FIRST SECTION.

SIMPLE MICROSCOPES.

CHAPTER I

SIMPLE MICROSCOPES CONSTRUCTED FROM A SINGLE LENS

BOTH WITH THE THICKNESS OF THE LENS IGNORED,

AS WELL AS HAVING A GIVEN MAGNIFICATION

PROBLEM 1

§ 38. *To construct a simple microscope, so that an object may be increased according to a given ratio, with the thickness of the lens ignored.*

SOLUTION

The prescribed magnification shall be $= m$, which evidently may refer to some arbitrary assumed distance *h* , with *h* commonly denoting a distance 8 in., and the focal length of the lens shall be *p*, which alone constitutes the microscope. Therefore since there must become $\alpha = \infty$ also there will become $A = \infty$; from which there becomes $\mathfrak{A} = 1$ and thus $a = p$, thus so that the distance of the object before the lens must be precisely equal to its focal length p. Therefore since in general there shall be $m = PQR \ldots Z.\frac{h}{a}$; for our case, where a single lens is present, this formula must be divided by all the letters *P, Q, R, ... Z* , thus so that there may become $m = \frac{h}{a}$, which also is understood most easily in this manner: indeed since this image shall be formed at the distance $\alpha = Aa$, its radius shall be $A\zeta$ with ζ being the radius of the object, this image will be discerned by the eye to be within the angle $\frac{\zeta}{a}$, while likewise the object to be present at a distance *h* to the naked eye may be appearing to be within the angle $= \frac{\zeta}{h}$, from which that angle divided by this gives the same magnification, thus so that there shall become $m = \frac{h}{a}$.

Whereby since this magnification *m* shall be given, hence there is deduced $a = p = \frac{h}{m}$ and thus both the focal length of the lens as well as distance of the object before the lens may be able to be determined from the prescribed magnification alone, from which the construction of the microscope now becomes known. Therefore it remains finally, that we may run through the remaining conditions pertaining to that. Indeed in the first place the radius of the aperture of this lens shall be $= x$, which hence will be required for the

determination of the confusion, and from the third problem of the introduction, the order of the clarity is apparent to be $y = \frac{hx}{ma} = x$ on account of $a = \frac{h}{m}$, since indeed it is itself evident, as the pencil of rays transmitted clearly shall be equal to the aperture itself. From the fourth problem, since besides the objective lens no other shall be present, here the letters $\mathfrak{q}, \mathfrak{r}, \mathfrak{s}$ etc. do not occur ; but for the place of the eye we will have here $O = 0$, or the eye will be required to be placed in contact with the lens and the apparent field of view plainly here cannot be determined, thus so that the view of the eye is unbounded. Again, from the fifth problem, it is understood there need be no concern here about a colored margin, since this is produced only by a series of lenses.
Truly the sixth problem does not pertain here. Thereafter the seventh problem gives this equation $0 = N \cdot \frac{1}{p}$, which since it shall be unable to occur, this confusion cannot generally be removed, but thus rather it may become greater, where p will be smaller or where a greater magnification may be desired. Finally, from the eighth problem, we deduce this equation for our case :

$$\frac{max^3}{h} \cdot \frac{\mu}{p^3}\left(\lambda + \frac{\mathfrak{A}^2}{A} \cdot \nu\right) < \frac{1}{k^3},$$

which on account of $A = \infty$ and $ma = h$ will be changed into this :

$$\frac{\mu\lambda x^3}{p^3} < \frac{1}{k^3},$$

from which there becomes

$$x = \frac{p}{k}\sqrt[3]{\frac{1}{\mu\lambda}},$$

and thus the aperture of the lens is known and likewise also the order of clarity and thus in this way everything pertaining to microscopes has been defined.

COROLLARY 1

§ 39. Therefore since for the aperture of the lens its radius shall be found to be $x = \frac{p}{k}\sqrt[3]{\frac{1}{\mu\lambda}}$, it is evident that the clarity, however great it may become, we may increase without detriment of the distinction, there must be taken $\lambda = 1$, and since μ does not differ much from unity, there will become $x = y = \frac{p}{k}$; from which, since there shall be approximately $k = 50$, there is no doubt, why the lens may not permit this aperture. Moreover we have seen before to be $p = \frac{h}{m}$, thus so that now we will have $x = y = \frac{h}{km}$. Whereby if we may put in place $k = 48$ and $h = 8$ in., there will be $x = y = \frac{1}{6m}$ in., and thus at once the magnification m will rise above 8 , the degree of clarity will emerge smaller than that, which we have agreed on for telescopes.

SCHOLIUM 1

§ 40. But the lens, which only magnifies eight times, scarcely merits the name of microscope, since a focal length p of one inch may be produced; yet meanwhile we see hence the radius of the aperture ought not to increase beyond $\frac{1}{48}$ in., if indeed we wish to enjoy such a great degree of distinction, as great as is accustomed to be required in telescopes. But it is agreed from trial and error for lenses of this kind to be granted a much greater aperture nor thus to be usual for the measurement to be defined, but truly also likewise it is agreed images of this kind to be rendered impure with a high order of confusion; which thus prevails also for all kinds of microscopes, of which the image represented generally is much more confused, than is accustomed to be tolerated in telescopes. Whereupon it is seen, while it is concerned with microscopes, a much smaller value of the letter k can be attributed than 50 can be attributed without risk, which thus with certain microscopes not to be scorned lest they be found to rise to 20. Yet meanwhile there is no doubt, why these instruments may not become much more useful, if they may be able to be freed from so much confusion; whereby here indeed I am going to assign the value 20 for the letter k, yet I shall let no occasion pass, whenever it will have been allowed to happen, for this degree of confusion being required to be diminished.

COROLLARY 2

§ 41. Therefore on taking $k = 20$ the radius of the aperture of the microscope must become $x = \frac{2}{5m}$ in.; for which since it shall be equal to the measure of the clarity, and at once m may exceed 8, in which case there becomes $y = \frac{1}{20}$ in., so that we can see no greater an object with full clarity, but where the ratio $m:8$ were greater, thus we must be content with a smaller clarity.

COROLLARY 3

§ 42. But since no greater clarity is required to be expected, unless there may be taken $\lambda = 1$, it is clear, a figure must be attributed to the size of the microscope lens present, and since there shall be $\mathfrak{A} = 1$, it will be convenient to have constructed this lens, so that the radius of the anterior face shall be $= \frac{p}{\rho}$ and the radius of the posterior face $= \frac{p}{\varepsilon}$.
Indeed if we wish to use a lens with each face equally convex, the confusion will be greater than a half.

SCHOLIUM 2

§ 43. Therefore so that the construction of this kind of telescopes may be seen more easily and clearer for any magnification, we have adjoined this table, in which we shall present, for particular values of the letter *m*, initially the distance of the object from the lens, which is the same as the focal length; then truly we will designate the radius of each face of the lens expressed in inches in two columns ; then truly we will assign the radius of the aperture and thus the order of clarity, indeed we will thus assign the radius of the aperture and the degree of clarity, so that with the full clarity put equal to $=1$ and the radius of the pupil $= \frac{1}{20}$ in. we may express the order of the clarity by $20y = \frac{8}{m}$, even if properly the square of this fraction must be taken, since the clarity does not depend on the diameter of the pencils, but rather on the total width of these. Which moreover it suffices to remind once. For the refraction of the glass we may take $n = 1,55$, so that there shall become

$$\rho = 0,1907, \quad \sigma = 1,6274,$$

and there will become

$$\text{radius of the} \begin{cases} \text{anterior face} = \ 5,2488p = 41,9504 \cdot \frac{1}{m} \\ \text{posterior face} = 0,6145p = \ \ 4,9160 \cdot \frac{1}{m} \end{cases}$$

on account of $p = \frac{8}{m}$ in. $= a$; then truly the radius of the aperture $x = \frac{2}{5m}$ in. and the measure of the clarity, as we have seen in the manner indicated, $= \frac{8}{m}$, from which the following table is constructed easily:

