PROPOSITION XII.

To ascertain the Manner in which the Flame of Powder impels a Ball, which is laid at a considerable Distance from the Charge.

We have, in many of the experiments cited by us above, laid the ball not immediately contiguous to the powder, but a small distance from it; the greatest interval, however, has not amounted to more than about $\frac{1}{2}$ inch, from the hinder part of the bullet to the nearest part of the powder ; and, in these cases, we have seen, that the theory agreed very; well with the experiments: but if a bullet be placed at a greater distance from the powder, suppose at 12, 18, or 24 inches, we can not then apply to the motion of this ball the same principles which, in the 7th proposition, we have applied to such as are contiguous to the powder, or nearly so; for we have seen, in the last proposition, that, when the surface of the fired powder is not confined by a heavy body, which it is obliged to impel before it, the flame dilates itself with a velocity much beyond what it can at any time communicate to a bullet by its continued pressure ; consequently, as in the distance of 12, 18, or 24 inches, the powder will have acquired a considerable degree of this velocity of expansion, the first motion of the ball will not be produced by the continued pressure of the powder, but by the actual percussion of the flame ; and it will therefore begin to move with a quantity of motion proportioned to the quantity of this flame; and the velocities of its respective parts.

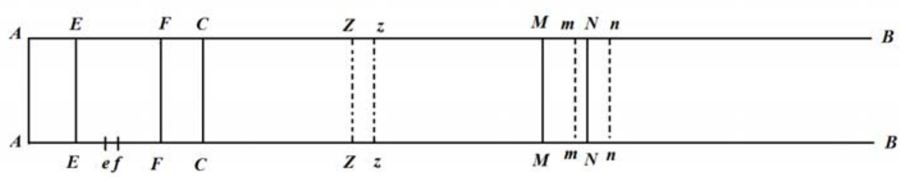
From hence then it follows, that the velocity of a bullet, laid a considerable distance before the charge, ought to be greater, than what would be communicated to it by the pressure of the powder acting in the manner described in the 7th proposition ; and this deduction from our theory we have confirmed by manifold experience ; by which we have found, that a ball laid in the barrel A, with its hinder part $11\frac{1}{4}$ inches from its breech, and impelled by 12 dw. of powder, has acquired on its discharge a velocity of about 1400 feet in 1"; when, if it had been acted on by the pressure of the flame only, it would not have acquired a velocity of 1200 feet in 1". The same we have found to hold true in all other greater distances, (and also in lesser, though not to the same degree) and in all quantities of powder. And we have likewise found, that these effects nearly correspond with what was laid down in the last proposition about the velocity of expansion, and the elastic and unelastic parts of the flame.

And from hence too arises another consideration of great consequence in the practice of Gunnery ; which is, that no bullet should at any time be placed at any considerable distance before the charge, unless the piece be extremely fortified; for a moderate charge of powder, which it has expanded itself through the vacant space, and reaches the ball, will, by the velocity each part has acquired, accumulate itself behind the ball, and will thereby be condensed prodigiously ; whence, if the barrel be not of an extraordinary firmness in that part, it must by this reinforced elasticity of the powder, infallably burst. The truth of this reasoning I have experienced in an exceeding good *Tower-musquet,* forged of very tough il'on ; for charging it with 12dw. of powder, and placing the ball 16 inches from the breech, on the firing it, the part of the barrel just behind the bullet was

swelled out to double its diameter, like a blown bladder, and two large pieces, of two inches long, were burst out of it. Having seen that the entire motion of a bullet, laid at a considerable distance from the charge, is acquired by two different methods, in which the powder acts on it; the first being the percussion of the parts of the flame, with the velocity they had respectively acquired by expanding ; the second, the continued pressure of the flame through the remaining part of the barrel ; I endeavoured to separate these different actions, and to retain that only, which arose from the continued pressure of the flame. For this purpose, I no longer placed the powder at the breech, from whence it would have full scope for its expansion, but I scattered it as uniformly as I could, through the whole cavity lest behind the bullet; imagining that, by this means, the progressive velocity of the flame in each part would be prevented by the expansion of the neighbouring parts : and I found, that the ball being laid $11\frac{1}{4}$ inches from the breech, its velocity, instead of 1400. feet in 1", which it acquired in the last experiments, was now no more than 1100 feet in 1"; which is 100 feet short of what, according to the theory, should arise from the continued pressure of the powder only.
The reason of this deficiency was, doubtless, the intestine motion of the flame; for the ascension of the powder, thus distributed through so much a larger space than what it could fill, must have produced many reverberations and pulsations of the flame ; and from these internal agitations of the fluid, its pressure on the containing surface will (as is the case in all other fluids) be considerably diminished ; and it has been in order to avoid this irregularity, that in all the experiments I have made, 1 have taken particular care to have the powder closely confined in as small a space as possible, even when the bullet lay at some little distance from it.

REMARK

The investigation, which is dealt with in this proposition, is one of the most difficult matters that can be come across in the theory of motion of fluid bodies ; and if this proposition were to be accomplished incisively, neither thus would there be the time to become familiar with the benefit of the art of resolution, nor would the foundations of the theory itself be sufficient, to which this investigation pertains, whereby all the difficulties to be overcome could be accomplished. It is here namely that the case is to be considered, in which the ball is not immediately next to the powder charge, but an appreciable space is left free between the powder and the ball. Now if this space were completely empty and within it so little air were to be found, that the compressed air released by the ignition were able to expand equally to reach the ball as in the manner described in the previous proposition: and in this case thus it would not be hard, after the ignition, for the foremost part of the compressed air to begin acting on the ball as well in an instant, as the speeds of all the different parts can be determined by the rules declared hitherto. However, a two-fold force acts on the ball, the first of which consists of the force due to the expansion of the compressed air, and the ball is pushed forwards until it is finally be driven from the barrel, but the other force arises from the force of the shock, and as it were, instantly exerts a force on the ball : thus the action of the first force can also be determined from the principles cited here, since such an effect has been seen above.The other effect is found from the action of another much greater force, which for the time being still cannot be explained way very well. Indeed this cause appears to be founded in the theory of the motion, thus assumed by a shock [*i.e.* a sudden force rather than a continued one] running through the motion, and consequently for the first force, since this theory has been advanced sufficiently explained already, it should not throw too great obstacles in the way ; however, for the second force, since on its own in this case, thus, the size of the proper force acting on the speed of the adjoining body must be known, and as one can see easily in the present circumstances, both these parts are completely unknown. For since here the adjoining body is constituted from the expanded air, so neither the whole circumstances of the force of the adjoining body nor the speed of the foremost small part can be assumed. Accordingly to make these clearer, one may consider thus, that presently only the foremost small part of the compressed air must impinge on the ball, while the latter part thus more distant will be inhibited from acting equally [We have to admit that the theory presented by Euler is not correct ; for the extra confined space has provided enough air for a more complete and instaneous combustion of the gunpowder which thus produces more exhaust gases and so a much higher pressure, which can rupture the casing. However, he has indicated the existence of a shock wave as the cause, and that itself is of interest.] Since now the same must still continue with their motion forwards to some extent, thus it is even so much, as if a part of



the same did not belong to the substance of the adjoining body. But how great this part may be, is precisely that in which the greatest difficulty lies. Meanwhile is still seems, that one may not be far from the truth, if the half or a third is assumed for this part required. Therefore if we put in place, that the ball initially is at *ZZ*, and the charge has filled the space *AC*, so calling $AC = b,\ AZ = f:$ thus the weight of the charge would be almost equal to a natural air column, of which the height $=1000b;$ if now we assume namely, that the powder is 1000 heavier than the air, and therefore the half of this quantity will push on the ball, deren Gewicht durch eine Luft-Säule, of which the height to be expressed $=k$. But likewise after the ignition the flame spreads as far as *ZZ*, as this will acquire a speed, as has been shown before, which comes from the height,

$$=\frac{2\alpha}{n(\alpha-\lambda)+244\alpha\lambda}\cdot 244\beta\lambda h\, l\frac{\alpha f-(\alpha-\lambda)b}{\lambda b}\ ;$$

where $\alpha=\frac{3}{2},\ n=1000,\ 244\beta=1000$ and λ comes from the ignited part of the powder, for which we will assume $\lambda=\frac{9}{10}$; moreover *h* is the height of a natural column of air, of which the weight is equal to the elasticity of the natural air. Therefore the above height, through which the speed of the flame impressed at *Z* will be ,

$$=\frac{90}{31}hl\frac{5f-2b}{3b}$$

and consequently the speed itself

$$=\sqrt{\frac{90}{31}hl\frac{5f-2b}{3b}}.$$

Now since at the beginning the ball is at rest, thus one must put in place this equation from the known rules : As the ratio of the sum itself from the two bodies $500b+k$ to the driving force [providing the shock] 500*b*, so the ratio of the speeds of the bodies being shocked $=\sqrt{\frac{90h}{31}l\frac{5f-2b}{3b}}$ to the common speed arising from the driving force ; therefore the speed of the ball arising from the driving force will be

$$=\frac{500b}{k+500b}\sqrt{\frac{90h}{31}l\frac{5f-2b}{3b}}.$$

If one had taken a fraction smaller than a half from the matter of the powder, so also must one assume a smaller number to be put in place for 500*b*. For that we will take 310*b*, so giving $k+310b$ above that same denominator $\alpha(2k+nb)-(n-244\alpha)\lambda b$, which has been found above in the VIIth remark of the above proposition, and now this agreement also seems to be a sharper pulse, so that the number 310*b* indicates the correct part to use. Therefore this expression can be found for the height, which expresses the speed of the ball :

$$\frac{310\cdot 900b^2h}{(k+310b)^2}l\frac{5f-2b}{3b}=\frac{279000b^2h}{(k+310b)^2}l\frac{5f-2b}{3b}.$$

