SECTION THREE.

CONCERNING COMPOSITE MICROSCOPES,

IN WHICH A SINGLE REAL IMAGE OCCURS;

TO WHICH ALL MICROSCOPES USED SO FAR ARE REFERRED.

CAPTER I

CONCERNING SIMPLER MICROSCOPES OF THIS KIND

134. N.B. Since generally in this kind of microscopes objects shall be represented inverted, in our general formulas there must be written $-m$ in place of m everywhere and besides the letters $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$ etc. all must be assumed negative.

PROBLEM 1

135. *To construct the most simple microscope of this kind, which consists of two lenses only, and to describe its qualitates.*

SOLUTION

Therefore since here only two lenses are present and a real image may be found in the interval between these, the letter P will have a negative value, which shall be $=-k$, thus so that for the magnification there may be had $m=\frac{kh}{a}$ or $k=\frac{ma}{h}$; clearly with a the distance of the object, α the distance of the image after the objective lens, and b the distance of the eyepiece lens after the image, and also there will be $k=\frac{\alpha}{b}$. Then truly with the letter $A=\frac{\alpha}{a}$ introduced the focal length of the first lens will be

$$p=\mathfrak{A}a$$

and of the second lens ,

$$q=\frac{Aa}{k}=\frac{Ah}{m}$$

and the interval between these lenses

$$=Aa\left(1+\frac{1}{k}\right)=aA\left(1+\frac{h}{ma}\right);$$

which therefore as it shall be positive, the number A must be positive and thus also $\mathfrak{A}=\frac{A}{A+1}$ will be positive, thus so that both lenses must be convex. Thence the radius of the area viewed in the object will be

$$z=\frac{\mathfrak{q}}{ma+h}\cdot ah\xi$$

where it is accustome to assume $\xi=\frac{1}{4}$, and so that there may be taken $\mathfrak{q}=1$, each side of the eyepiece lens is agreed to be made convex, thus so that there may be had

$$z=\frac{1}{4}\cdot\frac{ah}{ma+h}$$

Moreover for the position of the eye we will find the distance

$$O=\frac{q}{Ma}\cdot\frac{h}{m}=q\left(1+\frac{h}{ma}\right)=\frac{Ah}{m}\left(1+\frac{h}{ma}\right),$$

since in this case there shall become

$$M=\frac{h}{ma+h} \text{ on account of } \mathfrak{q}=1.$$

With which known we will examine the equation [§ 23], by which the coloured margin is destroyed, which demands that there shall become

$$0=\frac{N'\mathfrak{q}}{k} \quad \text{or} \quad 0=\frac{h}{ma},$$

which since it shall be unable to be done, it is evident in this case the coloured margin cannot be removed. So that if we may wish to tolerate this margin, we will consider also the equation for removing the other coloured margin [§ 31]

$$\frac{mx^3}{a^2h}\left(\mu\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{v}{A\mathfrak{A}}\right)+\frac{\mu'\lambda'}{A^3k}\right)=\frac{1}{k^3},$$

which confusion therefore is unable to be reduced to zero ; from which no account induces these two kinds of glass to be used ; but since the eyepiece lens must be equally convex on both sides, there must become $\lambda'=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2$; thence for the first lensit will be agreed to take $\lambda=1$, so that the total confusion may be rendered smaller and hence the radius of the aperture of the first lens x ;with which understood there will be $y=\frac{hx}{ma}$ and the measure of the clarity $=20y=\frac{20hx}{ma}$. Certainly for microscopes it is customary to take $k=20$. Truly here at this point we may define nothing, since without doubt it may be better, if the value of k may be able to be increased as far as 50 , as in the telescopes we have made.

COROLLARY 1

136. Since there shall be $\mathfrak{A} = \frac{A}{A+1}$ and thus less than unity, it is evident, where where $\mathfrak{A}$ may approach unity, with that the confusion to become smallerand thus thus there a greater value is going to be found for x . Therefore since it may eventuate that if A shall be a large number, in the above case another term may become a minimum in the expression for the confusion.

COROLLARY 2

137. Therefore since at this point A shall be allowed to be by our choice, its value will be agreed to be large enough. Yet the length of the instrument prevents, lest we may attribute an exceedingly large value to the letter A ; evidently this length is observed, which will be approximately $Aa;$ on account of which the letter A may be defined from the maximum length we may wish to allow.

COROLLARY 3

138. Since then also the distance of the object a may be left to our choice, on this account it may not be agreed to put an exceedingly large distance in place, but rather that will be assumed to be as small as the circumstances permit; but it will be seen this distance a scarcely can be diminished less than half an inch.

SCHOLIUM 1

139. But if we may not attend to these circumstances, two lenses will be assumed as it pleases, and then it will be allowed to define the interval of these, so that they will produce the given magnification; so that which may be made clearer, we will consider both the focal lengths p and q as given for a given magnification m. Therefore since there shall be $q = \frac{Ah}{m}$ we will find at once $A = \frac{mq}{h}$ and hence $\mathfrak{A} = \frac{mq}{mq+h}$. Then since there shall be $p = \mathfrak{A}a$, hence we deduce the distance of the object

$$a = \frac{p}{\mathfrak{A}} = \frac{mq+h}{mq} \cdot p = \left(1 + \frac{h}{mq}\right) p.$$

Moreover the interval of these two lenses taken must become :

$$Aa\left(1+\tfrac{h}{ma}\right)=p+q+\tfrac{mpq}{h}.$$

Then truly for the place of the eye there will be :

$$O=\tfrac{hq}{(mq+h)p}\left(p+q+mpq\right)$$

and

$$z=\tfrac{1}{4}\cdot\tfrac{(mq+h)p}{m\left(p+q+\frac{mpq}{h}\right)}$$

Finally since A shall be a large enough number, the aperture of the objective lens will be allowed to be assumed so large, so that its radius shall be

$$x=\tfrac{1}{k}\cdot\sqrt[3]{\tfrac{hp^2q}{\mu\lambda(mq+h)}},$$

from which the measure of the clarity is concluded

$$=\tfrac{20hq}{k(mq+h)}\cdot\sqrt[3]{\tfrac{hq}{\mu\lambda(mq+h)p}},$$

from which it is understood the clarity thus to become greater, where a smaller focal length of the first lens p may be taken and where a greater focal length of the second lens q may be taken .

EXAMPLE

140. With the distance of the object put $=a$, which shall be either of one inch or smaller, the choice may be left to the artificier, and lest the second lens may become exceedingly small with greater magnifications, we may assume $A=40$ and the separation of the lenses will become

$$=40a\left(1+\tfrac{h}{ma}\right)$$

then truly there will be $\mathfrak{A}=\tfrac{40}{41}$, from which we will have this equation for the aperture of the lens:

$$\tfrac{mx^3}{a^2h}\left(\mu\left(\tfrac{41^3}{40^3}\lambda+\tfrac{41\nu}{40^3}\right)+\tfrac{\mu'\lambda'h}{40^3ma}\right)=\tfrac{1}{k^3},$$

where the other term evidently before the first can be removed. Therefore we may assume $\lambda=1$, and since $\mu\left(\tfrac{41^3}{40^3}\lambda+\tfrac{41\nu}{40^3}\right)$ shall be approx. $=1$, there will be

$$x=\frac{1}{k}\cdot\sqrt[3]{\frac{a^2h}{m}} \text{ and hence } y=\frac{h}{km}\cdot\sqrt[3]{\frac{h}{ma}}$$

and the measure of the clarity

$$=\frac{20h}{km}\cdot\sqrt[3]{\frac{h}{ma}}.$$

So that if now therefore, as in microscopes is almost accustomed to happen, there may be taken $k=20$, the measure of the clarity will be

$$=\frac{h}{m}\cdot\sqrt[3]{\frac{h}{ma}},$$

thus so that the clarity shall decrease in the ratio $m^{\frac{4}{3}}$, as with simple microscopes it will decrease only in the ratio *m*.Finally for the location of the eye the distance will be

$$O=\frac{40h}{m}\left(1+\frac{h}{ma}\right).$$

SCHOLIUM 2

141. The diminution of the clarity, which has appeared in this case, may also disturb the situation a little, but only if the distance of the object *a* may be taken small enough; truly the paricular fault, under which these microscopes labour, consists of this, because the objects shall be going to appear with a significant surrounding coloured margin. Whereby before everything will be looked after, so that these microscopes may be freed from this fault, since that cannot be performed in any other way, unless an additional lens shall be introduced, thus so that microscopes of this kind must be constructed with a minimum of three lenses, and since here a different kind of the glass can be of some help, indeed at first we have assumed all these lenses to be made from the same kind of glass. Then truly it will be agreed to subject two cases to be examined here, the one, where this new lens may be located before the real image, truly the other, where it may be located after that; which two cases we will examine further in the two following problems.

PROBLEM 2

142. *Thus to construct a microscope from three lenses, so that the coloured margin may vanish, and the middle lens may lie before the real image.*

SOLUTION

Therefore in this case three lenses shall be had, of the lenses *P* and *Q* the first *P* will retain a positive value, the latter truly *Q* will be given a negative value to be put in place. Therefore we may put $Q=-k$, so that the magnification will become $m=Pk\cdot\frac{h}{a}$ and thus $Pk=\frac{ma}{h}$; from which the focal legths of the lenses will be

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{B}}{P} \cdot a, \quad r = -\frac{AB}{Pk} \cdot a = -AB \cdot \frac{h}{m}.$$

Then the intervals of the lenses

$$\text{I and II} = Aa\left(1 - \frac{1}{P}\right), \quad \text{II and III} = -\frac{AB}{P} \cdot a\left(1 + \frac{1}{k}\right);$$

which so that both may become positive, the first $A\left(1 - \frac{1}{P}\right)$ must be positive, then also $-AB > 0$ or AB shall be a negative quantity, thus so that, if A were a positive number, then there must become $P > 1$ and $B < 0$, but if there shall be $A < 0$, then there must become $P<1$ and $B>0$.

