lambda'''=1+0,6298\left(1-2\mathfrak{D}\right)^2=16,745,$$

from which values we may deduce

$$p=m\sqrt[3]{\left(1,0931m+188\right)} \text{ in.}$$

PROBLEM 4

177. *In place of the objective lens to substitute four convex lenses of this kind contiguous to each other, so that for the two remaining lenses put in place following the above precepts a greater degree of clarity may be obtained.*

SOLUTION

Since here six lenses may be had and thus five intervals, also just as many letters *P, Q, R, S, T* are going to be introduced into the calculation ; of which the first three *P, Q* and *R* shall be approximately equal to one, since the first four lenses are placed in close proximity to each other; truly the final letter *T* must be negative or $T=-k$, since a real image lies in the final interval, and thus there will be had

$$PQRSk=\tfrac{ma}{h},$$

thence the focal lengths of the lenses are now expressed thus :

$$p = \mathfrak{A}a,\ q = -\tfrac{A\mathfrak{B}a}{P},\ r = \tfrac{AB\mathfrak{C}a}{PQ},$$
$$s = -\tfrac{ABC\mathfrak{D}a}{PQR},\ t = \tfrac{ABCD\mathfrak{E}a}{PQRS}\ \text{ and }\ u = ABCDE\cdot\tfrac{h}{m}.$$

Truly the intervals of the lenses themselves will be had thus :

$$\text{First} = Aa(1-\tfrac{1}{P}),$$
$$\text{second} = -ABa\left(\tfrac{1}{P}-\tfrac{1}{PQ}\right),$$
$$\text{third} = ABCa\left(\tfrac{1}{PQ}-\tfrac{1}{PQR}\right),$$
$$\text{fourth} = -ABCDa\left(\tfrac{1}{PQR}-\tfrac{1}{PQRS}\right)$$
$$\text{fifth} = ABCDEa\left(\tfrac{1}{PQRS}-\tfrac{h}{ma}\right).$$

Truly the distance of the eye will be as before

$$O = \tfrac{1}{2}u\left(1+\tfrac{h}{ma}\right)$$

and likewise the radius of the area viewed

$$z = \tfrac{1}{2}\cdot\tfrac{ah}{ma+h}.$$

On this account truly the field itself, so that it may extend out so much, there will be required to be $\mathfrak{E} = -2$ and hence $E = -\tfrac{2}{3}$. Finally moreover, so that the colored margin may vanish, there will be required to be $k = 1$, thus so that $PQRS = \tfrac{ma}{h}$. Finally in order that the smallest confusion arising from the aperture of the lenses may be produced, from the above it will be permitted to deduce this to become, if these four expressions

$$\mathfrak{A},\ -A\mathfrak{B},\ AB\mathfrak{C},\ -ABC\mathfrak{D}$$

may be returned equal to each other; from which deduce these determinations :

$$1.\ \mathfrak{B} = -\tfrac{\mathfrak{A}}{A} = \mathfrak{A}-1,$$
$$2.\ \mathfrak{C} = -\tfrac{\mathfrak{B}}{B} = \mathfrak{B}-1 = \mathfrak{A}-2,$$
$$3.\ \mathfrak{D} = -\tfrac{\mathfrak{C}}{C} = \mathfrak{C}-1 = \mathfrak{A}-3.$$

Thence truly there may be put $-ABCD = \theta$, so that the focal length of the fifth lens may become

$$t = +2\theta \cdot \tfrac{h}{m}$$

and of the sixth

$$u = \tfrac{2}{3}\theta \cdot \tfrac{h}{m} = \tfrac{1}{3}t$$

and the fifth interval $= \tfrac{2}{3}t = 2u.$

Now in this equation on assuming $ABCD = -\theta$ the corresponding Germanic letters are introduced in place of the letters *A, B, C, D* and there will be

$$\tfrac{\mathfrak{A}}{1-\mathfrak{A}} \cdot \tfrac{\mathfrak{B}}{1-\mathfrak{B}} \cdot \tfrac{\mathfrak{C}}{1-\mathfrak{C}} \cdot \tfrac{\mathfrak{D}}{1-\mathfrak{D}} \cdot = -\theta$$

or

$$\theta = \tfrac{\mathfrak{A}}{1-\mathfrak{D}} = \tfrac{\mathfrak{A}}{4-\mathfrak{A}},$$

from which these letters will be defined in terms of θ in this manner :

$$\mathfrak{A} = \tfrac{4\theta}{\theta+1}, \quad \mathfrak{B} = \tfrac{3\theta-1}{\theta+1}, \quad \mathfrak{C} = \tfrac{2\theta-2}{\theta+1} \quad \text{and} \quad \mathfrak{D} = \tfrac{\theta-3}{\theta+1},$$
$$A = -\tfrac{4\theta}{3\theta-1}, \quad B = -\tfrac{3\theta-1}{2\theta-2}, \quad C = -\tfrac{2\theta-2}{\theta-3} \quad \text{and} \quad D = \tfrac{\theta-3}{4};$$

from which values in the first place the focal lengths may be defined thus:

$$p = \tfrac{4\theta a}{\theta+1}, \quad q = \tfrac{4\theta}{\theta+1} \cdot \tfrac{a}{P}, \quad r = \tfrac{4\theta}{\theta+1} \cdot \tfrac{a}{PQ},$$
$$s = \tfrac{4\theta}{\theta+1} \cdot \tfrac{a}{PQR}, \quad t = 2\theta \cdot \tfrac{h}{m} \quad \text{and} \quad u = \tfrac{2}{3}\theta \cdot \tfrac{h}{m}$$

and in a similar manner the intervals of the lenses :

$$\text{First} = -\tfrac{4\theta}{3\theta-1} \cdot a\left(1 - \tfrac{1}{P}\right),$$
$$\text{second} = -\tfrac{4\theta}{2\theta-2} \cdot a\left(\tfrac{1}{P} - \tfrac{1}{PQ}\right),$$
$$\text{third} = -\tfrac{4\theta}{\theta-3} \cdot a\left(\tfrac{1}{PQ} - \tfrac{1}{PQR}\right),$$
$$\text{fourth} = \theta a\left(\tfrac{1}{PQR} - \tfrac{h}{ma}\right),$$
$$\text{fifth} = \tfrac{2\theta}{3} \cdot a \cdot \tfrac{2h}{ma} = \tfrac{4}{3}\theta \cdot \tfrac{h}{m} = 2u.$$

So that now if we may wish, so that any of the first three intervals may become

$$= \zeta p = \tfrac{4\theta}{1+\theta} \cdot \zeta a,$$

the letters *P, Q, R* and *S* may be determined in the following manner:

$$\tfrac{1}{P}=1+\tfrac{(3\theta-1)}{1+\theta}\cdot\zeta,\quad \tfrac{1}{PQ}=1+\tfrac{(5\theta-3)}{1+\theta}\cdot\zeta,\quad \tfrac{1}{PQR}=1+\tfrac{(6\theta-6)}{1+\theta}\cdot\zeta.$$

With these in place the equation for the given order of distinction required can be expressed in the following form:

$$\frac{1}{k^3}=\frac{\mu mx^3}{a^2h}\left\{\begin{array}{l}\frac{1}{\mathfrak{A}^3}\left(\lambda+\frac{\lambda'}{P}+\frac{\lambda''}{PQ}+\frac{\lambda'''}{PQR}\right)\\ -\frac{\nu}{\mathfrak{A}^3}\left(\mathfrak{A}(\mathfrak{A}-1)+\frac{\mathfrak{B}(\mathfrak{B}-1)}{P}+\frac{\mathfrak{C}(\mathfrak{C}-1)}{PQ}+\frac{\mathfrak{D}(\mathfrak{D}-1)}{PQR}\right)\\ +\frac{h}{8\theta^3ma}\left(\lambda''''-6\nu\right)+\frac{27h\lambda'''''}{8\theta^3ma}\end{array}\right\},$$

for the sake of brevity we may put

$$\frac{1}{k^3}=\frac{\mu mx^3}{a^2h}\cdot\Lambda,$$

thus so that Λ will denote that quantity enclosed within the brackets, for which it is agreed to give the letters $\lambda,\ \lambda',\ \lambda''$ and λ''' the value $=1$, clearly so that these quantities may emerge minimally, and since both sides of the two last lenses must be equally convex, there will be

$$\lambda''''=1+\left(\tfrac{\sigma-\rho}{2\tau}\right)^2\left(1-2\mathfrak{C}\right)^2=1+25\left(\tfrac{\sigma-\rho}{2\tau}\right)^2$$

and

$$\lambda'''''=1+\left(\tfrac{\sigma-\rho}{2\tau}\right)^2.$$

Therefore the quantity Λ, if the values found may be substituted in place of $\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C}$ and $\mathfrak{D}$, thus may be expressed :

$$\Lambda=$$

$$\frac{(1+\theta)^3}{16\theta^3}-\frac{\nu(\theta+1)(5\theta^2-6\theta+5)}{16\theta^3}+\frac{(1+\theta)^2(7\theta-5)}{32\theta^3}\cdot\zeta$$
$$-\frac{\nu(\theta-1)(7\theta^2-18\theta+23)}{16\theta^3}\cdot\zeta+\frac{h}{8\theta^3ma}\left(\lambda''''-6\nu\right)+\frac{27h\lambda'''''}{8\theta^3ma}.$$

And in this matter it is intended mainly, that the value of Λ itself clearly may be reduced either to zero or at least to some very small amount, so that from this equation the number k may be produced much greater than 20, even if the aperture of the first lens may be taken as large as its figure permits; but then there will be had $y=\frac{hx}{ma}$ for this value assumed for x for the greatest clarity and the measure of the clarity will become

$$=\frac{20hx}{ma}=\frac{160x}{ma}.$$

COROLLARY 1

178. Since for microscopes θ always is a number great enough, unless perhaps a very small magnification may be required, which case here we exclude deservedly, the two final members of Λ evidently are so small, that they may be neglected without risk, and thus here the value will have to be estimated from the first three members only

$$\Lambda = \frac{(1+\theta)^3}{16\theta^3} - \frac{v(\theta+1)(5\theta^2-6\theta+5)}{16\theta^3} + \frac{(1+\theta)^2(7\theta-5)}{32\theta^3}\cdot\zeta - \frac{v(\theta-1)(7\theta^2-18\theta+23)}{16\theta^3}\cdot\zeta.$$

COROLLARY 2

179. But since θ will be a very large number, this expression is reduced to the following approximately correct form:

$$\Lambda = \frac{1}{16} - \frac{5}{16}v + \frac{7}{32}\zeta - \frac{7v}{16}\cdot\zeta + \frac{3}{16\theta} + \frac{v}{16\theta} + \frac{9}{32\theta}\cdot\zeta + \frac{25}{16\theta}\cdot\zeta;$$

which expression if it may be equal to zero, the true value of Λ without doubt will become so small, so that the maximum value of the letter x may be able to be given which the figure of the lens permits.

COROLLARY 3

180. Since the letter v depends on the nature of the glass, of which the value, as the refraction is increased from $n = 1,50$ as far as to 1,58, increases from $\frac{1}{5}$ as far as to $\frac{1}{4}$. On taking $v = \frac{1}{5}$ there will become

$$\Lambda = \frac{21}{160}\zeta + \frac{19}{32\theta}\cdot\zeta + \frac{1}{5\theta};$$

since all the terms shall be positive it is apparent, if the lenses were made from such a glass, the value Λ cannot be reduced to zero. But if there were $v = \frac{1}{4}$, there will be had

$$\Lambda = -\frac{1}{64} + \frac{13}{64\theta} + \frac{7}{64}\zeta + \frac{43}{64\theta}\cdot\zeta,$$

which value certainly will be able to be equal to zero, which clearly will happen in the case $\theta = \infty$, if there were $\zeta = \frac{1}{7}$, which value is adapted well enough in practice; but if we may take $\theta = 50$, then there will become $\Lambda = 0$, if there were $\zeta = 9\frac{37}{393}$ or $\zeta = \frac{1}{11}$, which also agrees very well in practice.

COROLLARY 4

181. Therefore in order that the value of Λ may be reduced to zero, it will be agreed to be used with glass producing the greatest refractive index, crown glass is of this kind, for which $n = 1,58$; and if perhaps it may be less successful in practice, this will come about in use, so that may be able to be satisfied by the intended aim with a little change of the lenses of the first interval; which remedy is easier to be used in practice there, so that in that construction of the lens no change is required.

SCHOLION 1

182. In order that θ always shall be a large enough number, is seen easily from the above treatment ; indeed since the penultimate focal length t shall be unable to be put less than one inch on account of $h = 8$ in., there will be $\theta = \frac{m}{16}$ in.; whereby, since the magnification m scarcely may be used to be considered less than 500 or 480, hence there will be had $\theta = 30$ in.; but the maximum magnification, which certainly on account of the deficiency of clarity we can desire at this point, it will be permitted to estimate $m = 960$, in which case therefore there will be $\theta = 60$ in., thus so that the values of θ contained between 30 and 60 shall be required to be estimated. But with this observed if the first member of the formula Λ were $= 0$, it will be easily understood the latter members in no way to be disturbed; for this final terms certainly now will be smaller than $\frac{125}{\theta^3 m}$; from which, if the first members may actually vanish, the equation will be produced :

$$\frac{1}{k^3} < \frac{\mu m x^3}{a^2 h} \cdot \frac{125}{\theta^3 m} < \frac{125 \mu x^3}{a^2 h \theta^3}$$

or if on assuming $\theta = 30$ there will become,

$$k^3 > \frac{30^3 a^2 h}{125 \mu x^3} \quad \text{or} \quad k > \frac{30}{x} \sqrt[3]{\frac{a^2 h}{125 \mu}}.$$

Now so that the value of x itself may be reached, we will note, if the objective lens were simple and thus its focal length $p = a$ approximately, then on account of its figure there can be taken $x = \frac{1}{6} a$ or certainly not more; and if here moreover four convex lenses may be substituted in place of the objective, of which the individual focal lengths are almost four times greater, yet, since the anterior face of the first is concave and thus the radius of the posterior face is very small, that scarcely allows a greater aperture than a simple lens, thus so that also in this case x shall be unable to be taken greater than $\frac{1}{6} a$; therefore there shall be $x = \frac{1}{6} a$ and on taking $a = \frac{1}{2}$ there shall always be $k > 90$, from which value a significant degree of distinctness may be indicated, since also for the best telescopes this value shall not be accustomed to be increased beyond 50 ; from which it will be able to conclude for that not to be necessary, as also the first terms of Λ may vanish completely, provided that multiplied by m they will not exceed the latter terms by much; but then the first terms must vanish almost completely; but with these put equal to zero the value of the number ζ thus in this kind will be determined, so that there shall be

$$\zeta=\frac{+5\nu-1-\frac{(3+\nu)}{\theta}-\frac{(3+\nu)}{\theta^2}+\frac{5\nu-1}{\theta^3}}{\frac{7}{2}-7\nu+\frac{9+50\nu}{2\theta}-\frac{(3+82\nu)}{2\theta^2}-\frac{(5-46\nu)}{2\theta^3}},$$

where it is required to be beware especially, lest the letter ζ may become exceedingly small, so that the interval ζp may be able in practice to be found conveniently, as that will be obtained, provided ζ may not be produced notably smaller than $\frac{1}{20}$; on account of which it will be worth the effort to investigate, whether also it may be allowed to make use of common glass to accomplish this aim, since now we have seen crystal glass to be suitable enough; since there fore for common glass there shall be $n=1,55$ and $\nu=0,2326$, there will become

$$\zeta=\frac{0,1630-\frac{3,2326}{\theta}-\frac{3,2326}{\theta^2}+\frac{0,1630}{\theta^3}}{1,8718+\frac{10,3150}{\theta}-\frac{11,0366}{\theta^2}-\frac{2,8498}{\theta^3}};$$

but here it will be agreed to be observed first, if there shall be $\theta=\infty$, to become almost $=\frac{1}{11}$, which value especially must certainly be able to be applied in practice ; but if we may suppose $\theta=30$, there will arise $\zeta=\frac{1}{43}$, which value is exceedingly small; from which it is apparent a greater value must be taken for θ. But with $\theta=50$ taken there is found $\zeta=\frac{0,0971}{2,0340}=\frac{1}{22}$ approx., which value will be able to be allowed conveniently. But with $\theta=60$ there is deduced $\zeta=\frac{0,1082}{2,0407}=\frac{1}{18}$ approximately, which value is seen to agree in practice very well. Therefore we will set out this case further in the following example.

EXAMPLE 1

183. If all the lenses may be made from common glass, for which there is $n=1,55$, and may be assumed $\theta=60$, so that thus a microscope for the magnification $m=1000$ may be able to be used, the following parts of the construction will themselves be had in the following manner.

