CHAPTER IV

CATADIOPTRIC TELESCOPES CONSTRUCTED

WITH A CONVEX SMALLER MIRROR

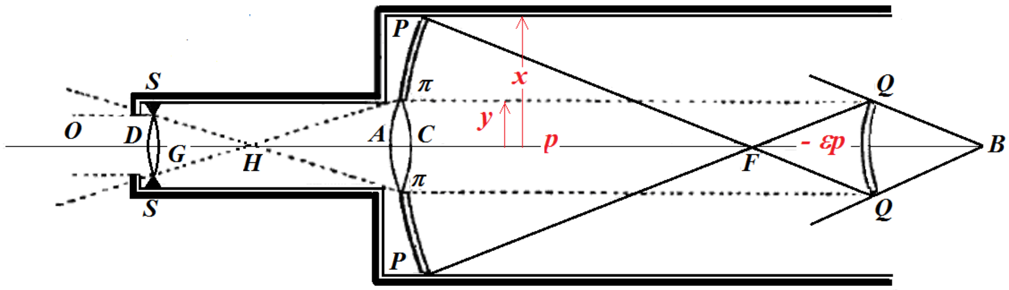

Fig. 8 modified

PROBLEM 1

50. *To describe the construction of telescopes of this kind, for which objects will be represented situated inverted or where a single real image may occur.*

SOLUTION

Since in this kind the separation of both the mirrors shall be $AB = (1-\varepsilon)p$ and thus $b = -\varepsilon p$, on account of $\alpha = p$ there will become $P = -\frac{\alpha}{b} = +\frac{1}{\varepsilon}$, where ε may designate some fraction smaller than the ratio of the hole to the greater mirror $\frac{y}{x}$ may designate, thus so that on putting $y = \delta x$ there shall become $\varepsilon < \delta$ on account of the ratio provided before in § 49, from which clearly it prevails, that the oblique rays also may be removed by the smaller mirror. Yet meanwhile the radius of the aperture of the smaller mirror may remain $= \delta x = y$, thus so that the radius of this mirror is equal to that of the hole, as we have assumed initially.

Now we may consider at once the equation, by which the colored margin is removed; which, if as well as the two mirrors two lenses may be used, is reduced to this form:

$$0 = \mathfrak{r} + \frac{\mathfrak{s}}{R},$$

from which, since both the letters $\mathfrak{r}$ and $\mathfrak{s}$ may be able to have positive values, as the account of the field may demand, it will be agreed to attribute a negative value to the letter R and certainly not to be smaller than unity, so that by assuming $\mathfrak{s} = 1$ the first lens *C*, of which the aperture is determined now by the hole, may not restrict the field. Therefore we may put $R = -i$, and since from the given magnification *m* on account of the inverse representation there shall be $PQR = -m$, hence there will become $PQ = \frac{m}{i}$ and $Q = \frac{\varepsilon m}{i}$. Truly there is $Q = -\frac{\beta}{c}$, and since there is

$$\beta + c = BC = AB = (1-\varepsilon)p,$$

we deduce hence

$$c = -\frac{i(1-\varepsilon)p}{\varepsilon m - i} \text{ and } \beta = \frac{\varepsilon m(1-\varepsilon)p}{\varepsilon m - i};$$

whereby, since in general $\frac{1}{q} = \frac{1}{b} + \frac{1}{\beta}$, there will be

$$\frac{1}{q} = \frac{-(1-2\varepsilon)m - i}{\varepsilon m(1-\varepsilon)p} \text{ and hence } q = -\frac{m\varepsilon(1-\varepsilon)p}{(1-2\varepsilon)m + i}.$$

Again from the values b and β we gather

$$B = \frac{\beta}{b} = \frac{-m(1-\varepsilon)}{\varepsilon m - i} \text{ and } \mathfrak{B} = \frac{+m(1-\varepsilon)}{(1-2\varepsilon)m + i}.$$

Then since there shall be $C = \frac{\gamma}{c}$ and $\mathfrak{C}c = r$, hence we find

$$\mathfrak{C} = \frac{r}{c} = -\frac{(\varepsilon m - i)r}{i(1-\varepsilon)p}$$

and thus

$$C = \frac{-(\varepsilon m - i)r}{i(1-\varepsilon)p + (\varepsilon m - i)r} = \frac{\gamma}{c},$$

from which again we deduce

$$\gamma = \frac{i(1-\varepsilon)pr}{i(1-\varepsilon)p + (\varepsilon m - i)r}.$$

Finally, since there shall be $R = -\frac{\gamma}{d} = -\frac{\gamma}{\varepsilon}$ on account of $s = d$, there will become

$$s = -\frac{\gamma}{R} = \frac{\gamma}{i} = \frac{(1-\varepsilon)pr}{i(1-\varepsilon)p + (\varepsilon m - i)r}$$

and hence the third interval

$$CD = \gamma + s = \frac{(1+i)(1-\varepsilon)pr}{i(1-\varepsilon)p + (\varepsilon m - i)r}.$$

Moreover now these equations give rise to the apertures :

$$1.\mathfrak{B}\mathfrak{q}=(P-1)M, \text{ from which there becomes } \mathfrak{q}=\frac{((1-2\varepsilon\ )+i)M}{\varepsilon m},$$

$$2.\ \mathfrak{C}\mathfrak{r}=(PQ-1)M-\mathfrak{q}=\frac{sm^2-(1-\varepsilon\ )im-i^2}{\varepsilon im}\cdot M$$

or

$$\mathfrak{C}\mathfrak{r}=\frac{(m+i)(\varepsilon m-i\ )}{\varepsilon im}\cdot M,$$

from which there is deduced

$$\mathfrak{r}=-\frac{(m+i)(1-\varepsilon\ )p}{\varepsilon mr}\cdot M,$$

from which, since there shall become $\mathfrak{s}=ir$ and therefore $\mathfrak{r}+\mathfrak{s}=(1+i)\mathfrak{r}$, there will become

$$\mathfrak{q}+\mathfrak{r}+\mathfrak{s}=-\frac{(1-2\varepsilon\ )mr+ir-(1+i)(m+i)(1-\varepsilon\ )p}{\varepsilon mr}\cdot M=M(m-1)$$

and thus on division by *M* we will find

$$r=\frac{-(m+i)(1+i)(1-\varepsilon\ )p}{\varepsilon m^2-(1-\varepsilon\ )m-i}\cdot M;$$

which value since it shall be negative, from that also the interval *CD* will be produced negative, from which it is clear that in practice this value cannot be used.

Truly since on many occasions the problems allow several solutions, likewise also here there arises in use besides the solution found here in addition another is included, which by division we have excluded from the calculation. So that which may be seen more easily, thus we may establish the calculation . Since initially there shall be

$$\mathfrak{q}=\frac{(1-2\varepsilon\ )m+i}{\varepsilon m}\cdot M, \text{ then } \mathfrak{s}=i\mathfrak{r},$$

there will become

$$\mathfrak{q}+\mathfrak{r}+\mathfrak{s}=\frac{(1-2\varepsilon\ )m+i}{\varepsilon m}\cdot M+(1+i)\mathfrak{r}=M(m-1),$$

from which there is deduced:

$$M=\frac{\varepsilon m(1+i)\mathfrak{r}}{\varepsilon m^2-(1-\varepsilon)m-i}.$$

and thus,

$$\mathfrak{q}=\frac{(1+i\ )((1-2\varepsilon)m+i)\mathfrak{r}}{\varepsilon m^2-(1-\varepsilon)m-i};$$

truly the other equation will give

$$\mathfrak{C}\mathfrak{r}=\frac{(m-i)\varepsilon m(1+i\ )\mathfrak{r}-i(1+i\ )((1-2\varepsilon)m+i)\mathfrak{r}}{\varepsilon im^2-(1-\varepsilon)im-i^2},$$

from which there becomes

$$\mathfrak{C} = \frac{(1+i)(m+i)(\varepsilon m-i)}{i(\varepsilon m^2-(1-\varepsilon)m-i)};$$

truly above we have found

$$\mathfrak{C} = \frac{-(\varepsilon m-i)r}{i(1-\varepsilon)p},$$

from which the equality of these two values can be found in two ways:

1. Evidently, if there were $i=\varepsilon m$, for which each value obviously vanishes,
2. moreover, so that with the division made by $\varepsilon m - i$, there becomes

$$\frac{(1+i)(m+i)}{\varepsilon m^2-(1-\varepsilon)m-i} = \frac{-r}{(1-\varepsilon)p},$$

and this is the incongruous solution found before. Therefore now we may put in place $i=\varepsilon m$ and there will become $\mathfrak{C}=0$, plainly hence the letter r truly is not found and our solution will be had in the following manner :

$$\alpha = p, \quad b=-\varepsilon p, \quad \beta=\infty, \quad c=-\infty, \quad \gamma = r, \quad d=\frac{r}{\varepsilon m},$$

where it may be observed to become $\beta + c = (1-\varepsilon)p$.
Hence therefore there will become:

$$B=\infty, \quad \mathfrak{B}=1, \quad C=0, \quad \mathfrak{C}=0,$$

then truly,

$$P=\tfrac{1}{\varepsilon}, \quad Q=1, \quad R=-\varepsilon m,$$

thus so that there shall become $PQR=-m.$

Truly since $B=\infty$ and $C=\mathfrak{C}=0$, the product formed remains undefined ; truly since there shall be

$$r=\frac{B\mathfrak{C}}{PQ}\cdot p=\varepsilon B\mathfrak{C}p,$$

hence in turn there will become

$$B\mathfrak{C}=\frac{r}{\varepsilon p}.$$

Truly besides the focal lengths will be

$$q=-\varepsilon p \text{ and } s=d=\frac{r}{\varepsilon m}$$

and the intervals

$$AB=BC=(1-\varepsilon)p \text{ and } CD=r\left(1+\frac{1}{\varepsilon m}\right).$$

Finally, since there shall become

$$\mathfrak{q} = \frac{(1+\varepsilon m)(1-\varepsilon)\mathfrak{r}}{\varepsilon m-1} \text{ and } \mathfrak{s} = \varepsilon m\mathfrak{r},$$

there will become

$$M = \frac{\varepsilon(1+\varepsilon m)\mathfrak{r}}{\varepsilon m-1} = \frac{(1+\varepsilon m)\mathfrak{s}}{m(\varepsilon m-1)}$$

and thus the radius of the apparent field of view

$$\Phi = \frac{1}{4}\cdot\frac{(1+\varepsilon m)\mathfrak{s}}{m(\varepsilon m-1)} = 859\frac{(\varepsilon m+1)\mathfrak{s}}{m(\varepsilon m-1)} \text{ minutes,}$$

where it will be allowed to assume $\mathfrak{s}=1$, but only if each side of the eyepiece lens may be made equally convex.

Truly the distance of the eye is found after the lens

$$O = \frac{\mathfrak{s}s}{Mm} = \frac{\varepsilon m-1}{\varepsilon m+1}\cdot s.$$

But since the radius of the aperture of the lens C cannot be greater than $y=\delta x$, we may put

$$\tfrac{1}{4}\mathfrak{r}r = \delta x \text{ or } \frac{\mathfrak{s}r}{4\varepsilon m} = \delta x;$$

from which on taking $\mathfrak{s}=1$ there is defined

$$r = 4\delta\varepsilon mx \text{ and hence } s = 4\delta x.$$

Truly also it is required to attend to the aperture of the smaller mirror, of which the radius actually is $=\delta x$; and which on account of the apparent field of view must be $=\frac{1}{4}\mathfrak{q}q$; which by this reason it is necessary that there shall be

$$\frac{(1-\varepsilon)(\varepsilon m+1)\mathfrak{s}p}{4m(\varepsilon m-1)} < \delta x$$

and thus

$$\mathfrak{s} < \frac{4m(\varepsilon m-1)\delta x}{(1-\varepsilon)(\varepsilon m+1)p}.$$

Therefore without risk it will be allowed to take $\mathfrak{s}=1$, but only if there were

$$4m(\varepsilon m-1)\delta x > (1-\varepsilon)(\varepsilon m+1)p.$$

Truly on the other hand $\mathfrak{s}$ ought to be taken less than unity.

Therefore it remains only, so that we may define the focal length of the principal mirror p from the formula of the radius of confusion, which is found to be expressed thus :

$$p = kx\sqrt[3]{m}\left(\tfrac{1-\varepsilon}{8} + \mu\tfrac{\varepsilon^4 p^3}{r^3}\lambda'' + \mu\tfrac{\varepsilon^3 p^3}{mr^3}\lambda'''\right)^{\frac{1}{3}}$$

if indeed a spherical figure may be induced in both mirrors; but if both may have a parabolic figure, there must become

$$r = k\varepsilon x\sqrt[3]{\mu m\left(\varepsilon\lambda'' + \tfrac{\lambda'''}{m}\right)},$$

thus so that otherwise now it may not be defined except by the magnitude of the mirror, since without doubt p must always be much greater than x. Truly since now as we have defined before $r = 4\delta\varepsilon mx$, we will have now

$$4\delta m = k\sqrt[3]{\mu m\left(\varepsilon\lambda'' + \tfrac{\lambda'''}{m}\right)}.$$

Now, since there shall be approx. $\mu = 1$ and there may be able to be taken $\lambda'' = 1$ and λ''' shall be less by 2, truly k may not be able to be taken below 50 , we will be able to estimate the value of ε ; indeed it must be taken so great, so that the number

$$\frac{4\delta m}{\sqrt[3]{(\varepsilon m+2)}}$$

may not be produced less than 50; from which it is apparent a very small fraction must be taken for ε; indeed if there shall be $\delta = \frac{1}{5}$ and $m = 100$, is it deduced $\varepsilon = \frac{1}{50}$ approximately.

EXAMPLE

51. We may put $m = 100$, $x = 2$ in., $y = \frac{1}{2}$ in. and thus $\delta = \frac{1}{4}$, and so that

$$\frac{4\delta m}{\sqrt[3]{(\varepsilon m+2)}}$$

may obtain a great enough value, we may assume $\varepsilon = \frac{1}{20}$; thus indeed $k = \frac{100}{\sqrt[3]{7}}$ or $k > 50$ arises ; hence therefore there will be $r = 10$ in. and $s = 2$ in. Then since for the smaller mirror there must be taken $8000 > 57p$, there will become

$$p < \tfrac{8000}{57}\cdot$$

From which without risk there can be taken $p = 25$ in. and thus there will become $q = -\frac{5}{4}$ in. and the interval $AB = BC = 23\frac{3}{4}$ in. and $CD = 12$ in. Truly the distance of the eye $O = \frac{4}{3}$ in., moreover the radius of the apparent field $\Phi = 12' \, 53''$, where properly it is required to observe here both mirrors assumed to be parabolic.

SCHOLIUM

52. Although this construction may succeed perfectly, yet such a telescope is predisposed to the most significant fault, so that it may be destitute from all use ; for since the rays reflected from the smaller mirror may become parallel to each other again, the stray rays passing around this mirror and incident on the lens *C* may suffer a common refraction and in short may be sent into the eye mixed likewise with these, thus so that the true object will be represented to the vision with the nearby rays, nor in any way will they be able to be separated. Therefore since the cause of this defect may be accommodated there, so that the rays reflected from the smaller mirror may become parallel or the interval $\beta = \infty$, lest this may be done, it will be required to take precautions carefully, so that it may be done, if the distance β were smaller than the interval *BC,* thus so that in this interval a real image may be incident and the letter *Q* may obtain a negative value. Truly besides, since *R* ought to have a negative value on account of the colored margin, now two real images will be had and the object will be perceived to be placed erect. Nor truly with only two lenses being used will we be able to satisfy our goal, but it will be required to call into aid in addition a lens, which will be able to be constructed most conveniently, so that it may require as a minimum aperture, if indeed it may be able to succeed in separating the stray rays in this manner, just as we may show in the following problem.

PROBLEM 2

52[*a*] *To construct a telescope of this kind with a smaller convex mirror and with three glass lenses, which object situated upright will be represented distinctly.*

SOLUTION

There may remain as before $y = \delta x$ and the separation of the mirrors $AB = \left(1-\varepsilon\right)p = BC$, so that there shall become $b = -\varepsilon p$. Now since there must become $\beta < (1-\varepsilon)p$ and yet it must exceed the half of this $\frac{1}{2}(1-\varepsilon)p$, we may put

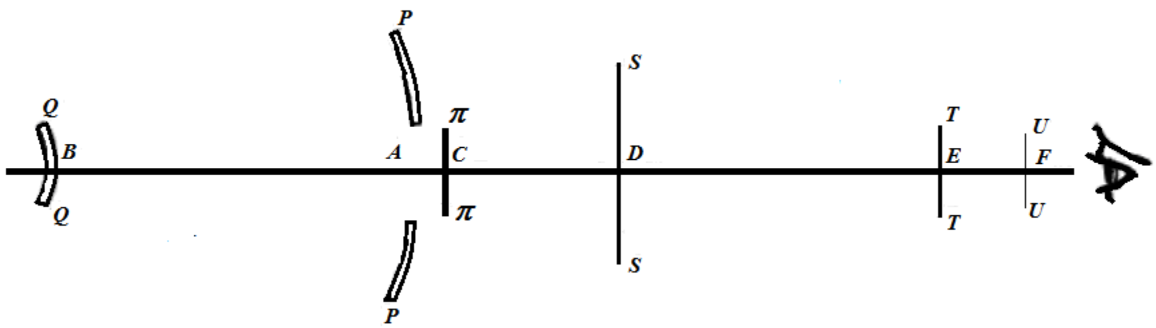

Fig. 9.

$\beta=\zeta(1-\varepsilon)p$, thus so that ζ may be contained within the bounds 1 and $\frac{1}{2}$; hence there will become therefore

$$q=\frac{\zeta\varepsilon(1-\varepsilon)}{\varepsilon-\zeta(1-\varepsilon)}\cdot p \;=\frac{-\zeta\varepsilon(1-\varepsilon)}{\zeta-\varepsilon(\zeta+1)}\cdot p.$$

Then truly there will be

$$B=\frac{\beta}{b}=\frac{-\zeta(1-\varepsilon)}{\varepsilon} \text{ and } \mathfrak{B}=\frac{\zeta(1-\varepsilon)}{\zeta-\varepsilon(\zeta+1)}.$$

Truly again there will be

$$c=(1-\varepsilon)(1-\zeta)p$$

and thus we will have

$$P=\frac{1}{\varepsilon},\quad Q=\frac{-\beta}{\varepsilon}=\frac{-\zeta}{1-\zeta}.$$

Therefore in addition we may put $R=-k$ and there shall become

$$PQRS=m=\frac{\zeta k}{\varepsilon(1-\zeta)}\cdot \mathrm{S},$$

from which the remaining focal distances will be

$$r=(1-\varepsilon)(1-\zeta)\mathfrak{C}p,\quad s=\frac{(1-\varepsilon)(1-\zeta)C\mathfrak{D}}{k}\cdot p$$

and

$$t=\frac{-\zeta(1-\varepsilon)CD}{\varepsilon m}\cdot p$$

and the remaining intervals

$$CD=(1-\varepsilon)(1-\zeta)\left(1+\frac{1}{k}\right)Cp,$$
$$DE=(1-\varepsilon)(1-\zeta)\left(1-\frac{1}{S}\right)CDp,$$

from which we will understand there must become
$C>0$ and thus $\mathfrak{C}<1$ and ($1-\frac{1}{S})D>0.$ Truly so that there may become $t>0$, there must become $D<0$ and thus $S<1$.

Now we will consider the equation for the colored margin being required to be removed, which is

$$0=\mathfrak{r}+\frac{\mathfrak{s}}{R}+\frac{\mathfrak{t}}{RS} \quad \text{or} \quad \mathfrak{r}=\frac{\mathfrak{s}}{k}+\frac{\mathfrak{t}}{kS};$$

so that now the second lens shall need a zero aperture, there may be put $\mathfrak{s}=0$ and there will become

$$\mathfrak{r}=\frac{\mathfrak{t}}{kS};$$

moreover the equations for the letters $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$, by putting

$$M=\frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t}}{m-1}=\frac{\mathfrak{q}+(1+kS)\mathfrak{r}}{m-1},$$

are :

$$1.\ \mathfrak{B}\mathfrak{q}=\frac{1-\varepsilon}{\varepsilon}\cdot M,$$

$$2.\ \mathfrak{C}\mathfrak{r}=\frac{-((1-\varepsilon)\zeta+\varepsilon)}{\varepsilon(1-\zeta)}\cdot M-\mathfrak{q},$$

$$3.\ 0=\frac{\zeta(\varepsilon+k)-\varepsilon}{\varepsilon(1-\zeta)}\cdot M-\mathfrak{q}-\mathfrak{r}.$$

From the first there will be had

$$\mathfrak{q}=\frac{\zeta-\varepsilon(\zeta+1)}{\varepsilon\zeta}\cdot M.$$

But from the third there becomes

$$\mathfrak{q}=\frac{\zeta(\varepsilon+k)-\varepsilon}{\varepsilon(1-\zeta)}\cdot M-\mathfrak{r},$$

which two values equated to each other give

$$M=\frac{\varepsilon\zeta(1-\zeta)\mathfrak{r}}{\zeta^2(1+k)-\zeta(1+\varepsilon)+\varepsilon}$$

and hence

$$\mathfrak{q}=\frac{(1-\zeta)(\zeta-\varepsilon(1+\zeta))\mathfrak{r}}{\zeta^2(1+k)-\zeta(1+\varepsilon)+\varepsilon}.$$

Then truly on account of $M=\frac{\mathfrak{q}+\mathfrak{r}(1+kS)}{m-1}$ there will be found also

$$M=\frac{kS\mathfrak{r}}{m-1+\frac{-\zeta(\varepsilon+k)+\varepsilon}{\varepsilon(1-\zeta)}},$$

from the equality of which values there is found finally

$$\zeta\left(\zeta kS-\varepsilon(1-\zeta)-\zeta(\varepsilon+k)+\varepsilon\right)=kS\left(\zeta^2(1+k)-\zeta(1+\varepsilon)+\varepsilon\right)$$

or

$$\zeta^2=S\left(\zeta(1+\varepsilon)-k\zeta^2-\varepsilon\right),$$

from which we conclude there must become

$$\zeta(1+\varepsilon)>k\zeta^2+\varepsilon \quad \text{or} \quad k<\frac{\zeta(1+\varepsilon)-\varepsilon}{\zeta^2}.$$

Truly in addition, so that the second equation for $\mathfrak{C}$ may produce a positive value, it is necessary, that there shall be $\mathfrak{q}<0$ and thus also $\mathfrak{B}<0$, from which the smaller mirror may become less concave; truly so that $\mathfrak{B}<0$, there must be $\zeta<\varepsilon(\zeta+1)$ or $\varepsilon>\frac{\zeta}{\zeta+1}$. Truly this does not suffice, but in addition it is necessary, that there shall be

$$-\mathfrak{q}>\frac{(1-\varepsilon)\zeta+\varepsilon}{\varepsilon(1-\zeta)}\cdot M$$

or

$$\frac{-\zeta+\varepsilon(\zeta+1)}{\zeta}>\frac{(1-\varepsilon)\zeta+\varepsilon}{(1-\zeta)},$$

from which it follows that $\varepsilon>\frac{\zeta}{1-\zeta}$; but since in now way may this be done, since ζ will be contained between the limits 1 and $\frac{1}{2}$ and ε must be less than unity, now at last we understand that this case cannot happen.

