CONCERNING TELESCOPES OF THE THIRD KIND,

FOR WHICH OBJECTS

SHALL AGAIN BE REPRESENTED SITUATED ERECT

CHAPTER I

SIMPLER TELESCOPES OF THE THIRD KIND

PREPARED FROM A SINGLE KIND OF GLASS

PROBLEM 1

294. *To construct the simplest telescope of this kind, which depends on three lenses only, so that the object may be represented increased in a given ratio and situated erect.*

We may put as always for the two intervals which arise here the fractions $\frac{\alpha}{b}=-P$ and $\frac{\beta}{c}=-Q,$ and since here the two images are had as being real, of which the one falling in the first interval is inverted, and truly the other in the second interval falling erect, thus so that the radius of the one shall be $=\alpha\Phi,$ truly the radius of the other shall be $=B\alpha\Phi,$ both letters P and Q must be negative, from which we may put $-P=k$ and $-Q=k',$ thus so that the magnification shall be $m=kk'$. Hence our [*i.e.* positive] terms themselves will be had thus :

$$b=\frac{\alpha}{k},\ \ \beta=\frac{B\alpha}{k}\ \text{ and }\ c=\frac{B\alpha}{kk'}$$

and the focal lengths

$$p=\alpha,\ \ q=\frac{\mathfrak{B}\alpha}{k}\ \text{ and }\ r=\frac{B\alpha}{kk'}=\frac{B\alpha}{m},\left[\begin{array}{l}\text{assuming the second image falls on the}\\ \text{focal point of the third lens}\end{array}\right]$$

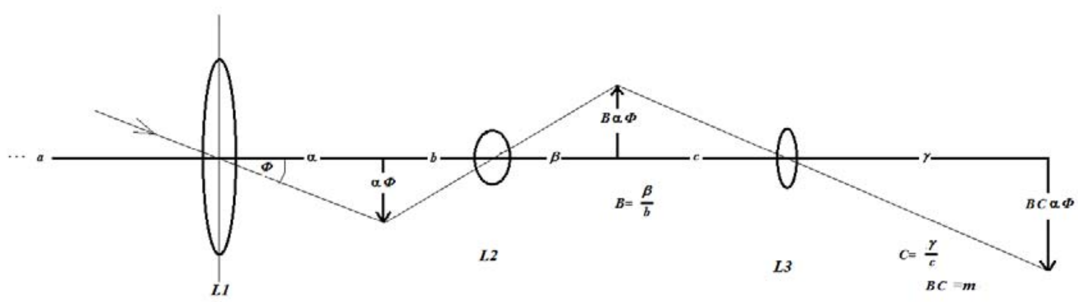
then truly the two intervals

$$\alpha+b=\alpha\left(1+\tfrac{1}{k}\right),\ \ \beta+c=\frac{B\alpha}{k}\left(1+\tfrac{1}{k'}\right),$$

which themselves are positive, if indeed there must become $B>0$ and thus also $\mathfrak{B}$.

[We may review again how these results arise from simple applications of the thin lens formula for successive lenses in an iterative manner, starting with the objective lens $L1$ written in the customary form $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$: an inverted image of a distant object is produced at the focal point of $L1$ at the distance α from this lens, whose purpose is to collect as much light as possible from a distant object with points on the object and image in a one to one correspondence as far as possible, and for which the rays are paraxial, thus designed to produce as sharp an image as possible, where we ignore all the possible deficiencies in the image ; this inverted image becomes the object of the second lens $L2$, at the distance $u = b$, for which we assume an erect image is formed at $v = \beta$, and for which lens the focal length f' is found from $\frac{1}{b} + \frac{1}{\beta} = \frac{1}{f}$, and the magnification $B = \frac{\beta}{b}$, and hence $f' = \frac{b\beta}{b+\beta} = \frac{\beta}{1+\frac{\beta}{b}} = \frac{B}{1+B}b = \mathfrak{B}b$; subsequently the image of the second lens becomes the object of the third lens $L3$, and we find $f'' = \frac{c\gamma}{c+\gamma} = \frac{\gamma}{1+\frac{\gamma}{c}} = \frac{C}{1+C}c = \mathfrak{C}c$. Hence the whole magnification m of the latter two lenses becomes BC, and we can write $m = BC = \frac{\beta\gamma}{bc}$, which generally does not agree with the above, but satisfies the case where the image is formed at the focal point of the third lens, which hence transmits a parallel beam to the eyepiece, being the situation of the greatest symmetry.

A complete discussion of these matters can be found in Ch. 7, § 411, Book I; a word of caution is necessary, however, as Euler had the occasional habit of choosing the same symbol for different quantities as defined here in the diagram and in parts of his *Dioptricae*. A notable feature in Euler's later optical works is a complete lack of ray diagrams, due presumably to his blindness, but which has the effect of making the purely algebraic proceedings occasionally less than transparent. Here we insert a diagram which may help explain what is going on, merely putting in place the various images, where the positions are found algebraically, and not from the convergence of rays geometrically:



Certainly Φ may be taken as the objective field of view, while the apertures of the following lenses are adjusted successively to enlarge a smaller part of this field, to become the apparent field of view. Finally, if the field of view Φ be magnified m times, or increased by an amount $(m-1)\Phi$, the apparent field as viewed shall be diminished in the same ratio, and this to be contained in the apertures of the first two lenses expressed as positive angles which are fractions of the possible maximum value ξ, and Φ is a fraction of this amount, as Euler now asserts.]

For the apparent field of view, since its mean radius shall be $\Phi = \frac{-\pi+\pi'}{m-1}$, we may put

$$\pi = -i\xi \quad \text{and} \quad \pi' = \xi,$$

with ξ denoting the maximum value, which the letters π and π' can take, and i a fraction less than unity, there will be

$$\Phi = \frac{i+1}{m-1}\xi$$

and hence for the position of the eye there will become:

$$O=\frac{\pi'}{\Phi}\cdot\frac{r}{m}=\frac{m-1}{i+1}\cdot\frac{B\alpha}{mm},$$

which distance also is essentially positive. With these in place the equations for the letters π, π' given above will give

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi}=\frac{\alpha}{b}=k \text{ or } \mathfrak{B}\cdot\frac{-i(m-1)}{i+1}=k+1,$$

from which

$$i=\frac{-k-1}{k+1+(m-1)\mathfrak{B}},$$

which value must be less than unity. Therefore since the value of *i* by necessity hence shall be negative and less than unity, the radius of the field of view will be

$$\Phi=\frac{\mathfrak{B}\xi}{k+1+(m-1)\mathfrak{B}},$$

which therefore certainly is smaller than $\frac{\xi}{m-1}$, where *k* is greater and $\mathfrak{B}$ is smaller. Therefore so that we may acquire a greater field of view, it is required to realize, that the least value of the letter *k* be applied, and truly the maximum value of the letter $\mathfrak{B}$ may be agreed on as well; but since there shall be $B=\frac{\mathfrak{B}}{1-\mathfrak{B}}$ and there must become $B>0$, hence it is clear $\mathfrak{B}$ cannot be increased beyond unity. But in the case, where there becomes $\mathfrak{B}=1$, there becomes $\Phi=\frac{\xi}{k+m}$. Then truly on account of $B=\infty$ the length of the tube may become infinite. Truly the diminution of the number *k* to become so small may add to the increase of the field, we may see now also, whether the color margin may be able to be removed, as in the end there must become [§ 49]

$$0=\frac{\pi}{\Phi}\cdot\frac{b}{p}+\frac{\pi'}{\Phi}\cdot\frac{c}{Bp}, \quad 0=-i\cdot\frac{1}{k}+\frac{1}{kk'},$$

which equation on account of $i<0$ in no manner can remain in place; from which these kind of telescopes will be troubled greatly by the flaw of the colored margin. Moreover we will have this equation for the radius of confusion [§ 42]:

$$\frac{\mu m x^3}{\alpha^3}\left(\lambda+\frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right)+\frac{\lambda''}{B^2 m}\right)=\frac{1}{k^3},$$

[It is required to be observed the letter *k* used on the right hand side of the equation differs from the quantity *k* in the left hand side of the equation. Noted by Emil Cherbuliez *O.O*. edition]
from which there is deduced :

$$\alpha = kx\sqrt[3]{\mu m\left(\lambda + \frac{\lambda'}{\mathfrak{B}^3 k} + \frac{\lambda''}{B^3 m} + \frac{\nu}{B\mathfrak{B}k}\right)},$$

which on taking $x = \frac{m}{50}$ in. and [outside the root] $k = 50$ will be changed into this value:

$$\alpha = m\sqrt[3]{\mu m\left(\lambda + \frac{\lambda'}{\mathfrak{B}^3 k} + \frac{\lambda''}{B^3 m} + \frac{\nu}{B\mathfrak{B}k}\right)},$$

in which expression since all the terms shall be positive, there is no doubt, why the focal distance α may not become much greater than in the case of two lenses.

COROLLARY 1

295. Since now it may be noticed, if $\mathfrak{B}$ may be taken $=1$, the length of the instrument to increase indefinitely and thus $\mathfrak{B}$ must be taken less than unity, the following term in the equation will be increased equally greatly and finally, by which the distance α will be augmented.

COROLLARY 2

296. But if we shall wish to remedy this inconvenience by increasing the number *k,* then the apparent field of view may be restricted.

SCHOLIUM 1

297. Therefore there is no doubt, why this kind of telescope may not be completely rejected, not only, because it shows an exceedingly small field of view and that a very long tube may be required, but also on account of that particular reason, so that the representation of the colored margin shall become messy; nor also at any time do we find a use for telescopes of this kind. Yet meanwhile we may propose a certain case in the following example.

EXAMPLE

298. If there may be put $\mathfrak{B} = \frac{4}{5}$ and $k = 2$, to describe a telescope of this kind for some magnification *m.*
Therefore since here there shall become $B = 4$, the terms will become

$$b = \tfrac{1}{2}\alpha, \quad \beta = 2\alpha, \quad c = \frac{4\alpha}{m}$$

and the focal distances

$$p = \alpha, \quad q = \tfrac{2}{5}\alpha \text{ and } r = \frac{4\alpha}{m}$$

and the intervals

$$\alpha + b = \tfrac{3}{2}\alpha, \quad \beta + c = 2\alpha + \tfrac{4\alpha}{m},$$

the sum of which gives the length of the tube $\frac{7}{2}\alpha + \frac{4\alpha}{m}$.

Then truly there is found $i = \frac{-15}{11+4m}$ and the radius of the field $\Phi = \frac{4\xi}{11+4m}$ or in terms of the measure of the angle $\Phi = \frac{3437}{11+4m}$ minutes of arc, which is not more of less than the ordinary field. Truly for the position of the eye there will be $O = \frac{4m+11}{mm} \cdot \alpha$. And finally we will have for the focal length α

$$\alpha = m\sqrt[3]{\mu m\left(\lambda + \tfrac{\lambda' \cdot 125}{128} + \tfrac{\lambda''}{4m} + \tfrac{5\nu}{32}\right)},$$

where there is approximately $\mu = 1$ and $\nu = \frac{1}{5}$; whereby, if the minimum values of the letters λ, λ', λ'' may be attributed [*i.e.* suitable constants are adopted for the lens], namely 1, there will become

$$\alpha = m\sqrt[3]{m\left(2 + \tfrac{1}{128} + \tfrac{1}{64m}\right)},$$

$$\alpha = m\sqrt[3]{\left(\left(2 + \tfrac{1}{128}\right)m + \tfrac{1}{64}\right)} \text{ in.};$$

hence, if there shall be $m = 25$, there will become

$$\alpha = 25\sqrt[3]{50\tfrac{27}{128}} = 92,23 \text{ dig.}$$

and hence the total length of the tube will be $= 340$ in. $= 28$ ft. 4 in., which length on account of the magnification generally is so great, so that it shall not be allowed in practice, even if the defect of the colored margin may not be present.

SCHOLIUM 2

299. Therefore since here nothing of practical use can be extracted and this most simple kind must be rejected completely, we may progress to simpler kinds which evidently arise, if three lenses may be added in addition to as single fourth lens, from which various kinds will arise, provided this new lens may be put in place either between the objective and the first image, between the first and the last image, or between this last lens and the eyepiece lens; therefore which cases it will be convenient to set out here separately.

PROBLEM 2

300. *If a new lens may be put between the objective lens and the first image, to investigate the nature of these telescopes and to describe the construction of these.*

SOLUTION

Since here there shall be four lenses, as always we may put in place three fractions :

$$\tfrac{\alpha}{b} = -P, \quad \tfrac{\beta}{c} = -Q \text{ and } \tfrac{\gamma}{d} = -R,$$

and since no image falls in the first interval, P will retain a positive value, truly the remaining two Q and R will become negative.

Whereby $Q = -k$ and $R = -k'$, so that by multiplication there may become $m = Pkk'$ and our terms shall be

$$b = -\tfrac{\alpha}{P}, \; c = -\tfrac{B\alpha}{Pk}, \; d = -\tfrac{BC\alpha}{Pkk'}, \; \beta = -\tfrac{B\alpha}{P}, \; \gamma = -\tfrac{BC\alpha}{Pk};$$

$$p = \alpha, \; q = -\tfrac{B\alpha}{P}, \; r = -\tfrac{BC\alpha}{Pk}, \; s = -\tfrac{BC\alpha}{m},$$

from which the intervals arise :

$$\alpha + b = \alpha\left(1 - \tfrac{1}{P}\right), \; \beta + c = -\tfrac{B\alpha}{P}\left(1 + \tfrac{1}{k}\right), \; \gamma + d = -\tfrac{BC\alpha}{Pk}\left(1 + \tfrac{1}{k'}\right)$$

and thus it is apparent $B\alpha$ ought to be negative so that also $BC\alpha$, and thus C must be positive ; from which, if $\alpha > 0$, there must become $P > 1$, $B < 0$ and $C > 0$, but if $\alpha < 0$, there must become $P < 1$, $B > 0$ and $C > 0$.

Now, since for the apparent field of view there shall be

$$\Phi = \tfrac{-\pi + \pi' - \pi''}{m-1},$$

there may be put

$$\pi = -\omega\xi, \quad \pi' = i\xi, \quad \pi'' = -\xi,$$

so that there shall be

$$\Phi = \tfrac{\omega + i + 1}{m-1}\xi = M\xi$$

with M being $= \tfrac{\omega + i + 1}{m-1}$. And it follows at once for the position of the eye :

$$O = \tfrac{\pi''}{\Phi} \cdot \tfrac{s}{m} = -\tfrac{1}{M} \cdot \tfrac{BC\alpha}{mm},$$

which distance is now positive by the above conditions. Moreover the equations for the letters π given above produce:

$$\frac{\mathfrak{B}\pi}{\Phi}-1=-\frac{\mathfrak{B}\omega}{M}-1=-P,\ \frac{\mathfrak{C}i}{M}+\frac{\omega}{M}+1=-Pk,$$

from which there is deduced

$$\omega=\frac{(P-1)(i+1)}{\mathfrak{B}(m-1)-P+1},$$

so that $\mathfrak{B}$ may remain indefinite, and

$$\mathfrak{C}=\frac{-(1+Pk)M-\omega}{i};$$

which quantity since it may be positive, there must be either i negative, or if i shall be positive, there must be

$$-(1+Pk)M-\omega\ >0\ \text{ or }\ -(1+Pk)\left(\frac{\omega+i+1}{m-1}\right)-\omega\ >0$$

or

$$-\omega(m+Pk)-(1+Pk)(i+1)>0,$$

from which it is apparent the fraction ω must be negative, hence so the apparent field of view may be diminished.

