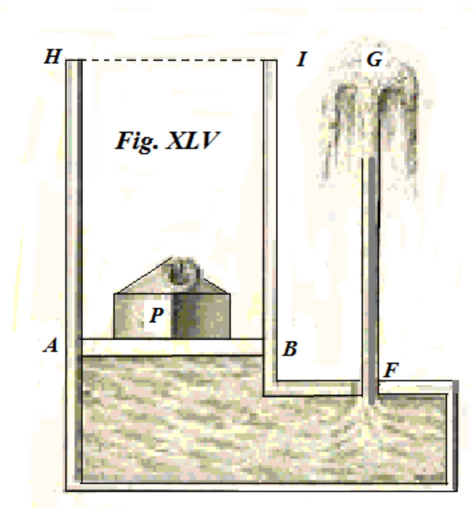
HYDRODYNAMICS SECTION NINE

Concerning the motion of fluids which are not ejected by their own weight but by certain other forces, and which concern hydraulic machines, especially where the highest degree of perfection of the same can be given, and how they can be perfected further both by the mechanics of solids as well as of fluids.

§. 1. In this section, where hydraulic machines are to be examined, I have decided mainly to perfect the use of these, as far as it can be done ; we will disregard the variations of the motions which originate from the forces or inertia of the internal part of the fluid because as we have seen, the internal motion of the water is generally so much different from the first flow, if the opening shall be small, as it is with most hydraulic machines by reason of the internal cross-sections. Indeed the business would be laughable in practical matters, to be concerned with the changes which happen with the first moments of the flow, and which now we have determined in section four, because there a need was satisfied so that the strength of the whole theory could be shown. Therefore during the whole motion, for the sake of brevity, we may consider the water to be expelled with a constant velocity, which itself may be considered as the root of a pressing internal force, after this force has been reduced to the weight of a water cylinder lying above the opening: for whatever this force should be, the weight of a vertical cylinder of water resting above the surface of the internal water will be required to be considered, and the height of this cylinder itself will give the height corresponding to the velocity of the water leaping out, but only if no external obstacles shall be present, and water from the widest vessel may be ejected. Thus it is required to understood this, that if the cover *AB* with the weight *P* pressing (Fig. 45) expels the water through the opening *F*, moreover the weight *P* shall be equal to the weight of a cylinder of water *HABI*, and then the jet of water *FG* must reach the height *HI*.



Definitions.

§. 2. By *moving force* I shall understand henceforth that principle acting, which agrees with the weight in pressing down in an animated way [*i.e.* related to the *living force* of Leibniz, capable of moving with a surface and so changing mechanical work into kinetic energy], and with the other dead forces of this kind [*i.e.* action-reaction type forces], as they are accustomed to be called.

Moreover the product which arises from the multiplication of this *moving force* by the velocity of the same during equal times in which it may exert its pressing force, I will designate by the *absolute potential* [*i.e*. the *work done* in a certain time]. Or because the product from the velocity and the proportional time is simply the interval traveled through, it will be agreed also to deduce the *absolute potential* from the *moving force* multiplied by the distance which the same has traveled through. Truly thus I call this product the *absolute potential*, because from that finally the labors of all working men are required to be estimated in the raising of the water drained, which I will give soon as shown in the rules which were observed by me in this matter. Meanwhile the hydraulic machines seen by me themselves commonly appeared to be reduced to two kinds, of which the one ejected water with force, while the other quietly transported water from place to place. I will deal with each in order according to its kind, and finally at the end I add something about the different moving forces.
[As with the rest of the presentation up to this stage, it is tentative as to what exactly Bernoulli means by his definitions; questions are resolved by the comparison of ratios as no units as such existed at the time, as many of his concepts rely on quantities involving a number of units. These special phrases in italics we have retained here in translation in italics.]

(A) *Concerning machines projecting water to a height by a force.*

Rule 1.

§. 3. The labors of working men, who are appointed for the raising of water by hydraulic machines, are required to be judged by the *absolute potential* [or, the mechanical work done], that is, from the *moving force* or pressure they exert, from the time and from the velocity of a point, to which the *moving force* is applied.

Demonstration.

(α) It is evident regarding the *moving force* situation [called here the *potentia movens*] : the work, indeed with all else equal, is everywhere proportional to the number of laborers, or to the *moving force.* (β) The situation on account of the time is just as clear with the repetition of all the circumstances, which arises from the doubling of the time. (γ) Finally according to the velocity it may reach, because the business is required to be deduced from that, so that you may double either the moving force, or its velocity, the effect arising will not be different, evidently with each part doubled. Imagine the weight *P* by its descent ejecting water through the opening *F* to the height *FG:* then with all else remaining constant, consider the doubling of the opening *F,* and you see twice the amount of water is going to be ejected to the same height *FG* by the same moving force *P* in the same time, but that weight is falling twice as fast. Equally the amount of water will be doubled with the rest remaining constant, if you double both the opening *F* and the cross-section *AB* as well as the weight or *moving force P*, then truly the velocity of this force doubled remains unchanged. Therefore in each way the effect is doubled. Q. E. D.

Scholium.

§. 4. The preceding proposition is not to be interpreted in a physiological sense but is required to be interpreted morally [*i.e.* the value depends on the effect produced only] : characteristically I estimate the work of the man who exercises twice the force with the same speed, neither more nor less than the one who doubles the speed with the same exertion, because evidently each has produced the same effect ; yet it can happen, that the work of the former, as the effort in the other to be not less hard, in the physiological sense shall be certainly greater. If by a trial anyone can exert a weight of 20 pounds through a distance of 200 *ft.* in a minute, this trial can easily be doubled [to twice the exertion], but truly the velocity with difficulty [to be doubled for the same weight]. From this consequence it is required to be noted especially for all kinds of machinery, how they ought to be composed, so that for the same time with the minimum tiredness of the men the product of these [exertions] from the trials of all the men by the velocity shall be a maximum: and from that it will be apparent, what length ought to be attributed to the levers in windlasses, how great the radius should be made in wheels or of the drum for treadmills, with oars how great a length should be agreed on, and so on for other machines.

Moreover the use in the account of the drums of treadmills, which may be used most frequently, so that a moment of our attention may make matters clearer there, that may be understood by this experiment:

Fig. XLVI

We may consider a vertical height of several thousand [ft.] in Fig. 46, to which height a man must ascend in a given time : but we may assume a time of ten hours, because for such laborers the end is accustomed to be daily, then we may imagine several ways *AC, AD* &c. inclined differently to the horizontal *BD*: With these in place we understand there the progression of the traveler there may be faster, where he chooses a less inclined path, so that in the same time he may reach the height of the mountain *A*, and it is apparent there shall be another way such as *AC,* upon which with the minimum weariness he may finish the journey, since no one either sets out up a vertical plane nor in a given time can finish an infinite distance ; we may put in place this path of the least fatigue to make an angle to the horizontal *ACB* of 30 degrees.

So that if it may be thus, a treadmill will be required to be made, so that the weight may be overcome with the desired velocity, since the person treading will be separated always by thirty degrees from the lowest point of the drum [on the circumference of the drum].

From the same principle also one is to be selected from machines of different kinds : thus e.g. if one may exert a force on a windlass, or a horizontal pressure, which may account for a quarter part of his own weight, and by this force pressing he may accomplish a distance of 200 ft. in a minute, thus as I have thought to become wearied in almost the same way, as if he trod with the same velocity on a rotating treadmill at an angle of 30 degrees; yet meanwhile double the weight in the same time will be carried by the treader to the same height in this manner, because with everything else equal he will exert twice the force [vertically downwards].

[Thus, according to Bernoulli in modern units, an 80 kg person exerts a force of 200N approx. for a time of 60 sec. and travels a distance of some 50 m., and so develops a power of around 200 watts.]

Rule 2.

§. 5. With the same *work done* I say all machines, which allow no friction and which generate no motions to a useless end, perform the same effect nor thus shall one be preferred before another.

Demonstration.

From mechanics it is agreed any composite machine can be reduced to a simple lever : therefore it will be allowed to represent all hydraulic machines by a simple pump fitted with a lever as in Fig. 47, where surely the piston is pushed down with the aid of the lever *MN* moveable about the point *M*, and thus the water is expelled through the opening *F*. But however if the moving force *P* is understood to be applied to the lever at *N*, we see from the preceding proposition nothing is to be gained to increase the work done from the increase or decrease in length of the lever *MN:* and certainly whatever that length shall become, so that the same moving force and moving with the same velocity expels the same amount of water with the same force, but only if the cross-section of the pump *AB* may have a constant ratio to the length of the lever *MN*. From which it is seen, all machines produce the same effect from the same work done, but only if it may be free from friction and with all motions considered useless abstracted according to the required end.

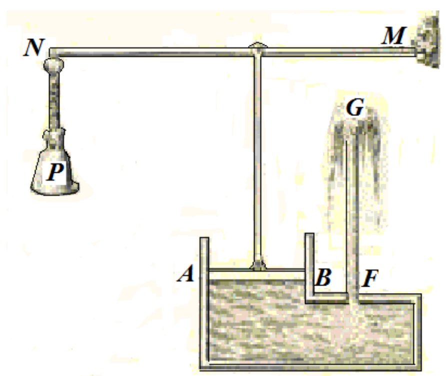


Fig. XLVII

Scholium.

§. 6. People are not lacking who believe a machine can be constructed, with the aid of which with the minimum labour the maximum amount of water shall be able to be raised to any height, and they torture the mind with wheels, levers, and appended weight required to be found [*i.e.* the idea of a perpetual motion machine]: but they waste their effort, nor are they required to listen to those who promise, when they themselves seem to have discovered something great: The best machine is that one, if we may look only at its effect, which tolerates the minimum friction, and which generates no useless motion, concerning which we will examine below the precepts requiring to be avoided by each.