ANOTHER SOLUTION

53. Therefore since this inconvenience thence arises, because we have assumed *R* negative, we will consider the other case, where *S* becomes negative with *R* remaining positive, and because *Q* has been put negative, we may put

$$Q=-i \text{ and } S=-k,$$

so that there shall become

$$PQRS=\frac{iRk}{\varepsilon}=m;$$

but the calculation may emerge more conveniently, if the letter *i* may be retained, and since there shall be $i = \frac{\beta}{\varepsilon}$ and $\beta + c = (1-\varepsilon)p$, it is evident there must be taken $i > 1$ and there will become

$$B = \frac{i(1-\varepsilon)}{1+i} \cdot p \quad \text{and} \quad c = \frac{1-\varepsilon}{1+i} \cdot p,$$

from which there becomes

$$B = +\frac{\beta}{b} = -\frac{i(1-\varepsilon)}{\varepsilon(1+i)} \quad \text{and} \quad \mathfrak{B} = +\frac{i(1-\varepsilon)}{i(1-2\varepsilon)-\varepsilon}$$

and hence

$$q = -\frac{i\varepsilon(\mathrm{i}-\varepsilon)}{i(1-2\varepsilon)-\varepsilon} \cdot p.$$

Truly the remaining focal distances will be

$$r = +\frac{(1-\varepsilon)\mathfrak{C}}{(1+i)} \cdot p, \quad s = -\frac{(1-\varepsilon)C\mathfrak{D}}{(1+i)R} \cdot p \quad \text{and} \quad t = -\frac{(1-\varepsilon)CD}{(1+i)Rk} \cdot p$$

and the two remaining distances will be

$$CD = +\frac{(1-\varepsilon)C}{1+i}\left(1-\frac{1}{R}\right) \cdot p,$$

$$DE = -\frac{(1-\varepsilon)CD}{(1+i)R}\left(1+\frac{1}{k}\right) \cdot p.$$

Therefore so that there may become $t > 0$, CD must be negative, so that the final interval itself is positive. Therefore so that the penultimate interval may become positive, $C\left(1-\frac{1}{R}\right)$ must be positive.

Again with the condition for the colored margin assumed $\mathfrak{s} = 0$ there becomes $\mathfrak{t} = \frac{1}{Rk}$ or $\mathfrak{t} = Rk\mathfrak{r}$, and since there shall be

$$M = \frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{t}}{m-1} = \frac{\mathfrak{q}+(1+Rk)\mathfrak{r}}{m-1},$$

it will be required for these three equations to be satisfied:

1. $\mathfrak{B}\mathfrak{q} = \frac{1-\varepsilon}{\varepsilon} \cdot M,$

2. $\mathfrak{C}\mathfrak{r} = -\frac{(1+\varepsilon)}{\varepsilon} \cdot M - \mathfrak{q},$

3. $0 = +\frac{(iR+\varepsilon)}{\varepsilon} \cdot M + \mathfrak{q} + \mathfrak{r}.$

Therefore from the third

$$\mathfrak{q}+\mathfrak{r}=-\tfrac{(iR+\varepsilon)}{\varepsilon}\cdot M$$

and hence

$$\mathfrak{q}+\mathfrak{r}(1+Rk)=-\tfrac{(iR+\varepsilon)}{\varepsilon}\cdot M+Rk\mathfrak{r}=M\left(m-1\right),$$

from which there is deduced

$$M=\frac{Rk\mathfrak{r}}{m+\frac{iR}{\varepsilon}}$$

and likewise

$$\mathfrak{q}=-\frac{Rk(iR+\varepsilon)}{m\varepsilon+iR}\cdot\mathfrak{r}-\mathfrak{r}=-\frac{\left(kiR^2+R(i+k\varepsilon)+m\varepsilon\right)}{m\varepsilon+iR}\cdot\mathfrak{r},$$

from which the value of $\mathfrak{q}$ will itself be produced negative; which since from the first form may be produced positive, if indeed there shall be $\mathfrak{B}>0$, it is apparent also this solution also cannot occur, if indeed the second mirror is convex, as we have assumed.

THIRD SOLUTION

54. Therefore the representation of an erect image only a single case remains, where with Q taken positive both the letters R and S obtain negative values. Therefore we may put

$$Q=+i,\ R=-k\ \text{ et }\ S=-k',$$

so that there shall become

$$PQRS=m=\tfrac{ikk'}{\varepsilon}\ \text{ and hence }\ k'=\tfrac{\varepsilon m}{ik}.$$

Again there will become

$$\beta=\tfrac{i(1-\varepsilon)}{i-1}\cdot p,\quad c=-\tfrac{(1-\varepsilon)}{i-1}\cdot p,$$

from which there becomes

$$B=\frac{-i(1-\varepsilon)}{\varepsilon(i-1)},\ \text{ and }\ \mathfrak{B}=\frac{i(1-\varepsilon)}{i(1-2\varepsilon)+\varepsilon};$$

whereby the focal lengths will be obtained in the following manner :

$$q=\frac{-\varepsilon i(1-\varepsilon)}{i(1-2\varepsilon)+\varepsilon}\cdot p,\quad r=\frac{-(1-\varepsilon)\mathfrak{C}}{i-1}\cdot p,\quad s=\frac{-(1-\varepsilon)C\mathfrak{D}}{(i-1)k}\cdot p,$$

and

$$t=\frac{-(1-\varepsilon)CD}{(i-1)kk'}\cdot p=\frac{-i(1-\varepsilon)CD}{\varepsilon(i-1)m}\cdot p$$

Truly the separations of the lenses will be

$$CD = \frac{-(1-\varepsilon)C}{i-1}\left(1+\frac{1}{k}\right)p,$$

$$DE = \frac{-(1-\varepsilon)CD}{(i-1)k}\left(1+\frac{1}{k'}\right)p,$$

from which there is understood to become $C<0$ and $D>0$ and thus $\mathfrak{D}<1$ and $\mathfrak{D}>0$.
But now the condition of the colored margin will give

$$0 = \mathfrak{r} + \frac{\mathfrak{t}}{kk'},$$

from which it is apparent there must be $\mathfrak{r}<0$ or on account of the lens *C* the field to be diminished. There here we may put $\mathfrak{r} = -\omega$, so that there may become $\mathfrak{t} = \omega kk' = \frac{\varepsilon m}{i}\cdot\omega$, since also here we have assumed $\mathfrak{s} = 0$: therefore for the apparent field, there will be

$$M = \frac{i\mathfrak{q}+\omega(\varepsilon m-i)}{i(m-1)},$$

to which the three following equations are required to be added:

$$1.\ \ \mathfrak{B}\mathfrak{q} = \frac{1-\varepsilon}{\varepsilon}\cdot M,$$

$$2.\ \ -\mathfrak{C}\omega = \frac{1-\varepsilon}{\varepsilon}\cdot M - \mathfrak{q},$$

$$3.\ \ 0 = -\frac{(ik+\varepsilon)}{\varepsilon}\cdot M - \mathfrak{q} + \omega.$$

Therefore from the third we conclude :

$$\mathfrak{q} = \omega - \frac{(ik+\varepsilon)}{\varepsilon}\cdot M;$$

and with $\omega\left(\frac{\varepsilon m}{i}-1\right)$ added to each side there will become:

$$\mathfrak{q} + \omega\left(\frac{\varepsilon m}{i}-1\right) = \frac{\varepsilon m}{i}\cdot\omega - \frac{(ik+\varepsilon)}{\varepsilon}\cdot M = M\,(m-1),$$

from which there is deduced:

$$M = \frac{\varepsilon^2 m\omega}{i(m\varepsilon+ik)},$$

from which in turn,

$$\mathfrak{q} = \frac{\varepsilon m(i-ik-\varepsilon)+i^2k}{i(m\varepsilon+ik)}\cdot\omega.$$

Truly from the first equation there becomes

$$\mathfrak{q} = \frac{\left(\varepsilon i(1-2\varepsilon)+\varepsilon^2\right)m\omega}{i^2\left(m\varepsilon+ik\right)},$$

the equality of which two equations provides this equation:

$$\varepsilon im(i-ik-\varepsilon)+i^3k = \varepsilon m\left(i\left(1-2\varepsilon\right)+\varepsilon\right)$$

or

$$\varepsilon m(i^2\left(1-k\right)-i\left(1-\varepsilon\right)-\varepsilon)+i^3k = 0.$$

from which equation we find

$$k = \frac{\varepsilon m\left(i^2-i(1-\varepsilon)-\varepsilon\right)}{i^2\left(\varepsilon m-i\right)}$$

or

$$k = \frac{\varepsilon m\left(i+\varepsilon\right)\left(i-1\right)}{i^2\left(\varepsilon m-i\right)},$$

which value must be positive; finally there must be taken $i > 1$ and $i < \varepsilon m$.
Now with the value of k substituted there is found :

$$M = \frac{\varepsilon\left(\varepsilon m-i\right)\omega}{\varepsilon im-i\left(1-\varepsilon\right)-\varepsilon}.$$

Finally from the second equation we deduce

$$\mathfrak{C} = -\frac{k\left(\varepsilon m-i\right)}{\varepsilon m+ik}.$$

Truly the second equation $k = \frac{\varepsilon m(i+\varepsilon)(i-1)}{i^2(\varepsilon m-i)}$, gives

$$\mathfrak{C} = -\frac{\varepsilon m\left(i+\varepsilon\right)\left(i-1\right)}{i^2\left(\varepsilon m+ik\right)},$$

which value therefore is negative and thus $C < 0$, just as was required now above. But the letters $\mathfrak{D}$ and D remain to be chosen as we please, provided D may be taken positive ; so that finally it may pertain to that quantity p, it will be agreed to define that from the confusion with the aid of the formula noted, where at once it will be required to be seen , that each spherical figure for the mirrors shall be induced into a parabolic form.

COROLLARY 1

55. Therefore if the letter $\mathfrak{t}$ may be introduced into the calculation, as it will be allowed to be taken equal to unity, we will have for the apparent field

$$M = \frac{i(\varepsilon m-i)}{m(\varepsilon im-i(1-\varepsilon)-\varepsilon)}\cdot \mathfrak{t},$$

which certainly gives the value for the radius of the field Φ on multiplying by 859 min. But we have seen the letter *i* must be taken between the values 1 and *εm*.

COROLLARY 2

56. If there may be taken $i=1$, there shall become $\beta=\infty$ and the rays reflected from the smaller mirror become parallel to each other, from which the above fault mentioned above will arise, so that clearly the stray rays which thus will become mixed with the true rays, in no way will be able to be separated; which case since it will be required to be avoided carefully, it will be agreed to take the letter *i* much greater than unity, nor yet can the other limit be assumed to be equal to *εm*, since otherwise the field will vanish at once.

COROLLARY 3

57. It will be readily apparent for the calculation not to have a maximum value for the expression *M* and its value there to be diminished more, where a greater letter *i* may be accepted. Whereby, since there must be $i>1$, if we assume $i=2$, there will become

$$M = \frac{2(\varepsilon m-2)\mathfrak{t}}{m(2\varepsilon m+\varepsilon-2)}$$

and thus for the greater magnifications $M=\frac{1}{m}\cdot\mathfrak{t}$, which value also will be produced, if there may be taken $i=3$ or 4 etc., provided i shall be much less than *εm,* which simple field is accustomed to be considered. But if the mean between the two limits may be assumed to be taken $i=\frac{\varepsilon m+1}{2}$, there will become

$$M = \frac{(\varepsilon m+1)\mathfrak{t}}{2m(\varepsilon m+\varepsilon+1)}$$

and the field for the greatest magnification will be reduced by half.

COROLLARY 4

58. Likewise also it is apparent from the first value of *M,* which is

$$M = \frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{t}}{m-1},$$

for which $\mathfrak{r}=-\omega=-\frac{i\mathfrak{t}}{\varepsilon m}$. For even if $\mathfrak{q}$ must be added, yet from the above it is apparent that $\mathfrak{q}<\omega$*;* indeed there was from the third equation

$$\mathfrak{q} = \omega - \frac{(ik+\varepsilon)}{\varepsilon} \cdot M.$$

SCHOLIUM 1

59. But with regard to the field of view it is required especially to examine, whether or not it may be permitted to write unity in place of $\mathfrak{t}$, so that a judgement can be considered for the first lens C, of which the radius of the aperture is actually $= \delta x$, but on account of the field of view it must become $= \frac{1}{4}\mathfrak{r}r$.

Therefore since there shall be $\mathfrak{r} = -\frac{i\mathfrak{t}}{\varepsilon m}$ and

$$r = \frac{(1-\varepsilon)\mathfrak{C}}{i-1} \cdot p \text{ or } r = \frac{\varepsilon m(1-\varepsilon)(i+\varepsilon)}{i^2(\varepsilon m+ik)} \cdot p,$$

but now above we will have found $\varepsilon m + ik$ to become

$$= \frac{\varepsilon m(i(\varepsilon m+\varepsilon-1)-\varepsilon)}{i(\varepsilon m-i)} = \frac{\varepsilon m(\varepsilon im-i(1-\varepsilon)-\varepsilon)}{i(\varepsilon m-i)},$$

on account of which there will become

$$r = \frac{(\varepsilon m-i)(1-\varepsilon)(1+\varepsilon)}{i(\varepsilon im-i(1-\varepsilon)-\varepsilon)} \cdot p;$$

from which, unless there were

$$\frac{(\varepsilon m-i)(1-\varepsilon)(1+\varepsilon)}{\varepsilon m(\varepsilon im-i(1-\varepsilon)-\varepsilon)} \cdot p > 4\delta x,$$

then it will be allowed to assume $\mathfrak{t} = 1$. On the other hand truly $\mathfrak{t}$ must be taken so much smaller than unity, where it will be of help to note $\delta > \varepsilon$. But since these formulas are exceedingly complicated, so that in general all the considerations for the construction of a telescope may be able to be expressed conveniently, we may put $i = \frac{1}{2}(\varepsilon m + 1)$, so that the distance CD may emerge smaller, and if the field of view may be reduced to half ; for then we will see, how the field may be able to be enlarged. For on putting $i = \frac{\varepsilon m+1}{2}$ there will become

$$k = \frac{2\varepsilon m(\varepsilon m+2\varepsilon+1)}{(\varepsilon m+1)^2},$$

which value will become $k = 2$ for large magnifications.

Then truly

$$\mathfrak{C}=\frac{-(\varepsilon M+2\varepsilon+1)(\varepsilon m-1)^2}{2(\varepsilon m+1)((\varepsilon m+1)(\varepsilon m+\varepsilon-1)-2\varepsilon)}\quad\left[=-\frac{(\varepsilon m+2\varepsilon+1)(\varepsilon m-1)}{2(\varepsilon m+1)(\varepsilon m+\varepsilon+1)}\right],$$

from which C is found.

SCHOLIUM 2

60. Truly since the value $i=\frac{\varepsilon m+1}{2}$ can be seen to deserve an exceedingly large value, we may assume the geometric mean rather for i and there shall be $i=\sqrt{\varepsilon m}$, and at first for the apparent field there will become

$$M=\frac{\varepsilon}{\varepsilon m+\sqrt{\varepsilon m}+\varepsilon}\cdot \mathfrak{t}.$$

Then truly we will have

$$k=\frac{\varepsilon+\sqrt{\varepsilon m}}{\sqrt{\varepsilon m}}$$

and hence

$$B=\frac{-(1-\varepsilon)\sqrt{\varepsilon m}}{\varepsilon\left(\sqrt{\varepsilon m}-1\right)}\quad\text{and}\quad \mathfrak{B}=\frac{(1-\varepsilon)\sqrt{\varepsilon m}}{(1-2\varepsilon)\sqrt{\varepsilon m}+\varepsilon},$$

$$\mathfrak{C}=\frac{-\left(\varepsilon+\sqrt{\varepsilon m}\right)\left(\sqrt{\varepsilon m}-1\right)}{\varepsilon m+\sqrt{\varepsilon m}+\varepsilon}\quad\text{and}\quad C=\frac{-\left(\varepsilon+\sqrt{\varepsilon m}\right)\left(\sqrt{\varepsilon m}-1\right)}{2\varepsilon m+\varepsilon\sqrt{\varepsilon m}}.$$

From these, if we may put $D=\theta$, so that there shall become $\mathfrak{D}=\frac{\theta}{1+\theta}$, the focal lengths will be found :

$$p=p,\quad q=\frac{-\varepsilon(1-\varepsilon)\sqrt{\varepsilon m}}{(1-2\varepsilon)\sqrt{\varepsilon m}+\varepsilon}\cdot p,\qquad r=\frac{(1-\varepsilon)\left(\varepsilon+\sqrt{\varepsilon m}\right)}{\varepsilon m+\sqrt{\varepsilon m}+\varepsilon}\cdot p,$$

$$s=\frac{\theta}{1+\theta}\cdot\frac{(1-\varepsilon)}{2\sqrt{\varepsilon m}+\varepsilon}\cdot p,\qquad t=\frac{\theta(1-\varepsilon)\left(\varepsilon+\sqrt{\varepsilon m}\right)}{\varepsilon m\left(\varepsilon+2\sqrt{\varepsilon m}\right)}\cdot p.$$

Truly the separations of the lenses will be

$$AB=BC=(1-\varepsilon)p,$$

$$CD=\frac{(1-\varepsilon)\left(\varepsilon+2\sqrt{\varepsilon m}\right)}{2\varepsilon m+\varepsilon\sqrt{\varepsilon m}}\cdot p,$$

$$DE=\frac{\theta(1-\varepsilon)\left(\varepsilon+\sqrt{\varepsilon m}+\varepsilon\right)}{\varepsilon m\left(\varepsilon+2\sqrt{\varepsilon m}\right)}\cdot p.$$

But for the position of the eye there will become

$$O = \frac{\mathfrak{t}t}{Mm} = \frac{\varepsilon m + \sqrt{\varepsilon m} + \varepsilon}{\varepsilon m} \cdot \mathfrak{t} = \mathfrak{t}\left(1 + \frac{1}{\sqrt{\varepsilon m}} + \frac{1}{m}\right).$$

Moreover, for the apertures we have found

$$\mathfrak{q} = \frac{(1-2\varepsilon)\sqrt{\varepsilon m} + \varepsilon}{\left(\varepsilon m + \sqrt{\varepsilon m} + \varepsilon\right)\sqrt{\varepsilon m}} \cdot \mathfrak{t}, \quad \mathfrak{r} = -\frac{\mathfrak{t}}{\sqrt{\varepsilon m}} \quad \text{and} \quad \mathfrak{s} = 0.$$

But it will be allowed to take $\mathfrak{t} = 1$, unless there may be produced

$$\frac{(1-\varepsilon)\left(\varepsilon + \sqrt{\varepsilon m}\right)}{\left(\varepsilon m + \sqrt{\varepsilon m} + \varepsilon\right)\sqrt{\varepsilon m}} \cdot p > 4\delta x.$$

But for the lens at *D,* for which there is $\mathfrak{s} = 0$, an aperture must be given, the radius of which shall be $= \frac{x}{PQR} = \frac{\varepsilon x}{\varepsilon + \sqrt{\varepsilon m}}$, thus so that the aperture of this lens shall be so small, so that at first it shall be adapted for keeping marginal rays away. Yet meanwhile, since the apparent field here is exceedingly small, certainly it will be worth the effort to obtain a greater field of view for telescopes of this kind, which we will establish in the following problem. [Diffraction by the small opening would render this approach useless.]

PROBLEM 3

61. *To add a new order of perfection to the kind of telescopes described in the preceding problem, while its apparent field of view may be increased.*

SOLUTION

This becomes with the addition of a new lens, thus so that now the telescope may be composed from two mirrors and four lenses. But there shall remain as before

$$P = \frac{1}{\varepsilon}, \quad Q = i, \quad R = -k \quad \text{and} \quad S = -k',$$

with which agreeing the letter *T* shall be

$$\frac{ikk'T}{\varepsilon} = m;$$

then there shall be also as before

$$B = \frac{-i(1-\varepsilon)}{\varepsilon(i-1)} \quad \text{and hence} \quad \mathfrak{B} = \frac{i(1-\varepsilon)}{i(1-2\varepsilon)+\varepsilon},$$

from which the focal lengths will be formed thus :

$$q = -\frac{\mathfrak{B}}{P}\cdot p = -\varepsilon\mathfrak{B}p,\quad r = \frac{B\mathfrak{C}}{PQ}\cdot p = \frac{\varepsilon B\mathfrak{C}}{ik}\cdot p,\quad s = \frac{\varepsilon BC\mathfrak{D}}{ik}\cdot p,\quad t = \frac{\varepsilon BCD\mathfrak{E}}{ikk'}\cdot p$$

and

$$u = -\frac{\varepsilon BCDE}{ikk'T}\cdot p = -\frac{\varepsilon BCDE}{m}\cdot p$$

and the intervals

$$AB = BC = \left(1-\varepsilon\right)p,\quad CD = \frac{\varepsilon BC}{i}(1+\tfrac{1}{k})p,\quad DE = \frac{\varepsilon BCD}{ik}(1+\tfrac{1}{k'})p$$

and

$$EF = \frac{\varepsilon BCDE}{ikk'}(1-\tfrac{1}{T})p,$$

where, since there shall be $B<0$, there must become $C<0$, thence $D>0$. Again since u shall become positive, there must become $E<0$ and hence on account of the final interval $T<1$. Now also we may put in place also $\mathfrak{r} = -\omega,\ \mathfrak{s} = 0$, and so that the field may emerge a maximum, $\mathfrak{u} = \mathfrak{t}$, thus so that there shall be

$$M = \frac{\mathfrak{q}-\omega+2\mathfrak{t}}{m-1}.$$

Truly so that the colored margin may vanish, there must become

$$\omega = \frac{\mathfrak{t}}{kk'} + \frac{\mathfrak{u}}{kk'T} = \frac{\mathfrak{t}}{kk'}\left(1+\frac{1}{T}\right),$$

and since there must become $T<1$, there may be taken at once $T = \frac{1}{2}$, so that there shall become $m = \frac{ikk'}{2\varepsilon}$ and hence $kk' = \frac{2\varepsilon m}{i}$; therefore then $\omega = \frac{3i}{2\varepsilon m}\mathfrak{t}$ and in turn $\mathfrak{t} = \frac{2\varepsilon m\omega}{3i}$; from which there becomes

$$M = \frac{\mathfrak{q}+\omega\left(\frac{4\varepsilon m}{3i}-1\right)}{m-1}.$$

But now it will be required to consider the four following equations:

1. $\mathfrak{B}\mathfrak{q} = \frac{1-\varepsilon}{\varepsilon} \cdot M$,

2. $\mathfrak{C}\omega = \frac{i-\varepsilon}{\varepsilon} \cdot M - \mathfrak{q}$,

3. $0 = -\left(\frac{ik+\varepsilon}{\varepsilon}\right) \cdot M - \mathfrak{q} + \omega$,

4. $\mathfrak{E}\mathfrak{t} = \frac{ikk'-\varepsilon}{\varepsilon} \cdot M - \mathfrak{q} + \omega$.

Therefore we will have from the third equation:

$$\mathfrak{q} - \omega = -\left(\frac{ik+\varepsilon}{\varepsilon}\right) \cdot M;$$

$\frac{4\varepsilon m\omega}{3i}$ may be added to each side and there will be produced

$$M\left(m-1\right) = \frac{4\varepsilon m\omega}{3i} - \left(\frac{ik+\varepsilon}{\varepsilon}\right)M,$$

from which there is found

$$M = \frac{4\varepsilon m\omega}{3i\left(m+\frac{ik}{\varepsilon}\right)} = \frac{4\varepsilon^2 m\omega}{3i\left(m\varepsilon+ik\right)}$$

or with the value of ω substituted

$$M = \frac{2\varepsilon}{m\varepsilon+ik} \cdot \mathfrak{t},$$

and in addition from the same equation there will become :

$$\mathfrak{q} = \frac{3i\left(m\varepsilon+ik\right)-4\varepsilon m\left(ik+\varepsilon\right)}{3i\left(m\varepsilon+ik\right)} \cdot \omega;$$

but truly the first equation gives

$$\mathfrak{q} = \frac{4\left(1-\varepsilon\right)\varepsilon m\omega}{3i\left(m\varepsilon+ik\right)\mathfrak{B}}$$

of which the equality of the values provides

$$3i\left(m\varepsilon+ik\right) - 4\varepsilon m\left(ik+\varepsilon\right) = \frac{4\varepsilon\left(1-\varepsilon\right)m}{\mathfrak{B}} = \frac{4\varepsilon m\left(i\left(1-2\varepsilon\right)+\varepsilon\right)}{i},$$

from which there becomes

$$ik\left(4\varepsilon m - 3i\right) = \varepsilon m\left(3i - 4\varepsilon\right) - \frac{4\varepsilon m\left(i\left(1-2\varepsilon\right)+\varepsilon\right)}{i} = \frac{\varepsilon m}{i}\left(3i^2 - 4i\left(1-\varepsilon\right) - 4\varepsilon\right)$$

or

$$ik=\frac{\varepsilon m\left(3i^2-4i(1-\varepsilon)-4\varepsilon\right)}{i(4\varepsilon m-3i)};$$

so that which value shall be positive, there must become

$$i<\tfrac{4}{3}\varepsilon m \text{ and likewise } i>\tfrac{2}{3}\left(1-\varepsilon+\sqrt{1+\varepsilon+\varepsilon^2}\right)$$

Hence moreover the second equation will be defined from the value of k :

$$\mathfrak{C}=\frac{-4\varepsilon m\left(i^2-i(1-\varepsilon)-\varepsilon\right)}{3i^2(m\varepsilon+ik)}=\frac{-4\varepsilon m(i-1)(i+\varepsilon)}{3i^2(m\varepsilon+ik)},$$

or from the other value of $\mathfrak{q}$:

$$\mathfrak{C}=\frac{-\varepsilon m(1+4k)+3ik}{3(m\varepsilon+ik)};$$

therefore on account of $\mathfrak{C}<0$ there will be also $C<0$, as it is required to use ; from the equality of which values the same value for *k* arises as before. Moreover there may be noted here to be :

$$\varepsilon m+ik=\frac{4\varepsilon m\left(\varepsilon im-i(1-\varepsilon)-\varepsilon\right)}{i(4\varepsilon m-3i)},$$

from which there becomes

$$M=\frac{i(4\varepsilon m-3i)\mathfrak{t}}{2m\left(\varepsilon im-i(1-\varepsilon)-\varepsilon\right)}.$$

Then truly we are permitted for us to choose our letter *D*, provided it may be taken positive.

