DIOPTRICS PART THREE,

CONTAINING BOOK THREE,

CONCERNED WITH THE CONSTRUCTION

OF BOTH SIMPLE AND COMPOSITE MICROSCOPES.

INTRODUCTION

MICROSCOPES IN GENERAL,

OR THE GENERAL PRECEPTS

REGARDING THE CONSTRUCTION OF MICROSCOPES

DEFINITION

1. *The microscope is a dioptric instrument, by which nearby objects will be enabled to be seen much more clearly and distinctly than by the naked eye, and which is usually constructed from one or more lenses placed together on the same axis.*

COROLLARY 1

2. So that it may reach a visible magnitude, indeed it is agreed the same object, where it may be moved nearer to the eye, accordingly to appear with a greater angle, truly if it were exceedingly close, cannot be viewed without the greatest confusion; whereby so that the object may appear distinctly, thus it must be represented by the microscope, as if it may be present at the true distance from the eye [*i.e*. the point of distinct near vision]. Hence, because the eye may be well constituted so that it may view distinctly at the greatest distances, that true distance, as we have put in the first book $=l$, likewise we have assumed to be infinity in the book about telescopes.

COROLLARY 2

3. Therefore whether a microscope may depend on a single or on more lenses, thus these must be properly ordered, so that the rays from any point of the object transmitted through all the lenses to be produced parallel to each other and thus the determinable distance after the eyepiece lens will become infinite; from which the first distance will be equal to the focal length of this lens.

COROLLARY 3

4. But the magnification, that we will indicate here also by the letter *m*, thus so that it must be understood, that the object, which we may view through the microscope, may appear to us to be greater by *m* times the angle, than if we might look at the same object at a certain distance $=h$ moved back from the naked eye; which distance h commonly is assumed to be eight inches.

COROLLARY 4

5. Moreover truly also it is required the lenses thus to be set out in order, so that the image of the object may become distinct enough or so that the confusion may not exceed a certain limit, which finally the radius of confusion generally found above must be suppressed below a certain limit; and indeed besides also it may come about that this image may be freed from the colored margin, if it can be done, clearly all the confusion arising from the diverse refractions of the rays will be able to be removed.

SCHOLIUM

6. But when a particular magnification may be wished, indeed that will be able to be done either quickly or not, as the clarity may be determined according to our choice, just as that has been done in telescopes, but generally for greater magnifications we must be content with lesser clarity; moreover the remedy for this defect is accustomed to be found by illuminating the object with a stronger light, as that can happen without difficulty, so that the objects in our vicinity are strong. Then also it is maximally incumbent, so that these instruments and likewise telescopes they may obtain a notable field of view, or so that an exceedingly small part of the object may not be presented to be viewed ; which part can be defined no simpler than by the angle formed by the objective lens, which also the smallest portion, if the proximity of the objective lens may be made less, may be able to form a large angle, but the true radius of this part seen, as above we have put $=z$, must enter into the calculation; also finally, since the distance of the object from the objective lens, that we have put $=a$, may depend on our choice, this treatment will differ in several ways from the preceding, if indeed not only the degree of clarity, but also the evidence of the apparent field of view may require another tedious investigation. On account of which in this first chapter it will be necessary to adapt the formulas found in the first book to these circumstances, before we may examine that in the construction of microscopes themselves.

PROBLEM 1

7. *To show the individual elements from however many lenses the microscope may have been composed, from which both the arrangement of the lenses as well as the distances between these and the focal lengths may be determined.*

SOLUTION

We shall set out the determinable distances of the individual lenses to be considered in the following manner :

Distance of	Distance of
object from lens $1 = a$,	from lens 1 to image $1 = \alpha$,
from image 1 to lens $2 = b$	from lens 2 to image $2 = \beta$
from image 2 to lens $3 = c$	from lens 3 to image $3 = \gamma$
from image 3 to lens $4 = d$	from lens 4 to image $4 = \delta$
:	:
:	:
from penult. image to final lens. $= l$	from last lens to last image $= \lambda = \infty$.

Clearly here it is required to be understood the images produced by the individual lenses to be either real or virtual [imaginary] , of which the difference, as we have observed now, in that it is allowed, that real images fall within the lens, but virtual ones fall outside this space.
Truly thence, so that we may compare these elements more conveniently, we may introduce capital letters of two kinds :

$$\alpha = Aa,\ \beta = Bb,\ \gamma = Cc,\ \delta = Dd,\ \varepsilon = Ee \text{ etc.}$$
$$\tfrac{\alpha}{b} = -P,\ \ \tfrac{\beta}{c} = -Q,\ \tfrac{\gamma}{d} = -R,\ \tfrac{\delta}{e} = -S \text{ etc.,}$$

where of the letters *A, B, C, D* etc. the final shall be $L = \infty$, and truly of the letters *P, Q, R* etc. the final shall be $= Z$, corresponding to the interval between the two final lenses.
From these letters introduced all the elements are expressed by a in the following manner:

$$\alpha = Aa,\ \beta = -\tfrac{AB}{P}\cdot a,\ \gamma = \tfrac{ABC}{PQ}\cdot a,\ \delta = -\tfrac{ABCD}{PQR}\cdot a \quad \text{etc.,}$$
$$b = -\tfrac{A}{P}\cdot a,\ c = \tfrac{AB}{PQ}\cdot a,\ d = -\tfrac{ABC}{PQR}\cdot a,\ e = -\tfrac{ABCD}{PQRS}\cdot a \text{ etc.,}$$

and the last of the letters *b, c, d* etc.

$$l = \mp \tfrac{ABC\ \dots\ K}{PQR\ \dots\ Z}\cdot a$$

and the last of the letters $\alpha,\ \beta,\ \gamma$ etc. etc.

$$\lambda = \pm \tfrac{ABC\ \ldots\ L}{PQR\ \ldots\ Z} \cdot a \ = \ \infty,$$

from which the intervals of the lenses are represented in order thus :

$$\begin{aligned}
&\text{First} && \alpha + b = Aa\left(1 - \tfrac{1}{P}\right),\\
&\text{second} && \beta + c = -\tfrac{AB}{P} a\left(1 - \tfrac{1}{Q}\right),\\
&\text{third} && \gamma + d = \tfrac{ABC}{PQ} a\left(1 - \tfrac{1}{R}\right),\\
&\text{fourth} && \delta + e = -\tfrac{ABCD}{PQR} a\left(1 - \tfrac{1}{S}\right), \text{ etc.;}
\end{aligned}$$

where since everything must be positive, also any quotient on dividing by the preceding must be made positive, and thus there will be required to be :

$$1.\ -\tfrac{B}{Q} \cdot \tfrac{Q-1}{P-1} > 0, \qquad 2.\ -\tfrac{C}{R} \cdot \tfrac{R-1}{Q-1} > 0,$$
$$3.\ -\tfrac{D}{S} \cdot \tfrac{S-1}{R-1} > 0, \qquad 4.\ -\tfrac{E}{T} \cdot \tfrac{T-1}{S-1} > 0, B\ Q\text{-}1$$
$$\text{etc.}$$

Where finally the focal lengths of the individual lenses, which we have indicated by the small letters *p, q, r, s, t* etc., so that we may express more succinctly, we may introduce the capital Germanic letters $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{D}$, etc., thus so that there shall be

$$\mathfrak{A} = \tfrac{A}{A+1},\ \mathfrak{B} = \tfrac{B}{B+1},\ \mathfrak{C} = \tfrac{C}{C+1},\ \mathfrak{D} = \tfrac{D}{D+1}, \text{ etc.}$$

and hence in turn:

$$A = \tfrac{\mathfrak{A}}{1-\mathfrak{A}},\ B = \tfrac{\mathfrak{B}}{1-\mathfrak{B}},\ C = \tfrac{\mathfrak{C}}{1-\mathfrak{C}},\ D = \tfrac{\mathfrak{D}}{1-\mathfrak{D}}, \text{ etc.}$$

thus so that for the last of these letters there shall be

$$\mathfrak{L} = \tfrac{L}{L+1} = 1 \text{ on account of } L = \infty \text{ and } L = \tfrac{\mathfrak{L}}{1-\mathfrak{L}} = \infty.$$

Therefore, from these letters the focal lengths may be expressed thus :

$$p = \mathfrak{A}a,\ \ q = -\tfrac{A\mathfrak{B}}{P} \cdot a,\ \ r = \tfrac{AB\mathfrak{C}}{PQ} \cdot a,\ \ s = -\tfrac{ABC\mathfrak{D}}{PQR} \cdot a, \text{ etc.,}$$

moreover, the focal length of the ultimate lens will become $= l$.

COROLLARY 1

8. Therefore the letters *A, B, C, D* etc. correspond to the individual lenses in the order of the first, second, third, etc; but the letters *P, Q, R* etc. correspond to the individual intervals, in the order of the first, second, third, etc.; on account of which the number of these latter letters will be one less than the former.

COROLLARY 2

9. In as much as if the letters *P, Q, R* etc. are observed as being positive, the images will be virtual, thus so that, if all these letters were positive, no real image will occur in the microscope, but if all these letters were negative, in the individual intervals a real image will be found; from which there will be just as many real images, as the number of these letters will be given negative values.

COROLLARY 3

10. Since these letters *P, Q, R* etc. shall be determined by the two elements for lenses shall pertain to lenses following each other, if a letter of this kind were positive, of the two elements from which it arises, the one will be positive, the other negative, but if such a letter were negative, both elements, for which it arises will be positive, indeed since all the intervals must be positive.

PROBLEM 2

11. *From however many lenses the microscope will have been made, to define a quantity from the individual images, whether they shall be real or virtual, and hence the magnification that the instrument produces, to be assigned both for an erect as well as an inverted image.*

SOLUTION

With the radius of the object [recall that all objects and images are regarded as circular], in as much as that is visible through the microscope, $=z$ the radii of the individual images are expressed by these elements in the following manner:

Radius of first image $=\frac{\alpha}{a}\cdot z = Az\,(\text{inverted})$

Radius of second image $=\frac{\alpha\beta}{ab}\cdot z = ABz\,(\text{erect})$

Radius of third image $=\frac{\alpha\beta\gamma}{abc}\cdot z = ABCz\,(\text{inverted})$

Radius of fourth image $=\frac{\alpha\beta\gamma\delta}{abcd}\cdot z = ABCDz\,(\text{erect})$

etc.,

from which the image of the final radius will be $= ABC \ldots Lz$; which image will be erect, if the number of the letters *A, B, C, ... L* were even, but inverted, if this shall be odd ; which final image, since the view of the object after the final lens may lie at an infinite distance $\lambda = Ll$, as the eye may be considered to be placed near the final lens and thus the object viewed at a distance Ll , so that it will appear to be within the angle $ABC \ldots K\frac{z}{l}$. Now so that we may define this magnification, which shall be equal to $= m$, we must compare this angle with the angle, within which the object itself z is going to appear to the eye at the distance $= h$; which angle since it shall be z , evidently the magnification shall become

$$m = ABC \ldots K \cdot \frac{h}{l}.$$

But as this image shall become either erect or inverted, two cases are required to be considered.