Now we may see, whether the colored margin to be removed or we may be able to satisfy this equation:

$$0=+\omega\cdot\frac{1}{P}-\frac{i}{Pk}+\frac{1}{Pkk''},$$

from which we gather

$$0=\omega-\frac{i}{k}+\frac{1}{kk'},\ \text{and thus}\ k'=\frac{-1}{k\omega-i}=\frac{1}{i-k\omega},$$

which value must be positive and thus $k\omega-i<0$, concerning which we will see henceforth. Now we will consider at this point the equation for the removal of the confusion, which will be shown in the following manner:

$$\alpha=kx\sqrt[3]{\mu m\left(\lambda-\frac{1}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right)-\frac{1}{B^3\mathfrak{C}Pk}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu}{C}\right)-\frac{\lambda'''}{B^3Cm}\right)},$$

for which expression we assume at this point $x=\frac{m}{50}$ in. and [outside the root sign] $k=50$.

COROLLARY 1

301. For the letters ω and i requiring to be estimated , whether they may have positive values, these two formulas are required to be considered:

$$\text{I.}\quad \mathfrak{C} = \frac{-(1+Pk)M-\omega}{i},$$
$$\text{II.}\quad k' = \frac{1}{i-k\omega},$$

from the first of which it is clear both the letters *i* and ω cannot both be taken positive, since otherwise $\mathfrak{B}$ may become negative, since the letter still must have a positive value. It is evident the second cannot happen, so that there shall be $\omega > 0$ and $i < 0$, since otherwise k' will become negative.

COROLLARY 2

302. From these two cases it follows that at no time can the letter ω be positive, which condition can be enunciated thus, so that the second lens must always have a diminished field of view.

COROLLARY 3

303. Therefore since ω must be negative always, there may be put $\omega = -\zeta$, so that there shall become

$$B\zeta = (1-P)M \quad \text{and} \quad M = \frac{1+i-\zeta}{m-1}.$$

Indeed our formulas, by necessity will be positive :

$$\text{I.}\quad \mathfrak{C} = \frac{-(1+Pk)M+\zeta}{i},$$
$$\text{II.}\quad k' = \frac{1}{i+k\zeta},$$

from which, if *i* shall be a positive fraction, there must be $\zeta > (1+Pk)M$. But if *i* shall be a negative fraction, on putting $i = -y$, by the first condition there must become $\zeta < (1+Pk)M$ and likewise $\zeta > \frac{y}{k}$.

COROLLARY 4

304. Besides also it is evident that $\zeta = -\omega$ at no time can vanish ; if indeed there shall be $i > 0$, there must become $\zeta > (1+Pk)M$. But if there shall be $i < 0$ or $i = -y$, there must be $\zeta > \frac{y}{k}$.

COROLLARY 5

305. Since in the case $i=-y$ we have found a twofold condition, the former $\zeta<(1+Pk)M$ and the latter $\zeta>\frac{y}{k}$, from a comparison of these it is necessary, that there shall be $(1+Pk)M>\frac{y}{k}$ or $y<(1+Pk)kM$.

SCHOLlUM

306. But so many different foremost cases thus can be found, so that in the solution of the problem it is not defined, whether the objective lens may have its own focal length α to be either positive or negative. But in each case it happen usually, if indeed nothing other can be assumed about the letter P, except that it shall be positive and that its value may be increased from a cipher as far as to infinity.

But while the letter P may be contained within the limits 0 and 1, α must have a negative value, or the objective lens will be concave and the letter B and likewise $\mathfrak{B}$ will be positive; from which there becomes $\zeta=\frac{(P-1)M}{\mathfrak{B}}$ and thus is positive. But if there may be taken $P=1$, in which case the two first lenses are joined in contact to each other, there becomes $\zeta=0$, which case, as we have seen, is excluded entirely, thus so that the objective lens shall be unable to be doubled. But if P shall be greater than unity, by necessity α shall be positive or the objective lens convex; from which B becomes negative and nor hence is $\mathfrak{B}$ defined. But since we know ω to be negative or ζ positive, it is apparent on account of $\zeta=-\frac{(P-1)M}{\mathfrak{B}}$ the letter $\mathfrak{B}$ must be negative, and hence again it is concluded $-B$ to be less than unity. Finally if P shall be an infinite number, the second lens may be put in place of the first image, and from its focal length q it is concluded

$$\mathfrak{B}=-P\cdot\frac{q}{\alpha}=-\infty \text{ and hence } B=-1;$$

and thus incidentally we have considered all the cases for the letter *P,* but which now deserve to be considered with more care. But before everything else it will be agreed to be unable to take $P=0$, since now the first interval will become infinite, unless the distance α shall be infinitely small, but that would become equally absurd, since the first lens must allow a definite aperture.

1. Establishing the case where $P<1$

307. Now for this case we notice to become $\alpha<0$; lest this negative may disturb us, we may put $\alpha=-a$ and there will become

$$b=\frac{a}{P},\quad c=\frac{Ba}{Pk},\quad d=\frac{BCa}{m},\quad \beta=\frac{Ba}{P},\quad \gamma=\frac{BCa}{Pk},$$

from which it is apparent both the letters B and C must be positive; for which the Germanic letters $\mathfrak{B}$ et $\mathfrak{C}$ not only will be positive too, but also less than one; whereby, since there shall be $\mathfrak{B}\zeta=(1-P)M$, clearly it follows to become $\zeta>(1-P)M$. Thence on account of

$$\mathfrak{C}=\frac{-(1+Pk)M+\zeta}{i} \text{ and } k'=\frac{1}{i+k\zeta}$$

not only must there be $\frac{-(1+Pk)M+\zeta}{i}>0$, but also $\frac{-(1+Pk)M+\zeta}{i}<1$; where so that it may be explained more clearly, it shall be agreed to examine these two cases.

1. If i shall be positive, from the value $\mathfrak{C}$ we arrive at these conditions:

$$\zeta>(1+Pk)M \text{ and } \zeta<(1+Pk)M+i;$$

moreover the condition of the letter k' thus is fulfilled at once. But since now we have found $\zeta>(1-P)M$, thence now it is apparent there must be

$$(1+Pk)M+i>(1-P)M \text{ and thus } i>-P(k+1)M\,;$$

so that it is true always, provided i shall be positive, as we have presumed.

2. If i shall be negative, there may be put $i=-y$ and there will become

$$\mathfrak{C}=\frac{(1+Pk)M-\zeta}{y},\quad k'=\frac{1}{k\zeta-y}.$$

Thence therefore these conditions follow:

$$\zeta<(1+Pk)M,\quad \zeta>(1+Pk)M-y, \text{ hence truly } \zeta>\frac{y}{k};$$

but we have found above $\zeta>(1-P)M$, from which there follows to become

$$(1+Pk)M>\frac{y}{k} \text{ or } y<(1+Pk)kM.$$

Therefore in this same case, where $P<1$, the fraction i will be able to be taken both positive as well as negative, and if the positive may be accepted, its value may be restricted without limit. Whereby, since i may not exceed one, at once it will be able with hesitation to put $i=1$, thus so that for the apparent field of view $\Phi=\frac{2-\zeta}{m-1}\cdot\xi$ provided ξ may not exceed unity.

But there is no proposal to take *i* negative, since then the field may be diminished exceedingly.

II. Establishing the case where $P>1$

308. Since here α is a positive quantity and thus with *b* negative, *B* must be negative, but *C* positive as before. Then also we have seen $\mathfrak{B}$ to be negative and thus $-B<1$; from which there becomes $\zeta=\frac{(1-P)M}{\mathfrak{B}}$ and thus positive, where it may only be noted $\mathfrak{B}$ must not be taken so small that ζ will exceed unity. Then there is had

$$\mathfrak{C}=\frac{-(1+Pk)M+\zeta}{i} \text{ and } k'=\frac{1}{i+k\zeta};$$

from which formulas clearly the same follow as have been advanced in the preceding case; from which it is seen also $i=1$ can be put in place, provided that from the value for ζ given before there shall be

$$\frac{(1-\mathrm{P})}{\mathfrak{B}}>1+Pk \text{ or } -\mathfrak{B}<\frac{P-1}{Pk+1} \text{ and } -\mathfrak{B}>\frac{(P-1)M}{(1+Pk)M+1}.$$

III. Establishing the case where $P=\infty$

309. Therefore in this case, as now we have observed above, there will be

$$B=-1 \text{ and } \mathfrak{B}=-\frac{Pq}{\alpha}.$$

But now it is evident there must be put in place $k=0$, thus yet, so that there shall be $Pk=\theta$, from which the terms are

$$b=0,\ \beta=0,\ c=\frac{\alpha}{\theta},\ \gamma=\frac{C\alpha}{\theta},\ d=\frac{C\alpha}{m}.$$

From which, since there shall be $\mathfrak{B}\zeta=(1-P)M$, there will be had now $\zeta=\frac{M\alpha}{q}$, and $q=\frac{M\alpha}{\zeta}$.

Then our two formulas will be

$$\mathfrak{C}=\frac{-(1+\theta)M+\zeta}{i} \text{ and } k'=\frac{1}{i},$$

where, since nothing may stand in the way, why we may not put $i=1$, thus so that in the case $k'=1$ and $Pk=\theta=m$, there shall become $\mathfrak{C}=-(1+m)M+\zeta$; from which value these limits are deduced :

$$\zeta>(1+m)M, \quad \zeta<(1+m)M+1;$$

but truly there becomes

$$M=\tfrac{2-\zeta}{m-1} \text{ and thus } \zeta>\tfrac{(1+m)(2-\zeta)}{m-1} \text{ and hence } \zeta>\tfrac{m+1}{m},$$

which value, even if it may exceed unity, yet can be used in practice, provided the letter ξ may be diminished in the same ratio, thus so that $\zeta\xi$ may not exceed the value $\frac{1}{4}$, if indeed $\frac{1}{4}$ may be taken for the maximum aperture. But if we may assume $i=\frac{1}{2}$, there shall be produced $k'=2$ and hence $m=2\theta$ or $\theta=\frac{m}{2}$ and thus we will have

$$\zeta>\left(1+\tfrac{1}{2}m\right)M \text{ and } \zeta<\left(1+\tfrac{1}{2}m\right)M+\tfrac{1}{2};$$

but since there is $M=\frac{3-2\zeta}{2(m-1)}$, the first condition gives

$$\zeta>\left(1+\tfrac{1}{2}m\right)\left(\tfrac{3-2\zeta}{2(m-1)}\right) \text{ or } \zeta>\tfrac{2+m}{2m},$$

and thus much more than $\zeta>\frac{1}{2}$. From which it is apparent the apparent field to be diminished more on account of the value ζ than from the value i to be increased and thus since always to become a little less than in the common astronomical tubes. Now above we have observed the place of such lenses to be avoided in practice.

IV. Establishing the particular case arising where $i=0$.

310. Since there shall be $i=0$ and $\mathfrak{C}$ may not exceed unity, on account of

$$\mathfrak{C}i=-(1+Pk)M+\zeta$$

there will be

$$\zeta=(1+Pk)M \text{ and hence } Pk=\tfrac{\zeta}{M}-1;$$

but $k'=\frac{1}{k\zeta}$; on account of $Pkk'=m$ there will become $Pk=mk\zeta$ and thus $P=m\zeta$. Whereby this value found for Pk put equal to this equation will give

$$mk\zeta=\tfrac{\zeta}{M}-1$$

and hence

$$k=\frac{\zeta}{Mm}-\frac{1}{m\zeta}$$

from which there is had again

$$k'=\frac{Mm}{\zeta-M};$$

truly since there is $M=\frac{1-\zeta}{m-1}$, there will become

$$k=\frac{m-1}{(1-\zeta)m}-\frac{1}{m\zeta}=\frac{m\zeta-1}{m(1-\zeta)\zeta},\quad k'=\frac{m(1-\zeta)}{m\zeta-1}$$

and

$$P=m\zeta \text{ and } Pk=\frac{m\zeta-1}{1-\zeta};$$

since which values by no means shall depend on $\mathfrak{C}$, we have gained this significant advantage, that the letter C may be chosen as we wish, and thus we will be able to effect, that the latter determinable distances and the latter lenses themselves, which up to this point generally have been found to be exceedingly small, now may be able to be found of a given magnitude, in which certainly the maximum convenience is present ; which finally so that it may extend to the letters $\mathfrak{B}$ and B, these two cases will be required to be considered, according as $P=m\zeta$ were either less than or greater than unity.

I. There shall be $m\zeta<1$ or $\zeta<\frac{1}{m}$ and there will be had

$$\mathfrak{B}=\frac{(1-m\zeta)M}{\zeta}=\frac{(1-m\zeta)(1-\zeta)}{\zeta(m-\zeta)};$$

but here we have seen $\mathfrak{B}$ must be positive and less than one; whereby in this case, where

$\zeta<\frac{1}{m}$ on account of $\mathfrak{B}=\frac{(1-m\zeta)(1-\zeta)}{(m-1)\zeta}$, there must become

$$(1-m\zeta)(1-\zeta)<(m-1)\zeta \text{ or } m\zeta^2-2m\zeta+1<0;$$

from which it is deduced ζ must be taken within the limits $\frac{1}{m}$ and $\frac{1}{2m}$.

But since by necessity the letters k and k' shall be positive, according to this necessity it is required, that there shall be $m\zeta>1$ or $\zeta>\frac{1}{m}$: on account of which condition the first case ought to be excluded at once.

II. Therefore there shall be $P(=m\zeta)>1$ or $\zeta>\frac{1}{m}$, just as the values k and k' postulate, and for which we must have recourse for the second case ; for since again for which there shall be

$$\mathfrak{B}=\frac{(1-m\zeta)(1-\zeta)}{(m-1)\zeta}$$

and likewise it may be observed $\mathfrak{B}$ must be negative without any other condition, except that there must be $\zeta<1$, as indeed the account of the absolute field of view demands, thus so that now it may be contained within the limits 1 and $\frac{1}{m}$, moreover it is clear how to arrange, that ζ will exceed the minimum limit $\frac{1}{m}$. According to which it may be seen to be worth the effort to add two examples, in the first of which ζ may be taken closer to the limit $\frac{1}{m}$, in the other 1 may be taken for the latter limit.

EXAMPLE 1

311. For the last case, where $i=0$, if there may be put $\zeta=\frac{2}{m}$, to describe the telescope thence arising.
Therefore in this case we will have

$$\mathfrak{B}=\frac{-(m-2)}{2(m-1)} \text{ and } B=\frac{-(m-2)}{3m-4}.$$

Again

$$P=2,\ \ k=\frac{m}{2(m-2)},\ \ k'=m-2,\ M=\frac{m-2}{m(m-1)},$$

from which our determinable distances on account of positive α will be

$$b=\frac{-\alpha}{2},\ \ c=\frac{(m-2)^2}{m(3m-4)}\alpha,\ \ d=\frac{m-2}{m(3m-4)}C\alpha,$$

$$\beta=\frac{m-2}{2(3m-4)}\alpha,\ \ \gamma=\frac{(m-2)^2}{m(3m-4)}C\alpha$$

and the focal lengths

$$p=\alpha,\ \ q=\frac{m-2}{4(m-1)}\alpha,\ \ r=\frac{(m-2)^2}{m(3m-4)}\mathfrak{C}\alpha,\ \ s=\frac{m-2}{m(3m-4)}(m-2)C\alpha.$$

Then truly the distances between the lenses :

$$\alpha+b=\frac{1}{2}\alpha,\ \ \beta+c=\frac{(m-2)(3m-4)}{2m(3m-4)}\alpha=\frac{m-2}{2m}\alpha,\ \ \gamma+d=\frac{(m-1)(m-2)}{m(3m-4)}C\alpha$$

and the distance of the eye

$$O=\frac{m-1}{m(3m-4)}C\alpha$$

and the radius of the field

$$\Phi = \frac{m-2}{m(m-1)} \cdot \xi \;;$$

which if desired in the measure of angles , can be taken to be
$\xi = 859$ min. on account of $\xi = \frac{1}{4}$.