Finally the fourth equation provides us with the value of the letter

$$\mathfrak{E}=\frac{2(2\varepsilon m+ik)}{\varepsilon m+ik}=\frac{4\left(2\varepsilon im-(1-\varepsilon)i-\varepsilon\right)-3i^2}{2\left(\varepsilon im-(1-\varepsilon)i-\varepsilon\right)},$$

which value is at once positive and greater than unity; whereby $\varepsilon<0$ is required to be used. [A lengthy explanatory correction is added at this point by the *O.O.* editor *Emil Cherbuliez,* which has not been added here.]

Finally for the position of the eye we will have

$$O=\frac{\mathfrak{t}u}{Mm}=\frac{2\left(\varepsilon im-i(1-\varepsilon)-\varepsilon\right)}{i(4\varepsilon m-3i)}\cdot u$$

or

$$O=\tfrac{1}{2}u\left(1+\tfrac{3i^2-4i(1-\varepsilon)-4\varepsilon}{i(4\varepsilon m-3i)}\right).$$

Again it remains, so that we may decide, whether or note we may be able to accept unity for $\mathfrak{t}$, which will be allowed, if there were

$$r<\tfrac{4\delta x}{\omega} \quad \text{or} \quad r<\tfrac{8\delta\varepsilon mx}{3i}.$$

Truly on the other hand there must be taken $\mathfrak{t}=\frac{8\delta\varepsilon mx}{3ir}$, in which case the field will be diminished in the same ratio, in which $\mathfrak{t}$ is less than one. But so that it may extend to the quantity *p*, that must be defined from the noted equation had in the ratio of the mirrors ; whether they shall be spherical or parabolic.

COROLLARY 1

62. Since the lens at *D,* as it suffices to be bored through by the smallest tiny hole, is removed from the lens *C* by the distance

$$CD=\tfrac{\varepsilon BC}{i}\left(1+\tfrac{1}{k}\right)p,$$

but the stray rays incident on the lens *C* are collected together at the distance $r=\frac{\varepsilon B\mathfrak{C}}{i}\cdot p$, so that these rays may be excluded, it is necessary, that these two distances in turn differ from each other, or the must be a notable difference between these quantities $C\left(1+\frac{1}{k}\right)$ and $\mathfrak{C}$, that is between $1+\frac{1}{k}$ and $1-\mathfrak{C}$ or between $\frac{1}{k}$ and $-\mathfrak{C}$.
Truly there is

$$\tfrac{1}{k}=\frac{i^2(4\varepsilon m-3i)}{\varepsilon m\left(3i^2-4i(1-\varepsilon)-4\varepsilon\right)} \quad \text{and} \quad -\mathfrak{C}=\frac{(4\varepsilon m-3i)(i-1)(i+\varepsilon)}{3i\left(\varepsilon im-i(1-\varepsilon)-\varepsilon\right)};$$

whereby, since the ratio between these quantities must be exceedingly unequal, this fraction:

$$\frac{3i^3\left(\varepsilon im-i(1-\varepsilon)-\varepsilon\right)}{\varepsilon m(i-1)(i+\varepsilon)\left(3i^2-4i(1-\varepsilon)-4\varepsilon\right)}$$

must differ the most from unity; but the difference between the numerator and the denominator is great enough, so that equality shall not be required to be a concern.

COROLLARY 2

63. But if moreover we may assume $i=2$, a fraction will emerge different from unity $=\frac{6(2\varepsilon m+\varepsilon-2)}{\varepsilon m(1+\varepsilon)(2+\varepsilon)}$, which may differ from unity on both sides well enough, so that the passage of the stray rays by no means will be a source of concern. But the maximum ratio of the field emerges, so that we may attribute as small value to *i* itself, as far as

circumstance permit. Moreover that passage will be avoided much more, if there may be taken $i < 2$.

EXAMPLE 1
For the magnification $m = 50$

64. Here we may put $\delta = \frac{1}{4}$, $\varepsilon = \frac{1}{5}$, and since this magnification postulates $x = 1$ in., there will become $y = \frac{1}{4}$ in. Then we may put $i = 3$; there will become

$$(i+\varepsilon)(i-1) = 6,4, \quad 3i^2 - 4i(1-\varepsilon) - 4\varepsilon = 16,6, \quad \varepsilon m = 10, \quad 4\varepsilon m - 3i = 31, B = -6, \quad \mathfrak{B} = \tfrac{6}{5},$$

$$ik = \tfrac{166}{93} = 1,785, \quad [k = 0,595], \quad em + ik = 15,355,$$

$$\mathfrak{C} = -0,6175, \quad C = -0,3817, \quad \mathfrak{E} = 2,4921, \quad E = -1,6702,$$

[Because of incorrect values assumed, and variables defined in more than one way, the results from § 64 to § 72 are inaccurate. See the *O.O.* volume, p. 176.]
from which the fundamental elements will be defined in the following way by putting θ in place of D, so that there shall become $\mathfrak{D} = \frac{\theta}{1+\theta}$:

$$\alpha = p, \quad \beta = 1,2p, \quad \gamma = 0,1526p,$$

$$b = -\tfrac{1}{5}p = -0,2p, \quad c = -0,4p, \quad d = 0,0855p,$$

$$\delta = 0,0855\theta p, \quad \varepsilon = 0,0229\theta p,$$

$$\varepsilon = -0,0382\theta p, \quad f = 0,07648p,$$

from which the intervals are deduced :

$$AB = BC = 0,8p, \quad CD = 0,2381p, \quad DE = 0,1084\theta p, \quad EF = 0,0382\theta p,$$

and thus the tube requiring to be connected to the opening of the mirrors will be around $= \frac{1}{3}p$.
Truly the focal lengths will become

$$q = \mathfrak{B}b = -0,24p, \quad r = \mathfrak{C}c = 0,247p, \quad s = \mathfrak{D}d = 0,0855 \ \tfrac{\theta}{1+\theta} \cdot p,$$

$$t = \mathfrak{E}e = 0,0571\theta p, \quad u = f = 0,0764\theta p.$$

In addition for this case we will have

$$M = \tfrac{93}{2740}\mathfrak{t} = 0,0339\mathfrak{t} \ \left(8,5307323\right).$$

Then again,

$$\mathfrak{q} = 0,113\mathfrak{t}, \quad \mathfrak{r} = -\omega = -0,45\mathfrak{t}.$$

Therefore now we may see, whether or not unity may be taken for $\mathfrak{t}$. Which in the end we will consider the value

$$\mathfrak{t}r = 4\delta x \quad \text{or} \quad 0,111p\mathfrak{t} = 1 \text{ dig.},$$

from which there becomes $\mathfrak{t} = \frac{1}{0,111p} = \frac{9}{p}$, from which it is clear, if p were of 9 inches or less, then there can be assumed $\mathfrak{t} = 1$, but if there were $p > 9$ in., then there must be taken $\mathfrak{t} = \frac{9}{p}$, and p will become a smaller field by so much an amount. Concerning the position of the eye, truly it is required to not the become :

$$O = \tfrac{1}{2}u\left(1 + \tfrac{16,6}{93}\right) = 0,59u.$$

Now truly there remains the particular investigation of the focal distance *p,* which is deduced from the measure of the confusion,

$$p = kx\sqrt[3]{50}\left\{\begin{matrix} 0,125 - 0,0283 + 0,00131\mu\left(\lambda + \nu\mathfrak{C}(1-\mathfrak{C})\right) \\ +\ 0,0031\mu\left(\frac{(1+\theta)^3\lambda'}{\theta^3} + \frac{\nu(1+\theta)}{\theta^2}\right) + \frac{0,00005\mu}{\theta^3}\left(\lambda'' + \nu\mathfrak{E}\left(1-\mathfrak{E}\right)\right) + \frac{0,00036\mu}{\theta^3}\lambda''' \end{matrix}\right\}$$

Concerning this expression truly we will observe the following :

I. If the main mirror shall be parabolic, the first term 0,125 after the root sign must be omitted ; and if also the smaller mirror may be parabolic, then also the second term will be allowed to be omitted. But with greater consideration it will be seen only the first mirror need be made parabolic, truly a perfect spherical figure may be induced for the other figure ; then truly the following members may be constructed or thus the letters $\lambda,\ \lambda',\ \lambda''$ together with θ will be assumed thus, so that these same members can be removed completely from the second, which is negative, and thus the whole confusion will be reduced to zero. So that if it will be successful, it will be sufficient to define the letter *p* form the opening alone , clearly by taking $p = 4x,\ 6x$ or $7x$, just as will have been seen. Therefore on account of $x = 1$ in. the focal distance *p* will be able to be taken less than 9 in. without risk.

II. But since there may be taken $p < 9$ in., it will be allowed to put $\mathfrak{t} = 1$ and the radius of the apparent field will be $= 859M$ minutes $= 29'$. But then the two latter lenses will be required to be made equally convex on each side, from which, if the lenses may be made from common glass, for which there is $n=1,55$, there will become

$$\lambda'''=1+\left(\tfrac{\sigma-\rho}{2\tau}\right)^2=1,6299,$$

but

$$\lambda''=1+0,6299\ \left(1-2\mathfrak{C}\right)^2=10,9991.$$

III. Since thus it will be allowed to take $p=4$ in., lest the distance of the final focal may become exceedingly small, it will suffice to put in place $\theta=1$ and hence the final term of our formula will be $=0,00055$. For the penultimate term there will be

$$\nu\mathfrak{C}\left(1-\mathfrak{C}\right)=-\ 0,8649$$

and thus

$$\lambda''+\nu\mathfrak{C}\left(1-\mathfrak{C}\right)=10,1342$$

and therefore the whole term $=0,00047$. On account of which both terms taken together will give 0,00102.

IV. But for the first lens there will be

$$\nu\mathfrak{C}\left(1-\mathfrak{C}\right)=-0,2323,$$

from which the whole term may become thence

$$=0,00123\lambda-0,00028.$$

Moreover for the second lens there will be

$$\tfrac{(1+\theta)^3}{\theta^3}\lambda'+\tfrac{\nu(1+\theta)}{\theta^2}=8\lambda'+2\nu$$

and hence the whole term will become

$$=0,0232\lambda'+0,00135.$$

V. Therefore with these letters λ and λ' found thus it will be necessary to be defined, so that there may become

$$0,0283=0,00123\lambda+0,0232\lambda'+0,00209$$

or

$$0,0262 = 0,00123\lambda + 0,0232\lambda',$$

where it is required to be observes the letters λ and λ' cannot be less than unity; therefore we may put $\lambda' = 1$ and there must become $0,0030 = 0,00123\lambda$ and hence

$$\lambda = \frac{0,00300}{0,00123} = \frac{300}{123} = 2,44.$$

Hence therefore we follow with the following construction:

A CATADIOPTRIC TELESCOPE FOR THE MAGNIFICATION $m = 50$

65. From these, which we have just established, we will obtain the following determinations:

I. For the principal mirror, which must be figured exactly into the shape of a parabola, the focal distance must be taken $p = 4$ in. Yet meanwhile we will retain the letter p as if undetermined in the calculation.
The radius of the aperture of this mirror will be $x = 1$ in. and the radius of the hole

$$y = \delta x = \tfrac{1}{4} \text{ dig.}$$

II. Before this mirror may be put in place for the interval $= 0,8p$ the second mirror *QBQ,* for which the focal length must be $q = -0,24p$, thus so that this mirror must be convex and made precisely into a spherical shape.
The radius of this aperture $= \frac{1}{4}$ in.

III. After this mirror in the opening of the greater mirror itself at the distance $BC = \frac{4}{5}p = 0,8p$ the first lens may be put in place prepared from common glass with $n = 1,55$, of which the focal length shall be $r = 0,247p$, by taking

$$\text{radius of the}\begin{cases}\text{anterior face } = \dfrac{r}{\sigma - \mathfrak{C}(\sigma-\rho) \mp \tau\sqrt{(\lambda-1)}} = \dfrac{r}{1,4385} = 0,1729p \\ \text{posterior face} = \dfrac{r}{\sigma + \mathfrak{C}(\sigma-\rho) \pm \tau\sqrt{(\lambda-1)}} = \dfrac{r}{0,3896} = 0,6339p.\end{cases}$$

The radius of the aperture $= \frac{1}{4}$ in. as of the opening, and the interval as far as to the second lens

$$= 0,2381p = CD.$$

IV. For the second lens *SDS,* of which the focal length $s = 0,0427p$, on account of $\mathfrak{D} = \frac{1}{2}$ and $\lambda' = 1$ may be taken

$$\text{radius of the}\begin{cases}\text{anterior face } = \frac{s}{\sigma-\frac{1}{2}(\sigma-\rho)} = \frac{s}{0,9090} = 0,04697p \\ \text{posterior face} = \frac{r}{\sigma+\frac{1}{2}(\sigma-\rho)} = \frac{s}{0,9090} = 0,04697p.\end{cases}$$

The radius of each aperture $= \frac{x}{PQR} = \frac{1}{26,775} = 0,037$ in.

and the distance to the third lens $DE = 0,1084p$.

V. For the third lens, of which the focal length $t = 0,0571p$, the radius of each face may be taken $= 0,0628p$.

The radius of its aperture $= \frac{1}{4}t = 0,0142p$ and the interval to the fourth lens $= 0,0382p$.

VI. For the fourth lens, of which the focal distance $u = 0,0764p$, the radius of each face may be taken $= 0,0840p$.

The radius of its aperture $= \frac{1}{4}u = 0,0191p$ and the distance to the eye $= 0,58u = 0,0443p$.

VII. Therefore the length of the anterior tube containing both the mirrors is a little greater than 0,8*p*. Truly the length of the posterior tube containing the lenses will be $= 0,4292p$ and thus the length of the whole instrument will be around 1,4292*p*, thus so that by taking $p = 5$ in. this length will be going to be 7 in.

VIII. But the radius of the apparent field now has been indicated above $= 29'$, which is notable enough for the magnification $m = 50$.

IX. Plainly here there is no need for diaphragms or cloths in place arranged in the places of the real image, since the second lens may have such a small aperture, which may exclude all the stray rays. Yet meanwhile, if in the place of the first real image, which falls after the first lens for the interval $\gamma = 0,1526p$, a diaphragm may be located, the radius of its opening must be taken to be $= 0,127p$; truly there will be scarcely a need for this diaphragm, since the image of the stray rays incident on the first lens may fall on an image after this lens for the distance $r = 0,247p$, while that of the more appropriate rays falls at the distance $\gamma = 0,1526p$, which distinction is notable enough.

X. If anyone may fear, lest from such a small mirror, of which the radius is $= 1$ in. and which thus has been bored with a hole, an exceedingly small amount of light may be transmitted to the eye, this measure of the inches may be increased as it pleases ; indeed nothing prevents, why the measure of the inches may not be doubled. For in this way it will be able to increase the clarity as desired, nor yet the length of the instrument to be increased, which itself is small, on this account avoids being very large.

EXAMPLE 2

For the magnification $m = 100$

66. Here we may put $\delta = \frac{1}{4}$ and $\varepsilon = \frac{1}{5}$ so that there shall be $\varepsilon m = 20$. Then truly we may assume $i = 4$, from which a shorter tube may arise, and we will have

$$P = \tfrac{1}{\varepsilon} = 5, \quad Q = i = 4, \quad R = -k = -\tfrac{43}{68} = -0,63235$$

on account of which

$$3i^2 - 4i(1-\varepsilon) - 4\varepsilon = 34\tfrac{2}{5} \text{ and } 4\varepsilon m - 3i = 68,$$

again

$$S = -k' = -\tfrac{680}{43} = -15,814 \text{ and } T = \tfrac{1}{2} = 0,5.$$

From which there becomes

$$PQ = 20, \quad PQR = -12,647, \quad PQRS = 200 \quad \text{and} \quad PQRST = 100.$$

Truly the remaining letters will be found :

$$\mathfrak{B} = \tfrac{16}{13} = 1,231, \quad B = -\tfrac{16}{3} - 5,333,$$
$$\mathfrak{C} = -\tfrac{17 \cdot 21}{383} = -\ 0,93211 (9,9694694), \quad C = -\tfrac{357}{740} = -\ 0,4824\ (9,6884898)$$

and

$$\mathfrak{D} = \tfrac{\theta}{1+\theta}, \quad D = \theta,$$

$$\mathfrak{E} = \tfrac{17 \cdot 783}{2 \cdot 5 \cdot 383} = 3,4755 (0,5410119), \quad E = -\tfrac{3,4755}{2,4755} = -1,4039\ (0,1473490),$$

from which we deduce

$$\log.B\mathfrak{C} = 0,6964410, \quad \log.BC\mathfrak{E} = 0,9514233,$$
$$\log.BC = 0,4104114, \quad \log.BCE = 0,5577604\ (-).$$

With these established our elements will become :

$$\alpha = p, \qquad b = -\tfrac{\alpha}{P} = -\tfrac{1}{5}\alpha = -0,2p,$$
$$\beta = Bb = 1,0666p, \qquad c = -0,2666p,$$
$$\gamma = Cc = 0,1286p, \qquad d = 0,20844p,$$
$$\delta = Dd = 0,20844\theta\, p, \qquad e = 0,01286\theta p$$
$$\varepsilon = E\varepsilon = -0,01806\theta p, \qquad f = 0,08612\theta\, p,$$

from which we obtain the intervals at once :

$$AB = 0,8p, \quad BC = 0,8p, \quad CD = 0,3320p,$$
$$DE = 0,2163\theta p, \quad EF = 0,01806\theta p.$$

Truly the focal lengths will themselves be had :

$$q = \mathfrak{B}b = -0,246p, \quad r = \mathfrak{C}c = 0,2485p,$$

$$s = \mathfrak{D}d = 0,2034\tfrac{\theta}{1+\theta}p, \quad t = \mathfrak{E}e = 0,0447\theta p, \quad u = f = 0,0361\theta p.$$

Truly in addition there will be $\omega = 0,3\mathfrak{t} = -\mathfrak{r}$, from which the equation $\mathfrak{r}r = 4\delta x$ will be changed into this :
$0,07455\mathfrak{t}p = x$; whereby, if there may be taken $x = 2$ in., hence there will become $\mathfrak{t} = \frac{2}{0,07455p}$. Therefore provided there were $p < 26$ in., it will be allowed to take $\mathfrak{t} = 1$ and the two final lenses will become equally convex on each side. Truly also if this may be allowed the total confusion to be reduced to zero, on account of $x = 2$ in. thus it will be able to take $p = 8$ in., even if it may be better to give a greater value to p ; from which it is evident to assume $\theta = 1$ without risk.

Besides truly for the apparent field of view there will be had $M = \frac{34}{1915}\mathfrak{t}$; whereby, if there can be taken $\mathfrak{t} = 1$, the radius of the field of view will become

$$\Phi = \tfrac{859\cdot 34}{1915}\text{min.} = 15\tfrac{1}{4} \text{ min.}$$

and for the position of the eye we will have

$$O = 0,563u = 0,02037p.$$

Finally so that the total confusion may vanish, it will be necessary for the first mirror to be made perfectly parabolic and then there must be

$$\frac{s(1+B)(1-B)^2}{8B^3} = \frac{\mu}{B^3\mathfrak{C}^3PQ}\left(\lambda + \nu\mathfrak{C}(1-\mathfrak{C})\right) - \frac{\mu}{B^3\mathfrak{C}^3PQR}\left(8\lambda' + 2\nu\right)$$

$$+\frac{\mu}{B^3C^3\mathfrak{E}^3PQRS}\left(\lambda'' + \nu\mathfrak{E}(1-\mathfrak{E})\right) - \frac{\mu}{B^3C^3E^3m}\lambda''',$$

where as before, if the refraction of the glass shall be $n = 1,55$, there will be

$$\lambda''' = 1,6299$$

and

$$\lambda'' = 1 + 0,6299(1-2\mathfrak{E})^2 = 23,308,$$

from which our equation will produce

$$\begin{aligned} 0,02864 = 0,000382\lambda & - 0,00016 \\ + 0,034843\lambda' & + 0,00200 \\ & - 0,00001 \\ & + 0,00015 \\ & + 0,00032 \end{aligned}$$

or

$$0,02634 = 0,000382\lambda + 0,03484\lambda',$$

which equality, since λ and λ' cannot be smaller than one, cannot exist. On which account we are forced to attribute a greater value to θ; therefore there will become $\theta = 2$ and our equation will become :

$$\begin{aligned} 0,02809 = 0,000382\lambda & - 0,00016 \\ + \ 0,01143\lambda' & + 0,00075 \\ & - 0,00001 \\ & + 0,00002 \\ & + 0,00004 \end{aligned}$$

or

$$0,02744 = 0,000382\lambda + 0,01143\lambda'.$$

[A more accurate value of this last number is 0,014699λ′, but the equation $0,02744 = 0,000382\lambda + 0,01143\lambda'$ with $\lambda' = 2$ gives rise to a negative value of λ, hence θ must be taken larger, for example $\theta = 3$. Emil Cherbuliez *O.O.* edition.]

Hence lest the value of λ may be produced exceedingly great, we may assume $\lambda' = 2$ and there will become $0,00458 = 0,000382\lambda$ and hence $\lambda = \frac{4580}{382} = 12$. But if we had assumed $\lambda' = 2\frac{1}{3}$ we would have obtained $\lambda = \frac{770}{382} = 2$.

Therefore we may use these latter values $\lambda = 2$ et $\lambda' = 2\frac{1}{3}$ with there being $\theta = 2$ and hence $\mathfrak{D} = \frac{2}{3}$; from which we deduce the following

THE CONSTRUCTION OF A CATADIOPTRIC TELESCOPE FOR $m = 100$

67. Therefore this construction will depend on the following determinations :

I. The first mirror will have to be worked following a perfectly parabolic figure, of which the focal length shall be $= p$, which will be required to be put at a minimum 8 in.;

of which the radius of the aperture $= x = 2$ in., but the radius of the hole $= \frac{1}{2}$ in. and the distance from the smaller mirror $AB = 0,8p$.

II. The small mirror to have a spherical figure, of which the focal length shall be $q = -0,246p$ and the radius of the aperture $= \frac{1}{2}$ in. and thence the distance to the first lens $BC = 0,8p$.

III. For the first lens, of which the focal length $r = 0,2485p$, truly the numbers

$$\mathfrak{C} = -0,9321 \text{ and } \lambda = 2,$$

there may be taken

$$\text{radius of the}\begin{cases}\text{anterior face } = \frac{r}{\sigma - \mathfrak{C}(\sigma-\rho) \pm \tau\sqrt{(\lambda-1)}} = \frac{r}{2,9666} = 0,1205p \\ \text{posterior face} = \frac{r}{\sigma + \mathfrak{C}(\sigma-\rho) \mp \tau\sqrt{(\lambda-1)}} = \frac{r}{-1,1485} = -1,0210p.\end{cases}$$

The radius of the aperture of the opening equals $= \frac{1}{2}$ in. and the distantance to the second lens $CD = 0,3320p$.