Rule 3.

§. 7. In machines for raising water, such as are shown by Figures 45 & 47, in which the inner surface of the water *AB* is almost at the same height with the opening *F*, the work done in the same times are in the triple [*i.e.* cubic] ratio of the velocity of the water leaping up.

Demonstration.

For the moving forces are in the duplicate [*i.e.* square] ratio of the speeds, with which the water erupts through the opening *F*, and the velocities of the moving forces are following the same ratio of the leaping water: But for the same times the works done are as the moving forces multiplied by their velocities, therefore the proposition is apparent.

[Thus the weight exerted by the piston takes the place of the head of water assumed previously, leading to $P \propto v^2$ and $Pv \propto v^3$. Thus the power of the machine is as the cube of the velocity.]

Scholium.

§. 8. It follows from this rule, if we have in mind that the water passing through the opening *F* shall be able to be raised to the height *FG*, a great part of the work done is to be lost without reward, when the water erupts with a greater force than what may correspond to the height *FG*; for example, in order to make the water be expelled with twice the velocity, an eightfold increase in the amount of work done is required, yet nor on account of the last proposition is the effect agreed to be more than doubled, because evidently twice as much water is being raised in the same time: and this effect could be obtained by a quarter of the work done by expressing the water passing through the opening by simply doubling the velocity; therefore this nominates three quarters of the work expended shall be said to be useless. I have indicated the origin of this detriment in §.5, and that consists of the useless motion which may be generated, according to the final part of the proposition : evidently all the motion which remains with the water being said to be superfluous in our case, after it reaches the height *G*.

[Thus, doubling the speed of the water passing though *F* leads to a four fold increase in the kinetic energy, and a similar increase in the height reached, so that $\frac{3}{4}$ of the work required to be done is wasted.]

Rule 4.

§. 9. When water is expelled through the channel *DF* (Fig. 48 [The original diagram has *AB* labeled at the top of the chamber, as the height fallen is negligible in comparison with *FD*.]) it has a velocity at the opening *F* which must correspond [*i.e.* be owed or due] to the vertical height *GF*, the work expended [*potentia absoluta*] in the same time is proportional to the velocity of the water at *F* multiplied by the height *G* above *AB.*

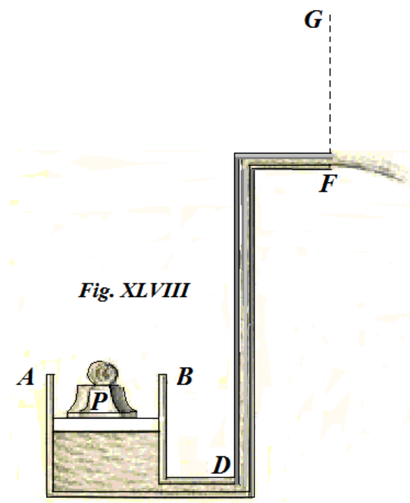


Fig. XLVIII

Demonstration.

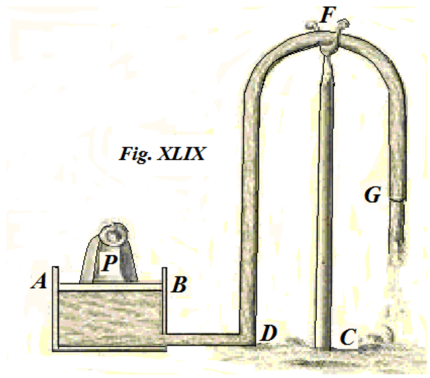
For the moving force *P* is proportional to the before mentioned height and the velocity of this force is as the velocity of the water at *F*.
[As before, the power or the work done in unit time is proportional to the force *P* by the speed or distance *v* moved in unit time, the force *P* in turn is proportional to the whole head of water *DFG*, giving the required result.]

Scholium.

§. 10. The *absolute potentials* [*i.e.* the amounts of work done] increase in a greater ratio than the velocities of the water out flowing, that is, than the amounts ejected in the same times: yet the differences of the ratios is almost undetectable, since the height *FG* certainly is small on account of the height of the channel *FD:* For example let *FG* be equal to $\frac{1}{4}FD$ (by ignoring the height *BD):* however soon the water may be ejected with the velocity doubled, thus, so that now there shall be $FD = FG$*;* thus the *absolute potential* [*i.e.* the works done] shall be as $1\times\frac{5}{4}$ to 2×2 or as 5 ad 16, thus so that for a twofold quantity of water being ejected, a three-fold *absolute potential* may be required. Indeed at first *FG* may be put $=\frac{1}{100}FD$, and then the water again may be able to be expressed with twice the velocity, now the *absolute potentials* shall be as 1×101 to 2×104 or as 101 to 208, which ratio is a little short of a half. Thus it follows, that with the water being drawn off with a smaller velocity, there the *absolute potential* to be expended with a greater reward [*i.e.* the work is done with greater efficiency], and then at last almost all the work is to be expended usefully, when the water flows out through the opening *F* with an almost undetectable velocity: but then the size of the opening will have to compensate for the smallness of the velocity, so that in a given time the amount of water withdrawn shall be notable. Thus the loss of the *absolute potential* may be defined.

Rule 5.

§.11. With the help of the pump *ABDF* set up, with a valve in place at the base and put into water, water flows from the lower place *AD* to the higher *F*, and the mean velocity of the water flowing out from *F* must correspond to the height *FG*, the loss of the *absolute potential* [*i.e.* work done which has been lost] will be to this whole work as *FG* to the height *G* above *AB*.



Demonstration.

We may imagine the small opening to be increased exceedingly with *F* decreased in the same ratio with the velocity of the water flowing out at *F*; thus the amount of water will not be changed in a given time of the outflow, if the velocity of the *moving force* shall be the same, and therefore the effect will be the same. But if the velocity may be diminished thus, so that the height itself shall be unmeasurable, the *moving force* may be expressed by the height *F* above *AB*, since before the *moving force* was equal to the height *G* above *AB;* and thus in each case the velocity of the *moving force* shall be the same, the work done for these same times will be as the height *G* to the height *F* above the common *AB*. Therefore the difference of the heights *G* and *F* will express the loss, since the whole height *G* above *AB* shall represent the whole *work done*.
[Thus, the loss in work is proportional to the difference of the heights : the loss being equal to the kinetic energy acquired by the water falling from *F* to *G*, or to that head of water.]

§.12. The same reasoning prevails for every kind of machines: Evidently as often as the water has a known velocity, carried away to the place to which it is to be raised, a great amount of the *work done* shall be lost: for with the height risen put $= A$, with the height corresponding to the velocity of the water at the place where it flows out $= B$, with the whole work done $= P$, $\frac{B}{A+B} \times P$ will be lost.
[Note: $P \times 1$ represents the work done in moving a unit distance.]

Also it can be observed, when the water is conveyed across a little at the height, at which *F* shall be put, the base must be constructed with the help of the pump inserted in a tube, that may be permitted by the lower tube *DF* being continued some distance towards that, nor being ended abruptly at *F*, just as that is apparent from Fig. 49. Now if, for example, the point *F* shall be put twice as high as the end of the tube *G*, the *absolute potential* required will be two-fold greater for the transfer of the water from the abrupt channel end at *F*, rather than by the continuation as far as *G*; if the water may flow out with a little velocity at both places, evidently the generating height of which shall be small with the ratio of the heights *FD* or *GD*.

Rule 6.

§. 13. Since the covers or rather the pistons *AB* for the pumps which we have considered up to this stage may not correspond very well with the sides of the machines, a gap is left, and from that another kind of loss of the absolute potential arises, which in pumps, in which the height of the opening above the piston can be ignored, may be determined thus. As the sum from the efflux opening and from the aforementioned gap, is to the same gap, thus the *absolute potential*, which is expended, to the part of that which is useless, or lost to the same.

Demonstration.

For the water is pressed equally through the opening and the gap, and it flows out with an equal velocity; but all the *absolute potential* which the water forces through the gap is lost, and this itself is had to the whole *absolute potential,* as the gap to the sum of the openings and the gap.

Scholium.

§. 14. Certainly it is agreed for well made and polished pistons to be used; also it is necessary that the cavity of the pump shall be completely cylindrical, and the walls of the same equally polished. But I would scarcely have believed, unless that were done for another end, to be from that, so that the cavity of the piston could be filled up with complete accuracy, because perhaps thus more losses arise from frictional forces, than if a small gap were left all around: Indeed if that gap were made for example, a hundredth part of the efflux opening, the part from friction will be scarcely more and thence no more than around a one hundredth part is lost from the *absolute potential,* and perhaps from the friction of the piston fitting the cavity exactly more friction arises. Therefore in this respect there is no exceeding concern that we should avoid the passage of water through the gap left by the piston. But this observation does not consider these machines, in which be the retraction of the piston water is drawn into the pump. Indeed here a proper and full size of piston generally is necessary.

Rule 7.

§. 15. In machines which have several openings sending water from one cavity to another, some *absolute potential* is lost, of which matter we have given an account in the previous section, because the *ascent potential* of individual drops is lost by the flow through a common opening from one cavity to another.

Where there are several openings and where the openings of this kind are smaller, there a greater loss of the *absolute potential* arises, which is usually of great effect, and that perhaps as well as the common opinion for the machines which Vitruvius called after the inventor Ctesibius. But I am talking about openings put in place thus, so that all the water flowing out must be transferred through these. Now that kind of such a loss may be set down by calculation.