I. If the number of lenses and thus also the number of letters *A, B, C, ... L* were odd, the final image will be inverted; which since it may fall at an infinite distance past the eyepiece, that image before the eyepiece will be considered to be situated erect. Whereby if in our formula for *m* the number of letters found *A, B, C, ... K* were even, the object is considered to be situated erect, evidently as long as this formula obtains a positive value.

II. But if the number of lenses and also likewise the number of letters *A, B, C, D, ... L* were even, it is understood readily the opposite must be the case. Whereby if in the expression of *m* the number of letters *A, B, C, ... K* were odd, the object may be considered to be in the inverted situation, evidently as long as this expression were kept negative.

Truly so that if we may introduce the letters *P, Q, R* etc. into the above formulas, there will be found :

radius of the first image $= \alpha \cdot \frac{z}{a}$

radius of the second image $= P\beta \cdot \frac{z}{a}$

radius of the third image $= PQ\gamma \cdot \frac{z}{a}$

radius of the fourth image $= PQR\delta \cdot \frac{z}{a}$

etc.

radius of the final image $= PQR \ldots Z\lambda \cdot \frac{z}{a}$;

which images are all inverted, if indeed these formulas have positive values. Whereby since here it may be freed from all ambiguity and this final image may fall at an infinite distance $= \lambda$ past the eyepiece, that before the eyepiece may be observed to be erect, within the angle $= PQR \ldots Z\lambda \cdot \frac{z}{a}$; from which it follows the magnification to become

$$m = PQR...Z \cdot \tfrac{h}{a}$$

for the erect position, clearly if this formula were positive; but if this may have a negative value, the image will be inverted; then truly in this case also the letter m will be agreed to be taken negative. But it is easily understood this latter expression for the former magnification has been preferred for a long time, since it not troubled by any ambiguity, and we will use this always in the following.

COROLLARY 1

12. But therefore if at the positions of real images it may be agreed to put diaphragms in place, from these formulas we understand at once, how great an opening it shall be required to introduce in these, clearly after we have understood, how great a part of the object, the radius of which we call here $= z$, the instrument may offer to be viewed.

COROLLARY 2

13. If all the letters P, Q, R etc. were positive and thus plainly no real image shall be present, then the instrument will show the object situated erect always ; but if a single real image may occur and thus a single one of these letters were negative, then the representation always shall become inverted, in which case the letter m must be introduced into the calculation with the opposite sign; but if two real images may occur, the representation again will be erect.

COROLLARY 3

14. Hence it will be apparent, how great the effect of introducing these letters P, Q, R, S etc. shall be, since these may show clearly both the distinction between real and virtual images, especially since this treatment may be applied equally to the division according to the real images for telescopes, certainly in which the essential difference between the diverse kinds of microscopes may be presented.

PROBLEM 3

15. *From however many lenses the microscope were composed, if the aperture of the first objective lens may be given, through which the rays may be transmitted as if from the centre, to define the apertures of the individual lenses necessary for the further transmission and the degree of clarity, by which the eye will view the object.*

SOLUTION

These apertures will be defined most readily from the fundamental principles established above from the known aperture of the first lens, from which the radii of the individual apertures following may be expressed by the letters *P, Q, R* etc. :

Radius of the aperture of the first lens $= x$
Radius of the aperture of the second lens $= \frac{b}{\alpha}\cdot x = \frac{1}{P}\cdot x$
Radius of the aperture of the third lens $= \frac{bc}{\alpha\beta}\cdot x = \frac{1}{PQ}\cdot x$
Radius of the aperture of the fourth lens $= \frac{bcd}{\alpha\beta\gamma}\cdot x = \frac{1}{PQR}\cdot x$ etc.,

from which we conclude the radius of the aperture of the final lens

$$= \frac{x}{PQR...Z};$$

but since we have found before:

$$m = PQR...Z\cdot\frac{h}{a}$$

this formula will become:

$$= \frac{h}{ma}\cdot x.$$

Truly the aperture of lens of the eyepiece must have such a minimum size, in order that it may transmit the rays entering through the objective lens, and since now the rays may be parallel to each other, these will be represented as if by a pencil of rays, which may enter the eye from the centre of the object ; from which, if the radius of this pencil $\frac{hx}{ma}$ were equal to the radius of the pupil, then the vision would enjoy full clarity ; but however much this expression is smaller than the radius of the pupil, the degree of clarity provided will be smaller. From which, since the degree of clarity was expressed above by the letter y, this will become $y = \frac{hx}{ma}$; which value as many times smaller as it were than the radius of the pupil, which may be estimated as being around $\frac{1}{20}^{\text{th}}$ of an inch, by just as much the clarity will be considered to be smaller than the natural or full state, or rather in the square ratio, just as is itself evident .

COROLLARY 1

16. Therefore with the given clarity y with the magnification m there is found $x = \frac{may}{h}$; from which the aperture of the objective lens becomes known, which with all else being equal therefore must be greater, when the distance of the object from the objective lens or a were greater. Therefore since the distance x shall depend on the focal length of the objective lens, hence it may be understood, how this lens may be prepared from the account of the distance a .

COROLLARY 2

17. Equally hence it will be apparent from the preceding problem, how the magnification *m* may be referred to that distance *h,* which commonly is assumed to be 8 inches, since in this operation the magnification *m* cannot be defined absolutely, and thus $\frac{m}{h}$ may denote that properly, which is presented under the notion of magnification mentioned.

PROBLEM 4

18. *From however many lenses the microscope were composed, the details which must be brought together from the individual lenses for the apparent field of view of these and the aperture they define, and the position of the eye may be assigned.*

SOLUTION

For this we have introduced special letters into the calculation ; indeed since the aperture of each lens thus shall depend on its focal length, in order that a certain part of it may not be exceeded, the radius of the aperture of each lens after the first has been established in the following manner from its focal length:

$$\text{second} = \pi q, \quad \text{third} = \pi' r, \quad \text{fourth} = \pi'' s, \quad \text{fifth} = \pi''' t \quad \text{etc.;}$$

from which, if the radius of the object shall be seen to be $= z$ and there may be called $\frac{z}{a} = \Phi,$ we have shown to be

$$z = a\Phi = \frac{-\pi + \pi' - \pi'' + \pi''' - \pi'''' \text{ etc.}}{ma - h} \cdot ah,$$

from which it is required to be understood with regard to the erect situation; for the inverse in place the magnification *m* must be taken negative.

But now, so that we may be able to judge the size of the field more easily, it shall be of the maximum aperture, as some lens, of which the focal length shall be for example $= \mathfrak{q}$, it can receive the radius $= \xi\mathfrak{q}$, evidently of which each lens shall be the maximum, if each may be equal, with ξ denoted by the fraction $\frac{1}{4}$; for the individual lenses, however smaller the apertures they may have, we may introduce new letters and put

$$\pi = -\mathfrak{q}\xi, \quad \pi' = +\mathfrak{r}\xi, \quad \pi'' = -\mathfrak{s}\xi, \quad \pi''' = +\mathfrak{t}\xi \quad \text{etc.,}$$

so that there may become

$$z = a\Phi = \frac{\mathfrak{q} + \mathfrak{r} + \mathfrak{s} + \mathfrak{t} \text{ etc.}}{ma - h} \cdot ah\xi,$$

in which for the sake of brevity we may put

$$M = \frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t} \text{ etc.}}{ma-h} \cdot h,$$

so that there

$$z = a\Phi = Ma\xi \text{ seu } \Phi = M\xi;$$

with which put in place these new letters $\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t}$ etc. will be referred to the following manner introduced before :

$$1. \mathfrak{B}\mathfrak{q} = (P-1)M,$$
$$2. \mathfrak{C}\mathfrak{r} = (PQ-1)M - \mathfrak{q},$$
$$3. \mathfrak{D}\mathfrak{s} = (PQR-1)M - \mathfrak{q} - \mathfrak{r},$$
$$4. \mathfrak{E}\mathfrak{t} = (PQRS-1)M - \mathfrak{q} - \mathfrak{r} - \mathfrak{s}$$
etc.,

the differences of which forms also are noteworthy, evidently

$$1. \mathfrak{C}\mathfrak{r} - \mathfrak{B}\mathfrak{q} = P(Q-1)M - \mathfrak{q},$$
$$2. \mathfrak{D}\mathfrak{s} - \mathfrak{C}\mathfrak{r} = PQ(R-1)M - \mathfrak{r},$$
$$3. \mathfrak{E}\mathfrak{t} - \mathfrak{D}\mathfrak{s} = PQR(S-1)M - \mathfrak{s},$$
etc.,

Therefore the final of these equations thus will be the expression:

$$\mathfrak{L}\mathfrak{z} = (PQR..Z-1)M - \mathfrak{q} - \mathfrak{r} \ldots - \mathfrak{h},$$
$$= \left(\tfrac{ma}{h}-1\right)M - \mathfrak{q} - \mathfrak{r} \ldots - \mathfrak{h}.$$

Now before we have shown to be $\mathfrak{L} = 1$; from which there becomes

$$\mathfrak{q} + \mathfrak{r} + \mathfrak{s} \ldots + \mathfrak{z} = \left(\tfrac{ma}{h}-1\right)M$$

which is the equation itself, from which the letter M is determined.

Now therefore it remains, that we may define the location of the eye or its distance past the final lens, which above we have called $= O$; certainly from above in the first place according to the number of lenses we shall find:

For one lens:

$$O = 0.$$

For two lenses :

$$O = \frac{\mathfrak{B}b\pi}{\pi-\Phi} = \frac{\mathfrak{q}b}{Ma}\cdot\frac{h}{m} = \frac{\mathfrak{q}q}{M}\cdot\frac{h}{ma}.$$

For three lenses:

$$O = \frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi} = \frac{\mathfrak{r}c}{Ma}\cdot\frac{h}{m} = \frac{\mathfrak{r}r}{M}\cdot\frac{h}{ma}.$$

For four lenses :

$$O = \frac{\mathfrak{D}d\pi''}{\pi''-\pi'+\pi-\Phi} = \frac{\mathfrak{s}d}{Ma}\cdot\frac{h}{m} = \frac{\mathfrak{s}s}{M}\cdot\frac{h}{ma}$$

etc.,

from which we conclude for any number of lenses the distance of the eye to become:

$$O = \frac{\mathfrak{z}l}{M}\cdot\frac{h}{ma}.$$

COROLLARY 1

19. Hence therefore we have followed new determinations for the apertures of the individual lenses, which clearly the provision of the field requires and which are not to be confused with the above, which are concerned with the order of clarity ; but that aperture which is greater must be attributed to any lens ; from which the following formulas are required to be observed properly :

Radius of the aperture for the first lens $= 0\xi p \cdots x,$,

radius of the aperture for the second lens $= \mathfrak{q}\xi q \cdots \frac{x}{P},$

radius of the aperture for the third lens $= \mathfrak{r}\xi r \cdots \frac{x}{PQ},$

radius of the aperture for the fourth lens $= \mathfrak{s}\xi s \cdots \frac{x}{PQR},$

from which

for the final lens $= \mathfrak{z}\xi l \cdots \frac{hx}{ma},$

where it is to be observed the letters $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$ etc. to be fractions less than one, the values of which cannot exceed unity.