Now we will consider the area seen in the object, of which the radius will be

$$z = \frac{\mathfrak{q} + \mathfrak{r}}{ma + h} \cdot ah\xi = Ma\xi,$$

thus so that there shall become

$$M = \frac{\mathfrak{q} + \mathfrak{r}}{ma + h} \cdot h,$$

where there will be taken $\mathfrak{r} = 1$, if indeed the eyepiece lens may be made with both sides equal. Moreover for $\mathfrak{q}$ this equation is had $-\mathfrak{B}\mathfrak{q} = (P-1)M$. Then for the place of the eye the distance shall become

$$O = \frac{\mathfrak{t}\mathfrak{r}}{Ma} \cdot \frac{h}{m},$$

which in order that it may become positive, it is necessary, that r shall be a positive quantity and thus AB becomes a negative quantity, as we have now indicated.

Moreover then the coloured margin will be destroyed, if there were

$$0 = \frac{\mathfrak{q}}{P} + \frac{\mathfrak{r}}{PQ} = \frac{\mathfrak{q}}{P} - \frac{\mathfrak{r}}{Pk}$$

and thus

$$k = \frac{\mathfrak{r}}{\mathfrak{q}} = \frac{1}{\mathfrak{q}} \text{ on account of } \mathfrak{r} = 1 \text{ or } \mathfrak{q} = \frac{1}{k}.$$

Therefore here a new way is offered beyond expectation for making these microscopes much more complete and thus by doubling the field of view, which is done, if the value of the letter $\mathfrak{q}$ equal to unity likewise may be given to the letter $\mathfrak{r}$, for which it is necessary, so that both the second as well as the third lens may become equally convex; on account of which we may put $k = 1$, so that there may become $\mathfrak{q} = \mathfrak{r} = 1$ and hence

$$z = \frac{2}{ma + h} \cdot ah\xi = \frac{1}{4} \cdot \frac{2ah}{ma + h}.$$

Then truly there will become $P=\frac{ma}{h}$; from which, since $P>1$, there will become $A>0$ and $\mathfrak{A}>0$, and $\mathfrak{A}<1$, and thus $B<0$. Moreover there will become

$$-\mathfrak{B}=\left(\tfrac{ma-h}{h}\right)M=\tfrac{2(ma-h)}{ma+h}$$

on account of $M=\frac{2}{ma+h}\cdot h$, from which on account of the very large number $\frac{ma}{h}$ there will be approx. $\mathfrak{B}=-2$ and $B=-\frac{2}{3}$, clearly as it is required. But without risk we will be able to put $\mathfrak{B}=-2$; even if indeed then $\mathfrak{q}$ may be produced a little smaller than unity and thus the coloured margin may not be removed perfectly, evidently with $k=1$ remaining, yet the former defect in the field of view will be scarcely perceptible, especially for large magnifications ; then now as we have observed more often there is no need for the coloured margin to be removed completely, since the place of the eye, to which it refers, may be quite a large length. For the position of the eye we will have now

$$O=\tfrac{r(ma+h)}{2ma}.$$

Therefore since now there shall be

$$\mathfrak{B}=-2,\quad B=-\tfrac{2}{3},\quad P=\tfrac{ma}{h}\ \text{ and }\ k=1,$$

truly the letter A thus may be allowed to be chosen by us, so that it must be taken positive only, the focal lengths of the lenses thus will themselves be had:

$$p=\mathfrak{A}a,\quad q=\tfrac{2Ah}{m},\quad r=\tfrac{2}{3}\cdot\tfrac{Ah}{m}=\tfrac{1}{3}q$$

and the distance of the eye

$$O=\tfrac{A(ma+h)h}{3m^2a}=\tfrac{A\left(1+\frac{h}{ma}\right)h}{3m}$$

and thus approx.

$$O=\tfrac{Ah}{3m}=\tfrac{1}{2}r.$$

The distances between the lenses now are found

$$\text{I and II}=Aa\left(1-\tfrac{h}{ma}\right),\quad \text{II and III}=\tfrac{4}{3}\cdot\tfrac{Ah}{m}$$

and thus the total length

$$=Aa+\tfrac{2Ah}{3m}$$

with there being $z=\frac{1}{4}\cdot\frac{2ah}{ma+h}$.

Therefore nothing other remains, except that we may define the aperture x of the first lens from this equation :

$$\frac{\mu mx^3}{a^2h}\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}+\frac{h}{A^3ma}\left(\frac{\lambda'}{8}-\frac{3\nu'}{4}\right)+\frac{27\lambda''h}{8A^3ma}\right)=\frac{1}{k^3},$$

where it is required to note, since both the posterior lenses must be equally convex on both sides, to become

$$\lambda'=1+\left(\tfrac{\sigma-\rho}{2\tau}\right)^2(1-2\mathfrak{B})^2=1+25\left(\tfrac{\sigma-\rho}{2\tau}\right)^2$$

and

$$\lambda''=1+\left(\tfrac{\sigma-\rho}{2\tau}\right)^2.$$

But for λ it is agreed to take unity ; then truly on account of $\mathfrak{A}=\frac{p}{a}$ this equation will be transformed conveniently thus :

$$\frac{\mu max^3}{p^3h}\left(1+\frac{\mathfrak{A}^2\nu}{A}+\frac{\mathfrak{A}^3h}{A^3ma}\left(\frac{\lambda'}{8}-\frac{3\nu'}{4}\right)+\frac{27\mathfrak{A}^3h\lambda''}{8A^3ma}\right)=\frac{1}{k^3}.$$

Now we may put for the sake of brevity

$$1+\frac{\mathfrak{A}^2\nu}{A}+\frac{\mathfrak{A}^3h}{A^3ma}\left(\frac{\lambda'}{8}-\frac{3\nu'}{4}\right)+\frac{27\mathfrak{A}^3h\lambda''}{8A^3ma}=\Lambda,$$

the value of which will exceed unity by a small amount, provided that a small number may be assumed for A . Whereby, since μ always shall be a number a little smaller than one, thus so that there may be taken $\Lambda\mu=1$, on account of which we will have

$$x=\frac{p}{k}\sqrt[3]{\frac{h}{ma}},$$

where for k the number 20 or rather a greater number can be assumed at this point, so that objects may be represented more distinctly. Then truly the measure of the clarity will be

$$=\frac{20ph}{kma}\sqrt[3]{\frac{h}{ma}}.$$

COROLLARY 1

143. Since at this point the letter A shall be left to our choice, it will be agreed to assume that so great, that the focal length r may not become exceedingly small for the maximum magnification; clearly so that for the magnification $m=1000$ the focal length of the eyepiece lens may not be taken beyond $\frac{1}{2}$ in. , it wll be required to take $A>94$; from which there will become [$\mathrm{A}=100$ and] $\mathfrak{A}=\frac{100}{101}$.

COROLLARY 2

144. Nor in any case will it be advised for the letter A to be granted a greater value, since then the length of the instrument will increase exceedingly; for if with $\mathrm{A} = 100$ the distance of the object a may be taken of only one inch, the length of the shall be greater than eight feet; whereby if we may wish to pust $\mathrm{A} = 100$, it will be necessary, that the distance of the object a may be reduced to half or even a quarter of an inch.

COROLLARY 3

145. But if the distance $a = \frac{1}{4}$ in. may be seen to be exceedingly small, it will be better certainly to assume $A = 50$ in which case the eyepiece lens, even if it may magnifiy a thousand times, yet it shall be reduced scarcely below $\frac{1}{4}$ in., which magnitude can be allowed easily in practise, since the aperture of such a lens now may allow a greater pupil.

COROLLARY 4

146. But if there may be assumed $A = 50$ only, then there will be $\mathfrak{A} = \frac{50}{51}$, thus so that the focal length of the objective lens p may be able to be taken a little smaller than the distance of the object a, which according to the circumstances may be allowed to be taken conveniently $= \frac{1}{2}$ in. Besides the true value of the letter A may be retained much closer to unity, while the two latter members of this letter clearly may be had as vanishing.

SCHOLIUM 1

147. This kind of microscopes and generally instruments, which today are circulating under the title of composite microscopes, are complete within themselves, which therefore on that account are to be had with the better microscopes, since they differ less from the construction prescribed here. Moreover a particular property consists in that, because the focal length of the middle lens shall be three times greater than that of the eyepiece lens and these lenses may be set out thus, so that a real image may be put in place between the two eyepiece lenses, or, which amounts to the same, so that the separation of these two lenses shall be twice as great as the focal length of the latter lens. Therefore so that we may put a clearer construction of these microscopes in place on account of the eye, initially we will consider the objective lens, of which its construction depends only on the distance of the object a, as it is allowed to be assumed as it pleases, if it may be prepared from common glass, for which there is $n = 1,55$, thus itself may be had :

Construction of the objective lens for a given distance a of the object, being prepared from common glass : the radius

$$\begin{cases}\text{of the anterior face} = \frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)} = \frac{p}{0,2194} = 4,5579p \\ \text{of the posterior face} = \frac{p}{\sigma+\mathfrak{A}(\sigma-\rho)} = \frac{p}{1,5987} = 0,6255p;\end{cases}$$

CONSTRUCTION OF MICROSCOPES OF THIS KIND COMPOSED FROM THREE LENSES, FOR ANY MAGNIFICATION

148. These individual lenses may be prepared from common glass, of which the refraction is $n = 1,55$, and on putting the object distance $= a$, as conveniently it will be allowed to suppoe $= \frac{1}{2}$ in., there will become :

I. For the first lens, of which the focal length $p = \frac{50}{51}a$, the radius

$$\begin{cases}\text{of the anterior face} = 4,4668a \\ \text{of the posterior face} = 0,6130a,\end{cases}$$

the radius of its aperture may be put in place $x = \frac{0,0980a}{\sqrt[3]{ma}}$
and the distance to the second lens $= 50a - \frac{400}{m}$ dig.