Finally the focal length α of the objective lens must be defined from the formula given in § 20, where it is to be observed the coefficient of λ' to become approx. 4, and the coefficient of λ'' always to be greater than 27; which terms since they all shall be positive, it is evident for a greater value to be found for α, thus so that these telescopes may emerge rather long

EXAMPLE 2

312. For the latter case, where $i = 0$, if there may be taken $\zeta = \frac{1}{2}$, to describe the telescope thence arising.
Therefore in this case there will be

$$\mathfrak{B} = \frac{-(m-2)}{2(m-1)} \text{ and } B = \frac{-(m-2)}{3m-4},$$

$$P = \frac{m}{2}, \quad k = \frac{2(m-2)}{m}, \quad k' = \frac{m}{m-2}, \quad M = \frac{1}{m(m-1)},$$

from which the determinable distances :

$$b = -\frac{2\alpha}{m}, \quad c = \frac{\alpha}{m(3m-4)}, \quad \beta = \frac{2(m-2)\alpha}{m(3m-4)}, \quad \gamma = \frac{C\alpha}{3m-4}, \quad d = \frac{m-2}{(3m-4)m} C\alpha \;;$$

and the focal lengths

$$p = \alpha, \quad q = \frac{m-2}{m(m-1)}\alpha, \quad r = \frac{\mathfrak{C}\alpha}{3m-4}, \quad s = \frac{m-2}{(3m-4)m} \;;$$

and the lens separations

$$\alpha + b = \frac{m-2}{m}\alpha, \quad \beta + c = \frac{\alpha}{m} = \frac{m-2}{2m}\alpha, \quad \gamma + d = \frac{2(m-1)C\alpha}{m(3m-4)}$$

and

$$O = \frac{2(m-1)(m-2)C\alpha}{mm(3m-4)};$$

now truly the radius of the field of view will become only

$$\Phi = \frac{430}{m-1} \text{ minutes.}$$

Moreover in the formula for defining the angle α it is required to be noted the coefficient λ' to become $\frac{16}{m}$, truly that of $\lambda'' > \frac{27}{m\mathfrak{C}^3}$, if indeed the magnification shall be very large ; from which it is clear a much smaller value to be summoned for the value α, thus so that hence suitable telescopes may be obtained, only if the field of view may not be too small.

COROLLARY 1

313. Since for the third lens we may assume both i and $\pi' = 0$, its aperture must be defined from the general formulas [§ 23], of which the radius will be $= \frac{rx}{B\mathfrak{C}\alpha}$, which therefore for the first example will be $\frac{m-2}{m}x$, but for the second $\frac{x}{m-2}$; from which, if there may be assumed $x = \frac{m}{50}$ inch., here the radius will be around $\frac{1}{50}$ inch, therefore which lens must conveniently will hold the location of the diaphragm.

COROLLARY 2

314. If as it were an average between the two examples introduced there may be put in place $\zeta = \frac{1}{\sqrt{m}}$, there will become

$$P = \sqrt{m},\ k = 1 \text{ and } k' = \sqrt{m},$$

again

$$\mathfrak{B} = -\frac{(\sqrt{m}-1)}{\sqrt{m}+1},\ B = \frac{-(\sqrt{m}-1)}{2\sqrt{m}},\ M = \frac{1}{m+\sqrt{m}}$$

and hence

$$b = -\frac{\alpha}{\sqrt{m}},\ \beta = \frac{+(\sqrt{m}-1)\alpha}{2m},\ c = \frac{\sqrt{m}-1}{2m}\alpha,\ \gamma = \frac{\sqrt{m}-1}{2m}C\alpha,\ d = \frac{\sqrt{m}-1}{2m\sqrt{m}}C\alpha,$$

$$\alpha + b = \left(1 - \frac{1}{\sqrt{m}}\right)\alpha,\ \beta + c = \frac{\sqrt{m}-1}{m}\alpha,\ \gamma + d = \frac{\sqrt{m}-1}{2m}C\alpha\left(1 + \frac{1}{\sqrt{m}}\right) = \frac{m-1}{2m\sqrt{m}}C\alpha$$

and the distance of the eye

$$O = +\frac{(m-1)}{2mm}\alpha;$$

whereby the length of the telescope will become

$$\frac{m-1}{m}\left(1 + \frac{1+\sqrt{m}\cdot C}{2m}\right)\alpha$$

and finally the radius of the field of view

$$\Phi = \frac{\xi}{m+\sqrt{m}} = \frac{859}{m+\sqrt{m}} \text{ minutes of arc.}$$

and the radius of the aperture of the third lens

$$= \frac{x}{\sqrt{m}} = \frac{\sqrt{m}}{50} \text{ dig.}$$

SCHOLIUM

315. In a similar manner, where here we have set out the case $i = 0$, also the question in general for any value of i will be able to be resolved; indeed from the equation

$\mathfrak{C}i = -(1+Pk)M + \zeta$ since there may be deduced

$$Pk = \frac{\zeta - \mathfrak{C}i}{M} - 1$$

and because there is $M = \frac{1+i-\zeta}{m-1}$, there will become

$$Pk = \frac{-\mathfrak{C}i(m-1)+m\zeta-i-1}{1+i-\zeta}.$$

Truly on account of $k' = \frac{1}{i+k\zeta}$ there will be also

$$Pk = \frac{m}{k'} = m(i+k\zeta),$$

from which we gather

$$-\frac{\mathfrak{C}i}{M} + \frac{m\zeta-i-1}{1+i-\zeta} = mi + mk\zeta$$

and hence

$$k = -\frac{\mathfrak{C}i}{Mm\zeta} + \frac{m\zeta+mi\zeta-mii-mi-i-1}{(1+i-\zeta)m\zeta},$$

and since there is

$$k + k\zeta = -\frac{\mathfrak{C}i}{Mm} + \frac{m\zeta-i-1}{(1+i-\zeta)m},$$

there will become

$$k' = \frac{m(1+i-\zeta)}{m\zeta-\mathfrak{C}i(m-1)-i-1}$$

and thus

$$Pk = \frac{m\zeta-\mathfrak{C}i(m-1)-i-1}{1+i-\zeta} \text{ and } P = \frac{m}{kk'}$$

because now k' must be a positive quantity, it is necessary, that there shall be

$$m\zeta > \mathfrak{C}i(m-1)+i+1 \;;$$

from which with the calculation made there will always be found to be $P > 1$, thus so that, even if there shall not be $i = 0$, yet the second case mentioned above may have a place in our considerations. But since the hypothesis $i = 0$ has supplied so convenient and concise a resolution, plainly there is no reason, why we may wish to assume the letter i to be either positive or negative, since thence there shall be no expectation of gain for the convenience. But besides the shortening of the calculation there are two convenient factors given to us by this same hypothesis $i=0$, they are of the greatest importance, of which the one, as we have seen, consists in this, so that the letters $\mathfrak{C}$ and C can be chosen arbitrarily by us and in this way the excessive smallness of the ocular lens may be able to be avoided; truly for the other convenience nothing further remains to be considered, therefore so that both the very small aperture may be attributed to the third lens without any detriment either to the field of view or to the clarity, so that with care all the stray light as well may be excluded, other than the regular light passing through the diaphragm.

PROBLEM 3

316. *If a telescope of this kind thus may be assembled from four lenses, so that both the two middle lenses may be put in place between the first and last images, to investigate its nature and describe its construction.*

SOLUTION

Therefore as before with our fractions in place

$$\tfrac{\alpha}{b}=-P,\quad \tfrac{\beta}{c}=-Q \text{ and } \tfrac{\gamma}{d}=-R,$$

here the letters P and R must be negative with Q remaining positive ; whereby if there may be put $P=-k$ and $R=-k'$, so that there shall be $m=Qkk'$, thus our elements themselves will be had:

$$b=\tfrac{\alpha}{k},\ \ \beta=\tfrac{B\alpha}{k},\ \ c=\tfrac{-B\alpha}{Qk},\quad \gamma=\tfrac{-BC\alpha}{Qk},\ \ d=\tfrac{-BC\alpha}{Qkk'}=\tfrac{-BC\alpha}{m}$$

and hence the intervals :

$$\alpha+b=\alpha\left(1+\tfrac{1}{k}\right) \text{ and thus } \alpha>0,$$
$$\beta+c=\tfrac{B\alpha}{k}\left(1-\tfrac{1}{Q}\right), \text{hence } B\left(1-\tfrac{1}{Q}\right)>0,$$
$$\gamma+d=\tfrac{-BC\alpha}{Qk}\left(1+\tfrac{1}{k'}\right), \text{ hence } BC<0.$$

For the apparent field we may put in place :

$$\pi=-\omega\xi,\quad \pi'=+i\xi \text{ and } \pi''=-\xi,$$

so that there may become

$$\Phi=\tfrac{\omega+i+1}{m-1}\cdot\xi=M\xi$$

with

$$M=\tfrac{\omega+i+1}{m-1}$$

being present, and hence in the first place the distance of the eye will be :

$$O=\tfrac{-\pi''}{\Phi}\cdot\tfrac{d}{m}=\tfrac{d}{Mm};$$

[recall that at this time the accommodation of the eye was not understood, to be discovered later by Thomas Young, so that we need not read too much into this formula], then the colored margin will vanish, if there were

$$0=\tfrac{\omega}{P}+\tfrac{i}{PQ}+\tfrac{1}{PQR} \text{ seu } 0=-\tfrac{\omega}{k}-\tfrac{i}{Qk}+\tfrac{1}{Qkk'},$$

[This formula is an algebraic construction, and ignores the physical principles needed to be applied to remove rays of different colors: the optical properties of glasses were not known well enough by Euler for him to make a proper assessment, and indeed eventually five independent types of aberration for lenses were described in 1853 by von Seidl, while the achromatic objective which was found originally by Chester Moor Hall around 1729-33 in Euler's time, in difference to Newton's ideas, and perfected (and patented) by Dollond, was largely ignored or unknown to Euler. See, e.g. Vol. 7, p. 177 of the Encycl. of Physics; there is an interesting article at http://www.mhs.ox.ac.uk/sphaera/index.htm?issue8/articl5],

from which we conclude

$$k'=\tfrac{1}{i+Q\omega} \text{ and } m=\tfrac{Qk}{i+Q\omega};$$

then truly it will be required for the following equations to be considered :

$$-\tfrac{\mathfrak{B}\omega}{M}=1+k,\quad \tfrac{\mathfrak{C}i}{M}+\tfrac{\omega}{M}=-1-Qk$$

or

$$\mathfrak{C}i=-(1+Qk)M-\omega$$

and

$$\mathfrak{B}\omega=-(1+k)M$$

of which a convenient evolution cannot generally be put in place, but it will be convenient to consider more particular cases. Truly two extreme cases will be had, the one, where the lens lies on the same first image, the other truly, where it falls on the second image. For the first evidently there shall become $Q=0$, for the other $Q=\infty$; but between these as if a certain mean deserves especially to be examined arising from the value $Q=1$; which cases themselves henceforth we will set out. Here therefore it remains only for the formula to be added for the removal of the confusion, from which evidently the distance α may be determined [recall that the k outside the root is not the same as the k within the root],

$$\alpha = kx\sqrt[3]{\mu m\left(\lambda + \frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) - \frac{\lambda'}{\mathfrak{B}^3\mathfrak{C}Qk}\left(\frac{\lambda''}{\mathfrak{C}^2 m} + \frac{\nu}{C}\right) - \frac{\lambda'''}{B^3C^3m}\right)}.$$

COROLLARY 1

317. Because we have found $k' = \frac{1}{i+Q\omega}$ on account of $Q > 0$ it is evident both the letters i and ω cannot both be negative. Neither truly also can both be positive ; for if ω were positive, $\mathfrak{B}$ and thus B would be negative and hence on account of $BC < 0$ there would have to be C and thus $\mathfrak{C}$ both positive and hence $\mathfrak{C}i$ would be positive, but it is evident that cannot happen from the value proposed for these given above.

COROLLARY 2

318. Therefore since the letters ω and i neither shall be able to be both positive or both negative, it is necessary for one to be positive and the other negative. If there shall be $\omega > 0$, we have seen in this manner there must be $\mathfrak{B} < 0$ and $B < 0$ and hence $C > 0$. But if there shall be $\omega < 0$, there will be $\mathfrak{B} > 0$; therefore hence nothing is defined about B. Truly from the other equation on putting $\omega = -\zeta$ there will be

$$\mathfrak{C}i = -(1+Qk)M + \zeta$$

from which it is understood, if there were $\zeta > (1+Qk)M$, to become $\mathfrak{C} > 0$, but if there were $\zeta < (1+Qk)M$, to become $\mathfrak{C} < 0$. But it will eventuate in the first case, if there were $1+k > (1+Qk)\mathfrak{B}$ or $\mathfrak{B} < \frac{1+k}{1+Qk}$, truly in the latter case, if $\mathfrak{B} > \frac{1+k}{1+Qk}$; but from this for the latter case since $\mathfrak{C}$ and C shall be negative, B must be positive and thus $\mathfrak{B} < 1$, from which it follows that $Q > 1$.

Establishing the first case where $Q = 0$

319. Since there shall be $Q = 0$, the second interval will become $= -\frac{B\alpha}{Qk} = c$ and thus $\beta = 0$, therefore either $B = 0$ or $k = \infty$. But the first cannot happen; for there would become $\mathfrak{B} = 0$ and q or the focal length of the second lens $= 0$, which is absurd. Hence it remains, that there shall be $k = \infty$, and since there shall be $q = \frac{\mathfrak{B}\alpha}{k}$ there will become $\mathfrak{B} = \frac{kq}{\alpha} = \infty$ and hence $B = -1$. From which it follows on account of $BC < 0$ to become $C > 0$ and $\mathfrak{C} < 1$. Therefore since there shall b $Q = 0$ and $k = \infty$, the

product Qk must be finite ; whereby there may be put in place $Qk = l$, so that there shall be $b = 0$, $\beta = 0$, $c = \frac{+\alpha}{l}$, $\gamma = \frac{C\alpha}{l}$, $d = \frac{C\alpha}{m}$ and again $O = \frac{C\alpha}{Mm^2}$.

Truly the destruction of the colored fringe demands $k' = \frac{1}{i}$, thus so that now i surely shall be a positive fraction and $m = \frac{1}{i}$. Moreover both our fundamental equations will give, in the first place

$$\mathfrak{B}\omega = -kM \text{ or } \frac{kq\omega}{\alpha} = -kM \text{ and thus } \omega = -\frac{M\alpha}{q},$$

truly the latter

$$\mathfrak{C}i = -(1+l)M + \frac{M\alpha}{q};$$

which since it must be positive, there will be required to be $\frac{\alpha}{q} > l+1$ or $q < \frac{\alpha}{1+l}$. Since $\omega < 0$, there may be written $\omega = -\zeta$ and we may maintain the letters i and ζ in the calculation and the will be $l = mi$, $q = \frac{M\alpha}{\zeta}$ and from that $\mathfrak{C}i = -(1+mi)M + \zeta$.
From which, since there shall be $\mathfrak{C} > 0$ and likewise $\mathfrak{C} < 1$, we arrive at these limits:

$$1.\ \zeta > (1+mi)M, \quad 2.\ \zeta < (1+mi)M + i;$$

since now there shall be

$$M = \frac{1+i-\zeta}{m-1},$$

with this value substituted from these limits the following are deduced:

$$1.\ \zeta > \frac{1+mi}{m}$$

and

$$2.\ \zeta < \frac{1+mi}{m} + \frac{(m-1)i}{(1+i)} \text{ or } \zeta < \frac{1+2mi+mii}{m(1+i)},$$

from which, if the letter i may be taken as it pleases and thence ζ is duly assumed, everything will be determined for the telescope ; but where we may be able to pass a better judgement on the field of view, we may substitute each of these limits in turn in place of ζ, and indeed the first limit will give $M = \frac{1}{m}$, truly the other greater limit $M = \frac{1}{m(1+i)}$; between which values the letter M and thus the apparent field of view will be contained.