In the first place evidently we will have

$$\mathfrak{A} = \tfrac{240}{61} = 4 - \tfrac{4}{61} = -- = 4 - \tfrac{1}{15} \text{ approximately,}$$

$$\mathfrak{B} = 3 - \tfrac{1}{15}, \quad \mathfrak{C} = 2 - \tfrac{1}{15}, \quad \mathfrak{D} = 1 - \tfrac{1}{15}$$

and again

$$\tfrac{1}{P} = 1 + \left(3 - \tfrac{1}{15}\right)\zeta;$$

and just as we have seen there must be taken $\zeta = \frac{1}{18}$, there will be

$$\begin{aligned}
\tfrac{1}{P} &= 1 + \tfrac{1}{6} - \tfrac{1}{270} = 1,1629,\\
\tfrac{1}{PQ} &= 1 + \tfrac{5}{18} - \tfrac{2}{270} = 1,2703,\\
\tfrac{1}{PQR} &= 1 + \tfrac{1}{3} - \tfrac{1}{90} = 1,3222\ ;
\end{aligned}$$

from which the focal lengths of the lenses will be

$$p = \left(4 - \tfrac{1}{15}\right)a = 3,9333a, \quad q = \tfrac{p}{P} = 4,5740a,$$
$$(0,5947571) \qquad (0,6602996)$$
$$r = \tfrac{p}{PQ} = 4,9965a, \qquad s = \tfrac{p}{PQR} = 5,2006a.$$
$$(0,6986634) \qquad (7160543)$$

Then truly

$$t = \tfrac{960}{m} \text{ in. and } u = \tfrac{320}{m} \text{ in.}$$

Again the common interval of these four lenses is

$$= \tfrac{1}{18}p = 0,2185a.$$

Truly the fourth interval will be

$$= 79,332a - \tfrac{480}{m} \text{ in.}$$

Truly the fifth

$$= 2u = \tfrac{640}{m} \text{ in.}$$

and the distance of the eye

$$O = \tfrac{1}{2}u = \tfrac{160}{m} \text{ in.}$$

Therefore now the construction of the individual lenses is required to be described :

I. For the first lens,

the focal length of which is $p = -\,3,9333a$ and the numbers

$$\lambda = 1,\ \mathfrak{A} = 4 - \tfrac{1}{15},$$

the radius

$$\begin{cases} \text{of the anterior face } = \frac{p}{\sigma - \mathfrak{A}(\sigma-\rho)} = -\frac{p}{4,0236} = -0,97756a \\ \text{of the posterior face} = \frac{p}{\rho + \mathfrak{A}(\sigma-\rho)} = +\frac{p}{5,8417} = +0,67332a, \end{cases}$$

which allows an aperture, of which the radius $x = 0,16833a$.

II. For the second lens,

the focal length of which is $q = 4,5740a$ and the numbers

$$\lambda = 1 \text{ and } \mathfrak{B} = 3 - \tfrac{1}{15},$$

the radius

$$\begin{cases} \text{of the anterior face} = \frac{q}{\sigma - \mathfrak{B}(\sigma-\rho)} = -\frac{q}{2,5869} = -1,7682a \\ \text{of the posterior face} = \frac{q}{\rho + \mathfrak{B}(\sigma-\rho)} = +\frac{q}{4,4050} = +0,0384a. \end{cases}$$

III. For the third lens,

the focal length of which is $r = 4,9965a$ and the numbers

$$\lambda = 1 \text{ and } \mathfrak{C} = \mathfrak{A} - 2,$$

the radius

$$\begin{cases} \text{of the anterior face } = \frac{r}{\sigma - \mathfrak{C}(\sigma-\rho)} = -\frac{r}{1,1503} = -4,3440a \\ \text{of the posterior face} = \frac{r}{\rho + \mathfrak{C}(\sigma-\rho)} = +\frac{r}{2,9683} = +1,6833a. \end{cases}$$

IV. For the fourth lens,

the focal length of which $s = 5,2006a$ and the numbers

$$\lambda = 1 \text{ and } \mathfrak{D} = \mathfrak{A} - 3,$$

the radius

$$\begin{cases} \text{of the anterior face } = \frac{s}{\sigma-\mathfrak{D}(\sigma-\rho)} = \frac{s}{0,2865} = 18,1522a \\ \text{of the posterior face} = \frac{s}{\rho+\mathfrak{D}(\sigma-\rho)} = \frac{s}{1,5316} = 3,3955a. \end{cases}$$

Hence there is deduced the following

CONSTRUCTION OF MICROSCOPES COMPOSED FROM SIX LENSES

WITH THE REFRACTION OF THE GLASS BEING $n = 1,55$

184. For this microscope the number m is taken arbitrarily very large, evidently on which only the two last lenses depend.

I. For the first lens,

the focal length of which $p = 3,9333a$, the radius

$$\begin{cases} \text{of the anterior face} = -0,97756a \\ \text{of the posterior face} = +0,67332a, \end{cases}$$

the radius of this aperture $= 0,16833a$
and the distance to the second lens $= 0,2185a$.

II. For the second lens
of which the focal length $q = 4,5740a$, the radius

$$\begin{cases} \text{of the anterior face} = -1,7682a \\ \text{of the posterior face} = +1,0384a; \end{cases}$$

the aperture and distance to the following lens are as before.

III. For the third lens
of which the focal length $r = 4,9965a$, the radius

$$\begin{cases} \text{of the anterior face} = -4,3440a \\ \text{of the posterior face} = +1,6833a, \end{cases}$$

the aperture and distance to the following lens are as before.

IV. For the fourth lens,

of which the focal length $s = 5,2006a$, the radius

$$\begin{cases}\text{of the anterior face} = 18,1522a \\ \text{of the posterior face} = 3,3955a,\end{cases}$$

with the aperture as before;
truly the distance to the fifth lens will be $= 79,332a - \frac{480}{m}$ in.

V. For the fifth lens

the focal length of which is $t = \frac{960}{m}$ in., the radius of each face may be taken

$$= \frac{1056}{m} \text{ in.},$$

the radius of this aperture $= \frac{240}{m}$ in.
and the distance to the sixth lens $= \frac{640}{m}$ in.

VI. For the sixth lens,

of which the focal length $u = \frac{320}{m}$ in., the radius of each face

$$= \frac{352}{m} \text{ in.},$$

the radius of this aperture $= \frac{80}{m}$ in.
and the distance to the eye $= \frac{160}{m}$ in.

VII. The radius of the area viewed within the object will be $= \frac{4a}{ma+8}$ in. and the measure of the clarity, by which objects will be represented, will be $= \frac{26,9328}{m}$ which, even if the magnification $m = 1000$ may be put in place, at this point is great enough.

VIII. Yet in this kind of microscopes this will be a strong source of displeasure, since the length of these, which evidently shall be almost equal to $80a$, becomes so great and thus will be seen to be less suitable; but since the distance of the object may be allowed easily to be diminished to half an inch or thus to a third, nothing stands in the way, why these microscopes may not be put for any use.

IX. Even if here some magnification may demand special fifth and sixth lenses, yet it is understood easily, if an instrument of this kind were to be applied to a particular magnification, then likewise also it will be able to be used with the best success both for greater as well as for lesser magnifications, while evidently its length either will be diminished or increased.

X. Finally since the four first lenses must be greater than the aperture of the first lens, it will be able for the artificer to establish from the precepts that the disks of these lenses may be contained in the diameter $\frac{2}{5}a$, thus so that, if there were $a=\frac{1}{2}$ in., the diameter of these disks would be $\frac{1}{5}$ in.

EXAMPLE 2

185. If all the lenses may be prepared from crystal glass, to describe all the moments which pertain to the construction of microscopes, thus so that there may become $\Lambda=0$. Since in this case there shall be $n=1,58$, there will be $\nu=0,2529$; from which we will deduce from the above formula

$$\zeta=\frac{0,2645-\frac{3,2529}{\theta}-\frac{3,2529}{\theta^2}+\frac{0,2645}{\theta^3}}{1,7297+\frac{10,8225}{\theta}-\frac{11,8689}{\theta^2}-\frac{3,3167}{\theta^3}};$$

now if θ shall be infinite, there will become $\zeta=\frac{0,2645}{1,7297}=\frac{1}{6}$ approx. ; but if we may assume as before $\theta=60$, there will be produced $\zeta=\frac{0,2094}{1,9068}=\frac{1}{9}$ approx.; from which it is apparent, if a smaller value may be attributed to ζ, then Λ is going to be obtaining a negative value, which we will be able to convert conveniently in our favor; indeed since then for the principal equation there may become for this case

$$\Lambda=\frac{-0,2094+1,9068\zeta}{16},$$

so that if we may put $\zeta=\frac{1}{18}$ into the preceding equation, there will become in the preceding example

$$\Lambda=-0,0065.$$

But since for the first lens we will assume $\lambda=1$, it is easily understood, if a greater value may be attributed to this λ, to be able to happen, that this expression for Λ may vanish completely; in the end we will have this formula $\lambda=1+\omega$, and since in the computation of the confusion arising from the letter $\lambda=1$ the formula shall be $\frac{1}{\mathfrak{A}^2}$, now from the value $\lambda=1+\omega$ there will arise $\frac{1+\omega}{\mathfrak{A}^2}$, thus so that the increased Λ value may be taken

$$=\frac{\omega}{\mathfrak{A}^2}=\frac{\omega(1+\theta)^3}{64\theta^3}=0,0164\omega,$$

thus so that there shall become

$$\Lambda=0,0164\omega-0,0065.$$

Whereby, since there may become $\Lambda = 0$, there should be taken $\omega = \frac{0,0065}{0,0164} = \frac{65}{164}$ and thus for the first lens established there must be put in place $\lambda = 1 + \frac{65}{164}$, with $\lambda' = 1 = \lambda'' = \lambda'''$ remaining for the following three lenses ; where it will be able to be effected, so that the first lens width may be provided with a little increase of the aperture. Therefore since there shall become $\theta = 60$ and $\zeta = \frac{1}{18}$ as in the preceding example, both the focal lengths as well as the intervals likewise will retain the same values and so much remains, so that the construction of the individual lenses may be led through.

I. For the first lens the focal length of which, $p = 3,9333a$

and the numbers $\lambda = 1 + \omega$ and $\mathfrak{A} = 4 - \frac{1}{15}$,

the radius

$$\begin{cases} \text{of the anterior face } = \frac{p}{\sigma - \mathfrak{A}(\sigma - \rho) + \tau\sqrt{\omega}} = \frac{p}{-4,0865 + 0,5524} = -1,1129a \\ \text{of the posterior face} = \frac{p}{\rho + \mathfrak{A}(\sigma - \rho) - \tau\sqrt{\omega}} = +\frac{p}{5,8106 - 0,5524} = +0,7480a, \end{cases}$$

from which this lens allows an aperture, the radius of which , $x = 0,1870a$.

II. For the second lens, the focal length of which is $q = 4,5740a$, the radius

$$\begin{cases} \text{of the anterior face } = -\frac{q}{2,6452} = -1,7292a \\ \text{of the posterior face} = +\frac{q}{4,3693} = +1,0469a. \end{cases}$$

III. For the third lens, the focal length of which, $q = 4,9965a$, the radius

$$\begin{cases} \text{of the anterior face } = \frac{r}{-1,2039} = -4,1503a \\ \text{of the posterior face} = +\frac{r}{2,9280} = +1,7065a. \end{cases}$$

IV. For the fourth lens, the focal length of which $s = 5,2006a$, the radius

$$\begin{cases} \text{of the anterior face} = \frac{s}{0,2375} = 21,8973a \\ \text{of the posterior face} = \frac{s}{1,48663} = 3,4983a. \end{cases}$$

Hence therefore there follows:

THE CONSTRUCTION OF MICROSCOPES COMPOSED FROM SIX LENSES

186. With the construction from crystal glass, for which $n=1,58$, for the first four lenses, since the procedure has been established above, for the given distance of the object $=a$, the intervals between these lenses may be put in place $=\frac{1}{18}p=0,2185a$ and the aperture of the first lens may be given, the radius of which $x=0,1870a$, and the interval from the fourth as far as to the fifth lens

$$=79,332a-\frac{480}{m}\text{ in.}$$

V. For the fifth lens,

the focal length of which $t=\frac{960}{m}$ in., and together with the sixth which will be permitted to be made from common glass:
the radius of each face may be taken $=\frac{1056}{m}$ in.,
the radius of its aperture $=\frac{240}{m}$ in..
and the distance to the sixth lens $=\frac{640}{m}$ in.

VI. For the sixth lens,

the focal length of which $u=\frac{320}{m}$ in.,
the radius of each face will be $=\frac{352}{m}$ in.,
the radius of its aperture $=\frac{80}{m}$ in.
and the distance of the eye $=\frac{160}{m}$ in.

VII. The radius of the area viewed in the object will be $\frac{4a}{ma+8}$ in.; but the measure of the clarity will become $=\frac{29,920a}{m}$, notably greater enough than in the preceding example.
Here the rest will be required to be observed the same as have been introduced above.

COROLLARY 5

187. With these two kinds of microscopes requiring to be compared between each other we attend to that particular convenience, so that, if perhaps a glass may occur, the refraction of which may hold a place somewhere between the refractions $n=1,55$ and $n=1,58$, then the construction of the lenses may be able to be defined readily by the interpolation rule .

SCHOLIUM 2

188. We may adapt the formulas, which we have found in this problem, to telescopes also, since here we are lead to somewhat different determinations. Therefore since there shall be $a = \infty$ and $h = a$, there will become $\theta = 0$, but yet still, so that $\theta a =$ to a finite quantity, and there may be put $\theta a = l$; then truly our elements become

$$\mathfrak{A} = 4\theta,\ \ \mathfrak{B} = -1,\ \ \mathfrak{C} = -2,\ \ \mathfrak{D} = -3 \text{ and } \mathfrak{E} = -2$$

and hence

$$A = 4\theta,\ \ B = -\tfrac{1}{2},\ \ C = -\tfrac{2}{3},\ \ D = -\tfrac{3}{4} \text{ and } E = -\tfrac{2}{3},$$

then truly

$$\tfrac{1}{P} = 1-\zeta,\ \ \tfrac{1}{PQ} = 1-3\zeta \text{ and } \tfrac{1}{PQR} = 1-6\zeta.$$

Whereby the focal lengths of the lenses will be :

$$p = 4l,\ \ q = 4l(1-\zeta),\ r = 4l(1-3\zeta),\ s = 4l(1-6\zeta),$$
$$t = \tfrac{2l}{m} \text{ and } u = \tfrac{2}{3}\cdot\tfrac{2l}{m}$$

and the intervals of the lenses

$$\text{first} = \text{second} = \text{third} = \xi p = 4\zeta l,$$
$$\text{fourth} = l(1-6\zeta) - \tfrac{l}{m},\quad \text{fifth} = \tfrac{4}{3}\cdot\tfrac{l}{m};$$

and finally the distance of the eye $O = \frac{1}{3}\cdot\frac{l}{m}\left(2+\frac{1}{m}\right)$.

Again for the radius of the apparent field

$$\Phi = \tfrac{z}{a} = \tfrac{1718}{m+1} \text{ min.}$$

Then the equation for sufficient distinctness requiring to be prepared will be

$$\tfrac{1}{k^3} = \tfrac{\mu m x^3}{l^3}\left\{\tfrac{1}{16} - \tfrac{5}{32}\zeta - \tfrac{\nu}{16}\left(5-23\zeta\right) + \tfrac{1}{8m}\left(\lambda'''' - 6\nu\right) + \tfrac{27\lambda'''''}{8m}\right\},$$

where indeed we may assume $\lambda = \lambda' = \lambda'' = \lambda''' = 1$; then truly the numbers λ'''' and λ''''' must be taken thence, so that the two final lenses must become equally convex. But if now we may wish, so that this expression finally may be reduced to zero, it will be required to put

$$m(1-\tfrac{5}{2}\zeta) - m\nu(5-23\zeta) + 2(\lambda'''' - 6\nu) + 54\lambda''''' = 0.$$

But the two final lenses will be allowed always to be constructed from common glass, where there is $n = 1,55$; but then there will be $\lambda'''' = 16,74\frac{1}{2}$ and $\lambda''''' = 1,6298$ and hence both the latter members will become 118,7080, thus so that there must become

$$m(1 - \tfrac{5}{2}\zeta) - mv(5 - 23\zeta) + 118,7080 = 0;$$

but if now the four first lenses may be prepared from the same common glass, on account of $v = 0,2326$ there will be found

$$-0,1630m + 2,8498\zeta m + 118,7080 = 0$$

and thus

$$\zeta = \frac{0,1630m - 118,7080}{2,8498m} \quad \text{or} \quad \zeta = \frac{1680m - 1187080}{28498m};$$

hence therefore a positive value for ζ cannot be produced, unless there shall become

$$m > \frac{1187080}{1630} \quad \text{or} \quad m > 728 \text{ approximately };$$

truly with so great a magnification the power of the telescopes will increase with the length and then indeed there must become $\zeta = 0$, since yet it must become greater than $\frac{1}{50}$; which inconvenience also shall be present, if the first lenses may be made from crystal glass, even if it may become a little negative. From which it is evident the formulas found here for telescopes by no means can be able to be applied with so much success than for microscopes, as we have shown just now.