Let the cross-section of the final opening sending water into the air be $=n$, moreover the cross-sections of the remaining openings, through which the water is trajected within the machine, may be designated by the letters α,β,γ &c. and there will be, on putting the same *moving force* everywhere, the height corresponding to the velocity of the water out flowing to the similar height with no obstructions from the internal holes, as 1 to $1+\frac{nn}{\alpha\alpha}+\frac{nn}{\beta\beta}+\frac{nn}{\gamma\gamma}+\&c.$ (by §.11 Sect. VIII); thence it follows with these heights made equal to each other, the *moving force* to be as $1+\frac{nn}{\alpha\alpha}+\frac{nn}{\beta\beta}+\frac{nn}{\gamma\gamma}+\&c.$ to 1, and because the *moving forces* are the same everywhere, also for the same times a similar ratio will be had for the *work done*. Therefore the part of this $\frac{nn}{\alpha\alpha}+\frac{nn}{\beta\beta}+\frac{nn}{\gamma\gamma}+\&c.$ is superfluous by which the loss of the *work done* will be to that whole work as $\frac{nn}{\alpha\alpha}+\frac{nn}{\beta\beta}+\frac{nn}{\gamma\gamma}+\&c.$ to $1+\frac{nn}{\alpha\alpha}+\frac{nn}{\beta\beta}+\frac{nn}{\gamma\gamma}+\&c.$

[It may be helpful to re-establish this result following KF, in modern notation : initially we have the relations from continuity for the water passing through the openings of cross-section area α,β,γ etc. and the corresponding speeds $V_\alpha,V_\beta,V_\gamma$:

$$\alpha V_\alpha=\beta V_\beta=\gamma V_\gamma=.....=nV_n,$$

and consequently :

$$\alpha^2\frac{V_\alpha^2}{2g}=\beta^2\frac{V_\beta^2}{2g}=\gamma^2\frac{V_\gamma^2}{2g}=.....=n^2\frac{V_n^2}{2g}.$$

From which it follows that :

$$V_\alpha^2=\frac{n^2}{\alpha^2}\cdot V_n^2;\ V_\beta^2=\frac{n^2}{\beta^2}\cdot V_n^2;\ V_\gamma^2=\frac{n^2}{\gamma^2}\cdot V_n^2;\text{etc.}$$

The corresponding heights for a freely falling body are :

$$H_\alpha=\frac{V_\alpha^2}{2g}=\frac{V_n^2}{2g}\cdot\frac{n^2}{\alpha^2};\ H_\beta=\frac{V_\beta^2}{2g}=\frac{V_n^2}{2g}\cdot\frac{n^2}{\beta^2};\ H_\gamma=\frac{V_\gamma^2}{2g}=\frac{V_n^2}{2g}\cdot\frac{n^2}{\gamma^2};\ \text{etc.}$$

Hence, the total height fallen H_{eq} corresponding to the stacked chambers is given by :

$$H_{eq}=H_\alpha+H_\beta+H_\gamma+\text{etc.}=\frac{V_n^2}{2g}\cdot\left[1+\frac{n^2}{\alpha^2}+\frac{n^2}{\beta^2}+\frac{n^2}{\gamma^2}+...\right].$$

If no intermediate internal openings are present, the equivalent height is just for the single final opening *n*, in which case we have for the full exit velocity V' :

$H_{eq} = \frac{V'^2_n}{2g}$. Since these heights represent the same amount of work initially, we see that in the latter case, this total amount of work or potential energy is still available, while in the former case, work has been done internally passing through the openings, and that

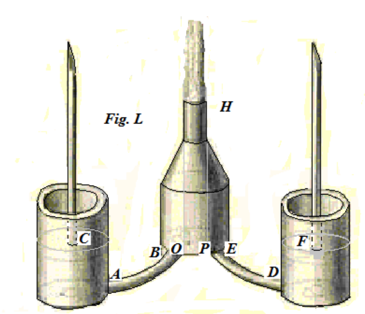
$$V'^2_n = V_n^2 \cdot \left[1 + \frac{n^2}{\alpha^2} + \frac{n^2}{\beta^2} + \frac{n^2}{\gamma^2} + \ldots\right], \text{etc.}$$

Note especially, following Pascal's Theorem, the pressure change due to the actual water or fluid in the pump is considered negligible in these calculations, as the impressed force P on the piston is considered much larger than this force. In addition it may be mentioned the resemblance of these results to the corresponding formulae for resistors in series or in parallel, or in some combination of these: The initial ascent potential corresponds to a constant voltage such as from a battery, the fluid flow corresponds to the electric current, while the resistance corresponding to a simple single opening of cross-section n is given by $\frac{1}{n^2}$; thus, if the Ohm's law equivalent formula may be put in place : $\frac{V}{I} = R$, then the equivalent flow problem has the form $R \propto \frac{1}{n^2}$, two resistors in series gives the corresponding resistance $R \propto \frac{1}{n^2} + \frac{1}{\alpha^2} \propto 1 + \frac{n^2}{\alpha^2}$, and so on as above, while the corresponding chambers in parallel gives the simple sum of the cross-sections. We may also note the Poiseuille's Law has a similar dependence on the area of cross-section.]

Scholium.

§.16. As often as the idea of a machine with holes may be required, through which water from a single container may be transferred to another (which happens in any kind of pump; such as suction or aspirator pumps, from the French *aspirantes,* or pressure pumps, *foulantes,* &c.), these are the holes, as many that the remaining circumstance allow, being made the largest, thus so that the cross-section of the opening of the efflux certainly shall be small with respect of the other interior openings: So that truly the use of the rules may be made clearer, we will consider other machines of no less common use.

Example 1.

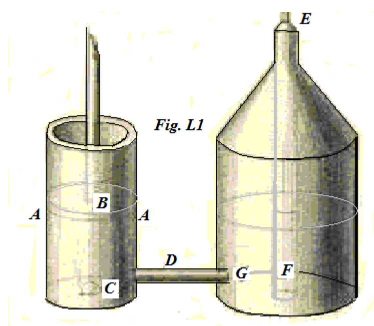


A machine shall be proposed (as Figure 50 shows) in which the pistons C and F are depressed alternately, and by the tubes AB, DE water is introduced into the container BEH, thus so that a jet is made continuously through the opening H. Since here the pistons act alternatively, we will consider either as if acting alone but continually; thus truly the efflux opening H is required to be considered, with the cross-section area n, and each of the openings o, p, with which individually the cross-section shall be a; thus the loss of the *absolute potential* [or work done which is lost] will be $\frac{nn}{\alpha\alpha}$, with the whole work put $=1+\frac{nn}{\alpha\alpha}$, which amounts are as *nn* to $nn+\alpha\alpha$. Certainly this loss is considerable, if it is possible to trust the images of these machines, in which often the openings *o* and *p* shall be small in comparison with the opening of the efflux *H,* because if it should arise, more than half the *work done* may be lost. But the channels *AB* and *DE* drawn through the whole are to be made as large as can be allowed, so that the machine may lose little of its worth.

Moreover this machine has been devised, so that the jet from *H* should be continuous. Because it can happen still, that a certain time interval may intercede between the final point of the elevation of the piston, and with the instant of the first depression of the same, and generally the jet would not be continuous and equal. Truly the author brings forth the best remedy for this inconvenience of that machine, of which Perrault made mention in his *Comment. ad Vitruvium, page* 318, 2nd edition, Paris, and which machine was said to be kept in the Royal Library in Paris; this machine will serve to provide us with another example in place : I will choose the figure together with its description from Perrault.
[C. Perrault: *Les dix livres d'architecture de Viturve*….Paris, 1684.]

Example 2.

" It is a machine" by referring to Perrault's preface, "in which water is expelled from the central container *A* (Fig. 51) by the piston *B* into the main vessel *FG,* from which the air is unable to escape, but only if some water may be present, because the tube *EF* descends almost as far as to the bottom : thus indeed it happens, that water propelled from the container *A* by the tube *D* and occupying the deepest part of the chamber hides the opening of the tube at *F,* and denies passage to the air. Therefore when the piston forces new water into the container, it is filled partially with air and partially with water, this water flowing in from the beginning exerts a force on each fluid, and since the water may not be able to leap out through the tube *FE* with the same velocity by which it has intruded from the pump through the pipe *D,* because evidently (in the words of Perrault) the tube *FE* in its extremity is perforated by a much smaller opening *E*, than is the opening *D* of the tube, the water in the cavity accumulates compressed air, and with the same compressed inversely, it will erupt out through the tube *FE,* even while the piston may be rising."



In this machine a large part of the *work done* is lost in the transition of water by the pipe *D,* and that loss will be greater there, when this tube is narrower: therefore it may be made wider or also several tubes may be constructed transmitting water: but in the present case this annotation is of the greatest concern, because a much greater loss arises from the narrow pipe *D*, than from other machines ; for make the cross-section of this tube the same as it is for the opening *E,* and in addition put the piston to be depressed and raised at equal time intervals : now not only half of the work done shall be lost, as previously, by plainly four fifths will be made useless. However because there are many parts in this machine which require a calculation, it pleases to go through these separately.

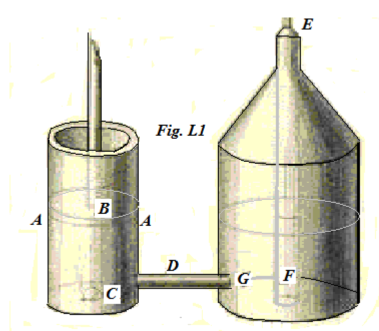
Digression containing some comments on the hydraulic machine that Fig. 51 represents.

(α) The water jet through *E* cannot be entirely equal, during the whole agitation of the piston : For while the piston is being raised, new water cannot be introduced, and thus the amount of water in contained in the air vessel *GE* is diminished, and the air superimposed is dilated and in fact its height is diminished: hence also the water ejected is continually diminished in velocity, then again it is accelerated by the intrusion of the piston.