II. For the second lens, of which the focal length $q = \frac{880}{m}$ in., the radius of each face $= \frac{880}{m}$ in.,
the radius of the aperture $= \frac{200}{m}$ in., and the distance to the third lens $= \frac{533}{m}$ in.

III. For the third lens, of which the focal length $r = \frac{267}{m}$ in.,
the radius of each face $= \frac{293}{m}$ in.,
the radius of its aperture $= \frac{67}{m}$ in.
and the distance to the eye $= \frac{133}{m}$ in.

IV. The radius of the area viewed within the object will be $z = \frac{4a}{ma+8}$ in.
and the length of the instrument $= 50a + \frac{267}{m}$ in.,
and the measure of its clarity $= \frac{16}{m\sqrt[3]{ma}}$.

Here it deserves to be observed first lens depends only on the distance of the object a and for that to be kept for all magnifications, truly the two posterior lenses depend only on the magnification for all the object distances a that can be put in place; from which it will be agreed to construct these two lenses only for a few particular orders of the magnification, just as the adjoining table will show:

	focal lenth		Distance	
m	lens II	lens III	II and III	of the eye
50	16 in.	5,33 in.	10,67 in.	2,67 in.
100	8	2,67	5,33	1,33
200	4	1,83	2,67	0,67
300	2,66	0,89	1,77	0,44
400	2	0,67	1,33	0,33
500	1,60	0,53	1,07	0,27
600	1,33	0,44	0,88	0,22
800	1	0,33	0,67	0,17
1000	0,8	0,27	0,53	0,13

Yet meanwhile we will show thence, how also these same three lenses may be able to be constructed to include all the magnifications.

SCHOLIUM 2

149. The formulas, from which we have deduced the construction of these microscopes, thus are general, so that also they may be applied to telescopes. Indeed since then there shall be $a=\infty$ and $\mathfrak{A}a$ will denote the focal length of the objective lens, cleary there must be in place $\mathfrak{A}=0$ and thus also $A=0$, yet still, so that there shall be $\mathfrak{A}a=p$; whereby since on account of $h=a$ the focal lengths of the two remaining lenses will be

$$q=-\tfrac{\mathfrak{B}p}{P} \quad \text{and} \quad r=-\tfrac{Bp}{Pk}=-\tfrac{Bp}{m},$$

or if on account of

$$\mathfrak{B}=-2 \quad r=-\tfrac{2}{3}$$

there will become

$$q=+\tfrac{2p}{m} \quad \text{and} \quad r=+\tfrac{3p}{2m}=\tfrac{1}{3}q,$$

Again the separation of the lenses

$$\text{I and II} \ =p(1-\tfrac{1}{m}), \qquad \text{II and III} \ =\tfrac{4p}{3m}=2r$$

and the distance of the eye

$$O = \frac{p(m+1)}{3m^2} = \frac{m+1}{2m} \cdot r,$$

from which the length of the whole telescope

$$= p\left(1 + \frac{2}{3m} + \frac{1}{3m^2}\right).$$

Then truly the radius of the field of view

$$\frac{z}{a} = \Phi = \frac{1}{4} \cdot \frac{2}{m+1}$$

Finally for the aperture requiring to be determinded will be had

$$x = \frac{p}{k}\sqrt[3]{\frac{1}{m}} \quad \text{and hence} \quad y = \frac{p}{km}\sqrt[3]{\frac{1}{m}}$$

and the measure of the clarity

$$= \frac{20p}{km}\sqrt[3]{\frac{1}{m}}.$$

Besides truly here it will help to have noted initially the radius of the real image ; which since in general it shall be $= ABz$ [§ 11], this will become $= -\frac{p}{3(m+1)}$; and indeed if a diaphragm must be inserted, hence its opening size must be determined ; thence since the radius of the pencil of rays entering into the eye shall be

$$= y = \frac{p}{km}\sqrt[3]{\frac{1}{m}},$$

as in place of the pupil of the eye a small hole is accustomed to be bored, the diameter of which hence will be determined. But nothing prevents, why this small opening may not be established larger. Truly since with telescopes there may be put $x = my$, there will be

$$p = kmy\sqrt[3]{m}.$$

Hence therefore, if we may use common glass, of which the refraction $n = 1,55$, the following arises:

Construction of an astronomical telescope constructed from three lenses :

I. For the first objective lens, of which the focal length is $p = kmy\sqrt[3]{m}$, and thus the radius may be given,

$$\begin{cases} \text{of the anterior face} = \frac{p}{1,6274} = 0,6145p \\ \text{of the posterior face} = \frac{p}{0,1907} = 5,2438p; \end{cases}$$

of which the radius of the aperture $x = my$, and the distance to the second lens $= p\left(1 - \frac{1}{m}\right)$.

II. For the second lens, of which the focal length is $q=\frac{2p}{m}$,

$$\text{the radius of each face} = \tfrac{11}{10}q=\tfrac{11p}{5m},$$

of which the radius of the aperture $=\frac{1}{4}q$
and the distance to the eyepiece lens $=\frac{4p}{3m}=\frac{2}{3}q=2r.$

III. For the eyepiece lens, of which the focal length is $r=\frac{2p}{3m}=\frac{1}{3}q$, the radius of each face will be

$$=\tfrac{11}{10}r=\tfrac{11p}{15m}=\tfrac{11}{30}q,$$

the radius of its aperture $=\frac{1}{4}r$
and the distance to the eye $=\frac{m+1}{2m}\cdot r.$

IV. Therefore the length of the whole telescope will be :

$$p\left(1+\tfrac{2}{3m}+\tfrac{1}{3m^2}\right)$$

and the radius of the apparent field

$$\Phi=\tfrac{1}{4}\cdot\tfrac{2}{m+1}=\tfrac{1718}{m+1}\text{ minutes.}$$

V. If in place of the real image, which is present at the middle point between the two posterior lenses, a diaphragm may be put in place, of which the radius will be required to be

$$=-\tfrac{p}{3(m+1)}.$$

PROBLEM 3

150. *With three convex lenses given, of which the focal length of the third lens shall be three times smaller than the focal length of the second lens, to construct a microscope from these, which shall be suitable for all magnifications required to be produced.*

SOLUTION

The focal length of the first lens shall be $=p$, of the second lens $=q$, and of the third lens $=r=-\frac{1}{3}q$, which shall be present in the objective; all three distance shall be positive and which may be given by a single magnification $=m$; whereby, if we may consider the formulas found in the above problem, we deduce at once from the two latter lenses :

$$A = \frac{mq}{2h} \quad \text{and thus} \quad \mathfrak{A} = \frac{mq}{mq+2h}.$$

Again the distance of the object from the first lens will become known

$$a = \frac{mq+2h}{mq} \cdot p = p\left(1 + \frac{2h}{mq}\right).$$

Hence our lenses must maintain the following distances between each other:

$$\text{I and II} = \frac{(mq+2h)p - hq}{2h} = p - \tfrac{1}{2}q + \frac{mpq}{2h},$$

$$\text{II and III } = 2r = \tfrac{2}{3}q.$$

Then as these lenses both produce the same field as we have assigned above, as well as the same clarity, concerning which these three given lenses are required as above :

I. So that the first lens shall be as near as possible plano-convex and its plane face shall be turned towards the object, or yet it will be much better, if the anterior radius shall be around six or seven times greater than the posterior radius.

II. So that the two remaining lenses shall be equally convex on both sides.
Then truly the radius of the area viewed in the object will be

$$z = \frac{h}{2m} \cdot \frac{mq+2h}{(mq+2h)p + hq} \cdot p,$$

for which it is required, that the distance of the eye from the eyepiece lens shall be

$$O = \tfrac{1}{2}r\left(1 + \frac{hq}{(mq+2h)p}\right) = \tfrac{1}{2}r \text{ approx.}$$

Praeterea vero pro apertura lentis obiectivae eius semidiameter reperitur

$$x = \frac{p}{k}\sqrt[3]{\frac{hq}{(mq+2h)p}},$$

where it is to be noted, where the number k may accept a value greater than 20, there the confusion to become smaller, and thus the measure x of the clarity found will be

$$\frac{20h}{ma} \cdot x = \frac{20hqx}{(mq+2h)p}.$$

For the diaphragm in place in loco constituting a real image the radius of the opening will be

$$-\frac{2}{3}Az = \frac{1}{2}r \cdot \frac{(mq+2h)p}{(mq+2h)p+hq}.$$

COROLLARY 1

151. In the first place so that it may extend to the distance of the object, that always will be a little greater than the focal length of the objective lens, where the magnification were smaller. Moreover if the magnification may become infinite, this distance must be taken $a = p$.

COROLLARY 2

152. Truly the separation of the first and second lenses will depend mainly on the magnification *m*, thus so that for an infinite magnification this separation thus shall be required to be taken infinite. Therefore lest for greater magnifications this separation may be produced exceedingly large, this inconvenience will be avoided, of the quantities p and q both may be taken so small, that they are allowed to be observed in practice.

COROLLARY 3

153. With the value substituted in place of x the measure of the clarity shall be

$$= \frac{20hq}{hq(mq+2h)k}\sqrt[3]{\frac{hq}{(mq+2h)p}},$$

we understand the clarity thus to be greater, where the focal length p were smaller; as still it may be agreed to be so great, so that the distance of the object *a,* which is approximately equal to this, may not become exceedingly small; truly besides also the clarity shall be proportional to this formula :

$$= \left(\frac{q}{mq+2h}\right)^{\frac{1}{3}}$$

which circumstances propose a not too small value for q.

EXAMPLE

154. We may take three given lenses to be prepared thus, so that there shall be:

1. The focal length of the first lens $p = \frac{1}{2}$ in. and that as far as possible plano-convex and the planer face turned towards the object. Thus indeed the distance of the object will not be required to be considered very small.

2. Moreover the second lens, which shall be equally convex on both sides, the focal length shall be $q = 1$ in., as the aperture may allow, its radius $x = \frac{1}{4}$ in.