COROLLARY 2

20. If perhaps the image produced were inverted, in which case, as we have just mentioned above, the magnification *m* is taken as negative, or $-m$ must be written in place of *m*, therefore in which case also a negative sign must be attributed to the individual letters $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$, thus so that then there may become

$$M = \frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t} \text{ etc.}}{ma+h}\cdot h.$$

COROLLARY 3

21. Because certain circumstances are accustomed to demand, so that for each case of the letters $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$ etc., one or another negative value may be able to be obtained, with this especially, as we have seen with telescopes, comes to be used in the first of these letters ; truly the latter are assumed always to be positive and thus without harm can be assumed to be equal to unity, thus so that the final or these certainly may be taken as one ; from which it is evident the distance of the eye O always to become positive, as often as the final lens were convex ; but if this lens were concave , then also the distance O will be produced negative. [Presumably a virtual image is produced in this case.]

SCHOLIUM

22. Finally this is required to be considered, since in the first book the letter l was used signifying the true distance of the eye, which in this case always is regarded as infinite, but here with same letter for a long time used for indicating otherwise, here always it shall signify the focal length of the final lens or eyepiece, which is the same as the distance of the penultimate image before the final lens; from which it follows, if the final lens were convex, the penultimate image certainly must be present before that; on account of which certainly a real image must fall before the final lens. Hence it is evident therefore, that which may not appear so clear above, plainly if there may be no real image present, then a convex final lens is not going to be able to produce a negative distance for the location O of the eye, for which case also we must provide a singular formula for the removal of the colored margin to be treated, which is quite different from that, which is used, whenever O is positive; here it will be agreed also which two cases to be treated separately.

PROBLEM 5

23. *From however many lenses a microscope were constructed, if the distance of the eye past the final lens O were produced positive, to remove the colored margin, from whatever kind of glass the individual lenses were prepared.*

SOLUTION

Since thus we may postulate this general solution, which also may be prepared for lenses from different kinds of glass, we may put the ratio of refraction for the first lens $= n$, for the second $= n'$, for the third $= n''$ etc., as we have done in the above books; and hence we may put the differential formulas, by which the dispersion of the rays is expressed, in the following manner :

$$\frac{dn}{n-1} = N,\ \frac{dn'}{n'-1} = N',\ \frac{dn''}{n''-1} = N'' \text{ etc.;}$$

with which observed above, [Book I. Suppl. VII], we have shown for the destruction of the colored margin for this equation to be satisfied:

$$0 = \frac{N'\cdot b\pi}{Aa\Phi} + \frac{N''\cdot c\pi'}{ABa\Phi} + \frac{N'''\cdot d\pi''}{ABCa\Phi} + \frac{N''''\cdot e\pi'''}{ABCDa\Phi} \text{ etc.;}$$

which equation, if both in place of the letters $\pi,\ \pi'$ etc. as well as in place of *b, c, d* etc. the values assigned before may be substituted, will be changed into this form:

$$0 = \frac{N'\cdot \mathfrak{q}}{P} + \frac{N''\cdot \mathfrak{r}}{PQ} + \frac{N'''\cdot \mathfrak{s}}{PQR} + \frac{N''''\cdot \mathfrak{t}}{PQRS} \text{ etc.;}$$

in which equation the final term will be expressed thus $\frac{N^{\cdots\cdots}\mathfrak{z}h}{ma}$.

[As various commentators have indicated, Euler had not produced a viable theory for correcting spherical aberration, at least for lenses of differing materials : a glance at the dispersion curves of various kinds of glass, i.e. refractive index versus wavelength, does reveal a certain similarity in form for these curves leading to the absorption edge in the UV, but they are all non-linear in form and their variations cannot be expressed in the uniform linear manner adopted by Euler.]

COROLLARY 1

24. Therefore it is apparent the colored margin cannot be removed completely, unless either one or more of the letters $\mathfrak{q},\ \mathfrak{r},\ \mathfrak{s}\ \ \mathfrak{t}$ etc. or *P, Q, R* etc. were negative, because otherwise all the terms shall be positive and the sum of these will not be able to be equal to zero.

COROLLARY 2

25. Therefore if no real image may be present, which happens, if all the letters *P, Q, R* etc. were positive, then by necessity one or several of the letters $\mathfrak{q},\ \mathfrak{r},\ \mathfrak{s}$ etc. must be negative ; moreover from whichever of there were negative, the apparent field is diminished.

PROBLEM 6

26. *From however many lenses the microscope were composed, if the distance of the eye O may be produced negative and thus the eyepiece of the final lens must be doubled, to remove the colored margin, from whatever kind of glass the individual lenses may have been prepared.*

SOLUTION

With the same put in place as in the preceding problem concerning the diversity of the glass above [Book. I. Suppl. VII], for this case the following number of special lens formulas are given, which according to our procedure thus may be put in place :
For one lens

$$0 = 0.$$

For two lenses

$$0 = N(A+1)\mathfrak{q}.$$

For three lenses

$$0 = N\left(A+1\right)B\mathfrak{t} - \tfrac{N'}{P}\left((B+1)\mathfrak{r}+\mathfrak{q}\right).$$

For four lenses

$$0 = N\left(A+1\right)BC\mathfrak{s} - \tfrac{N'}{P}\left((B+1)C\mathfrak{s}-\mathfrak{q}\right) + \tfrac{N''}{PQ}\left(\left(C+1\right)\mathfrak{s}+\mathfrak{r}\right).$$

For five lenses

$$0 = N\left(A+1\right)BCD\mathfrak{t} - \tfrac{N'}{P}\left((B+1)CD\mathfrak{t}+\mathfrak{q}\right) + \tfrac{N''}{PQ}\left(\left(C+1\right)D\mathfrak{t}-\mathfrak{r}\right) - \tfrac{N'''}{PQR}\left(\left(D+1\right)\mathfrak{t}+\mathfrak{s}\right).$$

For six lenses

$$0 = N\left(A+1\right)BCDE\mathfrak{u} - \tfrac{N'}{P}\left((B+1)CDE\mathfrak{u}-\mathfrak{q}\right) + \tfrac{N''}{PQ}\left(\left(C+1\right)DE\mathfrak{u}+\mathfrak{r}\right)$$
$$-\tfrac{N'''}{PQR}\left(\left(D+1\right)E\mathfrak{u}-\mathfrak{s}\right) + \tfrac{N''''}{PQRS}\left(\left(E+1\right)\mathfrak{u}+\mathfrak{t}\right).$$

For seven lenses

$$0 = N\left(A+1\right)BCDEF\mathfrak{v} - \tfrac{N'}{P}\left((B+1)CDEF\mathfrak{v}+\mathfrak{q}\right) + \tfrac{N''}{PQ}\left(\left(C+1\right)DEF\mathfrak{v}-\mathfrak{r}\right)$$
$$-\tfrac{N'''}{PQR}\left(\left(D+1\right)EF\mathfrak{v}+\mathfrak{s}\right) + \tfrac{N''''}{PQRS}\left(\left(E+1\right)F\mathfrak{v}-\mathfrak{t}\right) - \tfrac{N'''''}{PQRST}\left(\left(F+1\right)\mathfrak{v}+\mathfrak{u}\right);$$

which formulas cannot be shown more neatly, and thus from these any case presented will be able to be used

PROBLEM 7

27. *From however many lenses the microscope were composed, plainly all the confusion, which on account of the different refrangibility of the rays besides the colored margin is to be feared, to be reduced to nothing, from whatever kind of glass the individual lenses were prepared.*

SOLUTION

Also with the letters N, N' etc. introduced, as has been done in the preceding problems, the equation found in book one [Book. I. Suppl. VII.], which is required to be satisfied, generally will be found expressed in the following manner for any number of lenses:

$$0 = N \cdot \tfrac{A+1}{A} - \tfrac{N'}{P} \cdot \tfrac{B+1}{AB} + \tfrac{N''}{PQ} \cdot \tfrac{C+1}{ABC} - \tfrac{N'''}{PQR} \cdot \tfrac{D+1}{ABCD} \text{ etc.,}$$

which also can be shown in the following manner:

$$0 = N \cdot \tfrac{1}{p} + \tfrac{N'}{P^2} \cdot \tfrac{1}{q} + \tfrac{N''}{P^2Q^2} \cdot \tfrac{1}{r} + \tfrac{N'''}{P^2Q^2R^2} \cdot \tfrac{1}{s} \text{ etc.,}$$

or also, if it pleases, in this manner:

$$0 = N \cdot \tfrac{1}{\mathfrak{A}} - \tfrac{N'}{P} \cdot \tfrac{1}{A\mathfrak{B}} + \tfrac{N''}{PQ} \cdot \tfrac{1}{AB\mathfrak{C}} - \tfrac{N'''}{PQR} \cdot \tfrac{1}{ABC\mathfrak{D}} \text{ etc.}$$

COROLLARY 1

28. Since the product of all the letters P, Q, R, S ... may provide the magnification, if this were very great, the terms of this equation soon will become so small, that it may suffice to be assuming the initial two or three terms, from which conveniently either the letter $\mathfrak{B}$ or $\mathfrak{C}$ will be able to be defined.

COROLLARY 2

29. Now above [Book. II] it has been shown moreover, unless the letters N, N' etc. were different from each other, this final equation would not be able to be implemented ; from which as far as for this condition to be satisfied, that lenses may not be made from the same kind of glass.

SCHOLIUM

30. Indeed we have shown so much the same above for telescopes, moreover the same can be demonstrated for the present case in this manner. Clearly for this we may use the first form of our equations and in that case the letters *N* may be put equal to each other, whose individual terms may be separated into two parts, so that this form may be produced:

$$0 = 1 + \frac{1}{A} - \frac{1}{ABP} + \frac{1}{ABCPQ} - \frac{1}{ABCDPQR}$$
$$- \frac{1}{AP} + \frac{1}{ABPQ} - \frac{1}{ABCPQR} + \frac{1}{ABCDPQRS} \text{ etc.,}$$

which may be agreed to be multiplied by a, and since from the elements there shall become

$$a = \frac{\alpha}{A} = -\frac{Pb}{AB} = -\frac{P\beta}{AB} = \frac{PQc}{AB} = \frac{PQ\gamma}{ABC} \text{ etc.,}$$

these values may be substituted successively into our equation, and our equation will go into this form:

$$0 = a + \frac{\alpha + b}{A^2} + \frac{\beta + c}{A^2B^2} + \frac{\gamma + d}{A^2B^2C^2} + \frac{\delta + e}{A^2B^2C^2D^2} \text{ etc.;}$$

where since the numerators will designate the separations of the lenses, all the denominators shall be square numbers, truly all these terms by necessity are positive. Only the final part alone may be able to remain in doubt; clearly this, until whenever we will have continued these terms, in addition there must be added the term

$$\frac{\varepsilon}{A^2B^2C^2D^2E^2},$$

which is the case of five lens , for which ε indeed is ∞; moreover it is required to be observed also to be $E = \infty$, since there shall become $\varepsilon = Ee$; with which value substituted it is evident this same term required to be added vanishes at once. In addition, as we have mentioned often now, also by no means is it necessary to be using different kinds of glass, so that this final equation may be satisfied most accurately, since now it may be treated outstandingly well by us, but only if its value may be able to be reduced small enough, which also is required to be established for the two preceding equations ; nor indeed does the nature of an investigation of this kind permit a rigorous solution, since at no time shall it be able to hope the experimental values of the letters *N, N′* etc. thus be able to be defined exactly, so that they may not notably stray from the truth ; and since a single kind of glass will be required to be used, always we are forced to tolerate this final source of confusion, but only if that may be allowed to be reduced smaller, that certainly will be required to be had for the maximum gain.