However if a space is put in place, which the air occupies in the vessel, far greater with the space occupied by the water, which is ejected during a single lifting of the piston, nearly all the inequalities will cease, the piston to be performing uniformly and to have been working for a long time before, because it is necessary according to the latter hypothesis, that the first motions will be very different from the following. Therefore for brevity we will satisfy everything with these hypotheses, that is, we may put everything to be in what is called a state of permanence.

(β) Therefore, with the first agitations of the piston the velocity of the water flowing out through *E* may be increased slowly, soon it comes about that the jet of water may reach a velocity, yet not a whole one ; where with everything put in place, it is apparent only an amount of water to be forced into the vessel by the depression of the piston, as great as that ejected by the agitation of the piston, from the same total amount. But with the first agitations more is thrust in than ejected, and thus not, as Mr. Perrault thought, because the opening at *E* shall be smaller than the other at *G* (for it will succeed likewise even if it may be greater), but because the effecting cause cannot at once exert all its effort in the ejection of the water.

(γ) It may be considered perhaps that the business has not been gone through in a satisfactory manner, as with everything now in place permanently, with no outside obstacles present, the water may leap out from the opening *E* with the velocity, by which it is possible to ascend to the height of a water column placed in equilibrium with the pressure of the piston: and thus certainly it shall be the case, if the pressure of the piston may be present without interruption, and nothing may be lost in the *ascent potential* of the water: because truly in both situations the matter may be had otherwise, it is possible for other considerations to arise in the judgment of the velocity of the jet of water : Hence no one is seen obscurely turning their thoughts towards the consideration of an account of the time, in which the piston is depressed and withdrawn, while also towards an account of the cross-sections in the small channel *D* and in the opening *E*.



(δ) Therefore we may put the time in which the piston is depressed $=\theta$ *;* the time of one complete agitation $=t$, the cross-section of the opening $E=\mu$, and of the tube $D=m$*:* then with the force of the intruding piston compared with the overlying column of water, we may make the height of this column $=a$, truly the height corresponding to the velocity of the leaping water $=x$. Thus from these preparations it is possible to investigate the calculation by two methods, the ratio which there shall become between the velocities of the water at the opening *E* and the tube *D,* and hence the value of the unknown *x* elicited.

For in the first place it is apparent with the time θ (in which one may consider the piston is inserted) just as much water flows through the pipe *D*, as in the time *t* (in which the piston is depressed and withdrawn again) flows out through *E.* Therefore the velocity at *D* to the velocity at *E* is as $\frac{1}{m\theta}$ to $\frac{1}{\mu t}$: and since this latter velocity shall $=\sqrt{x}$, the former will be $=\frac{\mu t}{m\theta}\sqrt{x}$.

[In the time θ a volume of water $mV_D\times\theta$ passes across the area *m* in *D* with a velocity V_D; while in the time *t*, a like volume $\mu V_E\times t$ emerges from *E* with the velocity $V_E=\sqrt{x}$. Hence the result follows.]

In the second place, because the velocity of the water flowing out must be due to the pressure of the air in the vessel, it follows that this pressure be equivalent to a column of water of height *x;* but if from the pressure of the piston you may take the pressure of the air, you will have the pressure, which generated the velocity of the water at *D* ; hence because the difference of the pressures is expressed by $a-x$, the velocity of the water in *D* will be represented by $\sqrt{a-x}$; therefore now the velocity of the water at *D* to the velocity of the water at the opening *E* will be as $\sqrt{a-x}$ to $\sqrt{x}$. With these ratios combined found in each way, there becomes

$$\sqrt{a-x}:\sqrt{x}=\frac{1}{m\theta}:\frac{1}{\mu t},$$

or

$$x=\frac{mm\theta\theta}{mm\theta\theta+\mu\mu tt}\times a .$$

It is apparent from this equation that the height thrown to be deficient from the pressure of the column *a* on two accounts ; evidently it will be deficient more, when the piston is depressed faster [*i.e.* θ is decreased] or raised slower [*i.e. t* is increased], then also when the opening *E* increases in the ratio to the cross-section of the small channel *D*. For example, were the cross-section of this opening equal to the cross-section of the tube *D* and the piston to be depressed and raised with equal speed, there will be produced $x=\frac{1}{5}a$, thus so that the jet flowing up will rise only to a fifth part of the height *a.*

(ε) The loss of the *work done* now can be elicited in this way, first by requiring to put no work into raising the pressed piston. Let the velocity with which the piston is depressed $=v$, and the *work done* in a time of one whole agitation to be expended $=av\theta$ (by the third paragraph); because truly the effect is consistent in that, so the flow made through *E* during the time *t* and the water itself may be raised to a height $x = \frac{mm\theta\theta}{mm\theta\theta + \mu\mu tt} \times a$, that would have been able to be effected by the simple pump of Fig. 44, if for the *pressing force* in that a cylinder of water were assumed of height $\frac{mm\theta\theta}{mm\theta\theta + \mu\mu tt} \times a$, and this force may have acted for a time t with a velocity $\frac{\theta}{t} v$, from which the *work done* in this simple machine, in which nothing is lost would be required to become

$$= \frac{mm\theta\theta}{mm\theta\theta + \mu\mu tt} \times a \times \frac{\theta}{t} v \times t = \frac{mm\theta\theta}{mm\theta\theta + \mu\mu tt} \times av\theta.$$

Therefore the total work done to its useless part shall be as
$av\theta$ to $av\theta - \frac{mm\theta\theta}{mm\theta\theta + \mu\mu tt} \times av\theta$ or as $mm\theta\theta + \mu\mu tt$ to $\mu\mu tt$. Therefore if the whole *work done* may be designated by *P*, its loss will be $= \frac{\mu\mu tt}{mm\theta\theta + \mu\mu tt} \times P$.

Therefore it is necessary with this besides other pumps, that certainly the tube shall be greater in cross-section than the opening *E*, or so that several shall be present. For if one shall be present, and this equal to the cross-section of the opening *E*, and likewise with a uniform velocity may be put to agitate the piston upwards and downwards, a loss of four fifths of the total part shall arise: and if even it becomes larger by doubling, even still half the *work done* is lost.

(ζ) Finally it is evident the walls of the vessel *GE* sustains a smaller pressure, than of the smaller vessel *AA*, naturally these pressures are as *x* to *a*, that is, as $mm\theta\theta + \mu\mu tt$ to $mm\theta\theta$, from which ratio the craftsmen will make a judgment about the firmness of the walls, which is required for each.

Rule 8.

§. 17. When the piston is drawn out in pumps and water flows into the small vessel, not only shall it be moved by its own weight but also for the greater part drawn in by the piston, then all the *work done* in this attraction is expended in the case come upon, because a pump, under water so that it may be filled itself at once if sufficient time for this to be implemented may be conceded ; and thus neither that attraction may be relevant as there with the water being ejected with a certain velocity, since the whole may be avoided, and that is said by me in that expended to be labour in name only.

Because truly a part of the inflow of water shall be proportional to its weight, part also by the raising of the piston, it is not possible for the loss in the *work done* to be estimated from the effect: But rather a calculation thus is required to be put in place, so that the work of putting the piston in a certain place by raising $=\pi$, with the velocity of the piston $=v$, and with the short time *dt* corresponding to the quantities $\pi \,\&\, v$, it may be said that all the *work* expended in lifting of the piston $=\int \pi v dt$ or $=\int \pi dx$, if by *dx* the element of the small distance ran through in the short time *dt* may be understood. Thus it follows, if the attempt by which the piston may be raised shall be with a constant magnitude, as it almost is, the *work done* shall be equal to the *moving force* multiplied by the space traversed : moreover by like reasoning since it may prevail also for the depression of the piston, and likewise the piston is raised just as much as it is depressed, it may appear the works to be expended, both in the drawing in as well as alternately in the water being expelled out, to be everywhere approximately as the *moving force* ; from which a loss arises which is $=\frac{\pi}{\pi+p}\times P$, clearly with the raising force $=\pi$, with the depressing force $=p$ and the *work* expended in the raising and lowering of the piston $=P$.

Otherwise the loss of the work done can be estimated approximately from that, because the generation ought to be considered of all the *ascent potential* of the useless inflow of water into the pump. But if from the same times, or if the piston may be moving up and down with the same speed, the velocity with which the water is being admitted to the velocity with which it is being ejected will be reciprocally as the corresponding openings, and the ascent potentials themselves everywhere will be in the inverse square ratio of the corresponding openings. If then the raising and lowering of the piston happen in different times, the velocities are inversely as the times, and the ascent potentials inversely as the squares of the times . Therefore the *ascent potential* of the influx generated to the ascent potential which arises from the outflow and alone is extended, is composed in the reciprocal square ratio from the ratio of the influx opening to the efflux opening and of the time, in which the water is being drawn in, to the time, in which it is being expelled.

Scholium.

§. 18. From each estimated account it follows that the piston is to be drawn up slowly : for thus the *moving force* shall be small in the account of the first method or the time of rising shall be great in the second account, and thus the laborers in the individual intervals of raising the piston will be repairing the drainage from the endeavour of the preceding depression. Again the latter method indicates the openings, through which the water may be drawn, are required to be wider and to be increased in number; indeed that is in accord with the first method, because thus almost a sufficient amount of water flows in by itself at once, and thus there is less need for a *moving force*.

Rule 9.