But if the ball were initially placed immediately in front of the powder, thus the same would have obtained a speed, from which the force itself acting as far as at *ZZ* , which arises from the height

$$=\frac{900bh}{k+310b}l\frac{5f-2b}{3b};$$

consequently this speed for that ratio will be as 1 to $\sqrt{\frac{310b}{k+310b}}$: and thus the force has imparted a far smaller speed to the ball than that, which it would have acquired at *Z*, if it were loaded immediately in front of the powder. If this is now the correct speed acquired from the force, thus it would be easy to determine the following increase itself through the force of expansion. To this end we must only take the above equation found, which, since one has ignored the small terms, and for the letters α,λ,β and *n* put in place the associated values, so that there is provided

$$v = \frac{900bh}{k+310b} l \frac{5f-2b}{3b},$$

and from that still a constant quantity be put in place, which must be provided, that if one puts $x = f$, the height v will turn into

$$\frac{279000bbh}{(k+310b)^2} l \frac{5f-2b}{3b}.$$

Thus it implies also that v which due height for the speed of the ball, after which the ball shall be driven forwards as far as *MM*, since $AM = x$, and since one will have such an equation :

$$v = \frac{900bh}{k+310b} l \frac{5f-2b}{3b} + C.$$

Now in order to determine the quantity *C*, thus one puts

$$x = f \quad \text{and} \quad v = \frac{279000bbh}{(k+310b)^2} l \frac{5f-2b}{3b},$$

there will be

$$\frac{279000bbh}{(k+310b)^2} l \frac{5f-2b}{3b} = \frac{900bh}{k+310b} l \frac{5f-2b}{3b} + C$$

and also

$$C = \frac{-900bkh}{(k+310b)^2} l \frac{5f-2b}{3b}.$$

Consequently one will obtain

$$v = \frac{900bh}{k+310b} l \frac{5x-2b}{3b} - \frac{900bkh}{(k+310b)^2} l \frac{5f-2b}{3b}.$$

But if the ball at *ZZ* had obtained no force, and as the mere expansion force alone were driving the ball forwards, so would

$$C = \frac{-900bh}{k+310b} l \frac{5f-2b}{3b}$$

and also

$$v=\frac{900bh}{k+310b}l\frac{5x-2b}{3b}-\frac{900bkh}{k+310b}l\frac{5f-2b}{3b}.$$

But if the ball were loaded initially immediately in front of the powder, to from this there would come :

$$v=\frac{900bh}{k+310b}l\frac{5x-2b}{3b}.$$

Now if for *x* the length of the whole barrel *a*, and for the example cited by the author there is put in place :

$$a=45,\ b=2,625,\ \ f=11,25,\ k=4900 \text{ and } lh=7,463893,$$

so we can consequently determine the form of the three speeds :

$$\frac{900b}{k+310b}=\frac{236250}{571375},\quad \frac{900bk}{(k+310b)^2}=\frac{490000}{571375}\cdot\frac{236250}{571375}$$

or

$$\frac{900b}{k+310b}=0,41348,$$
$$\frac{900bk}{(k+310b)^2}=0,35459$$

and

$$\frac{5x-2b}{3b}=27{,}905,\quad \frac{5f-2b}{3b}=6{,}4762,$$

consequently

$$l27,905=1,445682$$
$$l6,476=0,811307$$
$$l1,445682=0,160073$$
$$l0,811307\ \ =9,909185.$$

One adds 0,362216 , so there becomes:

$$ll\frac{5x-2b}{3b}=0,522289$$
$$ll\frac{5f-2b}{3b}=0,271401;$$

thus one finds :

$$\frac{900b}{k+310b} l \frac{5x-2b}{3b} = 1,37639,$$
$$\frac{900b}{k+310b} l \frac{5f-2b}{3b} = 0,77243$$
$$\frac{900bk}{(k+310b)^2} l \frac{5f-2b}{3b} = 0,66242.$$

Therefore if one initially should load the ball in the vicinity of the powder, thus there becomes $v = 1,37639h$, and the speed works out at 1582 ft.per second. But if the ball were put initially at *ZZ*, and the powder also dispersed through the space *AZ* , so that the ball endured no shock, thus there will be $v = 0,60395h$ and this speed works out at 1048 ft. per second. But if the ball placed at *ZZ* then the shock together with the force of the powder is obtained, then there will be $v = 0,71396h$ and this speed works out as 1140 ft. per second.

However it is to be observed that the formula, from which the speed is determined, is a little too small, because the number found still must be increased somewhat. But notwithstanding which, it is clear that the speed of the ball in the last case, since the same begins through the shock of the powder on the motion to be put in place, because it is smaller, then the experiments have identified. For if we have paid attention to the foregoing circumstances, thus one sees easily that the subtile matter must have accumulated very quickly behind the ball by the force, and thus has become a far greater elasticity, from which consequently the ball has a far greater degree of speed impressed on it.But whatsoever the swelling and the course of dispersion attained, the cause of which is easily seen. Since there the ball as it were in an instant will be impressed with a rather greater degree of speed by the force, thus it is apparent, that a pressing force, which even in this short time were able to impress even this degree of speed, must be astonishing; and it must be perhaps a ten fold stronger force than the expansion force of the powder alone is able to exercise on the ball to produce such an effect. Now since a cannon or musket barrel in a similar place can take care only of such a force made a little stronger than the ordinary expansion force, so one has little to wonder at, if an ordinary barrel falls apart or another would be stretched due to this greater increase of the force, as has been seen in the example brought forwards by the author.

If the ball had not had this shock, thus the same would have obtained from our calculation a speed of 1048 Rh. ft. per second, which works out as 1080 English feet, and consequently as with 1100, so obtained through the experiments, thus so much better a match than the formula which has been used here, which is a little too small. Also if author for this case takes a speed of 1200 ft., so either an error must have crept into his reckoning or which is more likely the cause apparent from this, in the incorrect nature of the author's theory itself ; in that he had not considered the force, which allowed the driving force of the flame. Since now the proposition behaves so, thus he fails in his reasoning, from which he wants to explain away the decrease in speed of the ball in this case ; as which also the same thus must be driven out, that from the action, which the author indicates, cannot follow. For, notwithstanding the truth fact that the proposition is true, that the expanding force of a flowing elastic matter will be smaller, if within itself an inner motion between the parts themselves is found, so it is still easy to explain, that this internal motion in the flame, if the powder through the whole space behind the ball were stretched out, must suddenly be stopped, if the same acts on the ball initially.

But we have in this case, as we have seen before in the motion of the ball, if the same were not loaded directly on the powder, to be performed assuming that the space between the powder and the ball to be completely void. Now since within this natural air is found, so a small change must be made for this in the conclusion. Because, at once after the ignition, the flame begins to extend itself, so all the air situated between the powder and the ball becomes compressed, and therefore becomes a driving force on the ball, of such a kind, that really sets the ball in motion, even before the flame itself extends there and exerts a force on that. And in this circumstance there appears to be the most likely cause why the ball obtains a greater speed than was found from the above calculation.