PROBLEM 8

31. *From however many lenses the microscope were composed, the radius of confusion which arises from the aperture of the lens, to be assigned and this whole confusion to be reduced below a certain limit, so that it will no longer impede the image formation.*

SOLUTION

In order that this may excel new letters $\lambda,\ \lambda'$ etc. are required to be introduced into the calculation, just as has been explained sufficiently in the first book. Then truly, if we may consider the individual lenses to be made from a particular kind of glass, the expression for the radius of confusion found above [Book I. Suppl. VII] with the letters *P, Q, R* etc. required to be used for the following form will be recalled:

$$\frac{\mu m x^3}{4aah}\left(\begin{array}{l}\mu\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}\right)-\frac{\mu'}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right)+\frac{\mu''}{A^3B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu''}{C\mathfrak{C}}\right)\\ -\frac{\mu'''}{A^3B^3C^3PQR}\left(\frac{\lambda'''}{\mathfrak{D}^3}+\frac{\nu'''}{D\mathfrak{D}}\right)+\frac{\mu''''}{A^3B^3C^3D^3PQRS}\left(\frac{\lambda''''}{\mathfrak{E}^3}+\frac{\nu''''}{E\mathfrak{E}}\right)\text{ etc.}\end{array}\right)$$

which formula may be rendered more succinctly by introducing the focal lengths ; indeed since there shall be

$$\mathfrak{A}=\frac{p}{a},\ \ A\mathfrak{B}=-\frac{Pq}{a},\ \ AB\mathfrak{C}=\frac{PQr}{a},\ \ ABC\mathfrak{D}=-\frac{PQRs}{a}\ \text{ etc.},$$

with these substituted our formula will become :

$$\frac{max^3}{4h}\left(\frac{\mu}{p^3}\left(\lambda+\frac{\mathfrak{A}^2}{A}\nu\right)+\frac{\mu'}{P^4q^3}\left(\lambda'+\frac{\mathfrak{B}^2}{B}\nu'\right)+\frac{\mu''}{P^4Q^4r^3}\left(\lambda''+\frac{\mathfrak{C}^2}{C}\nu''\right)\text{ etc.}\right).$$

Now the limits shall be, which the value of the formula must not exceed, $=\frac{1}{4k^3}$, where it is required to observe for the telescopes obtained above to be around $k=50$; whereby, if for the sake of brevity we may put

$$\Lambda=\frac{\mu}{p^3}\left(\lambda+\frac{\mathfrak{A}^2}{A}\nu\right)+\frac{\mu'}{P^4q^3}\left(\lambda'+\frac{\mathfrak{B}^2}{B}\nu'\right)+\frac{\mu''}{P^4Q^4r^3}\left(\lambda''+\frac{\mathfrak{C}^2}{C}\nu''\right)\text{ etc.},$$

there ought to become:

$$\frac{max^3}{h}\Lambda<\frac{1}{k^3},$$

from which the radius of the objective lens may be defined most conveniently to become

$$x < \frac{1}{k}\sqrt[3]{\frac{h}{ma\Lambda}};$$

and if it will have pleased this formula Λ finally may be reduced to zero, then this radius x offers no impedance, we may put in place of such a size that it permits the figure of the objective lens.

COROLLARY 1

32. Therefore when this quantity x were defined, then finally we will be able to assign the order of the clarity ; for from the equation found above $y = \frac{hx}{ma}$ we know the radius of the pencil of rays, which are being transmitted from the points of the object into the eye, which will determine the order of the clarity related to the pupil.

COROLLARY 2

33. Indeed with telescopes we have seen a sufficient degree of clarity to be produced, only if y shall be not much smaller than $\frac{1}{50}$ inch, but in microscopes it will be required for a much smaller degree of clarity to be present.

COROLLARY 3

34. But if in place of x we may substitute the value found, we will have for the order of clarity

$$y = \frac{1}{k}\left(\frac{h}{ma}\right)^{\frac{4}{3}}\sqrt[3]{\frac{1}{m\Lambda}};$$

from which it is understood, so that the further we may wish to remove the object from the microscope, thus the object is going to appear with lesser clarity, the cause of which being, that in all microscopes the distance of the object from the objective lens must be taken extremely small.

COROLLARY 4

35. It is evident from the final form of our expression, if all the lenses were convex or the letters p, q, r etc. positive, all the terms containing the letters λ, λ', λ'' etc. to be positive also; from which, since the letters v, v', v'' etc. shall be extremely small, in no way will it be possible that the radius of confusion Λ be reduced to zero; but if some lens or another were concave, then certainly it will be able to happen, so that this quantity Λ may vanish.

SCHOLIUM 1

36. Therefore this formula for Λ can be used, especially with the letters $\lambda,\ \lambda',\ \lambda''$ etc. conveniently defined, since the remaining letters now generally arrive at their determined values by the preceding conditions . But it will be required to remember whatever lens to be added itself to have a number λ, which certainly cannot be less than one, from which since with two determinable distances both faces are required to be determined. Moreover above the formulas for the radii of the faces now have been given, but these we may show here conveniently changed for a while. For example it may be the first lens, of which the determinable distances are a and α, but the number required to be added to these $=\lambda$, from which both its faces above [Book I, § 55] are defined thus, so that there shall become

$$\text{radius of the}\begin{cases}\text{anterior face } = \dfrac{a\alpha}{\alpha\rho+a\sigma\pm\tau(a+\alpha)\sqrt{\lambda-1}}\\[2ex] \text{posterior face } = \dfrac{a\alpha}{a\rho+\alpha\sigma\mp\tau(a+\alpha)\sqrt{\lambda-1}}.\end{cases}$$

But since there shall become $\alpha = Aa$, these formulas will become :

$$\text{radius of the}\begin{cases}\text{anterior face } = \dfrac{Aa}{A\rho+\ \sigma\pm\tau(1+A)\sqrt{\lambda-1}}\\[2ex] \text{posterior face } = \dfrac{a\alpha}{\rho+\ A\sigma\mp\tau(1+A)\sqrt{\lambda-1}}.\end{cases}$$

Now the numerators and denominators of each fraction may be divided by $1+A$, and since there shall be $\frac{A}{A+1}=\mathfrak{A}$ thus $\mathfrak{A}a=p$ and $\frac{1}{1+A}=1-\mathfrak{A}$, our formulas will be changed into the following:

$$\text{radius of the}\begin{cases}\text{anterior face } = \dfrac{p}{\sigma-\mathfrak{A}(\sigma-\rho)\pm\tau(1+A)\sqrt{\lambda-1}}\\[2ex] \text{posterior face } = \dfrac{p}{\rho+\mathfrak{A}(\sigma-\rho)\mp\tau(1+A)\sqrt{\lambda-1}},\end{cases}$$

where the letters ρ,σ and τ are required to be chosen from the refractive index, which has been agreed for any lens, and equally these letters μ and ν, as we have shown in the first book [§ 55] . But lest we may have a need thence to show these, we have added the table given here [Book II,§ 15] :

n	ρ	σ	τ	μ	ν	$\mu\nu$
1,50	0,2858	1,7143	0,9583	1,0714	0,2000	0,2143
1,51	0,2653	1,6956	0,9468	1,0420	0,2065	0,2151

1,52	0,2456	1,6776	0,9358	1,0140	0,2129	0,2159
1,53	0,2267	1,6601	0,9252	0,9875	0,2196	0,2168
1,54	0,2083	1,6434	0,9149	0,9622	0,2260	0,2176
1,55	0,1907	1,6274	0,9051	0,9381	0,2326	0,2182
1,56	0,1737	1,6119	0,8956	0,9151	0,2393	0,2192
1,57	0,1573	1,5970	0,8864	0,8932	0,2461	0,2199
1,58	0,1414	1,5827	0,8775	0,8724	0,2529	0,2206
1,59	0,1259	1,5689	0,8689	0,8525	0,2597	0,2214
1,60	0,1111	1,5555	0,8607	0,8333	0,2666	0,2221

SCHOLIUM 2

37. From these principles established it is easily understood, how this treatise concerned with the doctrine of microscopes may be agreed to be subdivided into sections. Clearly in the first place we will consider simple microscopes, which depend on a single lens, and which we will consider in a twofold manner, just as the thickness of the lens is negligible or an account of this is had in the calculation. Then we will consider three kinds of microscopes composed, as we have done in telescopes; clearly in the first no straight forwards real image will occur, or all the letters *P, Q, R* etc. will be positive; but in the second kind a single real image will occur and thus a single one of the letters will have a negative value, whichever that may become; finally in the third kind two real images will be found and thus two of these letters, whichever ones these may become, will be allocated negative values. But to introduce more real images evidently will become superfluous. Truly it is to be observed both the simple microscopes as well as the composite ones of the first and third kind the object is going to be represented as situated erect, while microscopes composed of the second kind will refer to these images inverted. On account of which this treatise will be resolved into the four following sections.

DE MICROSCOPIIS IN GENERE

VEL PRAECEPTA GENERALIA

CIRCA CONSTRUCTIONEM MICROSCOPIORUM

DEFINIIIO

1. *Microscopium est instrumentum dioptricum, per quod obiecta propinqua multo maiora quam nudis oculis clare et distincte conspicere licet quodque una pluribusve lentibus super eodem axe constitutis constare solet.*

COROLLARIUM 1

2. Quod ad magnitudinem visam attinet, constat quidem idem obiectum, quo propius oculo admoveatur, sub eo maiore angulo apparere, verum si nimis fuerit propinquum, non sine maxima confusione conspici posse; quare ut obiectum distincte appareat, per microscopium ita debet repraesentari, quasi in iusta ab oculo distantia existeret. Hinc, quia oculus bene constitutus in distantia maxima distincte cernere solet, iustam illam distantiam, quam in primo libro posuimus $= l$, perinde ac in libro de telescopiis infinitam assumemus.

COROLLARIUM 2

3. Sive igitur microscopium una sive pluribus lentibus constet, eae ita dispositae esse debent, ut radii ex quolibet obiecti puncto per omnes lentes transmissi inter se reddantur paralleli ideoque pro lente oculari distantia determinatrix posterior fiat infinita; ex quo prior ipsi huius lentis distantiae focali erit aequalis.

COROLLARIUM 3

4. Multiplicatio autem, quam hic etiam littera m indicabimus, ita intelligi debet, ut obiectum, quod per microscopium contemplamur, nobis sub angulo m vicibus maiore appareat, quam si idem obiectum ad certam distantiam $= h$ remotum nudis oculis intueremur; quae distantia h vulgo octo digitorum assumi solet.

COROLLARIUM 4

5. Tum vero etiam lentes ita dispositas esse oportet, ut repraesentatio obiecti fiat satis distincta seu ut confusio certum quendam limitem non excedat, quem in finem semidiameter confusionis supra in genere inventa infra certum limitem deprimi debet;

praeterea vero etiam hanc repraesentationem a margine colorato liberari conveniet ac, si fieri potest, omnis plane confusio a diversa radiorum refractione oriunda tolli debebit.