§. 19. Finally the water jet surging upwards never is observed to reach that height which should correspond to the initial velocity of the water, that is, if the jet of water may begin to surge upwards from its origin with such a velocity, as may be acquired by a weight falling freely from the height *a*, the fluid will not be able to rise to the whole height *a*, even if you may remove the resistance of the air, or whatever you may wish to think out, because in any case the motion must be retarded. For from the nature of the matter itself by necessity some reduction emerges, this is, on account of its physical nature: Evidently any droplet even if beginning its vertical ascent, is unable still not to be deflected to the side and finally, when it reaches the top, it may be carried by a horizontal motion, which must be noteworthy, because all the water passes through the upper limb or part of the water jet which flowed out from the opening: therefore consider for any droplet from that point of the time when it may be present moving with a horizontal velocity, as it has acquired by a weight falling freely through a height *b* : thus you see the jet cannot rise up beyond the height $a-b$: And with this ascribed the loss arises to the ratio of the total *work done* as *b* to *a.*

Scholium.

§. 20. It was observed amongst all the water ejected with a common velocity from tubes of different forms some rose higher than others : Therefore here it is required to consider the most suitable shapes for the emission of water from the final tubes (*des ajutages*). Concerning this matter Mr. Mariotte has conducted experiments in his *Traité du movement des eaux.*

General Scholium.

§. 21. Until this point we have examined the impediments, which arise in the case of hydraulic machines with the ejection of water with impetus: I consider these to be the precepts that I have set out ; yet there can be others in addition to be thought out, but, as I believe, to be certainly of less concern. Almost everywhere we have given generally geometric measures and in a like manner we have indicated, where it may be possible to go for the greater part against the same impediments. Anyone who reaches out, thinking to be able with the minimum labour or (which I have shown in §. 3 to recede from the same) with the minimum *work done* to some greater effect to be desired in the raising of water, is deceived in belief, and wastes [midnight-] oil and effort. For if from these impediments set out or perhaps from others drawn out from consideration, the most perfect machine in the nature of the matter will be the simple pump of Figure 45, and if water with its help projected to a height may be gathered at *G*, I say it may not be able to happen that with less work the same amount of water may be raised to the same height *FG*.

There is then another kind of machine, which differs from the machines treated up to this stage in that, because while those eject water with impetus, these quietly transfer water without noticeable motion. But with these which can be given the ultimate grade of perfection, it recoils from the same. Moreover for most they are with many obstacles and with these of the greatest degree of inconvenience. Therefore concerning these it will be required for us now to act directly.

(B) *Concerning hydraulic machines in the transportation water without notable force from a lower to a higher place.*

Rule 10.

§. 22. If some weight may be raised through a given vertical height (*a*) by a *moving force* to be variable but applied directly in some manner, and the body shall remain with no motion at the peak of the proposed motion, the same will be constantly expended in the raising of the body by the same *absolute force*, surely equal to that produced from the weight of the body and by the height of the elevation *a*.

Demonstration.

For if the weight, which I will call *A*, will rise through the height *y*, and there may be put in place of the living force, the moving force of the variable *P* directly applied, and to be moving with the velocity *v*, the short time interval will be, in which the weight may be raised by the element *dy*, $=\frac{dy}{v}$, which multiplied by the *moving force P* and of the same velocity *v* gives the element of the work done (by defin. §. 2) $=Pdy$, therefore $\int Pdy$ will give the total *work done*, if after the integration there becomes $y=a$; truly in all increments of the motion the increment of the velocity *dv* is equal to the animated or moving force [evidently the acceleration], which here is $\frac{P-A}{A}$, multiplied into the element of time which now is $\frac{dy}{v}$;

[Here we run into the difficulty of distinguishing between mass and weight : evidently $P-A$ is a force and hence *A* is a weight; while on dividing by *A* the assumption is made that *A* is also a mass, and in which case we no longer have $2g=1$ as before, but instead $g=1$ for the acceleration of gravity. Then we have simply Newton's second law of motion, and $\frac{P-A}{A}=acc.$]

therefore we will have

$$dv=\left(\frac{P-A}{A}\right)\times\frac{dy}{v}$$

or

$$Avdv=Pdy-Ady,$$

that is

$$\tfrac{1}{2}Avv=\int Pdy-Ay,$$

or $$\int Pdy = \tfrac{1}{2}Avv + Ay,$$

[*i.e*. the usual formula for the conservation of kinetic plus potential energy, assuming the *A*'s have differing meanings; the first *A* is the mass and the second *A* is a weight where gravity is taken as 1. Note that in this case the familiar half fraction is present. There was a continual process going on at this time to establish once and for all what was conserved in collisions between particles, elastic and inelastic: by the turn of the century, Thomas Young was using the term 'energy' as an alternative to *vis viva*, to describe that quantity of a body which could be conserved in elastic collisions, or in its motion up or down with gravity: see his *Lectures on Natural Philosophy*, Vol. 1, page 78 and thereabouts.]

where it is required to make $y = a$ & $v = 0$ (by hypoth.) thus so that there shall be $\int Pdy = Aa$.

But because, as we have seen, $\int Pdy$ expresses the whole work expended in raising the weight, this same force will remain the same always, and by name equal to the product from the weight *A* with the altitude *a,* as the proposition had done . Q. E. D.

Corollary.

§. 23. From our demonstration it is apparent, the *work done* also to be the same, as often as the velocity at the summit is the same, that is, as often as the height to which the body is able to ascend with its residual velocity, evidently $\tfrac{1}{2}vv$ is constant: and if this corresponding height may be called *b,* the *work done* will be $= A(a+b)$. Therefore now it is apparent, how great a part of the work may be lost, when the moving force shall be the weight *A* to be lifted to a height *a*, and likewise at the summit the velocity to be had remaining corresponds to the height *b;* clearly the loss of the work done to the whole work shall be as *b* to $b+a$.
[Thus the point is being made, that to lift a body through a certain height *a*, a slightly greater force is required than the weight to be raised, and so some residual kinetic energy remains at the summit *a*, which corresponds to an extra height *b*.]

Scholium l.

§. 24. And thus it is required to beware, lest machines thus shall be constructed, so that water and may be carried to the required place with a violent motion. But a little of this kind is accustomed to be lost in most machines.

Scholium 2.

§. 25. All things themselves are had likewise if the body may not be raised vertically, but upon some inclined plane, or even by a curve of some kind: for always the total *work done* [or the *absolute potential*] will be equal to $A(a+b)$, that is, by the product from the weight into the increase in height augmented by the height corresponding to the velocity of the body remaining at the summit, the demonstration of which I shall pass over, because it differs little from the previous demonstration.

General Scholium.

§. 26. Because the effect of all machines of whatever construction can be reduced to the nature of an inclined plane, it is evident all machines, if from frictions and from these losses in *work done*, which we have reviewed up to the present, we may have thought to remove, with the same reduced, because the work done simply depends on the height to which the body is required to be raised by the weight of the same. The *work done* [*potentia absoluta*] has this property in common with the *living force* [*vis viva*] or with the *actual ascent* or *descent*. And this is the ultimate level of the perfect machine, which is not possible to be transgressed, nor indeed even able to be reached; for always with all the frictions and losses removed the same moving force would be able to raise a greater weight to the same height. So that now it should be able to put a certain comparison in place about the deficiencies of machines, both of these which project water to a desired height, as well as those which just transport the same, now of these of the latter kind we will indicate also the most remarkable failures.

(I) There are frictional forces from many obstacles in most machines of this kind, so that they alone may absorb the maximum part of the work, especially moreover when they raise water passing through square paddles or oval bowls, connected to a chain returning in a circle, through a channel to which they are adapted.
[Diagram taken from the Frontispiece to the *Hydrodynamicae*]

(II) And most machines, especially truly which we have just mentioned, again are usually designated by the name of pertaining to rose gardens, these are prepared thus, so that while the water is raised continually a part of that may be sprinkled, clearly it may fall either at the place from which it was drawn or perhaps from some higher place to a lower place, as in rose gardens; if in these the rounded or square paddles are well adjusted to the channel, the friction may become almost insurmountable, but if it were less, the greatest amount of water left drips through the gaps from the above divisions to the lower ones, thus so that the minimum part of the water may remain in these, when they have reached to top, of the amount which they have received from the whole passage. And thus it may be seen, even by this name alone, these machines certainly are to be disapproved, especially if indeed pure water shall be required to be raised, which could be raised by pumps.

(III) The machines too are accustomed to be of this kind, so that they raise water below the proposed height: But the force which corresponds is lost, and if the water is required to be carried across the structure, as I have indicated in §.12, that may prevail with difficulty .

(IV) And there are machines, which do not allow the direct application of the moving force, from which obliquity again some loss arises.

§. 27. These are almost all the obstacles, which were considered by me to be of notable concern; but I do not know either from these to what extent it shall stand in the way, just as with the first kind of machines we have explained : mechanics have got to know certain tricks for diminishing the friction: I would have preferred machines which raised the water in buckets rather than by the rose-garden kind : but buckets thus shall be made, only if it can happen, they may be filled at once that in the lowest place and they may discharge nothing before arriving at the highest place. When water is required to be carried across from a higher to a lower place, work is required to be given, so that the force of the water working may move the drums or wheels forwards in a rotary action, although much may be lacking, so that thus all the useful work may be expended usefully, exactly as happened with the pump we have shown in Fig. 49 (§. 12). The principle of the action may consist, if I judge correctly, to be most suitable for treadmills : for men are best suited for this work ; this is relevant, as I have pointed out in §.4 on the occasion of the first rule about the angle of the slope, under which the traveler in a given time, with the minimum tiredness, shall be able to reach a certain vertical height. I might suppose a man of medium height, healthy and robust, advancing upon a way inclined at 30 degrees, to be going to achieve 3600 feet without difficulty in a single hour, and therefore to be raising the weight of his body to a vertical height of 1800 ft., which I may put to be 144 pounds, or two cubic feet of water. Therefore such a man with the aid of a treadmill being turned around, and of the most perfect construction (in which clearly no *work done* may be lost) raises two cubic feet of water every hour to a vertical height of 1800 feet, or what is the same, every second one cubic foot to a height of one foot : machines which are of much smaller effect, with the workers performing the task, I consider have little to commend them : Meanwhile with an experiment performed in the household of the most illustrious *General* de Coulon with pumps, which I shall put in place at the end of the section, I have tested the effect to be not much less, from which I have been confirmed in my opinion that the operators of treadmills to be better for the most part : but I can foresee a lesser effect produced with machines composed exceedingly long, because with these the greatest part of the *work done* is expended without use. Of this matter I may present a notable example from the very well-known Marly machine, to be shown as incredible the loss arising of almost all the *work done* from all the impediments brought together. [There is a discussion plus a painting of this machine in Wikipedia.]