Magnification	Focal length	Radius anterior face	Radius posterior face	Radius of aperture	Degree of clarity
10	0,800	4,195	0,492	0,040	0,800
20	0,400	2,097	0,246	0,020	0,400
30	0,266	1,398	0,164	0,013	0,266
40	0,200	1,048	0,123	0,010	0,200
50	0,160	0,839	0,098	0,008	0,160
60	0,133	0,699	0,082	0,007	0,133
70	0,114	0,599	0,070	0,006	0,114
80	0,100	0,524	0,062	0,005	0,100
90	0,089	0,466	0,055	0,004	0,089
100	0,080	0,419	0,049	0,004	0,080
120	0,066	0,349	0,041	0,003	0,066
140	0,057	0,299	0,035	0,003	0,057
160	0,050	0,262	0,031	0,002	0,050

Hence it is clear these magnifications cannot be continued further, since then the radii of the faces of the lens will have become exceedingly small, so that they may be able to be made in practice, then truly the aperture as well must become so very small, so that on account of the defect of the clarity objects will scarcely be able to be seen. Moreover since the aperture of these lenses must be so very small, these lenses themselves also will be allowed to be made so small, so that the thicknesses of these besides the focal lengths, however small that may have been, may be able to be ignored without error, since evidently in these lenses and in larger lenses the reasoning is the same ; truly it will be required to make these most thin lenses, so that the margins around these may agree amongst themselves as it were. But since generally it is customary to attribute much greater thicknesses to these lenses, which may maintain a notable enough ratio to the focal length and thus will exceed that, as happens in glass globes, which are accustomed to be used in place of small lenses of this kind, generally it will be worth the effort in the determination of such microscopes, to give an account of the thickness.

PROBLEM 2

§ 44. *If the thickness of the lens may not be permitted to be ignored, to construct microscopes, which may represent objects increased according to the following given ratio.*

SOLUTION

For solving this problem, the solution of the problem in Book I, § 329 is to be advanced, the solution of which may be transferred here by putting in place $O=\alpha+l=0$, thus so that there shall be $l=-\alpha$ with there being $\alpha=\infty$, as we assume here. Therefore then, if the distance of the object before the lens shall be $=a$, the thickness of the lens $=v$, the radius of the anterior face $=f$ and that of the posterior face $=g$, with that indefinite quantity introduced $=k$ we have found these formulas there:

$$f=\frac{(n-1)a(k+v)}{k+v+2na},\quad g=\frac{(n-1)(v-k)}{2n},$$

from which the lens is determined. So that if now there may be put $\frac{k-v}{k+v}=i$, there we define the magnification

$$m=\frac{1}{i}\cdot\frac{h}{a}=\frac{k+v}{k-v}\cdot\frac{h}{a}.$$

Thence if the radius of the aperture in the anterior face shall be *x*, that must be no smaller than $ix=\frac{k-v}{k+v}\cdot x$ in the posterior face, and it will give rise to the order of clarity

$$y = -ix = \tfrac{v-k}{v+k} \cdot x.$$

Truly later the following formula has been found for the radius of confusion [Book. I, § 210] :

$$+\frac{1}{4}ix^3 \cdot \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2-\frac{2}{k-v}\cdot\frac{4}{(k+v)^2}\right),$$

the value of which must not exceed the limit put in place before of $\frac{1}{4k^2}$ with $k = 20$; moreover that is reduced to this form :

$$\frac{n}{8(n-1)^2}\cdot\frac{x^3}{k+v}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{k+v}{a}+2\right)^2\cdot\frac{1}{k-v}-\frac{8}{(k+v)^2}\right),$$

from which the following equation is deduced:

$$\frac{n}{2(n-1)^2}\cdot x^3\left(\frac{n(k+v)}{a^3(k-v)}+\frac{4n+2}{a^2(k-v)}+\frac{4n+8}{a(k^2-v^2)}+\frac{16v}{(k+v)^2(k-v)}\right)=\frac{1}{k^3},$$

from which the quantity x can be defined. Again above the position of the eye was defined thus, so that now there may become:

$$O = \frac{(v-k)v}{n(v+k)};$$

from which a portion within the object may be viewed , of which the radius

$$z = \frac{na}{v}\cdot\frac{k-v}{k+v}\cdot x.$$

So that the colored margin may be removed, there must become $k = \infty$, if indeed $O > 0$; but since there may become $O < 0$, there must become $k = -2a - v$,and since for this confusion to be removed completely there must become

$$(\alpha + v)(a + \alpha + v) = 0,$$

but on account of $\alpha = \infty$, it is evident this confusion to become very large.

COROLLARY 1

§ 45. Since we have found

$$\frac{k+v}{k-v}\cdot\frac{h}{a}=m,$$

in order that this value shall be positive or the image represented upright, it is necessary, that the quantity k may be contained beyond the bounds $+v$ et $-v$; indeed if it may be contained within these limits, the magnification m shall become negative and thus the image is represented inverted, while clearly a real image shall be formed by the lens.

COROLLARY 2

§ 46. Therefore two cases are required to be considered, for which the magnification m shall be positive, the one, where not only $k > 0$, but also $k > v$; the anterior face of the lens will be convex, the posterior truly concave and $m>\frac{h}{a}$; truly for the other case, where $k < 0$ and likewise $k<-v$, or if on putting $k=-l$ if there were $l>v$, there will become

$$f=\frac{(n-1)a(l-v)}{l-v-2na} \quad \text{and} \quad g=\frac{(n-1)(l+v)}{2n}$$

and thus the posterior face will be convex always, anterior truly concave, unless $l>v+2na$; for if $l>v+2na$, the anterior face also will be convex and in this case again there will become $m<\frac{h}{a}$.

COROLLARY 3

§ 47. So that it may reach to the position of the eye, for which there is

$$O=\frac{(v-k)v}{n(v+k)},$$

since $\frac{k+v}{k-v}$ is a positive quantity, evidently $=\frac{ma}{h}$, there will become $O=-\frac{vh}{mna}$ and thus always to be negative on account of the thickness of the lens, nor will this value vanish unless likewise the thickness of the lens were vanishing.

COROLLARY 4

§ 48. This expression is noteworthy for the order of the clarity, because $my=\frac{hx}{a}$ and thus $y=\frac{hx}{ma}$, from which it is apparent the thickness of the lens changes nothing in the clarity order, clearly if for the same magnification *m* the aperture *x* may arrive at the same value.

COROLLARY 5

§ 49. Since in the first case, where there was $v=0$, the apparent field would have been indefinite on account of the thickness of the lens this will be determined thus, so that there shall be

$$mz=\frac{nhx}{v} \text{ or } z=\frac{nhx}{mv},$$

from which it is apparent, so that the smaller were the thickness, thus the greater the field would become and if in place of *x* the clarity *y* may be introduced, it will produce $z=\frac{nay}{v}$, thus so that for the same ratio of the thickness to the distance *a* , the radius of the field shall be proportional to the clarity.