SCHOLION

6. Quando autem insignia multiplicatio desideratur, vix ac ne vix quidem effici poterit, ut claritas ad nostrum arbitrium determinetur, quemadmodum id in telescopiis est factum, sed plerumque pro maioribus multiplicationibus minore claritatis gradu contenti esse debemus; cui defectui autem remedium adferri solet ipsum obiectum forti lumine illuminando, quod, quia obiecta vicina in nostra sunt potestate, sine difficultate fieri potest. Deinde etiam in id maxime est incumbendum, ut haec instrumenta perinde ac telescopia notabilem campum apparentem obtineant seu ut non nimis exigua portio obiecti obtutui repraesentetur; quae portio non simpliciter per angulum ad lentem obiectivam formatum definiri potest, quia etiam minima portiuncula, si lenti obiectivae proxima admoveatur, ingentem angulum formare posset, sed vera semidiameter huius portionis visae, quam supra posuimus $= z$, in computum duci debet; denique etiam, cum distantia obiecti a lente obiectiva, quam ponimus $= a$, ab arbitrio nostro pendeat, haec tractatio plurimum a praecedente discrepabit, siquidem non solum gradus claritatis, sed etiam campi apparentis indicium longe aliam investigationem requirat. Quamobrem in hoc primo capite formulas generales in primo libro inventas ad has circumstantias accommodari necesse erit, ante quam in ipsam constructionem microscopiorum inquiramus.

PROBLEMA 1

7. *Ex quotcunque lentibus microscopium fuerit compositum, singula elementa exhibere, quibus tam lentium dispositio quam earum intervalla et distantiae focales determinantur.*

SOLUTIO

Distantias determinatrices singularum lentium sequenti modo conspectui exponamus:

Distantiae	Distantiae
obiecti a lente $1^{ma} = a$,	obiecti a lente $1^{ma} = \alpha$,
ab imagine 1^{ma} ad lentem $2^{dam} = b$	ab imagine 1^{ma} ad lentem $2^{dam} = \beta$
ab imagine 2^{da} ad lentem $3^{tiam} = c$	ab imagine 2^{da} ad lentem $3^{tiam} = \gamma$
ab imagine 3^{tia} ad lentem $4^{tam} = d$	ab imagine 3^{tia} ad lentem $4^{tam} = \delta$
:	:
:	:
ab imag. penult. ad lent. ult. $= l$	ab imag. penult. ad lent. ult. $= \lambda = \infty$.

Hic scilicet intelligendum est a singulis lentibus imagines proiici, sive eae sint reales sive fictae, quarum discrimen, uti iam observavimus, in eo est situm, ut imagines reales intra lentem, a qua formantur, et lentem sequentem cadant, fictae vero extra hoc spatium.

Deinde vero, quo commodius haec elementa inter se comparemus, litteras maiusculas duplicis generis introducamus:

$$\alpha = Aa,\ \beta = Bb,\ \gamma = Cc,\ \delta = Dd,\ \varepsilon = Ee \text{ etc.}$$
$$\tfrac{\alpha}{b} = -P,\ \ \tfrac{\beta}{c} = -Q,\ \tfrac{\gamma}{d} = -R,\ \tfrac{\delta}{e} = -S \text{ etc.,}$$

ubi litterarum *A, B, C, D* etc. ultima sit $L = \infty$ litterarum vero *P, Q, R* etc. ultima sit $= Z$ intervallo inter binas ultimas lentes respondens.

His litteris introductis omnia elementa sequenti modo per primum *a* exprimentur:

$$\alpha = Aa,\ \beta = -\tfrac{AB}{P}\cdot a,\ \gamma = \tfrac{ABC}{PQ}\cdot a,\ \delta = -\tfrac{ABCD}{PQR}\cdot a \quad \text{etc.,}$$
$$b = -\tfrac{A}{P}\cdot a,\ c = \tfrac{AB}{PQ}\cdot a,\ d = -\tfrac{ABC}{PQR}\cdot a,\ e = -\tfrac{ABCD}{PQRS}\cdot a \text{ etc.,}$$

et litterarum *b, c, d* etc. ultima

$$l = \mp \tfrac{ABC\ \ldots\ K}{PQR\ \ldots\ Z}\cdot a$$

et litterarum $\alpha,\ \beta,\ \gamma$ etc. etc. ultima

$$\lambda = \pm \tfrac{ABC\ \ldots\ L}{PQR\ \ldots\ Z}\cdot a\ =\ \infty,$$

ex quibus intervalla lentium ita ordine repraesentantur:

Primum $\alpha + b = Aa\left(1-\frac{1}{P}\right)$,

secundum $\beta + c = -\frac{AB}{P}a\left(1-\frac{1}{Q}\right)$,

tertium $\gamma + d = \frac{ABC}{PQ}a\left(1-\frac{1}{R}\right)$,

quartum $\delta + e = -\frac{ABCD}{PQR}a\left(1-\frac{1}{S}\right)$ etc.;

quae cum omnia debeant esse positiva, etiam quodlibet per praecedens divisum quotum dare debet positivum sicque esse oportet

$$1.\ -\frac{B}{Q}\cdot\frac{Q-1}{P-1} > 0, \quad 2.\ -\frac{C}{R}\cdot\frac{R-1}{Q-1} > 0,$$
$$3.\ -\frac{D}{S}\cdot\frac{S-1}{R-1} > 0, \quad 4.\ -\frac{E}{T}\cdot\frac{T-1}{S-1} > 0, B\ Q\text{-}1$$
etc.

Quo denique distantias focales singularum lentium, quas litteris minusculis *p, q, r, s, t* etc. indicamus, concinnius exprimamus, litteras maiusculas germanicas $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, etc. introducamus, ita ut sit

$$\mathfrak{A}=\frac{A}{A+1},\ \mathfrak{B}=\frac{B}{B+1},\ \mathfrak{C}=\frac{C}{C+1},\ \mathfrak{D}=\frac{D}{D+1}, \text{ etc.}$$

hincque vicissim

$$A=\frac{\mathfrak{A}}{1-\mathfrak{A}},\ B=\frac{\mathfrak{B}}{1-\mathfrak{B}},\ C=\frac{\mathfrak{C}}{1-\mathfrak{C}},\ D=\frac{\mathfrak{D}}{1-\mathfrak{D}}, \text{ etc.}$$

ita ut pro ultima harum litterarum sit

$$\mathfrak{L}=\frac{L}{L+1} \text{ ob } L=\infty \text{ et } L=\frac{\mathfrak{L}}{1-\mathfrak{L}}=\infty.$$

Ex his ergo litteris distantiae focales ita exprimentur:

$$p=\mathfrak{A}a,\ \ q=-\frac{A\mathfrak{B}}{P}\cdot a,\ \ r=\frac{AB\mathfrak{C}}{PQ},\ \ s=-\frac{ABC\mathfrak{D}}{PQR}, \text{ etc.,}$$

ultimae autem lentis distantia focalis fiet $=l$.

COROLLARIUM 1

8. Litterae ergo *A, B, C, D* etc. singulis lentibus, primae, secundae, tertiae etc., ordine respondent; at litterae *P, Q, R* etc. ad singula intervalla, primum, secundum, tertium etc., ordine referuntur; quam ob causam numerus harum posteriorum litterarum unitate minor erit quam priorum.

COROLLARIUM 2

9. Quatenus litterae *P, Q, R* etc. ut positivae spectantur, imagines erunt fictae, ita ut, si omnes istae litterae essent positivae, nulla imago realis in microscopio occurreret, sin autem omnes hae litterae essent negativae, in singulis intervallis imago realis reperiretur; unde quot fuerint imagines reales in microscopio, tot istarum litterarum valores sortientur negativos.

COROLLARIUM 3

10. Cum istae litterae *P, Q, R* etc. per bina elementa ad lentes sibi succedentes pertinentia determinentur, si huiusmodi littera fuerit positiva, binorum elementorum, ex quibus oritur, alterum erit positivum, alterum negativum, sin autem talis littera fuerit negativa, ambo elementa, ex quibus oritur, erunt positiva, quippe quia omnia intervalla debent esse positiva.

PROBLEMA 2

11. *Ex quotcunque lentibus microscopium fuerit compositum, singularum imaginum, sive sint fictae sive reales, quantitatem definire hincque multiplicationem, quam instrumentum producit, assignare tam pro repraesentatione erecta quam inversa.*

SOLUTIO

Posita semidiametro obiecti, quatenus id per microscopium est conspicuum, $=z$ semidiametri singularum imaginum per ipsa elementa sequenti modo supra sunt expressae:

Semi diameter imaginis primae $=\frac{\alpha}{a}\cdot z=Az\,(\text{inversa})$

semidiameter imaginis secundae $=\frac{\alpha\beta}{ab}\cdot z=ABz\,(\text{erecta})$

semidiameter imaginis tertiae $=\frac{\alpha\beta\gamma}{abc}\cdot z=ABCz\,(\text{inversa})$

semidiameter imaginis quartae $=\frac{\alpha\beta\gamma\delta}{abcd}\cdot z=ABCDz\,(\text{erecta})$

etc.,

unde imaginis ultimae semidiameter erit $=ABC\ ...\ Lz$; quae imago erit erecta, si litterarum *A, B, C, ... L* numerus sit par, inversa autem, si is sit impar; quae ultima imago, cum fiat obiectum visionis post ultimam lentem ad distantiam infinitam $\lambda=Ll$ cadens, quam oculus circa ultimam lentem constitutus ideoque in distantia *Ll* contemplatur, ei apparebit sub angulo $ABC\ ...\ K\frac{z}{l}$. Ut nunc hinc multiplicationem, quae sit $=m$, definiamus, istum angulum comparare debemus cum angulo, sub quo ipsum obiectum *z* ad distantiam $=h$ oculo esset appariturum; qui angulus cum sit z, manifestum est fore multiplicationem

$$m = ABC \ldots K \cdot \tfrac{h}{l}.$$

An autem haec repraesentatio futura sit erecta sive inversa, duo casus sunt perpendendi.

I. Si numerus lentium ideoque etiam litterarum *A, B, C, ... L* fuerit impar, ultima imago erit inversa; quae cum post oculum ad distantiam infinitam cadat, eam oculus ante se in situ erecto conspiciet. Quare si in formula nostra pro *m* inventa numerus litterarum *A, B, C, ... K* fuerit par, obiectum situ erecto cernetur, quatenus scilicet haec formula positivum valorem obtinet.

II. Sin autem numerus lentium ideoque etiam litterarum *A, B, C, D, ... L* fuerit par, facile intelligitur contrarium locum habere debere. Quare si in expressione ipsius *m* numerus litterarum *A, B, C, ... K* fuerit impar, obiectum situ inverso cernetur, quatenus scilicet ista expressio fuerit negativa.