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CAPUT II

DE ULTERIORI HORUM MICROSCOPIORUM PERFECTIONE

DUM IIS MAIOR CLARITATIS GRADUS

DUAS PLURESVE LENTES CONVEXAS

LOCO OBIECTIVAE SUBSTITUENDO COMPARATUR

PROBLEMA 1

157. *Loco lentis obiectivae eiusmodi duas lentes convexas proxime sibi iunctas substituere, ut binis reliquis lentibus secundum praecepta in superiore capite data constitutis maior claritatis gradus obtineatur.*

SOLUTIO

Cum hic quatuor lentes sint considerandae, quarum binae priores minimo intervallo sint seiunctae, tertia vero ante imaginem realem cadat, littera P minima ab unitate discrepabit, Q vero adhuc erit positiva, tertia vero R negativa; quam ob causam statuamus $R=-k$, unde distantiae focales harum lentium erunt

$$p=\mathfrak{A}a,\quad q=-\frac{A\mathfrak{B}a}{P},\quad r=\frac{AB\mathfrak{C}a}{PQ}\ \text{ et }\ s=\frac{ABC\mathfrak{C}a}{PQk},$$

Cum vero sit

$$PQk=\frac{ma}{h},$$

erit

$$s=\frac{ABC}{m}\cdot\frac{h}{m}.$$

Intervalla autem lentium erunt

$$\text{I et II} = Aa\left(1-\tfrac{1}{P}\right),$$
$$\text{II et III} = -\tfrac{AB}{P}\cdot a\left(1-\tfrac{1}{Q}\right),$$
$$\text{III et IV} = \tfrac{ABC}{PQ}\cdot a\left(1+\tfrac{1}{k}\right).$$

Cum autem omnes lentes sint convexae, erit

$$1.\ \mathfrak{A} > 0,\quad 2.\ -A\mathfrak{B} > 0,\quad 3.\ \mathrm{AB}\mathfrak{C} > 0 \ \text{ et }\ 4.\ ABC > 0$$

ideoque etiam

$$\tfrac{C}{\mathfrak{C}} > 0 \ \text{ seu }\ 1+C > 0\,.$$

Ratione intervallorum autem tenendum est, quia primum debet esse minimum, litteram P parum ab unitate discrepare, ita ut, si hoc intervallum ponatur $=\eta a$, futurum sit $P=\frac{A}{A-\eta}$ existente η fractione satis parva. Deinde debet esse $-AB(Q-1)>0$ et $ABC>0$.

Consideremus nunc spatium in obiecto conspicuum, cuius semidiameter est

$$z=\tfrac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}}{ma+h}\cdot ah\xi\,,$$

ubi $\mathfrak{q}$ tam erit parvum, ut reiici possit; deinde vero tam $\mathfrak{r}$ quam $\mathfrak{s}$ unitati aequales sumi poterunt, siquidem binae postremae lentes utrinque aequaliter convexae conficiantur. Hoc enim modo maximus campus visionis obtinebitur, uti in capite praecedente est ostensum. Ponamus igitur

$$M=\tfrac{2h}{ma+\mathrm{h}}$$

fietque

$$z=Ma\xi\,;$$

unde pro loco oculi habebitur

$$O=\tfrac{s}{Ma}\cdot\tfrac{h}{m}\,,$$

quae, ut iam assumsimus, est positiva. Ex quo pro tollendo margine colorato reperitur haec aequatio

$$0=\tfrac{\mathfrak{q}}{P}+\tfrac{\mathfrak{r}}{PQ}+\tfrac{\mathfrak{s}}{PQR}\,;$$

cum autem sit

$$\mathfrak{q}=0 \ \text{ et }\ \mathfrak{r}=\mathfrak{s}=1 \ \text{ et }\ R=-k$$

habebitur

$$0=1-\tfrac{1}{k} \ \text{ seu }\ k=1$$

ut ante, ita ut iam sit $PQ=\frac{ma}{h}$, et quia proxime $P=1$, fiet proxima $Q=\frac{ma}{h}$ sive pro maioribus multiplicationibus erit Q valde magnum. His notatis aequationes fundamentales erunt

$$1.\ -\mathfrak{B}\mathfrak{q} = (P-1)M,$$
$$2.\ -\mathfrak{C}\mathfrak{r} = (PQ-1)\mathrm{M}-\mathfrak{q},$$

quarum prior non amplius in computum venit, quoniam tam $\mathfrak{q}$ quam $P-1$ sunt valde parva; altera vero dat

$$\mathfrak{C} = -\left(\tfrac{ma}{h}-1\right)M = -2\left(\tfrac{ma-h}{ma+h}\right),$$

unde pro maioribus multiplicationibus concluditur

$$\mathfrak{C} = -2 \text{ et } C = -\tfrac{2}{3},$$

quibus valoribus tuto uti licebit; etiamsi enim vel campus visionis parumper diminueretur vel etiam margo coloratus non perfecte tolleretur, id neutiquam turbare debet. Quare, cum hactenus invenerimus

$$k=1,\quad PQ=\tfrac{ma}{h}\quad \text{et}\quad \mathfrak{C}=-\tfrac{2}{3},$$

prodeunt distantiae focales

$$p=\mathfrak{A}a,\quad q=-\tfrac{A\mathfrak{B}a}{P},\quad r=-2AB\cdot\tfrac{h}{m}\quad \text{et}\quad s=-\tfrac{2}{3}\cdot AB\cdot\tfrac{h}{m},$$

ita ut sit $s=\frac{1}{3}r$, et nunc apparet *AB* esse debere negativum. Intervalla autem ita exprimentur:

$$\text{Primum} = Aa\left(1-\tfrac{1}{P}\right) = na \quad \text{existente} \quad P=\tfrac{A}{A-\eta},$$
$$\text{secundum} = -\tfrac{AB}{P}\cdot a\left(1-\tfrac{1}{Q}\right),$$

quod ob $AB<0$ per se fit positivum,

$$\text{tertium} = -\tfrac{4}{3}AB\cdot\tfrac{h}{m} = 2s$$

atque distantia oculi

$$O=\tfrac{s}{Ma}\cdot\tfrac{h}{m}=s\left(\tfrac{ma+h}{2ma}\right)=\tfrac{1}{2}s \text{ proxima.}$$

Tandem superest consideranda aequatio pro apertura *z* determinanda, quae est

$$\frac{1}{k^3}=\frac{\mu m x^3}{a^2h}\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}-\frac{1}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu}{B\mathfrak{B}}\right)-\frac{h}{A^3B^3ma}\left(\frac{\lambda''}{8}-\frac{3\nu}{4}\right)-\frac{27h\lambda''}{8A^3B^3ma}\right).$$

Statuatur brevitatis gratia

$$\Lambda=\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}-\frac{1}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu}{B\mathfrak{B}}\right)-\frac{h}{A^3B^3ma}\left(\frac{\lambda''}{8}-\frac{3\nu}{4}\right)-\frac{27h\lambda''}{8A^3B^3ma}$$

ut sit

$$x=\tfrac{1}{k}\sqrt[3]{\tfrac{a^2h}{\mu mA}}=\tfrac{a}{k}\sqrt[3]{\tfrac{h}{\mu maA}};$$

quae expressio ut eo maior prodeat quam casu praecedente, efficiendum est, ut valor Λ, quantum fieri potest, infra unitatem deprimatur, ad quod primo littera $\lambda=\lambda'=1$ capiatur; pro duabus posterioribus autem lentibus, quia utrinque aequaliter convexae esse debent, litterae λ'' et λ''' ita iam definiuntur, ut sit

$$\lambda''=1+25\left(\tfrac{\sigma-\rho}{2\tau}\right)^2 \quad \text{et} \quad \lambda'''=1+\left(\tfrac{\sigma-\rho}{2\tau}\right)^2;$$

tantum igitur restant definiendae litterae A et B, quia propemodum est $P=1$. At circa litteras A et B iam praescribitur esse 1. $\mathfrak{A}>0$ et 2. $AB<0$ pariter ac $A\mathfrak{B}<0$, ita ut sit $\frac{B}{\mathfrak{B}}>0$ seu $1+B>0$. Quamobrem omnia illa membra pro Λ erunt positiva, ita ut eius valor ad nihilum redigi nequeat, sed tantum ad minimum sit revocandus. Pro primo quidem termino is eo minor reddetur, quo maior capiatur $\mathfrak{A}$; quia autem tum A fit negativum, littera B fiet positiva ideoque $\mathfrak{B}<1$, ex quo secundum membrum solum iterum fit maius unitate. Simili modo, si $\mathfrak{B}$ statuatur numerus magnus, fiet B negativum et A capi debebit positivum; unde $\mathfrak{A}$ fiet unitate minus, ita ut nunc primus terminus solus unitatem sit superaturus. Deinde vero etiam inprimis cavendum est, ne productum illud negativum AB fiat nimis parvum, quoniam alioquin distantiae focales r et s quasi evanescerent, ex quo necesse est, ut formula $-AB$ non infra certum valorem deprimatur. Statuamus igitur $AB=-\theta$, ita ut θ denotet limitem illum pro hoc producto observandum; qui cum ut quantitas constans spectari queat, dum litterae A et B pro variabilibus habentur, erit

$$\tfrac{dB}{B}=-\tfrac{dA}{A}.$$

His ergo notatis expressio litteram Λ definiens erit

$$\Lambda=\tfrac{1}{\mathfrak{A}^3}+\tfrac{v}{A\mathfrak{A}}-\tfrac{1}{A^3\mathfrak{B}^3P}-\tfrac{v}{A^3B\mathfrak{B}P}+\tfrac{h}{\theta^3ma}\left(\tfrac{\lambda''}{8}-\tfrac{3v}{4}\right)+\tfrac{27h\lambda''}{8\theta^3ma},$$

in qua posteriora membra sunt constantia; unde ad minimum eius valorem inveniendum tantum opus erit priora membra differentiari, ubi quidem $P=1$. Hunc in finem notetur esse

$$\tfrac{1}{\mathfrak{A}}=1+\tfrac{1}{A} \quad \text{et} \quad \tfrac{1}{\mathfrak{B}}=1+\tfrac{1}{B}$$

hincque

$$\tfrac{d\mathfrak{A}}{\mathfrak{A}^2}=\tfrac{dA}{A^2} \quad \text{et} \quad \tfrac{d\mathfrak{B}}{\mathfrak{B}^2}=\tfrac{dB}{B^2}=-\tfrac{dA}{AB},$$

ex quo aequatio differentialis prodibit

$$0=-\frac{3B}{\mathfrak{A}^2}+\frac{3B}{A^2\mathfrak{A}^3}-\frac{3}{A^2\mathfrak{B}^2}-\nu\left(\frac{B}{A}+\frac{B}{\mathfrak{A}}-\frac{2}{A^2\mathfrak{B}}+\frac{1}{A^2B}\right),$$

quae per B divisa dat

$$0=3\left(\frac{1}{A^2\mathfrak{B}^3}-\frac{1}{\mathfrak{A}^2}-\frac{1}{A^2\mathfrak{B}^2B}\right)-\nu\left(\frac{1}{A}+\frac{1}{\mathfrak{A}}-\frac{2}{A^2\mathfrak{B}B}+\frac{1}{A^2B^2}\right),$$

atque hinc elisis litteris germanicis elicietur

$$0=3\left(\frac{1}{AB}-1\right)\left(1+\frac{2}{A}+\frac{1}{AB}\right)+\nu\left(\frac{1}{A^2B^2}+\frac{2}{A^2B}-\frac{2}{A}-1\right),$$

quae ergo reducitur ad hos factores:

$$0=(3+\nu)\left(\frac{1}{AB}-1\right)\left(1+\frac{2}{A}+\frac{1}{AB}\right);$$

ex qua, cum ob $AB=-\theta$ secundus factor evanescere nequeat, factor tertius praebet

$$B=-\frac{1}{A+2}\quad \text{et}\quad \mathfrak{A}=-\frac{1}{A+1}\ ;$$

sive etiam ambae litterae per θ sequenti modo definientur:

$$B=\frac{\theta-1}{2}\quad \text{et}\quad \mathfrak{B}=\frac{\theta-1}{\theta+1},$$

deinde

$$A=-\frac{2\theta}{\theta-1}\quad \text{et}\quad \mathfrak{A}=+\frac{2\theta}{\theta+1},$$

Ex his autem valoribus concludimus fore

$$\Lambda=\frac{(\theta+1)^3}{8\theta^3}\left(1+\frac{1}{P}\right)-\frac{\nu\left(\theta^2-1\right)}{4\theta^2}\left(1-\frac{1}{\theta P}\right)+\frac{h}{\theta^3ma}\left(\frac{\lambda''}{8}-\frac{3\nu}{4}\right)+\frac{27h\lambda''}{8\theta^3ma},$$

ubi θ plerumque erit numerus valde magnus, ut etiam pro maioribus multiplicationibus distantia focalis s non fiat nimis exigua Hinc igitur erit satis exacte

$$\Lambda=\tfrac{1}{4}(1-\nu),$$

et cum propemodum sit $\nu=\frac{1}{5}$, erit $\Lambda=\frac{1}{5}$, ita ut tuto sumi possit $\mu\Lambda=\frac{1}{5}$; unde obtinebitur

$$x=\frac{a}{k}\sqrt[3]{\frac{5h}{ma}},$$

qui valor eum, quem in capite praecedente habuimus, superat in ratione $\sqrt{5}:1$ seu proxime ut 17: 10. Quare etiam claritas in eadem ratione hic maior obtinetur.

COROLLARIUM 1

158. Per numerum igitur θ distantiae focales sequenti modo exprimuntur:

$$p = \mathfrak{A}a = \tfrac{2\theta}{\theta+1}\cdot a,\quad q = \tfrac{2\theta}{(\theta+1)P}\cdot a,\quad r = 2\theta\cdot\tfrac{h}{m}\quad \text{et}\quad s = \tfrac{2}{3}\theta\cdot\tfrac{h}{m},$$

ita ut sit proxime $q = p$ et exacte $s = \tfrac{1}{3}r$; tum vero lentium intervalla

$$\text{primum } = -\tfrac{2\theta}{(\theta-1)}\left(1-\tfrac{1}{P}\right)a = \eta a$$

ideoque

$$P = \tfrac{2\theta}{2\theta+\eta(\theta-1)} \quad \text{adeoque} \quad P < 1,$$

$$\text{secundum} = \theta\left(\tfrac{1}{P}-\tfrac{1}{PQ}\right)a = \tfrac{\theta a}{P}-\tfrac{\theta h}{m},$$

$$\text{tertium } = \tfrac{4}{3}\theta\cdot\tfrac{h}{m} = 2s.$$

COROLLARIUM 2

159. Quoniam intervallum binarum lentium sibi proximarum convenientissime ex earum distantia focali definitur, ponamus esse $\eta a = \zeta p$; hinc definietur

$$P = \tfrac{(\theta+1)}{(\theta+1)+\zeta(\theta-1)};$$

quare, cum sit θ numerus valde magnus, fiet $P = \tfrac{1}{1+\zeta}$; quare si capiatur $\zeta = \tfrac{1}{10}$, fiet $P = \tfrac{10}{11}$, qui valor ad praxin satis videtur accommodatus, cum hoc intervallum adhuc exiguam mutationem permittat. Quod ad campum visionis attinet, spatii in obiecto conspicui semidiameter erit eadem scilicet est uti in problemate 2 capitis praecedentis atque etiam distantia oculi perinde hic determinatur.

SCHOLION 1

160. En ergo iam insignem perfectionem eorum microscopiorum, quae in capite praecedente evolvimus, cum claritas hic inventa iam notabiliter maior sit quam ibi idque in ratione 12:7, et quia revera claritas secundum rationem duplicatam sentitur, hic triplo maior est censenda. Quocirca in his microscopiis multiplicatio multo longius proferri poterit quam in praecedentibus, antequam obscuritas fiat intolerabilis. Hinc si velimus, ut pro multiplicatione $m = 1000$ distantia focalis lentis ocularis non minor fiat quam $\tfrac{1}{4}$ dig.,

oportebit assumere $\theta = 47$, ita ut sumto $\theta = 50$ non sit metuendum, ut lente oculari nimis exigua opus habeamus. Hunc igitur casum in sequenti exemplo evolvisse operae pretium videtur.