IV. For the second lens, of which the focal distance $s = 0,1356p$ and the number $\mathfrak{D} = \frac{2}{3}$ and $\lambda' = 2,3333$, there may be taken

$$\text{radius of the}\begin{cases}\text{anterior face } = \frac{r}{\sigma - \mathfrak{D}(\sigma-\rho) \pm \tau\sqrt{(\lambda-1)}} = \frac{r}{1,7147} = 0,0791p \\ \text{posterior face} = \frac{r}{\sigma + \mathfrak{D}(\sigma-\rho) \mp \tau\sqrt{(\lambda-1)}} = \frac{r}{0,1034} = 1,3114p.\end{cases}$$

The radius of its aperture $= \frac{x}{PQR} = 0,16$ in. and the distance to the third lens $DE = 0,4326p$.

V. For the third lens, of which the focal length $t = 0,0894p$, there may be taken

$$\text{the radius of each face } = 0,0983p.$$

The radius of its aperture $= \frac{1}{4}t = 0,0224p$ and the distance to the fourth lens $EF = 0,03612p$.

VI. For the fourth lens, of which the focal distance $u = 0,0722p$, there may be taken the radius of each face $=0,0794p$.

The radius of its aperture $=\frac{1}{5}u=0,0181p$ and the distance to the eye

$$O = 0,563\,u = 0,0204p.$$

VII. Therefore the length of the first tube will be a little greater than 0,8*p*, but of the adjoined tube the length $=0,8211p$ and hence the length of the whole instrument will be around $=1,6211p$.

VIII. The radius of the apparent field of view $=15\frac{1}{4}$ min., which as we have observed above, this also is consistent as well.

EXAMPLE 3

For the magnification $m=150$

68. As before there shall remain $\delta=\frac{1}{4}$ and $\varepsilon=\frac{1}{5}$, so that there shall be $\varepsilon m=30$; moreover there may be taken $[Q]=i=5$, and so that we may enjoy sufficient clarity, there shall be $x=3$ in., so that there shall be $y=\frac{3}{4}$ in., and hence we deduce

$$P=5,\quad Q=5,\quad R=-k=-0,6652,$$
$$S=-k'=-18,040 \text{ and } T=\tfrac{1}{2};$$

hence

$$PQ=25,\quad PQR=-16,63,\quad PQRS=300 \quad\text{and}\quad PQRST=150;$$

then truly the remaining letters will be found

$$\mathfrak{B}=\tfrac{5}{4}=1,25,\quad B=-\,5,$$

$$\mathfrak{C}=\tfrac{33,28}{33,320}=-0,9986\ (9,9994001),\quad C=\tfrac{0,9986}{19986}=-0,49966(9,6986742),$$

$$\mathfrak{D}=\tfrac{\theta}{\theta+1},\ D=\theta,$$

$$\mathfrak{E}=\tfrac{118,32}{33,320}=3,5504(0,5502750),\quad E=\tfrac{3,5504}{2,5504}\ =-1,3921(0,1436667),$$

[Corrected values in brackets; recall the incorrect value of $\mathfrak{C}$ used.]

from which we deduce

$$\log B\mathfrak{C} = 0,6983701,\ \log BC = 0,3976442,$$

$$\log BC\mathfrak{E} = 0,9479192,\ \log BCE = 0,5413109(-).$$

From these established our elements will become :

$$\alpha = p,\quad b = -0,2p,\quad \beta = p,\quad c = -0,2p,\quad \gamma = 0,099932p,$$

$$d = 0,15023p,\quad \delta = 0,15023\theta p,\ e = 0,008338\,p.$$

$$\varepsilon = -0,01159\theta p,\quad f = +\ 0,02318\theta p,$$

from which we deduce the spacing

$$AB = 0,8p = BC,\quad CD = \ 0,25016p,$$

$$DE = 0,15856\theta p,\ \ EF = 0,01159\theta p.$$

Thus the focal lengths themselves will be had :

$$q = -0,25p,\quad r = 0,19972p,\ \ s = 0,15023\tfrac{\theta}{1+\theta}\,p,$$

$$t = 0,02956\theta p \quad \text{and} \quad u = \ 0,02318\theta p.$$

Again there becomes $\omega = \frac{1}{4}\mathfrak{t} = -\mathfrak{r}$, from which the equation $\mathfrak{r}\mathrm{r} = 4\delta x$ will give

$$\mathfrak{t} = \tfrac{12}{0,19972p} = \tfrac{60}{p}\ \text{approx.} ;$$

while p therefore shall be < 60, with care it will be allowed to assume $\mathfrak{t} = 1$, and because then there will be $M = \frac{2}{166,630}$, hence the radius of the field will be

$$\Phi = 10\tfrac{1}{3}\ \text{min.}$$

and for the position of the eye

$$O = 0,555u = 0,01285\theta\,p.$$

Finally if the first mirror may be made parabolic, all the confusion will be removed by satisfying this equation

$$0,0288 = 0,00030144\lambda - 0,0001394 + 0,0036177\ \tfrac{\lambda'(1+\theta)^3}{\theta^3}$$
$$+\ 0,00084146\tfrac{1+\theta}{\theta^2} + \tfrac{0,0001002}{\theta^3} + \tfrac{0,00024176}{\theta^3}$$

or

$$0,0289399 = 0,00030144\lambda + 0,0036177\frac{(1+\theta)^3}{\theta^3}\lambda' + 0,00084146\frac{1+\theta}{\theta^3} + \frac{0,003419}{\theta^3}.$$

Here it is apparent at once there cannot be taken $\theta = 1$; therefore it may be tried by putting $\theta = \frac{3}{2}$ and there will become

$$0,0289399 = 0,00030144\lambda + 0,0167487\lambda' + 0,00093495 + 0,0001013$$

or

$$0,0279037 = 0,00030144\lambda + 0,0167487\lambda';$$

whereby, if here there may be set $\lambda' = 1$, there will become

$$\lambda = \frac{0,01116500}{0,00030144} = \frac{11155}{301} = 37,$$

but if we may assume $\lambda = 1$, there will become

$$\lambda' = \frac{0,0276023}{0,0167487} = \frac{276023}{167487} = 1,648.$$

But if for λ there may be put in place either 2 or 3, the value of λ' thence will scarcely be changed, from which it will be seen to be outstanding for practical use, if λ' itself may be given some certain value; since then indeed on account of the trivial errors λ can be made to vary a lot, several lenses can be prepared with various values of λ, from which the most suitable can be indicated by experiment . Therefore we may put $\lambda' = \frac{3}{2}$ and there will be found

$$\lambda = \frac{0,0027807}{0,0003014} = \frac{27807}{3014} = 9,$$

from which in practice three lenses will be able to be prepared from the values $\lambda = 8, \ = 9, \ = 10.$

Therefore on putting $\theta = \frac{3}{2}$, so that there shall be $\mathfrak{D} = \frac{3}{5}$, there may be taken $\lambda' = \frac{3}{2}$ and $\lambda = 9$, from which the following is deduced :

CONSTRUCTION OF A CATADIOPTRIC TELESCOPE FOR $m = 150$

69. This construction contains the following determinations :

I. The objective mirror most be worked into the following parabolic figure most accurately, the focal length of this cannot be less than 12 in., which we will designate here by the letter p. The radius of the aperture shall be $x=3$ dig., truly of the hole the radius $=\frac{3}{4}$ in. and the distance to the smaller mirror $AB=0,8p$.

II. The smaller mirror will be ground exactly to a spherical figure, the focal length of which will be $q=-0,25p$, certainly which is convex. The radius of its aperture $=\frac{3}{4}$ in. and the distance to the first lens $BC=0,8p$.

III. For the first lens, of which the focal length is $r=0,19972p$ and with the numbers $\mathfrak{C}=-0,9986$ and $\lambda=9$, there may be taken

$$\text{radius of the}\begin{cases}\text{anterior face}=\frac{r}{\sigma-\mathfrak{C}(\sigma-\rho)\pm\tau\sqrt{8}}=\frac{r}{0,5025}=0,39745p\\ \text{posterioris}=\frac{r}{\rho+\mathfrak{C}(\sigma-\rho)\mp\tau\sqrt{8}}=\frac{r}{1,3156}=0,15181p.\end{cases}$$

But if there may be taken $\lambda=10$, there will be produced

$$\text{radius of the}\begin{cases}\text{anterior face }=\frac{r}{0,3468}=0,57589p\\ \text{posterior face}=\frac{r}{1,4713}=0,13574p,\end{cases}$$

from which we conclude in general there can be taken

$$\text{radius of the}\begin{cases}\text{anterior face }=(0,39745\pm0,17844\omega)p\\ \text{posteror face}=(0,15181\mp0,01607\omega)p,\end{cases}$$

where it will be agreed ω to be defined by experiment.

Moreover the radius of the aperture of this lens $=\frac{3}{4}$ in. and the distance to the second lens $CD=0,25016p$.

IV. For the second lens, of which the focal length is $s=0,090138p$ and the numbers $\mathfrak{D}=\frac{3}{5}$ and $\lambda'=1,5$, there may be taken

$$\text{radius of the}\begin{cases}\text{anterior face }=\frac{s}{\sigma-\mathfrak{D}(\sigma-\rho)\pm\tau\sqrt{0,5}}=\frac{s}{0,1254}=0,71880p\\ \text{posterior face}=\frac{s}{\rho+\mathfrak{D}(\sigma-\rho)\mp\tau\sqrt{0,5}}=\frac{s}{1,69281}=0,05325p.\end{cases}$$

The radius of the aperture $=\frac{x}{PQR}=\frac{2}{11}$ in. $=0,18$ in. and the distance to the third lens $DE=0,23784p$.

V. For the third lens, of which the focal length $t=0,04434p$,

the radius of each face may be taken $=0,048774p$.

The radius of its aperture $=\frac{1}{4}t=0,01108p$ and the distance to the fourth lens $EF=0,01738p$.

VI. For the fourth lens, of which the focal length $u=0,03477p$,

the radius of each face may be taken $=0,03824p$.

The radius of its aperture $=\frac{1}{4}u=0,00869p$ and the distance to the eye $O=0,555u=0,01929p$.

VII. Therefore the length of the first tube containing the mirrors will exceed 0,8 p by a small amount, truly the length of the latter will be $=0,52467\,p$, thus so that the length of the whole instrument will be around $=1,82467p$. Then truly the radius of the apparent field will be $=10\frac{1}{3}$ minutes.

SCHOLIUM

70. The remedy which we have advanced as an aid for the first lens, also may be adapted easily in practice to the preceding example. For we may put the radii of the two faces to be found for making this lens to be *f* and *g* and now the question is reduced to this, in what way may these radii be changed, so that the focal length may remain the same. For we may put the former $=f+x$, the latter $=g-y$ and it is by necessity, that there may become [for the thin lens formula]

$$\frac{fg}{f+g}=\frac{(f+x)(x-y)}{f+g+x-y},$$

from which with *x* assumed as it pleases taken either negative or positive there must become

$$y=\frac{g^2x}{f^2+(f+g)x};$$

whereby, since *x* and *y* shall be small enough, there will become $y=\frac{g^2x}{f^2}$; or

$$x:y=f^2:g^2,$$

thus so that by putting $x=f^2\omega$ there shall be going to become $y=g^2\omega$. Therefore for the first lens, of which the radii found above shall be *f* and *g,* it will be agreed to substitute others successively, of which the radii shall be $f\pm f^2\omega$ and $g\mp g^2\omega$. Then here also it will help to be observing for the first lens a smaller aperture can suffice, as we have assigned this equal to the opening. For the aperture will suffice, the radius of which $=\frac{1}{4}\mathfrak{r}=\frac{1}{16}r=0,01248p$; from which, if $p=12$ in., this same radius will become $=0,1497$ in. $=\frac{1}{7}$ in. approx. ; and if thus there may become $p=20$ in., this same radius will become $=\frac{1}{4}$ in., from which we may conclude it may suffice, if we may attribute an aperture to this lens, of which the radius shall be $\frac{1}{4}$ in.; with which agreed we will prevent a huge abundance of stray rays from entering and the remaining thus happily will be excluded from the second lens, even if its opening is not as small as in the preceding examples, the reason for this being the case, because we have increased the letter *i* in a much smaller ratio than the magnification *m*; for this reason we will attribute a much greater value to the letter *i* in the following example, since thence nothing other is to be concerned about, apart from a small diminution of the field.

EXAMPLE 4

For the magnification *m* = 200

71. With the letters remaining $\delta=\frac{1}{4}$ and $\varepsilon=\frac{1}{5}$ there may be taken $i=10$, and so that a sufficient order of the clarity may be obtained, we may assume $x=5$ in., so that the radius of the opening shall be $=\delta x=\frac{5}{4}$ in. and $\varepsilon m=40$. Hence therefore the values may be deduced

$$P=5,\quad Q=10,\quad R=-k=-0,8221,$$

$$S=-k'=-9,7312 \text{ and } T=\frac{1}{2}$$

and hence

$$PQ=50,\quad PQR=-41,105,\quad PQRS=400 \text{ and } PQRST=200;$$

truly the remaining letters will be determined [corrected] thus :

$$\mathfrak{B}=\frac{40}{31}=1,2903,\quad B=\frac{40}{9}=-\ 4,4444,$$

$$\mathfrak{C}=-1,0153(0,0066052)(-),\quad C=-0,50381\ (9,7022655)(-),$$

$$\mathfrak{E}=3,2841(0,5164093),\quad E=-1,4377\ (0,1576942),$$

from which the following logarithms are deduced:

$$\log.B\mathfrak{C} = 0,6544183, \quad \log.BC = 0,3500786,$$
$$\log.BC\mathfrak{E} = 0,8664879, \ \log.BCE = \ 0,5077728(-);$$

hence the elements are defined in the following way:

$$\begin{aligned} \alpha = p, \quad & b = -0,2p, & \beta &= 0,8889p, \\ & c = -0,0889p, & \gamma &= \ 0,04478, \\ & d = 0,054473p, & \delta &= \ \ 0,054473\theta p \\ & e = 0,005598\theta p, & \varepsilon &= -0,008048\theta p \\ \text{and} \quad & f = 0,0160960\theta p, \end{aligned}$$

from which the intervals are defined

$$AB = 0,8p = BC, \quad CD = 0,09925p,$$
$$DE = 0,060071\theta p, \quad EF = 0,008048\theta p,$$

truly the focal lengths :

$$q = -0,2581p, \qquad r = 0,09025p,$$

$$s = 0,05447\tfrac{\theta}{1+\theta}p, \quad t = 0,01838\theta p$$

and

$$u = 0,016096\theta p.$$

Again there is $\omega = -\mathfrak{r} = \frac{3}{8}\mathfrak{t}$; from which the equation $\mathfrak{r}r = 4\delta x$ gives $\mathfrak{t} = \frac{160}{p}$ in., from which it is clear, provided p shall be less than 160 in., $\mathfrak{t} = 1$ can be taken safely ; but if the confusion may be reduced to zero, thus it will be allowed to take $p = 20$ in.; but then thee will become $M = \frac{1}{120}$, from which the radius of the field will be $\frac{859}{120}$ min. $= 7\frac{1}{6}$ min. Truly besides for the position of the eye there will be had $O = 0,6u$.

Therefore finally it remains, that we may reduce the confusion to zero, which will be done by this equation:

$$0,029074 = 0,00020418\lambda - 0,0000972 + 0,0020329\tfrac{(1+\theta)^3}{\theta^3}\lambda'$$
$$+0,00047286\tfrac{1+\theta}{\theta^2} + \tfrac{0,000111}{\theta^3} + \tfrac{0,00000116}{\theta^3}$$

or

$$0,029171 = 0,00020418\lambda + 0,0020329\tfrac{(1+\theta)^3}{\theta^3}\lambda'$$
$$+ \ 00004729\tfrac{1+\theta}{\theta^2} + \tfrac{0,0001122}{\theta^3},$$

where now nothing stands in the way, why we may not place $\theta = 1$, and hence we will have

$$0,028113 = 0,0002042\lambda + 0,016264\lambda'.$$

Therefore lest this value of λ may appear exceedingly great, it will be convenient to put $\lambda' = 1\frac{1}{2}$ and there will be found $\lambda = \frac{3717}{204} = 18$ approx. Truly it will be more convenient to assume $\lambda' = 1\frac{2}{3}$, from which there will become $\lambda = \frac{1006}{204} = 5.$ Therefore we may retain the values $\theta = 1,\ \ \lambda' = 1\frac{2}{3}$, so that there may become $\lambda = 5$, to which we will be able to add the finite values $\lambda = 4$ and $\lambda = 6$, by which in practice it may be better deliberated, and hence we deduce the following :

CONSTRUCTION OF A CATADIOPTRIC TELESCOPE FOR THE MAGNIFICATION $m = 200$.

72. Here as thus far we may put in place the focal length of the principal mirror $= p$, which, as we have seen, it is agreed not to take less than 20 in. But it will be better not to assume that to be much greater.

I. Therefore the first mirror with great care must be fashioned into the parabolic form, of which the focal length $= p$; the radius of its aperture $x = 5$ in. and the radius of the opening $y = 1\frac{1}{4}$ in. Truly the distance to the smaller mirror $AB = 0,8p.$

II. For the second smaller mirror its figure will be made spherical most accurately, so that its focal length will be $q = 0,2581p$. The radius of its aperture $= 1\frac{1}{4}$ in. and the distance to the first lens in the opening $= BC = 0,8p.$

III. For the first lens, of which the focal length $r = 0,09025p$,
$\mathfrak{C} = -\ 1,0153$, and $\lambda = 5$, there may be taken

$$\text{radius of the}\left\{\begin{array}{l}\text{anterior face } = \frac{s}{\sigma - \mathfrak{C}(\sigma-\rho)\mp\tau\sqrt{4}} = \frac{r}{3,0861\mp 1,8102}\\ \text{posterior face} = \frac{s}{\sigma + \mathfrak{C}(\sigma-\rho)\pm\tau\sqrt{4}} = \frac{r}{-1,2680\pm 1,8102},\end{array}\right\}$$

hence

$$\text{radius of the}\left(\begin{array}{l}\text{anterior face } = 0,070734p\\ \text{posterior face} = 0,16639p.\end{array}\right.$$

But if we shall have taken $\lambda = 4$, there will be produce

$$\text{radius of the}\begin{cases}\text{anterior face} = \frac{r}{3,0861\mp 1,5677} = 0,05944p\\ \text{posterior face} = \frac{r}{-1,2680\pm 1,5677} = 0,30113p.\end{cases}$$

But if we may have taken $\lambda = 6$, there will become

$$\text{radius of the}\begin{cases}\text{anterior face} = \frac{r}{3,0861\mp 2,0239} = 0,08496p\\ \text{posterior face} = \frac{r}{-1,2680\pm 2,0239} = 0,11940p.\end{cases}$$

From which cases we deduce in the following conclusions to assist in practice:
In the first case: If $\lambda = 5-\omega$ with ω denoting some arbitrary fraction, there will become

$$\text{radius of the}\begin{cases}\text{anterior face} = (0,07073-0,01129\omega)\,p\\ \text{posterior face} = \left(0,16639+0,13474\omega\right)p.\end{cases}$$

In the second case: But if $\lambda = 5+\omega$, there will become

$$\text{radius of the}\begin{cases}\text{anterior face} = (0,07073+0,01423\omega)\,p\\ \text{posterior face} = \left(0,16639-0,04699\omega\right)p.\end{cases}$$

The radius of its aperture $= 1\frac{1}{4}$ in., and the distance to the second lens $CD = 0,09925p$.

IV. For the second lens, its focal length is $s = 0,02723p$ and the numbers $\mathfrak{D} = \frac{1}{2}$ and $\lambda' = 1,6667$, there may be taken

$$\text{radius of the}\begin{cases}\text{anterior face} = \frac{s}{\frac{1}{2}(\sigma+\rho)\mp\tau\sqrt{0,6667}} = \frac{s}{0,9090\mp 0,7390}\\ \text{posterior face} = \frac{s}{\frac{1}{2}(\sigma+\rho)\pm\tau\sqrt{0,6667}} = \frac{s}{0,9090\pm 0,7390}\end{cases}$$

or

$$\text{radius of the}\begin{cases}\text{anterior face} = 0,01652p\\ \text{posterior face} = 0,16018p.\end{cases}$$

The radius of its aperture $= \frac{x}{PQR} = \frac{1}{8}$ in. and the distance to the third lens $DE = 0,06007p$.

V. For the fourth lens, of which the focal length $t = 0,01838p$, there may be taken

the radius of each face $= 0,02022p$.

The radius of its aperture $=\frac{1}{4}t=0,00459p$ and the distance from the fourth lens $EF=0,007798p$.

VI. For the fourth lens, of which the focal length $u=0,015596p$, the radius of each face may be taken $=0,01715p$. The radius of its aperture $=\frac{1}{4}u=0,0039p$ and the distance to the eye $=0,6u=0,00936p$.

VII. Hence therefore the length of the first tube will be as if $=p$, since it must be greater than $\frac{4}{5}p$, truly the lengths contained the latter lenses $=0,17648p$, so that the whole length shall be going to be around $=1,17648p$. Truly the radius of the apparent field of view will be $=7\frac{1}{6}$ minutes.

VIII. If we may consider the first lens only for clarity, the radius of its aperture must be $=\frac{x}{PQ}=\frac{1}{10}$ in., but if we may consider that for field, this radius shall be

$$=\frac{1}{4}\mathfrak{r}r=\frac{3}{32}r=0,00846p,$$

which, if thus there shall become $p=40$ in., it will become

$$0,8384 \text{ dig.}=\frac{1}{3} \text{ dig.}$$

Whereby, since the radius of the opening $=1\frac{1}{4}$ in., with care it will be allowed cover over the edge of this lens, while the radius of the aperture may become $\frac{1}{3}$ dig., with which agreed on the stray rays now will be excluded for the greater part.

IX. Therefore since indeed there may be no need to attribute so great a magnification to the first lens, it will be permitted for a much smaller hole than $1\frac{1}{4}$ in. of the greater mirror to be put in place, while this mirror itself will gain a greater surface, also the degree of clarity will be increased, and nor thus truly will it be necessary to diminish the size of the smaller mirror, since a sufficiently copious supply of rays will be able to fall on the mirror. The stray rays are gathered together after the lens C an the distance $r=0,09025p$, truly the proper rays at the distance $\gamma=0,0448p$.

X. Since then the first real image falls after the first lens at the distance $r=0,0448p$, but the stray rays falling on this lens will form their own image at the distance $r=0,09025p$, which since that shall be greater than twice as great, at no time will it be a concern, lest the stray rays may propagate as far as to the eye.

CAPUT IV

DE TELESCOPIIS CATADIOPTRICIS MINORE SPECULO CONVEXO INSTRUCTIS

PROBLEMA 1

50. *Constructionem huiusmodi telescopiorum describere, quibus obiecta situ inverso repraesententur seu ubi unica imago realis occurrat.*

SOLUTIO

Cum in hoc genere distantia amborum speculorum sit $AB=(1-\varepsilon)p$ ideoque $b=-\varepsilon p$, ob $\alpha=p$ erit $P=-\frac{\alpha}{b}=+\frac{1}{\varepsilon}$, ubi ε designat fractionem aliquanto minorem, quam ratio foraminis ad speculum maius $\frac{y}{x}$ designat, ita ut posito $y=\delta x$ sit $\varepsilon<\delta$ ob rationem ante allegatam § 49, qua scilicet obtinetur, ut etiam radii obliqui a minore speculo excipiantur. Interim tamen semidiameter aperturae minoris speculi maneat $=\delta x=y$, ita ut hoc speculum foramini aequetur, uti initio assumsimus.