Quodsi vero in superiores formulas litteras *P, Q, R* etc. introducamus, invenietur

semidiameter imaginis primae $=\alpha \cdot \tfrac{z}{a}$
semidiameter imaginis secundae $=P\beta \cdot \tfrac{z}{a}$
semidiameter imaginis tertiae $=PQ\gamma \cdot \tfrac{z}{a}$
semidiameter imaginis quartae $=PQR\delta \cdot \tfrac{z}{a}$
etc.
semidiameter imaginis ultimae $=PQR...Z\lambda \cdot \tfrac{z}{a}$;

quae imagines omnes sunt inversae, siquidem istae formulae valores habuerint positivos. Quare cum hic omnis ambiguitas cesset haecque ultima imago ad distantiam infinitam $=\lambda$ post oculum cadat, oculus eam ante se situ erecto conspiciet sub angulo $=PQR...Z\lambda \cdot \tfrac{z}{a}$; ; unde sequitur multiplicationem fore

$$m = PQR...Z \cdot \tfrac{h}{a}$$

pro situ erecto, si scilicet haec formula fuerit positiva; sin autem ea valorem habeat negativum, repraesentatio erit inversa; tum vero hoc casu ipsam litteram *m* negative capi conveniet. Facile autem intelligitur hanc posteriorem expressionem pro multiplicatione priori longe esse anteferendam, quia nulla ambiguitate laborat, eaque in sequentibus perpetuo utemur.

COROLLARIUM 1

12. Quodsi ergo in locis imaginum realium diaphragmata constitui conveniat, ex his formulis statim intelligimus, quantum foramen iis induci oporteat, postquam scilicet cognoverimus, quantam obiecti portionem, cuius semidiametrum hic vocamus $=z$, instrumentum spectandam offerat.

COROLLARIUM 2

13. Si omnes litterae P, Q, R etc. fuerint positivae ideoque nulla plane imago realis occurat, tunc instrumentum semper obiecta situ erecto repraesentabit; sin autem unica occurrat imago realis ideoque unica istarum litterarum fuerit negativa, tum repraesentatio semper fiet situ inverso, quo casu ipsa littera m signo contrario in calculum introduci debebit; at si duae imagines reales locum habeant, repraesentatio iterum erit erecta.

COROLLARIUM 3

14. Hinc adparet, quanti momenti sit introductio harum litterarum P, Q, R, S etc., cum eae tam perspicue distinctionem inter imagines reales et fictas commonstrent, praecipue cum hunc tractatum aeque ac praecedentem de telescopiis secundum imagines reales dividi conveniat, quippe in quo essentiale discrimen inter diversa microscopiorum genera continetur.

PROBLEMA 3

15. *Ex quotcunque lentibus microscopium fuerit compositum, si detur apertura primae lentis obiectivae, per quam radii ex obiecti quasi centro transmittantur, definire aperturas singularum lentium ad ulteriorem transmissionem necessarias et gradum claritatis, quo oculus obiectum contuebitur.*

SOLUTIO

Ex principiis fundamentalibus supra satis expositis hae aperturae facillime definiuntur ex apertura primae lentis cognita, unde semidiametri singularum aperturarum sequenti modo per litteras P, Q, R etc. exprimentur:

Semidiameter aperturae lentis primae $= x$
semidiameter aperturae lentis secundae $= \frac{b}{\alpha} \cdot x = \frac{1}{P} \cdot x$
semidiameter aperturae lentis tertiae $= \frac{bc}{\alpha\beta} \cdot x = \frac{1}{PQ} \cdot x$
semidiameter aperturae lentis quartae $= \frac{bcD}{\alpha\beta\gamma} \cdot x = \frac{1}{PQR} \cdot x$
etc.,

unde concludimus pro ultima lente requiri semidiametrum aperturae

$$= \frac{x}{PQR...Z};$$

cum autem ante invenerimus

$$m = PQR...Z \cdot \frac{h}{a}$$

erit ista formula

$$= \frac{h}{ma} \cdot x.$$

Tantam nempe aperturam lens ocularis ad minimum habere debet, ut radios per lentem obiectivam ingressos transmittat, et cum nunc radii inter se sint paralleli, ii quasi penicillum radiosum repraesentabunt, qui a centro obiecti in oculum intrat; ex quo, si semidiameter huius penicilli $\frac{hx}{ma}$ semidiametro pupillae aequaretur, tunc visio plena claritate frueretur; quatenus autem ista expressio minor est quam semidiameter pupillae, eatenus gradus claritatis evadit minor. Unde, cum supra gradus claritatis littera *y* fuerit expressus, erit hic $y = \frac{hx}{ma}$; qui valor quoties fuerit minor semidiametro pupillae, quae circiter $\frac{1}{20}$ dig. aestimatur, toties claritas minor erit censenda quam naturalis seu plena, vel potius in ratione duplicata, prouti per se est manifestum.

COROLLARIUM 1

16. Data igitur claritate *y* cum multiplicatione *m* reperitur $x = \frac{may}{h}$; unde apertura lentis obiectivae innotescit, quae ceteris paribus eo maior esse debet, quo maior fuerit distantia obiecti a lente obiectiva sive *a.* Cum igitur *x* a distantia focali lentis obiectivae pendeat, hinc colligere licet, quomodo haec lens ratione distantiae *a* debeat esse comparata.

COROLLARIUM 2

17. Tam hinc quam ex praecedente problemate etiam patet, quomodo multiplicatio *m* ad distantiam illam *h,* quae vulgo 8 dig. assumitur, referatur, quandoquidem in hoc negotio multiplicationem *m* non absolute definire licet, sicque $\frac{m}{h}$ proprie id denotat, quod sub notione multiplicationis menti offertur.

PROBLEMA 4

18. *Ex quotcunque lentibus microscopium fuerit compositum, momenta, quae a singulis lentibus ad campum apparentem conferuntur earumque aperturam definiunt, exponere locumque oculi assignare.*

SOLUTIO

Ad hoc supra litteras peculiares in calculum introduximus; cum enim cuiusque lentis apertura ita ab eius distantia focali pendeat, ut certam eius partem superare non debeat, semidiameter aperturae cuiusque lentis post primam sequenti modo per eius distantiam focalem est stabilita:

$$\text{secundae} = \pi q,\quad \text{tertiae} = \pi' r,\quad \text{quartae} = \pi'' s,\quad \text{quintae} = \pi''' t \quad \text{etc.;}$$

unde, si semidiameter obiecti conspicui sit $= z$ voceturque $\frac{z}{a} = \Phi$, ostendimus esse

$$z = a\Phi = \frac{-\pi+\pi'-\pi''+\pi'''-\pi'''' \text{ etc.}}{ma-h}\cdot ah,$$

quod intelligendum est de situ erecto; pro inverso enim situ multiplicatio *m* negative accipi debet.

Nunc autem, quo facilius de quantitate campi iudicare queamus, sit aperturae maximae, quam quaepiam lens, cuius distantia focalis sit v. gr. $= \mathfrak{q}$, recipere potest, semidiameter $= \xi\mathfrak{q}$, cuius scilicet haec lens foret capax, si esset utrinque aequalis, denotante ξ vulgo $\frac{1}{4}$; pro singulis lentibus, quatenus minores habere possunt aperturas, introducamus novas litteras et ponamus

$$\pi = -\mathfrak{q}\xi,\quad \pi' = +\mathfrak{r}\xi,\quad \pi'' = -\mathfrak{s}\xi,\quad \pi''' = +\mathfrak{t}\xi \quad \text{etc.,}$$

ut fiat

$$z = a\Phi = \frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t} \text{ etc.}}{ma-h}\cdot ah\xi,$$

in qua porro brevitatis gratia ponamus

$$M = \frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t} \text{ etc.}}{ma-h}\cdot h,$$

ut fiat

$$z = a\Phi = Ma\xi \quad \text{seu} \quad \Phi = M\xi;$$

quibus positis novae hae litterae $\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t}$ etc. sequenti modo ad ante introductas referentur:

$$1. \mathfrak{B}\mathfrak{q} = (P-1)M,$$
$$2. \mathfrak{C}\mathfrak{r} = (PQ-1)M - \mathfrak{q},$$
$$3. \mathfrak{D}\mathfrak{s} = (PQR-1)M - \mathfrak{q} - \mathfrak{r},$$
$$4. \mathfrak{E}\mathfrak{t} = (PQRS-1)M - \mathfrak{q} - \mathfrak{r} - \mathfrak{s}$$
etc.,

quarum formarum differentiae etiam notatu dignae sunt, nimirum

$$1. \mathfrak{C}\mathfrak{r} - \mathfrak{B}\mathfrak{q} = P(Q-1)M - \mathfrak{q},$$
$$2. \mathfrak{D}\mathfrak{s} - \mathfrak{C}\mathfrak{r} = PQ(R-1)M - \mathfrak{r},$$
$$3. \mathfrak{E}\mathfrak{t} - \mathfrak{D}\mathfrak{s} = PQR(S-1)M - \mathfrak{s},$$
etc.,

Illarum igitur aequationum ultima ita erit expressa:

$$\mathfrak{L}\mathfrak{z} = (PQR..Z-1)M - \mathfrak{q} - \mathfrak{r} \ldots - \mathfrak{h},$$
$$= \left(\tfrac{ma}{h} - 1\right)M - \mathfrak{q} - \mathfrak{r} \ldots - \mathfrak{h}.$$

Ante vero ostendimus esse $\mathfrak{L} = 1$; unde fiet

$$\mathfrak{q} + \mathfrak{r} + \mathfrak{s} \ldots + \mathfrak{z} = \left(\tfrac{ma}{h} - 1\right)M$$

quae est ipsa illa aequatio, qua littera M determinatur.

Nunc igitur superest, ut locum oculi seu eius distantiam post ultimam lentem, quam supra vocavimus $= O$, definiamus; quod quidem primo secundum lentium numerum ex superioribus repetamus:

Pro una lente

$$O = 0.$$

Pro duabus lentibus

$$O = \frac{\mathfrak{B}b\pi}{\pi - \Phi} = \frac{\mathfrak{q}b}{Ma} \cdot \frac{h}{m} = \frac{\mathfrak{q}q}{M} \cdot \frac{h}{ma}.$$

Pro tribus lentibus

$$O = \frac{\mathfrak{C}c\pi'}{\pi' - \pi + \Phi} = \frac{\mathfrak{r}c}{Ma} \cdot \frac{h}{m} = \frac{\mathfrak{r}r}{M} \cdot \frac{h}{ma}.$$

Pro quatuor lentibus

$$O = \frac{\mathfrak{D}d\pi''}{\pi'' - \pi' + \pi - \Phi} = \frac{\mathfrak{s}d}{Ma} \cdot \frac{h}{m} = \frac{\mathfrak{s}s}{M} \cdot \frac{h}{ma}$$
etc.,

unde concludimus pro lentium numero quocunque fore distantiam oculi

$$O = \frac{\mathfrak{z}l}{M} \cdot \frac{h}{ma}.$$

COROLLARIUM 1

19. Hinc igitur novas determinationes pro aperturis singularum lentium sumus consecuti, quas scilicet adparitio campi postulat et quae non sunt confundendae cum superioribus, quas gradus claritatis postulat; cuilibet autem lenti ea apertura, quae est maior, tribui debet; unde sequentes formulae probe sunt observandae:

Semidiameter aperturae pro prima lente $= 0\xi p \cdot \cdot \cdot x,$,
semidiameter aperturae pro secunda lente $= \mathfrak{q}\xi q \cdot \cdot \cdot \frac{x}{P},$
semidiameter aperturae pro tertia lente $= \mathfrak{r}\xi r \cdot \cdot \cdot \frac{x}{PQ},$
semidiameter aperturae pro quarta lente $= \mathfrak{s}\xi s \cdot \cdot \cdot \frac{x}{PQR},$

unde

pro ultima lente $= \mathfrak{z}\xi l \cdot \cdot \cdot \frac{hx}{ma},$

ubi notetur litteras $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$ etc. fractiones esse unitate minores, quarum valores unitatem superare nequeant.