3. Truly both sides of the third convex eyepiece lens shall be equally convex with the focal length $r=\frac{1}{3}$ in., so that it will allow an aperture, of which the radius $=\frac{1}{12}$ in. or the diameter $=\frac{1}{6}$ in.

Therefore the main parts are prepared from these given considerations, so that by no means may they depend on a certain magnification, truly the remaining parts must themselves be defined for some magnification. Of the first kind of the parts there are:

1. The distance of the third lens from the second, which will be constantly $\frac{2}{3}$ in, with the prevailing eyepiece, thus so that the eyepiece lens may stand apart from the the real image by the distance $=\frac{1}{3}$ in., clearly equal to its own focal length. But for the sake of short and long sighted people it will be agreed this interval may be made changeable with the aid of a screw, by which the eyepiece lens may be moved closer or further away by the twentieth part of an inch.

2. The distance of the eye from the eyepiece lens likewise can be considered constantly $=\frac{1}{2}r=\frac{1}{6}$ in.; even if indeed actually that may depend a little on the magnification and must become a little greater, yet the position of the eye is not capable of so great a precision.

Momenta autem pro varia multiplicatione varibilia sunt sequentia:

1. The distance of the object from the objective lens, which in this case will be

$$a=\left(\tfrac{1}{2}+\tfrac{8}{m}\right) \text{ in.}$$

evidently on taking $h=8$ in., thus so that this distance will always exceed $\frac{1}{2}$ in.

2. But the interval between the first and second lens depends mainly on the magnification ; so that if the interval may be indicated by the letter L, there will become $L=\frac{m}{32}$ in., thus so that for the magnification $m=320$ L may become ten inches and, if it may be increased to two feet, now it may serve for a magnification $m=768$, for producing such change there is a need for a moveable tube.

3. Especially also the aperture of the first lens depends on the magnification, of which the radius will be

$$x=\frac{1}{20}\sqrt[3]{\frac{2}{m+16}} \text{ in.,}$$

where we have assumed $k=20$. But it is clear, if we may put the value of x smaller, the confusion to be going to be diminished in the triplicate ratio.

4. But if at the place of the real image or within the distance $=\frac{1}{3}$ in. after the middle lens we may wish to put in place a diaphragm, the aperture of the opening must be

$$=\frac{1}{6}\cdot\frac{m+16}{m+32}\text{ in.},$$

whereby, if the magnification shall be very great, this radius will be $\frac{1}{6}$ in. and therefore its diameter $=\frac{1}{3}$ in., which measure also can be used with smaller magnifications, thus so that there shall be no need to change the diaphragm.

5. So that it may be extended to small openings, to which the eye is required to be applied, if necessary it may be seen its radius may be established equal to y itself, that is found to be $=\frac{16x}{m+16}$; which therefore for the case $m=112$ will be produced $=\frac{1}{640}$ in. Truly since a very small hole may not be able to be seen through, it will suffice to be forewarned that this small hole become as small as possible.

Moreover with this microscope constructed, just as has been descibed here, with its help an area may be viewed within the object, of which the radius will be

$$z=\frac{4}{m}\cdot\frac{m+16}{m+32}\text{ in.},$$

which therefore for the greater magnifications will become $z=\frac{4}{m}$ in., which area certainly is quite remarkable. Finally indeed the measure of the clarity, by which objects will be discerned, will become

$$\frac{320x}{m+16}=\frac{16}{m+16}\sqrt[3]{\frac{2}{m+16}},$$

of which the square properly speaking is considered to be proportional to that clarity. But if the object alone may be illuminated and we may wish to increase the magnification as far as that, so that the clarity thus may be made a hundred thousand times smaller (since then at this point it will be twice as great, than if the object were illuminated by a full moon [The sun is approx. 400,000 time brighter than the full moon using modern data.]), this will happen for the magnification $m=700$; which magnification is so great, that scarcely any greater would be desired.

PROBLEM 4

155. *Thus to construct a composite microscope from three lenses, so that the colored margin may vanish and the real image may lie after the middle lens.*

SOLUTION

Therefore in this case since three lenses will be present, of the letters P and Q the first P will have a negative value with the second Q remaining positive. Therefore there will be $P=-k$, thus so that the magnification shall be

$$m=kQ\cdot\frac{h}{a}\ \text{ or }\ kQ=\frac{ma}{h};$$

from which the focal lengths of the lenses will be

$$p = \mathfrak{A}a, \quad q = +\tfrac{A\mathfrak{B}}{k}a, \quad r = -\tfrac{ABh}{m}$$

and the separation of the lenses

$$\text{I and II} = Aa\left(1+\tfrac{1}{k}\right), \qquad \text{II and III} = \tfrac{AB}{k}a\left(1-\tfrac{1}{Q}\right),$$

from which it is apparent there must be $A>0$ and thus also $\mathfrak{A}>0$ and $\mathfrak{A}<1$, then truly also $B(Q-1)>0$, thus so that, if B shall be >0, then there must be $Q>1$, but if $B<0$, then $Q<1$.

Now the area observed will be considered, of which the radius is

$$z = \tfrac{\mathfrak{q}+\mathfrak{r}}{ma+h}\cdot ah\xi,$$

thus so that on putting

$$M = \tfrac{\mathfrak{q}+\mathfrak{r}}{ma+h}\cdot h$$

there shall become

$$z = Ma\xi,$$

where there can be taken $\mathfrak{r}=1$, if indeed both sides of the eyepiece lens shall be equal and $\mathfrak{q}$ may not exceed unity, either negative or positive. But this equation may be had for $\mathfrak{q}$:

$$\mathfrak{B}\mathfrak{q} = (k+1)M.$$

Then the distance for the eye will become

$$O = \tfrac{\mathfrak{r}r}{Ma}\cdot\tfrac{h}{m};$$

which so that it shall be positive, there must become $r>0$ or $AB<0$ and thus B is negative and $Q<1$. Hence therefore for the colored margin requiring to be removed this equation will be had:

$$0 = \tfrac{\mathfrak{q}}{P} + \tfrac{\mathfrak{r}}{PQ} = \tfrac{\mathfrak{q}}{k} + \tfrac{1}{kQ},$$

from which there is deduced $\mathfrak{q} = -\tfrac{1}{Q}$; whereby, since there shall be $Q<1$, $\mathfrak{q}$ will be produce not only negative, but also greater than unity; since this shall be absurd, it is evident in this case clearly the colored margin cannot be taken away ; nor truly is there a need that we may pursue this case further.

SCHOLION

156. Therefore in this case I reject and we will finish this same chapter, in which we have considered the more simple microscopes of this kind, and we will investigate in what way these microscopes may be able to be carried to greater perfection ; moreover a particular inconvenience under which microscopes labour even now, consists in this, that for the greater magnifications the clarity may become exceedingly small, the account of this matter evidently is placed in the smallness of the aperture x, which moreover is unable to accept a greater value, unless that expression for the radius of the confusion found may arrive at a smaller value ; we will be able to obtain that in two ways, clearly in the first, provided that in place of the simple objective lens two or three or even four convex lenses are substituted ; certainly we will gain in that way, as these lenses may be allowed greater focal lengths than a simple lens and also may arrive at a greater aperture. Then truly, if we may wish to make use of concave lenses, that expression for the radius of confusion will be able to be reduced to zero, thus so that the other bounds of the first lens may not be described, except those which the figure of the lens itself demands. Then truly, also if we may use different kinds of glass, thus it will be able to be effected, so that also the other confusion arising from the diverse refractions will be able to be completely removed ; in which even if it may be seen to consist of the highest order of perfection, yet that at this point may contain inconveniences, since these lenses substituted in place of the objective lens may become much smaller ; which otherwise may eventuate with the first method, certainly it will be worth the effort to pursue both these ways. Finally indeed, even if the field of view here has been notable enough, yet, so that we may resolve this argument fully, we will show also, how this field may be agreed to become greater at this point and to be increased as desired.

SECTIO TERTIA.

DE MICROSCOPIIS COMPOSITIS, IN QUIBUS UNICA IMAGO REALIS OCCURRIT; QUO OMNIA MICROSCOPIA HUCUSQUE USITATA SUNT REFERENDA.

CAPUT I

DE MICROSCOPIIS SIMPLICIORIBUS HUIUS GENERIS PRAEMONITUM

134. Quoniam in hoc microscopiorum genere obiecta situ inverso repraesentantur, in formulis nostris generalibus ubique loco m scribi debet $-m$ ac praeterea etiam litterae $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$ etc. omnes negative sumi debent.