SCHOLIUM I

§ 50. Since here two principal cases have arisen requiring to be considered, the one, where $k>v$, the other truly, where $l>v$, with there being $l=-k$, for the first equation it has been shown the solution rendering the confusion insensible has been shown in the solution; truly for the second that will have to be come thus :

$$\frac{n}{2(n-1)^2}\cdot x^3\left(\frac{n(l-v)}{a^3(l+v)}-\frac{4n+2}{a^2(l+v)}+\frac{4n+8}{a(l^2-v^2)}+\frac{16v}{(l-v)^3(l+v)}\right)=\frac{1}{k^3}.$$

So that if now we may introduce the magnification *m* , truly we may eliminate *l* or *k*, hence the equation will adopt this form :

$$\frac{n}{2(n-1)^2a^2h}\cdot x^3\left(mn-\frac{(2n+1)(h-ma)}{v}+\frac{(n+2)(h-ma)^2}{mv^2}+\frac{(h-ma)^4}{m^3av^3}\right)=\frac{1}{k^3},$$

which, if for the sake of brevity there may be put $\frac{h-ma}{v}=s$, will adopt this form:

$$\frac{n}{2(n-1)^2a^2h}\cdot x^3\left(mn-(2n+1)s+\frac{(n+2)s^2}{m}-\frac{s^2}{m^2}+\frac{hs^3}{m^3a}\right)=\frac{1}{k^3};$$

which again may be changed into this form:

$$\frac{n}{2(n-1)^2 a^2 h}\cdot x^3\left(m\left(1-\frac{s}{m}\right)^2\left(n-\frac{s}{m}\right)+\frac{hs^3}{m^3 a}\right)=\frac{1}{k^3},$$

which will be applied much more easily to any case. Finally since between the two cases mentioned there may be put $k=\infty$ as if a mean, there will become $k=\infty$

$$f=(n-1)a,\ \ g=\infty,\ m=\frac{h}{a},\ y=+x,\ O=-\frac{v}{n},\ z=\frac{nax}{v};$$

truly in addition this equation will be required to be resolved:

$$\frac{n^2 x^3}{2a^3(n-1)^2}=\frac{1}{k^3},$$

from which there is deduced

$$x=\frac{a}{k}\sqrt[3]{\frac{2(n-1)^2}{n^2}},$$

thus so that here the value shall be notably smaller than in the first problem ; since in the first problem the face was almost plane, from which this case arises, if that lens may be inverted, with which done, there a much smaller aperture may be allowed; and finally thus it is noteworthy, that neither the thickness of the lens nor the magnification shall change in any confusion.

SCHOLIUM 2

§ 51. Since our set up is not able to handle all possible cases, but only these, for which it may be allowed to attribute one or perhaps several lenses of excellent quality, here this single case is seen to be mainly worth our attention in the problem mentioned, where it is free from a colored margin; which happens, as we have seen, if there may be taken $k=-2a-v$. Yet meanwhile it is necessary for us to include setting out that special case, where in place of a small lens it is customary to make use of a whole glass sphere, since microscopes of this kind may be prepared easily and are called into use frequently; according to which we resolve the two following problems for these two cases of this chapter .

PROBLEM 3

§ 52. *To construct a given microscope without ignoring the thickness of the lens thus, so that it may magnify in a given ratio and likewise may represent a given object without a colored margin.*

SOLUTION

Since on account of the thickness, the distance of the eye O always may be produced negative and thus it may be required to apply the eye directly to the lens, so that the colored margin may be reduced to zero, now we have seen there must be taken $k=-2a-v$; from which both radii of the lens will be expressed thus:

$$f=-a \text{ and } g=\frac{(n-1)}{2}(a+v),$$

thus so that the first face shall be concave and the object itself must be located at its very centre ; from which the rays plainly will undergo no refraction in the first face; hence we will have this equation for the magnification : $m=\frac{h}{a+v}$; from which, since m is given, there is deduced $a+v=\frac{h}{m}$; but for the degree of clarity there will be $y=\frac{a+v}{a}x$; from which , if as above the radius of the pupil may be considered to be $\frac{1}{20}$ in., the measure of the clarity will be able to be estimated to be

$$20y=\frac{20(a+v)}{a}\cdot x,$$

since clearly x is expressed in inches. But for the position of the eye there is found

$$O=-\frac{(a+v)v}{na};$$

which since it shall be negative, the eye will be required to be applied directly to the lens. Since the radius of the anterior aperture has been put $=x$, that in the posterior face will be $=\frac{a+v}{a}\cdot x$, which itself is the value of y. Hence for the apparent field there will be,

$$z=\frac{n(a+v)}{v}\cdot x.$$

Finally in order that the quantity x may be defined from the condition of distinction, we may use from the first equation, which on account of

$$i=\frac{k-v}{k+v}=\frac{a+v}{a} \text{ and } k+v=-2a, \quad k-v=-2(a+v)$$

and hence

$$\frac{1}{ia}+\frac{2}{k-v}=0 \text{ and } (k-v)(k+v)^2=-8a^2(a+v)$$

will be changed into this:

$$\frac{n}{2(n-1)^2}\cdot\frac{x^3}{a^3}=\frac{1}{k^3},$$

from which there is deduced:

$$x = \frac{a}{k}\sqrt[3]{\frac{2(n-1)^2}{n}},$$

from which value all the remaining may be deduced.

EXAMPLE

We may set the thickness $v = a$ and there shall be common glass, of which the refractive index $n = 1,55$, and there will be found

$$x = 0,73081 \cdot \frac{a}{20} = 0,0365a$$

clearly on putting $k = 20$ as before ; moreover since the magnification m may be given and thus there shall be $a = \frac{h}{2m}$, there will be

$$x = 0,0183 \cdot \frac{h}{m}$$

from which the following construction is deduced :

I. Distance of the object from the lens $a = \frac{h}{2m}$.
II. Radius of the anterior face $= -a = -\frac{h}{2m}$.
III. Thickness of the lens $v = \frac{h}{2m}$.
IV. Radius of the posterior face $= \frac{n-1}{n} \cdot \frac{h}{m} = 0,35484 \cdot \frac{h}{m}$.
V. Radius of the anterior aperture $= 0,0183 \cdot \frac{h}{m}$.
VI. Radius of the posterior aperture $= 0,0366 \cdot \frac{h}{m}$ in.
VII. Measure of the clarity $= 0,732 \cdot \frac{h}{m}$ or by taking $h = 8$ in.,
this measure will be $= \frac{5,856}{m}$.
VIII. Radius of the area seen in the object $z = 2nx = 0,0567 \cdot \frac{h}{m}$.

Which so that it may be compared more easily with the case of the first problem, we may establish the case, where with the magnification $m = 100$, and by taking h = 8 in. the following determinations will be produced:

I. Distance of the object from the lens $= 0,04$ in.
II. Radius of the anterior lens $= -0,04$ in.
III. Thickness of the lens $= 0,04$ in.
IV. Radius of the posterior face $= 0,0284$ in.
V. Radius of the anterior aperture $= 0,0015$ in.

VI. Radius of the posterior aperture $=0,0030$ in.
VII. Measure of the clarity = 0,0585 in.
VIII. Radius of the area viewed in the object $=0,0045$ in.

SCHOLIUM

§ 53. Therefore these microscopes are much inferior to the preceding ones, where the thickness was a minimum, nor therefore can any advantage come to mind for making microscopes of this kind.

PROBLEM 4

§ 54. *If a glass globule may be used in place of the lens, to describe the construction of the microscope, so that a given magnification may be produced.*

SOLUTION

Here therefore there will be 1. $f=g$, then truly 2. $v=2f$. We deduce at once from the first condition

$$\frac{a(k+v)}{k+v+2na}=\frac{v-k}{2n},$$

from which it follows:

$$k^2+4nak=v^2.$$

But since again there shall be $v=2f$, the value of g will give

$$f=\frac{(n-1)(2f-k)}{2n} \text{ or } 2f+(n-1)k=0,$$

from which there becomes $k=-\frac{2f}{n-1}$, which value substituted in the above equation provides

$$(2-n)f-2(n-1)a=0,$$

thus so that there shall become

$$f=\frac{2(n-1)}{2-n}\cdot a \text{ or } a=\frac{2-n}{2(n-1)}\cdot f.$$

Now truly the magnification m gives

$$m=\frac{2-n}{n}\cdot\frac{h}{a}=\frac{2(n-1)}{n}\cdot\frac{h}{f}$$

on account of

$$k+v=-\frac{2(2-n)}{n-1}\cdot f \quad \text{and} \quad k-v=-\frac{2n}{n-1}\cdot f;$$

from which we come upon

$$f=\frac{2(n-1)}{n}\cdot\frac{h}{m} \quad \text{and} \quad a=\frac{2-n}{n}\cdot\frac{h}{m}$$

and hence

$$k=-\frac{4}{n}\cdot\frac{h}{m} \quad \text{and} \quad v=\frac{4(n-1)}{n}\cdot\frac{h}{m},$$

from which there is found for the position of the eye

$$O=-\frac{4(n-1)}{n(2-n)}\cdot\frac{h}{m};$$

since which distance shall be negative, the eye will have to be applied directly to the lens.