But for requiring to define the distance α the above formula will adopt this form

$$\alpha = kx\sqrt[3]{\mu m\left(\lambda + * + \frac{1}{\mathfrak{C}l}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{\nu}{C}\right) + \frac{\lambda'''}{C^3 m}\right)}.$$

[The letter k before the root may designate the number 50; the asterisk sign * denotes the term $\frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right)$ which vanishes on account of $\mathfrak{B}=\infty$ and $k=\infty$.]
Moreover concerning which case again it will prevail, as we have mentioned above in [§ 232], evidently the representation of the image in place to be fouled on account of a small amount of impurities of the objective lens.

But with regard to the rest, the greater field is always half of the simple field, truly which deficiency may be easily rectified by adding a new lens.

Establishing the case where $Q=\infty$

320. Therefore in this case the second interval shall become $\beta+c=\frac{B\alpha}{k}$; from which it follows B to be positive and thence C negative. Then truly, since $c=-\frac{\beta}{Q}$, there will become $c=0$ and $\gamma=0$. But since the focal length of this lens shall be $r=\mathfrak{C}c$, there will become $\mathfrak{C}=\infty$ and hence $C=-1$, and since $B>0$, there will become $\mathfrak{B}>0$ but <1.

Again, since there shall be $m=Qkk'$ nor truly $k=0$, it is necessary that there shall be $k'=0$, from which there may be put $Qk'=l$ [*i.e.*, where l is the height of the ray parallel to the axis], so that there shall be $m=kl$. Now truly from the colored margin we have $k'=\frac{1}{i+Qm}=\frac{l}{Q}$; from which there follows $\omega=\frac{1}{l}$ and hence positive. But since $\mathfrak{B}$ shall be positive, from the first fundamental equation it follows that

$$\omega=\frac{-(1+k)M}{\mathfrak{B}},$$

from which ω will be required to be a negative quantity ; since it may be concluded adversely to that, it is evident this is an impossible case or rather in this case the colored margin cannot be removed. Otherwise in this case the third lens may be put in place of the second image; since it may involve a contradiction, hence it is easily understood notably the third lens interval must be put in place before the latter image.

Establishing the straight forwards case where $Q=1$ and the rays transmitted by the two first lenses shall become parallel again.

321. Therefore in this case the telescope will be composed as it were from two astronomical tubes certainly themselves removed in turn from the same axis by a certain distance, for which the kind being referred to are commonly called terrestrial telescopes. Therefore since there shall be $Q=1$, lest the second interval $\beta+c$ on account of $\beta=-Qc$ may vanish, both β as well as c must be infinite, since that may arise both if $k=0$, as will as if $B=\infty$; but the first case cannot be considered here, since the first interval also would become infinite; from which it is necessary, that there shall be $B=\infty$ and $\mathfrak{B}=1$. But lest our third interval may emerge $=\infty$, the product BC must be a finite and negative quantity ; whereby there may be put

$BC = -\theta$ and thus $C = -\frac{\theta}{B} = 0.$ But so that the middle interval may remain finite, by putting it $= \eta\alpha$, the quantity B may be obtained truly not as if infinite, but only considered to be very large, while evidently we will be satisfied by the prescribed conditions, from which also the value of Q will be found to differ only a little from one ; for since there must be

$$\frac{B\alpha}{k}\left(1-\frac{1}{Q}\right) = \eta\alpha,$$

since there shall be

$$Q = \frac{B}{B-\eta\alpha} = 1 + \frac{\eta k}{B};$$

then truly also there will be

$$\mathfrak{B} = \frac{B}{1+B},\ C = -\frac{\theta}{B},\ \text{and}\ \mathfrak{C} = \frac{-\theta}{B-\theta} = \frac{-\theta}{B}.$$

With these noted our fundamental equations will be :

$$\omega = \frac{-(1+k)M(1+B)}{B}\ \text{ and }\ +\frac{\theta i}{B} = +(1+k)M + \frac{\eta k^2 M}{B} + \omega,$$

in which if in place of ω from above the value found may be substituted, there will be obtained

$$\frac{\theta i}{B} = \frac{\eta k^2 M - (1+k)M}{B}\ \text{ and hence }\ i = \frac{(\eta k^2 - 1 - k)M}{\theta},$$

and now it will be allowed to put $B = \infty$, $\mathfrak{B} = 1$, $C = \mathfrak{C} = 0$, yet thus, so that there shall be $BC = \theta$.

But the removal of the colored margin provides $k' = \frac{1}{i+\omega}$, and on account of $kk' = m$ there may be deduced $i + \omega = \frac{k}{m}$; since then there is $M = \frac{1+i+\omega}{m-1}$, now there will become $M = \frac{m+k}{m(m-1)}$, and if the values found for i and ω may be substituted into the formula $i + \omega = \frac{k}{m}$, this equation will arise:

$$\frac{k}{m} = \frac{\left(\eta k^2 - (1+k)(1+\theta)\right)M}{\theta};$$

and with the value substituted for M

$$\theta(m-1)k = \left(\eta k^2 - (1+k)(1+\theta)\right)(m+k),$$

from which there is gathered

$$\theta = \frac{(\eta k^2 - k - 1)(m+k)}{k^2 + 2mk + m},$$

and since η must be a positive number, it is necessary, that there shall be $\eta > \frac{k+1}{k^2}$ and indeed thus, so that θ may not become exceedingly small ; whereby indeed now our terms may be expressed thus :

$$b = \frac{\alpha}{k},\ \beta = \infty,\ \ c = \infty,\ \gamma = \frac{\theta\alpha}{k},\ d = \frac{\theta\alpha}{m},$$

$$\alpha + b = \alpha(1 + \tfrac{1}{k}),\ \ \beta + c = \eta\alpha,\ \ \gamma + d = \theta\alpha\left(\tfrac{1}{k} + \tfrac{1}{m}\right)$$

and thence the distance of the eye

$$O = \frac{\theta\alpha}{Mm^2} = \frac{(m-1)\theta\alpha}{m(m+k)}$$

and the focal lengths

$$p = \alpha,\ \ q = \frac{\alpha}{k},\ \ r = \frac{\theta\alpha}{k},\ \ s = \frac{\theta\alpha}{m}$$

But the distance α must be defined from the following equation:

$$\alpha = kx\sqrt[3]{\mu m}\left(\lambda + \frac{\lambda'}{k} + \frac{\lambda''}{\theta^2 k} + \frac{\lambda'''}{\theta^3 m}\right),$$

whereby, lest the value of α may become exceedingly large, it is agreed k [within the root sign] to be assumed large, then truly θ will be not much less than one; so that for the first concern, also the apparent field induces to give the maximum value of the letter k , since then M continually increases more ; truly it is to be observed properly in the formula $\Phi = M\xi$ for the letter ξ to be able to be assumed only as far as the value $\frac{1}{4}$, in as much as the letters i and ω do no exceed unity, thus so that, if either i or ω may exceed unity, then ξ is to be diminished in the same ratio. On account of being a cause of the greatest importance to enquire into the maximum value of k, from which $i = 1$ may be produced. Moreover on putting $i = 1$ we find

$$1 + \omega = \frac{k}{m} \ \text{ or } \ m(m-2) = k^2 + 2mk,$$

the resolution of this equation gives rise to

$$k = -m + \sqrt{2m(m-1)}.$$

Clearly this value of k provides us with $i = 1$, and

$$\omega = \frac{k-m}{m} = \frac{-2m + \sqrt{2m(m-1)}}{m},$$

which value is negative and less than one, from which there will be had for the apparent field of view

$$\Phi = \frac{\sqrt{2m(m-1)}}{m(m-1)} \cdot \xi = \sqrt{\frac{2}{m(m-1)}} \cdot \xi;$$

but if k may attain a greater value at this time, it may produce a certain i greater than unity, but then ξ must be taken thus, so that there may become $i\xi = \frac{1}{4}$ or $\xi = \frac{1}{4i}$, and thus $\Phi = \frac{1+i+\omega}{m-1} \cdot \frac{1}{4i}$ will be produced, from which calculation put in place it becomes known the field to be more diminished there continually, so that the value of k will have exceeded that term . Therefore this case has the most gain, if there may be taken

$$k = -m + \sqrt{2m(m-1)},$$

from which there becomes

$$k' = \frac{m+\sqrt{2m(m-1)}}{m-2}$$

SCHOLIUM

322. Since in the preceding problem an especially memorable case has been deduced by putting $i = 0$, which may lead one to suspect such a position may be agreed to be put in place here also. On account of which we may show neither the position $i = 0$ nor $\omega = 0$ can be used here in this problem .

For in the first place even if there shall be $\omega = 0$, on account of $k' = \frac{1}{i+Q\omega}$ there must be $i > 0$; but because $\omega = 0$ the first equation

$$\mathfrak{B}\omega = -(1+k)M$$

is unable to remain, unless $\mathfrak{B} = \infty$ and thus $B = -1$; now on account of $BC < 0$, C must be positive and thus $\mathfrak{C}$ also > 0, from which it is apparent the other equation

$$\mathfrak{C}i = -(1+Qk)M$$

plainly cannot remain ; and thus it is not possible to assume $\omega = 0$.

It will be shown in a similar manner that the number i cannot vanish; then also on account of $k' = \frac{1}{i+Q\omega}$ there must be $\omega > 0$ and hence the latter equation

$$\mathfrak{C}i = -(1+Qk)M - \omega$$

cannot remain, unless $\mathfrak{C}i$ shall be a finite negative quantity, and thus $\mathfrak{C}=\infty$; from which there becomes $C=-1$ and hence on account of $BC<0$, there will become $B>0$ and likewise $\mathfrak{B}>0$, which clearly is in contradiction to the first equation

$$\mathfrak{B}\omega=-(1+k)M ;$$

from which also it is clear the number i cannot be taken $=0$.

Nor therefore besides the three cases mentioned here does any other case deserve to considered and thus with such convenience may come before all the others, so that that alone may be seen to be noteworthy, which may be deduced in practice ; for not only does it uncover the maximum field, but also for the value α is not provided exceedingly great, because in that cubic formula the terms after λ all become very small and smaller than that, where the greater were the magnification, since there shall become approximately $k=m\left(\sqrt{2}-1\right)=\frac{1}{2}m$. Then truly here also the number θ is allowed to be chosen by us, by which we may be able to effect, so that the latter lenses may not become exceedingly small ; but with θ taken as it pleases the quantity η will be defined by the following equation; since indeed we have found above :

$$\theta=\frac{(\eta k^2-k-1)(m+k)}{k^2+2mk+m},$$

on account of $m\left(m-2\right)=2mk+k^2$ and $m+k=\sqrt{2}m(m-1)$ there will be

$$\theta=\frac{(\eta k^2-k-1)\sqrt{2}}{\sqrt{m\left(m-1\right)}}$$

and hence

$$\eta=\frac{k+1}{k^2}+\frac{\theta\sqrt{m\left(m-1\right)}}{k^2\sqrt{2}},$$

from which the value of the interval between the second and third lenses becomes known.

PROBLEM 4

323. *If a telescope of this kind thus may be composed from four lenses, so that one lens may be put in place between the second image and the eyepiece, to investigate its nature and to describe its construction.*

SOLUTION

Therefore since here the first image falls between the first lens and the second, truly the second between the second lens and the third, the letters P and Q will be negative with R alone positive. Whereby if there may be put $P=-k$ and $Q=-k'$, our terms will be :

$$b=\frac{\alpha}{k},\ \beta=\frac{B\alpha}{k},\ c=\frac{B\alpha}{kk'},\ \gamma=\frac{BC\alpha}{kk'},\ \text{and}\ d=\frac{-BC\alpha}{kk'R}=\frac{-BC\alpha}{m}.$$

Hence the interval

$$\alpha+b=\alpha\left(1+\frac{1}{k}\right)\ \text{and thus}\ \alpha\ \text{is positive,}$$

$$\beta+c=\frac{B\alpha}{k}\left(1+\frac{1}{k'}\right),\ \text{therefore}\ B>0,\ \text{and}\ \mathfrak{B}>0\ \text{and likewise}\ \mathfrak{B}<1,$$

$$\gamma+d=\frac{Bk\alpha}{kk'}\left(1-\frac{1}{R}\right),\ \text{therefore}\ C\left(1-\frac{1}{R}\right)>0.$$

Moreover, for the position of the eye there will be $O=\frac{d}{Mm}$; which so that it shall be positive, there must be $d>0$, from which this new condition may arise, so that there shall be $C<0$, which condition with the preceding taken jointly gives $1-\frac{1}{R}<0$ and thus $R<1$. So that if now we may put

$$\pi=-\omega\xi,\quad \pi'=i\xi\ \text{and}\ \pi''=-\xi,$$

so that there may become

$$\Phi=\frac{\omega+i+1}{m-1}\cdot\xi=M\xi$$

with

$$M=\frac{\omega+i+1}{m-1},$$

our fundamental equations will be

$$\mathfrak{B}\omega=-(1+k)M\ \text{and}\ \mathfrak{C}i=-(1+kk')M-\omega,$$

from the first of which on account of $\mathfrak{B}>0$ is clear at once to become $\omega<0$.

Moreover, the removal of the colored fringe demands, that there shall be

$$0=\frac{\omega}{P}+\frac{i}{PQ}+\frac{1}{PQR}$$

and thus

$$0=-\frac{\omega}{k}+\frac{i}{kk'}+\frac{i}{kk'R}\ \text{from which}\ R=\frac{1}{\omega k'-i};$$

therefore so that R may be produced positive, by necessity I must be a negative number. Therefore we may put in place $\omega=-\zeta$; and $i=-y$, so that now for the apparent field of view :

$$M=\frac{1-y-\zeta}{m-1}\ \text{and thus}\ y+\zeta<1.$$

Therefore since there shall be

$$R=\frac{1}{y-k'\zeta} \text{ and hence } m=\frac{kk'}{y-k'\zeta},$$

it is required to note on account of $R<1$ and $R=\frac{m}{kk'}$ there must be $kk'>m$; hence, since $y=k'\zeta+\frac{kk'}{m}$, there will become $y>1$ and thus $y+\zeta$ must become much greater than 1; which since it shall be absurd, it is apparent the case of this problem cannot be considered.

SCHOLlUM

324. Therefore since this problem shall be required to be excluded completely, as little equally may be able to be resolved for the condition of the colored fringe by the first used only from the three lenses, the second and third lenses leave so great a problem for us. But since a single straight forward case arises there from the second, noted to prevail especially over all the rest, as also the final third case deserves the greatest attention before the others, hence we may set up two particular kinds of telescopes of the third kind and we will treat these separately, so that we may show initially just as for each, one or several lenses will be required to be added from the same kind of glass, then also with a different kind of glass, they may be able to be carried to a higher order of perfection. Truly of these two kinds the latter thus is to be noted chiefly, since the said common terrestrial telescope is itself included among these ; for actually it differs the most from these, in as much as it has been freed from the glasses, from which these instruments labour, as commonly accustomed to be made; for which also if we may be unwilling to call more lenses in to help, hence rules will be given thus required for the perfection of this telescope, so that a further perfection may not be expected. But the first kind, as long as it may demand the disposition of another ocular lens, at one time in short was unknown and recently at last has been introduced into practice by the most clever Dollond. Just as clearly the minimum aperture endowed for lenses has been used; nor yet from experience alone was the highest order of perfection able to be hoped for, of which this kind is capable. Yet this is easily brought to mind, unless in addition one lens may be added, the field may become exceedingly small, as we may be able to agree to that. For we have seen the field of view always to be a little less than in common astronomical tubes, to which remedy we will return in the following. Finally, concerning this kind, it will be agreed to note we are going to be using these measures in the following, which have been put in place in § 314, where clearly we have put $\zeta=\frac{1}{\sqrt{m}}$, since thence they may be seen to be more suitable for obtaining the determinations in practice.