PROBLEMA 1

135. *Microscopium huius generis simplicissimum, quod tantum ex duabus lentibus constet, construere eiusque qualitates describere.*

SOLUTIO

Cum ergo hic duae tantum lentes occurrant inque earum intervallo imago realis reperiatur, habebit littera P valorem negativum, qui sit $=-k$, ita ut pro multiplicatione habeatur $m=\frac{kh}{a}$ seu $k=\frac{ma}{h}$; scilicet denotante a distantiam obiecti, α distantiam imaginis post lentem obiectivam et b distantiam lentis ocularis post imaginem erit quoque $k=\frac{\alpha}{b}$. Tum vero introducta littera erit $A=\frac{\alpha}{a}$ distantia focalis primae lentis

$$p=\mathfrak{A}a$$

et secundae lentis

$$q=\frac{Aa}{k}=\frac{Ah}{m}$$

harumque lentium intervallum

$$=Aa\left(1+\frac{1}{k}\right)=aA\left(1+\frac{h}{ma}\right);$$

quod ergo ut sit positivum, numerus A debet esse positivus ideoque etiam $\mathfrak{A} = \frac{A}{A+1}$ erit positivus, ita ut ambae lentes debeant esse convexae. Deinde erit spatii inobiecto conspicui semidiameter

$$z = \frac{\mathfrak{q}}{ma+h} \cdot ah\xi$$

ubi sumi solet $\xi = \frac{1}{4}$, et ut capi possit $\mathfrak{q} = 1$, lentem ocularem utrinque aeque convexam statui convenit, ita ut habeatur

$$z = \frac{1}{4} \cdot \frac{ah}{ma+h}$$

Pro loco autem oculi inveniemus distantiam

$$O = \frac{q}{Ma} \cdot \frac{h}{m} = q\left(1 + \frac{h}{ma}\right) = \frac{Ah}{m}\left(1 + \frac{h}{ma}\right),$$

quoniam hoc casu fit

$$M = \frac{h}{ma+h} \text{ ob } \mathfrak{q} = 1.$$

Quo cognito examinemus aequationem [§ 23], qua margo coloratus destruitur, quae postulat, ut sit

$$0 = \frac{N'\mathfrak{q}}{k} \quad \text{seu} \quad 0 = \frac{h}{ma},$$

quod cum fieri nequeat, evidens est marginem coloratum hoc casu tolli non posse. Quodsi ergo hunc marginem tolerare velimus, consideremus etiam aequationem pro altera confusione tollenda [§ 31]

$$\frac{mx^3}{a^2h}\left(\mu\left(\frac{\lambda}{\mathfrak{A}^3} + \frac{\nu}{A\mathfrak{A}}\right) + \frac{\mu'\lambda'}{A^3k}\right) = \frac{1}{k^3},$$

quae ergo confusio ad nihilum redigi nequit; unde nulla ratio suadet duas vitri species adhibere; cum autem lens ocularis debeat esse utrinque aequalitar convexa, sumi debebit $\lambda' = 1 + \left(\frac{\sigma-\rho}{2\tau}\right)^2$; deinde pro priori lente sumi convenit $\lambda = 1$, quo tota confusio minor reddatur hincque definatur semidiameter aperturae primae lentis x; qua cognita erit $y = \frac{hx}{ma}$ et mensura claritatis $= 20y = \frac{20hx}{ma}$. Pro microscopiis quidem sumi solet $k = 20$. Verum hic nihil adhuc definiamus, cum sine dubio praestaret, si valor ipsius k ad 50 usque augeri posset, uti in telescopiis fecimus.

COROLLARIUM 1

136. Cum sit $\mathfrak{A} = \frac{A}{A+1}$ ideoque unitate minus, manifestum est, quo magis ad unitatem accedat, eo minorem fore confusionem ideoque pro x eo maiorem

valorem inventum iri. Cum igitur hoc eveniat, si *A* sit numerus magnus, hoc casu insuper alterum membrum in expressione pro confusione fiet minimum.

COROLLARIUM 2

137. Cum igitur *A* adhuc arbitrio nostro sit permissa, eius valorem satis magnum assumi conveniet. Interim tamen longitudo instrumenti prohibet, ne litterae *A* valorem nimis magnum tribuamus; longitudo haec scilicet est spectanda, quae proxima erit *Aa;* quocirca ex maxima longitudine, quam admittere voluerimus, littera *A* definiatur.

COROLLARIUM 3

138. Cum deinde etiam distantia obiecti *a* arbitrio nostro relinquatur, ob rationem iam allatam non conveniet hanc distantiam nimis magnam statuere, sed potius praestabit eam tam parvam assumere, quam circumstantiae permittunt; videtur autem haec distantia *a* vix infra dimidium digitum commode diminui posse.

SCHOLION 1

139. Quodsi ad has circumstantias non attendamus, binae lentes pro lubitu assumi poterunt atque tum adeo earum intervallum definire licebit, ut datam multiplicationem producant; quod quo clarius reddatur, spectemus ambas distantias focales *p* et *q* tanquam datas una cum multiplicatione *m.*

Cum igitur sit $q=\frac{Ah}{m}$ inveniemus statim $A=\frac{mq}{h}$ hincque $\mathfrak{A}=\frac{mq}{mq+h}$. Deinde cum sit $p=\mathfrak{A}a$, hinc elicimus distantiam obiecti

$$a=\frac{p}{\mathfrak{A}}=\frac{mq+h}{mq}\cdot p=\left(1+\frac{h}{mq}\right)p.$$

Intervallum autem harum duarum lentium capi debebit

$$Aa\left(1+\frac{h}{ma}\right)=p+q+\frac{mpq}{h}.$$

Tum vero pro loco oculi erit

$$O=\frac{hq}{(mq+h)p}\left(p+q+mpq\right)$$

et

$$z=\frac{1}{4}\cdot\frac{(mq+h)p}{m\left(p+q+\frac{mpq}{h}\right)}$$

Denique cum A sit numerus satis magnus, aperturam lentis obiectivae tantam assumere licebit, ut sit eius semidiameter

$$x=\frac{1}{k}\cdot\sqrt[3]{\frac{hp^2q}{\mu\lambda(mq+h)}},$$

unde concluditur mensura claritatis

$$=\frac{20hq}{k(mq+h)}\cdot\sqrt[3]{\frac{hq}{\mu\lambda(mq+h)p}},$$

unde intelligitur claritatem fieri eo maiorem, quo minor capiatur distantia focalis primae lentis p et quo maior capiatur distantia secundae lentis q.

EXEMPLUM

140. Posita distantia obiecti $=a$, quae sive sit unius digiti sive minor, arbitrio artificis relinquatur, ac ne pro maioribus multiplicationibus secunda lens fiat nimis parva, sumamus $A=40$ fietque intervallum lentium

$$=40a\left(1+\frac{h}{ma}\right)$$

tum vero erit $\mathfrak{A}=\frac{40}{41}$, unde pro apertura lentis obiectivae habebimus hanc aequationem:

$$\frac{mx^3}{a^2h}\left(\mu\left(\frac{41^3}{40^3}\lambda+\frac{41\nu}{40^3}\right)+\frac{\mu'\lambda'h}{40^3ma}\right)=\frac{1}{k^3},$$

ubi alterum membrum manifesto prae priori reiici potest. Sumamus igitur

$\lambda = 1,$ et cum $\mu\left(\frac{41^3}{40^3}\lambda + \frac{41\nu}{40^3}\right)$ sit proxime $=1$, erit

$$x = \frac{1}{k}\cdot\sqrt[3]{\frac{a^2h}{m}} \text{ hincque } y = \frac{h}{km}\cdot\sqrt[3]{\frac{h}{ma}}$$

et mensura claritatis

$$= \frac{20h}{km}\cdot\sqrt[3]{\frac{h}{ma}}.$$

Quodsi ergo nunc, ut in microscopiis fere fieri solet, sumatur $k = 20$, erit mensura claritatis

$$= \frac{h}{m}\cdot\sqrt[3]{\frac{h}{ma}},$$

ita ut claritas decrescat in ratione $m^{\frac{4}{3}}$, cum in microscopiis simplicibus tantum decreverit in ratione *m.* Denique pro loco oculi erit distantia

$$O = \frac{40h}{m}\left(1 + \frac{h}{ma}\right).$$

SCHOLION 2

141. Diminutio claritatis, quae hoc casu prodiit, parum negotium turbaret, si modo distantia obiecti *a* satis parva caperetur; verum praecipuum vitium, quo haec microscopia laborant, in hoc consistit, quod obiecta insigni margine colorato cincta sint adparitura. Quare ante omnia erit curandum, ut ista microscopia ab hoc vitio liberentur, id quod alio modo praestari nequit, nisi insuper lentem introducendo, ita ut huiusmodi microscopia ad minimum tribus lentibus constare debeant, et quoniam vitri diversitas hic parum subsidii adferre potest, primo quidem omnes has lentes ex eodem vitro parari assumamus. Tum vero duos casus hic examini subiici conveniet, alterum, quo nova ista lens ante imaginem realem, alterum vero, quo post eam collocatur; quos duos casus in sequentibus problematibus fusius pertractemus.

PROBLEMA 2

142. *Microscopium compositum ita ex tribus lentibus conficere, ut margo coloratus evanescat et lens media ante imaginem realem cadat.*

SOLUTIO

Hoc ergo casu cum habeantur tres lentes, litterarum *P* et *Q* prior *P* positivum retinebit valorem, posterior vero *Q* negativa statui debebit. Ponatur igitur $Q = -k$, ut sit multiplicatio $m = Pk\cdot\frac{h}{a}$ ideoque $Pk = \frac{ma}{h}$; unde distantiae focales lentium erunt

$$p = \mathfrak{A}a, \quad q = -\frac{A\mathfrak{B}}{P}\cdot a, \quad r = -\frac{AB}{Pk}\cdot a = -AB\cdot\frac{h}{m}.$$

Deinde intervalla lentium

$$\text{I et II} = Aa\left(1-\tfrac{1}{P}\right), \quad \text{II et III} = -\tfrac{AB}{P}\cdot a\left(1+\tfrac{1}{k}\right);$$

quae ut ambo fiant positiva, primo $A\left(1-\tfrac{1}{P}\right)$ debet esse positivum, deinde etiam $-AB > 0$ sive AB quantitas negativa, ita ut, si A fuerit numerus positivus, tum debeat esse $P > 1$ et $B < 0$, sin autem sit $A < 0$, tum esse debeat $P<1$ et $B>0$.

Nunc consideremus spatium in obiecto conspicuum, cuius semidiameter erit

$$z = \tfrac{\mathfrak{q}+\mathfrak{r}}{ma+h}\cdot ah\xi = Ma\xi,$$

ita ut sit

$$M = \tfrac{\mathfrak{q}+\mathfrak{r}}{ma+h}\cdot h,$$

ubi sumi poterit $\mathfrak{r} = 1$, si quidem lens ocularis utrinque fiat aequalis. Pro $\mathfrak{q}$ autem habetur ista aequatio $-\mathfrak{B}\mathfrak{q} = (P-1)M$. Deinde pro loco oculi fiat distantia

$$O = \tfrac{\mathfrak{tr}}{Ma}\cdot\tfrac{h}{m},$$

quae ut fiat positiva, necesse est, ut r sit quantitas positiva ideoque AB quantitas negativa, ut iam notavimus.