ANMERKUNG

Die Untersuchung, wovon in diesem Satz gehandelt wird, ist eine von den schwehresten Materien, welche immer in der Lehre von der Bewegung flüßiger Körper vorkommen können; und wenn dieser Satz nach aller Schärfe ausgeführet werden sollte, so würden noch zur Zeit weder die bekannten Vortheile der Auflösungskunst, noch die Grundsätze der Wissenschaft selbst, wohin diese Untersuchung gehöret, hinreichend seyn, alle dabey vorkommenden Schwierigkeiten zu überwinden. Es wird nehmlich hier der Fall betrachtet, wenn die Kugel nicht unmittelbar auf das Pulver geladen, sondern ein merklicher Raum zwischen dem Pulver und der Kugel ledig gelassen wird. Wenn nun dieser Raum völlig leer wäre und sich darinne auch so gar keine Luft befände, so würde sich gleich nach der Entzündung die dadurch befreyte zusammen gepreßte Luft nach der im vorigen Satze beschriebenen Art so lange gantz frey ausdehnen, biß die vördersten Theile derselben die Kugel erreichten. Und in diesem Fall würde es so schwehr nicht seyn, so wohl die Ausdehnungs-Kraft der zusammen gedruckten Luft, in dem Augenblick, da dieselbe auf die Kugel zu würken anfängt, als auch die Geschwindigkeit aller Theile derselben nach den bißher erklärten Regeln zu bestimmen. Und da alsdenn auf die Kugel eine doppelte Gewalt würket, wovon die erste in der Ausdehnungs-Kraft der zusammengedruckten Luft bestehet, und die Kugel so lange fortstößt, biß dieselbe gänzlich zum Lauf hinaus getrieben worden, die andere aber aus der Gewalt des Stoßes entspringt, und gleichsam nur einen Augenblick ihre Kraft auf die Kugel ausübet: so könnte auch noch die Würkung der ersten Kraft nach den hier angeführten Grundsätzen bestimmet werden, wie solches denn auch oben würklich geschehen ist. Allein es finden sich bey Bestimmung der andern Würkung um so viel grössere Schwierigkeiten, welche noch zur Zeit nicht wohl aus dem Wege geräumet werden können. Diese Sache scheinet zwar in die Lehre von der Mittheilung der Bewegung, so durch den Stoß geschieht, zu lauffen, und folglich, da diese Lehre schon genugsam ausgeführt ist, keinen so grossen Hindernissen unterworfen zu seyn; allein da in diesem Falle so wohl die Schwehre, als die Geschwindigkeit des anstossenden Körpers bekannt sein muß, ·so siehet man leicht, daß in den gegenwärtigen Aufgaben diese beyden Stücke gänzlich ungewiß sind. Denn weil hier die sich ausdehnende Luft den anstossenden Körper ausmacht, so kann weder das ganze Wesen derselben für die Schwehre des anstossenden Körpers, noch die Geschwindigkeit der vordersten Theilchen für die Geschwindigkeit desselben angenommen werden. Um dieses deutlicher zu machen, so darf man nur betrachten, daß, so bald die vordersten Theilchen der zusammen gepreßten Luft auf die Kugel stossen, die hintern nur in so ferne zugleich mit würken, als die Bewegung derselben gehemmet wird. Da nun dieselben ihre Bewegung noch einiger maßen fortsetzen können, so ist es eben so viel, als wenn ein Theil derselben gar nicht zu dem Wesen des anstossenden Körpers gehörte. Wie groß aber dieser Theil sey, ist eben dasjenige, worinn die gröste Schwierigkeit bestehet. Inzwischen scheinet es doch, daß man von der Wahrheit nicht allzusehr abweiche, wenn man die Helfte, oder ein Drittel für diesen gesuchten Theil annimmt. Wenn wir demnach setzen, daß (Fig. 9) die Kugel anfänglich in *ZZ* gesetzt worden, und die Ladung den Raum *AC* erfüllet habe, so nenne man $AC = b,\ AZ = f$:so wird das Gewicht der Ladung ungefehr einer natürlichen Luft-Säule gleichen, deren Höhe $=1000b;$ wenn wir nehmlich annehmen, daß das Pulver 1000 mal schwehrer ist, als die Luft, und hiervon wird also die Helfte an die Kugel stossen, deren Gewicht durch eine Luft-Säule, welcher Höhe $=k$, ausgedrückt .worden. Indem sich aber nach der Entzündung die Flamme biß *ZZ* ausbreitet, so wird dieselbe, wie aus dem vorhergehenden erhellet, eine Geschwindigkeit bekommen, welche aus der Höhe

$$= \frac{2\alpha}{n(\alpha-\lambda)+244\alpha\lambda}\cdot 244\beta\lambda h\ l\,\frac{\alpha f-(\alpha-\lambda)b}{\lambda b}$$

herkömmt, wo $\alpha = \frac{3}{2},\ n = 1000,\ 244\beta = 1000$ und λ den entzündeten Theil des Pulvers bedeutet, wofür wir annehmen wollen $\lambda = \frac{9}{10}$; aber *h* ist die Höhe einer natürlichen Luft-Säule, deren Gewicht der Elasticität der natürlichen Luft gleich ist. Dahero wird die obige Höhe, wodurch die Geschwindigkeit der Flamme in *Z* ausgedruckt wird, seyn

$$= \frac{90}{31} hl\,\frac{5f-2b}{3b}$$

und folglich die Geschwindigkeit selbst

$$= \sqrt{\frac{90}{31} hl\,\frac{5f-2b}{3b}}$$

Da nun die Kugel anfänglich stille gestanden, so muß man nach den bekannten Regeln diese Vergleichung anstellen: Wie sich verhält die Summa der beyden Körper $500b+k$ zu dem anstossenden *500b*, also verhält sich die Geschwindigkeit des anstossenden Körpers $= \sqrt{\frac{90h}{31} l\frac{5f-2b}{3b}}$ zur Geschwindigkeit beyder nach dem Stoß; dahero die Geschwindigkeit der Kugel nach dem Stoß seyn wird

$$=\frac{500b}{k+500b}\sqrt{\frac{90h}{31}l\frac{5f-2b}{3b}}.$$

Wenn man von der Materie des Pulvers einen kleineren Theil als die Helfte genommen hätte, so müßte man auch für 500b eine kleinere Zahl setzen. Wollen wir dafür 310b annehmen, so giebt $k+310b$ eben denjenigen Nenner $\alpha(2k+nb)-(n-244\alpha)\lambda b$, welcher oben in der Vllten Anmerkung zum vorigen Satz gefunden worden, und eben diese Uebereinstimmung scheinet auch eine starke Probe zu seyn, dass diese Zahl 310b den richtigen Theil anzeigt. Demnach bekommen wir für die Höhe, welche die Geschwindigkeit der Kugel ausdrückt, diese Expression:

$$\frac{310\cdot 900b^2h}{(k+310b)^2}l\frac{5f-2b}{3b}=\frac{279000b^2h}{(k+310b)^2}l\frac{5f-2b}{3b}.$$

Wenn aber die Kugel anfänglich unmittelbar vor das Pulver wäre gesetzt worden, so würde dieselbe, nach dem sie biß in *ZZ* fortgestossen worden, eine Geschwindigkeit erhalten haben, welche aus dieser Höhe

$$=\frac{900bh}{k+310b}l\frac{5f-2b}{3b}$$

entstehet; folglich wird sich diese Geschwindigkeit zu jener verhalten, wie $\sqrt{\frac{310b}{k+310b}}$: und also wird die der Kugel durch den Stoß mitgetheilte Geschwindigkeit weit kleiner seyn, als diejenige, welche sie in *Z* erlangt haben würde, wenn sie unmittelbar vor das Pulver wäre geladen worden. Wenn nun diese durch den Stoß erlangte Geschwindigkeit richtig ist, so wird es leicht seyn, die folgende Vermehrung derselben durch die Ausdehnungskraft zu bestimmen. Wir dürfen zu diesem Ende nur die oben gefundene Aequation nehmen, welche, nachdem man die kleinen Terminos weggelassen, und für die Buchstaben α, λ, β und n die gehörigen Werthe gesetzt, also beschaffen ist

$$v=\frac{900bh}{k+310b}l\frac{5f-2b}{3b},$$

und dazu noch eine unveränderliche Quantität setzen, welche so beschaffen seyn muß, daß, wenn man $x=f$ setzt, die Höhe v in

$$\frac{279000bbh}{(k+310b)^2}l\frac{5f-2b}{3b}$$

verwandelt werde. Es bedeute also *v* die für die Geschwindigkeit der Kugel gebührende Höhe, nachdem die Kugel biß in *MM,* da $AM = x$, fortgestossen worden, und da wird man eine solche Aequation haben

$$v = \frac{900bh}{k+310b} l \frac{5f-2b}{3b} + C.$$

Um nun die Quantität *C* zu bestimmen, so setze man

$$\frac{279000bbh}{(k+310b)^2} l \frac{5f-2b}{3b} = \frac{900bh}{k+310b} l \frac{5f-2b}{3b} + C$$

da wird

$$x = f \quad \text{und} \quad v = \frac{279000bbh}{(k+310b)^2} l \frac{5f-2b}{3b},$$

und also

$$C = \frac{-900bkh}{(k+310b)^2} l \frac{5f-2b}{3b}.$$

Folglich wird man erhalten

$$v = \frac{900bh}{k+310b} l \frac{5f-2b}{3b} - \frac{900bkh}{(k+310b)^2} l \frac{5f-2b}{3b}.$$

Wenn aber die Kugel in *ZZ* keinen Stoß bekommen hätte, sondern bloß allein von der Ausdehnungs-Kraft wäre fort getrieben worden, so würde

$$C = \frac{-900bh}{k+310b} l \frac{5f-2b}{3b}$$

und also

$$v = \frac{900bh}{k+310b} l \frac{5x-2b}{3b} - \frac{900bkh}{k+310b} l \frac{5f-2b}{3b}.$$