Nunc statim consideremus aequationem, qua margo coloratus destruitur; quae, si praeter specula duae lentes adhibeantur, reducitur ad hanc formam:

$$0=\mathfrak{r}+\frac{\mathfrak{s}}{R},$$

unde, ut ambae litterae $\mathfrak{r}$ et $\mathfrak{s}$ valores positivos habere queant, uti ratio campi postulat, conveniet litterae *R* valorem tribui negativum et quidem unitate non minorem, ut sumto $\mathfrak{s}=1$ prior lens *C,* cuius apertura iam per foramen deteminatur, campum non restringat. Ponamus igitur $R=-i$, et cum ex data multiplicatione *m* ob repraesentationem inversam sit $PQR=-m$, fiet hinc $PQ=\frac{m}{i}$ et $Q=\frac{\varepsilon m}{i}$. Est vero $Q=-\frac{\beta}{c}$, et quia est

$$\beta+c=BC=AB=(1-\varepsilon)p,$$

hinc colligimus

$$c=-\frac{i(1-\varepsilon)p}{\varepsilon m-i} \quad \text{et} \quad \beta=\frac{\varepsilon m(1-\varepsilon)p}{\varepsilon m-i};$$

quare, cum in genere $\frac{1}{q}=\frac{1}{b}+\frac{1}{\beta}$, erit

$$\frac{1}{q}=\frac{-(1-2\varepsilon)m-i}{\varepsilon m(1-\varepsilon)p} \quad \text{hincque} \quad q=-\frac{m\varepsilon(1-\varepsilon)p}{(1-2\varepsilon)m+i}.$$

Porro ex valoribus *b* et β colligimus

$$B=\frac{\beta}{b}=\frac{-m(1-\varepsilon)}{\varepsilon m-i} \quad \text{et} \quad \mathfrak{B}=\frac{+m(1-\varepsilon)}{(1-2\varepsilon)m+i}.$$

Deinde cum sit $C=\frac{\gamma}{c}$ et $\mathfrak{C}c=r$, hinc invenimus

$$\mathfrak{C}=\frac{r}{c}=-\frac{(\varepsilon m-i)r}{i(1-\varepsilon)p}$$

ideoque

$$C=\frac{-(\varepsilon m-i)r}{i(1-\varepsilon)p+(\varepsilon m-i)r}=\frac{\gamma}{c},$$

ex quo porro colligitur

$$\gamma=\frac{i(1-\varepsilon)pr}{i(1-\varepsilon)p+(\varepsilon m-i)r}.$$

Denique, cum sit $R=-\frac{\gamma}{d}=-\frac{\gamma}{\varepsilon}$ ob $s=d$, erit

$$s=-\frac{\gamma}{R}=\frac{\gamma}{i}=\frac{(1-\varepsilon)pr}{i(1-\varepsilon)p+(\varepsilon m-i)r}$$

hincque tertium intervallum

$$CD=\gamma+s=\frac{(1+i)(1-\varepsilon)pr}{i(1-\varepsilon)p+(\varepsilon m-i)r}.$$

Nunc autem aperturae praebent has aequationes:

$$1.\ \mathfrak{B}\mathfrak{q}=(P-1)M,\ \text{unde fit}\ \mathfrak{q}=\frac{((1-2\varepsilon)+i)M}{\varepsilon m},$$

$$2.\ \mathfrak{C}\mathfrak{r}=(PQ-1)M-\mathfrak{q}=\frac{sm^2-(1-\varepsilon)im-i^2}{\varepsilon im}\cdot M$$

seu

$$\mathfrak{C}\mathfrak{r}=\frac{(m+i)(\varepsilon m-i)}{\varepsilon im}\cdot M,$$

unde elicitur

$$\mathfrak{r}=-\frac{(m+i)(1-\varepsilon)p}{\varepsilon mr}\cdot M,$$

unde, cum sit $\mathfrak{s}=ir$ ideoque $\mathfrak{r}+\mathfrak{s}=(1+i)\mathfrak{r}$, erit

$$\mathfrak{q}+\mathfrak{r}+\mathfrak{s}=-\frac{(1-2\varepsilon)mr+ir-(1+i)(m+i)(1-\varepsilon)p}{\varepsilon mr}\cdot M=M(m-1)$$

sicque facta divisione per M inveniemus

$$r=\frac{-(m+i)(1+i)(1-\varepsilon)p}{\varepsilon m^2-(1-\varepsilon)m-i}\cdot M;$$

qui valor cum sit negativus, ex eo etiam prodibit intervallum *CD* negativum, unde patet hunc valorem in praxi locum habere non posse.

Verum cum saepenumero problemata duas pluresve solutiones admittant, idem etiam hic usu venit hocque problema praeter solutionem hic inventam insuper aliam complectitur, quam per divisionem ex calculo expulimus. Quod quo facilius appareat, calculum ita instituamus. Cum primo sit

$$\mathfrak{q}=\frac{(1-2\varepsilon)m+i}{\varepsilon m}\cdot M,\ \text{deinde}\ \mathfrak{s}=i\mathfrak{r},$$

erit

$$\mathfrak{q}+\mathfrak{r}+\mathfrak{s}=\frac{(1-2\varepsilon)m+i}{\varepsilon m}\cdot M+(1+i)\mathfrak{r}=M(m-1),$$

unde colligitur

$$M=\frac{\varepsilon m(1+i)\mathfrak{r}}{\varepsilon m^2-(1-\varepsilon)m-i}.$$

ideoque

$$\mathfrak{q}=\frac{(1+i)((1-2\varepsilon)m+i)\mathfrak{r}}{\varepsilon m^2-(1-\varepsilon)m-i};$$

altera vero aequatio dabit

$$\mathfrak{C}\mathfrak{r}=\frac{(m-i)\varepsilon m(1+i)\mathfrak{r}-i(1+i)((1-2\varepsilon)m+i)\mathfrak{r}}{\varepsilon m^2-(1-\varepsilon)im-i^2},$$

unde fit

$$\mathfrak{C}=\frac{(1+i)(m+i)(\varepsilon m-i)}{i(\varepsilon m^2-(1-\varepsilon)m-i)};$$

supra vero iam invenimus

$$\mathfrak{C}=\frac{-(\varepsilon m-i)r}{i(1-\varepsilon)p},$$

unde patet aequalitatem horum duorum valorum duplici modo obtineri posse:
1. scilicet, si fuerit $i=\varepsilon m$, quo quippe uterque valor evanescit, 2. autem, quo facta divisione per $\varepsilon m-i$ fit

$$\frac{(1+i)(m+i)}{\varepsilon m^2-(1-\varepsilon)m-i}=\frac{-r}{(1-\varepsilon)p},$$

haecque est solutio incongrua ante inventa. Statuamus igitur nunc $i=\varepsilon m$ fietque $\mathfrak{C}=0$, littera vero *r* hinc plane non determinatur et nostra solutio sequenti modo se habebit:

$$\alpha=p,\quad b=-\varepsilon p,\quad \beta=\infty,\quad c=-\infty,\quad \gamma=r,\quad d=\frac{r}{\varepsilon m},$$

ubi notetur fore $\beta+c=(1-\varepsilon)p.$

Hinc porro erit

$$B=\infty,\quad \mathfrak{B}=1,\quad C=0,\quad \mathfrak{C}=0,$$

tum vero

$$P=\tfrac{1}{\varepsilon},\quad Q=1,\quad R=-\varepsilon m,$$

ita ut sit $PQR=-m.$

Quia vero $B=\infty$ et $C=\mathfrak{C}=0,$ productum in se manet indefinitum; verum cum sit

$$r=\tfrac{B\mathfrak{C}}{PQ}\cdot p=\varepsilon B\mathfrak{C}p,$$

hinc vicissim erit

$$B\mathfrak{C}=\tfrac{r}{\varepsilon p}.$$

Praeterea vero erunt distantiae focales

$$q=-\varepsilon p \;\text{ et }\; s=d=\tfrac{r}{\varepsilon m}$$

atque intervalla

$$AB=BC=(1-\varepsilon)p \;\text{ et }\; CD=r\left(1+\tfrac{1}{\varepsilon m}\right).$$

Denique, cum sit

$$\mathfrak{q}=\frac{(1+\varepsilon m)(1-\varepsilon)\mathfrak{r}}{\varepsilon m-1} \;\text{ et }\; \mathfrak{s}=\varepsilon m\mathfrak{r},$$

erit

$$M=\frac{\varepsilon(1+\varepsilon m)\mathfrak{r}}{\varepsilon m-1}=\frac{(1+\varepsilon m)\mathfrak{s}}{m(\varepsilon m-1)}$$

ideoque semidiameter campi apparentis

$$\Phi=\frac{1}{4}\cdot\frac{(1+\varepsilon m)\mathfrak{s}}{m(\varepsilon m-1)}=859\frac{(\varepsilon m+1)\mathfrak{s}}{m(\varepsilon m-1)} \;\text{ minut.},$$

ubi sumere licebit $\mathfrak{s}=1$, si modo lens ocularis utrinque fiat aeque convexa.

Oculi vero post hanc lentem distantia reperitur

$$O=\tfrac{\mathfrak{s}s}{Mm}=\tfrac{\varepsilon m-1}{\varepsilon m+1}\cdot s.$$

Quia autem lentis C semidiameter aperturae maior esse nequit quam $y=\delta x$, ponamus

$$\tfrac{1}{4}\mathfrak{r}r=\delta x \;\text{ sive }\; \tfrac{\mathfrak{s}r}{4\varepsilon m}=\delta x;$$

unde sumto $\mathfrak{s}=1$ definitur

$$r = 4\delta\varepsilon mx \text{ hincque } s = 4\delta x.$$

Verum etiam ad aperturam minoris speculi est attendendum, cuius semidiameter revera est $= \delta x$; et quae ob campum esse deberet $= \frac{1}{4}\mathfrak{q}q$; quam ob causam necesse est sit

$$\frac{(1-\varepsilon)(1+\varepsilon m)\mathfrak{s}p}{4m(\varepsilon m-1)} < \delta x$$

ideoque

$$\mathfrak{s} < \frac{4m(\varepsilon m-1)\delta x}{(1-\varepsilon)(\varepsilon m+1)p}.$$

Tuto igitur sumere licebit $\mathfrak{s} = 1$, si modo fuerit

$$4m(\varepsilon m-1)\delta x > (1-\varepsilon)(\varepsilon m+1)p.$$

Contra vero $\mathfrak{s}$ unitate minus accipi deberet.

Tantum igitur superest, ut ex formula semidiametri confusionis definiamus distantiam focalem speculi principalis *p,* quae ita reperitur expressa:

$$p = kx\sqrt[3]{m}\left(\frac{1-\varepsilon}{8} + \mu\frac{\varepsilon^4 p^3}{r^3}\lambda'' + \mu\frac{\varepsilon^3 p^3}{mr^3}\lambda'''\right)^{\frac{1}{3}}$$

siquidem ambobus speculis figura sphaerica inducatur; at si ambo habeant figuram parabolicam, debebit esse

$$r = k\varepsilon x\sqrt[3]{\mu m\left(\varepsilon\lambda'' + \frac{\lambda'''}{m}\right)},$$

ita ut iam aliter non definiatur nisi ex quantitate speculi, cum sine dubio semper esse debeat *p* multo maius quam *x*. Quia vero iam ante definivimus $r = 4\delta\varepsilon mx$, habebitur nunc

$$4\delta m = k\sqrt[3]{\mu m\left(\varepsilon\lambda'' + \frac{\lambda'''}{m}\right)}.$$

Cum, nunc sit proxima $\mu = 1$ sumique possit $\lambda'' = 1$ et λ''' binario sit minus, *k* vero infra 50 capi non debeat, valorem ipsius ε aestimare poterimus; tantus enim esse debet, ut numerus

$$\frac{4\delta m}{\sqrt[3]{(\varepsilon m+2)}}$$

non minor prodeat quam 50; unde patet pro ε sumi debere fractionem valde parvam; si enim esset $\delta = \frac{1}{5}$ et $m = 100$, colligitur circiter $\varepsilon = \frac{1}{50}$.

EXEMPLUM

51. Ponamus $m=100$, $x=2$ dig., $y=\frac{1}{2}$ dig. ideoque $\delta=\frac{1}{4}$, et ut

$$\frac{4\delta m}{\sqrt[3]{(\varepsilon m+2)}}$$

satis magnum obtineat valorem, sumamus $\varepsilon=\frac{1}{20}$; sic enim prodit $k=\frac{100}{\sqrt[3]{7}}$ seu $k>50$; hinc ergo erit $r=10$ dig. et $s=2$ dig . Deinde cum pro speculo minore debeat esse $8000>57p$, erit

$$p<\frac{8000}{57}\cdot$$

Unde tuto sumi poterit $p=25$ dig. sicque erit $q=-\frac{5}{4}$ dig. et intervallum $AB=BC=23\frac{3}{4}$ dig. et $CD=12$ dig. Oculi vero distantia $O=\frac{4}{3}$ dig., at campi apparentis semidiameter $\Phi=12'\ 53''$, ubi probe notandum hic ambo specula assumi perfecte parabolica

SCHOLION

52. Quamvis autem haec constructio perfecte succedat, tamen tale telescopium tam insigni vitio erit praeditum, ut omni usu destituatur; cum enim radii a minore speculo reflexi iterum fiant inter se paralleli, radii peregrini circa hoc speculum transeuntes et in lentem *C* incidentes cum illis refractionem communem patientur simulque cum iis in oculum deferentur, ita ut verum obiectum cum vicinis prorsus permixtum visioni repraesentetur, neque ullo modo separari poterunt. Cum igitur huius vitii causa in eo sit sita, quod radii a minore speculo reflexi fiant paralleli seu intervallum $\beta=\infty$, ne hoc fiat, diligenter erit cavendum, quod fiet, si distantia β minor fuerit intervallo *BC,* ita ut in hoc intervallum imago realis incidat litteraque *Q* negativum obtineat valorem. Praeterea vero, quia etiam *R* negativum valorem habere debet ob marginem coloratum, duae iam habebuntur imagines reales et obiecta situ erecto cernentur. Neque vero duabus tantum lentibus adhibendis scopo nostro satisfacere poterimus, sed tertiam insuper lentem in subsidium vocari oportebit, quae commodissime ita instrui poterit, ut aperturam quam minimam requirat, siquidem hoc modo segregatio radiorum peregrinorum felicissime suceedet, quemadmodum in sequente problemate ostendemus.

PROBLEMA 2

52[a] *Huiusmodi telescopium cum speculo minore convexo et tribus lentibus vitreis construere, quod obiecta situ erecto distincte repraesentet.*

SOLUTIO

Maneat ut ante $y = \delta x$ et intervallum speculorum $AB = (1-\varepsilon)p = BC$, ut sit $b = -\varepsilon p$. Iam cum debeat esse $\beta < (1-\varepsilon)p$ et tamen superare

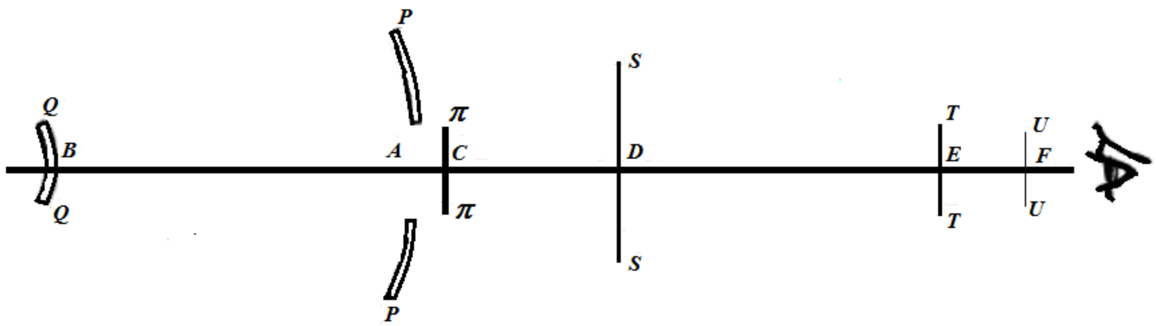

Fig. 9.

debeat eius semissem $\frac{1}{2}(1-\varepsilon)p$, statuamus $\beta = \zeta(1-\varepsilon)p$, ita ut ζ inter limites 1 et $\frac{1}{2}$ contineatur; hinc ergo fiet

$$q = \frac{\zeta\varepsilon(1-\varepsilon)}{\varepsilon-\zeta(1-\varepsilon)}\cdot p = \frac{-\zeta\varepsilon(1-\varepsilon)}{\zeta-\varepsilon(\zeta+1)}\cdot p.$$

Tum vero erit

$$B = \frac{\beta}{b} = \frac{-\zeta(1-\varepsilon)}{\varepsilon} \quad \text{et} \quad \mathfrak{B} = \frac{\zeta(1-\varepsilon)}{\zeta-\varepsilon(\zeta+1)}.$$

Porro vero erit

$$c = (1-\varepsilon)(1-\zeta)p$$

sicque habebimus

$$P = \frac{1}{\varepsilon}, \quad Q = \frac{-\beta}{\varepsilon} = \frac{-\zeta}{1-\zeta}.$$

Statuatur igitur praeterea $R = -k$ fiatque

$$PQRS = m = \frac{\zeta k}{\varepsilon(1-\zeta)}\cdot \mathrm{S},$$

unde reliquae distantiae focales erunt

$$r = (1-\varepsilon)(1-\zeta)\mathfrak{C}p, \quad s = \frac{(1-\varepsilon)(1-\zeta)C\mathfrak{D}}{k}\cdot p$$

et

$$t = \frac{-\zeta(1-\varepsilon)CD}{\varepsilon m}\cdot p$$

reliquaque intervalla

$$CD = \left(1-\varepsilon\right)(1-\zeta)\left(1+\tfrac{1}{k}\right)Cp,$$
$$DE = \left(1-\varepsilon\right)(1-\zeta)\left(1-\tfrac{1}{S}\right)CDp,$$

unde intelligemus esse debere $C>0$ ideoque $\mathfrak{C}<1$ et ($1-\frac{1}{S})D>0$. Ut vero fiat $t>0$, debet esse $D<0$ ideoque $S<1$.

Consideretur nunc aequatio pro margine colorato tollendo, quae est

$$0=\mathfrak{r}+\tfrac{\mathfrak{s}}{R}+\tfrac{\mathfrak{t}}{RS} \quad \text{sive} \quad \mathfrak{r}=\tfrac{\mathfrak{s}}{k}+\tfrac{\mathfrak{t}}{kS};$$

ut iam secunda lens nulla apertura indigeat, statuatur $\mathfrak{s}=0$ eritque

$$\mathfrak{r}=\tfrac{\mathfrak{t}}{kS};$$

aequationes autem pro litteris $\mathfrak{r}$, $\mathfrak{s}$, $\mathfrak{t}$, posito

$$M=\tfrac{\mathfrak{q}+\mathfrak{r}+\mathfrak{s}+\mathfrak{t}}{m-1}=\tfrac{\mathfrak{q}+(1+kS)\mathfrak{r}}{m-1},$$

sunt

$$1.\ \mathfrak{B}\mathfrak{q}=\tfrac{1-\varepsilon}{\varepsilon}\cdot M,$$

$$2.\ \mathfrak{B}\mathfrak{r}=\frac{-\left((1-\varepsilon)\zeta+\varepsilon\right)}{\varepsilon\left(1-\zeta\right)}\cdot M-\mathfrak{q},$$

$$3.\ 0=\frac{\zeta\left(\varepsilon+k\right)-\varepsilon}{\varepsilon\left(1-\zeta\right)}\cdot M-\mathfrak{q}-\mathfrak{r}.$$

Ex prima autem habetur

$$\mathfrak{q}=\frac{\zeta-\varepsilon\left(\zeta+1\right)}{\varepsilon\zeta}\cdot M.$$

Ex tertia autem fit

$$\mathfrak{q}=\frac{\zeta\left(\varepsilon+k\right)-\varepsilon}{\varepsilon\left(1-\zeta\right)}\cdot M-\mathfrak{r},$$

qui duo valores inter se aequati dant

$$M=\frac{\varepsilon\zeta\left(1-\zeta\right)\mathfrak{r}}{\zeta^2\left(1+k\right)-\zeta\left(1+\varepsilon\right)+\varepsilon}$$

hincque

$$\mathfrak{q}=\frac{\left(1-\zeta\right)\left(\zeta-\varepsilon\left(1+\zeta\right)\right)\mathfrak{r}}{\zeta^2\left(1+k\right)-\zeta\left(1+\varepsilon\right)+\varepsilon}.$$

Tum vero ob $M=\frac{\mathfrak{q}+\mathfrak{r}(1+kS)}{m-1}$ reperietur etiam

$$M=\frac{kS\mathfrak{r}}{m-1+\frac{-\zeta(\varepsilon+k)+\varepsilon}{\varepsilon(1-\zeta)}},$$

ex quorum valorum aequalitate ob reperitur tandem

$$\zeta\left(\zeta kS-\varepsilon(1-\zeta)-\zeta(\varepsilon+k)+\varepsilon\right)=kS\left(\zeta^2(1+k)-\zeta(1+\varepsilon)+\varepsilon\right)$$

seu

$$\zeta^2=S\left(\zeta(1+\varepsilon)-k\zeta^2-\varepsilon\right),$$

unde concludimus esse debere

$$\zeta(1+\varepsilon)>k\zeta^2+\varepsilon \quad \text{sive} \quad k<\frac{\zeta(1+\varepsilon)-\varepsilon}{\zeta^2}.$$

Praeterea vero, ut ex secunda aequatione pro $\mathfrak{C}$ prodeat valor positivus, necesse est, ut sit $\mathfrak{q}<0$ ideoque etiam $\mathfrak{B}<0$, unde speculum minus foret concavum; verum ut fiat $\mathfrak{B}<0$, debet esse $\zeta<\varepsilon(\zeta+1)$ seu $\varepsilon>\frac{\zeta}{\zeta+1}$. Hoc vero non sufficit, sed insuper necesse est, ut sit

$$-\mathfrak{q}>\frac{(1-\varepsilon)\zeta+\varepsilon}{\varepsilon(1-\zeta)}\cdot M$$

seu

$$\frac{-\zeta+\varepsilon(\zeta+1)}{\zeta}>\frac{(1-\varepsilon)\zeta+\varepsilon}{(1-\zeta)},$$

unde sequitur $\varepsilon>\frac{\zeta}{1-\zeta}$;; quod cum nullo modo fieri queat, quia ζ intra limites 1 et $\frac{1}{2}$ continetur et ε unitate minus esse debet, nunc demum intelligimus hunc casum locum habere non posec.

ALlA SOLUTIO

53. Quoniam igitur hoc incommodum inde nascitur, quod sumsimus R negativum, consideremus alterum casum, quo S fit negativum manente R positivo, et quoniam Q positum est negativum, ponamus

$$Q=-i \text{ et } S=-k,$$

ut sit

$$PQRS=\frac{iRk}{\varepsilon}=m;$$

calculus autem commodior evadet, si littera i retineatur, et cum sit

$i=\frac{\beta}{\varepsilon}$ et $\beta+c=(1-\varepsilon)p$, evidens est capi debere $i>1$ eritque

$$B=\tfrac{i(1-\varepsilon)}{1+i}\cdot p \quad \text{et} \quad c=\tfrac{1-\varepsilon}{1+i}\cdot p,$$

unde fit

$$B=+\tfrac{\beta}{b}=-\tfrac{i(1-\varepsilon)}{\varepsilon(1+i)} \quad \text{et} \quad \mathfrak{B}=+\tfrac{i(1-\varepsilon)}{i(1-2\varepsilon)-\varepsilon}$$

hincque

$$q=-\tfrac{i\varepsilon(\mathrm{i}-\varepsilon)}{i(1-2\varepsilon)-\varepsilon}\cdot p.$$

Reliquae vero distantiae focales erunt

$$r=+\tfrac{(1-\varepsilon)\mathfrak{C}}{(1+i)}\cdot p, \quad s=-\tfrac{(1-\varepsilon)C\mathfrak{D}}{(1+i)R}\cdot p \ \text{ et } \ t=-\tfrac{(1-\varepsilon)CD}{(1+i)Rk}\cdot p$$

et duo reliqua intervalla erunt

$$CD=+\tfrac{(1-\varepsilon)C}{1+i}\left(1-\tfrac{1}{R}\right)\cdot p,$$

$$DE=-\tfrac{(1-\varepsilon)CD}{(1+i)R}\left(1+\tfrac{1}{k}\right)\cdot p.$$

Ut igitur fiat $t>0$, debet esse CD negativum, quo ipso etiam ultimum intervallum fit positivum. Ut vero et penultimum fiat positivum, debet esse $C\left(1-\frac{1}{R}\right)$ positivum.