Tum autem margo coloratus destruetur, si fuerit

$$0 = \tfrac{\mathfrak{q}}{P} + \tfrac{\mathfrak{r}}{PQ} = \tfrac{\mathfrak{q}}{P} - \tfrac{\mathfrak{r}}{Pk}$$

adeoque

$$k = \tfrac{\mathfrak{r}}{\mathfrak{q}} = \tfrac{1}{\mathfrak{q}} \;\text{ ob }\; \mathfrak{r} = 1 \;\text{ seu }\; \mathfrak{q} = \tfrac{1}{k}.$$

Hic igitur praeter exspectationem novus modus se offert ista microscopia multo magis perficiendi atque adeo campum visionis duplicandi, id quod fit, si litterae $\mathfrak{q}$ valor unitati aequalis perinde ac litterae $\mathfrak{r}$ tribuatur, ad quod necesse est, ut tam secunda quam tertia lens fiant utrinque aeque convexae; quamobrem ponamus $k = 1$, ut fiat $\mathfrak{q} = \mathfrak{r} = 1$ hincque

$$z = \tfrac{2}{ma+h}\cdot ah\xi = \tfrac{1}{4}\cdot\tfrac{2ah}{ma+h}.$$

Tum vero erit $P = \tfrac{ma}{h}$; unde, quia $P > 1$, erit $A > 0$ et $\mathfrak{A} > 0$ et $\mathfrak{A} < 1$ ideoque $B < 0$. Fiet autem

$$-\mathfrak{B} = \left(\tfrac{ma-h}{h}\right)M = \tfrac{2(ma-h)}{ma+h}$$

ob $M = \tfrac{2}{ma+h}\cdot h$, ex quo ob $\dfrac{ma}{h}$ numerum praemagnum erit proxime $\mathfrak{B} = -2$ et $B = -\tfrac{2}{3}$, plane ut requiritur. Tuto autem statuere poterimus $\mathfrak{B} = -2$; etsi enim tum $\mathfrak{q}$ aliquanto minor unitate prodeat ideoque margo coloratus non perfecte tollatur, manente scilicet $k = 1$, tamen defectus prior in campo visionis vix erit sensibilia, praecipue pro magnis multiplicationibus; deinde iam saepius annotavimus non opus esse, ut margo coloratus penitus destruatur, quoniam locus oculi, ad quem refertur, non exiguam patitur latitudinem. Pro loco oculi vero nunc habebimus

$$O=\frac{r(ma+h)}{2ma}.$$

Cum igitur nunc sit

$$\mathfrak{B}=-2,\quad B=-\tfrac{2}{3},\quad P=\tfrac{ma}{h}\quad \text{et}\quad k=1,$$

littera vero A ita arbitrio nostro permittatur, ut tantum positiva accipi debeat, distantiae focales lentium ita se habebunt:

$$p=\mathfrak{A}a,\quad q=\tfrac{2Ah}{m},\quad r=\tfrac{2}{3}\cdot\tfrac{Ah}{m}=\tfrac{1}{3}q$$

et distantia oculi

$$O=\frac{A(ma+h)h}{3m^2a}=\frac{A\left(1+\frac{h}{ma}\right)h}{3m}$$

ideoque proxime

$$O=\tfrac{Ah}{3m}=\tfrac{1}{2}r.$$

Intervalla autem lentium nunc reperiuntur

$$\text{I et II}=Aa\left(1-\tfrac{h}{ma}\right),\quad \text{II et III}=\tfrac{4}{3}\cdot\tfrac{Ah}{m}$$

sicque tota longitudo

$$=Aa+\tfrac{2Ah}{3m}$$

existente $z=\frac{1}{4}\cdot\frac{2ah}{ma+h}$.

Nihil igitur alius restat, nisi ut aperturam primae lentis definiamus ex hac aequatione :

$$\frac{\mu mx^3}{a^2h}\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}+\frac{h}{A^3ma}\left(\frac{\lambda'}{8}-\frac{3\nu'}{4}\right)+\frac{27\lambda''h}{8A^3ma}\right)=\frac{1}{k^3},$$

ubi notandum est, quia ambae posteriores lentes debent esse utrinque aequaliter convexae, fore

$$\lambda'=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2(1-2\mathfrak{B})^2=1+25\left(\frac{\sigma-\rho}{2\tau}\right)^2$$

et

$$\lambda''=1+\left(\frac{\sigma-\rho}{2\tau}\right)^2.$$

At pro λ unitatem sumi convenit; tum ergo ob $\mathfrak{A}=\frac{p}{a}$ haec aequatio commode ita transformabitur:

$$\frac{\mu max^3}{p^3h}\left(1+\frac{\mathfrak{A}^2v}{A}+\frac{\mathfrak{A}^3h}{A^3ma}\left(\frac{\lambda'}{8}-\frac{3v'}{4}\right)+\frac{27\mathfrak{A}^3h\lambda''}{8A^3ma}\right)=\frac{1}{k^3}.$$

Ponamus nunc brevitatis gratia

$$1+\frac{\mathfrak{A}^2v}{A}+\frac{\mathfrak{A}^3h}{A^3ma}\left(\frac{\lambda'}{8}-\frac{3v'}{4}\right)+\frac{27\mathfrak{A}^3h\lambda''}{8A^3ma}=\Lambda,$$

cuius valor unitatem non nisi parum superabit, dummodo pro A numerus modicus assumatur. Quare, cum μ semper sit numerus ab unitate parum deficiens, ita ut sumi possit $\mu\Lambda=1$, quocirca habebimus

$$x=\frac{p}{k}\sqrt[3]{\frac{h}{ma}},$$

ubi pro k sumi potest 20 vel potius numerus adhuc maior, quo obiecta distinctius repraesententur. Tum vero erit mensura claritatis

$$=\frac{20ph}{kma}\sqrt[3]{\frac{h}{ma}}.$$

COROLLARIUM 1

143. Cum adhuc littera A arbitrio nostro sit relicta, eam tantam assumi conveniet, ut distantia focalis r non fiat nimis exigua etiam pro maximis multiplicationibus; scilicet ut pro multiplicatione $m=1000$ distantia focalis lentis ocularis non infra $\frac{1}{2}$ dig. capi debeat, oportebit sumere $A>94$; unde erit [$\mathrm{A}=100$ et] $\mathfrak{A}=\frac{100}{101}$.

COROLLARIUM 2

144. Neutiquam vero consultum erit litterae A multo maiorem valorem tribuere, quia tum longitudo instrumenti nimium excresceret; si enim posito $\mathrm{A}=100$ distantia obiecti a unius tantum digiti sumeretur, longitudo instrumenti octo pedes esset superatura; quare si velimus statuere $\mathrm{A}=100$, necesse erit, ut distantia obiecti a ad dimidium digitum. vel etiam $\frac{1}{4}$ dig. reducatur.

COROLLARIUM 3

145. At si distantia $a=\frac{1}{4}$ dig. nimis parva videatur, praestabit utique assumere $A=50$ quo casu lens ocularis, etiamsi millies multiplicemus, tamen vix infra $\frac{1}{4}$ dig. sit reducenda, quae magnitudo in praxi facile admitti potest, cum talis lens aperturam adhuc patiatur pupilla maiorem.

COROLLARIUM 4

146. At si tantum sumatur $A = 50$, tum erit $\mathfrak{A} = \frac{50}{51}$, ita ut distantia focalis lentis obiectivae p tantillo minor accipi debeat quam distantia obiecti $a,$ quam pro circumstantiis commode $= \frac{1}{2}$ dig. sumere licebit. Praeterea vero valor litterae A multo propius ad unitatem revocabitur, dum bina posteriora membra huius litterae plane pro evanescentibus haberi poterunt.

SCHOLION 1

147. Haec microscopiorum species pleraque instrumenta, quae hodie sub titulo microscopiorum compositorum circumferuntur, in se complectitur, quae igitur pro eo melioribus sunt habenda, quo minus a constructione hic praescripta discrepant. Praecipua autem proprietas in hoc consistit, quod distantia focalis lentis mediae triplo maior sit quam lentis ocularis haeque lentes ita disponantur, ut imago realis media interiaceat inter binas lentes oculares, sive, quod eodem redit, ut intervallum harum duarum lentium duplo maius sit quam distantia focalis postremae lentis. Quo igitur constructionem horum microscopiorum clarius ob oculos ponamus, primo considere- mus lentem obiectivam, quae tantum a distantia obiecti $a,$ quam pro lubitu assumere licet, pendet, cuius constructio, si ex vitro communi, pro quo est $n = 1,55$, paretur, ita se habet:

Constructio lentis obiectivae pro data distantia obiecti a ex vitro communi parandae

$$\text{Radius faciei}\begin{cases}\text{anterioris} = \frac{p}{\sigma - \mathfrak{A}(\sigma-\rho)} = \frac{p}{0,2194} = 4,5579p \\ \text{posterioris} = \frac{p}{\sigma + \mathfrak{A}(\sigma-\rho)} = \frac{p}{1,5987} = 0,6255p;\end{cases}$$

CONSTRUCTIO HUIUSMODI MICROSCOPIORUM EX TRIBUS LENTIBUS COMPOSITORUM PRO QUAVIS MULTIPLICATIONE

148. Singulae hae lentes ex vitro communi, cuius refractio est $n = 1,55$, parentur et posita obiecti distantia $= a,$ quam commode $= \frac{1}{2}$ dig. assumere licebit, erit:

I. Pro lente prima, cuius distantia focalis est $p = \frac{50}{51}a$, sumatur

$$\text{radius faciei}\begin{cases}\text{anterioris} = 4,4668a \\ \text{posterioris} = 0,6130a,\end{cases}$$

eius aperturae semidiameter statuatur $x = \frac{0,0980a}{\sqrt[3]{ma}}$

et distantia ad lentem secundam $= 50a - \frac{400}{m}$ dig.