Wenn aber die Kugel anfänglich unmittelbar vor das Pulver wäre geladen worden, so würde heraus kommen

$$v = \frac{900bh}{k+310b} l \frac{5x-2b}{3b}.$$

Wenn wir nun für *x* die Länge des ganzen Laufs *a*, und für das von dem Autore angeführte Exempel setzen

$$a = 45,\ b = 2{,}625,\ \ f = 11{,}25,\ k = 4900 \text{ und } lh = 7{,}463893,$$

so können wir die dreyerley Geschwindigkeiten folgender Gestalt bestimmen:

$$\frac{900b}{k+310b} = \frac{236250}{571375},\quad \frac{900bk}{(k+310b)^2} = \frac{490000}{571375}\cdot\frac{236250}{571375}$$

oder

$$\frac{900b}{k+310b} = 0{,}41348,$$
$$\frac{900bk}{(k+310b)^2} = 0{,}35459$$

und

$$\frac{5x-2b}{3b} = 27{,}905,\quad \frac{5f-2b}{3b} = 6{,}4762,$$

folglich

$$l27{,}905 = 1{,}445682$$
$$l6{,}476 = 0{,}811307$$
$$l1{,}445682 = 0{,}160073$$
$$l0{,}811307 = 9{,}909185.$$

Man addiere 0,362216 , so kommt

$$ll\frac{5x-2b}{3b} = 0{,}522289$$
$$ll\frac{5f-2b}{3b} = 0{,}271401;$$

so bekommt man

$$\frac{900b}{k+310b}l\frac{5x-2b}{3b} = 1{,}37639,$$
$$\frac{900b}{k+310b}l\frac{5f-2b}{3b} = 0{,}77243$$
$$\frac{900bk}{(k+310b)^2}l\frac{5f-2b}{3b} = 0{,}66242.$$

Wenn also die Kugel anfänglich unmittelbar vor das Pulver geladen wird, so wird $v = 1{,}37639h$, und die Geschwindigkeit beträgt in einer Secunde 1582 Schuh. Wird aber die Kugel anfänglich in *ZZ* gesetzt, und das Pulver durch den Raum *AZ* also zerstreuet, daß die Kugel keinen Stoß leidet, so wird $v = 0{,}60395h$ und diese Geschwindigkeit beträgt 1048 Schuh in einer Secunde. Wenn aber die Kugel in *ZZ* zugleich den Stoß von dem Pulver erhält, so wird $v = 0,71396h$ und diese Geschwindigkeit beträgt 1140 Schuh in einer Secunde.

Es ist aber zu merken, daß die Formul, aus welcher diese Geschwindigkeiten bestimmet worden, etwas zu klein ist, dahero die gefundenen Zahlen noch um etwas vermehret werden müssen. Dem ungeachtet aber ist klar, daß die Geschwindigkeit der Kugel im letzten Fall, da dieselbe anfänglich durch den Stoß des Pulvers in Bewegung gesetzet worden, weit kleiner ist, als die Erfahrung ausgewiesen. Wenn wir aber auf die dabey vorgefallenen Umstände Acht haben, so sieht man leicht, daß sich die subtile Materie bey dem Stoß hinter der Kugel sehr stark müsse gehäuffet, und also eine weit grössere Elasticität bekommen haben, wodurch folglich der Kugel auch ein weit grösserer Grad der Geschwindigkeit eingedruckt worden. Was aber die Aufschwellung und Zersprengung des Laufs anlangt, so ist die Ursache davon leicht einzusehen. Denn da der Kugel gleichsam in einem Augenblick ein ziemlicher Grad der Geschwindigkeit durch den Stoß eingedruckt wird, so ist klar, daß eine druckende Kraft, welche in eben dieser kurzen Zeit der Kugel eben diesen Grad der Geschwindigkeit einzudrücken vermögend wäre, erstaunlich groß seyn müsse; und es dürfte vielleicht eine 10 mahl stärkere Kraft, als die ausdehnende Gewalt des Pulvers allein auf die Kugel auszuüben pflegt, nicht hinlänglich seyn, eine solche Würkung hervorzubringen. Da nun eine Canone oder ein Mußketen-Lauf an einem jeglichen Ort nur so stark gemacht zu werden pflegt, als zu Aushaltung der ordentlichen Ausdehnungs-Kraft des Pulvers erfordert wird, so hat man sich nicht zu verwundern, wenn von diesem grossen Zuwachs der Gewalt ein gewöhnlicher Lauf zerspringet oder auseinander gedehnet wird, wie in dem von dem Autors angeführten Exempel geschehen ist. Wenn die Kugel diesen stoß nicht bekommen hätte, so würde dieselbe nach unserer Rechnung eine Geschwindigkeit von 1048 Rheinl. Schuhen in einer Secunde erhalten, welche in Englischen Schuhen 1080 beträgt, und folglich mit 1100, so durch die Experimente heraus gekommen, um so viel mehr genau überein trifft, da die Formul, welche hier gebraucht worden, etwas zu klein ist. Wenn also der Autor für diesen Fall eine Geschwindigkeit von 1200 Schuhen heraus bringt, so muß entweder in seine Rechnung ein Fehler eingeschlichen seyn, oder die Ursache davon steckt, welches wahrscheinlicher ist, in der Unrichtigkeit der Theorie des Autoris selbst; indem er nicht auf die Kraft gesehen, welche zu Forttreibung der Flamme erfordert wird. Da sich nun die Sache also verhält, so fällt auch seine Erklärung, wodurch er die Verminderung der Geschwindigkeit der Kugel in diesem Fall erklären will, weg; als welche außer diesem auch so beschaffen ist, daß daraus die Würkung, welche der Autor angiebt, nicht folgen kann. Denn ungeachtet dieser Satz unstreitig wahr ist, daß die ausdehnende Kraft einer flüßigen elastischen Materie geringer wird, wenn sich darinne eine innerliche Bewegung unter den Theilen derselben befindet, so ist doch leicht zu erachten, daß diese innerliche Bewegung in der Flamme, wenn das Pulver durch den gantzen Raum hinter der Kugel zerstreuet worden, alsobald aufhören müsse, wenn dieselbe auf die Kugel zu würken anfängt. Wir haben aber in demjenigen, was wir vorher von der Bewegung der Kugel, wenn dieselbe nicht unmittelbar auf das Pulver geladen wird, ausgeführet haben, angenommen, daß der Raum zwischen dem Pulver und der Kugel völlig leer sey. Da sich nun darinn eine natürliche Luft befindet, so entstehet daher in den gemachten Schlüßen eine geringe Veränderung. Denn, so bald sich nach der Entzündung die Flamme auszubreiten anfängt, so wird die zwischen dem Pulver und der

Kugel befindliche Luft zusammen gedruckt, und bekommt folglich eine Kraft die Kugel fortzustossen, dergestalt, daß die Kugel würklich in Bewegung gesetzt wird, ehe die Flamme dieselbe unmittelbar erreichet und darauf den Stoß ausübet. Und in diesem Umstande scheinet die Ursache grösten theils zu stecken, daß die Kugel eine schnellere Bewegung erhält, als durch die obige Rechnung gefunden worden.

PROPOSITION XIII.

To enumerate the various Kinds of Powder, and to describe the properest Methods of examining its Goodness.

The powder, we have hitherto considered, is supposed to be such as is made for the service of the government ; but, besides this, there are many other kinds, some better and some worse, which I here propose to enumerate, as far as they have come to my knowledge.

But, first, I must premise, that the government powder, if properly wrought, is, I believe, nearly as good as any powder made for general use. I have examined it with great care, and have compared it with other powders made here in *England*, which are esteemed the best, such as the *Battle,&*c. and I cannot find any sensible difference between them. I have likewise compared it, in frequent trials, with some *Spanish* powder, taken out of the *St. Jago* prize; and though, if I were to give my opinion, I should rather believe the *Spanish* powder the better of the two, yet so small an inequality as a fiftieth or sixtieth part, which is the most, that the difference between them can amount to, is too little to be ascertained with absolute certainty. I conceive too, by comparing the experiments of others with my own, that the *French* powder is little different from ours; though I cannot be so certain on this head as I could wish, having never been able to procure any of their powder myself. But it must be remembered, that when I speak of our government powder, it must be what is Supposed to be made of the standard proportion of materials, and properly wrought; for such was the powder I made use of. The stl'ongest powder, I have yet met with, is some which I am told was made in *Holland;* its force, compared with that of our government powder, is nearly as 5 to 4. But this powder is undoubtedly made of the choicest picked materials, and is probably wrought up with spirits; so that quantities of it could not be made, but at a much greater expence, than what would be repaid by its additional strength.

The next best powder, that has come to my hands, is a powder made in *Portugal*, under the direction of a *Dutchman,* who some years since established powder-mills near *Lisbon.* This is in strength inferior to the *Dutch* powder last mentioned ; but is however nearer to that than to our government powder.