The tract Weidler produced : *Tractatus de machinis hydraulicis….* in which he makes a full description of the Marly machine, and it concerns all the water to be raised by the motion of 14 wheels, of which the blades are propelled by the force of the Seine : hence the force is made for all the wheels equal to a weight of 1000594 pounds, and this is what we have designated by the name *moving force.* Moreover the paddles to be carried in some motion to be gathered from some circumstances, where they complete $3\frac{3}{4}$ feet per second, and this velocity being taken for the velocity of the *moving force ;* then he adds, in individual days to have raised by the force of that machine 11,700,000 pounds of water to a height of 500 *ft.* Thus with these in place we may consider now the simplest machine in Fig. 45, where nothing concerning the *work done* may be understood to be lost, with such a size for the effective force *P,* equally with the velocity so that $3\frac{3}{4}$ is required for the motion. But the height will be $FG = 500$ *ft.* and because in the time of 24 hours 11,700,000 pounds must be ejected through the opening *F*, that is, 162,500 *cubic ft.*, the magnitude of this hole required to be put in place will be $= 0,0108$ parts of one square foot: The velocity of the water at *F* is so much, that it may release 173 *ft.* in a single second. Therefore the velocity $3\frac{3}{4}$ is contained 46 times, which the weight *P* it assumed to have, and the cross-section of the pump *AB* must increase just as many times with the cross-section of the hole *F:* Therefore the cross-section *AB* will require to be considered 0, 4968 *parts of a sq.ft.*, from which it follows, the weight *P* is going to be equal to the weight of a cylinder of water on the base *AB* constructed to a height 500 *ft.*, or to a weight of 248,4 *cubic ft.* of water, that is, to a weight of 17,885 pounds, which only a fifty sixth part effects the *moving force*, as Weidler has shown to be applied to the water moved with the same velocity. Therefore in the whole machine thus the loss shall become as equal to $\frac{55}{56}$ of the whole work done [*potentia absoluta*].

Thus after we have examined the nature of hydraulic machines, so much as that can be done in general, not without reason a somewhat special example will be examined more accurately, and because the screw of Archimedes is endowed with many outstanding properties, which no one has uncovered well enough, as far as I know, from this example I may choose and that therefore more willingly, because there shall be many, contrary to our rules, who think the single virtue of the screw present to be for the raising a great amount of water in the shortest time and by a small force : but they are mistaken who think thus: for if no account of the accidental obstacles may be had, the same performs the same *work*, as all other machines do.

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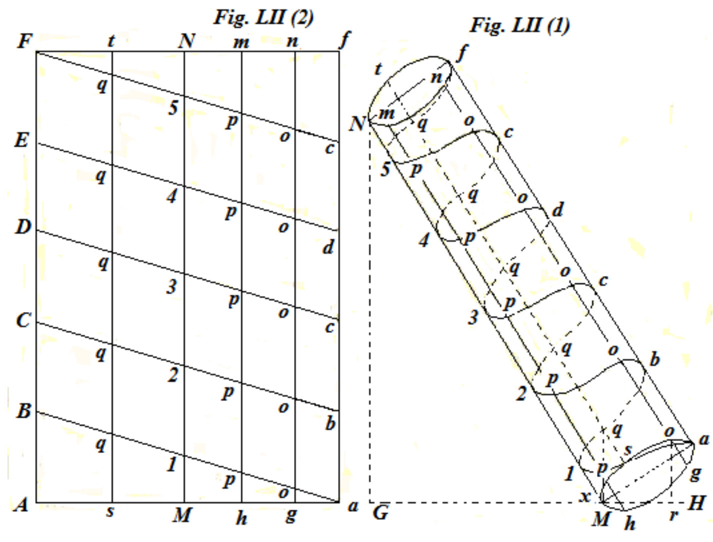
A special dissertation on the Screw of Archimedes.

(I) There are various authors, who have taught the way this screw is required to be constructed : the most useful returns to this point, that a certain channel or several are turned around the surface of a cylinder, and indeed thus so that the channel everywhere may have the same inclination on account of the axis of the cylinder, as Vitruvius beyond necessity ordered in all screws to be at a semi-right angle. Therefore before everything else, so that a spiral line may be drawn on the surface of the cylinder to the normal of which the channel shall be required to be put in place, that which most easily by my judgment certainly can arise on a highly polished surface (especially since spirals may be some distance from each other) by turning a thin rope around the same a number of times: for this stretched out by itself shall make the desired line [on the outer surface], for neither can the spiral be similar to itself everywhere, nor can it have a constant inclination to the axis, unless the arc intercepted between two points shall be the same for all the arcs having minimum end points, as the nature of which is clearly to agree with the extended rope: truly if friction shall arise from an impediment, the string will be required to be extended to smaller intervals. But it is not why we should be most careful in explaining the matter in several of the most easy ways.

In the first place, the law of the spiral is, that everywhere it will be inclined equally to the axis of the cylinder, on which law the following construction depends, as I have put to be discussed in the example below.

Imagine the right cylinder *MafN* (Fig. 52, (1)), on the surface of which the spiral



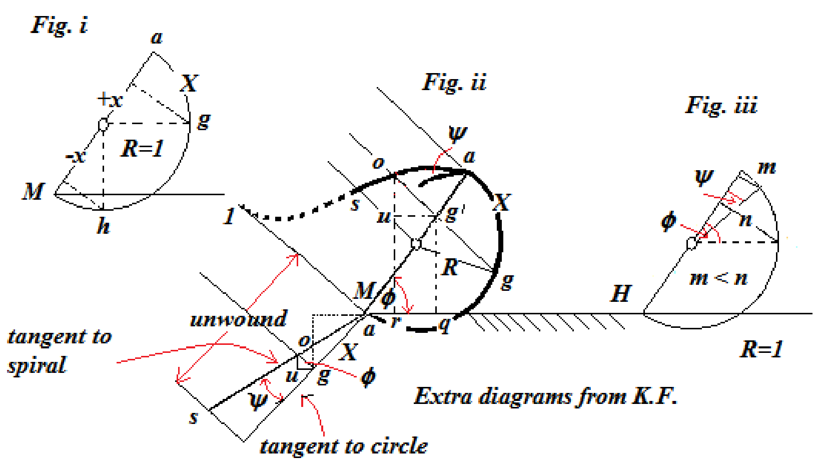
a1b2c3d &c. shall be required to be inscribed; and consider the same surface unfolded into the plane figure of the rectangular parallelogram *AafF* (Fig. 52, (2)); here there is taken from one part *AB, BC, CD, DE* & *EF,* from the other *ab, bc, cd, de* & *ef,* the individual parts equal to the individual parts; the points *B, C, D, E* & *F* are joined by the right lines *a, b, c, d* & *e:* thus with these made, if the plane surface again may be rolled into a cylinder, with the lines *AF* & *af* joined, and with the points *A* & *a, B* & *b* &c. coinciding, it arises that the lines *aB, bC, cD* &c. on the cylindrical surface form a continuous line, which itself will be the desired spiral. Towards aiding understanding I have distinguished the homologous points in each figure with common letters.

(II) Now the cylinder *MafN* (Fig. 52, (1)) will have been proposed having the channel described bent around according to the manner of the spiral, the diameter of which we will consider as infinitely small in the ratio of the diameter pertaining to the cylinder: and thus the screw of Archimedes will be had, so that if we wished to use for the raising of water from *M* to *N*, the cylinder will be required to be inclined to the horizontal, and thus indeed so that the angle *aMH* (intercepted between the base diameter *Ma,* which is in the vertical plane, and the horizontal *MH*) shall be greater than the angle *sao*, which the tangents of the circle and of the spiral make at the common point *a.* Then with the cylinder turned about its axis in the direction *aghMs* water flows in through the lower opening of the long drawn out channel and flowing out through the upper opening. [*i.e.* clockwise looking in from the base end.]

(III) So that we may understand correctly the nature of the elevation, three points present themselves to us requiring to be examined in any winding of the spiral, namely the points *o*, *p* & *q*, of which the first *o* stands furthest from the horizontal, the other *p* is nearest to the same [horizontal], and *q* has been placed at the same height approximately as the point *o* in the winding taken below [*i.e. op* is approx. horizontal; *s* and the *q*'s are the inflection point]: through the individual points *o* the right line *gn* has been drawn, through the points *p* the line *hm*, and through the points *q* the right line *st.* Truly in the following we well determine the position of these lines.