EXEMPLUM

161. Statuamus igitur $\theta = 50$ sumtoque $h = 8$ dig. et, ut modo notavimus, $P = \frac{10}{11}$ pro data obiecti distantia $= a$ nanciscimur sequentes distantias focales:

$$p = \frac{100}{51}a,\quad q = \frac{110}{51}a,\quad r = \frac{800}{m}\text{dig.}\ \text{ et }\ s = \frac{800}{3m}\text{dig.}$$

lentiumque intervalla

$$\text{primum} = \frac{10}{49}a,\quad \text{secundum} = 55a - \frac{400}{m}\text{dig.}\ \text{ et tertium} = \frac{1600}{3m}\ \text{dig.}$$

et distantiam oculi

$$= \frac{400}{3m}\left(1 + \frac{8}{ma}\right)\ \text{dig.}$$

Quoniam porro est $\mathfrak{A} = \frac{100}{51}$ et $\mathfrak{B} = \frac{49}{51}$, ob $\lambda = 1$ et $\lambda' = 1$ constructio lentium duarum priorum, si quidem ex vitro communi conficiantur, ita se habebit:

I. Pro lente prima

erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{p}{\sigma - \mathfrak{A}(\sigma - \rho)} = -\frac{p}{1{,}1896} = -0{,}84062p\\ \text{posterioris} = \rho\frac{p}{\sigma + \mathfrak{A}(\sigma - \rho)} = \frac{p}{3{,}0077} = 0{,}33248p.\end{cases}$$

II. Pro secunda lente

erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{q}{\sigma - \mathfrak{B}(\sigma - \rho)} = \frac{q}{0{,}2470} = 4{,}0486q\\ \text{posterioris} = \frac{q}{\rho + \mathfrak{B}(\sigma - \rho)} = \frac{q}{1{,}5711} = 0{,}6365q.\end{cases}$$

His notatis evolvamus valorem litterae Λ, qui erit $\Lambda = 0{,}221$, qui per $\mu = 0{,}9381$ multiplicatus dabit $\mu\Lambda = 0{,}2073$; qui ergo a supra assumto $\frac{1}{5}$ vix differt; hinc ergo colligimus pro apertura lentis obiectivae

$$x = \frac{a}{k}\sqrt[3]{\frac{40}{ma}} = \frac{0{,}17099}{\sqrt[3]{ma}},$$

ex quo valore prodit mensura claritatis

$$\frac{160}{ma}\cdot x=\frac{27,3584}{m\sqrt[3]{ma}};$$

denique pro campo visionis erit

$$z=\frac{4a}{ma+8}\text{ dig.},$$

ac si tandem in loco imaginis realis velimus diaphragma constituere, foraminis eius semidiameter debet esse

$$=ABCz=\frac{133a}{ma+8}\text{ dig.};$$

unde conficitur sequens

CONSTRUCTIO HUIUSMODI MICROSCOPIORUM EX QUATUOR LENTIBUS COMPOSITORUM PRO QUAVIS MULTIPLICATIONE

162. Singulae hae lentes ex vitro communi, cuius refractio est $n=1,55$, parentur et posita obiecti distantia $=a$, quam iterum $=\frac{1}{2}$ dig. assumi licebit, erit:

I. Pro lente prima, cuius distantia focalis est $p=\frac{100}{51}a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = & -1,6482a\\ \text{posterioris} = +0,6420a,\end{cases}$$

eius aperturae semidiameter

$$x=\frac{0,17099}{\sqrt[3]{ma}}\cdot a$$

et distantia ad lentem secundam

$$=\frac{10}{49}a=0,2040a.$$

II. Pro lente secunda, cuius distantia focalis $q=\frac{110}{51}a$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = 8,7323a\\ \text{posterioris} = 1,3730a,\end{cases}$$

cuius aperturae semidiameter aliquanto maior quam praecedentis, et distantia ad lentem tertiam $=55a-\frac{400}{m}$ dig.

III. Pro lente tertia, cuius distantia focalis est $r=\frac{800}{m}$ dig., erit

$$\text{radius faciei utriusque } r=\frac{800}{m}\text{ dig.},$$

aperturae semidiameter $=\frac{200}{m}$ dig.
et distantia ad lentem quartam $=\frac{1600}{3m}$ dig. $=\frac{533}{m}$ dig.

IV. Pro lente quarta, cuius distantia focalis $r=\frac{800}{m}$ dig., erit

radius faciei utriusque $r=\frac{293}{m}$ dig.,

eius aperturae semidiameter $=\frac{67}{m}$ dig.
et distantia ad oculum $=\frac{133}{m}$ dig.

V. Spatii in obiecto conspicui semidiameter erit

$$z=\frac{4a}{ma+8}\text{ dig.}$$

et instrumenti longitudo $=55,2040a+\frac{267}{m}$ dig.
et mensura claritatis $=\frac{27,358}{m\sqrt[3]{ma}}$,

ubi observandum est ut supra § 148, IV duas priores lentes pro omni multiplicatione, duas vero posteriores pro qualibet obiecti distantia retineri posse, pro quibus eadem inserviet tabula, quam ibi adiecimus.

SCHOLION 2

163. Eaedem formulae, quas hic invenimus, etiam ad telescopia transferri possunt; ubi cum sit $a=\infty$ et $h=a$, ne lentes in infinitum crescant, debet esse $\theta=0$, ita tamen, ut θa fiat quantitas finita; scilicet cum sit $p=\frac{2\theta}{\theta+1}\cdot a$, erit $\theta a=\frac{p}{2}$ sicque reliquae distantiae focales erunt

$$q=\frac{p}{P},\quad r=\frac{p}{m}\quad\text{et}\quad s=\frac{p}{3m},$$

deinde lentium intervalla

$$\text{primum }=\left(1-\frac{1}{P}\right)p,\quad\text{secundum }=\frac{p}{2P}-\frac{p}{2m},\quad\text{tertium }=\frac{2p}{3m}.$$

Quod nunc ad litteram P attinet, formula supra data hic locum habebit

$$P=\frac{\theta+1}{\theta+1+\zeta(\theta-1)},$$

quae hic dat

$$P=\frac{1}{1-\zeta};$$

quia autem hic de telescopiis agitur, sumi poterit $\zeta=\frac{1}{25}$, ita ut sit $P=\frac{25}{24}$;
tum vero erit distantia oculi

$$O = \frac{s(m+1)}{2m} = \frac{1}{2}s\left(1+\frac{1}{m}\right),$$

ita ut tota longitudo fiat

$$= p\left(1-\frac{1}{2P}+\frac{1}{3m}+\frac{1}{6m^2}\right)$$

ac porro semidiameter campi apparentis

$$\frac{z}{a} = \Phi = \frac{1}{2}\cdot\frac{1}{m+1} = \frac{1718}{m+1}$$

Nunc etiam consideretur aequatio postrema ex semidiametro confusionis deducta, in qua membrum vinculis inclusum per $8\theta^3$ multiplicetur factor vero communis per idem dividatur, et habebitur

$$\frac{1}{k^3} = \frac{\mu m x^3}{p^3}\left(1+\frac{1}{P}-\frac{2\nu}{P}+\frac{1}{m}\left(\lambda''-6\nu\right)-\frac{27\lambda''}{m}\right);$$

ubi, cum pro vitro communi sit $\nu = 0,2326$, si statuatur $P = \frac{25}{24}$ et termini per m divisi negligantur, ob $\mu = 1$ proxime fiet proxime

$$\frac{1}{k^3} = \frac{mx^3}{p^3}\cdot\frac{3}{2},$$

unde colligitur

$$p = kx\sqrt[3]{\frac{3}{2}\cdot m},$$

unde, cum claritatis gradus y dari soleat, ut sit $x = my$, tum vero assumatur $ky = 1$, siquidem statuatur $y = \frac{1}{50}$ dig. et $k = 50$, uti supra est factum, habebitur

$$p = m\sqrt[3]{\frac{3}{2}m}, \quad \text{sive} \quad p = \frac{8}{7}m\sqrt[3]{m}.$$

Cognito autem P erit $q = \frac{25}{24}p.$. Sumsimus autem hic $\lambda = 1$ et $\lambda' = 1$, et cum sit $\mathfrak{A} = 0$ et $\mathfrak{B} = -1$, constructio harum lentium pro vitro communi, ubi $n = 1,55$, erit:

I. Pro lente prima

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{p}{\sigma} = 0,6145p\\ \text{posterioris} = \frac{p}{\rho} = 5,2438p.\end{cases}$$

II. Pro lente secunda autem erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{q}{\sigma+(\sigma-\rho)} = +\frac{q}{3,0641} = +0,3264q\\ \text{posterioris} = \frac{q}{\rho-(\sigma-\rho)} = -\frac{q}{1,2460} = -0,8026q.\end{cases}$$

hinc ergo obtinetur sequens

CONSTRUCTIO TELESCOPII ASTRONOMICI
EX QUATUOR LENTIBUS COMPOSITI
PRO VITRO COMMUNI $n = 1,55$

164. Singula momenta pro constructione pro more recepto ita in ordinem redigantur, scilicet proposita multiplicatione m definiatur inde

$$p = \tfrac{8}{7}m\sqrt[3]{m}.$$

I. Pro prima lente, cuius distantia focalis $= p$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = 0,6145p\\ \text{posterioris} = 5,2438p.\end{cases}$$

eius aperturae semidiameter $= \frac{m}{50}$ dig.
et distantia ad lentem sequentem erit $= \frac{1}{25}p = 0,04p$.

II. Pro lente secunda, cuius distantia focalis est $\frac{25}{24}p$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = +0,3134p\\ \text{posterioris} = -0,7705p;\end{cases}$$

apertura non definitur, dummodo sit maior praecedente, et distantia ad lentem tertiam $= \frac{12}{25}p - \frac{p}{2m}$.

III. Pro tertia lente, cuius distantia focalis est $r = \frac{p}{m}$, erit

$$\text{radius utriusque faciei} = 1,1\cdot\frac{p}{m},$$

eius aperturae semidiameter $= \frac{p}{4m}$, et distantia ad lentem quartam $=\frac{2p}{3m}$.

IV. Pro quarta lente, cuius distantia focalis $=\frac{p}{3m}$, erit

$$\text{radius utriusque faciei } = 1,1\cdot\frac{p}{m},$$

eius aperturae semidiameter $=\frac{p}{12m}$

et distantia ad oculum $O=\frac{p}{6m}\left(1+\frac{1}{m}\right)$.

V. Tota ergo longitudo erit

$$=p\left(\frac{13}{25}+\frac{1}{3m}+\frac{1}{6m^2}\right)$$

et semidiameter campi apparenti $\Phi=\frac{1718}{m+1}$ min.

VI. Si in loco imaginis realis, quae inter binas posteriores lentes medium interiacet, diaphragma sit constituendum, eius foraminis radius erit

$$=ABCz=\frac{p}{6(m+1)}.$$

PROBLEMA 2

165. *Iisdem quaternis lentibus retentis microscopium conficere, quod ad omnes multiplicationes producendas sit accommodatum.*

SOLUTIO

Sint harum lentium distantiae focales *p, q, r* et *s,* quae, uti ex problemate praecedente perspicitur, ita debent esse comparatae, ut sit primo $s=\frac{1}{3}r$, tum vero $q=\frac{11}{10}p$; deinde etiam recordandum est ambas posteriores, lentes utrinque esse debere aeque convexas, de figura vero priorum mox videbimus. Formulas ergo supra inventas considerando erit:

1. $\theta=\frac{mr}{2h}$,

unde , cum sit $p=\frac{2\theta}{\theta+1}\cdot a$, hinc colligemus

$$a=\frac{\theta+1}{2\theta}\cdot p=\frac{(mr+2h)}{2mr}\cdot p,$$

quae ergo etiam a multiplicatione pendet, ita ut pro qualibet multiplicatione distantiam obiecti variari oporteat.

seu

2. Lentium intervalla ita se habebunt:

$$\text{Primum } =\frac{1}{10}p,$$

$$\text{secundum} = \frac{11mr+18h}{40h}\cdot p - \frac{1}{2}r \quad \text{ob} \quad P = \frac{10(mr+2h)}{11mr+18h}$$

seu

$$\text{secundum intervallum } = \frac{11mrp}{40h} + \frac{9}{20}p - \frac{1}{2}r,$$

$$\text{tertium } = \frac{4}{3}\cdot\frac{1}{2}r = \frac{2}{3}r$$

atque distantia oculi

$$O = s\left(\frac{1}{2} + \frac{hr}{(mr+2h)p}\right)$$

seu O proxima $= \frac{1}{2}s$; sicque pro varia multiplicatione tantum secundum intervallum fiet mutabile.

Porro vero erit spatii conspicui semidiameter

$$z = \frac{1}{2}\cdot\frac{(mr+2h)hp}{m(mr+2h)p+2mhr},$$

unde, si m sit numerus praemagnus, fiet $z = \frac{h}{2m}$

Ut nunc figuram duarum priorum lentium definiamus, pro quibus supra sumsimus $\lambda = 1$ et $\lambda' = 1$, perpendere oportet litteras

$$\mathfrak{A} = \frac{2mr}{mr+2h} \quad \text{et} \quad \mathfrak{B} = \frac{mr-2h}{mr+2h}.$$

Quia autem earum figura pro varia multiplicatione mutari non potest atque pro rei natura sufficit figuram tantum proxime definivisse, consideramus m ut numerum praemagnum sumamusque $\mathfrak{A} = 2$ et $\mathfrak{B} = 1$. Possumus etiam superioribus valoribus uti, ubi erat $\theta = 50$, quippe qui valor certe multiplicationi magnae respondet; facile enim intelligitur tum eandem figuram tam maioribus quam minoribus multiplicationibus satis exacte convenire; quare si vitrum commune adhibeamus, habebitur pro lente prima

$$\text{radius faciei}\begin{cases}\text{anterioris} & = -0,8406p\\ \text{posterioris} & = +0,3325p\end{cases}$$

et pro lente secunda

$$\text{radius faciei}\begin{cases}\text{anterioris} = 4,0486q\\ \text{posterioris} = 0,6365q.\end{cases}$$

Denique pro apertura primae lentis invenimus eius semidiametrum

$$x = \frac{0,171a}{\sqrt[3]{ma}}$$

indeque mensuram claritatis nacti sumus

$$= \frac{27{,}358}{m\sqrt[3]{ma}}$$

existente $a = \frac{mr+2h}{2mr} \cdot p = \frac{1}{2} p$ proxime.

EXEMPLUM

166. 1. Sumamus pro harum lentium distantiis focalibus

$$p = 1 \text{ dig.}, \quad q = \tfrac{11}{12} \text{ dig.}, \quad r = 1 \text{ dig.}, \quad \text{et } s = \tfrac{1}{3} \text{dig.},$$

quippe qui valores ad praxin maxime idonei videntur; ac si hae lentes ex vitro communi parentur, earum figura ita determinetur, ut sit

2. I. Pro lente prima

$$\text{radius faciei} \begin{cases} \text{anterioris} & = -0,84 \text{ dig.} \\ \text{posterioris} & = +0,33 \text{ dig.} \end{cases}$$

II. Pro secunda lente

$$\text{radius faciei} \begin{cases} \text{anterioris} & = 4,45 \text{ dig.} \\ \text{posterioris} & = 0,70 \text{ dig.} \end{cases}$$

III. Pro tertia lente

$$\text{radius faciei utriusque } = 1,1 \text{ dig.}$$

IV. Pro quarta lente

$$\text{radius faciei utriusque } = \tfrac{11}{30} \text{ dig.}$$

3. Quibus lentibus paratis prima et secunda ad intervallum $= \frac{1}{10}$ dig. firmentur, tertia vero et quarta ad intervallum $= \frac{2}{3}$ dig., ita tamen, ut pro indole oculi quarta lens tantillum mutari possit; ambo autem paria eiusmodi tubis inserantur, qui pro lubitu ad maius minusve spatium diduci queant, quemadmodum multiplicatio postulat, siquidem intervallum inter secundam et tertiam lentem esse debet $\left(\frac{11m}{320} - \frac{1}{20}\right)$ dig.