The common sale powder here in *England,* such as is to be had at every grocer's, is much worse than the government or the *battle* powder, and extremely various, according to the caprice of the maker. I have tried some, whose strength has been in proportion to the government powder, as l to 3 nearly, and other parcels have been still worse; but the worst of all is the powder made for the *African* trade, usually styled *Guinea* powder but these weaker powders are not worth examination, as there is no established standard for their composition.

Now these differences in the strength of powder may arise from three causes; either from the quality of the materials, from the proportion observed in their mixture, or from the manner of working them together.

Powder, as is generally known, is composed of saltpetre, sulphur, and charcoal; of these materials, the sulphur and charcoal are much the cheapest ; and though there are peculiar kinds of these, which are fittest for this purpose, yet the expence of having the very best is so small, when compared to the whole cost of the powder, that it is strange if powder, which would be otherwise good, is spoiled by bad sulphur or charcoal.

The most expensive part of the composition, and consequently the part to which the. defects of powder arc oftner owing, is the saltpetre. This is a substance imbibed by the earth from the air; for a quantity of earth, which has had its saltpetre washed out of it, will, when it has been exposed to the air for some time, produce saltpetre again ; and this as often as the experiment shall be repeated.

Saltpetre is of itself an uninflamable substance ; for if it be placed in the most violent fire, , it only melts and never flames, provided no combustible matter is permitted to mix with it: but though of itself, unmixed with other bodies, it will neither flame nor burn; yet, if it be joined with burning substances, it prodigiously augments the violence of their burning; performing, in this case, what the air, forcibly mixed with fire by the blast of a of bellows, does in a much inferior degree.

Powder then being a mixture of sulphur and charcoal, which are very inflammable substances, with saltpetre, which in itself is not, if the saltpetre be too much in quantity, when compared with the other two, their burning may not be sufficient to consume the whole of the saltpetre; whence the fire may be less violent, and consequently (according to what we have observed in the 10th proposition) the powder less vigorous, than if some of the saltpetre was taken away, and a like quantity of the other materials were added in its stead. On the other hand, if the saltpetre in the composition be less than what the burning of the other two substances can easily consume, the fire will be less active than it ought to be; because it is not augmented so much as it would be, if a larger quantity of saltpetre had been added to the composition.

Hence then it appears, that the goodness of powder is not to be estimated only from the quantity of saltpetre contained in it, although that substance seems to be the basis of the elastic fluid, in which its force consists; for since the converting of the saltpetre into that fluid, and the elasticity of the fluid afterwards, depend in some measure on the violence of the fire produced at the explosion, it is plain that there is a certain proportion in the mixture of the materials, which will best contribute to this purpose, and consequently to the perfection of the powder.

What this proportion is, has been ascertained by experience ; and it seems now to be generally agreed, that in any quantity of powder $\frac{3}{4}$ of it should be saltpetre, the remaining $\frac{1}{4}$ consisting of equal quantities of sulphur and charcoal. This is the proportion followed by the *French*, and I believe by most nations in Europe ; we indeed pretend to a greater degree of nicety in our proportions; though I am told they do not greatly differ from what I have mentioned; nor am I convinced that they are preservable : this I am sure of, that no methods of proving powder, hitherto generally practised in *England* , could at all ascertain the difference ; and other powders, made with the usual proportions, are no whit inferior to ours.

But it is not the due proportion of the materials only, which is necessary to the making of good powder; another circumstance, not less essential, is the mixing them well together; if this be not effectually done, some parts of the composition will have too much saltpetre in them, and others too little; and in either case there will be a loss of strength in the powder.

As the excellency of powder then depends on so many particulars, in the quality and quantity of the materials, and in the working them together ; it is doubtless of great importance, that those who receive the public stores should have it in their power to satisfy themselves about the goodness of what is delivered to them. The method most commonly followed for this purpose, here with us, is (if I am rightly informed) to fire a small heap of it on a clean board, and to attend nicely to the flame and smoke it produces, as likewise to the marks it leaves behind it on the table; from all which instructive particulars the merit of the powder is ascertained with great accuracy, as is pretended: but besides this uncertain method, which I presume (how much soever it may be practised) none will seriously undertake to defend, there are, on particular occasions, other contrivances made use of; all which bear some analogy to the common powder-triers, sold at the shops ; only they are more artfully fabricated, and instead of a spring they move a weight, which is a more certain and equable power.

But these machines, though more perfect than the common powder-triers, are yet liable to great irregularities ; for as they are all moved by the instantaneous stroke of the flame, and not by its continued pressure, they do not determine the force of the fired powder with that certainty and uniformity, which were to be desired in these kinds of trials : and therefore I cannot but think the method followed by the *French*, in the receiving of powders from the makers, to be much better. Their practice is thus.

They have, in each magazine, a small mortar cast, with its bed, according to a determined pattern, which is the same throughout the kingdom : this mortar is always pointed at 45°, and it contains just three ounces of powder; and it is a standing maxim, that no powder can be received into their stores, unless three ounces of it, placed in the chamber of this mortar, throws a solid ball, of $7\frac{1}{2}$ inches diameter, to the distance of at least 55 *French* fathom.

It has been objected to this method, that if each barrel of powder was to be proved in this manner, the trouble of charging the mortar, and bringing back the ball each time, would be intolerable, and the delay so great, that no business of this kind could ever be finished ; and if a number of barrels arc received on the merit of a few, it is great odds but some bad ones will be amongst them, which may prove a great disappointment in time of service. Add to this another exception, which to me has much more weight; and that is, the monstrous disproportion between the weight of the ball and the powder that projects it; so that the powder continues in action a longer time, and expands through a much larger space, in proportion to its quantities in these trials, than it ever does in any real service; whence it happens, that the vapour cools, and great part of it escapes through the touch-hole, or by the side of the bullet; so that the quantity of motion produced by the explosion is, in this instance, but little more than half of what it ought to be, if the powder acted on the ball with its full force undiminished by these accidents ; consequently, as this diminution of force may not be always constant, the action of the same powder, by the varying of these adventitious circumstances, may, at different times, convey the ball to different distances.

Now this last exception does no ways hold against the method by which I have tried the comparative strength of different kinds of powder, which has been by the actual velocity given to a bullet, by such a quantity of powder as is usually esteemed a proper charge for the piece : and as this velocity, however great; is easily discovered by the motion, which the pendulum acquires from the stroke of the bullet, (according to the principles laid down above) it might seem a good amendment to the method used by the *French*, to introduce this trial by the pendulum instead of it. But though I am satisfied, that· this would be much more accurate, less laborious, and readier than the other, yet, as there is some little attention and caution required in this practice, which might render it of less dispatch than might be convenient; when a great number of barrels were to be separately tried, I should myself chuse to practise another method not less certain, but prodigiously more expeditious ; so that I could engage, that the weighing out of a small parcel of powder from each barrel should be the greatest part of the labour; and, doubtless, three or four hands could, by this means, examine 500 barrels in a morning ; besides, the machines for this purpose, as they might be made of cast il'on, should be so very cheap, that they might be multiplied at pleasure. However, I shall defer the description of this method at present, and shall proceed to the consideration of the resistance of the air, a subject of the greatest importance to the perfection of gunnery.

REMARK

The power of the powder is based on these two points, as has been clearly shown above : firstly on the amount of the elastic matter, thus to be directed entirely from the air which arises from a given amount of powder through ignition, and secondly from the suddenness of the ignition itself. The more air thus contained in the powder, so the greater is the density of the same, and therefore also the expansive force, which constitutes the power of the powder. Herein again too is contained all that which has been advanced likewise above about the greater matter of the flame, from which the motion must be arise, and on account of which the driving force of the powder will become diminished. Since the more air becomes produced from a given quantity of powder the smaller must be the remaining part necessary, which constitutes the grosser matter. Therefore one has a double advantage to be maintained from such a powder, from within which a greater amount of compressed air is contained : likewise from that not only a stronger expansive force originates, but also the amount of the grosser matter which equally must be present in the motion, accordingly is so much smaller. Also but in particular there arises from the suddenness of the ignition in particular that through the confined air freed from its bondage, and will be put in place to exercise their force. The quicker thus the ignition itself spreads through the whole interior of the powder, so the greater also will be the force, thus from that put in place, likewise equally in the first instant a greater force is at hand, which acts on the ball, and subsequently acting to move forwards also. But through the suddenness of the ignition the spreading of the force also increases still the other principle ; for the quicker the fire begins in the powder, the greater is the heat too, thus to be created from that. Now since through the heat, as we have seen above, the elasticity of the air will be very markedly increased thus rising from that to a much greater increase in the strength of the powder.

Thus in order to gain a complete equal knowledge of the force from each kind of powder, thus it is necessary, firstly that one must know, how much air may be held in a given quantity of powder, and then secondly, how much time overall passes, while the ignition passes through all the powder. Now the first lets itself be determined through experiments, which the author has introduced in the first proposition of this chapter ; but one cannot rise to a full degree of understanding on the latter, as the time, in which the whole ignition happens, is all too short, and also is based on such a variable condition, that all the considerations will not be one and the same. Therefore we want from that, to take everything together into consideration that above has been made through experiment.