TELESCOPIORUM
SECTIO TERTIA.

DE
TELESCOPIIS TERTII GENERIS,
QUIBUS
OBJECTA ITERUM SITU ERECTO
REPRAESENTANTUR.

CAPUT I

DE TELESCOPIIS SIMPLICIORIBUS TERTII GENERIS EX UNICA VITRI SPECIE PARATIS

PROBLEMA 1

294. *Telescopium simplicissimum huius generis, quod tribus tantum constat lentibus, construere, quod obiecta secundum datam rationem aucta et situ erecto repraesentet.*

SOLUTIO

Pro duobus intervallis, quae hic occurrunt, ponamus ut semper fractiones $\frac{\alpha}{b}=-P$ and $\frac{\beta}{c}=-Q$, et quia hic dnae imagines reales habentnr, quarum altera in prius intervallum cadens est inversa, altera vero in posterius intervallum cadens erecta, ita ut sit semidiameter illius $=\alpha\Phi$, huius vero $=B\alpha\Phi$, ambae litterae P et Q debent esse negativae, unde statuamus $-P=k$ et $-Q=k'$, ut sit multiplicatio $m=kk'$. Hinc elementa nostra ita se habebunt:

$$b=\frac{\alpha}{k},\quad \beta=\frac{B\alpha}{k}\quad \text{et}\quad c=\frac{B\alpha}{kk'}$$

et distantiae focales

$$p=\alpha,\quad q=\frac{\mathfrak{B}\alpha}{k}\quad \text{and}\quad r=\frac{B\alpha}{kk'}=\frac{B\alpha}{m},$$

tum vero bina intervalla

$$\alpha+b=\alpha\left(1+\frac{1}{k}\right),\quad \beta+c=\frac{B\alpha}{k}\left(1+\frac{1}{k'}\right),$$

quae per se sunt positiva, siquidem esse debet $B>0$ ideoque et $\mathfrak{B}$.

Pro campo porro apparente, eum sit eius semidiameter $\Phi=\frac{-\pi+\pi'}{m-1}$, ponamus

$$\pi=-i\xi \quad \text{et}\quad \pi'=\xi,$$

denotante ξ maximum valorem, quem litterae π et π' recipere possunt, et i fractionem unitate minorem, eritque

$$\Phi = \frac{i+1}{m-1}\xi$$

atque hinc pro loco oculi fiet

$$O = \frac{\pi'}{\Phi}\cdot\frac{r}{m} = \frac{m-1}{i+1}\cdot\frac{B\alpha}{mm},$$

quae distantia etiam per se est positiva. His positis aequationes pro litteris π, π' supra datae dabunt

$$\frac{\mathfrak{B}\pi-\Phi}{\Phi} = \frac{\alpha}{b} = k \quad \text{seu} \quad \mathfrak{B}\cdot\frac{-i(m-1)}{i+1} = k+1,$$

unde

$$i = \frac{-k-1}{k+1+(m-1)\mathfrak{B}},$$

qui valor debet esse unitate minor. Cum igitur hinc valor ipsius *i* necessario sit negativus et unitate minor, erit campi semidiameter

$$\Phi = \frac{\mathfrak{B}\xi}{k+1+(m-1)\mathfrak{B}},$$

quae certe eo minor est quam $\frac{\xi}{m-1}$, quo *k* est maius et quo minus est $\mathfrak{B}$.
Quo igitur campum maiorem obtineamus, in id est incumbendum, ut litterae *k* quam minimus, litterae $\mathfrak{B}$ vero quam maximus valor concilietur; at cum sit $B = \frac{\mathfrak{B}}{1-\mathfrak{B}}$ et debeat esse $B > 0$, hinc evidens est $\mathfrak{B}$ non ultra unitatem augeri posse. Casu autem, quo fit $\mathfrak{B} = 1$, fit $\Phi = \frac{\xi}{k+m}$. Tum vero ob $B = \infty$ longitudo tubi fieret infinita. Diminutio vero numeri *k* quum parum conferat ad campum augendum, videamus nunc etiam, an margo coloratus destrui possit, quem in finem esse deberet [§ 49]

$$0 = \frac{\pi}{\Phi}\cdot\frac{b}{p} + \frac{\pi'}{\Phi}\cdot\frac{c}{Bp}, \quad 0 = -i\cdot\frac{1}{k} + \frac{1}{kk'},$$

quae aequatio ob $i < 0$ nullo modo subsistere potest; unde haec telescopiorum species vitio marginis colorati quam maxime laborabit. Ceterum pro semidiametro confusionis habebimus hanc aequationem [§ 42]:

$$\frac{\mu m x^3}{\alpha^3}\left(\lambda + \frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2} + \frac{\nu}{B}\right) + \frac{\lambda''}{B^2 m}\right) = \frac{1}{k^3},$$

[Notandum est litteram *k* dextro latere aequationis adhibitam plane diferre & quantitate *k* sinistro latere aequationis posita. E. Ch.]
unde colligitur

$$\alpha = kx\sqrt[3]{\mu m\left(\lambda + \frac{\lambda'}{\mathfrak{B}^3 k} + \frac{\lambda''}{B^3 m} + \frac{\nu}{B\mathfrak{B}k}\right)},$$

qui sumto $x = \frac{m}{50}$ dig. et [extra radicem] $k = 50$ abit in hunc valorem:

$$\alpha = m\sqrt[3]{\mu m\left(\lambda + \frac{\lambda'}{\mathfrak{B}^3 k} + \frac{\lambda''}{B^3 m} + \frac{\nu}{B\mathfrak{B}k}\right)},$$

in qua expressione cum omnia membra sint positiva, nullum est dubium, quin distantia focalis a multo fiat maior quam casu duarum lentium.

COROLLARIUM 1

295. Cum iam sit animadversum, si $\mathfrak{B}$ caperetur $=1$, longitudinem instrumenti in infinitum excrescere ideoque $\mathfrak{B}$ capi debere minus unitate, secundum membrum in aequatione valde increscet pariter ac ultimum, ex quo distantia α augebitur.

COROLLARIUM 2

296. Sin autem huic incommodo mederi vellemus augendo numerum *k,* tunc campus apparens restringeretur.

SCHOLION 1

297. Nullum igitur est dubium, quin haec prima istiusmodi telescopiorum species penitus sit repudianda, non solum, quod nimis exiguum campum ostendat tubusque fiat valde longus, sed eam ob causam praecipue, quod repraesentatio margine colorato sit inquinata; neque etiam reperimus huiusmodi telescopia unquam usu fuisse recepta. Interim tamen casum quendam in sequente exemplo proponamus.

EXEMPLUM

298. Si sumatur $\mathfrak{B} = \frac{4}{5}$ et $k = 2$, telescopium huius generis describere pro multiplicatione quacunque *m.*
Cum igitur hinc sit $B = 4$, erunt elementa

$$b = \tfrac{1}{2}\alpha, \quad \beta = 2\alpha, \quad c = \frac{4\alpha}{m}$$

et distantiae focales

$$p = \alpha, \quad q = \tfrac{2}{5}\alpha \quad \text{et} \quad r = \frac{4\alpha}{m}$$

et intervalla

$$\alpha + b = \tfrac{3}{2}\alpha, \quad \beta + c = 2\alpha + \frac{4\alpha}{m},$$

quorum summa $\frac{7}{2}\alpha+\frac{4\alpha}{m}$, dat tubi longitudinem.

Tum vero reperitur $i=\frac{-15}{11+4m}$ et semidiameter campi $\Phi=\frac{4\xi}{11+4m}$ seu in mensura anguli $\Phi=\frac{3437}{11+4m}$ minut., qui non multo est minor quam campus ordinarius. Pro 1oco oculi vero erit $O=\frac{4m+11}{mm}\cdot\alpha$. Denique vero pro distantia focali α habebimus

$$\alpha=m\sqrt[3]{\mu m\left(\lambda+\frac{\lambda'\cdot 125}{128}+\frac{\lambda''}{4m}+\frac{5\nu}{32}\right)},$$

ubi circiter est $\mu=1$ et $\nu=\frac{1}{5}$; quare, si litteris λ, λ', λ'' valor minimus, scilicet 1, tribuatur, erit

$$\alpha=m\sqrt[3]{m\left(2+\frac{1}{128}+\frac{1}{64m}\right)},$$

$$\alpha=m\sqrt[3]{\left(\left(2+\frac{1}{128}\right)m+\frac{1}{64}\right)}\ \text{dig.};$$

hinc, si esset $m=25$, erit

$$\alpha=25\sqrt[3]{50\tfrac{27}{128}}=92,23\ \text{dig.}$$

hincque tota longitudo erit $=340$ dig. $=28$ ped. 4 dig., quae longitudo ratione multiplicationis utique tam est magna, ut in praxi nullo modo admitti possit, etiamsi vitium marginis colorati non adesset.

SCHOLION 2

299. Cum igitur hinc nihil in usum practicum trahi possit haecque species simplicissima penitus reiici debeat, ad species simpliciores progrediamur, quae scilicet oriuntur, si tribus lentibus insuper una lens quarta adiungatur, ex quo variae species nascentur, prouti haec nova lens vel inter obiectivam et priorem imaginem vel inter priorem et posteriorem vel inter hanc posteriorem et lentem ocularem constituatur; quos ergo casus seorsim hic evolvi conveniet.

PROBLEMA 2

300. *Si inter lentem obiectivam et primam imaginem nova lens ponatur, indolem horum telescopiorum indagare eorumque constructionem describere.*

SOLUTIO

Cum hic quatuor lentes sint, statuantur ternae fractiones ut semper

$$\frac{\alpha}{b}=-P,\quad \frac{\beta}{c}=-Q \text{ et } \frac{\gamma}{d}=-R,$$

et quia in primum intervallum nulla imago cadit, retinebit P valorem positivum, reliquae vero Q et R fient negativae.

Quare ponatur $Q=-k$ et $R=-k'$, ut fiat multiplicatio $m=Pkk'$ elementaque nostra sint

$$b=-\frac{\alpha}{P},\ c=-\frac{B\alpha}{Pk},\ d=-\frac{BC\alpha}{Pkk'},\ \beta=-\frac{B\alpha}{P},\ \gamma=-\frac{BC\alpha}{Pk};$$

$$p=\alpha,\ q=-\frac{B\alpha}{P},\ r=-\frac{B\mathfrak{C}\alpha}{Pk},\ s=-\frac{BC\alpha}{m},$$

unde prodeunt intervalla

$$\alpha+b=\alpha\left(1-\frac{1}{P}\right),\ \beta+c=-\frac{B\alpha}{P}\left(1+\frac{1}{k}\right),\ \gamma+d=-\frac{BC\alpha}{Pk}\left(1+\frac{1}{k'}\right)$$

sicque patet $B\alpha$ esse debere negativum ut et $BCa,$ ideoque C debet esse positivum; unde, si $\alpha>0$, debet esse $P>1,\ B<0$ et $C>0$, sin autem $\alpha<0$, debet esse $P<1,\ B>0$ et $C>0.$

Nunc, cum pro campo apparente sit

$$\Phi=\frac{-\pi+\pi'-\pi''}{m-1},$$

statuatur

$$\pi=-\omega\xi,\quad \pi'=i\xi,\quad \pi''=-\xi,$$

ut sit

$$\Phi=\frac{\omega+i+1}{m-1}\xi=M\xi$$

existente $M=\frac{\omega+i+1}{m-1}$. Atque statim pro loco oculi sequitur

$$O=\frac{\pi''}{\Phi}\cdot\frac{s}{m}=-\frac{1}{M}\cdot\frac{BC\alpha}{mm},$$

quae distantia per conditiones superiores iam est positiva. Aequationes autem pro litteris π supra datae praebent

$$\frac{\mathfrak{B}\pi}{\Phi}-1=-\frac{\mathfrak{B}\omega}{M}-1=-P,\ \frac{\mathfrak{C}i}{M}+\frac{\omega}{M}+1=-Pk,$$

unde colligitur

$$\omega=\frac{(P-1)(i+1)}{\mathfrak{B}(m-1)-P+1},$$

ut maneat $\mathfrak{B}$ indefinitum, et

$$\mathfrak{C} = \frac{-(1+Pk)M-\omega}{i};$$

quae quantitas cum debeat esse positiva, debet esse vel *i* negativum vel, si esset *i* positivum, deberet esse

$$-(1+Pk)M-\omega > 0 \text{ sive } -(1+Pk)\left(\frac{\omega+i+1}{m-1}\right)-\omega > 0$$

seu

$$-\omega\left(m+Pk\right)-(1+Pk)\left(i+1\right) > 0,$$

unde patet fractionem ω negativam esse debere, ita ut hinc campus apparens diminuatur.

Videamus iam, an marginem coloratum tollere vel huic aequationi satisfacere possimus:

$$0 = +\omega\cdot\frac{1}{P} - \frac{i}{Pk} + \frac{1}{Pkk''},$$

unde colligimus

$$0 = \omega - \frac{i}{k} + \frac{1}{kk''}, \text{ adeoque } k' = \frac{-1}{k\omega - i} = \frac{1}{i-k\omega},$$

qui valor debet esse positivus adeoque $k\omega - i < 0$, de quo deinceps videbimus. Nunc adhuc aequationem pro confusione aperturae tollenda contemplemur, quae sequenti modo exhibebitur:

$$\alpha = kx\sqrt[3]{\mu m\left(\lambda - \frac{1}{\mathfrak{B}P}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right) - \frac{1}{B^3\mathfrak{C}Pk}\left(\frac{\lambda''}{\mathfrak{C}^2}+\frac{\nu}{C}\right) - \frac{\lambda'''}{B^3Cm}\right)},$$

pro qua expressione hactenus sumsimus $x = \frac{m}{50}$, dig. et [extra radicem] $k = 50$.

COROLLARIUM 1

301. Pro diiudicandis litteris ω et *i,* utrum valores habere queant positivos, considerandae sunt hae duae formulae:

$$\text{I.}\quad \mathfrak{C} = \frac{-(1+Pk)M-\omega}{i},$$
$$\text{II.}\quad k' = \frac{1}{i-k\omega},$$

ex quarum prima patet ambas litteras *i* et ω simul positivas esse non posse, quia alioquin $\mathfrak{B}$ foret negativum, quae littera tamen valorem positivum habere debet. Ex secunda vero evidens est fieri non posse, ut sit $\omega > 0$ et $i < 0$, quia alioquin k' prodiret negativum.

COROLLARIUM 2

302. Ex his duobus casibus sequitur litteram ω nunquam positivam esse posse, quae conditio ita enunciari potest, ut secunda lens semper campum apparentem imminuere debeat.

COROLLARIUM 3

303. Cum igitur ω semper debeat esse negativum, ponatur $w=-\zeta$, ut sit

$$B\zeta=(1-P)M \quad \text{et} \quad M=\frac{1+i-\zeta}{m-1}.$$

Nostrae vero formulae, necessario positivae, erunt

$$\text{I.} \quad \mathfrak{C}=\frac{-(1+Pk)M+\zeta}{i},$$
$$\text{II.} \quad k'=\frac{1}{i+k\zeta},$$

unde, si sit i fractio positiva, debet esse $\zeta>(1+Pk)M$. Sin autem i sit fractio negativa, puta $i=-y$, per primam debet esse $\zeta<(1+Pk)M$ et simul $\zeta>\frac{y}{k}$.

COROLLARIUM 4

304. Praeterea etiam manifestum est fractionem $\zeta=-\omega$ nunquam evanescere posse; si enim sit $i>0$, debet esse $\zeta>(1+Pk)M$. Sin autem sit $i<0$ seu $i=-y$, debet esse $\zeta>\frac{y}{k}$.

COROLLARIUM 5

305. Quia casu $i=-y$ duplicem invenimus conditionem, priorem $\zeta<(1+Pk)M$ et posteriorem $\zeta>\frac{y}{k}$, ex earum comparatione necesse est, ut sit
$(1+Pk)M>\frac{y}{k}$ seu $y<(1+Pk)kM$.