4. Simili modo etiam distantia obiecti aliquantum erit variabilis et pro qualibet multiplicatione esse debet

$$a = \frac{m+16}{2m} \text{ dig. } = \left(\frac{1}{2} + \frac{8}{m}\right) \text{ dig.}$$

Deinde vero locus oculi ut constans spectari potest, ita ut sit eius distantia

$$O = \frac{1}{6} \text{ dig.}$$

5. Tertiae et quartae lenti tanta datur apertura, quantae sunt capaces.

6. Primae autem lentis apertura maxima a multiplicatione pendet, cum sit eius semidiameter

$$x = \frac{0,0855}{\sqrt[3]{\frac{1}{2}m}}$$

unde mensura claritatis prodit

$$x = \frac{27,36}{m\sqrt[3]{\frac{1}{2}m}}$$

7. Circa hoc microscopium haud abs re fore arbitror, si pro aliquot praecipuis multiplicationibus valores momentorum variabilium, quae sunt
1. distantia obiecti $= a$, 2. intervallum lentium secundae et tertiae, quod indicemus littera L, 3. semidiameter aperturae primae lentis x et 4. mensura claritatis $= 20y$, adiunxerimus; quem in finem sequens tabella est adiecta:

m	*a*	*L*	*x*	*Claritas*
50	0,660	1,669	0,0292	0,1871
100	0,580	3,387	0,0232	0,0743
200	0,540	6,825	0,0184	0,0295
300	0,527	10,262	0,0160	0,0172
400	0,520	13,700	0,0146	0,0117
500	0,516	17,137	0,0136	0,0087
600	0,613	20,576	0,0128	0,0068
700	0,611	24,012	0,0121	0,0065
800	0,510	27,450	0,0116	0,0047
900	0,509	30,887	0,0111	0,0040
1000	0,508	34,325	0,0108	0,0034

PROBLEMA 3

167. *Loco lentis obiectivae eiusmodi tres lentes convex proxime sibi iunctas substituere, ut binis reliquis lentibus secundum praecepta in capite superiore data constitutis maior claritatis gradus obtineatur.*

SOLUTIO

Cum hic quinque lentes sint considerandae et imago realis in quartum seu ultimum intervallum incidat, litterae P, Q, R erunt positivae, sequens vero S ponatur $= -k$, ita ut sit $PQRk = \frac{ma}{h}$. Hinc distantiae focales singularum lentium ita exprimentur:

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{B}a}{P}, \quad r = \frac{AB\mathfrak{C}a}{PQ},$$
$$s = \frac{ABC\mathfrak{D}a}{PQR} \quad \text{et} \quad t = -ABCD \cdot \frac{h}{m}.$$

Intervalla vero lentium ita se habebunt:

$$\text{I et II} = Aa\left(1-\tfrac{1}{P}\right), \ \text{II et III} = -\tfrac{AB}{P}\cdot\left(1-\tfrac{1}{Q}\right),$$
$$\text{III et IV} = \tfrac{ABC}{PQ}\cdot a\left(1-\tfrac{1}{R}\right), \ \text{IV et V} = \tfrac{ABCDa}{PQR}\cdot a\left(1+\tfrac{1}{k}\right);$$

quorum cum duo priora sint valde parva, litterae P et Q quam minima ab unitate recedere debent; quamobrem in expressione campi litterae $\mathfrak{q}$ et $\mathfrak{r}$ pro nihilo erunt habendae, posteriores vero $\mathfrak{s}$ et $\mathfrak{t}$ unitati aequales sumantur, siquidem binae postremae lentes utrinque fiant aeque convexae. Hinc ergo spatii in obiecto conspicui fiet semidiameter

$$z = \frac{2ah}{ma+h}\cdot\xi;$$

at vero littera

$$M = \frac{2h}{ma+h},$$

ex qua distantia oculi fit

$$O = \frac{\mathfrak{t}}{Ma}\cdot\frac{h}{m} = \tfrac{1}{2}t\left(1+\tfrac{h}{ma}\right) \ \text{seu proxima} = \tfrac{1}{2}t.$$

Aequationum porro fundamentalium prima et secunda omitti possunt,
quia ob litteras P et Q proxima $=1$ litterae $\mathfrak{q}$ et $\mathfrak{r}$ sponte fiunt minimae;
tertia vero dabit

$$\mathfrak{D}\mathfrak{s} = (PQR-1)M,$$
$$-\mathfrak{D} = \frac{2(ma-hk)}{k(ma+h)},$$

unde pro maioribus multiplicationibus fit $\mathfrak{D} = -\frac{2}{k}$; ex aequatione autem pro margine colorato, quae hoc casu erit

$$\frac{1}{PQR} - \frac{1}{PQRk} = 0,$$

colligimus ut ante $k=1$, ita ut sit $\mathfrak{D} = -2$ hincque $D = -\frac{2}{3}$; quibus inventis distantiae focales erunt

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{B}}{P}\cdot a, \quad r = \frac{AB\mathfrak{C}}{PQ}\cdot a,$$
$$s = 2ABC\cdot\frac{h}{m} \quad \text{et} \quad t = \tfrac{2}{3}ABC\cdot\frac{h}{m} = \tfrac{1}{3}s;$$

unde sequitur

$$\mathfrak{A} > 0, \quad A\mathfrak{B} < 0, \quad AB\mathfrak{C} > 0 \quad \text{et} \quad ABC > 0$$

et intervalla lentium

$$\text{primum} = Aa\left(1-\tfrac{1}{P}\right),\ \ \text{secundum} = -\tfrac{AB}{P}\cdot a\left(1-\tfrac{1}{Q}\right),$$
$$\text{tertium} = \tfrac{ABC}{PQ}\cdot a\left(1-\tfrac{1}{R}\right),\ \text{quartum} = \tfrac{4}{3}ABC\cdot\tfrac{h}{m} = 2t.$$

Denique pro apertura primae lentis seu littera x definienda habetur ista aequatio:

$$\frac{1}{k^3} = \frac{\mu m x^3}{a^2 h}\left\{\begin{array}{l}\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}} - \frac{1}{A^3 P}\left(\frac{\lambda'}{\mathfrak{B}^3} + \frac{\nu}{B\mathfrak{B}}\right) + \frac{1}{A^3B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3} + \frac{\nu}{C\mathfrak{C}}\right)\\ \frac{1}{8A^3B^3C^3ma}\left(\lambda''' - 6\nu\right) + \frac{27h\lambda''''}{8A^3B^3C^3ma}\end{array}\right\},$$

in qua expressione numeri λ''' et λ'''' inde dantur, quod hae lentes debent esse utrinque aeque convexae; priores vero λ, λ' et λ'' habent coefficientes positivos, quia $[\mathfrak{A} > 0,]\ A\mathfrak{B} < 0\ [, AB\mathfrak{C} > 0]$, cum ex hypothesi omnes lentes sint convexae. Quare cum totum negotium nunc eo redeat, ut huic expressioni valor minimus concilietur, primo his litteris λ, λ', λ'' tribuatur valor minimus, qui est unitas; deinde vero litterae *A, B, C* ita definiri debent, ut haec expressio minimum adipiscatur valorem; quem in finem ante omnia notari convenit, ne binae ultimae lentes pro maioribus multiplicationibus nimis fiant exiguae, quantitatem ABC semper certum limitem superare debere; quare, cum ea positiva esse debeat, statuamus

$$ABC = \theta,$$

ita ut θ tanquam numerus datus spectari possit; ex quo bina ultima membra per se determinantur; restant igitur tantum tria membra priora, quibus, quomodo minimus valor induci queat, est investigandum, ubi quidem pro litteris P et Q unitatem scribere licebit. Cum autem iam supra huiusmodi investigationes saepius expediverimus, inde concludere possumus a scopo nos minime esse aberraturos, si has tres formulas $\mathfrak{A}$, $-A\mathfrak{B}$ et $AB\mathfrak{C}$ inter se reddamus aequales, ita ut distantiae focales *p, q, r* eatenus tantum a ratione aequalitatis recedant, quatenus litterae P et Q ab unitate discrepant. Aequalitas autem primae et secundae harum expressionum dat

$$\mathfrak{B} = -\tfrac{\mathfrak{A}}{A} = \mathfrak{A} - 1 \text{ seu } \mathfrak{B} = -\tfrac{1}{A+1}$$

unde fit

$$B = \tfrac{\mathfrak{A}-1}{2-\mathfrak{A}} = -\tfrac{1}{2+A}.$$

Aequalitas antem secundae et tertiae dabit

$$\mathfrak{C} = -\tfrac{\mathfrak{B}}{B} = \mathfrak{B} - 1 = -\tfrac{1}{B+1};$$

quamobrem habebimus

$$\mathfrak{C} = \mathfrak{A} - 2 = -\tfrac{A-2}{A+1} \quad \text{hincque} \quad C = -\tfrac{A+2}{2A+3};$$

at vero debet esse $ABC = \theta$, unde omnes hae litterae per θ sequenti modo exprimentur:

$$A=\frac{3\theta}{1-2\theta},\quad B=-\frac{(1-2\theta)}{2-\theta}\quad \text{et}\quad C=-\frac{(2-\theta)}{3}$$

atque hinc porro

$$\mathfrak{A}=\frac{3\theta}{1+\theta},\quad \mathfrak{B}=-\frac{(1-2\theta)}{1+\theta}\quad \text{et}\quad \mathfrak{C}=-\frac{(2-\theta)}{1+\theta};$$

quibus valoribus adhibitis aequatio nostra ultima induet hanc formam:

$$\frac{1}{k^3}=\frac{\mu m x^3}{a^2 h}\left\{\begin{aligned}&\frac{(1+\theta)^3}{27\theta^3}+\frac{\nu(1-2\theta)(1+\theta)}{9\theta^2}+\frac{(1+\theta)^3}{27\theta^3 P}\left(1-\frac{\nu(1-2\theta)(2-\theta)}{(1+\theta)^2}\right)\\&+\frac{(1+\theta)^3}{27\theta^3 PQ}\left(1-\frac{3\nu(2-\theta)}{(1+\theta)^2}\right)+\frac{h}{8\theta^3 ma}\left(\lambda'''-6\nu\right)+\frac{27h\lambda''''}{8\theta^3 ma}\end{aligned}\right\},$$

Statuamus nunc brevitatis gratia expressionem uncinulis inclusam

$$A=\frac{(1+\theta)^3}{27\theta^3}\left(1+\frac{1}{P}+\frac{1}{PQ}\right)-\frac{\nu(1+\theta)}{27\theta^3}\left(3\theta(2\theta-1)+\frac{(2\theta-1)(\theta-2)}{P}+\frac{3(2-\theta)}{PQ}\right)$$
$$+\frac{h}{8\theta^3 ma}\left(\lambda'''-6\nu\right)+\frac{27h\lambda''''}{8\theta^3 ma};$$

quae formula, si θ fuerit numerus praemagnus et litterae P et Q unitati aequales reputentur, praebet

$$A=\frac{3-8\nu}{27};$$

qui valor utique multo minor est, quam si lens obiectiva esset vel simplex vel etiam duplicata; unde etiam x maiorem valorem sortietur, qui erit

$$x=\frac{1}{k}\sqrt[3]{\frac{a^2 h}{\mu m A}}$$

et dabit semidiametrum aperturae lentis obiectivae, dummodo ea non fuerit maior, quam figura lentis permittit. Invento autem x erit $y=\frac{h}{ma}\cdot x$ et mensura claritatis $=\frac{20h}{ma}\cdot x$.

COROLLARIUM 1

168. Hae formulae aeque patent ad telescopia atque ad microscopia hoc tantum discrimine intercedente, quod pro telescopiis, ubi $a=\infty$ et $h=a$, sit θ infinite parvum, pro microscopiis autem θ fiat numerus praemagnus.

COROLLARIUM 2

169. Pro microscopiis igitur erit proxima

$$\mathfrak{A}=3,\quad A=-\tfrac{3}{2},\quad \mathfrak{B}=2,\quad B=-2,\quad \mathfrak{C}=1 \text{ et } C=\infty$$

seu numerus praemagnus; tum vero

$$\Lambda=\tfrac{1}{27}\left(1+\tfrac{1}{P}+\tfrac{1}{PQ}\right)-\tfrac{\nu}{27}\left(6+\tfrac{2}{P}\right).$$

COROLLARIUM 3

170. At si numeri huius praemagni θ rationem quoque habere velimus, habebimus adhuc propius

$$\mathfrak{A}=3-\tfrac{3}{\theta},\quad A=\tfrac{3}{2}-\tfrac{4}{3\theta},$$
$$\mathfrak{B}=2-\tfrac{3}{\theta},\quad B=-2-\tfrac{3}{\theta},$$
$$\mathfrak{C}=1-\tfrac{3}{\theta},\quad \mathrm{C}=\tfrac{\theta}{3}-1;$$

tum vero etiam adcuratius erit

$$\Lambda=\tfrac{1}{27}\left(1+\tfrac{1}{P}+\tfrac{1}{PQ}\right)+\tfrac{1}{9\theta}\left(1+\tfrac{1}{P}+\tfrac{1}{PQ}\right)-\tfrac{\nu}{27}\left(6+\tfrac{2}{P}\right)-\tfrac{\nu}{9\theta}\left(1-\tfrac{1}{P}-\tfrac{1}{PQ}\right).$$

COROLLARIUM 4

171. Quod nunc ad intervalla lentium priorum attinet, si sumamus utrumque eorum esse debere $=\zeta p\ =\zeta\mathfrak{A}a$, valoribus prioribus proxime veris adhibendis reperiemus

$$P=\tfrac{1}{1+2\zeta} \text{ et } Q=\tfrac{1}{1+P\zeta}=\tfrac{1+2\zeta}{1+3\zeta},$$

unde, si statuamus $\zeta=\tfrac{1}{10}$, erit

$$P=\tfrac{5}{6} \text{ et } Q=\tfrac{12}{13} \text{ hincque } PQ=\tfrac{10}{13}$$

et

$$\Lambda=\tfrac{7}{54}-\tfrac{14\nu}{45}+\tfrac{7}{18\theta}+\tfrac{\nu}{6\theta}.$$

COROLLARIUM 5

172. Cum autem valor ν a ratione vitri pendeat, notetur pro vitro coronario, quo est $n=1,53$, esse propemodum $\nu=\tfrac{1}{5}$ et pro vitro crystallino, quo $n=1,58$, esse $\nu=\tfrac{1}{4}$; hinc ergo colligitur pro vitro coronario fore

$$\Lambda = \tfrac{91}{1350} + \tfrac{19}{45\theta}.$$

Pro vitro autem crystallino erit

$$\Lambda = \tfrac{7}{135} + \tfrac{31}{72\theta};$$

ex quo perspicitur plurimum praestare, si tres priores lentes ex vitro crystallino parentur.

SCHOLION 1

173. Quod nunc ad lentium constructionem attinet, quia pro tribus prioribus numeri $\lambda,\ \lambda'$ et λ'' unitati aequales sunt positi, ut scilicet singulae minimam confusionem producant, sufficiet litteris $\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C}$ valores proximos tribuisse, ita ut tuto capere liceat $\mathfrak{A} = 3,\ \mathfrak{B} = 2$ et $\mathfrak{C} = 1;$ unde secundum praecepta generalia singulae hae lentes pro distantiis focalibus datis $p,\ q,\ r$ construi poterunt, ubi notasse iuvabit esse

$$q = \tfrac{p}{P} = \tfrac{6}{5}p \quad \text{et} \quad r = \tfrac{p}{PQ} = \tfrac{13}{10}p;$$

licebit enim nunc distantiam focalem p tanquam cognitam spectare ex eaque distantiam obiecti definire, quae erit

$$a = \tfrac{1+\theta}{3\theta} \cdot p = \tfrac{1}{3}p\left(1 + \tfrac{1}{\theta}\right);$$

tum vero littera θ commodissime definitur ex lente quarta, cuius distantia focalis $s,$ si itidem ut cognita spectetur, erit $\theta = \tfrac{ms}{2h}$, ita ut nunc habeatur

$$a = \tfrac{1}{3}p\left(1 + \tfrac{2h}{ms}\right).$$

Tum vero erit $t = \tfrac{1}{3}s$ et intervallum ultimum $= \tfrac{2}{3}s,$ dum duo priora intervalla sunt per hypothesin $= \tfrac{1}{10}p.$ Tertium vero intervallum maxime a multiplicatione pendebit; erit enim id

$$= \theta a\left(\tfrac{13}{10} - \tfrac{h}{ma}\right) = \tfrac{13msa}{20h} - \tfrac{1}{2}s = \tfrac{13mps}{60h} + \tfrac{13}{30}p - \tfrac{1}{2}s,$$

ex quo patet, quo maior multiplicatio desideretur, eo magis instrumentum elongari debere; tum vero etiam apertura primae lentis inprimis a multiplicatione pendet; ex formula enim supra inventa, cum sit proxima

$$\mu = 1,\quad a = \tfrac{1}{2}p \quad \text{et} \quad \Lambda = \tfrac{7}{135} \text{ pro vitro crystallino,}$$

si, ut supra fecimus, sumamus $k = 20,$ [$h =$ 8 dig.], obtinebimus

$$x=\frac{1}{10}\sqrt[3]{\frac{15p^2}{7m}}\text{ dig.}=\frac{1}{10}\sqrt[3]{\frac{135a^2}{7m}}\text{ dig.};$$

quare, si ut supra distantiam obiecti circiter dimidii digiti statuamus, ut sit $p=\frac{3}{2}$ dig., fiet

$$x=\frac{1}{10}\sqrt[3]{\frac{135}{28m}}\text{ dig.}=\frac{0,1689}{\sqrt[3]{m}}\text{ dig.}$$

Porro autem pro claritate erit

$$y=\frac{8}{10m}\sqrt[3]{\frac{135}{7ma}}$$

et mensura claritatis

$$=\frac{16}{m}\sqrt[3]{\frac{135}{7ma}}$$

et casu $a=\frac{1}{2}$ dig. erit ea

$$=\frac{54,060}{m\sqrt[3]{m}}.$$

Ex his igitur statim poterimus eiusmodi microscopium conficere, quod retentis iisdem lentibus ad omnes multiplicationes producendas sit accommodatum; utamur autem ut hactenus vitro communi, pro quo est $n=1,55$, ita ut valorem ipsius x aliquantillum imminui conveniat, uti cuique lubuerit; ac tum pro lente prima, cuius distantia focalis $=p$ et $\mathfrak{A}=3$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris } =\frac{p}{\sigma-3(\sigma-\rho)}=-\frac{p}{2,6827}=-0,3728p\\ \text{posterioris}=\frac{p}{\rho+3(\sigma-\rho)}=+\frac{p}{4,5008}=+0,2222p,\end{cases}$$

quae ergo aperturam admittit, cuius semidiameter erit circiter

$$x=0,055p=\frac{1}{18}p.$$

Pro secunda autem lente, cuius distantia focalis $q=\frac{6}{5}p$, erit radius

$$\text{radius faciei }(\text{ob } \mathfrak{B}=2)\begin{cases}\text{anterioris } =\frac{q}{\sigma-2(\sigma-\rho)}=-\frac{q}{1,2460}=-0,8026p\\ \text{posterioris}=\frac{q}{\rho+2(\sigma-\rho)}=+\frac{q}{3,0641}=+0,3274q.\end{cases}$$

Pro lente autem tertia, cuius distantia focalis est $r=\frac{13}{10}p$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris } = \frac{r}{\rho} = 5,2438r\\ \text{posterioris} = \frac{r}{\sigma} = 0,6145r.\end{cases}$$

Videtur autem hic commode sumi posse $p = 1\frac{1}{2}$ dig., ut fiat circiter $a = \frac{1}{2}$ dig., tum vero $s = 1$ dig., ut fiat $t = \frac{1}{3}$ dig.; unde orietur sequens

CONSTRUCTIO MICROSCOPII EX QUINQUE LENTIBUS COMPOSITI AD OMNES MULTIPLICATIONES IDONEI

174. Si omnes lentes ex vitro communi, pro quo est $n = 1,55$, parentur, habebitur:

I. Pro lente prima, cuius distantia focalis est $p = 1\frac{1}{2}$ dig.,

$$\text{radius faciei}\begin{cases}\text{anterioris } = -0,5592 \text{ dig.}\\ \text{posterioris} = +0,3333 \text{ dig.;}\end{cases}$$

cuius semidiameter aperturae posset esse x = 0,0833 dig., verum ob multiplicationem datam $= m$ sumi conveniet

$$x = \frac{0,15}{\sqrt[3]{m}} \text{ dig.}$$

et distantia ad lentem secundam $= 0,15$ dig.