The author had employed that kind of powder in his experiments, which in England had been made to comply with the government service regulations, and the amount of trapped air therein can be determined in a two-fold manner: in the first place in a consideration of the volumes, and in the second place in a consideration of the weights. By the first method he has found, that a cubic inch of powder contained air, that by expansion the same increased to be capable of filling a volume of 244 cubic inches of ordinary air, and with the same degree of density as ordinary air. Now since a cubic inch of powder holds in a cubic inch ordinary air in very compressed condition, so it is clear, that if in a cannon or musket barrel the length of the space AC , which will be placed filled with powder, $=b$ within which the enclosed air shall be equal to a cylinder of natural air, whose thickness is as one with the width of the barrel CC, but whose length is $=244b$. But from that the author compares the weight of this air held in the powder with the whole weight of the powder, as he had found that this itself to be to that in the proportion, as 3 to10. Now since in the following case the weight of the inclosed air was equal to the weight of, of which the height was $=244b$, so the weight of the whole charge of powder must be equal to the weight of a column of ordinary air, of which the height $=\frac{10}{3}\cdot 244b=813b$. Consequently the coarser part, from which the powder consists, from the weight equals an air column, of which the height $=569b$. Now since the coarser part is not expanded by the ignition, if we put in place that the air in the powder granules to be 800 times denser than ordinary air, and thus initially has taken up a part in the volume AC, of which the length $=\frac{244}{800}b$, so the remainder $=\frac{556}{800}b$, which partially is occupied by the air from the coarser material of the powder grains, and partially from the little found between the grains. If also we put in place, that the little space between the powder grains to work out to a fifth part of the whole space, so a space remains for the coarser material of the powder left alone, the length of which $=\frac{396}{800}b$; and also the coarser material must take up so much space besides from the ignition.

Now in order to this kind of powder to be equivalent to another kind, thus let us put in place, that which in one kind of powder contains air ot be equal to an air column of which the length $=mb$, namely if b indicates the length AC, and that which the weight of the whole powder equals a column of air, of which the height $=nb$. If now further we assume that air which is contained in the pores of the powder to be always 800 times denser than ordinary air, so the same must be for the ignition of a given volume, of which the length $=\frac{m}{800}b$ *:* consequently, if for which the fifth part of *AC* will be assumed again for the interstices between the grains of powder, so a space remains for the grosser matter left over, of which the length $=\frac{640-m}{800}b$. Now if we want to know, how the different values of the letters *m* and *n* can be worked out from the speed of the ball, so we must only calculate the correct fall of these. Thus the length of the whole barrel shall be $AB = a$ *;* *k* the height of an air column, of which the weight is equal to the weight of the ball ; and *h* the height of an air column, of which the weight is equal to the elasticity of ordinary air. One puts $AM = x$, and from which the ball will be aready driven forwards from *CC* as far as to *MM*, so that its speed shall be $=\sqrt{v}$ *;* in the same manner, just as the ball departs through $Mm = dx$, the height *v* will be greater by *dv*. Now this increase in the speed of the ball therefore is to be brought about in this manner, as there a force will be necessary, which is equal to the weight of an air column, thus $=\frac{kdv}{dx}$. [Thus, the work done on the equivalent air column rising by *dv*, *kdv*, is equal to the work done by the excess pressure in the barrel acting through *dx*]. Further the total weight of the coarser and subtle parts of the powder is equal to the weight of an air column, of which the height $= nb$, thus for the acceleration of this matter a force is required $=\frac{nbdv}{2dx}$ [see, *e.g.*, the note added in Remark 6 of Prop. XI], which formula finds a place, because the coarser matter may be spread out evenly throughout the whole space *AM*, or as we have assumed above, that a half of that will be driven forwards with the ball, but the other half remains behind on the base *AA*. If also meanwhile nothing has been lost of the combined matter going out through the touch-hole or by the play-space, thus the force which will be required of the powder itself for the acceleration of the ball, is

$$=\left(k+\tfrac{1}{2}nb\right)\frac{dv}{dx},$$

which the powder must have from a single sudden ignition, or not. We will put in place, that all the powder has been ignited initially, hence there is thus an air cylinder produced, of which the height $= mb$, which is held in the space *AM* with the coarse matter. Since now the coarse matter alone alone takes up a part

$$=\tfrac{640-m}{800}b,$$

thus there remains the space for the above air:

$$x-\tfrac{(640-m)b}{800}.$$

Thus as many times as this magnitude $x - \frac{(640-m)b}{800}$ is smaller than mb, thus just as many times will the air be denser than ordinary air. One puts for brevity :

$$\frac{800mb}{800x-(640-m)b} = s,$$

so the density of the compressed air in *AM* will be *s* times greater than ordinary air : and consequenty the elasticity will be expressed by a column of ordinary air, of which the height

$$= h\left(s + \frac{ss}{6q} \right) = h\left(s + \frac{ss}{4800} \right),$$

namely if we put 800 for *q*, as then the highest degree of the density of the air. But this height must be multiplied by the letter β on account of the ignition, the value of which is approximately 4. Also from this there is produced this equation, if the counter pressure and resistance of the other air are included in the motion :

$$\left(k+\tfrac{1}{2}nb\right)\frac{dv}{dx} = \beta h\left(s + \frac{ss}{4800} \right) - h - \tfrac{1}{2}v,$$

which, if one ignores the smallest terms, gives this integrated equation :

$$\left(k+\tfrac{1}{2}nb\right)v = \beta mbh\, l\,\frac{800x-640b+mb}{160b+mb}$$

or

$$v = \frac{\beta mbh}{k+\tfrac{1}{2}nb}\, l\,\frac{800x-640b+mb}{160b+mb}.$$

Thus one sees from this, that the speed can become so much greater and consequently the powder also to be so much better, the greater the number *m* shall be, and the smaller the number *n* : that is, the more air to have become trapped in a given quantity of powder, and at the same time the lighter the powder is itself. To be sure the last circumstance encountered, thus draws so little out, and can very well be completely left out of the consideration : because powders have nearly all the same weight, and the same cannot very well be diminished. But if also a small increase of the weight were itself possible, yet through that there would be hardly any increase in the speed of the ball established: thereby it is mainly from the magnitude of the number *m* arising, through which the amount of the trapped air in the powder will be indicated. Thus if by all kinds of powder the whole ignition happened in an instant, as our author wants to maintain, so one could not judge the best powder with certainty other than by the size of the number *m*. Now since by investigating the value of the government powder of these letters, $m = 244$ is found, so one must have a powder of this kind, in which *m* is still greater than 244 as better than these where *m* has a smaller value for an inferior kind. One can also derive from this the correct way to investigate the quality of powders, which in assigning that experiment to be adopted that the author suggested to determine this number *m* ; which alone of all the trials could be used because of so many circumstances, which thereby would have to be assumed in the consideration, becoming all too wordy and difficult. But if all the powder ignited at once, as we can assume here with the author, thus one would still attain this objective quite easily. Thus in this case it would be enough, for the very first action to be considered alone, which the powder exerted after the ignition : and to this end one would be able to estimate carefully, which were the best of the common powder samples, that can be dealt with to the greatest advantage, from the height to which a known weight will be driven by the force of the powder. The author also puts this common kind of powder to the test by no other way than through that which only indicates the first force of the same. But if, as the author himself claims, all the powder ignites at once, so the following expansion force is agreed based on the first ignition alone, of such a form, that for a greater or lesser force to be found in the first instant, so also the whole strength of the powder accordingly must be so much greater or smaller. If therefore the author wants to test the powder samples in this manner, he will be contradicting himself, for it is the intention to destroy the system of instantaneous ignition, which he still insists to delare, denies, or to be cast into doubt. [Presumably Robins had in mind that the impure samples did not have instantaneous combustion, which he presumed happened only for the pure form.] But we are unconcerned about having declared this contrary point of view, as we can still consider this ordinary proving method, if machines are manufactured with associated diligence, that do not deny all benefits. Because also, if not all the powder were ignited in the first instant, so the whole force still can be based for the most part on the strength of the first force, and if the time, which for a full a complete combustion an equal quantity of powder would necessitate, is just the same, so also the whole force can be properly assessed from the first thrust. But if herein a disparity should be found, so one must change the form of the powder sample machine, as the powder could be acting for some time on the body, which it was wished to set in motion, and such an improvement would still not be too difficult a work to render these machines susceptible to this degree of perfection.