COROLLARIUM 2

20. Si forte repraesentatio fuerit inversa, quo casu, ut supra iam monuimus, multiplicatio m negative accipitur seu $-m$ loco m scribi debet, eo casu quoque singulis litteris $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$ signum negativum tribui debet, ita ut
tum fiat

$$M = \frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t} \text{ etc.}}{ma + h} \cdot h.$$

COROLLARIUM 3

21. Quoniam circumstantiae quaedam postulare solent, ut pro utroque casu litterarum $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$ etc. una vel altera negativum valorem sortiri debeat, hoc praecipue, uti in telescopiis vidimus, in prioribus harum litterarum usu venit; posteriores vero semper positivae atque adeo ipsi unitati aequales tuto assumi possunt, ita ut earum ultima certo pro unitate haberi possit; ex quo perspicuum est distantiam oculi O semper fore positivam, quoties postrema lens fuerit convexa; sin autem haec lens fuerit concava, tum etiam distantia O prodibit negativa.

SCHOLION

22. Ceterum hic monendum est, cum in primo libro littera l usurpata sit ad iustam oculi distantiam significandam, quae hic perpetuo ut infinita spectatur, hic eandem litteram longe alio significatu adhiberi, siquidem hic semper significeat distantiam focalem lentis ultimae seu ocularis, quae eadem est distantia penultimae imaginis ante ultimam lentem; ex quo sequitur, si ultima lens fuerit convexa, penultimam imaginem certe ante eam repraesentari debere; quocirca ante ultimam lentem certe imago realis esset casura. Hine igitur perspicuum est, id quod supra non tam clare patebat, si nulla plane adsit imago realis, tum lentem ultimam convexam esse non posse ideoque pro loco oculi distantiam O semper prodire negativam, pro quo casu etiam coacti fuimus peculiarem formulam pro margine colorato destruendo tradere, quae penitus diversa est ab ea, quae locum habet, quoties quantitas O est positiva; quos ergo duos casus etiam hic seorsim tractari conveniet.

PROBLEMA 5

23. *Ex quotcunque lentibus microscopium fuerit compositum, si distantia oculi post ultimam lentem O prodierit positiva, destruere marginem coloratum, ex quacunque vitri specie singulae lentes fuerint paratae.*

SOLUTIO

Quoniam hic solutionem ita generalem postulamus, quae etiam ad lentes ex diversis vitri speciebus paratas pateat, rationem refractionis pro prima lente ponamus $=n$, pro secunda $=n'$, pro tertia $=n''$ etc., uti in superioribus libris fecimus; atque hinc statuamus formulas differentiales, quibus dispersio radiorum exprimitur, sequenti modo:

$$\frac{dn}{n-1}=N,\ \frac{dn'}{n'-1}=N',\ \frac{dn''}{n''-1}=N''\ \text{etc.;}$$

quibus notatis supra [Lib. I. Suppl. VII] ostendimus pro destructione marginis colorati satisfieri debere huic aequationi:

$$0=\frac{N'\cdot b\pi}{Aa\Phi}+\frac{N''\cdot c\pi'}{ABa\Phi}+\frac{N'''\cdot d\pi''}{ABCa\Phi}+\frac{N''''\cdot e\pi'''}{ABCDa\Phi}\ \text{etc.;}$$

quae aequatio, si tam loco litterarum $\pi,\ \pi'$ etc. etc. quam loco b, c, d etc. valores ante assignati substituantur, transibit in hanc formam:

$$0=\frac{N'\cdot\mathfrak{q}}{P}+\frac{N''\cdot\mathfrak{r}}{PQ}+\frac{N'''\cdot\mathfrak{s}}{PQR}+\frac{N''''\cdot\mathfrak{t}}{PQRS}\ \text{etc.;}$$

in qua aequatione terminus ultimus ita erit expressus $\frac{N^{\cdots\cdots}\mathfrak{z}h}{ma}$.

COROLLARIUM 1

24. Patet ergo marginem coloratum tolli non posse, nisi vel litterarum $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$ $\mathfrak{t}$ etc. vel *P, Q, R* etc. una pluresve fuerint negativae, quia alioquin omnes termini essent positivi eorumque aggregatum nihilo aequari non posset.

COROLLARIUM 2

25. Si ergo nulla adsit imago realis, quod evenit, si omnes litterae *P*, *Q, R* etc. fuerint positivae, tum necessaria litterarum $\mathfrak{q}$, $\mathfrak{r}$, $\mathfrak{s}$ etc. una vel altera debet esse negativa; quae autem earum fuerint negativae, iis campus apparens diminuitur.

PROBLEMA 6

26. *Ex quotcunque lentibus microscopium fuerit compositum, si distantia oculi O prodeat negativa ideoque oculus ultimae lenti immediate duplicari debeat, destruere marginem coloratum, ex quacunque vitri specie singulae lentes fuerint paratae.*

SOLUTIO

Manentibus iisdem, quae in praecedente problemate circa diversitatem vitri sunt posita, supra [Lib. I. Suppl. VII, p. 239] pro hoc casu secundum lentium numerum peculiares formulae sunt datae, quae ad nostrum institutum translatae ita se habent:
Pro una lente

$$0 = 0..$$

Pro duabus lentibus

$$0 = N(A+1)\mathfrak{q}.$$

Pro tribus lentibus

$$0 = N\left(A+1\right)B\mathfrak{t} - \tfrac{N'}{P}\left((B+1)\mathfrak{r}+\mathfrak{q}\right).$$

Pro quatuor lentibus

$$0 = N\left(A+1\right)BC\mathfrak{s} - \tfrac{N'}{P}\left((B+1)C\mathfrak{s}-\mathfrak{q}\right) + \tfrac{N''}{PQ}\left(\left(C+1\right)\mathfrak{s}+\mathfrak{r}\right).$$

Pro quinque lentibus

$$0 = N\left(A+1\right)BCD\mathfrak{t} - \tfrac{N'}{P}\left((B+1)CD\mathfrak{t}+\mathfrak{q}\right) + \tfrac{N''}{PQ}\left(\left(C+1\right)D\mathfrak{t}-\mathfrak{r}\right) - \tfrac{N'''}{PQR}\left(\left(D+1\right)\mathfrak{t}+\mathfrak{s}\right).$$

Pro sex lentibus

$$0 = N(A+1)BCDE\mathfrak{u} - \tfrac{N'}{P}\left((B+1)CDE\mathfrak{u} - \mathfrak{q}\right) + \tfrac{N''}{PQ}\left((C+1)DE\mathfrak{u} + \mathfrak{r}\right)$$
$$- \tfrac{N'''}{PQR}\left((D+1)E\mathfrak{u} - \mathfrak{s}\right) + \tfrac{N''''}{PQRS}\left((E+1)\mathfrak{u} + \mathfrak{t}\right).$$

Pro septem lentibus

$$0 = N(A+1)BCDEF\mathfrak{v} - \tfrac{N'}{P}\left((B+1)CDEF\mathfrak{v} + \mathfrak{q}\right) + \tfrac{N''}{PQ}\left((C+1)DEF\mathfrak{v} - \mathfrak{r}\right)$$
$$- \tfrac{N'''}{PQR}\left((D+1)EF\mathfrak{v} + \mathfrak{s}\right) + \tfrac{N''''}{PQRS}\left((E+1)F\mathfrak{v} - \mathfrak{t}\right) - \tfrac{N'''''}{PQRST}\left((F+1)\mathfrak{v} + \mathfrak{u}\right);$$

quas formulas concinnius exhibere non licet ideoque iis quovis casu oblato erit utendum.

PROBLEMA 7

27. *Ex quotcunque lentibus microscopium fuerit compositum, omnem plane confusionem, quae ob diversam radiorum refrangibilitatem praeter marginem coloratum est metuenda, ad nihilum redigere, ex quacunque vitri specie singulae lentes fuerint paratae.*

SOLUTIO

Introductis etiam litteris N, N' etc., uti in praecedentibus problematibus est factum, aequatio in libro primo inventa [Lib. I. Suppl. VII.], cui est satisfaciendum, sequenti modo generatim pro quovis lentium numero expressa reperietur:

$$0 = N \cdot \tfrac{A+1}{A} - \tfrac{N'}{P} \cdot \tfrac{B+1}{AB} + \tfrac{N''}{PQ} \cdot \tfrac{C+1}{ABC} - \tfrac{N'''}{PQR} \cdot \tfrac{D+1}{ABCD} \text{ etc.,}$$

quae etiam hoc modo exhiberi potest:

$$0 = N \cdot \tfrac{1}{p} + \tfrac{N'}{P^2} \cdot \tfrac{1}{q} + \tfrac{N''}{P^2Q^2} \cdot \tfrac{1}{r} + \tfrac{N'''}{P^2Q^2R^2} \cdot \tfrac{1}{s} \text{ etc.,}$$

vel etiam, si libuerit, hoc modo:

$$0 = N \cdot \tfrac{1}{\mathfrak{A}} - \tfrac{N'}{P} \cdot \tfrac{1}{A\mathfrak{B}} + \tfrac{N''}{PQ} \cdot \tfrac{1}{AB\mathfrak{C}} - \tfrac{N'''}{PQR} \cdot \tfrac{1}{ABC\mathfrak{D}} \text{ etc.}$$

COROLLARIUM 1

28. Cum productum omnium litterarum P, Q, R, S ... multiplicationem praebeat, si haec fuerit valde magna, termini huius aequationis mox fient tam parvi, ut sufficiat binos vel ternos terminos initiales assumsisse, ex quibus commode vel littera $\mathfrak{B}$ vel $\mathfrak{C}$ definiri poterit.

COROLLARIUM 2

29. Iam supra [Lib. II] autem ostensum est, nisi litterae *N, N'* etc. fuerint inter se diversae, hanc ultimam aequationem nullo modo adimpleri posse; unde eatenus tantum huic conditioni satisfieri poterit, quatenus lentes non ex eadem vitri specie conficiuntur.