SCHOLlON

306. Tot autem casus diversi ideo potissimum habent locum, quod in solutione problematis non definitur, utrum lens obiectiva habeat suam distantiam focalem α positivam an negativam. Utrumque autem usu venire potest, siquidem circa litteram P

nihil aliud praecipitur, nisi quod sit positiva ideoque eius valor a ciphra usque in infinitum augeri queat.

Quamdiu autem littera P intra limites 0 et 1 continetur, α valorem habere debet negativum seu lens obiectiva erit concava et littera B positiva ideoque et $\mathfrak{B}$; unde fit $\zeta = \frac{(P-1)M}{\mathfrak{B}}$ adeoque positivum. Sin autem statuatur $P = 1$, quo casu binae lentes priores sibi immediate iunguntur, fit $\zeta = 0$, qui casus, uti vidimus, penitus excluditur, ita ut lens obiectiva duplicata esse nequeat. At si sit P maior unitate, necessario fit α positivum seu lens obiectiva convexa; unde B fit negativum neque vero hinc definitur $\mathfrak{B}$. At quia novimus esse ω negativum seu ζ positivum, ob $\zeta = -\frac{(P-1)M}{\mathfrak{B}}$ patet litteram $\mathfrak{B}$ negativam esse debere, hincque porro concluditur $-B$ esse unitate minus. Si denique P sit numerus infinitus, secunda lens in ipso loco prioris imaginis constituetur et ex eius distantia focali q concluditur

$$\mathfrak{B} = -P \cdot \frac{q}{\alpha} = -\infty \text{ hincque } B = -1;$$

atque sic contemplati sumus obiter omnes casus pro littera P, qui autem nunc diligentius perpendi merentur. Ante omnia autem notari convenit sumi non posse $P = 0$, quia iam primum intervallum fieret infinitum, nisi distantia α esset infinite parva, quod autem foret aeque absurdum, quia prima lens aperturam definitam admittere debet.

1. EVOLUTIO CASUS QUO $P < 1$

307. Pro hoc casu iam animadvertimus fore $\alpha < 0$; quae negatio ne turbet, ponamus $\alpha = -a$ eritque

$$b = \frac{a}{P}, \quad c = \frac{Ba}{Pk}, \quad d = \frac{BCa}{m}, \quad \beta = \frac{Ba}{P}, \quad \gamma = \frac{BCa}{Pk},$$

unde patet ambas litteras B et C debere esse positivas; unde litterae germanicae $\mathfrak{B}$ et $\mathfrak{C}$ non solum erunt quoque positivae, sed etiam unitate minores; quare, cum sit $\mathfrak{B}\zeta = (1-P)M$, manifesto sequitur fore $\zeta > (1-P)M$.
Deinde ob

$$\mathfrak{C} = \frac{-(1+Pk)M+\zeta}{i} \text{ et } k' = \frac{1}{i+k\zeta}$$

non solum esse debet $\frac{-(1+Pk)M+\zeta}{i} > 0$, sed etiam $\frac{-(1+Pk)M+\zeta}{i} < 1$; quod quo clarius explicetur, duos casus examinari conveniet.

1. Si i sit positivum, ex valore $\mathfrak{C}$ nanciscimur has conditiones:

$$\zeta > (1+Pk)M \text{ et } \zeta < (1+Pk)M + i;$$

conditio autem litterae k' sic sponte impletur. Quia autem iam invenimus $\zeta > (1-P)M$, nunc inde patet esse debere

$$(1+Pk)M+i>(1-P)M \text{ ideoque } i>-P(k+1)M\ ;$$

id quod semper est verum, dummodo *i* sit positivum, uti supponimus.

2. Si *i* sit negativum, ponatur $i=-y$ eritque

$$\mathfrak{C}=\frac{(1+Pk)M-\zeta}{y},\ \ k'=\frac{1}{k\zeta-y}.$$

Inde igitur sequuntur hae conditiones:

$$\zeta<(1+Pk)M,\ \ \zeta>(1+Pk)M-y, \text{ hinc vero } \zeta>\tfrac{y}{k};$$

at supra iam invenimus $\zeta>(1-P)M$, unde sequitur fore

$$(1+Pk)M>\tfrac{y}{k} \text{ sive } y<(1+Pk)kM.$$

Isto igitur casu, quo $P<1$, fractio *i* tam positive capi poterit quam negative, ac si positive accipiatur, eius valor nulla limitatione restringi. Quare, cum *i* unitatem superare nequeat, poterit sine haesitatione statim poni $i=1$, ita ut pro campo apparente fiat $\Phi=\frac{2-\zeta}{m-1}\cdot\xi$ dummodo ξ non superet unitatem.

Nulla autem ratio suadet capere *i* negativum, quia tum campus nimium diminueretur.

II. EVOLUTIO CASUS QUO $P>1$

308. Quia hic est α quantitas positiva ideoque *b* negativa, debet esse *B* negativum, at *C* ut ante positivum. Deinde etiam vidimus esse $\mathfrak{B}$ negativum ideoque $-B<1$; unde fit $\zeta=\frac{(1-P)M}{\mathfrak{B}}$ adeoque positivum, ubi tantum notetur $\mathfrak{B}$ tam parvum accipi non debere, ut ζ superet unitatem. Deinde habetur

$$\mathfrak{C}=\frac{-(1+Pk)M+\zeta}{i} \text{ et } k'=\frac{1}{i+k\zeta};$$

ex quibus formulis plane eadem sequuntur, quae in casu praecedente sunt allata; unde videtur etiam statui posse $i=1$, dummodo ex valore pro ζ ante dato sit

$$\frac{(1-P)}{\mathfrak{B}}>1+Pk \text{ sive } -\mathfrak{B}<\frac{P-1}{Pk+1} \text{ et } -\mathfrak{B}>\frac{(P-1)M}{(1+Pk)M+1}.$$

III. EVOLUTIO CASUS QUO $P=\infty$

309. Hoc ergo casu, ut iam supra notavimus, erit

$$B=-1 \text{ et } \mathfrak{B}=-\tfrac{Pq}{\alpha}.$$

Nunc autem evidens est statui debere $k=0$, ita tamen, ut sit $Pk=\theta$, ex quo elementa erunt

$$b=0,\ \beta=0,\ \ c=\tfrac{\alpha}{\theta},\ \ \gamma=\tfrac{C\alpha}{\theta},\ \ d=\tfrac{C\alpha}{m}.$$

Deinde, cum sit $\mathfrak{B}\zeta=(1-P)M$, habebitur nunc $\zeta=\tfrac{M\alpha}{q}$, unde $q=\tfrac{M\alpha}{\zeta}$.
Deinde binae nostrae formulae erunt

$$\mathfrak{C}=\frac{-(1+\theta)M+\zeta}{i} \text{ et } k'=\tfrac{1}{i},$$

ubi, cum nihil impediat, quominus ponatur $i=1$, erit hoc casu $k'=1$ et $Pk=\theta=m$, ita ut sit $\mathfrak{C}=-(1+m)M+\zeta$; ex quo valore hi limites colliguntur:

$$\zeta>(1+m)M,\quad \zeta<(1+m)M+1;$$

at vero est

$$M=\tfrac{2-\zeta}{m-1} \quad\text{ideoque}\quad \zeta>\frac{(1+m)(2-\zeta)}{m-1} \quad\text{ideoque}\quad \zeta>\tfrac{m+1}{m},$$

qui valor, etsi unitatem superat, tamen in praxi locum habere potest, dummodo littera ξ in eadem ratione diminuatur, ita ut $\zeta\xi$ non superet valorem $\frac{1}{4}$, siquidem $\frac{1}{4}$ pro apertura maxima accipiatur. Sin autem sumsissemus $i=\frac{1}{2}$, prodiisset $k'=2$ hincque $m=2\theta$ seu $\theta=\frac{m}{2}$ sicque haberemus

$$\zeta>\left(1+\tfrac{1}{2}m\right)M \text{ et } \zeta<\left(1+\tfrac{1}{2}m\right)M+\tfrac{1}{2};$$

quia autem est $M=\frac{3-2\zeta}{2(m-1)}$, prior condito dat $\zeta>\left(1+\frac{1}{2}m\right)\left(\frac{3-2\zeta}{2(m-1)}\right)$ sive $\zeta>\frac{2+m}{2m}$ ideoque multo magis $\zeta>\frac{1}{2}$. Ex quo patet campum apparentem ob valorem ζ magis imminui quam ob valorem i augeri sicque cum semper aliquanto minorem fieri quam in tubis astronomicis communibus. Supra iam observavimus talem lentis locum in praxi vitari oportere.

IV. EVOLUTIO CASUS PRORSUS SINGULARIS QUO $i=0$

310. Cum sit $i=0$ et $\mathfrak{C}$ unitatem superare nequeat, ob

$$\mathfrak{C}i=-(1+Pk)M+\zeta$$

erit

$$\zeta = (1+Pk)M \text{ hincque } Pk = \tfrac{\zeta}{M} - 1;$$

at est $k' = \frac{1}{k\zeta}$; ob $Pkk' = m$ erit $Pk = mk\zeta$ ideoque $P = m\zeta$. Quare ille valor pro Pk inventus huic aequalis positus dabit

$$mk\zeta = \tfrac{\zeta}{M} - 1$$

hincque

$$k = \tfrac{\zeta}{Mm} - \tfrac{1}{m\zeta}$$

ex quo porro habetur

$$k' = \tfrac{Mm}{\zeta - M};$$

quia vero est $M = \frac{1-\zeta}{m-1}$, nascetur

$$k = \tfrac{m-1}{(1-\zeta)m} - \tfrac{1}{m\zeta} = \tfrac{m\zeta - 1}{m(1-\zeta)\zeta}, \quad k' = \tfrac{m(1-\zeta)}{m\zeta - 1}$$

et

$$P = m\zeta \text{ atque } Pk = \tfrac{m\zeta - 1}{1-\zeta};$$

qui valores cum neutiquam a $\mathfrak{C}$ pendeant, hoc insigne lucrum iam sumus adepti, ut littera C penitus arbitrio nostro relinquatur, sicque efficere poterimus, ut posteriores distantiae determinatrices ipsaeque lentes posteriores, quae hactenus plerumque nimis parvae sunt repertae, nunc datae magnitudinis fieri queant, in quo certe maximum commodum consistit; quod denique ad litteras $\mathfrak{B}$ et B attinet, duos casus considerari oportet, prouti $P = m\zeta$ fuerit vel unitate minor vel unitate maior.

I. Sit igitur $m\zeta < 1$ seu $\zeta < \frac{1}{m}$ et habebitur

$$\mathfrak{B} = \tfrac{(1-m\zeta)M}{\zeta} = \tfrac{(1-m\zeta)(1-\zeta)}{\zeta(m-\zeta)};$$

ibi autem vidimus $\mathfrak{B}$ esse debere positivum et unitate minus; quocirca hoc casu, quo

$$\zeta < \tfrac{1}{m} \text{ ob } \mathfrak{B} = \tfrac{(1-m\zeta)(1-\zeta)}{(m-1)\zeta} \text{ debet esse}$$

$$(1-m\zeta)(1-\zeta) < (m-1)\zeta \text{ seu } m\zeta^2 - 2m\zeta + 1 < 0;$$

unde colligitur ζ capi debere intra limites $\frac{1}{m}$ et $\frac{1}{2m}$.

Cum autem litterae k et k' necessario sint positivae, ad hoc necessario requiritur, ut sit $m\zeta > 1$ seu $\zeta > \frac{1}{m}$: ob quam conditionem casus primus statim excludi debuisset.

II. Sit igitur $P(= m\zeta) > 1$ seu $\zeta > \frac{1}{m}$, prouti valores k et k' postulant, atque ad casum secundum recurrere debemus; pro quo cum iterum sit

$$\mathfrak{B} = \frac{(1-m\zeta)(1-\zeta)}{(m-1)\zeta}$$

simulque notetur $\mathfrak{B}$ esse debere negativum sine ulla alia conditione, nisi quod esse debeat $\zeta < 1$, uti quidem ratio campi absolute postulat, ita ut iam contineatur intra limites 1 et $\frac{1}{m}$, manifestum autem est expedire, ut ζ quam minime limitem $\frac{1}{m}$ superet. Ex quo operae pretium videtur duo exempla adiungere, in quorum altero ζ limiti priori $\frac{1}{m}$, in altero vero limiti posteriori 1 propius accipiatur.

EXEMPLUM 1

311. Pro casu postremo, quo $i = 0$, si statuatur $\zeta = \frac{2}{m}$, telescopium inde oriundum describere.
Hoc igitur casu habebimus

$$\mathfrak{B} = \frac{-(m-2)}{2(m-1)} \quad \text{et} \quad B = \frac{-(m-2)}{3m-4}.$$

Porro

$$P = 2, \;\; k = \frac{m}{2(m-2)}, \;\; k' = m-2, \; M = \frac{m-2}{m(m-1)},$$

unde distantiae nostrae determinatrices ob α positivum erunt

$$b = \frac{-\alpha}{2}, \;\; c = \frac{(m-2)^2}{m(3m-4)}\alpha, \;\; d = \frac{m-2}{m(3m-4)}C\alpha,$$

$$\beta = \frac{m-2}{2(3m-4)}\alpha, \;\; \gamma = \frac{(m-2)^2}{m(3m-4)}C\alpha$$

et distantiae focales

$$p = \alpha, \;\; q = \frac{m-2}{4(m-1)}\alpha, \;\; r = \frac{(m-2)^2}{m(3m-4)}\mathfrak{C}\alpha, \;\; s = \frac{m-2}{m(3m-4)}(m-2)C\alpha.$$

Tum vero intervalla lentium

$$\alpha+b=\tfrac{1}{2}\alpha,\ \ \beta+c=\frac{(m-2)(3m-4)}{2m(3m-4)}\alpha=\frac{m-2}{2m}\alpha,\ \ \gamma+d=\frac{(m-1)(m-2)}{m(3m-4)}C\alpha$$

et distantia oculi

$$O=\frac{m-1}{m(3m-4)}C\alpha$$

et campi semidiameter

$$\Phi=\frac{m-2}{m(m-1)}\cdot\xi\ ;$$

quae si in mensura angulorum desideretur, sumi potest $\xi=859$ min. ob $\xi=\frac{1}{4}$.

Distantia denique focalis lentis obiectivae α definiri debet ex formula in problemate data [p. 13], ubi notandum est ipsius λ' coefficientem circiter fore 4, et ipsius λ'' coefficiens semper maior erit quam 27; qui termini cum omnes sint positivi, evidens est pro α semper ingentem valorem reperiri, ita ut haec telescopia valde longa evadant.

EXEMPLUM 2

312. Pro casu postremo, quo $i=0$, si sumatur $\zeta=\frac{1}{2}$, telescopium inde oriundum describere.
Hoc igitur casu erit

$$\mathfrak{B}=\frac{-(m-2)}{2(m-1)}\ \text{ et }\ B=\frac{-(m-2)}{3m-4},$$

$$P=\frac{m}{2},\ \ k=\frac{2(m-2)}{m},\ \ k'=\frac{m}{m-2},\ M=\frac{1}{m(m-1)},$$

unde distantiae determinatrices

$$b=-\frac{2\alpha}{m},\ \ c=\frac{\alpha}{m(3m-4)},\ \ \beta=\frac{2(m-2)\alpha}{m(3m-4)},\ \ \gamma=\frac{C\alpha}{3m-4},\ \ d=\frac{m-2}{(3m-4)m}C\alpha$$

et distantiae focales

$$p=\alpha,\ \ q=\frac{m-2}{m(m-1)}\alpha,\ \ r=\frac{\mathfrak{C}\alpha}{3m-4},\ \ s=\frac{m-2}{(3m-4)m}$$

et intervalla

$$\alpha+b=\frac{m-2}{m}\alpha,\ \ \beta+c=\frac{\alpha}{m}=\frac{m-2}{2m}\alpha,\ \ \gamma+d=\frac{2(m-1)C\alpha}{m(3m-4)}$$

et

$$O=\frac{2(m-1)(m-2)C\alpha}{mm(3m-4)};$$

nunc vero campi semidiameter erit tantum

$$\Phi = \frac{430}{m-1} \text{ minut.}$$

In formula autem pro distantia α definienda notandum est coefficientem λ' fore $\frac{16}{m}$, ipsius vero $\lambda'' > \frac{27}{m\mathfrak{C}^3}$, siquidem multiplicatio sit praemagna; unde patet pro α valorem multo minorem prodire, ita ut hinc telescopia satis idonea obtinerentur, si modo campus non esset tam exiguus.