Perhaps also the new way of the author that the powder is to be tested, which he has kept as a secret, is none other than some modest improvement of the common powder tester and it is probable that an able artisan after a few trials could easily produce the same. The whole concern would namely only assume from that, that one must make the lower part of the small cannon, in which the powder would be placed, somewhat deeper, and the same given the shape of a small cylinder, so that the powder does not completely fill the whole cavity. Furthermore, one must make the weight which it is desired to be projected to a height, to be made in the form of a plug that fits exactly into the lower end of the cylinder, and can be put as far back as to touch the powder. On account of which the powder will act on this weight, not only at the first instant of ignition, but also as long as the cylinder is driven forwards. And since this weight in consideration of the small amount of powder, which will be brought to test, is very great, and consequently can be put into so small a motion, that the weight in question : it is evident that before the weight can leave the cannon entirely, already so much time will have passed, that meanwhile all the powder or still at the least from a proportion so small, that in the cannon it can all be looked after to ignite as in an ordinary firearm.

ANMERKUNG

Die Gewalt des Pulvers beruhet, wie aus obigem zur Gnüge erhellet, auf diesen zwey Punckten: erstlich auf der Menge der subtilen elastischen Materie, so mit der Luft einerley zu seyn gewiesen worden, welche aus einer gegebenen Quantität Pulver durch die Entzündung erzeuget wird, und zweytens auf der Plötzlichkeit der Entzündung selbst. Je mehr Luft also in dem Pulver enthalten ist, je grösser ist auch die Dichtigkeit derselben, und folglich wird auch die Ausdehnungs-Kraft, worinne die Gewalt des Pulvers bestehet, um so viel grösser. Hierinne ist auch schon zugleich dasjenige enthalten, was oben von der gröberen Materie der Flamme, welche mit in Bewegung gesetzt werden muß, und weswegen die forttreibende Kraft des Pulvers vermindert wird, angeführet worden. Denn je mehr Luft aus einer gegebenen Quantität Pulver erzeuget wird, je kleiner muß nothwendig der Ueberrest, welcher die gröbere Materie ausmacht, seyn. Dahero hat man von einem solchen Pulver, worinne eine grössere Menge zusammen gedruckter Luft enthalten ist, einen doppelten Vortheil zu gewarten: indem daraus nicht nur eine stärkere Ausdehnungs-Kraft entstehet, sondern auch die Menge der gröberen Materie, welche zugleich in Bewegung gesetzt werden muß, um so viel geringer ist. Hiernechst kommt es aber insonderheit auf die Plötzlichkeif der Entzündung an, als wodurch die eingeschlossene Luft von ihren Banden befreyet, und in Stand gesetzet wird, ihre Kraft auszuüben. Je geschwinder sich also die Entzündung durch das gantze Wesen des Pulvers ausbreitet, je grösser wird auch die Kraft, so daraus entsteht, indem gleich im ersten Augenblick eine grössere Gewalt vorhanden ist, welche auf die Kugel würket, und auch nachgehends darauf zu würken fortfährt. Durch die Plötzliohkeit der Entzündung wird aber auch die ausbreitende Gewalt noch aus einem andern Grunde vermehret; denn je schneller das Pulver Feuer fängt, je grösser ist auch die Hitze, so dabey hervorgebracht wird. Da nun durch die Hitze, wie wir oben gesehen, die Elasticität der Luft sehr merklich vermehret wird so entspringt daher auch ein sehr grosser Zuwachs in der Gewalt des Pulvers.

Um also von der Stärke einer jeglichen Art von Pulver eine vollkommene Kenntniß zu erlangen, so ist nöthig, daß man erstlich wisse, wie viel Luft in einer gegebenen Quantität Pulver enthalten sey, und denn zweytens, wie viel Zeit vorbey gehe, indem sich die Entzündung durch alles Pulver ausbreite. Das erstere läßt sich nun auf diejenige Art, welche der Autor in den ersten Sätzen dieses Capitels ausgeführt, durch Versuche bestimmen; über das letztere aber kan man nicht wohl zu einer völligen Gewißheit gelangen, indem die Zeit, in welcher die gänzliche Entzündung geschieht, allzukurz, und auch auf solchen veränderlichen Umständen beruht, daß dieselbe allem Ansehen nach nicht immer einerley seyn wird. Wir wollen demnach dasjenige, was oben über den ersten Punkt durch Versuche ausgemacht worden, zusammen nehmen, und in Erwegung ziehen.

Der Verfasser hat seine Versuche über diejenige Art von Pulver, welche in Engelland zum Dienst der Regierung gemacht zu werden pflegt, angestellt, und die Menge der darinne eingeschlossenen Luft auf eine doppelte Art bestimmt: erstlich in Ansehung des Raums, und zweytens in Ansehung des Gewichts. Durch die erste Art hat er befunden, daß die in einem cubischen Zoll Pulver enthaltene Luft, nachdem sich dieselbe mit der natürlichen Luft auf einerley Grad der Dichtigkeit ausgebreitet, einen Raum von 244 cubischen Zollen auszufüllen vermögend sey. Da nun in einem cubischen Zoll Pulver 244 cubische Zoll natürliche Luft in einem sehr zusammen gepreßten Zustande enthalten, so ist klar, daß wenn in einer Canonen oder in einem Mußketen-Lauf die Länge des Raums *AC*, welche mit Pulver augefüllet worden, $=b$ gesetzt wird, die darinne eingeschlossene Luft einem Cylinder von natürlicher Luft gleich sey, dessen Dicke mit der Weite des Laufes *CC* einerley, die Länge aber $=244b$ ist. Nachdem aber der Verfasser das Gewicht dieser in dem Pulver enthaltenen Luft mit dem ganzen Gewicht des Pulvers verglichen, so hat er befunden, daß sich jenes zu diesem verhalte, wie 3 zu 10. Da nun in dem vorigen Fall das Gewicht der eingeschlossenen Luft dem Gewicht einer Luft-Säule gleich war, deren Höhe $=244b$, so muß das Gewicht der ganzen Ladung von Pulver dem Gewicht einer natürlichen Luft-Säule gleichen, deren Höhe $=\frac{10}{3}\cdot 244b=813b$. Folglich sind die gröbern Theile, woraus das Pulver besteht, dem Gewicht nach einer Luft-Säule gleich, deren Höhe $=569b$. Da sich nun diese gröbern Theile bey der Entzündung nicht ausdehnen, wenn wir setzen, daß die Luft in den Pulverkörnern 800 mahl dichter sey, als die natürliche, und folglich anfänglich in dem Raum *AC* einen Theil eingenommen habe, dessen Länge $=\frac{244}{800}b$, so ist der Ueberrest $=\frac{556}{800}b$, welcher theils von der gröbern Materie der Pulverkörner, theils von der zwischen den Körnern befindlichen Luft eingenommen wird. Wenn wir also setzen, daß die Zwischenräumlein zwischen den Pulver-Körnern den fünften Theil des ganzen Raums austragen, so bleibt für die gröbere Materie des Pulvers allein ein Raum übrig, dessen Länge $=\frac{396}{800}b$; und so viel Raum muß auch die gröbere Materie des Pulvers nach der Entzündung beständig einnehmen. Um nun mit dieser Art von Pulver andere Arten zu vergleichen, so laßt uns setzen, daß die in einer andern Art von Pulver enthaltene Luft einer natürlichen Luft-Säule gleich sey, deren Länge $=mb$, wenn nehmlich *b* die Länge des Raums *AC* andeutet, und daß die Schwehre des sämtlichen Pulvers einer Luft-Säule gleiche, deren Höhe $=nb$. Wenn wir nun ferner annehmen, daß die in den Poris des Pulvers enthaltene Luft beständig 800 mahl dichter sey, als die natürliche, so muß dieselbe vor der Entzündung einen Raum einnehmen, dessen Länge $=\frac{m}{800}b$: folglich, wenn für die Zwischen-Räumlein wiederum der fünfte Theil angenommen wird, so bleibt für die gröbere Materie ein Raum übrig, dessen Länge $=\frac{640-m}{800}b$. Wenn wir nun wissen wollen, was die verschiedenen Werthe der Buchstaben *m* und *n* in der Geschwindigkeit der Kugel austragen können, so dürfen wir nur die Rechnung auf diesen Fall richten. Es sey demnach die Länge des ganzen Laufs $AB=a$; *k* die Höhe einer Luft-Säule, deren Gewicht dem Gewicht der Kugel gleich ist; und *h* die Höhe einer Luft-Säule, deren Gewicht der Elasticität der natürlichen Luft gleich ist. Man setze $AM=x$, und nachdem die Kugel von *CC* schon biß in *MM* fortgetrieben worden, so sey ihre Geschwindigkeit $=\sqrt{v}$; dergestalt, daß, indem die Kugel durch $Mm=dx$ fortgehet, die Höhe *v* umd *v* grösser werde. Um nun diese Vermehrung der Geschwindigkeit in der Kugel hervor zu bringen, so wird dazu eine