Now so that we may obtain the maximum value of x , in the first place we may retain the quantity a in the calculation, and since there shall be

$$f=\frac{2(n-1)a}{2-n}, \quad k=-\frac{4a}{2-n}, \quad v=\frac{4(n-1)a}{2-n}$$

and hence

$$k-v=-\frac{4n}{2-n}\cdot a \quad \text{and} \quad k+v=-4a$$

and hence

$$\frac{k+v}{k-v}=\frac{2-n}{n}\cdot a \quad \text{and} \quad k^2-v^2=\frac{16n}{2-n}\cdot a^2,$$

the equation found above shall adopt this form:

$$\frac{n}{2(n-1)^2}\cdot x^3\left(\frac{2-n}{a^3}-\frac{(2n+1)(2-n)}{2na^3}+\frac{(n+2)(2-n)}{4na^3}+\frac{n-1}{4na^3}\right)=\frac{1}{k^3},$$

which again is reduced to this:

$$\frac{3n-n^2-1}{8(n-1)^2}\cdot\frac{x^3}{a^3}=\frac{1}{k^3},$$

from which there is found

$$x=\frac{2a}{k}\sqrt[3]{\frac{(n-1)^2}{3n-n^2-1}}=\frac{2(2-n)}{nk}\cdot\frac{h}{m}\sqrt[3]{\frac{(n-1)^2}{3n-n^2-1}}.$$

Therefore with the radius of the aperture x found in the anterior part of the globule, in the posterior part this will become $=\frac{n}{2-n}\cdot x$, to which the order of the clarity y is equal ; truly we express the measure of the clarity by $20y$, while clearly the distances are expressed in inches.

Then truly the radius of the area to be viewed in the object will be $z = \frac{n^2}{4(n-1)} \cdot x$. But since the distance of the eye O has emerged negative, so that the colored margin may vanish, there must become $k = -2a - v$ or $-4a + 2a = 0$; since which cannot be the case, also it is clear the colored margin cannot be removed, but to be notable enough. Therefore from all these we may deduce the following rules for the construction of microscopes of this kind, in which the magnification shall be $= m$:

I. A glass globule shall be prepared, of which the radius shall be $f = \frac{2(n-1)}{n} \cdot \frac{h}{m}$; if the refraction of which shall be $n = 1,55$ and there may be taken $h = 8$, there will become $f = \frac{5,6774}{m}$ in.

II. An object must be placed at a distance $a = \frac{2,3226}{m}$ in. before this globule.

III. Moreover in the anterior part an aperture may be attributed to the globule, of which the radius shall be

$$x = \frac{2(2-n)}{nk} \cdot \frac{h}{m} \sqrt[3]{\frac{(n-1)^2}{3n-n^2-1}},$$

which expression expanded numerically becomes

$$x = \frac{36}{155m} \sqrt[3]{\frac{3025}{12475}} \text{ in.}$$

or

$$x = \frac{0,14483}{m} \text{in.} \quad \text{and hence} \quad z = \frac{0,15816}{m} \text{ in.}$$

But in the posterior part the radius of the aperture must become

$$ix = \frac{2h}{km} \sqrt[3]{\frac{(n-1)^2}{3n-n^2-1}} = \frac{0,49887}{m} \text{ in.}$$

Now since there shall become $y = ix$, we will have the measure of the clarity $20y = \frac{9,9774}{m}$, which therefore is greater than in the case of the simple lens, where it was only $= \frac{1}{m}$, but since the gain by no means may compensate that defect, whereby the objects appear tainted by a colored margin. Therefore it will be worth the effort to adjoin a similar table, such as we have given in problem 1 above:

Magnification	Distance of object	Radius of globule	Anterior radius	Posterior radius	Measure of clarity	Radius of field
10	0,232	0,568	0,014	0,050	0,998	0,016
20	0,116	0,284	0,007	0,025	0,499	0,008
30	0,077	0,189	0,005	0,017	0,333	0,005
40	0,058	0,142	0,004	0,012	0,249	0,004
50	0,046	0,114	0,003	0,010	0,199	0,003
60	0,039	0,094	0,002	0,008	0,166	0,003

as it will not be convenient to continue further on account of the exceedingly small apparent field of view; but if a greater aperture may be assumed, the confusion produced clearly will become intolerable.

SCHOLIUM

§ 55. Now from these it is abundantly clear in this kind of simple microscopes, the first example introduced where the thinnest small lenses may be used, with all the remaining kinds long since fallen out of use; and yet meanwhile this kind labours under two conspicuous inconveniences, which we may consider here so that it may become clearer, what chiefly may be desired in the perfection of microscopes. The first inconvenience is the excessive proximity by which an object must be moved up to the lens, which happens, so that the distance must almost vanish completely for a greater magnification, which circumstance is the reason why, if objects may not be the smoothest, the smallest inequalities shall maintain either an exceedingly great or exceedingly small distance from the lens, and thus may appear with the greatest confusion. Therefore in the first place it will be a requirement chiefly for the greater magnifications microscopes of this kind may be invented, which do not demand so small a distance from the lens. The other inconvenience consists of the exceedingly small clarity that microscopes of this kind show in the greater magnifications ; indeed from the above table § 43 we have seen shown, if the magnification shall be $m = 100$, the clarity to be designated there to be 0,080, and since the clarity shall be proportional to the square of this number, that shall become 0,0064 and thus 156 times smaller than the nature clarity [which is equal to 1], which indeed at this stage is acceptable enough, unless the nature of the object itself shall be very obscure; but hence it is understood, if a much greater magnification may be desired, a stronger light should be used. Indeed the remedy of this same defect may be by increasing the aperture of the lens; but then the confusion may be increased so much, that finally it can no longer be tolerated, especially since we have added that same table, so that there may be thus only $k = 20$, while for telescopes it is customary to put $k = 50$, thus so that with these microscopes the order of distinction now shall be fifteen times smaller than in telescopes [following the inverse cube of *k*], thus so that rather it shall be required to attend to obtaining a greater order of distinction. But the greater part of that latter inconvenience will be removed from the central part by doubling or go as far as tripling the lens, but not to the extend where multiple lenses of this kind, such as we have described in the first book, will be able to be used, of which clearly the interval has been assumed to vanish completely; on account of which in this business it will be agreed to assume now so great an interval between these lenses, which may be able to be used in practice; which discussion we will subject to be investigated most carefully in the following chapters; truly finally we will have the thickness of the lenses always taken as zero, from which the greatest caution will be required, lest thicker lenses may be made than the forms and apertures of these demand.

SECTIO PRIMA.
DE
MICROSCOPIIS SIMPLICIBUS.