COROLLARIUM 1

313. Quia pro lente tertia sumsimus i hincque et $\pi' = 0$, eius apertura ex formulis generalibus [§ 23] definiri debet, cuius semidiameter erit $= \frac{rx}{B\mathfrak{C}\alpha}$, quae ergo pro priori exemplo fit $\frac{m-2}{m}x$, pro secundo antem $\frac{x}{m-2}$; unde, si sumatur $x = \frac{m}{50}$ dig., hic semidiameter erit circiter $\frac{1}{50}$ dig., quae ergo lens commodissime locum diaphragmatis tenebit.

COROLLARIUM 2

314. Si quasi medium sumendo inter duo exempla allata statuatur $\zeta = \frac{1}{\sqrt{m}}$, erit

$$P = \sqrt{m} \text{ et } k = 1 \text{ et } k' = \sqrt{m},$$

porro

$$\mathfrak{B} = -\frac{\left(\sqrt{m}-1\right)}{\sqrt{m}+1},\ B = \frac{-\left(\sqrt{m}-1\right)}{2\sqrt{m}},\ M = \frac{1}{m+\sqrt{m}}$$

atque hinc

$$b = -\frac{\alpha}{\sqrt{m}},\ \beta = \frac{+\left(\sqrt{m}-1\right)\alpha}{2m},\ c = \frac{\sqrt{m}-1}{2m}\alpha,\ \gamma = \frac{\sqrt{m}-1}{2m}C\alpha,\ d = \frac{\sqrt{m}-1}{2m\sqrt{m}}C\alpha,$$

$$\alpha + b = \left(1 - \frac{1}{\sqrt{m}}\right)\alpha,\ \beta + c = \frac{\sqrt{m}-1}{m}\alpha,\ \gamma + d = \frac{\sqrt{m}-1}{2m}C\alpha\left(1 + \frac{1}{\sqrt{m}}\right) = \frac{m-1}{2m\sqrt{m}}C\alpha$$

et distantia oculi

$$O = +\frac{(m-1)}{2mm}\alpha;$$

quare longitudo telescopii erit

$$\frac{m-1}{m}\left(1 + \frac{1+\sqrt{m}\cdot C}{2m}\right)\alpha$$

ac denique semidiameter campi

$$\Phi=\frac{\xi}{m+\sqrt{m}}=\frac{859}{m+\sqrt{m}}\ \text{minut.}$$

et semidiameter aperturae tertiae lentis

$$=\frac{x}{\sqrt{m}}=\frac{\sqrt{m}}{50}\ \text{dig.}$$

SCHOLION

315. Simili modo, quo hic casum $i=0$ expedivimus, etiam quaestio in genere pro quovis valore ipsius i resolvi poterit; ex aequatione enim

$\mathfrak{C}i=-(1+Pk)M+\zeta$ quum deducatur

$$Pk=\frac{\zeta-\mathfrak{C}i}{M}-1$$

et quia est $M=\frac{1+i-\zeta}{m-1}$, fiet

$$Pk=\frac{-\mathfrak{C}i(m-1)+m\zeta-i-1}{1+i-\zeta}.$$

Verum ob $k'=\frac{1}{i+k\zeta}$ erit etiam

$$Pk=\frac{m}{k'}=m(i+k\zeta),$$

unde colligimus

$$-\frac{\mathfrak{C}i}{M}+\frac{m\zeta-i-1}{1+i-\zeta}=mi+mk\zeta$$

hincque

$$k=-\frac{\mathfrak{C}i}{Mm\zeta}+\frac{m\zeta+mi\zeta-mii-mi-i-1}{(1+i-\zeta)m\zeta},$$

et quia est

$$k+k\zeta=-\frac{\mathfrak{C}i}{Mm}+\frac{m\zeta-i-1}{(1+i-\zeta)m},$$

erit

$$k'=\frac{m(1+i-\zeta)}{m\zeta-\mathfrak{C}i(m-1)-i-1}$$

ideoque

$$Pk=\frac{m\zeta-\mathfrak{C}i(m-1)-i-1}{1+i-\zeta}\quad\text{et}\quad P=\frac{m}{kk'}$$

quia nunc k' debet esse quantitas positiva, necesse est, ut sit

$$m\zeta>\mathfrak{C}i(m-1)+i+1\ ;$$

unde facto calculo semper reperietur esse $P>1$, ita ut, etiamsi non sit $i=0$, tamen solus casus secundus supra memoratus locum habeat. Quia autem hypothesis $i=0$ tam commodam et concinnam suppeditavit resolutionem, nulla plane est ratio, cur litteram i sive positivam sive negativam assumere vellemus, cum pro commodo nullum inde lucrum sit exspectandum. Praeter concinnitatem calculi autem duo commoda, quae nobis ista hypothesis $i=0$ largitur, maximi sunt momenti, quorum alterum, uti vidimus, in hoc consistit, ut litterae $\mathfrak{C}$ et C arbitrio nostro permittantur hocque modo nimia lentis ocularis parvitas evitari queat; alterum vero commodum huic nihil cedera est censendum, propterea quod tam exigua apertura lenti tertiae sine ullo sive campi sive claritatis detrimento tribui possit, ut omne lumen peregrinum tutius quam per diaphragmata ordinaria excludatur.

PROBLEMA 3

316. *Si telescopium huius generis ita ex quatuor lentibus sit componendum, ut binae mediae ambae inter imaginem priorem et posteriorem constituantur, indolem eius indagare eiusque constructionem describere.*

SOLUTIO

Positis igitur ut ante nostris fractionibus

$$\frac{\alpha}{b}=-P,\quad \frac{\beta}{c}=-Q \ \text{ et } \ \frac{\gamma}{d}=-R,$$

hic litterae P et R debent esse negativae manente Q positiva; quare si ponatur $P=-k$ et $R=-k'$, ut sit $m=Qkk'$, elementa nostra ita se habebunt:

$$b=\frac{\alpha}{k},\quad \beta=\frac{B\alpha}{k},\quad c=\frac{-B\alpha}{Qk},\quad \gamma=\frac{-BC\alpha}{Qk},\quad d=\frac{-BC\alpha}{Qkk'}=\frac{-BC\alpha}{m}$$

hincque intervalla

$$\alpha+b=\alpha\left(1+\frac{1}{k}\right) \text{ ideoque } \alpha>0,$$
$$\beta+c=\frac{B\alpha}{k}\left(1-\frac{1}{Q}\right), \text{hinc } B\left(1-\frac{1}{Q}\right)>0,$$
$$\gamma+d=\frac{-BC\alpha}{Qk}\left(1+\frac{1}{k'}\right), \text{ hinc } BC<0.$$

Pro campo apparente statuamus

$$\pi=-\omega\xi,\quad \pi'=+i\xi \ \text{ et } \ \pi''=-\xi,$$

ut fiat

$$\Phi=\frac{\omega+i+1}{m-1}\cdot\xi=M\xi$$

existente

$$M=\frac{\omega+i+1}{m-1},$$

atque hinc primo erit distantia oculi

$$O=\frac{-\pi''}{\Phi}\cdot\frac{d}{m}=\frac{d}{Mm};$$

deinde margo coloratus evanescet, si fuerit

$$0=\frac{\omega}{P}+\frac{i}{PQ}+\frac{1}{PQR} \text{ seu } 0=-\frac{\omega}{k}-\frac{i}{Qk}+\frac{1}{Qkk'},$$

unde concludimus

$$k'=\frac{1}{i+Q\omega} \quad \text{et} \quad m=\frac{Qk}{i+Q\omega};$$

tum vero considerari oportet sequentes aequationes:

$$-\frac{\mathfrak{B}\omega}{M}=1+k, \quad \frac{\mathfrak{C}i}{M}+\frac{\omega}{M}=-1-Qk$$

seu

$$\mathfrak{C}i=-(1+Qk)M-\omega$$

et

$$\mathfrak{B}\omega=-(1+k)M$$

quarum evolutio commode generaliter institui non potest, sed casus magis particulares contemplari conveniet. Verum casus extremi duo habentur, alter, quo lens in ipsam imaginem priorem, alter vero, quo in imaginem posteriorem cadit. Illo scilicet fit $Q=0$, hoc vero $Q=\infty$; inter hos autem quasi medius quidam praecipue perpendi meretur oriundus ex valore $Q=1$; quos casus deinceps seorsim evolvamus. Hic igitur tantum superest formulam adiungere pro confusione destruenda, ex qua scilicet distantia α determinatur,

$$\alpha=kx\sqrt[3]{\mu m\left(\lambda+\frac{1}{\mathfrak{B}k}\left(\frac{\lambda'}{\mathfrak{B}^2}+\frac{\nu}{B}\right)-\frac{\lambda'}{\mathfrak{B}^3\mathfrak{C}Qk}\left(\frac{\lambda''}{\mathfrak{C}^2 m}+\frac{\nu}{C}\right)-\frac{\lambda'''}{B^3C^3m}\right)}.$$

COROLLARIUM 1

317. Quoniam invenimus $k'=\frac{1}{i+Q\omega}$ ob $Q>0$ evidens est ambas litteras i et ω simul negativas esse non posse. Neque vero etiam ambae possunt esse positivae; si enim ω esset positivum, foret $\mathfrak{B}$ ideoque et B negativum hincque ob $BC<0$ deberet esse C positivum ideoque et $\mathfrak{C}$ positivum ac proinde $\mathfrak{C}i$ positivum, id quod fieri non posse ex valore pro ii supra dato manifestum est.

COROLLARIUM 2

318. Cum igitur ambae litterae ω et i nec positivae nec negativae esse queant, necesse est alteram esse positivam, alteram negativam. Si sit $\omega>0$, modo vidimus esse debere $\mathfrak{B}<0$ et $B<0$ hincque $C>0$. Sin autem sit $\omega<0$, erit $\mathfrak{B}>0$; de B vero hinc nihil definitur. Ex altera vero aequatione posito $\omega=-\zeta$ erit

$$\mathfrak{C}i = -(1+Qk)M + \zeta$$

unde intelligitur, si fuerit $\zeta > (1+Qk)M$, fore $\mathfrak{C} > 0$, sin autem sit $\zeta < (1+Qk)M$, fore $\mathfrak{C} < 0$. Prius autem evenit, si fuerit $1+k > (1+Qk)\mathfrak{B}$ seu $\mathfrak{B} < \frac{1+k}{1+Qk}$, posteriori vero, si $\mathfrak{B} > \frac{1+k}{1+Qk}$; hoc ipso autem posteriori casu cum sint $\mathfrak{C}$ et C negativa, debet esse B positivum ideoque $\mathfrak{B} < 1$, ex quo sequitur fore $Q > 1$.

EVOLUTIO CASUS PRIMI QUO $Q = 0$

319. Quia est $Q = 0$, erit secundum intervallum $= -\frac{B\alpha}{Qk} = c$ ideoque $\beta = 0$, ergo vel $B = 0$ vel $k = \infty$. At prius fieri nequit; foret enim $\mathfrak{B} = 0$ et q seu distantia focalis secundae lentis $= 0$, quod est absurdum. Restat ergo, ut sit $k = \infty$, et cum sit $q = \frac{\mathfrak{B}\alpha}{k}$ erit $\mathfrak{B} = \frac{kq}{\alpha} = \infty$ atque hinc $B = -1$. Ex quo sequitur ob $BC < 0$ fore $C > 0$ et $\mathfrak{C} < 1$. Cum vero sit $Q = 0$ et $k = \infty$, productum Qk debet esse finitum; quare statuatur $Qk = l$, ut sit $b = 0$, $\beta = 0$, $c = \frac{+\alpha}{l}$, $\gamma = \frac{C\alpha}{l}$, $d = \frac{C\alpha}{m}$ porroque $O = \frac{C\alpha}{Mm^2}$.

Destructio vero marginis colorati postulat $k' = \frac{1}{i}$, ita ut iam i certe sit fractio positiva et $m = \frac{1}{i}$. Ambae autem aequationes nostrae fundamentales dabunt, prior

$$\mathfrak{B}\omega = -kM \quad \text{sive} \quad \frac{kq\omega}{\alpha} = -kM \quad \text{ideoque} \quad \omega = -\frac{M\alpha}{q},$$

posterior vero

$$\mathfrak{C}i = -(1+l)M + \frac{M\alpha}{q};$$

quod cum debeat esse positivum, oportet esse $\frac{\alpha}{q} > l+1$ sive $q < \frac{\alpha}{1+l}$. Quia $\omega < 0$, scribatur $\omega = -\zeta$ et litteras i et ζ in calculo retineamus eritque $l = mi$, $q = \frac{M\alpha}{\zeta}$ ac proinde $\mathfrak{C}i = -(1+mi)M + \zeta$.

Unde, cum sit $\mathfrak{C} > 0$ simulque $\mathfrak{C} < 1$, nanciscimur hos limites:

$$1.\ \zeta > (1+mi)M, \quad 2.\ \zeta < (1+mi)M + i;$$

cum iam sit

$$M = \frac{1+i-\zeta}{m-1},$$

hoc valore substituto ex istis limitibus colliguntur sequentes:

$$1.\ \zeta > \frac{1+mi}{m}$$

et

$$2.\quad \zeta < \frac{1+mi}{m} + \frac{(m-1)i}{(1+i)} \text{ sive } \zeta < \frac{1+2mi+mii}{m(1+i)},$$

ex quibus, si littera i pro lubitu capiatur indeque ζ debite assumatur, omnia pro telescopio erunt determinata; quo autem melius de campo iudicare possimus, loco ζ seorsim utrumque limitem substituamus, ac prior quidem limes dabit $M = \frac{1}{m}$, alter vero limes maior $M = \frac{1}{m(1+i)}$; inter quos valores littera M ideoque et campus apparens continebitur.

Pro definienda autem distantia α formula superior hanc induet formam

$$\alpha = kx\sqrt[3]{\mu m\left(\lambda + * + \frac{1}{\mathfrak{C}l}\left(\frac{\lambda''}{\mathfrak{C}^2} + \frac{v}{C}\right) + \frac{\lambda'''}{C^3 m}\right)}.$$

De hoc autem casu iterum valet, quod supra [§ 232] commemoravimus, scilicet ob impuritates minimas lentis in loco imaginis constitutae repraesentationem obiectorum inquinari.

De cetero autem campus semper maior est semissi campi simplicis, quem vero defectum nova lente adiicienda facile supplere licet.

EVOLUTIO CASUS QUO $Q = \infty$

320. Hoc ergo casu fit secundum intervallum $\beta + c = \frac{B\alpha}{k}$; unde sequitur B positivum ideoque C negativum. Tum vero, quia $c = -\frac{\beta}{Q}$, erit $c = 0$ et $\gamma = 0$. Cum autem huius lentis distantia focalis sit $r = \mathfrak{C}c$, erit $\mathfrak{C} = \infty$ hincque $C = -1$, et quia $B > 0$, fiet $\mathfrak{B} > 0$ at < 1.