SCHOLION

30. Istud quidem tantum pro telescopiis supra demonstravimus, idem autem quoque pro casu praesente demonstrari potest hoc modo. Ad hoc scilicet utamur prima forma nostrae aequationis in eaque litterae *N* inter se ponantur aequales, cuius singuli termini in duas partes discerpantur, ut prodeat haec forma:

$$0 = 1 + \frac{1}{A} - \frac{1}{ABP} + \frac{1}{ABCPQ} - \frac{1}{ABCDPQR}$$
$$- \frac{1}{AP} + \frac{1}{ABPQ} - \frac{1}{ABCPQR} + \frac{1}{ABCDPQRS} \text{ etc.,}$$

quae per *a* multiplicata censeatur, et cum sit ex elementis

$$a = \frac{\alpha}{A} = -\frac{Pb}{AB} = -\frac{P\beta}{AB} = \frac{PQc}{AB} = \frac{PQ\gamma}{ABC} \text{ etc.,}$$

hi valores successive in nostra aequatione substituantur et aequatio nostra abibit in hanc formam:

$$0 = a + \frac{\alpha + b}{A^2} + \frac{\beta + c}{A^2B^2} + \frac{\gamma + d}{A^2B^2C^2} + \frac{\delta + e}{A^2B^2C^2D^2} \text{ etc.;}$$

ubi cum numeratores intervalla lentium designent, denominatores vero omnes sint numeri quadrati, omnes isti termini necessario sunt positivi. Tantum de ultima parte solitaria dubium superesse posset; scilicet hic, quousque hos terminos continuavimus, insuper adiungi deberet terminus

$$\frac{\varepsilon}{A^2B^2C^2D^2E^2},$$

qui est casus quinque lentium, pro quo ε quidem est ∞; notandum autem est esse etiam $\varepsilon = Ee$, cum sit *8 = Ee;* quo valore substituto istum terminum insuper adiungendum sponte evanescere manifestum est. Ceterum, uti iam saepius monuimus, etiam diversa vitra adhibendo neutiquam necesse est, ut huic ultimae aequationi accuratissime satisfiat, cum iam satis praeclare nobis agatur, si modo eius valor satis exiguus reddi queat, id quod etiam de duabus praecedentibus aequationibus est tenendum; neque enim natura rei ipsa huiusmodi solutionem rigorosam permittit, cum nunquam sit sperandum per experimenta valores litterarum. *N, N′* etc. ita exacte definiri posse, ut non notabiliter a veritate aberrent; et quia unicam vitri speciem usurpando semper coacti sumus hanc ultimam confusionem tolerare, si modo eam minorem reddere licuerit, id certe pro maximo lucro erit habendum.

PROBLEMA 8

31. *Ex quotcunque lentibus microscopium fuerit compositum, semidiametrum confusionis, quae a lentium apertura oritur, assignare totamque hanc confusionem infra datum limitem reducere, ut repraesentationi non amplius officiat.*

SOLUTIO

Ad hoc praestandum novae litterae λ, λ' etc. pro singulis lentibus in calculum sunt introducendae, quemadmodum in primo libro sufficienter est explicatum. Tum vero, si singulas lentes ex peculiari vitri specie factas consideremus, expressio pro semidiametro confusionis supra [Lib. I. Supplem. VII] inventa litteris *P, Q, R* etc. adhibendis ad sequentem formam revocabitur:

$$\frac{\mu m x^3}{4aah}\left(\begin{array}{l}\mu\left(\frac{\lambda}{\mathfrak{A}^3}+\frac{\nu}{A\mathfrak{A}}\right)-\frac{\mu'}{A^3P}\left(\frac{\lambda'}{\mathfrak{B}^3}+\frac{\nu'}{B\mathfrak{B}}\right)+\frac{\mu''}{A^3B^3PQ}\left(\frac{\lambda''}{\mathfrak{C}^3}+\frac{\nu''}{C\mathfrak{C}}\right)\\ -\frac{\mu'''}{A^3B^3C^3PQR}\left(\frac{\lambda'''}{\mathfrak{D}^3}+\frac{\nu'''}{D\mathfrak{D}}\right)+\frac{\mu''''}{A^3B^3C^3D^3PQRS}\left(\frac{\lambda''''}{\mathfrak{E}^3}+\frac{\nu''''}{E\mathfrak{E}}\right)\text{ etc.}\end{array}\right)$$

quae formula succinctior reddetur distantias focales introducendo; cum enim sit

$$\mathfrak{A}=\frac{p}{a},\quad A\mathfrak{B}=-\frac{Pq}{a},\quad AB\mathfrak{C}=\frac{PQr}{a},\quad ABC\mathfrak{D}=-\frac{PQRs}{a}\quad\text{etc.,}$$

his valoribus substitutis fiet nostra formula

$$\frac{max^3}{4h}\left(\frac{\mu}{p^3}\left(\lambda+\frac{\mathfrak{A}^2}{A}\nu\right)+\frac{\mu'}{P^4q^3}\left(\lambda'+\frac{\mathfrak{B}^2}{B}\nu'\right)+\frac{\mu''}{P^4Q^4r^3}\left(\lambda''+\frac{\mathfrak{C}^2}{C}\nu''\right)\text{ etc.}\right).$$

Sit nunc limes, quem valor huius formulae superare non debet, $=\frac{1}{4k^3}$, ubi notandum est pro telescopiis supra sumtum esse $k=50$ circiter; quare, si brevitatis gratia ponamus

$$\Lambda=\frac{\mu}{p^3}\left(\lambda+\frac{\mathfrak{A}^2}{A}\nu\right)+\frac{\mu'}{P^4q^3}\left(\lambda'+\frac{\mathfrak{B}^2}{B}\nu'\right)+\frac{\mu''}{P^4Q^4r^3}\left(\lambda''+\frac{\mathfrak{C}^2}{C}\nu''\right)\text{ etc.,},$$

debebit esse

$$\frac{max^3}{h}\Lambda<\frac{1}{k^3},$$

unde commodissime definitur semidiameter lentis obiectivae

$$x < \frac{1}{k}\sqrt[3]{\frac{h}{ma\Lambda}};$$

ac si licuerit formulam hanc Λ penitus ad nihilum redigere, tunc hanc semidiametrum x nihil impedit, quominus tantam statuamus, quam figura lentis obiectivae permittit.

COROLLARIUM 1

32. Quando ergo hinc quantitas x fuerit definita, tum demum gradum claritatis assignare poterimus; ex aequatione enim supra inventa $y = \frac{hx}{ma}$ cognoscimus semidiametrum penicillorum radiosorum, qui a singulis obiecti punctis in oculum transmittuntur, quae ad pupillam relata gradum claritatis determinabit.

COROLLARIUM 2

33. De telescopiis quidem vidimus sufficientem claritatis gradum produci, si modo y non multo minor sit quam $\frac{1}{50}$ dig., in microscopiis autem nos plerumque multo minore claritate contentos esse oportebit.

COROLLARIUM 3

34. At si loco x valorem inventum substituamus, pro gradu claritatis habebimus

$$y = \frac{1}{k}\left(\frac{h}{ma}\right)^{\frac{4}{3}}\sqrt[3]{\frac{1}{m\Lambda}};$$

unde intelligitur, quo longius obiectum a microscopio removere velimus, eo minore claritate obiectum esse appariturum, quae causa est, ut in omnibus microscopiis distantia obiecti a lente obiectiva tam exigua capi debeat.

COROLLARIUM 4

35. Ex ultima forma nostrae expressionis manifestum est, si omnes lentes fuerint convexae seu litterae p, q, r etc. positivae, omnes terminos litteras $\lambda, \lambda', \lambda''$ etc. continentes fore quoque positivos; unde, cum litterae v, v', v'' etc. sint valde parvae, quantitas illa Λ nullo modo ad nihilum redigi poterit; sin autem una vel altera lens fuerit concava, tum utique fieri poterit, ut haec quantitas Λ evanescat.

SCHOLION 1

36. Haec igitur formula praecipue litteris $\lambda,\ \lambda',\ \lambda''$ etc. convenienter definiendis inservit, quandoquidem reliquae litterae iam per conditiones praecedentes plerumque suas determinationes adipiscuntur. Meminisse autem oportet quamlibet lentem sibi adiunctum habere numerum λ, qui quidem uniate minor esse nequit, ex quo cum binis distantiis determinatricibus ambae facies determinantur. Supra autem formulae pro radiis facierum iam sunt datae, sed eas in calculi commodum hic aliquantisper mutatas exhibeamus. Exemplo sit lens prima, cuius distantiae determinatrices sunt a et α, numerus autem iis adiungendus $=\lambda$, ex quibus binae eius facies supra [Lib. I, § 55] ita sunt definitae, ut sit

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{a\alpha}{\alpha\rho+a\sigma\pm\tau(a+\alpha)\sqrt{\lambda-1}}\\ \text{posterioris} = \frac{a\alpha}{a\rho+\alpha\sigma\mp\tau(a+\alpha)\sqrt{\lambda-1}}.\end{cases}$$

Cum autem sit $\alpha = Aa$, fient istae formulae:

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{Aa}{A\rho+\ \sigma\pm\tau(1+A)\sqrt{\lambda-1}}\\ \text{posterioris} = \frac{a\alpha}{\rho+\ A\sigma\mp\tau(1+A)\sqrt{\lambda-1}}.\end{cases}$$

Dividantur nunc numeratores et denominatores utriusque fractionis per $1+A$, et cum sit $\frac{A}{A+1}=\mathfrak{A}$ ideoque $\mathfrak{A}a = p$ et $\frac{1}{1+A}=1-\mathfrak{A}$, nostrae formulae abibunt in sequentes:

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{p}{\sigma-\mathfrak{A}(\sigma-\rho)\pm\tau(1+A)\sqrt{\lambda-1}}\\ \text{posterioris} = \frac{p}{\rho+\mathfrak{A}(\sigma-\rho)\mp\tau(1+A)\sqrt{\lambda-1}},\end{cases}$$

ubi litterae ρ, σ et τ ex ratione refractionis, quae cuilibet lenti convenit, sunt desumendae, pariter atque litterae μ et ν, uti in primo libro [§ 55] ostendimus. Ne autem opus habeamus eas inde depromere, tabulam ibi [Lib. II, § 15] datam hic adiungamus:

n	ρ	σ	τ	μ	ν	$\mu\nu$
1,50	0,2858	1,7143	0,9583	1,0714	0,2000	0,2143
1,51	0,2653	1,6956	0,9468	1,0420	0,2065	0,2151
1,52	0,2456	1,6776	0,9358	1,0140	0,2129	0,2159
1,53	0,2267	1,6601	0,9252	0,9875	0,2196	0,2168
1,54	0,2083	1,6434	0,9149	0,9622	0,2260	0,2176
1,55	0,1907	1,6274	0,9051	0,9381	0,2326	0,2182
1,56	0,1737	1,6119	0,8956	0,9151	0,2393	0,2192

1,57	0,1573	1,5970	0,8864	0,8932	0,2461	0,2199
1,58	0,1414	1,5827	0,8775	0,8724	0,2529	0,2206
1,59	0,1259	1,5689	0,8689	0,8525	0,2597	0,2214
1,60	0,1111	1,5555	0,8607	0,8333	0,2666	0,2221

SCHOLION 2

37. His principiis praemissis facile intelligitur, quomodo hanc de microscopiis doctrinam tractari et in sectiones subdividi conveniat. Primo scilicet microscopia simplicia, quae unica constant lente, contemplabimur idque duplici modo, prout huius lentis crassities negligitur vel eius ratio in calculo habetur. Deinde tria genera microscopiorum compositorum considerabimus, pro uti in telescopiis fecimus; in primo scilicet genera nulla prorsus occurret imago realis seu omnes litterae *P, Q, R* etc. erunt positivae; in secundo autem genera unica occurret imago realis ideoque unica illarum litterarum negativum habebit valorem, quaecunque ea fuerit; in tertio denique genere duae imagines reales locum habebunt sicque binae illarum litterarum, quaecunque eae fuerint, valores sortientur negativos. Plures autem imagines reales introducere prorsus foret superfluum. Notandum vero est tam microscopia simplicia quam composita primi et tertii generis obiecta situ erecto esse repraesentatura, dum microscopia composita secundi generis ea situ inverso referent. Quamobrem haec tractatio quatuor sequentibus sectionibus absolvetur.