(IV) Let the radius, which pertains to the base of the cylinder, $=1$ and 1 is taken for the whole sine; the sine of the angle $sao = m = \sin\psi$, and the cosine of the same $= M[=\cos\psi]$, the sine of the angle $aMH = n[=\sin\phi]$, and the cosine of the same $= N[=\cos\phi]$; the arc $ag = X$; the cosine of that arc, $[\cos X = x]$, the perpendicular sent from *o* to the horizontal [level of the water], surely will be

$$or = \frac{mNX}{M} + n(1+x) = [\frac{\sin\psi\cos\phi X}{\cos\psi} + \sin\phi(1+x) = X\tan\psi\cos\phi + \sin\phi(1+x)].$$



[Following K.F. P.97 : Initially we may note two important angles, the first angle ϕ is that of the inclination of the cylinder to the vertical: in the extra diagrams $\phi = 0$ gives a vertical cylinder : essentially the complement of the angle of the inclined plane of the axis of the cylinder to the horizontal; the second angle ψ is the reduced plane inclination angle introduced by winding a spiral round the surface of the cylinder, the pitch of which, essentially leading to a longer slope for the same rise in height; this is the angle ψ for the equivalent unwound plane, and as Bernoulli has remarked, it is given by the angle between the tangents of the base circle and the winding at the start, and remains constant. In the action of the screw, any point on the generating circle moves at a constant rate around the central axis, while progressing along the axis at a constant rate, thus tracing out the spiral. Half the generating circle is shown above at some position in its rotation, where a small part of the cylinder at the bottom is below the horizontal level of the water.

We have : from the semicircle drawn in the plane infolded in *Fig. ii*, where initially the spiral and the semi-circle are almost coplanar: $R\cos arcX = R\cdot x$, as in *Fig. i* ; from which there arises $Mg' = R(1+x)$; and

$$g'q = ur = R(1+x)\sin\phi = R(1+x)\sin(aMH) = R(1+x)n.$$

In addition, the angle $sao = \psi$ represents the slope of the inclined plane followed by the helical winding around the cylinder, and X is equal to the stretched out length of the equivalent winding curve between *a* and *g*, as in the lower left part of *Fig. ii* :

$$X \cdot \tan\psi = X \cdot \frac{\sin(sao)}{\cos(sao)} = X \cdot \frac{m}{M} = og'.$$

This results in $ou = og \cdot \cos\varphi = og \cdot \cos(aMH) = og \cdot N = X \cdot \frac{m}{M} \cdot N$ and finally, the height of the highest point above the surface of the water $or = ou + ur = \frac{m \cdot N \cdot X}{M} + R(1+x)n,$ or with $R = 1,$ $or = \frac{m \cdot N \cdot X}{M} + n(1+x).$ Note : these triangles are not all coplanar.]

Truly because *or* is a maximum, there shall be

$$\frac{m \cdot N \cdot dX}{M} + ndx = 0,$$

and since from the nature of the circle there shall be $dX = \frac{-dx}{\sqrt{1-xx}}$,
[*i.e.* $\cos X = x$ then $\sin XdX = dx$ *etc.*] there will be

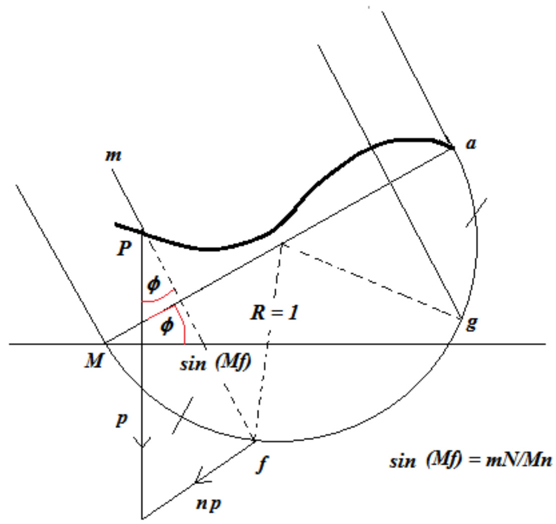
$$\frac{-mNdx}{M\sqrt{1-xx}} + ndx = 0,$$

therefore $$\sqrt{1-xx} = \frac{mN}{Mn}.$$

Therefore the sine of the angle sought $ag = \frac{mN}{Mn}$ or the cosine $x = \pm\frac{\sqrt{nn - mm}}{Mn}$: the upper sign gives the arc *ag*, the lower gives the arc *ah* determining the lowest point *p*.

And thus we have determined both the upper point *o*, as well as the lower *p*, and it is evident the arcs *Mh* and *ag* to be equal to each other, but likewise from the irrational affected quantity $\sqrt{nn - mm}$ the value of the letter *x* cannot be deduced, as *m* shall be greater than *n*: nor indeed in this case is the lower point given, because the whole of the spiral continually ascends everywhere: Nor also thus is this screw serviceable in raising water; from which the reasoning now is apparent, which I have warned about in the second article of this digression, concerning which for the required angle *aMH* to be in excess of the above angle *sao.*

(V) Now we may put a ball to be somewhere onside the channel, and the screw to be firmly in place: thus the ball will be minimally at rest, but unable to remain at some point *P*. Because if indeed it cannot be retained by the screw, the ball will descend, and in the descent it will rotate around the screw, and if further it may be imagined, the weight of the screw to be negligible and the motion of the ball to be made freely with no frictional obstacles, the ball will descend along the right line *mh* [Fig. LII] by no other law, then the ball is descending freely on an inclined plane. And thus it is apparent a force is required to impede the ball descending, and towards keeping the screw firmly in place. We may consider that force to be applied at the point *f* in the plane of the circle and perpendicularly to the radius to be sought in the ratio, that it may have to the weight of the ball at some point of rest *P*.



Let the weight of the ball $= p$: because indeed the action of the ball is vertical, it will be required to be resolved into two other parts standing perpendicular to each other, of which one shall have a common direction with the direction of the screw, with the other perpendicular to the axis of the same; the former to be rejected will contribute nothing to the rotation around the spiral, and the latter only will be required to be considered ; truly by that action the amount left is $= np$ and it acts on the lever, which is = to the sine of the arc *ag*, and here the sine (by *art.* IV) is $= \frac{mN}{Mn}$ [The clockwise torque acting into the plane at *f*]. Therefore the moment of the action is $= \frac{m \cdot N}{M \cdot n} \times n \cdot p = \frac{m \cdot N \cdot p}{M}$; if this is divided by the radius of the base, which with the relevant lever for the force applied at f put in equilibrium with the force of the ball, you will have that force sought $= \frac{mNp}{M}$. Thus such is allowed to be deduced from the nature of the lever directly, which others are accustomed to desire from a different principle. With these premises advanced we will now begin to consider the use of the machine, which it has for the raising of water.

Problem.

(VI) The maximum amount of water is sought which the screw can eject in some revolution.

Solution.

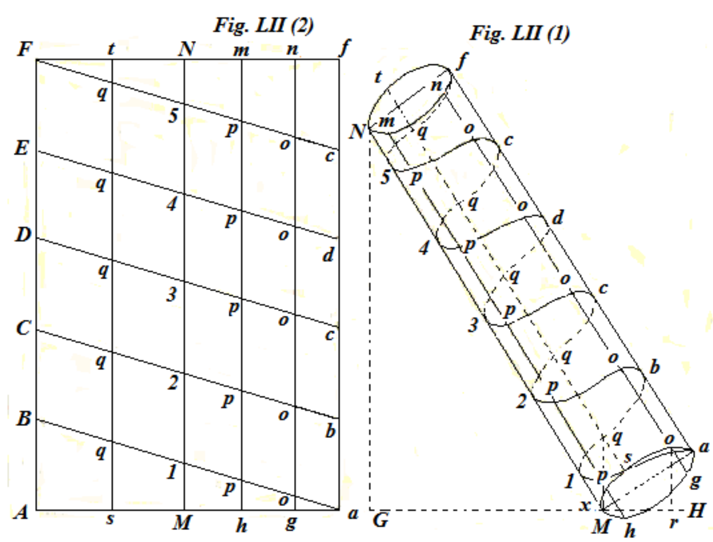
We will consider the whole spiral *alb*, and the amount of water which it may contain full shall be $= q$ *:* But it must be observed that the helix cannot be completely full of water, for if the channel were completely full, water would flow out of the lower opening, therefore any branch, such as *alb*, is partially filled with air and partially with water ; moreover there will be the one extremity at *o* or the highest point, but the other at *q* or the point at the level proposed initially: therefore the part filled with water is *opq*, and if this part may be put to the whole length of the spiral *alb* as *g* to *h*, the maximum amount of water requiring to be ejected in a single revolution will be $= \frac{gq}{h}$. Q. E. I.

Scholium 1.

(VII) Because, as we have said, it cannot happen that water shall be drawn together through the whole channel, it is required to be warned, lest it may be impeded by the separation of water, which can happen easily when the whole base of the cylinder is immersed in water, because thus the entry of air through the lower opening of the channel may be forbidden: Nor is it required to be made, so that an excessive part of the base may be jutting out of the water, so that thus not all the screw has drawn up water, as otherwise may be able in a single revolution ; indeed nothing will be drawn up, if the immersion does not reach the point *h* : But the immersion must happen as far as to the point *g*, because thus the arc of the spiral *opq*, which prevails to retain the water, shall become a maximum. For even if at no time had I caused any danger to the thing, and most other authors may have considered to talk otherwise about that, I may still prefer to reason, that the authority of these is to be believed, who have not turned their mind to the immersion. Therefore on account of the *rule of immersion* this will be observed, truly the base will be submerged, while the chord of the arc jutting out of the water shall be $=\frac{2mN}{Mn}$, where the letters m, N, M & n indicate the same, as in the fourth article.

Scholium 2.

(VIII) Indeed it is apparent after a light consideration of the matter there is a greater ratio between the arc of the helix *opq* and the whole helix *alb*, that is, between *g* and *h*, and hence there a greater amount of water to be ejected in individual revolutions with all else equal, when the angle *sao* is less, and when the angle *aMH* is greater, or when the distance between two nearby windings is less and where the screw may be inclined more towards the horizontal: But truly that ratio cannot be expressed algebraically :Yet in any



particular case that can easily be obtained approximately.