II. Pro lente secunda, cuius distantia focalis est $q = \frac{18}{10}$ dig., capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris } = -1,4447 \text{ dig.}\\ \text{posterioris} = +0,5875 \text{ dig.;}\end{cases}$$

apertura modo maior sit praecedente,
distantia ad lentem tertiam $= 0,15$ dig.

III. Pro lente tertia, cuius distantia focalis est $r = \frac{39}{20}$ dig., capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris } = 10,2255 \text{ dig.}\\ \text{posterioris} = \ \ 1,1983 \text{ dig.;}\end{cases}$$

de apertura idem tenendum quod ante
et distantia ad lentem quartam erit

$$= \left(\frac{13m}{320} - \frac{1}{2}\right) \text{ dig.}$$

IV. Pro quarta lente, cuius distantia focalis est $s=1$ dig., capiatur radius utriusque faciei $=1{,}1$ dig., aperturae semidiameter $=\frac{1}{4}$ dig.,
et distantia ad lentem quintam $=\frac{2}{3}$ dig. $=0{,}67$ dig.

V. Pro quinta lente, cuius distantia focalis $=\frac{1}{3}$ dig., capiatur
radius utriusque faciei $=0{,}37$ dig.,
eius aperturae semidiameter $=\frac{1}{12}$ dig.,et distantia oculi $=\frac{1}{6}$ dig. $=0{,}17$ dig.

VI. Semidiameter spatii in obiecto conspicui erit $z=\frac{1}{2}\frac{ah}{ma+h}$ existente distantia obiecti

$$a=\frac{1}{3}p\left(1+\frac{2h}{ms}\right)=\frac{1}{2}\left(1+\frac{16}{m}\right)\text{ dig.}$$

VII. Cum sit semidiameter aperturae lentis obiectivae

$$x=\frac{0{,}15}{\sqrt[3]{m}}\text{ dig.},$$

fiet hinc

$$y=\frac{hx}{ma}=\frac{2{,}40}{m\sqrt[3]{m}}$$

et mensura claritatis

$$=\frac{48}{m\sqrt[3]{m}},$$

ita ut, si fuerit $m=512$, mensura claritatis futura sit $=\frac{3}{256}=\frac{1}{85}$, quae adhuc 34 vicibus maior est quam claritas lunde plenae.

VIII. Subiungamus adhuc tabellam, in qua pro praecipuis multiplicationibus m exhibeantur

1. distantia obiecti a lente obiectiva $a=\frac{1}{2}\left(1+\frac{16}{m}\right)$ dig.,
2. intervallum lentium tertiae et quartae, quod sit $l=\left(\frac{13m}{320}-\frac{1}{2}\right)$ dig.
3. semidiameter aperturae lentis obiectivae $x=\frac{0{,}15}{\sqrt[3]{m}}$ dig. et 4. claritas $x=\frac{48}{m\sqrt[3]{m}}$ dig.

m	*a*	*l*	*x*	*Claritas*
50	0,660	1,531	0,041	0,261
100	0,580	3,563	0,032	0,103
200	0,540	7,625	0,026	0,041
300	0,527	11,688	0,022	0,024
400	0,520	15,750	0,020	0,016
500	0,516	19,813	0,019	0,012
600	0,513	23,875	0,018	0,009

700	0,511	27,938	0,017	0,008
800	0,510	32,000	0,016	0,006
900	0,509	36,068	0,016	0,006
1000	0,508	40,125	0,015	0,005

SCHOLION 2

175. Cum formulae nostrae inventae aeque ad telescopia ae microscopia pateant, dum illo casu ponitur $\theta = 0$, hoc vero $\theta =$ numero praemagno, operae pretium videtur adcuratius investigare, cuiusmodi instrumenta sint proditura et ad quemnam usum futura sint accommodata, si litterae θ valor mediocris veluti 1 vel 2 tribuatur; hunc in finem sumamus $\theta = 2$, ut sit

$$s = \tfrac{4h}{m} \quad \text{et} \quad t = \tfrac{4h}{3m} \quad \text{hincque} \quad m = \tfrac{4h}{s};$$

tum vero sequentes habebuntur valores:

$$\mathfrak{A} = 2, \quad \mathfrak{B} = 1, \quad \mathfrak{C} = 0$$

ideoque

$$A = -2, \quad B = \infty \quad \text{et} \quad C = 0,$$

ita tamen, ut sit

$$BC = -1 \quad \text{et} \quad \mathrm{B}\mathfrak{C} = -1;$$

ex quibus valoribus distantiae focales lentium priorum erunt

$$p = 2a, \quad q = \tfrac{2a}{P}, \; r = \tfrac{2a}{PQ}$$

et intervalla

$$\text{primum} = -2a(1 - \tfrac{1}{P}), \quad \text{secundum} = \infty(1 - \tfrac{1}{Q}),$$

$$\text{tertium } = \tfrac{2a}{PQ}\left(1 - \tfrac{1}{R}\right) = 2a\left(\tfrac{1}{PQ} - \tfrac{h}{ma}\right) \text{ et quartum } = \tfrac{8h}{3m}$$

manante distantia oculi $O = \frac{1}{2}t.$ Ut iam fiat primum intervallum $= \frac{1}{10}p$, sumi dabebit $P = \frac{10}{11}$, at Q semper debet esse $= 1$, quantumvis secundum intervallum accipiatur; conveniet autem primo aequale sumi, ita ut sit $q = \frac{11}{5}a$ et $r = \frac{11}{5}a$ tertiumque intervallum $= 2a\left(\frac{11}{10} - \frac{h}{ma}\right)$; deinde ut ante erit

$$z = \tfrac{1}{2} \cdot \tfrac{ah}{ma+h};$$

Verum nunc obtinebimus

$$\Lambda = \frac{2}{5} - \frac{\nu}{4} + \frac{h}{64ma}\left(\lambda''' - 6\nu\right) + \frac{27h\lambda''''}{64ma},$$

qui valor pro casu $\nu = \frac{1}{5}$ foret $\Lambda = \frac{7}{20}$, at pro casu $\nu = \frac{1}{4}$ foret $\Lambda = \frac{27}{80}$ seu utroque casu proxima $\Lambda = \frac{1}{3}$; hinc ergo colligimus:

$$x = \frac{1}{k}\sqrt[3]{\frac{3a^2h}{\mu m}}$$

seu

$$x = \frac{1}{20}\sqrt[3]{\frac{3a^2h}{m}}$$

indeque porro

$$y = \frac{h}{20m}\sqrt[3]{\frac{3h}{ma}}$$

et mensura claritatis

$$y = \frac{h}{m}\sqrt[3]{\frac{3h}{ma}}.$$

His notatis, quae ad instrumenti constructionem pertinent, sequentia observentur:

1. Si caperetur $s = 1$ dig., foret $m = 4h$ dig. seu $\frac{h}{m} = \frac{1}{4}$ dig., quem valorem ita interpretari oportet, quod instrumentum nobis obiecta *m* vicibus maiora repraesentet, quam si ea in distantia *h* spectaremus; unde patet obiecta nobis in eadem magnitudine repraesentari, quam si ea nudis oculis spectaremus in distantia $= \frac{h}{m}$, ex quo perspicuum est instrumentum, de quo hic agitur, nobis obiecta eadem magnitudine esse repraesentaturum, quam si ea cerneremus in distantia $= \frac{1}{4}$ dig. sublata scilicet summa confusione, qua obiecta tam vicina nos adficerent.

2. Si distantiam focalem *s* maiorem vel minorem uno digito assumeremus, multiplicatio etiam fieret vel minor vel maior; praxis autem minorem valorem pro *s* vix admittit, propterea quod $t = \frac{1}{3}s$, minorem vero multiplicationem nemo magnopere desiderabit; unde iste valor $s = 1$ dig. nostro scopo maxime accommodatus videtur.

3. Huiusmodi ergo instrumentum tum usum praestare poterit, quando obiecta ita spectare optamus, quasi ea in distantia $= \frac{1}{4}$ dig. intueremur, vel, quod eodem redit, 32 vicibus maiora, quam si ea in distantia octo digitorum aspiceremus, sicque hoc instrumentum idem praestabit, quod microscopium tricies et his multiplicans.

4. Quia autem in microscopiis distantia obiecti admodum parva sumi solet, hoc instrumentum tum potissimum usurpari poterit, quando ad obiecta non pro lubitu appropinquare licet; quamobrem, si distantia obiecti *a* aliquanto maior fuerit, quam in microscopiis admitti solet, videamus, quomodo nostrum instrumentum tum futurum sit

comparatum; statuamus igitur praeterea $a = 1$ ped. $= 12$ dig. manente $s = 1$ dig. et distantiae focales lentium hoc modo determinabuntur:

$$p = 24 \text{ dig.}, \quad q = 26,4 \text{ dig.}, \quad r = 26,4 \text{ dig.}, \quad s = 1 \text{ dig. et } t = \tfrac{1}{3} \text{ dig.}$$

Deinde vero intervalla

$$\text{primum} = 2,4 \text{ dig.}, \quad \text{secundum} = 2,4 \text{ dig.},$$
$$\text{tertium} = 25,9 \text{ dig.}, \quad \text{quartum} = 0,67 \text{ dig.}$$

Aperturae vero primae lentis semidiameter nunc erit

$$x = \tfrac{1}{20}\sqrt[3]{\tfrac{3\cdot 144}{4}} \text{ dig.} = \tfrac{1}{20}\sqrt[3]{108} \text{ dig.} = 0,238 \text{ dig.}$$

hincque mensura claritatis $= 0,099$ sicque ipsa claritas 100 vicibus minor erit, quam si idem obiectum nudis oculis aspicimus, quae sola circumstantia huiusmodi instrumenta ab usu practico excluderet, nisi longitudo eorum iam satis esset incommoda; quin etiam, si propius ad obiecta accedere liceat, nihil impedit, quominus microscopio ordinario utamur, praecipue si tam exigua multiplicatione contenti esse velimus; idem adeo praestaret microscopium simplex distantiae focalis $= \tfrac{1}{4}$ dig.

SCHOLION 3

176. Easdem igitur nostras formulas nunc etiam ad telescopia adplicemus, ubi est $a = \infty$ et sumitur $h = a$; tum igitur capi oportet $\theta = 0$, ita tamen, ut θa fiat quantitas finita; cum igitur sit

$$p = \tfrac{3\theta}{1+\theta}\cdot a = 3\theta a,$$

ita ut sit $\theta a = \tfrac{1}{3}p$, unde erit porro

$$q = \tfrac{p}{P} \text{ et } r = \tfrac{p}{PQ}, \text{ at } s = \tfrac{2p}{3m} \text{ et } t = \tfrac{2p}{9m};$$

tum vero intervalla lentium

$$\text{primum} = p\left(1 - \tfrac{1}{P}\right), \quad \text{secundum} = \tfrac{p}{2P}\left(1 - \tfrac{1}{Q}\right),$$
$$\text{tertium} = \tfrac{p}{3}\left(\tfrac{1}{PQ} - \tfrac{1}{m}\right), \quad \text{quartum} = \tfrac{4p}{9m}$$

pro loco oculi manente

$$O = \tfrac{1}{2}t\left(1 + \tfrac{1}{m}\right).$$

Faciamus nunc duo priora intervalla inter se aequalia, et quia lentes obiectivae iam sunt multo maiores, statuamus utrumque $\tfrac{1}{25}p$ et reperietur

$$P = \tfrac{25}{24} \text{ et } Q = \tfrac{12}{11} \text{ hincque } PQ = \tfrac{25}{22};$$

unde superiores valores erunt

$$q = \tfrac{24}{25}\, p \ \text{ et } \ r = \tfrac{22}{25}\, p$$

tertiumque intervallum

$$= \tfrac{p}{3}\left(\tfrac{22}{25} - \tfrac{1}{m}\right) = \tfrac{22}{75}\, p - \tfrac{p}{3m}.$$

Pro campo porro apparente fiet eius semidiameter

$$\Phi = \tfrac{z}{a} = \tfrac{2}{m+1}\cdot \xi = \tfrac{1}{4}\cdot \tfrac{2}{m+1} = \tfrac{1718}{m+1} \text{ min.}$$

Denique aequatio pro distinctione visionis erit

$$\tfrac{\mu m x^3}{p^3}\left(\tfrac{71}{25} - \tfrac{36}{5}\nu + \tfrac{27}{8m}\left(\lambda''' - 6\nu\right) + \tfrac{729\lambda''''}{8m}\right) = \tfrac{1}{k^3}.$$

Hic iam proponi solet gradus claritatis, quo obiecta repraesententur, qui sit $y = \tfrac{1}{50}$ dig ., sumique debet $x = my = \tfrac{m}{50}$ dig. et capiatur etiam ut in libro superiore $k = 50$; quibus positis reperietur

$$p = m\sqrt[3]{\mu m\left(\tfrac{71}{25} - \tfrac{36}{5}\nu + \tfrac{27}{8m}\left(\lambda''' - 6\nu\right) + \tfrac{729\lambda''''}{8m}\right)},$$

ubi, si vitro communi utamur, erit $\mu = 0,9381$ et $\nu = 0,2326$; at vero iam sumsimus $\lambda = \lambda' = \lambda'' = 1$, et quia binae postremae lentes debent esse utrinque aeque convexae, esse oportet

$$\lambda'''' = 1,6298 \ \text{ et } \ \lambda''' = 1 + 0,6298\left(1 - 2\mathfrak{D}\right)^2 = 16,745,$$

ex quibus valoribus colligimus

$$p = m\sqrt[3]{\left(1,0931m + 188\right)} \text{ dig.}$$

PROBLEMA 4

177. *Loco lentis obiectivae eiusmodi quatuor lentes convexas proxime sibi iunctas substituere, ut binis reliquis lentibus secundum praecepta superiora constitutis maior claritatis gradus obtineatur.*

SOLUTIO

Cum hic sex habeantur lentes ideoque quinque intervalla, totidem quoque litterae *P, Q, R, S, T* in calculum sunt introducendae; quarum tres priores *P, Q* et *R* unitati proxima sunt aequales, quia quatuor priores lentes sibi proxima iunctae ponuntur; ultima vero *T* debet esse negativa sive $T = -\,k$, quia imago realis in ultimum intervallum incidit, sicque habebitur

$$PQRSk = \frac{ma}{h}$$

deinde distantiae focales lentium nunc ita exprimentur:

$$p = \mathfrak{A}a,\ q = -\frac{A\mathfrak{B}a}{P},\ r = \frac{AB\mathfrak{C}a}{PQ},$$
$$s = -\frac{ABC\mathfrak{D}a}{PQR},\ t = \frac{ABCD\mathfrak{E}a}{PQRS} \text{ and } u = ABCDE \cdot \frac{h}{m}.$$

Intervalla vero lentium ita se habent:

$$\text{Primum} = Aa(1 - \tfrac{1}{P}),$$
$$\text{secundum } = -ABa\left(\tfrac{1}{P} - \tfrac{1}{PQ}\right),$$
$$\text{tertium } = ABCa\left(\tfrac{1}{PQ} - \tfrac{1}{PQR}\right),$$
$$\text{quartum} = -ABCDa\left(\tfrac{1}{PQR} - \tfrac{1}{PQRS}\right)$$
$$\text{quintum } = ABCDEa\left(\tfrac{1}{PQRS} - \tfrac{h}{ma}\right).$$

Distantia vero oculi erit ut ante

$$O = \tfrac{1}{2}u\left(1 + \tfrac{h}{ma}\right)$$

perinde ac spatii conspicui semidiameter

$$z = \tfrac{1}{2} \cdot \tfrac{ah}{ma+h}.$$

Ob hunc ipsum vero campum, ut tantus evadat, oportet esse $\mathfrak{E} = -2$ hincque $E = -\frac{2}{3}$.
Postea autem ut margo coloratus evanescat, debet esse $k = 1$, ita ut sit $PQRS = \frac{ma}{h}$.
Denique ut confusio ab apertura lentium oriunda prodeat minima, ex superioribus colligere licet hoc fieri, si istae expressiones quatuor

$$\mathfrak{A},\ -A\mathfrak{B},\ AB\mathfrak{C},\ -ABC\mathfrak{D}$$

inter se aequales reddantur; unde colligimus has determinationes:

$$1.\ \mathfrak{B} = -\frac{\mathfrak{A}}{A} = \mathfrak{A} - 1,$$
$$2.\ \mathfrak{C} = -\frac{\mathfrak{B}}{B} = \mathfrak{B} - 1 = \mathfrak{A} - 2,$$
$$3.\ \mathfrak{D} = -\frac{\mathfrak{C}}{C} = \mathfrak{C} - 1 = \mathfrak{A} - 3.$$

Deinde vero ponatur $-ABCD = \theta$, ut fiat quintae lentis distantia focalis

$$t = +2\theta \cdot \frac{h}{m}$$

sextaeque

$$u = \tfrac{2}{3}\theta \cdot \tfrac{h}{m} = \tfrac{1}{3}t$$

et intervallum quintum $= \tfrac{2}{3}t = 2u$.