CAPUT I

DE MICROSCOPIIS SIMPLICIBUS UNICA LENTE CONSTANTIBUS

TAM NEGLECTA LENTIS CRASSITIE QUAM EIUS RATIONE HABITA

PROBLEMA 1

§ 38. *Microscopium simplex conficere, quod obiecta secundum datam rationem aucta repraesentet neglecta lentis crassitie.*

SOLUTIO

Sit multiplicatio praescripta $=m$, quae scilicet ad distantiam pro arbitrio assumtam k referatur denotante k vulgo distantiam 8 dig., sitque p distantia focalis lentis, quae sola microscopium constituit. Cum igitur esse debeat $\alpha=\infty$ erit quoque $A=\infty$; unde fit $\mathfrak{A}=1$ ideoque $a=p$, ita ut distantia obiecti ante lentem praecise eius distantiae focali p aequalis esse debeat. Cum igitur sit in genera $m=PQR\ ...\ Z.\frac{h}{a}$; pro nostro casu, quo unica adest lens, haec formula per omnes litteras $P, Q, R, ...\ Z$ debet dividi, ita ut fiat $m=\frac{h}{a}$ quod etiam hoc modo facillime ostenditur: cum enim haec imago cadat ad distantiam $\alpha=Aa$, eius semidiameter sit $A\zeta$ existente ζ semidiametro obiecti, haec imago ab oculo cernetur sub angulo $\frac{\zeta}{a}$, dum idem obiectum ad distantiam b existens nudo oculo appariturum esset sub angulo $=\frac{\zeta}{b}$, ex quo ille angulus per hunc divisus ipsam dat multiplicationem, ita ut sit $m=\frac{h}{a}$; quare cum haec multiplicatio m sit data, hinc colligitur $a=p=\frac{h}{m}$ sicque tam distantia focalis lentis quam distantia obiecti ante lentem per solam multiplicationem praescriptam determinatur, ex quo constructio microscopii iam innotescit. Tantum igitur superest, ut reliquas conditiones eo pertinentes percurramus. Primo igitur sit semidiameter aperturae huius lentis $=x$, quam deinceps ex confusione determinari oportebit, et ex problemate tertio Introductionis patet fore gradum claritatis $y=\frac{\mathrm{h}x}{ma}=x=x$ ob $a=\frac{h}{m}$, quod quidem per se est perspicuum, cum penicillus radiosus transmissus ipsi aperturae manifesto sit aequalis. Ex quarto problemate, cum praeter lentem obiectivam nulla alia adsit, litterae $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$ etc. hic nullum locum inveniunt; at pro loco oculi hic habebimus $O=0$ sive oculum lenti immediate adplicari oportet campusque apparens hic plane non determinatur, ita ut visus oculi nusquam terminetur. Ex quinto porro problemate intelligitur hic nullum marginem coloratum esse pertimescendum, quia is tantum a lentibus sequentibus producitur. Sextum vero problema huc prorsus non pertinet. Septimum dein problema hanc dat aequationem $0 = N \cdot \frac{1}{p}$, quod cum fieri nequeat, haec confusio tolli omnino non potest, sed potius eo maior fiet, quo minus erit p seu quo maior desideretur multiplicatio. Ex octavo denique problemate deducimus pro nostro casu hanc aequationem:

$$\frac{max^3}{h} \cdot \frac{\mu}{p^3}\left(\lambda + \frac{\mathfrak{A}^2}{A} \cdot \nu\right) < \frac{1}{k^3},$$

quae ob $A = \infty$ et $ma = h$ abit in hanc:

$$\frac{\mu\lambda x^3}{p^3} < \frac{1}{k^3},$$

ex qua fit

$$x = \frac{p}{k}\sqrt[3]{\frac{1}{\mu\lambda}},$$

sicque apertura lentis innotescit simulque etiam gradus claritatis hocque modo omnia, quae ad microscopium pertinent, sunt definita.

COROLLARIUM 1

§ 39. Cum igitur pro apertura lentis inventa sit eius semidiameter $x = \frac{p}{k}\sqrt[3]{\frac{1}{\mu\lambda}}$, evidens est, ut claritatem, quantum fieri potest, sine detrimento distinctionis augeamus, sumi debere $\lambda = 1$, et quia μ non multum ab unitate differt, fiet $x = y = \frac{p}{k}$; unde, cum sit circiter $k = 50$, nullum est dubium, quin lens hanc aperturam admittat. Ante autem vidimus esse $p = \frac{h}{m}$, ita ut nunc habeamus $x = y = \frac{h}{km}$. Quare si statuamus $k = 48$ et $h = 8$ dig., erit $x = y = \frac{1}{6m}$ dig. sicque, statim ac multiplicatio m supra 8 excurrat, gradus claritatis minor evadet eo, quem supra telescopiis conciliavimus.

SCHOLION 1

§ 40. Lens autem, quae tantum octies multiplicat, vix nomen microscopii meretur, cum distantia focalis p prodeat unius digiti; interim tamen hinc videmus semidiametrnm aperturae non ultra $\frac{1}{48}$ dig. augeri debere, si quidem tanto distinctionis gradu frui velimus, quantus in telescopiis exigi solet. Experientia autem constat huiusmodi lentibus multo maiorem aperturam tribui neque adeo ad mensuram definiri solere, at vero etiam indidem constat huiusmodi repraesentationes non mediocri confusione esse inquinatas; quod adeo etiam de omnibus microscopiis valet, quorum repraesentatio plerumque multo magis confusa est, quam in telescopiis tolerari solet. Quocirca videtur, dum de microscopiis agitur, litterae k multo minor valor quam 50 tute tribui posse, quem adeo in quibusdam microscopiis non spernendis ne ad 20 quidem assurgere comperi. Interim tamen nullum est dubium, quin haec instrumenta multo maiorem utilitatem sint allatura, si a tam notabili confusione liberari queant; quare hic quidem litterae *k* valorem 20 sum assignaturus, nullam tamen occasionem praetermittam, quoties fieri licuerit, hunc confusionis gradum diminuendi.

COROLLARIUM 2

§ 41. Sumto ergo $k = 20$ semidiameter aperturae microscopii debebit esse $x = \frac{2}{5m}$ dig.; cui cum mensura claritatis sit aequalis, statim atque *m* superat 8, quo casu fit $y = \frac{1}{20}$ dig., non amplius obiecta plena claritate videmus, sed quo maior fuerit ratio $m:8$, eo minore claritate contenti esse debemus.

COROLLARIUM 3

§ 42. Quia autem ne tanta quidem claritas est exspectanda, nisi capiatur $\lambda = 1$, patet, quanti intersit lenti microscopicae debitam figuram tribuisse, et cum sit $\mathfrak{A} = 1$, hanc lentem ita construi conveniet, ut sit radius faciei anterioris $= \frac{p}{\rho}$ et posterioris $= \frac{p}{\varepsilon}$. Si enim lente utrinque aeque convexa uti vellemus, confusio ultra dimidium fieret maior.

SCHOLION 2

§ 43. Quo igitur constructio huiusmodi microscopiorum pro qualibet multiplicatione facilius et clarius perspiciatur, tabulam hic subiungamus, in qua pro praecipuis valoribus litterae *m* primo distantiam obiecti a lente, quae eadem est eius distantia focalis, exhibeamus; deinde vero radios utriusque lentis faciei in digitis expressos duabus columnis designemus; tum vero semidiametrum aperturae et gradum claritatis ita assignemus, ut posita claritate plena $= 1$ et pupillae semidiametro $= \frac{1}{20}$ dig. gradus claritatis per $20y = \frac{8}{m}$ exprimatur, etiamsi proprie quadratum huius fractionis sumi deberet, quoniam claritas pendet non a diametro penicillorum, sed a tota eorum crassitie. Quod autem semel monuisse sufficit. Pro refractione autem vitri sumamus $n = 1,55$, ut sit

$$\rho = 0,1907, \quad \sigma = 1,6274,$$

eritque

$$\text{radius faciei} \begin{cases} \text{anterioris} = 5,2488p = 41,9504 \cdot \frac{1}{m} \\ \text{posterioris} = 0,6145p = 4,9160 \cdot \frac{1}{m} \end{cases}$$

ob $p = \frac{8}{m}$ dig. $= a$; tum vero est semidiameter aperturae $x = \frac{2}{5m}$ dig. et mensura claritatis, ut modo vidimus, $= \frac{8}{m}$, unde facile sequens tabula conficitur:

Multiplicatio	Distantia focalis	Radius faciei anterioris	Radius faciei posteriori	Semidiameter aperturae	Mensura claritatis
10	0,800	4,195	0,492	0,040	0,800
20	0,400	2,097	0,246	0,020	0,400
30	0,266	1,398	0,164	0,013	0,266
40	0,200	1,048	0,123	0,010	0,200
50	0,160	0,839	0,098	0,008	0,160
60	0,133	0,699	0,082	0,007	0,133
70	0,114	0,599	0,070	0,006	0,114
80	0,100	0,524	0,062	0,005	0,100
90	0,089	0,466	0,055	0,004	0,089
100	0,080	0,419	0,049	0,004	0,080
120	0,066	0,349	0,041	0,003	0,066
140	0,057	0,299	0,035	0,003	0,057
160	0,050	0,262	0,031	0,002	0,050

Hinc evidens est has multiplicationes ulterius continuari non posse, cum tum radii facierum lentis nimis fierent exigui, quam ut in praxi elaborari possint, tum vero apertura tam parva fieri deberet, ut ob defectum claritatis obiecta vix conspici possent. Ceterum cum apertura harum lentium tam exigua esse debeat, eas quoque ipsas tam parvas conficere licebit, ut earum crassities prae distantia focali, quantumvis ea parva fuerit, sine errore negligi queat, quia scilicet in his lentibus eadem ac in maioribus ratio est; tenuissimas nempe has lentes elaborari oportet, ut margines circumquaque inter se quasi conveniant. Cum autem plerumque his lentibus multo maior crassities tribui soleat, quae ad distantiam focalem satis notabilem teneat rationem eamque adeo superet, uti fit in globulis vitreis, qui loco huiusmodi lenticularum usurpari solent, operae utique pretium erit in determinatione talium microscopiorum crassitiei rationem habere.