II. Pro secunda lente, cuius distantia focalis est $q = \frac{880}{m}$ dig., sumatur

radius utriusque faciei $= \frac{880}{m}$ dig.,

aperturae semidiameter $= \frac{200}{m}$ dig .

et distantia ad lentem tertiam $= \frac{533}{m}$ dig.

III. Pro lente tertia, cuius distantia focalis $r = \frac{267}{m}$ dig., erit

radius faciei utriusque $= \frac{293}{m}$ dig.,

eius aperturae semidiameter $= \frac{67}{m}$ dig.

et distantia ad oculum $= \frac{133}{m}$ dig.

IV. Spatii in obiecto conspicui semidiameter erit $z = \frac{4a}{ma+8}$ dig.

et instrumenti longitudo $= 50a + \frac{267}{m}$ dig.

atque mensura claritatis $= \frac{16}{m\sqrt[3]{ma}}$.

Notari hic meretur primam lentem tantum a distantia obiecti a pendere eamque pro omni multiplicatione retineri posse, duas vero posteriores lentes tantum a multiplicatione pendere easdemque pro omni distantia obiecti a locum habere posse; unde tantum pro variis aliquot multiplicationis gradibus praecipuis duas has lentes construi conveniet, veluti tabella subiuncta indicabit:

	Distantia focalis		Distantia	
m	lentis II	lentis III	II et III	oculi
50	16 dig.	5,33 dig.	10,67dig.	2,67 dig.
100	8	2,67	5,33	1,33
200	4	1,83	2,67	0,67
300	2,66	0,89	1,77	0,44
400	2	0,67	1,33	0,33
500	1,60	0,53	1,07	0,27
600	1,33	0,44	0,88	0,22
800	1	0,33	0,67	0,17
1000	0,8	0,27	0,53	0,13

Interim tamen deinceps ostendemus, quemadmodum etiam iisdem ternis lentibus retentis microscopia ad omnes multiplicationes accommodata construi possint.

SCHOLION 2

149. Formulae, ex quibus hanc microscopiorum constructionem deduximus, ita sunt generales, ut etiam ad telescopia accommodari queant. Cum enim tum sit $a=\infty$ et $\mathfrak{A}a$ distantiam focalem lentis obiectivae denotet, evidens est statui debere $\mathfrak{A}=0$ ideoque etiam $A=0$, ita tamen, ut sit $\mathfrak{A}a=p$; quare ob $h=a$ erunt distantiae focales duarum reliquarum lentium

$$q=-\frac{\mathfrak{B}p}{P} \quad \text{et} \quad r=-\frac{Bp}{Pk}=-\frac{Bp}{m},$$

sive ob

$$\mathfrak{B}=-2 \quad r=-\tfrac{2}{3}$$

erit

$$q=+\frac{2p}{m} \quad \text{et} \quad r=+\frac{3p}{2m}=\tfrac{1}{3}q,$$

lentium porro intervalla

$$\text{I et II} \;=p(1-\tfrac{1}{m}), \qquad \text{II et III} \;=\frac{4p}{3m}=2r$$

et distantia oculi

$$O=\frac{p(m+1)}{3m^2}=\frac{m+1}{2m}\cdot r,$$

unde longitudo telescopii tota

$$=p\left(1+\frac{2}{3m}+\frac{1}{3m^2}\right).$$

Tum vero semidiameter campi visionis

$$\frac{z}{a}=\Phi=\frac{1}{4}\cdot\frac{2}{m+1}$$

Denique pro apertura determinanda habebitur

$$x=\frac{p}{k}\sqrt[3]{\frac{1}{m}} \quad \text{hincque} \quad y=\frac{p}{km}\sqrt[3]{\frac{1}{m}}$$

et mensura claritatis

$$=\frac{20p}{km}\sqrt[3]{\frac{1}{m}}.$$

Praeterea vero hic notasse iuvabit primo semidiametrum imaginis realis; quae cum in genere sit $=ABz$ [§ 11], erit ea $=-\frac{p}{3(m+1)}$; quodsi enim in hoc loco diaphragma inseratur, eius foramen hinc determinari debet; deinde cum sit semidiameter penicillorum radiosorum in oculum ingredientium

$$=y=\frac{p}{km}\sqrt[3]{\frac{1}{m}},$$

quoniam in loco oculi operculum statui solet foraminulo pertusum, eius semidiameter hinc determinabitur. Nihil autem impedit, quominus hoc foraminulum maius statuatur. Cum vero in telescopiis detur $x = my$, erit

$$p=kmy\sqrt[3]{m}.$$

Hinc igitur, si vitro communi utamur, cuius refractio est $n=1,55$, sequens nascitur

Constructio telescopii astronomici tribus lentibus instructi

I. Pro prima lente obiectiva, cuius distantia focalis est $p=kmy\sqrt[3]{m}$, ideoque datur, erit

$$\text{radius faciei}\begin{cases}\text{anterioris}=\frac{p}{1,6274}=0,6145p\\ \text{posterioris}=\frac{p}{0,1907}=5,2438p;\end{cases}$$

eius aperturae semidiameter $x=my$

et distantia ad lentem secundam $=p\left(1-\frac{1}{m}\right)$.

II. Pro secunda lente, cuius distantia focalis est $q = \frac{2p}{m}$, erit

$$\text{radius utriusque faciei} = \tfrac{11}{10}q = \tfrac{11p}{5m},$$

eius aperturae semidiameter $= \frac{1}{4}q$

et distantia ad lentem ocularem $= \frac{4p}{3m} = \frac{2}{3}q = 2r.$

III. Pro lente oculari, cuius distantia focalis est $r = \frac{2p}{3m} = \frac{1}{3}q,$ erit

$$\text{radius utriusque faciei} = \tfrac{11}{10}r = \tfrac{11p}{15m} = \tfrac{11}{30}q,$$

eius aperturae semidiameter $= \frac{1}{4}r$

et distantia ad oculum $= \frac{m+1}{2m}\cdot r.$

IV. Longitndo ergo huius telescopii erit

$$p\left(1+\tfrac{2}{3m}+\tfrac{1}{3m^2}\right)$$

et campi apparentis semidiameter

$$\Phi = \tfrac{1}{4}\cdot\tfrac{2}{m+1} = \tfrac{1718}{m+1}\ \text{minut.}$$

V. Si in loco imaginis realis, quae in medio puncto inter binas posteriores lentes existit, diaphragma constitui debeat, eius semidiameter esse oportet

$$= -\tfrac{p}{3(m+1)}.$$

PROBLEMA 3

150. *Datis tribus lentibus convexis, quarum tertiae distantia focalis triplo sit minor quam secundae, ex iis microscopium componere, quod ad omnes multiplicationes producendas sit aptum.*

SOLUTIO

Sit primae lentis, quae locum obiectivae occupat, distantia focalis $= p$, secundae lentis $= q$ et tertiae lentis $= r = -\frac{1}{3}q$; quae omnes tres distantiae sint positivae et datae una cum multiplicatione $= m$; quare, si formulas in superiore problemate inventas contemplemur, ex binis posterioribus lentibus statim colligimus

$$A = \tfrac{mq}{2h} \quad \text{ideoque} \quad \mathfrak{A} = \tfrac{mq}{mq+2h}.$$

Porro ex prima lente innotescet distantia obiecti

$$a=\frac{mq+2h}{mq}\cdot p=p\left(1+\frac{2h}{mq}\right).$$

Hinc nostrae lentes sequentia inter se intervalla tenere debebunt:

$$\text{I et II}=\frac{(mq+2h)p-hq}{2h}=p-\tfrac{1}{2}q+\frac{mpq}{2h},$$

$$\text{II et III}=2r=\tfrac{2}{3}q.$$

Deinde ut hae lentes tam eundem campum producant, quem supra assignavimus, quam etiam eandem claritatem, circa has tres lentes datas insuper requiritur:

I. Ut lens prima propemodum sit plano-convexa eiusque facies plana obiecto obvertatur, vel adhuc magis praestabit, si radius anterior sexies vel septies circiter maior sit quam posterior.

II. Ut binae reliquae lentes utrinque sint aequaliter convexae.
Tum vero spatii in obiecto conspicui semidiameter erit

$$z=\frac{h}{2m}\cdot\frac{mq+2h}{(mq+2h)p+hq}\cdot p,$$

pro quo requiritur, ut oculi a lente oculari distantia sit

$$O=\tfrac{1}{2}r\left(1+\frac{hq}{(mq+2h)p}\right)=\tfrac{1}{2}r\ \text{ proxime.}$$

Praeterea vero pro apertura lentis obiectivae eius semidiameter reperitur

$$x=\frac{p}{k}\sqrt[3]{\frac{hq}{(mq+2h)p}},$$

ubi notetur, quo magis k numerum 20 superare accipiatur, eo minorem fore confusionem, atque sic invento x mensura claritatis erit

$$\frac{20h}{ma}\cdot x=\frac{20hqx}{(mq+2h)p}.$$

Pro diaphragmate in loco imaginis realis constituendo erit radius foraminis

$$-\tfrac{2}{3}Az=\tfrac{1}{2}r\cdot\frac{(mq+2h)p}{(mq+2h)p+hq}.$$

COROLLARIUM 1

151. Quod primo ad distantiam obiecti attinet, ea semper erit aliquanto maior quam distantia focalis lentis obiectivae idque eo magis, quo minor fuerit multiplicatio. Sin autem multiplicatio adeo fiat infinita, sumi debebit haec distantia $a = p$.

COROLLARIUM 2

152. Intervallum vero lentium primae et secundae potissimum a multiplicatione m pendet, ita ut pro multiplicatione infinita hoc intervallum adeo infinitum sit capiendum. Ne igitur pro maioribus multiplicationibus hoc intervallum nimis prodeat magnum, hoc incommodum evitabitur, si quantitates p et q tam parvae accipiantur, quam circumstantiae in praxi observandae permittunt.