Iam in hac aequatione assumta $ABCD = -\theta$ loco litterarum *A, B, C, D* introducantur litterae germanicae respondentes eritque

$$\tfrac{\mathfrak{A}}{1-\mathfrak{A}} \cdot \tfrac{\mathfrak{B}}{1-\mathfrak{B}} \cdot \tfrac{\mathfrak{C}}{1-\mathfrak{C}} \cdot \tfrac{\mathfrak{D}}{1-\mathfrak{D}} \cdot = -\theta$$

sive

$$\theta = \tfrac{\mathfrak{A}}{1-\mathfrak{D}} = \tfrac{\mathfrak{A}}{4-\mathfrak{A}},$$

unde per θ istae litterae hoc modo definientur:

$$\mathfrak{A} = \tfrac{4\theta}{\theta+1}, \quad \mathfrak{B} = \tfrac{3\theta-1}{\theta+1}, \quad \mathfrak{C} = \tfrac{2\theta-2}{\theta+1} \quad \text{et} \quad \mathfrak{D} = \tfrac{\theta-3}{\theta+1},$$
$$A = -\tfrac{4\theta}{3\theta-1}, \quad B = -\tfrac{3\theta-1}{2\theta-2}, \quad C = -\tfrac{2\theta-1}{\theta-3} \quad \text{et} \quad D = \tfrac{\theta-3}{4};$$

ex quibus valoribus primo distantiae focales ita definientur:

$$p = \tfrac{4\theta a}{\theta+1}, \quad q = \tfrac{4\theta}{\theta+1} \cdot \tfrac{a}{P}, \quad r = \tfrac{4\theta}{\theta+1} \cdot \tfrac{a}{PQ},$$
$$s = \tfrac{4\theta}{\theta+1} \cdot \tfrac{a}{PQR}, \quad t = 2\theta \cdot \tfrac{h}{m} \quad \text{et} \quad u = \tfrac{2}{3}\theta \cdot \tfrac{h}{m}$$

similique modo lentium intervalla:

$$\text{Primum} = -\tfrac{4\theta}{3\theta-1} \cdot a\left(1 - \tfrac{1}{P}\right),$$
$$\text{secundum} = -\tfrac{4\theta}{2\theta-2} \cdot a\left(\tfrac{1}{P} - \tfrac{1}{PQ}\right),$$
$$\text{tertium} = -\tfrac{4\theta}{\theta-3} \cdot a\left(\tfrac{1}{PQ} - \tfrac{1}{PQR}\right),$$
$$\text{quartum} = \theta a\left(\tfrac{1}{PQR} - \tfrac{h}{ma}\right),$$
$$\text{quintum} = \tfrac{2\theta}{3} \cdot a \cdot \tfrac{2h}{ma} = \tfrac{4}{3}\theta \cdot \tfrac{h}{m} = 2u.$$

Quodsi iam velimus, ut trium intervallorum priorum quodlibet fiat

$$= \zeta p = \tfrac{4\theta}{1+\theta} \cdot \zeta a,$$

litterae *P, Q, R* et *S* sequenti modo determinabuntur:

$$\tfrac{1}{P} = 1 + \tfrac{(3\theta-1)}{1+\theta} \cdot \zeta, \quad \tfrac{1}{PQ} = 1 + \tfrac{(5\theta-3)}{1+\theta} \cdot \zeta, \quad \tfrac{1}{PQR} = 1 + \tfrac{(6\theta-6)}{1+\theta} \cdot \zeta.$$

His praemissis aequatio pro dato distinctionis gradu obtinendo sequenti forma exprimi poterit:

$$\frac{1}{k^3}=\frac{\mu m x^3}{a^2 h}\left\{\begin{array}{l}\frac{1}{\mathfrak{A}^3}\left(\lambda+\frac{\lambda'}{P}+\frac{\lambda''}{PQ}+\frac{\lambda'''}{PQR}\right)\\ -\frac{\nu}{\mathfrak{A}^3}\left(\mathfrak{A}\left(\mathfrak{A}-1\right)+\frac{\mathfrak{B}(\mathfrak{B}-1)}{P}+\frac{\mathfrak{C}(\mathfrak{C}-1)}{PQ}+\frac{\mathfrak{D}(\mathfrak{D}-1)}{PQR}\right)\\ +\frac{h}{8\theta^3 ma}\left(\lambda''''-6\nu\right)+\frac{27h\lambda'''''}{8\theta^3 ma}\end{array}\right\},$$

pro qua brevitatis gratia ponamus

$$\frac{1}{k^3}=\frac{\mu m x^3}{a^2 h}\cdot\Lambda,$$

ita ut Λ denotet quantitatem illam uncinulis inclusam, pro qua notetur litteris λ, λ', λ'' et λ''' valorem $=1$ tribui convenire, ut scilicet haec quantitas minima evadat, et quia duae postremae lentes utrinque debent esse aeque convexae, erit

$$\lambda''''=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2\left(1-2\mathfrak{C}\right)^2=1+25\left(\frac{\sigma-\rho}{2\tau}\right)^2$$

et

$$\lambda'''''=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2.$$

Quantitas ergo Λ, si loco $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$ et $\mathfrak{D}$ valores inventi substituantur, ita exprimetur:

$$\Lambda=$$
$$\frac{(1+\theta)^3}{16\theta^3}-\frac{\nu(\theta+1)(5\theta^2-6\theta+5)}{16\theta^3}+\frac{(1+\theta)^2(7\theta-5)}{32\theta^3}\cdot\zeta$$
$$-\frac{\nu(\theta-1)(7\theta^2-18\theta+23)}{16\theta^3}\cdot\zeta+\frac{h}{8\theta^3 ma}\left(\lambda''''-6\nu\right)+\frac{27h\lambda'''''}{8\theta^3 ma}.$$

Atque in hoc negotio id potissimum intenditur, ut valor ipsius Λ vel plane ad nihilum redigatur vel saltim tam exiguus reddatur, ut ex hac aequatione numerus k multo maior prodeat quam 20, etiamsi apertura primae lentis tanta accipiatur, quam eius figura permittit; tum autem hoc valore pro x assumto pro gradu claritatis habebitur $y=\frac{hx}{ma}$ et mensura claritatis fiet

$$=\frac{20hx}{ma}=\frac{160x}{ma}.$$

COROLLARIUM 1

178. Quoniam pro microscopiis θ semper est numerus satis magnus, nisi forte multiplicatio exigua requiratur, quem casum hic merito excludimus, bina postrema membra ipsius Λ manifesto tam sunt parva, ut tuto negligi queant, sicque hic valor aestimari debebit ex prioribus tantum membris

$$\Lambda = \frac{(1+\theta)^3}{16\theta^3} - \frac{v(\theta+1)(5\theta^2-6\theta+5)}{16\theta^3} + \frac{(1+\theta)^2(7\theta-5)}{32\theta^3}\cdot\zeta - \frac{v(\theta-1)(7\theta^2-18\theta+23)}{16\theta^3}\cdot\zeta.$$

COROLLARIUM 2

179. Cum autem sit θ numerus praemagnus, haec expressio reducitur ad sequentem formam proxima veram:

$$\Lambda = \frac{1}{16} - \frac{5}{16}v + \frac{7}{32}\zeta - \frac{7v}{16}\cdot\zeta + \frac{3}{16\theta} + \frac{v}{16\theta} + \frac{9}{32\theta}\cdot\zeta + \frac{25}{16\theta}\cdot\zeta;$$

quae expressio si nihilo esset aequalis, verus valor ipsius Λ sine dubio tam foret exiguus, ut litterae x maximus valor, quem lentis figura permittit, tribui posset.

COROLLARIUM 3

180. Quoniam littera v ab indole vitri pendet, cuius valor, prouti refractio ab $n = 1,50$ usque ad 1,58 augetur, ab $\frac{1}{5}$ usque ad $\frac{1}{4}$ crescit. Sumto $v = \frac{1}{5}$ fiet

$$\Lambda = \frac{21}{160}\zeta + \frac{19}{32\theta}\cdot\zeta + \frac{1}{5\theta};$$

quae partes cum omnes sint positivae, patet, si lentes ex tali vitro parentur, valorem Λ ad nihilum redigi non posse. Sin autem fuerit $v = \frac{1}{4}$, habebitur

$$\Lambda = -\frac{1}{64} + \frac{13}{64\theta} + \frac{7}{64}\zeta + \frac{43}{64\theta}\cdot\zeta,$$

qui valor utique nihilo aequalis esse poterit, quod scilicet eveniet casu $\theta = \infty$, si fuerit $\zeta = \frac{1}{7}$, qui valor ad praxin satis est accommodatus; at si sumamus $\theta = 50$, tum fiet $\Lambda = 0$, si fuerit $\zeta = 9\frac{37}{393}$ seu $\zeta = \frac{1}{11}$, quod etiam praxi maxime convenit.

COROLLARIUM 4

181. Ut igitur valor ipsius Λ ad nihilum redigatur, vitro uti conveniet maiorem refractionem producente, cuiusmodi est vitrum crystallinum, pro quo $n = 1,58$; ac si forte praxis minus successerit, commode hic usu venit, ut lentium priorum intervallis tantillum

mutatis scopo intento satisfieri queat; quod remedium in praxi eo facilius adhibetur, quod in ipsa lentium constructione nulla mutatio exigitur.

SCHOLION 1

182. Quod θ semper sit numerus satis magnus, ex supra traditis facile perspicitur; cum enim penultimae lentis distantia focalis t uno digito minor statui nequeat ob $h=8$ dig., erit $\theta=\frac{m}{16}$ dig.; quare, cum multiplicatio m vix minor desiderari soleat quam 500 vel 480, habebitur hinc $\theta=30$ dig.; maximam autem multiplicationem, quam quidem ob defectum claritatis adhuc desiderare possumus, aestimare licet $m=960$, quo ergo casu erit $\theta=60$ dig., ita ut valores ipsius θ intra 30 et 60 contenti sint aestimandi. Hoc autem notato si priora membra formulae Λ fuerint $=0$, facile intelligetur posteriora membra neutiquam esse turbatura; haec enim ultima membra certe adhuc minora erunt quam $\frac{125}{\theta^3 m}$; unde, si priora membra actu evanescant, prodibit aequatio

$$\frac{1}{k^3}<\frac{\mu m x^3}{a^2 h}\cdot\frac{125}{\theta^3 m}<\frac{125\mu x^3}{a^2 h\theta^3}$$

sive sumto $\theta=30$ erit

$$k^3>\frac{30^3 a^2 h}{125\mu x^3} \quad \text{sive} \quad k>\frac{30}{x}\sqrt[3]{\frac{a^2 h}{125\mu}}.$$

Nunc quod ad valorem ipsius x attinet, observemus, si lens obiectiva esset simplex ideoque eius distantia focalis $p=a$ proxime, tum ob eius figuram capi posse $x=\frac{1}{6}a$ vel certe non maius; etsi autem hic quatuor lentes convexae in locum obiectivae substituantur, quarum singularum distantiae focales sunt fere quadruplo maiores, tamen, quia primae facies anterior est concava ideoque posterioris faciei radius valde parvus, ea vix maiorem aperturam admittet quam lens simplex, ita ut etiam hoc casu x maius capi nequeat quam $\frac{1}{6}a$; sit ergo $x=\frac{1}{6}a$ et sumto $a=\frac{1}{2}$ semper erit $k>90$, quo valore indicatur insignia gradus distinctionis, cum etiam pro optimis telescopiis hic valor non ultra 50 augeri soleat; ex quo concludere licet non ad eo necessarium esse, ut etiam priora membra ipsius Λ penitus evanescant, dummodo ea per m multiplicata non multum superent posteriora; tum autem priora membra fere penitus evanescere debebunt; at iis nihilo aequalibus positis valor numeri ζ ita in genere determinabitur, ut sit

$$\zeta=\frac{+5\nu-1-\frac{(3+\nu)}{\theta}-\frac{(3+\nu)}{\theta^2}+\frac{5\nu-1}{\theta^3}}{\frac{7}{2}-7\nu+\frac{9+50\nu}{2\theta}-\frac{(3+82\nu)}{2\theta^2}-\frac{(5-46\nu)}{2\theta^3}},$$

ubi inprimis cavendum est, ne littera ζ nimis fiat parva, quam ut intervallum ζp

commode in praxi locum habere queat, id quod obtinetur, dummodo ζ non notabiliter minor prodeat quam $\frac{1}{20}$; quamobrem operae pretium erit investigare, an etiam vitro communi ad hunc scopum uti liceat, quandoquidem iam vidimus crystallinum satis esse idoneum; cum igitur pro vitro communi sit $n=1,55$ et $v=0,2326$, fiet

$$\zeta=\frac{0,1630-\frac{3,2326}{\theta}-\frac{3,2326}{\theta^2}+\frac{0,1630}{\theta^3}}{1,8718+\frac{10,3150}{\theta}-\frac{11,0366}{\theta^2}-\frac{2,8498}{\theta^3}};$$

hic autem primum observari convenit, si esset $\theta=\infty$ fore $=\frac{1}{11}$ circiter, qui valor utique ad praxin maxime esset accommodatus; at si sumamus $\theta=30$, orietur $\zeta=\frac{1}{43}$, qui valor nimis est exiguus; unde patet pro θ maiorem valorem accipi debere. Sumto autem $\theta=50$ reperitur $\zeta=\frac{0,0971}{2,0340}=\frac{1}{22}$ proxime, qui valor adhuc admitti commode poterit. Sumto autem $\theta=60$ eruitur $\zeta=\frac{0,1082}{2,0407}=\frac{1}{18}$ proxime , qui valor praxi egregie convenire videtur. Hunc igitur casum sequenti exemplo fusius evolvamus.

EXEMPLUM 1

183. Si omnes lentes ex vitro communi, pro quo est $n=1,55$, conficiantur ac sumatur $\theta=60$, ut microscopium adeo ad multiplicationem $m=1000$ adhiberi possit, momenta constructionis sequenti modo se habebunt.