Conditio porro marginis colorati sumto $\mathfrak{s}=0$ praebet $\mathfrak{t}=\frac{1}{Rk}$ sive $\mathfrak{t}=Rk\mathfrak{r}$, et cum sit

$$M=\tfrac{\mathfrak{q}+\mathfrak{r}+\mathfrak{t}}{m-1}=\tfrac{\mathfrak{q}+(1+Rk)\mathfrak{r}}{m-1},$$

satisfieri oportet his tribus aequationibus:

$$1.\ \mathfrak{B}\mathfrak{q}=\tfrac{1-\varepsilon}{\varepsilon}\cdot M,$$

$$2.\ \mathfrak{C}\mathfrak{r}=-\tfrac{(1+\varepsilon)}{\varepsilon}\cdot M-\mathfrak{q},$$

$$3.\ 0=+\tfrac{(iR+\varepsilon)}{\varepsilon}\cdot M+\mathfrak{q}+\mathfrak{r}.$$

Ex tertia ergo fit

$$\mathfrak{q}+\mathfrak{r}=-\tfrac{(iR+\varepsilon)}{\varepsilon}\cdot M$$

hincque

$$\mathfrak{q}+\mathfrak{r}(1+Rk)=-\tfrac{(iR+\varepsilon)}{\varepsilon}\cdot M+Rk\mathfrak{r}=M\left(m-1\right),$$

unde colligitur

$$M=\frac{Rk\mathfrak{r}}{m+\frac{iR}{\varepsilon}}$$

simulque

$$\mathfrak{q}=-\tfrac{Rk(iR+\varepsilon)}{m\varepsilon+iR}\cdot\mathfrak{r}-\mathfrak{r}=-\tfrac{\left(kiR^2+R(i+k\varepsilon)+m\varepsilon\right)}{m\varepsilon+iR}\cdot\mathfrak{r},$$

ex quo valor ipsius $\mathfrak{q}$ prodit negativus; qui cum ex prima forma prodeat positivus, siquidem est $\mathfrak{B}>0$, patet etiam hanc solutionem locum habere non posse, siquidem secundum speculum est convexum, uti assumsimus.

TERTIA SOLUTIO

54. Pro repraesentatione igitur erecta unicus tantum casus superest, quo sumto Q positivo ambae litterae R et S negativos obtinent valores. Statuamus igitur

$$Q=+i,\ R=-k\ \text{ et }\ S=-k',$$

ut sit

$$PQRS=m=\tfrac{ikk'}{\varepsilon}\ \text{ hincque } k'=\tfrac{\varepsilon m}{ik}.$$

Porro erit

$$\beta=\tfrac{i(1-\varepsilon)}{i-1}\cdot p,\ \ c=-\tfrac{(1-\varepsilon)}{i-1}\cdot p,$$

unde fit

$$B=\tfrac{-i(1-\varepsilon)}{\varepsilon(i-1)},\quad\text{et}\quad\mathfrak{B}=\tfrac{i(1-\varepsilon)}{i(1-2\varepsilon)+\varepsilon};$$

quare distantiae focales sequenti modo se habebunt:

$$q=\tfrac{-\varepsilon i(1-\varepsilon)}{i(1-2\varepsilon)+\varepsilon}\cdot p,\ \ r=\tfrac{-(1-\varepsilon)\mathfrak{C}}{i-1}\cdot p,\ \ \ s=\tfrac{-(1-\varepsilon)C\mathfrak{D}}{(i-1)k}\cdot p,$$

et

$$t=\tfrac{-(1-\varepsilon)CD}{(i-1)kk'}\cdot p=\tfrac{-i(1-\varepsilon)CD}{\varepsilon(i-1)m}\cdot p$$

Intervalla vero lentium erunt

$$CD=\tfrac{-(1-\varepsilon)C}{i-1}\left(1+\tfrac{1}{k}\right)p,$$

$$DE=\tfrac{-(1-\varepsilon)CD}{(i-1)k}\left(1+\tfrac{1}{k'}\right)p,$$

unde intelligimus esse debere $C<0$ et $D>0$ ideoque $\mathfrak{D}<1$ et $\mathfrak{D}>0$..

Nunc autem conditio marginis colorati dabit

$$0=\mathfrak{r}+\frac{\mathfrak{t}}{kk'},$$

unde patet esse debere $\mathfrak{r}<0$ seu ob lentem C campum diminui. Ponamus ergo hic $\mathfrak{r}=-\omega$, ut fiat $\mathfrak{t}=\omega kk'=\frac{\varepsilon m}{i}\cdot\omega$, quandoquidem etiam hic assumsimus $\mathfrak{s}=0$: pro campo ergo apparente erit

$$M=\frac{i\mathfrak{q}+\omega(\varepsilon m-i)}{i(m-1)},$$

cui sequentes tres aequationes sunt adiungendae:

$$1.\ \mathfrak{B}\mathfrak{q}=\frac{1-\varepsilon}{\varepsilon}\cdot M,$$

$$2.\ -\mathfrak{C}\omega=\frac{1-\varepsilon}{\varepsilon}\cdot M-\mathfrak{q},$$

$$3.\ 0=-\frac{(ik+\varepsilon)}{\varepsilon}\cdot M-\mathfrak{q}+\omega.$$

Ex hac ultima ergo concludimus

$$\mathfrak{q}=\omega-\frac{(ik+\varepsilon)}{\varepsilon}\cdot M;$$

addatur utrinque $\omega\left(\frac{\varepsilon m}{i}-1\right)$ eritque

$$\mathfrak{q}+\omega\left(\frac{\varepsilon m}{i}-1\right)=\frac{\varepsilon m}{i}\cdot\omega-\frac{(ik+\varepsilon)}{\varepsilon}\cdot M=M(m-1),$$

ex quo colligitur

$$M=\frac{\varepsilon^2 m\omega}{i(m\varepsilon+ik)},$$

unde vicissim

$$\mathfrak{q}=\frac{\varepsilon m(i-ik-\varepsilon)+i^2k}{i(m\varepsilon+ik)}\cdot\omega.$$

Ex prima vero aequatione fit

$$\mathfrak{q}=\frac{\left(\varepsilon i(1-2\varepsilon)+\varepsilon^2\right)m\omega}{i^2(m\varepsilon+ik)},$$

quorum valorum aequalitas suppeditat hanc aequationem:

$$\varepsilon im(i-ik-\varepsilon)+i^3k=\varepsilon m\left(i(1-2\varepsilon)+\varepsilon\right)$$

seu

$$\varepsilon m(i^2(1-k)-i(1-\varepsilon)-\varepsilon)+i^3k=0.$$

ex qua aequatione invenimus

$$k=\frac{\varepsilon m\left(i^2-i(1-\varepsilon)-\varepsilon\right)}{i^2(\varepsilon m-i)}$$

seu

$$k=\frac{\varepsilon m(i+\varepsilon)(i-1)}{i^2(\varepsilon m-i)},$$

qui valor debet esse positivus; quem in finem sumi debet $i>1$ et $i<\varepsilon m$.
Iam substituto valore ipsius k reperitur

$$M=\frac{\varepsilon(\varepsilon m-i)\omega}{\varepsilon im-i(1-\varepsilon)-\varepsilon}.$$

Ex secunda denique aequatione colligimus

$$\mathfrak{C}=-\frac{k(\varepsilon m-i)}{\varepsilon m+ik}.$$

Secunda vero aequatio

$$\mathfrak{C}=-\frac{\varepsilon m(i+\varepsilon)(i-1)}{i^2(\varepsilon m+ik)},$$

qui valor ergo est negativus ideoque et $C<0$, uti supra iam requirebatur. Litterae autem $\mathfrak{D}$ et D arbitrio nostro manent permissae, dummodo D positive capiatur; quod tandem ad ipsam quantitatem p attinet, eam ex confusione definiri convenit ope formulae notae, ubi inprimis dispiciendum erit, utrum speculis figura sphaerica inducta sit an parabolica

COROLLARIUM 1

55. Si ergo littera $\mathfrak{t}$ in calculum introducatur, quam licebit unitati aequalem sumere, pro campo apparente habebimus

$$M=\frac{i(\varepsilon m-i)}{m\left(\varepsilon im-i(1-\varepsilon)-\varepsilon\right)}\cdot\mathfrak{t},$$

quippe qui valor per 859 min. multiplicatus dat semidiametrum campi Φ.
Vidimus autem litteram i intra limites 1 et εm capi debere.

COROLLARIUM 2

56. Si caperetur $i=1$, foret $\beta=\infty$ et radii a speculo minore reflexi fierent inter se paralleli, unde vitium supra memoratum oriretur, quod scilicet radii peregrini ita cum propriis permiscerentur, ut nullo modo separari possent; qui casus cum sit sollicite evitandus, litteram i unitate multo maiorem accipi conveniet, neque tamen alteri limiti εm aequalis assumi potest, quia alioquin campus prorsus evanesceret.

COROLLARIUM 3

57. Calculum instituenti facile patebit maximum in hac expressione *M* locum non habere et eius valorem eo magis diminutum iri, quo maior littera *i* accipiatur. Quare, cum esse debeat $i>1$, si sumamus $i=2$, erit

$$M=\frac{2(\varepsilon m-2)\mathfrak{t}}{m(2\varepsilon m+\varepsilon-2)}$$

sicque pro magnis multiplicationibus $M=\frac{1}{m}\cdot\mathfrak{t}$, qui valor etiam prodit, si capiatur $i=3$ vel 4 etc., dummodo *i* sit multo minus quam εm, qui campus simplex censeri solet. Sin autem medium inter limites sumendo capiatur $i=\frac{\varepsilon m+1}{2}$, fiet

$$M=\frac{(\varepsilon m+1)\mathfrak{t}}{2m(\varepsilon m+\varepsilon+1)}$$

et pro magnis multiplicationibus campus ad dimidium redigetur.

COROLLARIUM 4

58. Idem etiam patet ex primitivo valore ipsius *M,* qui est

$$M=\frac{\mathfrak{q}+\mathfrak{r}+\mathfrak{t}}{m-1},$$

pro quo $\mathfrak{r}=-\omega=-\frac{i\mathfrak{t}}{\varepsilon m}$. Etsi autem $\mathfrak{q}$ addi debet, tamen ex superioribus patet esse $\mathfrak{q}<\omega$; erat enim ex tertia aequatione

$$\mathfrak{q}=\omega-\frac{(ik+\varepsilon)}{\varepsilon}\cdot M.$$

SCHOLION 1

59. Circa campum autem inprimis est inquirendum, an loco $\mathfrak{t}$ scribere liceat unitatem, quod iudicium ex prima lente *C* est petendum, cuius semidiameter aperturae revera est $=\delta x,$ ob campum autem esse debet $=\frac{1}{4}\mathfrak{r}r.$

Cum igitur sit $\mathfrak{r}=-\frac{i\mathfrak{t}}{\varepsilon m}$ et

$$r=\frac{(1-\varepsilon)\mathfrak{C}}{i-1}\cdot p \quad \text{seu} \quad \frac{\varepsilon m(1-\varepsilon)(i+\varepsilon)}{i^2(\varepsilon m+ik)}\cdot p,$$

iam supra autem invenerimus esse

$$\varepsilon m+ik=\frac{\varepsilon m(i(\varepsilon m+\varepsilon-1)-\varepsilon)}{i(\varepsilon m-i)}=\frac{\varepsilon m(\varepsilon im-i(1-\varepsilon)-\varepsilon)}{i(\varepsilon m-i)},$$

quocirca erit

$$r=\frac{(\varepsilon m-i)(1-\varepsilon)(1+\varepsilon)}{i(\varepsilon im-i(1-\varepsilon)-\varepsilon)}\cdot p;$$

unde, nisi fuerit

$$\frac{(\varepsilon m-i)(1-\varepsilon)(1+\varepsilon)}{\varepsilon m(\varepsilon im-i(1-\varepsilon)-\varepsilon)}\cdot p>4\delta x,$$

tum sumere licebit $\mathfrak{t}=1$. Contra vero $\mathfrak{t}$ tanto minus unitate capi debebit, ubi notasse iuvabit esse $\delta>\varepsilon$. Quoniam autem hae formulae nimis sunt complicatae, quam ut in genere omnia momenta pro constructione telescopii commode exprimi queant, statuamus $i=\frac{1}{2}(\varepsilon m+1)$, ut intervallum *CD* minus evadat, etsi campus ad semissem redigitur; deinde enim videbimus, quomodo campus amplificari possit. Posito autem $i=\frac{\varepsilon m+1}{2}$ erit

$$k=\frac{2\varepsilon m(\varepsilon m+2\varepsilon+1)}{(\varepsilon m+1)^2},$$

qui valor abit in $k=2$ pro magnis multiplicationibus.

Deinde vero

$$\mathfrak{C}=\frac{-(\varepsilon M+2\varepsilon+1)(\varepsilon m-1)^2}{2(\varepsilon m+1)((\varepsilon m+1)(\varepsilon m+\varepsilon-1)-2\varepsilon)}\ \left[=-\frac{(\varepsilon m+2\varepsilon+1)(\varepsilon m-1)}{2(\varepsilon m+1)(\varepsilon m+\varepsilon+1)}\right],$$

unde *C* reperitur.

SCHOLION 2

60. Quia vero valor $i=\frac{\varepsilon m+1}{2}$ merito nimis magnus videri potest, pro *i* potius medium geometricum sumamus sitque $i=\sqrt{\varepsilon m}$, ac primo pro campo apparente fiet

$$M=\frac{\varepsilon}{\varepsilon m+\sqrt{\varepsilon m}+\varepsilon}\cdot\mathfrak{t}.$$

Deinde vero habebimus

$$k=\frac{\varepsilon+\sqrt{\varepsilon m}}{\sqrt{\varepsilon m}}$$

hincque

$$B=\frac{-(1-\varepsilon)\sqrt{\varepsilon m}}{\varepsilon(\sqrt{\varepsilon m}-1)}\quad\text{et}\quad\mathfrak{B}=\frac{(1-\varepsilon)\sqrt{\varepsilon m}}{(1-2\varepsilon)\sqrt{\varepsilon m}+\varepsilon},$$

$$\mathfrak{C}=\frac{-(\varepsilon+\sqrt{\varepsilon m})(\sqrt{\varepsilon m}-1)}{\varepsilon m+\sqrt{\varepsilon m}+\varepsilon}\quad\text{et}\quad C=\frac{-(\varepsilon+\sqrt{\varepsilon m})(\sqrt{\varepsilon m}-1)}{2\varepsilon m+\varepsilon\sqrt{\varepsilon m}}.$$

Ex his, si ponamus $D=\theta$, ut sit $\mathfrak{D}=\frac{\theta}{1+\theta}$, reperientur distantiae focales

$$p = p,\quad q = \frac{-\varepsilon(1-\varepsilon)\sqrt{\varepsilon m}}{(1-2\varepsilon)\sqrt{\varepsilon m}+\varepsilon}\cdot p,\qquad r = \frac{(1-\varepsilon)\left(\varepsilon+\sqrt{\varepsilon m}\right)}{\varepsilon m+\sqrt{\varepsilon m}+\varepsilon}\cdot p,$$

$$s = \frac{\theta}{1+\theta}\cdot\frac{(1-\varepsilon)}{2\sqrt{\varepsilon m}+\varepsilon}\cdot p,\quad t = \frac{\theta(1-\varepsilon)\left(\varepsilon+\sqrt{\varepsilon m}\right)}{\varepsilon m\left(\varepsilon+2\sqrt{\varepsilon m}\right)}\cdot p.$$

Intervalla vero lentium erunt

$$AB = BC = (1-\varepsilon)p,$$

$$CD = \frac{(1-\varepsilon)\left(\varepsilon+2\sqrt{\varepsilon m}\right)}{2\varepsilon m+\varepsilon\sqrt{\varepsilon m}}\cdot p,$$

$$DE = \frac{\theta(1-\varepsilon)\left(\varepsilon+\sqrt{\varepsilon m}+\varepsilon\right)}{\varepsilon m\left(\varepsilon+2\sqrt{\varepsilon m}\right)}\cdot p.$$

Pro loco autem oculi erit

$$O = \frac{\mathfrak{t}t}{Mm} = \frac{\varepsilon m+\sqrt{\varepsilon m}+\varepsilon}{\varepsilon m}\cdot\mathfrak{t} = \mathfrak{t}\left(1+\frac{1}{\sqrt{\varepsilon m}}+\frac{1}{m}\right).$$

Pro aperturis autem invenimus

$$\mathfrak{q} = \frac{(1-2\varepsilon)\sqrt{\varepsilon m}+\varepsilon}{\left(\varepsilon m+\sqrt{\varepsilon m}+\varepsilon\right)\sqrt{\varepsilon m}}\cdot\mathfrak{t},\quad \mathfrak{r} = -\frac{\mathfrak{t}}{\sqrt{\varepsilon m}}\ \text{ et }\ \mathfrak{s} = 0.$$

Licebit autem sumere $\mathfrak{t} = 1$, nisi prodeat

$$\frac{(1-\varepsilon)\left(\varepsilon+\sqrt{\varepsilon m}\right)}{\left(\varepsilon m+\sqrt{\varepsilon m}+\varepsilon\right)\sqrt{\varepsilon m}}\cdot p > 4\delta x.$$

Lenti autem in *D,* pro qua est $\mathfrak{s} = 0$, apertura tribui debet, cuius semidiameter sit $= \frac{x}{PQR} = \frac{\varepsilon x}{\varepsilon+\sqrt{\varepsilon m}}$, ita ut huius lentis apertura sit tam exigua, ut ad radios peregrinos arcendos apprime sit accommodata. Interim tamen, quia campus apparens hic nimis est exiguus, utique operae erit pretium huic generi telescopiorum maiorem campum procurare, quod in sequente problemate praestabimus.

PROBLEMA 3

61. *Telescopiorum generi in problemate praecedente descripto novum gradum perfectionis addere, dum eius campus apparens amplificatur.*

SOLUTIO

Fit hoc additione novae lentis, ita ut nunc telescopium ex duobus speculis et quatuor lentibus componatur. Maneat autem ut ante

$$P=\tfrac{1}{\varepsilon},\quad Q=i,\quad R=-k\quad \text{et}\quad S=-k',$$

quibus accedente littera T sit

$$\tfrac{ikk'T}{\varepsilon}=m;$$

deinde sit etiam ut ante

$$B=\frac{-i(1-\varepsilon)}{\varepsilon(i-1)}\ \text{ hincque }\ \mathfrak{B}=\frac{i(1-\varepsilon)}{i(1-2\varepsilon)+\varepsilon},$$

ex quibus distantiae focales ita formabuntur:

$$q=-\tfrac{\mathfrak{B}}{P}\cdot p=-\varepsilon\mathfrak{B}p,\quad r=\tfrac{B\mathfrak{C}}{PQ}\cdot p=\tfrac{\varepsilon B\mathfrak{C}}{ik}\cdot p,\quad s=\tfrac{\varepsilon BC\mathfrak{D}}{ik}\cdot p,\quad t=\tfrac{\varepsilon BCD\mathfrak{E}}{ikk'}\cdot p$$

et

$$u=-\tfrac{\varepsilon BCDE}{ikk'T}\cdot p=-\tfrac{\varepsilon BCDE}{m}\cdot p$$

et intervalla

$$AB=BC=(1-\varepsilon)\,p,\quad CD=\tfrac{\varepsilon BC}{i}(1+\tfrac{1}{k})p,\quad DE=\tfrac{\varepsilon BCD}{ik}(1+\tfrac{1}{k'})p$$

et

$$EF=\tfrac{\varepsilon BCDE}{ikk'}(1-\tfrac{1}{T})p,$$

ubi, cum sit $B<0$, debet esse $C<0$, deinde $D>0$. Porro ut fiat u positivum, debet esse $E<0$ hincque ob ultimum intervallum $T<1$. Nunc statuatur etiam $\mathfrak{r}=-\omega,\ \mathfrak{s}=0$, et ut campus maximus evadat, $\mathfrak{u}=\mathfrak{t}$, ut sit

$$M=\frac{\mathfrak{q}-\omega+2\mathfrak{t}}{m-1}.$$

Ut vero margo coloratus evanescat, debet esse

$$\omega=\tfrac{\mathfrak{t}}{kk'}+\tfrac{\mathfrak{u}}{kk'T}=\tfrac{\mathfrak{t}}{kk'}\left(1+\tfrac{1}{T}\right),$$

et quia debet esse $T<1$, sumatur statim $T=\frac{1}{2}$, ut sit $m=\frac{ikk'}{2\varepsilon}$ hincque $kk'=\frac{2\varepsilon m}{i}$; dum igitur $\omega=\frac{3i}{2\varepsilon m}\mathfrak{t}$ ac vicissime $\mathfrak{t}=\frac{2\varepsilon m\omega}{3i}$; unde fit

$$M=\frac{\mathfrak{q}+\omega\left(\frac{4\varepsilon m}{3i}-1\right)}{m-1}.$$

Nunc autem considerari oportet sequentes quatuor aequationes:

1. $\mathfrak{B}\mathfrak{q} = \frac{1-\varepsilon}{\varepsilon} \cdot M,$

2. $\mathfrak{C}\omega = \frac{i-\varepsilon}{\varepsilon} \cdot M - \mathfrak{q},$

3. $0 = -\left(\frac{ik+\varepsilon}{\varepsilon}\right) \cdot M - \mathfrak{q} + \omega,$

4. $\mathfrak{E}\mathfrak{t} = \frac{ikk'-\varepsilon}{\varepsilon} \cdot M - \mathfrak{q} + \omega.$

Ex tertia igitur habemus

$$\mathfrak{q} - \omega = -\left(\frac{ik+\varepsilon}{\varepsilon}\right) \cdot M;$$

addatur utrinque $\frac{4\varepsilon m\omega}{3i}$ ac prodibit

$$M\left(m-1\right) = \frac{4\varepsilon m\omega}{3i} - \left(\frac{ik+\varepsilon}{\varepsilon}\right) M,$$

unde invenitur

$$M = \frac{4\varepsilon m\omega}{3i\left(m+\frac{ik}{\varepsilon}\right)} = \frac{4\varepsilon^2 m\omega}{3i\left(m\varepsilon+ik\right)}$$

seu substituto valore ipsius ω

$$M = \frac{2\varepsilon}{m\varepsilon+ik} \cdot \mathfrak{t},$$

atque insuper ex eadem aequatione erit

$$\mathfrak{q} = \frac{3i\left(m\varepsilon+ik\right) - 4\varepsilon m\left(ik+\varepsilon\right)}{3i\left(m\varepsilon+ik\right)} \cdot \omega;$$

at vero prima aequatio dat

$$\mathfrak{q} = \frac{4\left(1-\varepsilon\right)\varepsilon m\omega}{3i\left(m\varepsilon+ik\right)\mathfrak{B}}$$

quorum valorum aequalitas praebet

$$3i\left(m\varepsilon+ik\right) - 4\varepsilon m\left(ik+\varepsilon\right) = \frac{4\varepsilon\left(1-\varepsilon\right)m}{\mathfrak{B}} = \frac{4\varepsilon m\left(i\left(1-2\varepsilon\right)+\varepsilon\right)}{i},$$

unde fit

$$ik\left(4\varepsilon m - 3i\right) = \varepsilon m\left(3i - 4\varepsilon\right) - \frac{4\varepsilon m\left(i\left(1-2\varepsilon\right)+\varepsilon\right)}{i} = \frac{\varepsilon m}{i}\left(3i^2 - 4i\left(1-\varepsilon\right) - 4\varepsilon\right)$$

seu

$$ik = \frac{\varepsilon m\left(3i^2 - 4i\left(1-\varepsilon\right) - 4\varepsilon\right)}{i\left(4\varepsilon m - 3i\right)};$$

qui valor ut sit positivus, debet esse

$$i < \tfrac{4}{3}\varepsilon m \quad \text{simulque} \quad i > \tfrac{2}{3}\left(1 - \varepsilon + \sqrt{1+\varepsilon+\varepsilon^2}\right)$$

Hinc autem valore ipsius k definito secunda aequatio dabit

$$\mathfrak{C}=\frac{-4\varepsilon m\left(i^2-i(1-\varepsilon)-\varepsilon\right)}{3i^2(m\varepsilon+ik)}=\frac{-4\varepsilon m(i-1)(i+\varepsilon)}{3i^2(m\varepsilon+ik)}$$

sive ex altero valore ipsius q

$$\mathfrak{C}=\frac{-\varepsilon m(1+4k)+3ik}{3(m\varepsilon+ik)};$$

erit ergo ob $\mathfrak{C}<0$ etiam $C<0$, uti requiritur; ex quorum valorum aequalitate idem valor pro k qui ante prodit. Notetur autem hic esse

$$\varepsilon m+ik=\frac{4\varepsilon m\left(\varepsilon im-i(1-\varepsilon)-\varepsilon\right)}{i(4\varepsilon m-3i)},$$

unde fit

$$M=\frac{i(4\varepsilon m-3i)\mathfrak{t}}{2m\left(\varepsilon im-i(1-\varepsilon)-\varepsilon\right)}.$$

Deinde vero littera D arbitrio nostro permittitur, dummodo sumatur positiva

Quarta denique aequatio nobis praebet valorem litterae

$$\mathfrak{E}=\frac{2(2\varepsilon m+ik)}{\varepsilon m+ik}=\frac{4\left(2\varepsilon im-(1-\varepsilon)i-\varepsilon\right)-3i^2}{2\left(\varepsilon im-(1-\varepsilon)i-\varepsilon\right)},$$

qui valor sponte positivus et unitate maior est; quare $E<0$, uti oportet.