Gewalt erfordert, welche dem Gewicht einer Luft-Säule, so $=\frac{kdv}{dx}$ gleichet. Da ferner die gröberen und subtilen Theile des Pulvers insgesammt ihrer Schwehre nach einer Luft-Säule gleichen, deren Höhe $=nb$, so wird zur Acceleration dieser Materie eine Kraft erfordert $=\frac{nbdv}{2dx}$, welche Formul Statt findet, die gröbere Materie mag durch den ganzen Raum *AM* gleich zerstreuet seyn, oder wie wir oben angenommen, die eine Hälfte davon mit der Kugel fortgestossen werden, die andere aber an dem Boden *AA* zurückbleiben. Wenn also inzwischen nichts von der sämtlichen Materie des Pulvers durch das Zündloch und Spielraum verlohren gegangen, so ist die Kraft, welche zur Acceleration sowohl der Kugel, als des Pulvers selbst erfordert wird,

$$=\left(k+\tfrac{1}{2}nb\right)\frac{dv}{dx},$$

das Pulver mag sich auf einmal plötzlich entzündet haben, oder nicht. Wir wollen setzen, das Pulver habe sich anfänglich alles auf einmahl entzündet, so ist, daraus ein Cylinder, dessen Höhe $=mb$, Luft entstanden, welche nebst der gröbern Materie im Raum *AM* enthalten ist. Da nun die gröbere Materie allein davon einen Platz

$$=\tfrac{640-m}{800}b$$

einnimmt, so bleibt für die Luft über

$$x-\tfrac{(640-m)b}{800}.$$

So vielmahl also diese Grösse $x-\tfrac{(640-m)b}{800}$ kleiner ist, als *mb*, so vielmahl wird die Luft dichter seyn, als die natürliche. Man setze der Kürze halber

$$\frac{800mb}{800x-(640-m)b}=s,$$

so wird die Dichte der in *AM* zusammen gedruckten Luft *s* mahl grösser seyn, als der natürlichen: und folglich ihre Elasticität durch die Höhe einer natürlichen Luft-Säule ausgedruckt werden, deren Höhe

$$=h\left(s+\frac{ss}{6q}\right)=h\left(s+\frac{ss}{4800}\right),$$

wenn wir nehmlich für *q*, als den höchsten Grad der Dichte der Luft, 800 setzen. Diese Höhe muß aber noch wegen der Erhitzung durch den Buchstaben β multipliciret werden, dessen Werth ungefehr 4 ist. Hieraus entspringt also, wenn der Gegendruck und Wiederstand der äusseren Luft mit in Betrachtung gezogen wird, diese Vergleichung:

$$\left(k+\tfrac{1}{2}nb\right)\frac{dv}{dx}=\beta h\left(s+\frac{ss}{4800}\right)-h-\tfrac{1}{2}v,$$

welche, wenn man die kleinsten Terminos wegläßt, diese Integral-Aequation dargiebt:

$$\left(k+\tfrac{1}{2}nb\right)v=\beta mbh\, l\,\frac{800x-640b+mb}{160b+mb}$$

oder

$$v=\frac{\beta mbh}{k+\frac{1}{2}nb}\, l\,\frac{800x-640b+mb}{160b+mb}.$$

Hieraus sieht man also, daß die Geschwindigkeit um so viel grösser beraus komme und folglich das Pulver um so viel besser sey, je grösser die Zahl *m*, und je kleiner die Zahl *n* ist: das ist, je mehr Luft in einer gegebenen Quantität Pulver eingeschlossen, und je leichter zugleich das Pulver selbst ist. Was zwar den letzteren Umstand betrift, so trägt derselbe sehr wenig aus, und kan gar wohl völlig aus der Acht gelassen werden: weil das Pulver fast alle einerley Schwehre hat, und dieselbe nicht wohl vermindert werden kan. Wenn aber auch eine kleine Verringerung der Schwelire desselben möglich wäre, so würde doch dadurch keine merkliche Vermehrung in der Geschwindigkeit der Kugel entstehen: dahero es hierbey hauptsächlich auf die Größe der Zahl *m*, wodurch die Menge der in dem Pulver eingeschlossenen Luft angezeigt wird, ankommt, Wenn dahero bey allen Arten von Pulver die sämmtliche Entzündung in einem Augenblick, wie unser Verfasser behaupten will, geschähe, so würde die Güte des Pulvers nicht richtiger, als aus der Größe der Zahl *m* beurtheilet werden können. Da nun derselbe bey Untersuchung des RegierungsPulvers den Werth dieses Buchstabens $m=244$ befunden, so muß man diejenigen Arten von Pulver, in welchen *m* noch grösser ist, als 244, für besser, diejenigen aber, wo *m* einen kleinern Werth hat, für schlechter halten. Hieraus könnte man also den richtigsten Weg, die Güte des Pulvers zu erforschen, herleiten, welcher in Anstellung derjenigen Versuche, welche der Autor zu Bestimmung dieser Zahl *m* vorgeschlagen, bestehen würde; allein die Probe würde wegen der vielen Umstände, welche dabey in Acht genommen werden müssen, allzu weitläuftig und beschwerlich fallen. Wenn sich aber alles Pulver auf einmahl entzündete, wie hier mit dem Verfasser angenommen worden, so würde man noch viel leichter zu diesem Zweck gelangen können. Denn in diesem Fall würde es gnug seyn, auf die allererste Würkung, welche das Pulver nach der Entzündung ausübet, allein zu sehen: und zu diesem Ende würde man sich der gemeinen Pulver-Proben, vermittelst welcher die Güte des Pulvers aus der Höhe, worauf ein Gewicht durch die Gewalt des Pulvers getrieben wird, beurtheilt zu werden pflegt, mit dem grösten Vortheil bedienen können. Der Autor setzt auch selbst an dieser gemeinen Art das Pulver zu probiren nichts anders aus, als daß dadurch nur die erste Kraft desselben angezeigt werde. Wenn sich aber, wie der Autor selbst behauptet, alles

Pulver auf einmahl entzündet, so beruhet die folgende Ausdehnungs-Kraft einig und allein auf der ersten, dergestalt, daß je grösser oder kleiner diese Kraft in dem ersten Augenblick befunden wird, auch die ganze Gewalt des. Pulvers um so viel grösser oder kleiner seyn muß. Wenn dahero der Verfasser diese gemeine Art der Pulver-Proben für unrichtig halten will, so wiederspricht er sich selbst, indem er dabey die plötzliche Entzündung des Pulvers, welche er doch vorher so hartnäckig behauptet, läugnet, oder zum wenigsten in Zweifel zieht. Ungeachtet wir aber in diesem Stücke das Gegentheil behauptet haben, so können wir doch dieser gemeinen Probirungs-Art, wenn die Maschinen mit gehörigem Fleiß verfertiget sind, nicht allen Nutzen absprechen. Denn, wenn sich auch nicht alles Pulver in dem ersten Augenblick auf einmahl entzündet, so beruht doch die ganze Gewalt meistentheils auf der Stärke des ersten Stosses, und wenn die Zeit, welche zur völligen Entzündung gleicher Quantitäten Pulver erfordert wird, einerley ist, so kann auch die ganze Gewalt richtig aus dem ersten Stoß beurtheilet werden. Wenn sich aber hierinne eine Ungleichheit befinden sollte, so müste man die gewöhnlichen Pulver-Proben dergestalt verändern, daß das Pulver einige Zeit auf den Körper, welcher in Bewegung gesetzt werden soll, würken könnte, und eine solche Verbesserung würde allem Ansehen nach nicht schwer ins Werk zu richten seyn.

Vielleicht bestehet auch die neue Manier des Verfassers, das Pulver zu probiren, welche er als ein Geheimniß verschweigt, in nichts anders, als in einer bequemen Verbesserung der gemeinen Pulverproben: und es dürfte ein tüchtiger Künstler dieselben nach einigen Versuchen leicht heraus bringen. Die ganze Sach würde nehmlich nur darauf ankommen, daß man das untere Gefäße, worein das Pulver gethan wird, etwas tiefer machte, und demselben die Gestalt eines kleinen Cylinders gäbe, damit das Pulver nicht die ganze Höhlung desselben ausfüllte. Ferner müste man das Gewicht, welches in die Höhe getrieben werden soll, in Gestalt eines Propfs verfertigen, daß dasselbe mit dem untern Ende genau in den Cylinder hinein paßte, und biß auf das Pulver hinein gesteckt werden könnte. Auf diese Art würde nicht nur die Gewalt des Pulvers im ersten Augenblick auf dieses Gewicht würken, sondern auch so lange fortdauren, biß dasselbe gänzlich aus dem Cylinder heraus getrieben worden. Und da dieses Gewicht in Ansehung der geringen Quantität Pulver, welche zu der Probe gebraucht wird, sehr groß ist, und folglich in keine so schnelle Bewegung gesetzt werden kann: so kann gnug seyn, wenn nur das unterste Ende sehr kurz in Gestalt eines Propfs formirt wird, indem, ehe dasselbe aus dem unteren Cylinder heraus getrieben wird, schon so viel Zeit vorbey geht, daß sich inzwischen alles Pulver, oder doch zum wenigsten nach Proportion so viel, als in Canonen zu geschehen pflegt, entzün kann.