PROBLEMA 2

§ 44. *Si lentis crassitiem negligere non liceat, microscopia conficere, quae obiecta secundum datam rationem aucta repraesentent.*

SOLUTIO

Ad hoc problema solvendum consideretur solutio problematis in Lib. I § 329 allati, cuius solutio huc transferetur statuendo $O=\alpha+l=0$, ita ut sit $l=-\alpha$ existente $\alpha=\infty$, uti hic assumimus. Tum igitur, si distantia obiecti ante lentem sit $=a$, crassities lentis $=v$, radius faciei anterioris $=f$ et posterioris $=g$, introducta quantitate adhuc indefinita $=k$ has ibi invenimus formulas:

$$f=\frac{(n-1)a(k+v)}{k+v+2na},\quad g=\frac{(n-1)(v-k)}{2n},$$

quibus lens determinatur. Quodsi nunc ponatur $\frac{k-v}{k+v}=i$, definimus ibi mutiplicationem

$$m=\frac{1}{i}\cdot\frac{h}{a}=\frac{k+v}{k-v}\cdot\frac{h}{a}.$$

Deinde si aperturae semidiameter in facie anteriore sit x, in facie posteriors ea debet esse non minor quam $ix=\frac{k-v}{k+v}\cdot x$ proditque gradus claritatis

$$y=-ix=\frac{v-k}{v+k}\cdot x.$$

Postea vero pro semidiametro confusionis inventa est [Lib. I, § 210] sequens formula:

$$+\frac{1}{4}ix^3\cdot\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2-\frac{2}{k-v}\cdot\frac{4}{(k+v)^2}\right),$$

cuius valor non superare debet limitem ante constitutum $\frac{1}{4k^2}$ existente $k=20$; reducitur autem ea hanc ad formam:

$$\frac{n}{8(n-1)^2}\cdot\frac{x^3}{k+v}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{k+v}{a}+2\right)^2\cdot\frac{1}{k-v}-\frac{8}{(k+v)^2}\right),$$

unde colligitur sequens aequatio:

$$\frac{n}{2(n-1)^2}\cdot x^3\left(\frac{n(k+v)}{a^3(k-v)}+\frac{4n+2}{a^2(k-v)}+\frac{4n+8}{a(k^2-v^2)}+\frac{16v}{(k+v)^2(k-v)}\right)=\frac{1}{k^3},$$

ex qua quantitas x definiri poterit. Porro supra locus oculi ita erat definitus, ut nunc fiat

$$O = \frac{(v-k)v}{n(v+k)};$$

unde conspicietur in obiecto portio, cuius semidiameter

$$z = \frac{na}{v} \cdot \frac{k-v}{k+v} \cdot x.$$

Ut margo coloratus tolleretur, deberet esse $k = \infty$, siquidem $O > 0$; at cum prodeat $O < 0$, debet esse $k = -2a - v$, et cum pro hac confusione penitus tollenda deberet esse

$$(\alpha + v)(a + \alpha + v) = 0,$$

ob $\alpha = \infty$ evidens eat hanc confusionem fore enormem.

COROLLARIUM 1

§ 45. Cum invenerimus

$$\frac{k+v}{k-v} \cdot \frac{h}{a} = m,$$

ut hic valor sit positivus sive repraesentatio erecta, necesse est, ut quantitas k extra limites $+v$ et $-v$ contineatur; si enim intra hos limites contineretur, multiplicatio m prodiret negativa ideoque repraesentatio inversa, dum scilicet imago realis intra lentem formaretur.

COROLLARIUM 2

§ 46. Duo ergo sunt casus considerandi, quibus multiplicatio m fit positiva, alter, quo non solum $k > 0$, sed etiam $k > v$; tum facies lentis anterior erit convexa, posterior vero concava et $m > \frac{h}{a}$; altero vero casu, quo $k < 0$ simulque $k < -v$, sive posito $k = -l$ si fuerit $l > v$, erit

$$f = \frac{(n-1)a(l-v)}{l-v-2na} \quad \text{et} \quad g = \frac{(n-1)(l+v)}{2n}$$

ideoque facies posterior semper convexa, anterior vero concava, nisi $l > v + 2na$; nam si $l > v + 2na$, etiam anterior facies erit convexa hocque porro casu erit $m < \frac{h}{a}$.

COROLLARIUM 3

§ 47. Quod ad locum oculi attinet, pro quo est

$$O = \frac{(v-k)v}{n(v+k)},$$

quia $\frac{k+v}{k-v}$ est quantitas positiva, scilicet $= \frac{ma}{h}$, erit $O = -\frac{vh}{mna}$ ideoque semper negativus propter lentis crassitiem neque hic valor evanescet nisi simul lentis crassities fuerit evanescens.

COROLLARIUM 4

§ 48. Pro gradu vero claritatis haec expressio est notatu digna, quod $my = \frac{hx}{a}$ ideoque $y = \frac{hx}{ma}$, , unde patet crassitiem lentis in gradu claritas nihil mutare, si scilicet pro eadem multiplicatione *m* apertura *a;* eundem adipiscatur valorem.

COROLLARIUM 5

§ 49. Cum priori casu, quo erat $v = 0$, campus apparens fuisset indefinitus hic ob lentis crassitiem ita determinabitur, ut sit

$$mz = \frac{nhx}{v} \quad \text{sive} \quad z = \frac{nhx}{mv},$$

unde patet, quo minor fuerit crassities, eo maiorem futurum esse campus ac si loco *x* introducatur claritas *y*, prodibit $z = \frac{nay}{v}$, ita ut pro eadem crassitiei ratione ad distantiam *a* semidiameter campi sit claritati proportionalis.