Cum porro sit $m = Qkk'$ neque vero $k = 0$, necesse est, ut sit $k' = 0$, ex quo ponatur $Qk' = l$, ut fiat $m = kl$. Iam vero ex margine colorato habemus $k' = \frac{1}{i+Qm} = \frac{l}{Q}$; unde sequitur $\omega = \frac{1}{l}$ hincque positivum. Cum autem $\mathfrak{B}$ sit positivum, ex prima aequatione fundamentali sequitur

$$\omega = \frac{-(1+k)M}{\mathfrak{B}},$$

unde oporteret esse ω quantitatem negativam; quod cum illi conclusioni adversetur, manifestum est hunc casum esse impossibilem seu potius hoc casu marginem coloratum destrui non posse. Ceterum hoc casu lens tertia in ipso loco secundae imaginis foret constituta; quod cum contradictionem involvat, hinc facile intelligitur tertiam lentem notabili intervallo ante imaginem posteriorem constitutam esse debere.

EVOLUTIO CASUS PRORSUS SINGULARIS QUO $Q = 1$ ET RADII PER BINAS LENTES PRIORES TRANSMISSI ITERUM FIUNT PARALLELI

321. Hoc ergo casu telescopium erit quasi ex duobus tubis astronomicis compositum certo quodam intervallo ab eodem axe a se invicem remotis, ad quod genus vulgaria telescopia terrestria dicta sunt referenda. Cum igitur sit $Q=1$, ne intervallum secundum $\beta+c$ ob $\beta=-Qc$ evanescat, debet esse tam β quam c infinitum, id quod eveniret, tam si $k=0$, quam si $B=\infty$; prius autem hic locum habere nequit, quia intervallum primum etiam fieret infinitum; ex quo necesse est, ut sit $B=\infty$ et $\mathfrak{B}=1$. Ne autem tertium intervallum evadat $=\infty$, productum BC debet esse quantitas finita et negativa; quare statuatur $BC=-\theta$ ideoque $C=-\frac{\theta}{B}=0.$ Ut autem intervallum medium valorem finitum, puta $=\eta\alpha$, obtineat, quantitas B non tanquam vere infinita, sed tantum praegrandis considerari debet, donec scilicet conditionibus praescriptis satisfecerimus, unde etiam valor ipsius Q aliquantillum ab unitate discrepare reperietur; quoniam enim esse debet

$$\frac{B\alpha}{k}\left(1-\frac{1}{Q}\right)=\eta\alpha,$$

inde fit

$$Q=\frac{B}{B-\eta\alpha}=1+\frac{\eta k}{B};$$

tum vero etiam erit

$$\mathfrak{B}=\frac{B}{1+B} \quad \text{et} \quad C=-\frac{\theta}{B} \quad \text{et} \quad \mathfrak{C}=\frac{-\theta}{B-\theta}=\frac{-\theta}{B}.$$

His notatis nostrae aequationes fundamentales erunt

$$\omega=\frac{-(1+k)M(1+B)}{B} \quad \text{et} \quad +\frac{\theta i}{B}=+(1+k)M+\frac{\eta k^2 M}{B}+\omega,$$

in qua si loco ω ex priore substituatur valor inventus, obtinebitur

$$\frac{\theta i}{B}=\frac{\eta k^2 M-(1+k)M}{B} \quad \text{hincque} \quad i=\frac{(\eta k^2-1-k)M}{\theta}$$

et nunc licebit ponere $B=\infty,\ \mathfrak{B}=1,\ C=\mathfrak{C}=0,$ ita tamen, ut sit $BC=\theta$.

Destructio autem marginis colorati praebet $k'=\frac{1}{i+\omega}$, et ob $kk'=m$ colligetur $i+\omega=\frac{k}{m}$; quia deinde est $M=\frac{1+i+\omega}{m-1}$, fiet nunc $M=\frac{m+k}{m(m-1)}$, et si valores pro i et ω inventi substituantur in formula $i+\omega=\frac{k}{m}$, orietur haec aequatio:

$$\frac{k}{m}=\frac{\left(\eta k^2-(1+k)(1+\theta)\right)M}{\theta}$$

et pro M substituto valore

$$\theta(m-1)k=\left(\eta k^2-(1+k)(1+\theta)\right)(m+k),$$

unde colligitur

$$\theta = \frac{(\eta k^2 - k - 1)(m+k)}{k^2 + 2mk + m},$$

et quia η debet esse numerus positivus, necesse est, ut sit $\eta > \frac{k+1}{k^2}$ et quidem ita, ut θ non fiat nimis exiguum; quandoquidem nunc elementa nostra ita exprimentur :

$$b = \frac{\alpha}{k},\ \beta = \infty,\ \ c = \infty,\ \gamma = \frac{\theta\alpha}{k},\ d = \frac{\theta\alpha}{m},$$

$$\alpha + b = \alpha(1 + \tfrac{1}{k}),\ \ \beta + c = \eta\alpha,\ \ \gamma + d = \theta\alpha\left(\tfrac{1}{k} + \tfrac{1}{m}\right)$$

indeque distantia oculi

$$O = \frac{\theta\alpha}{Mm^2} = \frac{(m-1)\theta\alpha}{m(m+k)}$$

atque distantiae focales

$$p = \alpha,\ \ q = \frac{\alpha}{k},\ \ r = \frac{\theta\alpha}{k},\ \ s = \frac{\theta\alpha}{m}$$

Distantia autem α definiri debet ex aequatione sequente:

$$\alpha = kx\sqrt[3]{\mu m}\left(\lambda + \frac{\lambda'}{k} + \frac{\lambda''}{\theta^2 k} + \frac{\lambda'''}{\theta^3 m}\right),$$

quare, ne valor ipsius α nimis fiat magnus, convenit k [sub radice] magnum assumi, tum vero θ non multo minus unitate; quod ad prius attinet, etiam campus apparens suadet litterae k quam maximum valorem dare, quia tum M continuo magis crescit; verum probe notandum est in formula $\Phi = M\xi$ pro littera ξ eatenus tantum valorem $\frac{1}{4}$ assumi posse, quatenus litterae i et ω unitatem non superant, ita ut, si vel i vel ω unitatem superaret, tum ξ in eadem ratione diminui deberet. Quaro ob causam maximi momenti est in eum valorem ipsius k inquirere, unde prodeat $i = 1$. Posito autem $i = 1$ reperimus

$$1 + \omega = \frac{k}{m}\ \text{ seu }\ m(m-2) = k^2 + 2mk,$$

cuius aequationis resolutio praebet

$$k = -m + \sqrt{2m(m-1)}.$$

Hic scilicet valor ipsius k nobis praebet $i = 1$ et

$$\omega = \frac{k-m}{m} = \frac{-2m + \sqrt{2m(m-1)}}{m},$$

qui valor est negativus et unitate minor, unde pro campo apparente habebitur

$$\Phi = \frac{\sqrt{2m(m-1)}}{m(m-1)} \cdot \xi = \sqrt{\frac{2}{m(m-1)}} \cdot \xi;$$

sin autem k adhuc maiorem adipisceretur valorem, prodiret quidam i maius unitate, sed tum ξ ita sumi deberet, ut fieret $i\xi = \frac{1}{4}$ seu $\xi = \frac{1}{4i}$, sicque pro campo prodiret $\Phi = \frac{1+i+\omega}{m-1} \cdot \frac{1}{4i}$; unde calculum instituenti innotescit campum continuo diminui eo magis, quo valor ipsius k illum terminum superaverit. Maxime igitur hic casus lucrosus est, si capiatur

$$k = -m + \sqrt{2m(m-1)},$$

unde fit

$$k' = \frac{m+\sqrt{2m(m-1)}}{m-2}$$

SCHOLION

322. Quia in antecedente problemate casus maxime memorabilis est deductus ponendo $i = 0$, suspicari quis posset etiam hic talem positionem institui convenire. Quamobrem hic ostendamus in hoc problemate neque positionem $i = 0$ neque $\omega = 0$ locum habere posse.

Primo enim si esset $\omega = 0$, ob $k' = \frac{1}{i+Q\omega}$ deberet esse $i > 0$; at ob $\omega = 0$ prima aequatio

$$\mathfrak{B}\omega = -(1+k)M$$

subsistere nequit, nisi sit $\mathfrak{B} = \infty$ ideoque $B = -1$; iam ob $BC < 0$ debet esse C positivum ideoque $\mathfrak{C}$ etiam > 0, ex quo patet alteram aequationem

$$\mathfrak{C}i = -(1+Qk)M$$

plane subsistere non posse; sicque evictum est sumi non posse $\omega = 0$.

Simili modo ostendetur numerum i evanescere non posse; tum enim ob $k' = \frac{1}{i+Q\omega}$ deberet esse $\omega > 0$ hincque posterior aequatio

$$\mathfrak{C}i = -(1+Qk)M - \omega$$

subsistere nequit, nisi sit $\mathfrak{C}i$ quantitas finita negativa ideoque $\mathfrak{C} = \infty$; unde fit $C = -1$ et hinc ob $BC < 0$ fiet $B > 0$ simulque $\mathfrak{B} > 0$, id quod primae aequationi

$$\mathfrak{B}\omega = -(1+k)M$$

manifesta contradicit; ex quo perspicuum est etiam numerum i non posse capi $=0$.

Neque ergo praeter tres casus hic commemoratos ullus alius hic perpendi meretur atque postremus adeo tantis commodis reliquos omnes antecedit, ut is solus dignus videatur, qui in praxin deducatur; non solum enim maximum campum aperit, sed etiam pro α valorem non nimis magnum largitur, quoniam in illa formula radicali cubica termini post λ sequentes omnes fiunt valde parvi eoque minores, quo maior fuerit multiplicatio, quoniam proxima fit $k=m\left(\sqrt{2}-1\right)=\frac{1}{2}m$. Tum vero hic etiam numerus θ arbitrio nostro permittitur, quo efficere possumus, ut lentes postremae non fiant nimis exiguae; sumto autem θ pro lubitu quantitas η sequenti aequatione definietur; quia enim supra invenimus

$$\theta=\frac{(\eta k^2-k-1)(m+k)}{k^2+2mk+m},$$

ob $m\left(m-2\right)=2mk+k^2$ et $m+k=\sqrt{2}m(m-1)$ erit

$$\theta=\frac{(\eta k^2-k-1)\sqrt{2}}{\sqrt{m\left(m-1\right)}}$$

hincque

$$\eta=\frac{k+1}{k^2}+\frac{\theta\sqrt{m\left(m-1\right)}}{k^2\sqrt{2}},$$

ex quo valore intervallum secundae et tertiae lentis innotescit.

PROBLEMA 4

323. *Si telescopium huius generis ita ex quatuor lentibus sit componendum, ut una lens inter imaginem secundam et ocularum constituatur, indolem eius indagare eiusque constructionem describere.*

SOLUTIO

Quia igitur hic prima imago inter lentem primam et secundam, secunda vero imago inter lentem secundam et tertiam cadit, litterae P et Q erunt negativae manente sola R positiva. Quare si statuatur $P=-k$ et $Q=-k'$, erunt elementa nostra

$$b=\frac{\alpha}{k},\ \beta=\frac{B\alpha}{k},\ c=\frac{B\alpha}{kk'},\ \gamma=\frac{BC\alpha}{kk'},\ \text{ et }\ d=\frac{-BC\alpha}{kk'R}=\frac{-BC\alpha}{m}.$$

Hincque intervalla

$$\alpha+b=\alpha\left(1+\frac{1}{k}\right)\ \text{ideoque }\ \alpha\ \text{ positivum,}$$

$$\beta + c = \frac{B\alpha}{k}\left(1 + \frac{1}{k'}\right), \text{ ergo } B > 0 \text{ et } \mathfrak{B} > 0 \text{ et simul } \mathfrak{B} < 1,$$

$$\gamma + d = \frac{Bk\alpha}{kk'}\left(1 - \frac{1}{R}\right), \text{ ergo } C\left(1 - \frac{1}{R}\right) > 0.$$

Pro loco autem oculi erit $O = \frac{d}{Mm}$; quae ut sit positiva, debet esse , $d > 0$, unde haec nova resultat conditio, ut sit $C < 0$, quae conditio cum antecedente coniuncta dat $1 - \frac{1}{R} < 0$ ideoque $R < 1$. Quodsi iam ponamus

$$\pi = -\omega\xi, \quad \pi' = i\xi \text{ et } \pi'' = -\xi,$$

ut fiat

$$\Phi = \frac{\omega + i + 1}{m - 1} \cdot \xi = M\xi$$

existente

$$M = \frac{\omega + i + 1}{m - 1}$$

aequationes nostrae fundamentales erunt

$$\mathfrak{B}\omega = -(1 + k)M \text{ et } \mathfrak{C}i = -(1 + kk')M - \omega,$$

ex quarum priore statim ob $\mathfrak{B} > 0$ liquet fore $\omega < 0$.

Destructio autem marginis colorati postulat, ut sit

$$0 = \frac{\omega}{P} + \frac{i}{PQ} + \frac{1}{PQR}$$

ideoque

$$0 = -\frac{\omega}{k} + \frac{i}{kk'} + \frac{i}{kk'R} \text{ unde } R = \frac{1}{\omega k' - i};$$

ut ergo R prodeat positivum, i necessario debet esse numerus negativus.
Statuamus ergo $\omega = -\zeta$; et $i = -y$, ut iam sit pro campo apparente

$$M = \frac{1 - y - \zeta}{m - 1} \text{ ideoque } y + \zeta < 1.$$

Cum igitur sit

$$R = \frac{1}{y - k'\zeta} \text{ atque hinc } m = \frac{kk'}{y - k'\zeta},$$

notandum est ob $R < 1$ et $R = \frac{m}{kk'}$ esse debere $kk' > m$; hinc, quia est $y = k'\zeta + \frac{kk'}{m}$, erit $y > 1$ ideoque multo magis $y + \zeta > 1$; quod cum sit absurdum, patet huius problematis casum locum habere non posse.

SCHOLlON

324. Cum igitur hoc problema penitus sit excludendum, cum aeque parum conditioni marginis colorati satisfacere possit atque primum tribus tantum lentibus adhibitis, relinquuntur nobis tantum problema secundum ac tertium. Quia autem ex secundo casus prorsus singularis ibi annotatus maxime reliquis omnibus antecellit, quemadmodum etiam ex tertio casus ultimus prae ceteris maximam attentionem meretur, hinc constituemus duas praecipuas species telescopiorum tertii generis easque seorsim ita pertractabimus, ut primo ostendamus, quemadmodum utraque una vel pluribus lentibus ex eodem vitro adiiciendis, deinde etiam ex diverso vitro, ad maiorem perfectionis gradum evehi queant. Harum duarum vero specierum posterior ideo potissimum est notanda, quia telescopia communia terrestria dicta quasi in se complectitur; revera enim ab iis differt plurimum, quatenus a vitiis, quibus haec instrumenta, uti vulgo fabricari soient, laborant, est liberata; unde si etiam plures lentes in subsidium vocare nolimus, hinc regulae dari poterunt haec telescopia terrestria ita perficiendi, ut maior perfectio exspectari nequeat. Prior autem species, quae longe aliam lentium ocularium dispositionem postulat, olim prorsus fuit ignota ac nuper demum a solertissimo Dollondo in praxin introduci est coepta. Quatenus scilicet lentibus minima apertura praeditis est usus; neque tamen a sola experientia summus perfectionis gradus, cuius haec species est capax, sperari poterat. Hoc tamen facile est animadversum, nisi insuper una lens adiungatur, campum nimis fore parvum, quam ut ei acquiescere queamus. Vidimus enim campum semper aliquanto esse minorem quam in tubis astronomicis vulgaribus, ad quod remedium etiam in sequentibus recurremus. Denique circa hanc speciem annotari convenit nos in posterum iis mensuris esse usuros, quae in paragrapho 314 sunt statutae, ubi scilicet posuimus $\zeta = \frac{1}{\sqrt{m}}$, cum inde aptissimae ad praxin determinationes obtineri videantur.