Tandem pro loco oculi habebimus

$$O=\frac{u\mathfrak{t}}{Mm}=\frac{2\left(\varepsilon im-i(1-\varepsilon)-\varepsilon\right)}{i(4\varepsilon m-3i)}\cdot u$$

sive

$$O=\frac{1}{2}u\left(1+\frac{3i^2-4i(1-\varepsilon)-4\varepsilon}{i(4\varepsilon m-3i)}\right).$$

Superest porro, ut diiudicemus, an pro $\mathfrak{t}$ unitas accipi queat, quod licebit, si fuerit

$$r<\frac{4\delta x}{\omega}\quad\text{seu}\quad r<\frac{8\delta\varepsilon mx}{3i}.$$

Contra vero accipi debet $\mathfrak{t}=\frac{8\delta\varepsilon mx}{3ir}$, quo casu campus in eadem ratione diminuetur, in qua $\mathfrak{t}$ ab unitate deficit. Quod autem ad quantitatem p attinet, ea ex aequatione nota definiri debet speculorum ratione habita; utrum sint sphaerica an parabolica

COROLLARIUM 1

62. Quia lens in *D,* quam minimo foraminulo pertundi sufficit, a lente *C* distat intervallo

$$CD = \frac{\varepsilon BC}{i}\left(1+\frac{1}{k}\right)p,$$

radii autem peregrini in lentem C incidentes post eam colliguntur ad distantiam $r = \frac{\varepsilon B\mathfrak{C}}{i}\cdot p$, ut hi radii excludantur, necesse est, ut hae duae distantiae a se invicem discrepent, seu notabilis differentia esse debet inter has quantitatas $C\left(1+\frac{1}{k}\right)$ et $\mathfrak{C}$, hoc est inter $1+\frac{1}{k}$ et $1-\mathfrak{C}$ seu inter $\frac{1}{k}$ et $-\mathfrak{C}$.

Est vero

$$\frac{1}{k} = \frac{i^2(4\varepsilon m-3i)}{\varepsilon m\left(3i^2-4i(1-\varepsilon)-4\varepsilon\right)} \quad \text{et} \quad -\mathfrak{C} = \frac{(4\varepsilon m-3i)(i-1)(i+\varepsilon)}{3i\left(\varepsilon im-i(1-\varepsilon)-\varepsilon\right)};$$

quare, cum ratio inter has quantitates debeat esse admodum inaequalis,
haec fractio:

$$\frac{3i^3\left(\varepsilon im-i(1-\varepsilon)-\varepsilon\right)}{\varepsilon m(i-1)(i+\varepsilon)\left(3i^2-4i(1-\varepsilon)-4\varepsilon\right)}$$

plurimum ab unitate discrepare debet; at differentia inter numeratorem et denominatorem satis est magna, ut aequalitas non sit metuenda

COROLLARIUM 2

63. Quodsi autem sumamus $i=2$, fractio illa ab unitate diversa evadet $=\frac{6(2\varepsilon m+\varepsilon-2)}{\varepsilon m(1+\varepsilon)(2+\varepsilon)}$, quae utique satis ab unitate discrepat, ut transitus radiorum peregrinorum neutiquam sit metuendus. Campi autem ratio maxime exigit, ut ipsi i tam parvum valorem tribuamus, quam circumstantiae permittunt. Ceterum multo magis ille transitus evitabitur, si capiatur $i<2$.

EXEMPLUM. 1
Pro multiplicatione $m=50$

64. Ponamus hic $\delta=\frac{1}{4}$, $\varepsilon=\frac{1}{5}$, et quia haec multiplicatio postulat $x=1$ dig., erit $y=\frac{1}{4}$ dig. Deinde statuamus $i=3$; erit

$(i+\varepsilon)(i-1)=6,4,\;\; 3i^2-4i(1-\varepsilon)-4\varepsilon=16,6,\;\; \varepsilon m=10,\;\; 4\varepsilon m-3i=31, B=-6,\;\; \mathfrak{B}=\frac{6}{5},$

$ik=\frac{166}{93}=1,785,\;\; [k=0,595],\;\; em+ik=15,355,$

$\mathfrak{C}=-0,6175,\;\; C=-0,3817,\;\; \mathfrak{E}=2,4921,\;\; E=-1,6702,$

unde elementa primitiva sequenti modo definientur ponendo θ loco D, ut sit $\mathfrak{D}=\frac{\theta}{1+\theta}$:

$$\alpha=p,\ \ \beta=1,2p,\ \ \gamma=0,1526p,$$

$$b=-\tfrac{1}{5}p=-0,2p,\ \ c=-0,4p,\ \ \ d=0,0855p,$$

$$\delta=0,0855\theta p,\ \ \ \ \varepsilon=0,0229\theta p,$$
$$\varepsilon=-0,0382\theta p,\ \ f=0,07648p,$$

ex quibus intervalla colliguntur

$$AB=BC=0,8p,\ \ CD=0,2381p,\ \ DE=0,1084\theta p,\ \ EF=0,0382\theta p,$$

sicque tubus foramini speculi annectendus erit circiter $=\frac{1}{3}p$.
Distantiae vero focales erunt

$$q=\mathfrak{B}b=-0,24p,\ \ r=\mathfrak{C}c=0,247p,\ \ s=\mathfrak{D}d=0,0855\ \frac{\theta}{1+\theta}\cdot p,$$

$$t=\mathfrak{E}e=0,0571\theta p,\ \ u=f=0,0764\theta p.$$

Praeterea pro hoc casu habebimus

$$M=\tfrac{93}{2740}\mathfrak{t}=0,0339\mathfrak{t}\ \ (8,530\ 7323).$$

Tum vero

$$\mathfrak{q}=0,113\mathfrak{t},\ \ \ \mathfrak{r}=-\omega=-0,45\mathfrak{t}.$$

Nunc igitur videamus, an pro $\mathfrak{t}$ sumi possit unitas nec ne? Quem in finem consideremus valorem

$$\mathfrak{t}r=4\delta x\ \ \ \text{seu}\ \ \ 0,111p\mathfrak{t}=1\ \text{dig.},$$

unde fit $\mathfrak{t}=\frac{1}{0,111p}=\frac{9}{p}$, unde apparet, si *p* fuerit novem digitorum vel minus, tum sumi posse $\mathfrak{t}=1$, sin antem fuerit $p>9$ dig., tum sumi debet $\mathfrak{t}=\frac{9}{p}$, et *p* campus tanto fiet minor. Circa locum oculi vero notandum est esse

$$O=\tfrac{1}{2}u\left(1+\tfrac{16,6}{93}\right)=0,59u.$$

Nunc vero restat praecipua investigatio distantiae focalis *p,* quae ex mensura confusionis colligitur,

$$p = kx\sqrt[3]{50}\begin{Bmatrix} 0,125 - 0,0283 + 0,00131\mu(\lambda + v\mathfrak{C}(1-\mathfrak{C}) \\ + 0,003\mu\left(\frac{(1+\theta)^3\lambda'}{\theta^3} + \frac{v(1+\theta)}{\theta^2}\right) + \frac{0,00005\mu}{\theta^3}\left(\lambda'' + v\mathfrak{E}\left(1-\mathfrak{E}\right)\right) + \frac{0,00036\mu}{\theta^3}\lambda''' \end{Bmatrix}$$

Circa hanc expressionem vero sequentia observemus:

I. Si speculum principale sit parabolicum, primum membrum post signum radicale 0,125 omitti debet; ac si etiam minus speculum esset parabolicum, tum quoque secundum terminum omittere liceret. Consultius autem videtur solum primum speculum parabolicum efficere, alteri vero figuram sphaerieam perfectam inducere; tum enim sequentia membra ita instrui sive litterae λ, λ', λ'' cum littera θ ita assumi poterunt, ut ista membra a secundo, quod est negativum, perfecte tollantur sicque tota confusio ad nihilum redigatur. Quod si successerit, sufficiet litteram p ex sola apertura definira, sumendo scilicet $p = 4x$ vel $6x$ vel $7x$, prouti visum fuerit. Hoc ergo casu ob $x = 1$ dig. distantia focalis p tuto minor quam 9 dig. accipi poterit.

II. Cum igitur sumi possit $p < 9$ dig., ponere licebit $\mathfrak{t} = 1$ et campi apparentis semidiameter erit $= 859M$ minut. $= 29$ minut. Tum autem binas postremas lentes utrinque aeque convexas confici oportet, unde, si lentes ex vitro communi, pro quo est $n = 1,55$, parentur, erit

$$\lambda''' = 1 + \left(\frac{\sigma-\rho}{2\tau}\right)^2 = 1,6299,$$

at

$$\lambda'' = 1 + 0,6299\ \left(1-2\mathfrak{E}\right)^2 = 10,9991.$$

III. Quia adeo capere liceret $p = 4$ dig., ne distantia focalis ultimae lentis fiat nimis parva, sufficiet statuera $\theta = 1$ atque hinc erit ultimum membrum nostrae formulae $= 0,00055$. Pro penultimo membro erit

$$v\mathfrak{E}\left(1-\mathfrak{E}\right) = -\ 0,8649$$

ideoque

$$\lambda'' + v\mathfrak{E}\left(1-\mathfrak{E}\right) = 10,1342$$

ac propterea totum membrum $= 0,00047$. Quocirca ambo postrema membra iunctim sumta dabunt 0,00102.

IV. Pro prima autem lente erit

$$v\mathfrak{C}\left(1-\mathfrak{C}\right) = -0,2323,$$

unde totum membrum inde natum fiet

$$= 0,00123\lambda - 0,00028.$$

Pro secunda autem lente erit

$$\frac{(1+\theta)^3}{\theta^3}\lambda' + \frac{\nu(1+\theta)}{\theta^2} = 8\lambda' + 2\nu$$

hincque totum membrum erit

$$= 0,0232\lambda' + 0,00135.$$

V. His ergo inventis litteras λ et λ' ita definiri oportet, ut fiat

$$0,0283 = 0,00123\lambda + 0,0232\lambda' + 0,00209$$

sive

$$0,0262 = 0,00123\lambda + 0,0232\lambda',$$

ubi notandum litteras λ et λ' unitate minores esse non posse; statuamus ergo $\lambda' = 1$ et esse debebit $0,0030 = 0,00123\lambda$ hincque

$$\lambda = \frac{0,00300}{0,00123} = \frac{300}{123} = 2,44.$$

Hinc igitur consequimur sequentem constructionem:

TELESCOPIUM CATADIOPTRICUM PRO MULTIPLICATIONE $m = 50$

65. Ex iis, quae modo evolvimus, obtinemus sequentes determinationes:

I. Pro speculo principali, quod exactissime secundum figuram parabolicam elaborari debet, distantia focalis accipi posset $p = 4$ dig. Interim tamen litteram p quasi indeterminatam in calculo retineamus.

Semidiameter aperturae huius speculi $x = 1$ dig. et semidiameter foramiis

$$y = \delta x = \tfrac{1}{4} \text{ dig.}$$

II. Ante hoc speculum ad intervallum $= 0,8p$ constituatur speculum secundum *QBQ,* pro quo debet esse distantia focalis $q = -0,24p$, ita ut hoc speculum debeat esse convexum et ad figuram sphaericam exacte elaboratum.
Eius aperturae semidiameter $= \frac{1}{4}$ dig.

III. Post hoc speculum in ipso foramine speculi maioris ad distantiam $BC = \frac{4}{5}p = 0,8p$ constituatur lens prima ex vitro communi $n = 1,55$ paranda, cuius distantia focalis sit $r = 0,247p$, capiendo

$$\text{radium faciei}\begin{cases}\text{anterioris} = \dfrac{r}{\sigma - \mathfrak{C}(\sigma-\rho) \mp \tau\sqrt{(\lambda-1)}} = \dfrac{r}{1,4385} = 0,1729p \\ \text{posterioris} = \dfrac{r}{\sigma + \mathfrak{C}(\sigma-\rho) \pm \tau\sqrt{(\lambda-1)}} = \dfrac{r}{0,3896} = 0,6339p.\end{cases}$$

Semidiameter aperturae $=\frac{1}{4}$ dig. ut foraminis et intervallum usque ad lentem secundam

$$=0,2381p=CD.$$

IV. Pro secunda lente *SDS,* cuius distantia focalis $s=0,0427p$, ob $\mathfrak{D}=\frac{1}{2}$ et $\lambda'=1$ capiatur

$$\text{radium faciei}\begin{cases}\text{anterioris}=\frac{s}{\sigma-\frac{1}{2}(\sigma-\rho)}=\frac{s}{0,9090}=0,04697p\\ \text{posterioris}=\frac{r}{\sigma+\frac{1}{2}(\sigma-\rho)}=\frac{s}{0,9090}=0,04697p.\end{cases}$$

Eius aperturae semidiameter

$$=\frac{x}{PQR}=\frac{1}{26,775}=0,037\text{ dig.}$$

et intervallum ad tertiam lentem

$$DE=0,1084p.$$

V. Pro tertia lente, cuius distantia focalis $t=0,0571p$, capiatur

radius utriusque faciei $=0,0628p$.

Eius aperturae semidiameter $=\frac{1}{4}t=0,0142p$ et intervallum ad quartam lentem $=0,0382p$.

Vl Pro quarta lente, cuius distantia focalis $u=0,0764p$, capiatur radius utriusque faciei $=0,0840p$.

Eius aperturae semidiameter $=\frac{1}{4}u=0,0191p$ et intervallum ad oculum $=0,58u=0,0443p.$

VII. Tubi ergo anterioris ambo specula continentis longitudo aliquanto maior est quam 0,8*p.* Tubi vero posterioris lentes continentis longitudo erit $=0,4292p$ sicque totius instrumenti longitudo erit circiter 1,4292*p*, ita ut sumto $p=5$ dig. haec longitudo futura sit 7 dig.

VIII. Campi autem apparentis semidiameter iam supra indicata est $=29$ minut., quae pro multiplicatione $m=50$ satis est notabilis.

IX. Diaphragmatis sive septis in locis imaginum realium collocandis hic plane non erit opus, cum secunda lens tam exiguam habeat aperturam, quae radios peregrinos omnes excludat. Interim tamen, si in loco primae imaginis realis, quae post primam lentem cadit, ad intervallum $\gamma=0,1526p$ collocetur diaphragma, eius foraminis semidiameter sumi debet $=0,127p$; hoc vero diaphragmate vix erit opus, cum radiorum peregrinorum in lentem primam incidentium imago cadat post hanc lentem ad distantiam $r = 0,247p$, dum ea radiorum propriorum cadit ad distantiam $\gamma = 0,1526p$, quod discrimen satis est notabile.

X. Si quis metuat, ne a tam exiguo speculo, cuius semidiameter est $=1$ dig. quodque adeo foramine est pertusum, nimis exigua luminis copia ad oculum transmittatur, is tantum mensuram digitorum pro lubitu augeat; nihil enim impedit, quominus mensura digiti adeo duplicetur. Hoc enim modo claritas ad lubitum augeri poterit neque tamen instrumenti longitudo, quae per se est parva, ob hanc causam enormis evadet.

EXEMPLUM 2

Pro multiplicatione $m = 100$

66. Statuamus hic $\delta = \frac{1}{4}$ et $\varepsilon = \frac{1}{5}$ ut sit $\varepsilon m = 20$. Tum vero sumamus $i = 4$, quo tubus brevior evadat, atque habebimus

$$P = \tfrac{1}{\varepsilon} = 5, \quad Q = i = 4, \quad R = -k = -\tfrac{43}{68} = -0,63235$$

ob

$$3i^2 - 4i(1-\varepsilon) - 4\varepsilon = 34\tfrac{2}{5} \text{ et } 4\varepsilon m - 3i = 68,$$

porro

$$S = -k' = -\tfrac{680}{43} = -15,814 \text{ et } T = \tfrac{1}{2} = 0,5.$$

Unde fit

$$PQ = 20, \; PQR = -12,647, \; PQRS = 200 \quad \text{et} \quad PQRST = 100.$$

Reliquae vero litterae reperientur

$\mathfrak{B} = \frac{16}{13} = 1,231, \quad B = -\frac{16}{3} - 5,333,$
$\mathfrak{C} = -\frac{17 \cdot 21}{383} = -\ 0,93211(9,9694694), \quad C = -\frac{357}{740} = -\ 0,4824\ (9,6884898)$
et
$\mathfrak{D} = \frac{\theta}{1+\theta}, \quad D = \theta,$

$\mathfrak{E} = \frac{17 \cdot 783}{2 \cdot 5 \cdot 383} = 3,4755(0,5410119), \quad E = -\frac{3,4755}{2,4755} = -1,4039\ (0,1473490),$

unde colligimus

$$\log.B\mathfrak{C} = 0,6964410, \quad \log.BC\mathfrak{E} = 0,9514233,$$
$$\log.BC = 0,4104114, \quad \log.BCE = 0,5577604\ (-).$$

His praemissis elementa nostra erunt

$$\alpha = p, \qquad b = -\tfrac{\alpha}{P} = -\tfrac{1}{5}\alpha = -0,2p,$$
$$\beta = Bb = 1,0666p, \qquad c = -0,2666p,$$
$$\gamma = Cc = 0,1286p, \qquad d = 0,20844p,$$
$$\delta = Dd = 0,20844\theta\, p, \qquad e = \ 0,01286\theta p$$
$$\varepsilon = E\varepsilon = -0,01806\theta p, \qquad f = 0,08612\theta\, p,$$

unde statim obtinemus intervalla

$$AB = 0,8p, \quad BC = 0,8p, \quad CD = 0,3320p,$$
$$DE = 0,2163\theta p, \quad EF = 0,01806\theta p.$$

Distantiae vero focales ita se habebunt:

$$q = \mathfrak{B}b = -0,246p, \quad r = \mathfrak{C}c = 0,2485p,$$

$$s = \mathfrak{D}d = 0,2034\tfrac{\theta}{1+\theta}p, \quad t = \mathfrak{E}e = \ 0,0447\theta p, \quad u = f = 0,0361\theta p.$$

Praeterea vero erit $\omega = 0,3\mathfrak{t} = -\mathfrak{r}$, unde aequatio $\mathfrak{r}r = 4\delta x$ abit in hanc:

$0,07455\mathfrak{t}p = x$; quare, si sumatur $x = 2$ dig., hinc fiet $\mathfrak{t} = \frac{2}{0,07455p}$. Dummodo igitur fuerit $p < 26$ dig., capere licebit $\mathfrak{t} = 1$ binasque ultimas lentes utrinque aeque convexas fieri oportet. Verum si etiam hic liceat totam confusionem ad nihilum redigere, ob $x = 2$ dig. sumi adeo posset $p = 8$ dig., etiamsi praestet ipsi p maiorem valorem tribuere; unde patet tuto assumi posse $\theta = 1$.

Praeterea vero pro campo apparente habebitur $M = \frac{34}{1915}\mathfrak{t}$; quare, si capi poterit $\mathfrak{t} = 1$, semidiameter campi apparentis erit

$$\Phi = \tfrac{859\cdot 34}{1915}\text{ min.} = 15\tfrac{1}{4}\text{ min.}$$

et pro loco oculi habebimus

$$O = 0,563u = 0,02037p.$$

Denique ut tota confusio evanescat, primum speculum perfecte parabolicum confici necesse est atque tum esse debebit

$$\frac{s(1+B)(1-B)^2}{8B^3} = \frac{\mu}{B^3\mathfrak{C}^3PQ}\big(\lambda + \nu\mathfrak{C}(1-\mathfrak{C})\big) - \frac{\mu}{B^3\mathfrak{C}^3PQR}(8\lambda' + 2\nu)$$

$$+\frac{\mu}{B^3C^3\mathfrak{C}^3PQRS}\big(\lambda'' + \nu\mathfrak{E}(1-\mathfrak{E})\big) - \frac{\mu}{B^3C^3E^3m}\lambda''',$$

ubi ut ante, si refractio vitri sit $n = 1,55$, erit

$$\lambda''' = 1,6299$$

et

$$\lambda'' = 1 + 0,6299(1 - 2\mathfrak{E})^2 = 23,308,$$

unde aequatio nostra praebebit

$$\begin{aligned} 0,02864 = 0,000382\lambda &- 0,00016 \\ +\, 0,034843\lambda' &+ 0,00200 \\ &- 0,00001 \\ &+ 0,00015 \\ &+ 0,00032 \end{aligned}$$

sive

$$0,02634 = 0,000382\lambda + 0,03484\lambda',$$

quae aequalitas, quia λ et λ' unitate minores esse nequeunt, subsistere non potest. Quamobrem coacti sumus ipsi θ maiorem valorem tribuere; sit ergo $\theta = 2$ et nostra aequatio fiet

$$\begin{aligned} 0,02809 = 0,000382\lambda &- 0,00016 \\ +\, 0,01143\lambda' &+ 0,00075 \\ &- 0,00001 \\ &+ 0,00002 \\ &+ 0,00004 \end{aligned}$$

sive

$$0,02744 = 0,000382\lambda + 0,01143\lambda'.$$

Ne hinc valor ipsius λ prodeat nimis magnus, sumamus $\lambda' = 2$ eritque $0,00458 = 0,000382\lambda$ hincque $\lambda = \frac{4580}{382} = 12.$ Sin autem sumsissemus $\lambda' = 2\frac{1}{3}$ abtinuissemus $\lambda = \frac{770}{382} = 2.$

Utamur ergo his postremis valoribus $\lambda = 2$ et $\lambda' = 2\frac{1}{3}$ existente $\theta = 2$ hincque $\mathfrak{D} = \frac{2}{3}$; unde colligitur sequens

CONSTRUCTIO TELESCOPII CATADIOPTRICI PRO $m = 100$

67. Haec ergo constructio constabit sequentibus determinationibus:

I. Primum speculum perfecte secundum figuram parabolicam elaboretur, cuius distantia focalis sit $= p$, quam ad minimum 8 dig. statui oportet; eius aperturae semidiameter $= x = 2$ dig., foraminis autem semidiameter $= \frac{1}{2}$ dig. et distantia a speculo minore $AB = 0,8p.$

II. Minus speculum figuram sphaericam habeto, cuius distantia focalis sit $q = -0,246p$ et semidiameter aperturae $= \frac{1}{2}$ dig. indeque distantia ad primam lentem $BC = 0,8p$.