SCHOLION 1

§ 50. Quoniam hic duo casus principales considerandi veniunt, alter, quo $k > v$, alter vero, quo $l > v$ existente $l = -k$, pro priore aequatio confusionem reddens insensibilem in solutione est exhibita; pro posteriore vero ea ita se habebit:

$$\frac{n}{2(n-1)^2} \cdot x^3 \left(\frac{n(l-v)}{a^3(l+v)} - \frac{4n+2}{a^2(l+v)} + \frac{4n+8}{a(l^2-v^2)} + \frac{16v}{(l-v)^3(l+v)} \right) = \frac{1}{k^3}.$$

Quodsi nunc etiam multiplicationem *m* introducamus, litteram vero *l* vel *k* eliminemus, haec aequatio induet hanc formam:

$$\frac{n}{2(n-1)^2 a^2 h}\cdot x^3\left(mn-\frac{(2n+1)(h-ma)}{v}+\frac{(n+2)(h-ma)^2}{mv^2}+\frac{(h-ma)^4}{m^3av^3}\right)=\frac{1}{k^3},$$

quae, si brevitatis gratia ponatur $\frac{h-ma}{v}=s$, induet hanc formam:

$$\frac{n}{2(n-1)^2 a^2 h}\cdot x^3\left(mn-(2n+1)s+\frac{(n+2)s^2}{m}-\frac{s^2}{m^2}+\frac{hs^3}{m^3a}\right)=\frac{1}{k^3};$$

quae porro mutatur in hanc:

$$\frac{n}{2(n-1)^2 a^2 h}\cdot x^3\left(m\left(1-\frac{s}{m}\right)^2\left(n-\frac{s}{m}\right)+\frac{hs^3}{m^3a}\right)=\frac{1}{k^3},$$

quae ad quosvis casus multo facilius adplicabitur. Ceterum cum inter binos casus memoratos quasi medius sit $k=\infty$ ponamus $k=\infty$ eritque

$$f=(n-1)a,\ \ g=\infty,\ m=\frac{h}{a},\ y=+x,\ O=-\frac{v}{n},\ z=\frac{nax}{v};$$

praeterea vero haec habebitur aequatio resolvenda:

$$\frac{n^2x^3}{2a^3(n-1)^2}=\frac{1}{k^3},$$

unde colligitur

$$x=\frac{a}{k}\sqrt[3]{\frac{2(n-1)^2}{n^2}},$$

ita ut hic valor notabiliter minor sit quam in problemate primo; quia in problemate primo facies anterior fere fuerat plana, hic casus inde oritur, si illa lens inverteretur, quo facto ea sine dubio multo minorem aperturam pateretur; ceterum et ideo est notatu dignus, quod crassities lentis neque in multiplicatione neque in confusione quidquam mutet.

SCHOLION 2

§ 51. Quoniam nostrum institutum non est omnes casus possibiles pertractare, sed eos tantum, quibus unam saltem vel plures excellentes qualitates lenti tribuere licuerit, hic unicus ille casus in problemate memoratus potissimum attentione nostra dignus videtur, quo marginis colorati est expers; quod, uti vidimus, evenit, si capiatur $k=-2a-v$. Interim tamen quaedam quasi necessitas nos cogit eum quoque casum evolvere, quo loco lenticulae globulus vitreus integer usurpari solet, quandoquidem huiusmodi microscopia facillime parantur et frequenter in usum sunt vocata; quamobrem his duobus casibus duo sequentia huius capitis problemata destinamus.

PROBLEMA 3

§ 52. *Non neglecta lentis crassitie eaque adeo data microscopium construere, quod in data ratione multiplicet simulque obiecta sine margine colorato repraesentet.*

SOLUTIO

Cum ob crassitiem distantia oculi *O* semper prodeat negativa ideoque oculum lenti immediate adplicari oporteat, ut margo coloratus ad nihilum redigatur, iam vidimus capi debere $k=-2a-v;$ unde ambo radii lentis ita erunt expressi:

$$f=-a \quad \text{et} \quad g=\tfrac{(n-1)}{2}(a+v),$$

ita ut prima facies sit concava et in ipso eius centro obiectum collocari debeat; unde radii in prima facie nullam plane refractionem patientur; deinde pro multiplicatione habebimus hanc aequationem $m=\frac{h}{a+v}$; unde, cum *m* detur, colligitur $a+v=\frac{h}{m}$; pro claritatis autem gradu erit $y=\frac{a+v}{a}x$ unde, si ut supra semidiameter pupillae aestimetur $\frac{1}{20}$ dig., mensura claritatis aestimari poterit

$$20y=\frac{20(a+v)}{a}\cdot x,$$

cum scilicet *x* in digitis exprimitur. Pro loco oculi autem reperitur

$$O=-\frac{(a+v)v}{na};$$

quae cum sit negativa, oculum lenti immediate adplicari oportet. Quia aperturae faciei anterioris semidiameter posita est $=x$, in facie posteriore ea erit $=\frac{a+v}{a}\cdot x$, qui est ipse valor ipsius *y*. Hinc pro campo apparente erit ,

$$z=\frac{n(a+v)}{v}\cdot x.$$

Denique ut ex conditione distinctionis quantitas *x* definiatur, utamur prima aequatione, quae ob

$$i=\frac{k-v}{k+v}=\frac{a+v}{a} \quad \text{et} \quad k+v=-2a, \quad k-v=-2(a+v)$$

hincque

$$\frac{1}{ia}+\frac{2}{k-v}=0 \quad \text{et} \quad (k-v)(k+v)^2=-8a^2(a+v)$$

abit in hanc:

$$\frac{n}{2(n-1)^2}\cdot\frac{x^3}{a^3}=\frac{1}{k^3},$$

ex qua elicitur

$$x=\frac{a}{k}\sqrt[3]{\frac{2(n-1)^2}{n}},$$

ex quo valore reliqua omnia determinantur.

EXEMPLUM

Statuamus crassitiem $v=a$ sitque vitrum commune, cuius refractio $n=1,55$, ac reperietur

$$x=0,73081\cdot\frac{a}{20}=0,0365a$$

posito scilicet $k=20$ ut ante; cum autem multiplicatio *m* detur ideoque sit $a=\frac{h}{2m}$, erit

$$x=0,0183\cdot\frac{h}{m}$$

unde sequens constructio colligitur:

I. Distantia obiecti a lente $a=\frac{h}{2m}$.
II. Radius faciei anterioris $=-a=-\frac{h}{2m}$.
III. Crassities lentis $v=\frac{h}{2m}$.
IV. Radius faciei posterioris $=\frac{n-1}{n}\cdot\frac{h}{m}=0,35484\cdot\frac{h}{m}$.
V. Semidiameter aperturae anterioris $=0,0183\cdot\frac{h}{m}$.
VI. Semidiameter aperturae posterioris $=0,0366\cdot\frac{h}{m}$ dig.
VII. Mensura claritatis $=0,732\cdot\frac{h}{m}$ seu sumto $h=8$ dig. erit ea mensura $=\frac{5,856}{m}$.
VIII. Semidiameter spatii visi in obiecto $z=2nx=0,0567\cdot\frac{h}{m}$.

Quae quo facilius cum casu problematis primi comparari queant, evolvamus casum, quo multiplicatio $m=100$, et sumto h = 8 dig. sequentes prodibunt determinationes:

I. Distantia obiecti a lente $=0,04$ dig.
II. Radius faciei anterioris $=-0,04$ dig.
III. Crassities lentis $=0,04$ dig.
IV. Radius faciei posterioris $=0,0284$ dig.
V. Semidiameter aperturae anterioris $=0,0015$ dig.
VI. Semidiameter aperturae posterioris $=0,0030$ dig.
VII. Mensura claritatis= 0,0585 dig.

VIII. Semidiameter spatii visi in obiecto $= 0,0045$ dig.

SCHOLION

§ 58. Haec ergo microscopia multo sunt inferiora praecedentibus, ubi crassities erat minima, neque ergo cuiquam in mentem veniet huiusmodi microscopia conficere.

PROBLEMA 4

§ 54. *Si loco lentis adhibeatur globulus vitreus, constructionem microscopii describere, quod datam multiplicationem producat.*

SOLUTIO

Hic ergo erit 1. $f = g$, tum vero 2. $v = 2f$. Ex priore conditione statim colligimus

$$\frac{a(k+v)}{k+v+2na} = \frac{v-k}{2n},$$

unde sequitur

$$k^2 + 4nak = v^2.$$

Cum autem porro sit $v = 2f$, valor ipsius g dabit

$$f = \frac{(n-1)(2f-k)}{2n} \text{ sive } 2f + (n-1)k = 0,$$

unde fit $k = -\frac{2f}{n-1}$, qui valor in superiore aequatione substitutus praebet

$$(2-n)f - 2(n-1)a = 0,$$

ita ut sit

$$f = \frac{2(n-1)}{2-n} \cdot a \text{ sive } a = \frac{2-n}{2(n-1)} \cdot f.$$

Nunc vero multiplicatio m dat

$$m = \frac{2-n}{n} \cdot \frac{h}{a} = \frac{2(n-1)}{n} \cdot \frac{h}{f}$$

ob

$$k+v = -\frac{2(2-n)}{n-1} \cdot f \text{ et } k-v = -\frac{2n}{n-1} \cdot f;$$

unde nanciscimur

$$f=\frac{2(n-1)}{n}\cdot\frac{h}{m}\quad\text{et}\quad a=\frac{2-n}{n}\cdot\frac{h}{m}$$

hincque

$$k=-\frac{4}{n}\cdot\frac{h}{m}\quad\text{et}\quad v=\frac{4(n-1)}{n}\cdot\frac{h}{m},$$

ex quibus pro loco oculi reperitur

$$O=-\frac{4(n-1)}{n(2-n)}\cdot\frac{h}{m};$$

quae distantia cum sit negativa, oculum lenti immediate adplicari oportet.