COROLLARIUM 3

153. Cum loco x valore substituto sit mensura claritatis

$$= \frac{20hq}{hq(mq+2h)k} \sqrt[3]{\frac{hq}{(mq+2h)p}},$$

intelligimus claritatem eo fore maiorem, quo minor fuerit distantia focalis p; quam tamen tantam esse convenit, ut distantia obiecti a, quae ipsi proxime est aequalis, non fiat nimis exigua; praeterea vero etiam claritas proportionalis est isti formulae:

$$= \left(\frac{q}{mq+2h}\right)^{\frac{1}{3}}$$

quae circumstantia suadet pro q valorem non nimis exiguum.

EXEMPLUM

154. Sumamus tres lentes datas ita esse comparatas, ut sit:

1. Lentis primae distantia focalis $p = \frac{1}{2}$ dig. eaque propemodum plano-convexa eiusque facies planior obiecto obvertatur. Sic enim distantia obiecti non nimis parva erit censenda.

2. Secundae autem lentis, quae utrinque sit aequaliter convexa, distantia focalis sit $q = 1$ dig., ut aperturam admittat, cuius semidiameter $x = \frac{1}{4}$ dig.

3. Tertiae vero lentis ocularis itidem utrinque aequaliter convexae sit distantia focalis $r=\frac{1}{3}$ dig., ut aperturam admittat, cuius semidiameter $=\frac{1}{12}$ dig. sive diameter $=\frac{1}{6}$ dig.

His igitur datis momenta constructionis ita sunt comparata, ut quaedam neutiquam a multiplicatione pendeant, reliqua vero pro qualibet multiplicatione seorsim definiri debeant. Prioris generis sunt:

1. Distantia tertiae lentis a secunda, quae constanter erit $\frac{2}{3}$ dig. pro oculis scilicet valentibus, ita ut lens ocularis ab imagine reali distet intervallo $=\frac{1}{3}$ dig., scilicet suae distantiae focali aequali. In gratiam autem myopum et presbytarum conveniet hoc intervallum mutabile reddi ope cochleae, qua lens ocularis circiter parte quadragesima digiti vel propius admoveri vel longius removeri possit.

2. Distantia oculi a lente oculari itidem censeri potest constanter $=\frac{1}{2}r=\frac{1}{6}$ dig.; etiamsi enim revera ea paulisper a multiplicatione pendeat et aliquantillo maior esse debeat, tamen locus oculi tantae praecisionis non est capax.

Momenta autem pro varia multiplicatione varibilia sunt sequentia:

1. Distantia obiecti a lente obiectiva, quae hoc casu erit

$$a=\left(\frac{1}{2}+\frac{8}{m}\right) \text{ dig.}$$

sumto scilicet $h=8$ dig., ita ut haec distantia semper superet $\frac{1}{2}$ dig.

2. Maxime autem a multiplicatione pendet intervallum lentium primae et secundae; quod si indicetur littera L, erit $L=\frac{m}{32}$ dig., ita ut pro multiplicatione $m=320L$ tantum fiat decem digitorum et, si ad duos pedes augeatur, iam multiplicationi $m=768$ inserviat, ad quam mutationem producendam evidens est tubo ductitio esse opus.

3. Inprimis quoque a multiplicatione pendet apertura primae lentis, cuius semidiameter erit

$$x=\frac{1}{20}\sqrt[3]{\frac{2}{m+16}} \text{ dig.,}$$

ubi sumsimus $k=20$. Perspicuum autem est, si valorem ipsius x minorem statuamus, confusionem in ratione triplicata diminutum iri.

4. Quodsi in loco imaginis realis seu in distantia $=\frac{1}{3}$ dig. post lentem mediam diaphragma velimus collocare, eius foraminis apertura esse debebit

$$=\frac{1}{6}\cdot\frac{m+16}{m+32} \text{ dig.,}$$

quare, si multiplicatio sit valde magna, haec semidiameter erit $\frac{1}{6}$ dig. eiusque ergo diameter $=\frac{1}{3}$ dig., quae mensura etiam minoribus multiplicationibus inservire potest, ita ut diaphragma mutare non sit opus.

5. Quod ad foraminulum attinet, cui oculus est adplicandus, si necesse videatur eius semidiametrum ipsi y aequalem statuere, reperietur ea $=\frac{16x}{m+16}$; quae ergo pro casu $m=112$ prodiret $=\frac{1}{640}$ dig. Quoniam vero tantillum foraminulum quasi sensus effugeret, sufficiet praecepisse hoc foraminulum quam minimum fieri.

Hoc autem microscopio constructo, quemadmodum hic est descriptum, eius ope in obiecto spatiolum conspicietur, cuius semidiameter erit

$$z=\frac{4}{m}\cdot\frac{m+16}{m+32}\text{ dig.},$$

quae ergo pro maioribus multiplicationibus erit $z=\frac{4}{m}$ dig., quod spatium certe satis est notabile. Denique vero mensura claritatis, qua obiecta cernentur, erit

$$\frac{320x}{m+16}=\frac{16}{m+16}\sqrt[3]{\frac{2}{m+16}},$$

cuius quadrato proprie loquendo ipsa claritas censenda est proportionalis. Quodsi ergo obiectum a sole collustretur et nos multiplicationem eo usque augere velimus, ut ipsa claritas centies millies adeo fiat minor (quandoquidem tum adhuc duplo maior erit, quam si obiectum a plena luna illustraretur), hoc eveniet pro multiplicatione $m=700$; quae multiplicatio tanta est, ut vix unquam maior desideretur.

PROBLEMA 4

155. *Microscopium compositum ita ex tribus lentibus conficere, ut margo coloratus evanescat et lens media post imaginem realem cadat.*

SOLUTIO

Hoc ergo casu cum tres habeantur lentes, litterarum P et Q prior P negativum habebit valorem posteriore Q manente positiva. Sit igitur $P=-k$, ut sit multiplicatio

$$m=kQ\cdot\frac{h}{a}\quad\text{seu}\quad kQ=\frac{ma}{h};$$

unde distantiae focales lentium erunt

$$p=\mathfrak{A}a,\quad q=+\frac{A\mathfrak{B}}{k}a,\quad r=-\frac{ABh}{m}$$

et lentium intervalla

$$\text{I et II} = Aa\left(1+\tfrac{1}{k}\right), \qquad \text{II et III} = \tfrac{AB}{k}a\left(1-\tfrac{1}{Q}\right),$$

unde patet esse debere $A>0$ ideoque etiam $\mathfrak{A}>0$ et $\mathfrak{A}<1$, tum vero etiam $B(Q-1)>0$, ita ut, si B sit >0, tum esse debeat $Q>1$, sin autem $B<0$, tum $Q<1$.

Nunc consideretur spatium conspicuum, cuius semidiameter est

$$z = \tfrac{\mathfrak{q}+\mathfrak{r}}{ma+h}\cdot ah\xi,$$

ita ut posito

$$M = \tfrac{\mathfrak{q}+\mathfrak{r}}{ma+h}\cdot h$$

fiat

$$z = Ma\xi,$$

ubi sumi potest $\mathfrak{r}=1$, siquidem lens ocularis sit utrinque aequalis et $\mathfrak{q}$ unitatem non superet, sive negative sive positive. Pro $\mathfrak{q}$ autem habebitur haec aequatio:

$$\mathfrak{B}\mathfrak{q} = (k+1)M.$$

Deinde pro loco oculi fiet distantia

$$O = \tfrac{\mathfrak{r}r}{Ma}\cdot\tfrac{h}{m};$$

quae ut sit positiva, debet esse $r>0$ seu $AB<0$ adeoque B negativum et $Q<1$. Hinc igitur pro margine colorato tollendo habebitur ista aequatio:

$$0 = \tfrac{\mathfrak{q}}{P}+\tfrac{\mathfrak{r}}{PQ} = \tfrac{\mathfrak{q}}{k}+\tfrac{1}{kQ},$$

unde colligitur $\mathfrak{q} = -\tfrac{1}{Q}$; quare, cum sit $Q<1$, prodiret $\mathfrak{q}$ non solum negativum, sed etiam unitate maius; quod cum sit absurdum, evidens est hoc casu marginem coloratum tolli plane non posse; neque ergo opus est, ut hunc casum ulterius prosequamur.

SCHOLION

156. Hoc igitur casu reiecto istud caput, in quo simpliciora huius generis microscopia sumus contemplati, finiemus et, quemadmodum haec microscopia ad maiorem perfectionis gradum evehi queant, indagabimus; praecipuum autem incommodum, quo haec microscopia etiam num laborant, in hoc consistit, quod pro maioribus multiplicationibus claritas nimis fiat exigua, cuius rei ratio manifesto sita est in parvitate aperturae *x,* quae autem maiorem valorem accipere non potest, nisi ipsa expressio pro semidiametro confusionis inventa minorem valorem adipiscatur; id quod duplici modo obtinere poterimus, priore scilicet, dum loco lentis obiectivae simplicis duae vel tres vel etiam quatuor lentes convexae substituuntur; quippe quo modo id lucramur, ut hae lentes maiores distantias focales consequantur quam lens simplex atque etiam maiorem aperturam adipiscantur. Deinde vero, si etiam lentibus concavis uti velimus, expressio illa pro semidiametro confusionis adeo ad nihilum redigi poterit, ita ut aperturae primae

lentis alii limites non praescribantur, nisi quos ipsa lentis figura postulat. Deinde vero, etiam si diversas vitri species adhibeamus, adeo effici poterit, ut etiam altera confusio a diversa refractione oriunda penitus tollatur; in quo etsi summus perfectionis gradus consistera videtur, tamen id adhuc incommodi se immiscet, quod lentes illae loco obiectivae substituendae multo fiant minores; quod cum priore modo secus eveniat, utique operae erit pretium hos ambos modos percurrere. Denique vero, etsi campus visionis hic est satis notabilis, tamen, ut argumentum hoc plene absolvamus, etiam monstrabimus, quomodo hunc campum adhuc magis atque adeo ad lubitum amplificari conveniat.