III. Pro prima lente, cuius distantia focalis $r = 0,2485p$, numeri vero

$$\mathfrak{C} = -0,9321 \text{ et } \lambda = 2,$$

capiatur

$$\text{radium faciei}\begin{cases}\text{anterioris} = \frac{r}{\sigma - \mathfrak{C}(\sigma-\rho) \pm \tau\sqrt{(\lambda-1)}} = \frac{r}{2,9666} = 0,1205p \\ \text{posterioris} = \frac{r}{\sigma + \mathfrak{C}(\sigma-\rho) \mp \tau\sqrt{(\lambda-1)}} = \frac{r}{-1,1485} = -1,0210p.\end{cases}$$

Semidiameter aperturae foramini aequalis $= \frac{1}{2}$ dig. et distantia ad lentem secundam $CD = 0,3320p$.

IV. Pro secunda lente, cuius distantia focalis $s = 0,1356p$ et numeri
$\mathfrak{D} = \frac{2}{3}$ et $\lambda' = 2,3333$,
capiatur

$$\text{radium faciei}\begin{cases}\text{anterioris} = \frac{r}{\sigma - \mathfrak{D}(\sigma-\rho) \pm \tau\sqrt{(\lambda-1)}} = \frac{r}{1,7147} = 0,0791p \\ \text{posterioris} = \frac{r}{\sigma + \mathfrak{D}(\sigma-\rho) \mp \tau\sqrt{(\lambda-1)}} = \frac{r}{0,1034} = 1,3114p.\end{cases}$$

Eius aperturae semidiameter $= \frac{x}{PQR} = 0,16$ dig. et distantia a lente tertia $DE = 0,4326p$.

V. Pro lente tertia, cuius distantia focalis $t = 0,0894p$, capiatur

$$\text{radius utriusque faciei } = 0,0983p.$$

Eius aperturae semidiameter $= \frac{1}{4}t = 0,0224p$ et distantia ad lentem quartam $EF = 0,03612p$.

VI. Pro lente quarta, cuius distantia focalis $u = 0,0722p$, capiatur

$$\text{radius utriusque faciei } = 0,0794p.$$

Eius aperturae semidiameter $=\frac{1}{5}u=0,0181p$ et distantia oculi

$$O = 0,563\ u = 0,0204p.$$

VII. Longitudo ergo tubi prioris aliquanto maior erit quam 0,8*p*, tubi autem affixi longitudo $=0,8211p$ hincque totius instrumenti eirciter $=1,6211p$.

VIII. Campi apparentis semidiameter $=15\frac{1}{4}$ min., et quae supra observavimus, praeterea etiam hic locum habent.

EXEMPLUM 3

Pro multiplicatione $m=150$

68. Maneant ut ante $\delta=\frac{1}{4}$ et $\varepsilon=\frac{1}{5}$, ut sit $\varepsilon m=30$; sumatur autem $i=5$, et ut claritate sufficiente fruamur, sit $x=3$ dig., ut sit $y=\frac{3}{4}$ dig., et hinc colligimus

$$P=5,\quad Q=5,\quad R=-k=-0,6652,$$
$$S=-k'=-18,040 \text{ et } T=\tfrac{1}{2};$$

hinc

$$PQ=25,\quad PQR=-16,63,\quad PQRS=300 \quad \text{et} \quad PQRST=150;$$

inde vero reliquae litterae reperientur

$$\mathfrak{B}=\tfrac{5}{4}=1,25,\quad B=-5,$$

$$\mathfrak{C}=\tfrac{33,28}{33,320}=-0,9986\ (9,9994001)\quad C=\tfrac{0,9986}{19986}=-0,49966(9,6986742),$$

$$\mathfrak{D}=\tfrac{\theta}{\theta+1},\ D=\theta,$$

$$\mathfrak{E}=\tfrac{118,32}{33,320}=3,5504(0,5502750),\quad E=\tfrac{3,5504}{2,5504}=-1,3921(0,1436667),$$

unde colligimus

$$\log B\mathfrak{C}=0,6983701,\ \log BC=0,3976442,$$

$$\log BC\mathfrak{E}=0,9479192,\ \log BCE=0,5413109(-).$$

His praemissis elementa nostra erunt

$$\alpha = p, \quad b = -0,2p, \quad \beta = p, \quad c = -0,2p, \quad \gamma = 0,099932p,$$

$$d = 0,15023p, \quad \delta = 0,15023\theta p, \quad e = 0,008338\, p.$$

$$\varepsilon = -0,01159\theta p, \quad f = +\, 0,02318\theta p,$$

unde colligimus intervalla

$$AB = 0,8p = BC, \quad CD = \; 0,25016p,$$

$$DE = 0,15856\theta p, \quad EF = 0,01159\theta p.$$

Distantiae vero focales ita se habebunt:

$$q = -0,25p, \quad r = 0,19972p, \quad s = 0,15023\tfrac{\theta}{1+\theta}\, p,$$

$$t = 0,02956\theta p \quad \text{et} \quad u = \; 0,02318\theta p.$$

Porro est $\omega = \frac{1}{4}\mathfrak{t} = -\mathfrak{r}$, unde aequatio $\mathfrak{r}r = 4\delta x$ dabit

$$\mathfrak{t} = \tfrac{12}{0,19972p} = \tfrac{60}{p} \text{ proxime;}$$

dum ergo p sit < 60, tuto sumere licebit $\mathfrak{t} = 1$, et quia tum erit $M = \frac{2}{166,630}$,
hinc semidiameter campi

$$\Phi = 10\tfrac{1}{3} \text{ min.}$$

et pro loco oculi

$$O = 0,555u = 0,01285\theta\, p.$$

Denique si primum speculum conficiatur parabolicum, omnis confusio tolletur huic aequationi satisfaciendo

$$0,0288 = 0,00030144\lambda - 0,0001394 + 0,0036177\; \tfrac{\lambda'(1+\theta)^3}{\theta^3}$$
$$+\; 0,00084146\tfrac{1+\theta}{\theta^2} + \tfrac{0,0001002}{\theta^3} + \tfrac{0,00024176}{\theta^3}$$

sive

$$0,0289399 = 0,00030144\lambda + 0,0036177\tfrac{(1+\theta)^3}{\theta^3}\lambda' + 0,00084146\tfrac{1+\theta}{\theta^3} + \tfrac{0,003419}{\theta^3}.$$

Hic patet statim sumi non posse $\theta = 1$; tentetur ergo positio $\theta = \frac{3}{2}$ eritque

$$0,0289399 = 0,00030144\lambda + 0,0167487\lambda' + 0,00093495 + 0,0001013$$

sive

$$0,0279037 = 0,00030144\lambda + 0,0167487\lambda';$$

quare, si hic statuatur $\lambda' = 1$, fiet

$$\lambda = \frac{0{,}01116500}{0{,}00030144} = \frac{11155}{301} = 37,$$

sin autem sumamus $\lambda = 1$, fiet

$$\lambda' = \frac{0{,}0276023}{0{,}0167487} = \frac{276023}{167487} = 1{,}648.$$

Sin autem λ statueretur 2 vel 3, valor ipsius λ' vix inde mutaretur, unde pro usu practico praestare videtur, si ipsi λ' certus quidam valor tribuatur; quia enim tum ob levissimos errores λ multum variare potest, plures lentes pro variis valoribus λ parari poterunt, ex quibus aptissimam experientia declarabit. Statuamus ergo $\lambda' = \frac{3}{2}$ ac reperietur

$$\lambda = \frac{0{,}0027807}{0{,}0003014} = \frac{27807}{3014} = 9,$$

unde in praxi ternae lentes parari poterunt ex valoribus $\lambda = 8, \ = 9, \ = 10.$
Posito ergo $\theta = \frac{3}{2}$, ut sit $\mathfrak{D} = \frac{3}{5}$, sumatur $\lambda' = \frac{3}{2}$ et $\lambda = 9$, unde colligitur sequens

CONSTRUCTIO TELESCOPII CATADIOPTRICI PRO $m = 150$

69. Haec constructio sequentibus determinationibus continetur:

I. Speculum obiectivum accuratissime secundum figuram parabolicam elaboretur, cuius distantia focalis minor non sit duodecim digitis, quam hic littera p designerons. Eius aperturae semidiameter vero sit $x = 3$ dig., foraminis vero semidiameter $= \frac{3}{4}$ dig. et distantia ad speculum minus $AB = 0{,}8p$.

II. Speculum minus exactissime ad figuraro sphaericam elaboretur, cuius distantia focalis sit $q = -0{,}25p$, quippe quod est convexum. Eius aperturae semidiameter $= \frac{3}{4}$ dig. et distantia ad primam lentem $BC = 0{,}8p.$

III. Pro prima lente, cuius distantia focalis est $r = 0{,}19972p$ numerique $\mathfrak{C} = -0{,}9986$ et $\lambda = 9$, capiatur

$$\text{radium faciei}\begin{cases}\text{anterioris} = \dfrac{r}{\sigma - \mathfrak{C}(\sigma-\rho) \pm \tau\sqrt{8}} = \dfrac{r}{0{,}5025} = 0{,}39745p \\ \text{posterioris} = \dfrac{r}{\rho + \mathfrak{C}(\sigma-\rho) \mp \tau\sqrt{8}} = \dfrac{r}{1{,}3156} = 0{,}15181p.\end{cases}$$

Sin autem sumeretur $\lambda = 10$, prodiret

$$\text{radium faciei}\begin{cases}\text{anterioris} = \frac{r}{3,3468} = 0,57589p \\ \text{posterioris} = \frac{r}{1,4713} = 0,13574p.\end{cases}$$

unde concludimus in genere sumi posse

$$\text{radium faciei}\begin{cases}\text{anterioris} = (0,39745 \pm 0,17844\omega)p \\ \text{posteroris} = (0,15181 \mp 0,01607\omega)p,\end{cases}$$

ubi ω per experientiam definiri conveniet.

Huius autem lentis semidiameter aperturae $= \frac{3}{4}$ dig. et distantia ad lentem secundam $CD = 0,25016p$.

IV. Pro secunda lente, cuius distantia focalis $s = 0,090138p$ et numeri $\mathfrak{D} = \frac{3}{5}$ et $\lambda' = 1,5$, capiatur

$$\text{radium faciei}\begin{cases}\text{anterioris} = \frac{s}{\sigma - \mathfrak{D}(\sigma - \rho) \pm \tau\sqrt{0,5}} = \frac{s}{0,1254} = 0,71880p \\ \text{posterioris} = \frac{s}{\sigma + \mathfrak{D}(\sigma - \rho) \mp \tau\sqrt{0,5}} = \frac{s}{1,69281} = 0,05325p.\end{cases}$$

Eius aperturae semidiameter $= \frac{x}{PQR} = \frac{2}{11}$ dig. $= 0,18$ dig. et distantia ad lentem tertiam $DE = 0,23784p$.

V. Pro tertia lente, cuius distantia focalis $t = 0,04434p$, sumatur

radius faciei utriusque $= 0,048774p$.

Eius aperturae semidiameter $= \frac{1}{4}t = 0,01108p$ et distantia ad lentem quartam $EF = 0,01738p$.

VI. Pro lente quarta, cuius distantia focalis $u = 0,03477p$, capiatur

radius utriusque faciei $= 0,03824p$.

Eius aperturae semidiameter $= \frac{1}{4}u = 0,00869p$ et distantia ad oculum $O = 0,555u = 0,01929p$.

VII. Longitudo ergo tubi prioris specula continentis aliquantum superabit 0,8 *p,* posterioris vero erit = 0,52467 *p,* ita ut totius instrumenti longitudo sit circiter $=1,82467p$. Tum vero semidiameter campi apparentis erit $=10\frac{1}{3}$ minut.

SCHOLION

70. Remedium in subsidium praxeos, quod hic pro prima lente attulimus, etiam facile ad exempla praecedentia accommodatur. Ponamus enim pro hac lente inventos esse radios facierum *f* et *g* et nunc quaestio eo redit, quomodo hos radios variari oporteat, ut distantia focalis maneat eadem. Ponatur prior $=f+x$, posterior $=g-y$ et necesse est, ut fiat

$$\frac{fg}{f+g}=\frac{(f+x)(x-y)}{f+g+x-y},$$

unde sumto *x* pro lubitu sive negative sive positive capi debebit

$$y=\frac{g^2x}{f^2+(f+g)x};$$

quare, cum *x* et *y* sint satis parva, erit $y=\frac{g^2x}{f^2}$; sive

$$x:y=f^2:g^2,$$

ita ut posito $x=f^2\omega$ futurum sit $y=g^2\omega$. Pro lente ergo prima, cuius radii supra inventi sint *f* et *g,* alias successive substitui conveniet, quarum radii sint $f\pm f^2\omega$ et $g+g^2\omega$.. Deinde hic etiam notasse iuvabit pro lente prima minorem aperturam sufficere posse, quam hic assignavimus foramini aequalem. Sufficiet enim apertura, cuius semidiameter $=\frac{1}{4}\mathfrak{t}r=\frac{1}{16}r=0,01248p$; unde, si $p=12$ dig., ista semidiameter foret $=0,1497$ dig. $=\frac{1}{7}$dig. circiter; ac si adeo esset $p=20$ dig., foret ista semidiameter $=\frac{1}{4}$ dig., ex quo concludimus sufficere, si huic lenti apertura tribuatur, cuius semidiameter sit $\frac{1}{4}$ dig.; quo pacto ingentem copiam radiorum peregrinorum ab introitu arcebimus sicque reliqui eo felicius a secunda lente excludentur, etsi eius apertura non tam est exigua ut in praecedentibus exemplis, cuius rei ratio est, quod litteram *i* in multo minore ratione auximus quam multiplicationem *m;* quam ob causam in sequente exemplo litterae *i* multo maiorem valorem tribuemus, quia inde nihil aliud est metuendum nisi exigua diminutio campi.

EXEMPLUM 4

Pro multiplicatione *m* = 200

71. Manentibus litteris $\delta=\frac{1}{4}$ et $\varepsilon=\frac{1}{5}$ capiatur $i=10$, et ut sufficians claritatis gradus obtineatur, sumamus $x=5$ dig., ut sit semidiameter foraminis $=\delta x=\frac{5}{4}$ dig. et $\varepsilon m=40$. Hinc ergo colliguntur valores

$$P=5,\quad Q=10,\quad R=-k=-0,8221,$$

$$S=-\,k'=-9,7312\quad \text{et}\quad T=\tfrac{1}{2}$$

hincque

$$PQ=50,\quad PQR=-41,105,\quad PQRS=400\quad \text{et}\quad PQRST=200;$$

reliquae vero litterae ita determinabuntur :

$$\mathfrak{B}=\tfrac{40}{31}=1,2903,\quad B=\tfrac{40}{9}=-\;4,4444,$$

$$\mathfrak{C}=-1,0153\left(0,0066052\right)(-),\quad C=-0,50381\;\left(9,7022655\right)(-),$$

$$\mathfrak{E}=3,2841\left(0,5164093\right),\quad E=-1,4377\;\left(0,1576942\right),$$

unde colliguntur sequentes logarithmi:

$$\log.B\mathfrak{C}=0,6544183,\qquad \log.BC=0,3500786,$$
$$\log.BC\mathfrak{E}=0,8664879,\quad \log.BCE=\;0,5077728(-);$$

hinc elementa sequenti modo definientur:

$$\begin{array}{lll} \alpha=p, & b=-0,2p, & \beta=0,8889p, \\ & c=-0,0889p, & \gamma=\;0,04478, \\ & d=0,054473p, & \delta=\;\;0,054473\theta p \\ & e=0,005598\theta p, & \varepsilon=-0,008048\theta p \\ \text{et} & f=0,0160960\theta p, & \end{array}$$

ex quibus colliguntur intervalla

$$AB=0,8p=BC,\qquad CD=0,09925p,$$
$$DE=0,060071\theta p,\quad EF=0,008048\theta p,$$

distantiae vero focales

$$q = -0,2581p, \qquad r = 0,09025p,$$

$$s = 0,05447\tfrac{\theta}{1+\theta}p, \quad t = 0,01838\theta p$$

et

$$u = 0,016096\theta p.$$

Porro est $\omega = -\mathfrak{r} = \frac{3}{8}\mathfrak{t}$; unde aequatio $\mathfrak{r}r = 4\delta x$ dabit $\mathfrak{t} = \frac{160}{p}$ dig., unde patet, dummodo p minor sit quam 160 dig., tuto sumi posse $\mathfrak{t} = 1$; at si liceat confusionem ad nihilum redigere, adeo sumere licebit $p = 20$ dig.; tum autem fiet $M = \frac{1}{120}$, unde semidiameter campi erit $\frac{859}{120}$ min. $= 7\frac{1}{6}$ min. Praeterea vero pro loco oculi habebitur $O = 0,6u$.

Tantum igitur superest, ut confusionem ad nihilum redigamus, quod fiet hac aequatione:

$$0,029074 = 0,00020418\lambda - 0,0000972 + 0,0020329\tfrac{(1+\theta)^3}{\theta^3}\lambda'$$
$$+0,00047286\tfrac{1+\theta}{\theta^2} + \tfrac{0,000111}{\theta^3} + \tfrac{0,00000116}{\theta^3}$$

sive

$$0,029171 = 0,00020418\lambda + 0,0020329\tfrac{(1+\theta)^3}{\theta^3}\lambda'$$
$$+\ 00004729\tfrac{1+\theta}{\theta^2} + \tfrac{0,0001122}{\theta^3},$$

ubi iam nihil obstat, quominus statuatur $\theta = 1$, hincque habebimus

$$0,028113 = 0,0002042\lambda + 0,016264\lambda'.$$

Ne igitur hinc valor ipsius λ prodeat nimis magnus, commode statui poterit $\lambda' = 1\frac{1}{2}$ atque reperietur $\lambda = \frac{3717}{204} = 18$ proxime. Commodius vero erit sumere $\lambda' = 1\frac{2}{3}$, unde fiet $\lambda = \frac{1006}{204} = 5.$ Retineamus igitur valores $\theta = 1,\ \ \lambda' = 1\frac{2}{3}$, ut fiat $\lambda = 5$, cui adiungere poterimus valores finitimos $\lambda = 4$ et $\lambda = 6$, quo praxi melius consulatur; atque hinc colligetur sequens

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72. Statuamus hic ut hactenus distantiam focalem speculi principalis $= p$, quam, ut vidimus, minorem quam 20 dig. assumi non convenit. Praestabit autem eam haud mediocriter maiorem assumere.

I. Speculum igitur primum ad curatissime forma parabolica elaboretur, cuius distantia focalis sit $= p$; eius aperturae semidiameter $x = 5$ dig. et semidiameter foraminis $y = 1\frac{1}{4}$ dig. Distantia vero ad speculum minus $AB = 0,8p$.

II. Pro secundo speculo minore convexo eius figura accuratissime sphaerice elaboretur, ut sit eius distantia focalis $q = 0,2581p$. Eius aperturae semidiameter $= 1\frac{1}{4}$ dig. et distantia ad primam lentem in foramine $= BC = 0,8p$.

III. Pro lente prima, cuius distantia focalis $r = 0,09025p$ et numeri $\mathfrak{C} = -1,0153$ et $\lambda = 5$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{s}{\sigma - \mathfrak{C}(\sigma-\rho)\mp\tau\sqrt{4}} = \frac{r}{3,0861\mp1,8102} \\ \text{posterioris} = \frac{s}{\sigma + \mathfrak{C}(\sigma-\rho)\pm\tau\sqrt{4}} = \frac{r}{-1,2680\pm1,8102},\end{cases}$$

hinc

$$\text{radius faciei}\begin{cases}\text{anterioris} = 0,070734p \\ \text{posterioris} = 0,16639p.\end{cases}$$

Sin autem sumeremus $\lambda = 4$, prodiret

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{r}{3,0861\mp1,5677} = 0,05944p \\ \text{posterioris} = \frac{r}{-1,2680\pm1,5677} = 0,30113p.\end{cases}$$

At si sumeretur $\lambda = 6$, foret

$$\text{radius faciei}\begin{cases}\text{anterioris} = \frac{r}{3,0861\mp2,0239} = 0,08496p \\ \text{posterioris} = \frac{r}{-1,2680\pm2,0239} = 0,11940p.\end{cases}$$

Ex quibus casibus deducimus in subsidium praxeos sequentes conclusiones:
Prior: Si $\lambda = 5 - \omega$ denotante ω fractionem arbitrariam, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = (0,07073 - 0,01129\omega)\,p \\ \text{posterioris} = (0,16639 + 0,13474\omega)\,p.\end{cases}$$

Posterior: Sin autem $\lambda = 5 + \omega$, erit

$$\text{radius faciei}\begin{cases}\text{anterioris} = (0,07073 + 0,01423\omega)\,p \\ \text{posterioris} = (0,16639 - 0,04699\omega)\,p.\end{cases}$$

Eius aperturae semidiameter $= 1\frac{1}{4}$ dig. et distantia ad lentem secundam $CD = 0,09925p$.

IV. Pro secunda lente, cuius distantia focalis est $s = 0,02723p$ et numeri $\mathfrak{D} = \frac{1}{2}$ et $\lambda' = 1,6667$, capiatur

$$\text{radius faciei}\begin{cases}\text{anterioris} = \dfrac{s}{\frac{1}{2}(\sigma+\rho)\mp\tau\sqrt{0,6667}} = \dfrac{s}{0,9090\mp0,7390}\\ \text{posterioris} = \dfrac{s}{\frac{1}{2}(\sigma+\rho)\pm\tau\sqrt{0,6667}} = \dfrac{s}{0,9090\pm0,7390}\end{cases}$$

seu

$$\text{radius faciei}\begin{cases}\text{anterioris} = 0,01652p\\ \text{posterioris} = 0,16018p.\end{cases}$$

Eius aperturae semidiameter $= \frac{x}{PQR} = \frac{1}{8}$ dig . et distantia a lente tertia $DE = 0,06007p$.

V. Pro lente tertia, cuius distantia focalis $t = 0,01838p$, capiatur

$$\text{radius faciei utriusque} = 0,02022p.$$

Eius aperturae semidiameter $= \frac{1}{4}t = 0,00459p$ et distantia a lente quarta $EF = 0,007798p$.

VI. Pro lente quarta, cuius distantia focalis $u = 0,015596p$, capiatur

$$\text{radius faciei utriusque} = 0,01715p.$$

Eius aperturae semidiameter $= \frac{1}{4}u = 0,0039p$ et distantia ad oculum $= 0,6u = 0,00936p$.

VII. Hinc ergo longitudo tubi prioris erit quasi $= p$, quia maior esse debet quam $\frac{4}{5}p$, posterioris vero lentes continentis $= 0,17648p$, ita ut tota longitudo futura sit circiter $= 1,17648p$. Campi vero apparentis semidiameter erit $= 7\frac{1}{6}$ minut.

VIII. Si pro lente prima tantum ad claritatem spectemus, eius aperturae semidiameter deberet esse $= \frac{x}{PQ} = \frac{1}{10}$ dig., sin autem ad campum spectemus, haec semidiameter esse debet

$$= \frac{1}{4}\mathfrak{r}r = \frac{3}{32}r = 0,00846p,$$

quae, si adeo esset $p = 40$ dig., fieret

$$0,8384 \text{ dig.} = \tfrac{1}{3} \text{ dig.}$$

Quare, cum semidiameter foraminis $= 1\frac{1}{4}$ dig., tuto oram huius lentis obtegere licebit, donec eius aperturae semidiameter fiat $\frac{1}{3}$ dig., quo pacto radii peregrini iam maximam partem excludentur.

IX. Cum igitur ne opus quidem sit tantam magnitudinem primae lenti tribuere, ipsum foramen maioris speculi multo minus statuera licebit quam $1\frac{1}{4}$ dig. hocque modo, dum ipsum hoc speculum maiorem superficiem adipiscetur, etiam claritatis gradus augebitur, neque vero ideo necesse erit et minoris speculi magnitudinem imminuere, cum sufficiens radiorum copia in speculum cadere possit. Radii peregrini colliguntur post lentem *C* in distantia $r = 0,09025p$, radii vero proprii in distantia $\gamma = 0,0448p$.

X. Cum deinde prima imago realis post lentem primam cadat ad distantiam $r = 0,0448p$, radii autem peregrini in hanc lentem incidentes suam imaginem forment ad distantiam $r = 0,09025p$, quae cum illa plus quam duplo sit maior, neutiquam metuendum erit, ne radii peregrini ad oculum usque propagentur.