Ut nunc valorem ipsius x obtineamus, retineamus primo in calculo quantitatem a, et cum sit

$$f=\frac{2(n-1)a}{2-n},\quad k=-\frac{4a}{2-n},\quad v=\frac{4(n-1)a}{2-n}$$

hincque

$$k-v=-\frac{4n}{2-n}\cdot a\quad\text{et}\quad k+v=-4a$$

hincque

$$\frac{k+v}{k-v}=\frac{2-n}{n}\cdot a\quad\text{et}\quad k^2-v^2=\frac{16n}{2-n}\cdot a^2,$$

aequatio supra inventa induet hanc formam:

$$\frac{n}{2(n-1)^2}\cdot x^3\left(\frac{2-n}{a^3}-\frac{(2n+1)(2-n)}{2na^3}+\frac{(n+2)(2-n)}{4na^3}+\frac{n-1}{4na^3}\right)=\frac{1}{k^3},$$

quae porro reducitur ad hanc:

$$\frac{3n-n^2-1}{8(n-1)^2}\cdot\frac{x^3}{a^3}=\frac{1}{k^3},$$

ex qua elicitur

$$x=\frac{2a}{k}\sqrt[3]{\frac{(n-1)^2}{3n-n^2-1}}=\frac{2(2-n)}{nk}\cdot\frac{h}{m}\sqrt[3]{\frac{(n-1)^2}{3n-n^2-1}}.$$

Inventa igitur semidiametro aperturae in parte anteriore globi x, in parte posteriore erit ea $=\frac{n}{2-n}\cdot x$, cui gradus claritatis y est aequalis; mensuram vero claritatis exprimimus per $20y$, dum scilicet distantiae in digitis exprimuntur.

Tum vero semidiameter spatii in obiecto conspicui erit $z=\frac{n^2}{4(n-1)}\cdot x$. Quia autem distantia oculi O prodiit negativa, ut margo coloratus evanesceret, debebat esse $k=-2a-v$ sive $-4a+2a=0$; quod cum non sit, etiam evidens est marginem coloratum non destrui, sed satis notabilem fore. Ex his igitur omnibus colliguntur sequentes regulae pro constructione huiusmodi microscopiorum, in quibus sit multiplicatio $=m$:

I. Paretur globulus vitreus, cuius radius sit $f=\frac{2(n-1)}{n}\cdot\frac{h}{m}$; cuius refractio si sit $n=1,55$ et capiatur h = 8, erit $f=\frac{5,6774}{m}$ dig.

II. Ante hunc globum obiectum exponi debet ad distantiam $a=\frac{2,3226}{m}$ dig.

III. Globulo autem in parte anteriore tribuatur apertura, cuius semidiameter sit

$$x=\frac{2(2-n)}{nk}\cdot\frac{h}{m}\sqrt[3]{\frac{(n-1)^2}{3n-n^2-1}},$$

quae expressio in numeros evoluta fiet

$$x=\frac{36}{155m}\sqrt[3]{\frac{3025}{12475}}\text{ dig.}$$

seu

$$x=\frac{0,14483}{m}\text{dig.}\quad\text{hincque}\quad z=\frac{0,15816}{m}\text{ dig.}$$

In parte posteriore autem semidiameter aperturae debet esse

$$ix=\frac{2h}{km}\sqrt[3]{\frac{(n-1)^2}{3n-n^2-1}}=\frac{0,49887}{m}\text{ dig.}$$

Cum nunc sit $y=ix$, habebimus mensuram claritatis $20y=\frac{9,9774}{m}$, quae ergo maior est quam casu lenticulae simplicia, ubi tantum erat $=\frac{1}{m}$, quod autem lucrum neutiquam compensat vitium illud, quo obiecta margine colorato inquinata adparent. Operae igitur pretium erit similem tabulam, qualem supra in problemate 1 dedimus, adiungere:

Multipli-catio	Distantia obiecti	Radius globi	Semidiameter anterioris	Semidiameter posterioris	Mensura claritatis	Semidiameter campi
10	0,232	0,568	0,014	0,050	0,998	0,016
20	0,116	0,284	0,007	0,025	0,499	0,008
30	0,077	0,189	0,005	0,017	0,333	0,005
40	0,058	0,142	0,004	0,012	0,249	0,004
50	0,046	0,114	0,003	0,010	0,199	0,003

60	0,039	0,094	0,002	0,008	0,166	0,003

quam ulterius continuare ob nimis exiguum campum apparentem non conveniet; sin autem apertura maior sumeretur, confusio prodiret plane intolerabilis.

SCHOLION

§ 55. Ex his iam abunde intelligitur in hoc genere microscopiorum simplicium speciem primo allatam, qua lenticulae tenuissimae usurpantur, reliquis omnibus palmam longe praeripere; interim tamen et ista species duobus insignibus incommodis laborat, quae hic fusius ob oculos ponamus, quo clarius appareat, quid potissimum in microscopiis perficiendum desideretur. Primum incommodum in nimia propinquitate, qua obiectum lenti admoveri debet, est situm, qua fit, ut pro maioribus multiplicationibus haec distantia fere penitus evanescere debeat, quae circumstantia in causa est, ut, obiecta si non sint laevissima, minimae inaequalitates vel a lente nimis magnam vel nimis parvam teneant distantiam ideoque summa confusione adpareant. Inprimis igitur in id erit incumbendum, ut pro maioribus potissimum multiplicationibus eiusmodi microscopia inveniantur, quae non tam exiguam a lente distantiam postulant. Alterum incommodum consistit in nimis parva claritate, quam ista microscopiorum species exhibet in maioribus multiplicationibus; ex tabula enim supra § 43 exhibita videmus, si multiplicatio sit $m = 100$, claritatem ibi designatam esse 0,080, et cum ipsa claritas huius quadrato sit proportionalis, ea fiet 0,0064 ideoque 156 vicibus minor quam claritas naturalis, quae quidem adhuc satis tolerabilis est, nisi ipsum obiectum sit natura sua valde obscurum; sed hinc intelligitur, si multo maior multiplicatio desideretur, tenebras non amplius esse ferendas. Isti quidem defectui remedium afferri posset aperturam lentis augendo; tum autem confusio tantopere augeretur, ut penitus tolerari non posset, praecipue cum istam tabulam ita adstruxerimus, ut tantum esset $k = 20$, dum pro telescopiis poni solet $k = 50$, ita ut in his microscopiis gradus distinctionis iam quindecies sit minor quam in telescopiis, ita ut potius curandum sit, ut maiorem gradum distinctionis obtineamus. Illud autem posterius incommodum maximam partem lentem duplicando atque adeo triplicando e medio tollere licebit, ubi autem non eiusmodi lentes multiplicatae, quales in primo libro descripsimus, usurpari poterunt, quarum scilicet intervallum penitus evanescens est assumtum; quamobrem in hoc negotio intervalla inter istas lentes iam tanta assumi conveniet, quae in praxi locum habere queant; quod argumentum in sequentibus capitibus diligentius examini subiiciemus; in posterum vero perpetuo crassitiem lentium pro nihilo habebimus, unde maxime erit cavendum, ne lentes minus tenues elaborentur, quam earum forma et apertura postulant.