Primo scilicet habebimus

$$\mathfrak{A}=\frac{240}{61}=4-\frac{4}{61}=--=4-\frac{1}{15}\text{ proxime,}$$

$$\mathfrak{B}=3-\frac{1}{15},\quad \mathfrak{C}=2-\frac{1}{15},\quad \mathfrak{D}=1-\frac{1}{15}$$

atque porro

$$\frac{1}{P}=1+\left(3-\frac{1}{15}\right)\zeta;$$

et quia modo vidimus sumi debere $\zeta=\frac{1}{18}$, erit

$$\frac{1}{P}=1+\frac{1}{6}-\frac{1}{270}\quad=1,1629,$$
$$\frac{1}{PQ}=1+\frac{5}{18}-\frac{2}{270}=1,2703,$$
$$\frac{1}{PQR}=1+\frac{1}{3}-\frac{1}{90}\quad=1,3222\ ;$$

unde distantiae focales lentium erunt

$$p=\left(4-\tfrac{1}{15}\right)a=3,9333a,\quad q=\tfrac{p}{P}=4,5740a,$$
$$(0,5947571)\qquad (0,6602996)$$
$$r=\tfrac{p}{PQ}=4,9965a,\qquad s=\tfrac{p}{PQR}=5,2006a.$$
$$(0,6986634)\qquad (7160543)$$

Tum vero
$$t=\tfrac{960}{m}\text{ dig. et } u=\tfrac{320}{m}\text{ dig.}$$

Harum porro quatuor priorum lentium intervallum commune est
$$=\tfrac{1}{18}p=0,2185a.$$
Quartum vero intervallum erit
$$=79,332a-\tfrac{480}{m}\text{ dig.}$$
Quintum vero
$$=2u=\tfrac{640}{m}\text{ dig.}$$
et distantia oculi
$$O=\tfrac{1}{2}u=\tfrac{160}{m}\text{ dig.}$$

Nunc igitur singularum lentium constructio est describenda

I. Pro prima lente

cuius distantia focalis est $p=-\ 3,9333a$ et numeri
$$\lambda=1,\ \mathfrak{A}=4-\tfrac{1}{15},$$
erit
$$\text{radius}\begin{cases}\text{anterioris }=\frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)}=-\frac{p}{4,0236}=-0,97756a\\ \text{posterioris}=\frac{p}{\rho+\mathfrak{A}(\sigma-\rho)}=+\frac{p}{5,8417}=+0,67332a,\end{cases}$$

quae aperturam admittit, cuius semidiameter $x=0,16833a.$

II. Pro secunda lente

cuius distantia focalis est $q=4,5740a$ et numeri
$$\lambda=1\text{ et }\mathfrak{B}=3-\tfrac{1}{15},$$
erit
$$\text{radius}\begin{cases}\text{anterioris }=\frac{q}{\sigma-\mathfrak{B}(\sigma-\rho)}=-\frac{q}{2,5869}=-1,7682a\\ \text{posterioris}=\frac{q}{\rho+\mathfrak{B}(\sigma-\rho)}=+\frac{q}{4,4050}=+0,0384a.\end{cases}$$

III. Pro lente tertia

cuius distantia focalis est $r = 4,9965a$ et numeri

$$\lambda = 1 \text{ et } \mathfrak{C} = \mathfrak{A} - 2,$$

erit

$$\text{radius} \begin{cases} \text{anterioris } = \frac{r}{\sigma - \mathfrak{C}(\sigma - \rho)} = -\frac{r}{1,1503} = -4,3440a \\ \text{posterioris} = \frac{r}{\rho + \mathfrak{C}(\sigma - \rho)} = +\frac{r}{2,9683} = +1,6833a. \end{cases}$$

IV. Pro quarta lente

cuius distantia focalis $s = 5,2006a$ et numeri

$$\lambda = 1 \text{ et } \mathfrak{D} = \mathfrak{A} - 3,$$

erit

$$\text{radius} \begin{cases} \text{anterioris } = \frac{s}{\sigma - \mathfrak{D}(\sigma - \rho)} = \frac{s}{0,2865} = 18,1522a \\ \text{posterioris} = \frac{s}{\rho + \mathfrak{D}(\sigma - \rho)} = \frac{s}{1,5316} = +3,3955a. \end{cases}$$

Hinc ergo deducitur sequens

CONSTRUCTIO MICROSCOPll EX SEX LENTIBUS COMPOSITI REFRACTIONE VITRI EXISTENTE $n = 1,55$

184. Pro hoc microscopio sumitur *m* numerus praemagnus arbitrarius, quippe a quo tantum binae lentes posteriores pendent.

I. Pro prima lente

cuius distantia focalis $p = 3,9333a$, erit

$$\text{radius faciei} \begin{cases} \text{anterioris } = -0,97756a \\ \text{posterioris} = +0,67332a, \end{cases}$$

eius aperturae semidiameter $= 0,16833a$

et distantia ad lentem secundam $= 0,2185a$.

II. Pro secunda lente

cuius distantia focalis $q = 4,5740a$, erit

$$\text{radius faciei} \begin{cases} \text{anterioris } = -1,7682a \\ \text{posterioris} = +1,0384a; \end{cases}$$

apertura et distantia ad lentem sequentem sunt ut ante.

III. Pro tertia lente

cuius distantia focalis $r = 4,9965a$, est

$$\text{radius faciei } \begin{cases} \text{anterioris } = -4,3440a \\ \text{posterioris} = +1,6833a, \end{cases}$$

apertura et distantia ad lentem sequentem ut ante.

IV. Pro lente quarta

cuius distantia focalis $s = 5,2006a$, erit

$$\text{radius faciei } \begin{cases} \text{anterioris } = 18,1522a \\ \text{posterioris} = 3,3955a, \end{cases}$$

apertura ut ante;
distantia ad lentem quintam vero erit $= 79,332a - \frac{480}{m}$ dig.

V. Pro quinta lente

cuius distantia focalis est $t = \frac{960}{m}$ dig., capiatur

$$\text{radius utriusque faciei } = \frac{1056}{m} \text{ dig.,}$$

eius aperturae semidiameter $= \frac{240}{m}$ dig .
et distantia ad lentem sextam $= \frac{640}{m}$ dig.

VI. Pro lente sexta

cuius distantia focalis $u = \frac{320}{m}$ dig., erit

$$\text{radius utriusque faciei } = \frac{352}{m} \text{ dig.,}$$

eius aperturae semidiameter $= \frac{80}{m}$ dig.
et distantia oculi $= \frac{160}{m}$ dig.

VII. Spatii in obiecto conspicui semidiameter erit $= \frac{4a}{ma+8}$ dig. et mensura claritatis, qua obiecta repraesentabuntur, erit $= \frac{26,9328}{m}$ quae, etiamsi multiplicatio statuatur $m = 1000$, adhuc satis est magna.

VIII. Hoc tantum in hoc genere microscopiorum displicebit forte, quod eorum longitudo, quippe quae fere aequalis est $80a$, tam fit enormis ideoque minus commoda videbitur; sed cum distantiam obiecti facile ad digiti dimidium vel adeo trientem diminuere liceat, nihil impedit, quominus haec microscopia ad quosvis usus adhiberi queant.

IX. Etiamsi hic quaelibet multiplicatio peculiares lentes quintam et sextam postulat, tamen facile intelligitur, si huiusmodi instrumentum ac certam multiplicationem fuerit accommodatum, tum idem etiam tam premaioribus quam pro minoribus multiplicationibus optimo successu adhiberi posse, dum scilicet eius longitudo sive minuitur sive augetur.

X. Denique cum quatuor lentes priores maiores esse debeant quam apertura primae lentis, artifici praecipi poterit, ut disci harum lentium in diametro contineant $\frac{2}{5}a$, ita ut, si fuerit $a=\frac{1}{2}$ dig., diameter horum discorum $\frac{1}{5}$ dig.

EXEMPLUM 2

185. Si omnes lentes ex vitro crystallino parentur, omnia momenta quae ad constructionem microscopiorum pertinent, describere, ita ut fiat $\Lambda=0$. Cum hoc casu sit $n=1,58$, erit $\nu=0,2529$; unde ex formula supra data colligemus

$$\zeta=\frac{0,2645-\frac{3,2529}{\theta}-\frac{3,2529}{\theta^2}+\frac{0,2645}{\theta^3}}{1,7297+\frac{10,8225}{\theta}-\frac{11,8689}{\theta^2}-\frac{3,3167}{\theta^3}};$$

iam si θ esset infinitum, foret $\zeta=\frac{0,2645}{1,7297}=\frac{1}{6}$ circiter; sin autem sumamus ut ante $\theta=60$, prodibit $\zeta=\frac{0,2094}{1,9068}=\frac{1}{9}$ circiter; unde patet, si ipsi ζ, minor valor tribuatur, tum Λ nacturum esse valorem negativum, quem commode in nostrum lucrum convertere poterimus; cum enim tum ex aequatione principali pro hoc casu fiat

$$\Lambda=\frac{-0,2094+1,9068\zeta}{16},$$

si ponamus ut in exemplo praecedente $\zeta=\frac{1}{18}$, fiet

$$\Lambda=-0,0065.$$

Cum autem pro prima lente sumserimus $\lambda=1$, facile intelligitur, si huic λ maior valor tribuatur, fieri posse, ut haec expressio pro Λ penitus evanescat; hunc in finem statuamus $\lambda=1+\omega$, et cum in computo confusionis ex littera $\lambda=1$ nata sit formula $\frac{1}{\mathfrak{A}^2}$, nunc ex valore $\lambda=1+\omega$ nascetur $\frac{1+\omega}{\mathfrak{A}^2}$, ita ut nunc valor Λ augmentum accipiat

$$=\frac{\omega}{\mathfrak{A}^2}=\frac{\omega(1+\theta)^3}{64\theta^3}=0,0164\omega,$$

ita ut fiat

$$\Lambda=0,0164\omega-0,0065.$$

Quare, ut fiat $\Lambda=0$, capi debebit $\omega=\frac{0,0065}{0,0164}=\frac{65}{164}$ sicque pro prima lente statui debebit $\lambda=1+\frac{65}{164}$, manentibus pro tribus lentibus sequentibus $\lambda'=1=\lambda''=\lambda'''$; quo effici poterit, ut prima lens aliquanto maioris aperturae capax reddatur. Cum igitur sit ut in exemplo praecedente $\theta = 60$ et $\zeta = \frac{1}{18}$, tam distantiae focales quam intervalla eosdem quoque valores retinebunt tantumque superest, ut singularum lentium constructio doceatur. erit

I. Pro prima autem lente

cuius distantia focalis $p = 3,9333a$ et numeri

$$\lambda = 1 + \omega \text{ et } \mathfrak{A} = 4 - \tfrac{1}{15},$$

erit

$$\text{radius}\begin{cases}\text{anterioris } = \frac{p}{\sigma - \mathfrak{A}(\sigma-\rho) + \tau\sqrt{\omega}} = \frac{p}{-4,0865+0,5524} = -1,1129a\\ \text{posterioris} = \frac{p}{\rho + \mathfrak{A}(\sigma-\rho) - \tau\sqrt{\omega}} = +\frac{p}{5,8106-0,5524} = +0,7480a,\end{cases}$$

unde haec lens aperturam admittit, cuius semidiameter $x = 0,1870a$

II. Pro secunda lente

cuius distantia focalis est $q = 4,5740a$, erit

$$\text{radius}\begin{cases}\text{anterioris } = -\frac{q}{2,6452} = -1,7292a\\ \text{posterioris} = +\frac{q}{4,3693} = +1,0469a.\end{cases}$$

III. Pro tertia lente

cuius distantia focalis $q = 4,9965a$, erit

$$\text{radius}\begin{cases}\text{anterioris } = \frac{r}{-1,2039} = -4,1503a\\ \text{posterioris} = +\frac{r}{2,9280} = +1,7065a.\end{cases}$$

IV. Pro lente quarta

cuius distantia focalis $s = 5,2006a$, erit

$$\text{radius}\begin{cases}\text{anterioris } = \frac{s}{0,2375} = 21,8973a\\ \text{posterioris} = +\frac{s}{1,48663} = 3,4983a.\end{cases}$$

Hinc ergo sequitur

CONSTRUCTIO MICROSCOPII EX SEX LENTIBUS COMPOSITI

186. Constructis ex vitro crystallino, pro quo $n=1,58$, quaternis lentibus prioribus, quemadmodum modo est praeceptum, pro data obiecti distantia $=a$ statuantur intervalla inter has lentes $=\frac{1}{18}p=0,2185a$ et priori lenti tribuatur apertura, cuius semidiameter $x=0,1870a$, et intervallum a quarta harum lentium usque ad quintam

$$=79,332a-\frac{480}{m}\text{ dig.}$$

V. Pro quinta lente

cuius distantia focalis $t=\frac{960}{m}$ dig.
et quam una cum sexta ex vitro communi conficere licebit, capiatur

radius utriusque faciei $=\frac{1056}{m}$ dig.,
eius aperturae semidiameter $=\frac{240}{m}$ dig.
et distantia ad lentem sextam $=\frac{640}{m}$ dig.

VI. Pro lente sexta

cuius distantia focalis $u=\frac{320}{m}$ dig., erit
radius utriusque faciei $=\frac{352}{m}$ dig.,
eius aperturae semidiameter $=\frac{80}{m}$ dig.
et distantia oculi $=\frac{160}{m}$ dig.

VII. Spatii in obiecto conspicui semidiameter erit $\frac{4a}{ma+8}$ dig.; at mensura claritas fiet $=\frac{29,920a}{m}$, satis notabiliter maior quam in exemplo praecedente.
Ceterum eadem hic erunt observanda, quae supra sunt allata.

COROLLARIUM 5

187. His duobus microscopiorum generibus inter se comparandis istud insigne commodum consequimur, quod, si forte vitrum occurrat, cuius refractio medium quodpiam teneat inter refractiones $n=1,55$ et $n=1,58$, tum per regulam interpolationum constructio lentium facile definiri queat.

SCHOLION 2

188. Accommodemus formulas, quas in hoc problemate invenimus, etiam ad telescopia, quandoquidem hic determinationes aliquantum differentes in duximus. Cum igitur sit $a=\infty$ et $h=a$, debebit esse $\theta=0$, sed ita tamen, ut fiat $\theta a=$ quantitati finitae, ponaturque $\theta a=l$; tum ergo fient elementa nostra

$$\mathfrak{A} = 4\theta,\ \ \mathfrak{B} = -1,\ \ \mathfrak{C} = -2,\ \ \mathfrak{D} = -3 \ \text{ et } \ \mathfrak{E} = -2$$

hincque

$$A = 4\theta,\ \ B = -\tfrac{1}{2},\ \ C = -\tfrac{2}{3},\ \ D = -\tfrac{3}{4} \ \text{ et } \ E = -\tfrac{2}{3},$$

tum vero

$$\tfrac{1}{P} = 1-\zeta,\ \ \tfrac{1}{PQ} = 1-3\zeta \ \text{ et } \ \tfrac{1}{PQR} = 1-6\zeta.$$

Quare distantiae focales lentium erunt

$$p = 4l,\ \ q = 4l(1-\zeta),\ r = 4l(1-3\zeta),\ s = 4l(1-6\zeta),$$
$$t = \tfrac{2l}{m} \ \text{ et } \ u = \tfrac{2}{3}\cdot\tfrac{2l}{m}$$

et lentium intervalla

$$\text{primum} = \text{secundo} = \text{tertio} = \xi p = 4\zeta l,$$
$$\text{quartum} = l(1-6\zeta) - \tfrac{l}{m},\quad \text{quintum} = \tfrac{4}{3}\cdot\tfrac{l}{m};$$

ac denique distantia oculi $O = \frac{1}{3}\cdot\frac{l}{m}\left(2+\frac{1}{m}\right)$.
Porro vero campi apparentis semidiameter

$$\Phi = \tfrac{z}{a} = \tfrac{1718}{m+1} \text{ min.}$$

Denique aequatio pro sufficiente distinctione comparanda erit

$$\tfrac{1}{k^3} = \tfrac{\mu m x^3}{l^3}\left\{\tfrac{1}{16} - \tfrac{5}{32}\zeta - \tfrac{v}{16}\left(5-23\zeta\right) + \tfrac{1}{8m}\left(\lambda'''' - 6v\right) + \tfrac{27\lambda'''''}{8m}\right\},$$

ubi quidem sumsimus $\lambda = \lambda' = \lambda'' = \lambda''' = 1$; tum vero numeri λ'''' et λ''''' inde sumi debent, quod binae postremae lentes utrinque debent esse aequaliter convexae. Quodsi iam velimus, ut haec expressio penitus ad nihilum redigatur, poni oportebit

$$m(1-\tfrac{5}{2}\zeta) - mv(5-23\zeta) + 2(\lambda'''' - 6v) + 54\lambda''''' = 0.$$

Binas autem postremas lentes semper licebit ex vitro communi construere, ubi est $n = 1{,}55$; tum autem erit $\lambda'''' = 16{,}74\frac{1}{2}$ et $\lambda''''' = 1{,}6298$ hincque bina membra postrema dabunt 118,7080, ita ut esse debeat

$$m(1-\tfrac{5}{2}\zeta) - mv(5-23\zeta) + 118{,}7080 = 0;$$

quodsi iam etiam quatuor priores lentes ex eodem vitro communi parentur, ob $v = 0{,}2326$ reperietur

$$-0{,}1630m + 2{,}8498\zeta m + 118{,}7080 = 0$$

adeoque

$$\zeta = \frac{-0,1630m - 118,7080}{2,8498m} \quad \text{seu} \quad \zeta = \frac{-1680m - 1187080}{28498m};$$

hinc ergo pro ζ valor positivus non prodit, nisi sit

$$m > \frac{1187080}{1630} \quad \text{seu} \quad m > 728 \text{ circiter;}$$

tanta vero multiplicatio vim telescopiorum longe superat ac tum quidem deberet esse $\zeta = 0$, cum tamen $\frac{1}{50}$ superare debeat; quod incommodum etiam locum habet, si priores lentes ex vitro crystallino conficiantur, etsi fiat aliquanto minus. Ex quo perspicuum est formulas hic inventas ad telescopia neutiquam tanto successu applicari posse quam ad microscopia, uti modo ostendimus.