I will choose an example of the preceding rule with the screw, such as Vitruvius used and taught how to construct. Moreover the angle *sao* will be made half a right angle and thus $\sin sao = m = \cos sao = M = \sqrt{\tfrac{1}{2}} = 0,70710$: then the ratio may be put in place between *NG* and *MG*, which is as 3 to 4; hence it is deduced that *GNM* or $aMH = 53°,\ 8'$, of which the sine $n = 0,80000$ and the cosine $N = 0,60000$: therefore (by *art.* III) the sine of the arc *ag* of the defining highest point $o = \dfrac{mN}{Mn} = \dfrac{3}{4}$, and the arc itself $ag = 48°,35'$. And thus by the strength of the rule *art.* VII, the arc emerging beyond the water must be $97°,10'$; and the immersed arc $262°,50'$.

Now so that in addition we may define the ratio between the arc of the winding *opq* and the whole winding *alb*, it is to be observed, that ratio to be the same, which lies between the arc of the circle *ghMs* and the circumference of the circle, which is seen from the associated figure. But the arc *ghMs* may now be determined in this way. Clearly there is the $\text{arc}.ghMs = \text{arc}.aghMs - \text{arc}.ag$. But we have seen in the third article, if from any point of the spiral, such as *o* and *q*, the perpendiculars may be dropped to the grazing horizontal point *M*, such as are *or* and *qx*, this perpendicular to be $= \dfrac{mN}{M} + n(1+x)$: or in our case $= 0,60000X + 0,80000(1+x)$, with *X* denoting the circular arc, assuming to correspond to the point on the spiral, clearly the arc *ag* or the arc. *aghMs*, and with *x* indicating the cosine of the same arc. Truly there is : arc. $ag = 48°,35'$ $=$ (because the radius is expressed by unity) 0,84797, the cosine of which $= 0,66153$: Therefore in our case there becomes $or = 0,50878 + 1,32922 = 1,83800$. Again because the points *o* and *q* have been placed at the same height, and the lines *or* and *qx* are equal to each other, it is apparent the question now can be reduced to that, so that another arc *aghMs* may be found corresponding to the point *q*, which if it may be called *X*, the cosine of which is *x*, there shall be $0,60000X + 0,80000(1+x) = or = 1,83800$: for this condition the arc *aghMs* is found to be approx. $175\tfrac{1}{2}$ degrees, with the point *s* lying in the region *agM:* And since the arc *ag* was $48°,35'$, finally the arc *ghMs* will be $126°,55'$, which therefore will be to the circumference of the circle approximately as 10 to 29: and a like ratio lies between the arc of the winding *opq* and the whole winding.

Therefore it follows, in single revolutions the screw described by Vitruvius ejected approximately $\tfrac{10}{29}$ of that amount, that the whole spiral contained filled with water, or a little beyond a third.

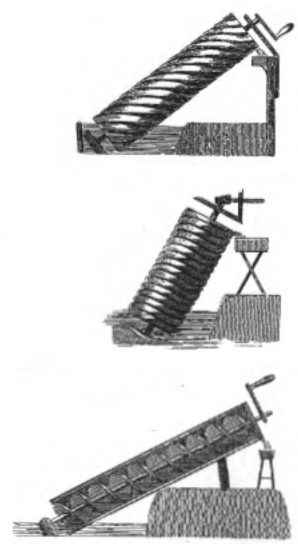
Scholium 3.

(IX) It is required to be observed still, whatever shall be the amount of water, which enters the lower channel of some rotating screw, and flows out from the same above, nothing, neither gain nor loss, falls on the *work done*, if no account may be had of friction, because the *moving force* is proportional to that quantity, with all else being equal. But truly because friction always obstructs, and is almost the same on account of the weight of the machine itself, whether a greater of lesser amount of water may be raised, the work is required to be given everywhere, so that this amount with all else the same may become a maximum: Concerning this matter I may act with a little more skill.

Scholium 4.

(X) Now above I have indicated, the ratio of the arc *ghMs* to increase to the circumference of the circle with the angle *sao* decreased [*i.e.* the pitch angle ϕ of the screw] and *NMG* [*i.e.* the angle ψ of the inclined plane]*:* therefore each shall be required to be constructed as small as possible, unless other inconveniences stand in the way, especially on account of the angle *NMG.* Which concerning the angle *sao*, that can be diminished almost as you wish, nor may hence another inconvenience result, unless because the sides of the channel being circumscribed are able to approach each other too much: On the contrary from the diminution of this angle another benefit may be obtained, namely because then it will be able therefore to raise the machine more vertically and the water itself therefore to be raised higher, and indeed the angle *aMH* must always be greater than the angle *sao:* moreover from the more vertical position of the screw likewise it may be found, that the machine shall be less inconvenient on account of its own weight and that may be supported more easily.

Thus considering these, I might believe it possible for an angle of around 5 degrees to be sufficient, which the channel makes with the base of the central part. Also Cardan [*De subtilitate* Lib. XXI] himself makes a smaller angle than that of Vitruvius [*De architectura*, Lib. X.6], and since there are fewer channels able to be wound around the same centre, when they have been put in place more obliquely, Vitruvius puts eight in place, Cardan putting in place only three : but the channels are longer in Cardan's spiral, thus so that it may agree with the length, what it falls short with the number of channels. With the account of the other angle *NMG* it deserves to be observed, the water can be raised higher, when that angle shall be made greater, but on the other hand a smaller amount of water is ejected in the individual revolutions. Perhaps a just mean position may be held, which will make the angle 60 degrees.



Adapted from Thomas Young Lectures on Natural Philosophy, Royal Institution

(XI) Now we will transfer also the calculation of our construction of this spiral to the standard of the last section, just as we have done regarding the precepts for the construction of the screw of Vitruvius, *art.* VIII. Because truly by the hypothesis the angle *sao* is 5^{o} and the angle $NMG = 60°$, by *art*. IV the arc *ag* will be found to be

$8^{o},43'$, and the vertical line $or = 1{,}00574$, to which the other vertical *qx* will be equal, if $284°,57'$ will be given to the arc *aghMs*, from which if the arc *ag* may be subtracted, the arc will remain $ghMs = 276°,14'$: which will correspond to the arc of the winding with the strength to retain an amount of water : therefore this part is to the whole winding as 16574 to 21600 or as 8287 to 10800, thus so that with the individual revolutions more than a four fifths part of the whole capacity shall be able to be ejected, and around two and a third times may be effected by this machine, than will be obtained by a similar machine made according to the plan of Vitruvius : also with the same centre water will be raised higher in the ratio as $\sqrt{3}$ to $\sqrt{2}$. Now I come to the *moving force* as well as the *work done*, which has to be expended in the raising water.

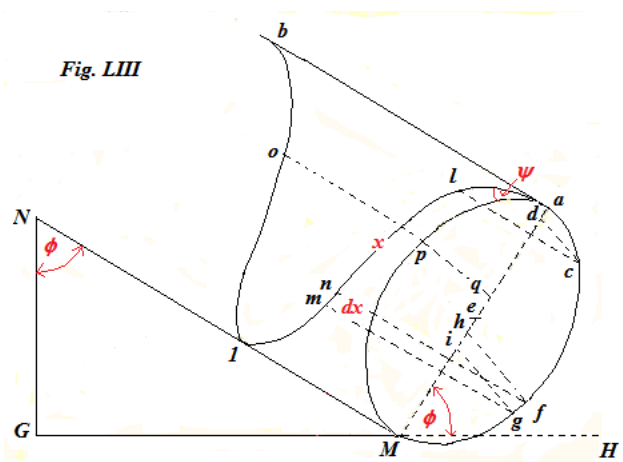
Problem.

(XII) With a given weight of water resting in a winding, to find the tangential force at f put in equilibrium with that weight.

Solution.

We have seen how this problem shall be solved geometrically in the account of a ball resting at the lowest point *p*. Indeed in the present case the matter itself is had a little otherwise, because the weight of water is distributed in the great arc of a winding, and not concentrated at some given point. Indeed it is easy to foresee in the preceding, in each case the forces to be the same as from the indirect rules of mechanics ; yet it pleases to give the desired demonstration of this matter from the nature of the lever, because mechanically minded people love to reduce everything to that.

We will consider the winding *alb* chosen from the fifty second figure, to avoid a



confusion of lines, with the denominations of *art.* IV remaining used.

Therefore in Figure 53 thus again the angle *NMG* will be the angle which the centre makes with the horizontal, the sine of which $= N$, [*i.e.* $\cos\phi = N$] and the sine of the angle $aMH = n$; [*i.e.* $\sin\phi = n$]; *alb* is a single revolution of the spiral: the base of the centre is the circle *acMpa;* the sine of the angle *pal* [*i.e.* ψ above] is as before $= m$, and of which the cosine is *M*

[*i.e.* $\sin\psi = m$ and $\cos\psi = M$; the justification being that the tangents of the circle and of the spiral make the small angle ψ close to the common point *a*, and near *a* it satisfies $\tan\psi \sim \psi =$ 'rise over run' or the gradient of the winding, which remains constant thereafter, and represents the effective gradient of the path];

truly the points *l* and *o* are the extremities of the water at rest in the winding and placed at the same height from the horizontal; from these points the right lines *lc* and *op* have been drawn perpendicular to the periphery of the base. In the part of the winding which the water occupies, two points *m* and *n* are taken infinitely close together, and through these the right lines *nf* and *mg* have been drawn, again perpendicular to the base. Consequently from the points *c, f, g, p* the perpendiculars *cd*, *fh*, *gi* and *pq* are drawn to the diameter *aM* ; and the centre of the base is put at *e*, and the radius $ea = 1$. Now the arc length of the spiral *l1o* full of water $= c$ and consequently the circular arcs corresponding to the same arcs are: $cMp = Mc = c\cos\psi$; the arc $al = e$; $ac = Me = e\cos\psi$; *ad* (or the versed sine of the arc *ac*) $= f$; $aq = g$; the weight of water in *l1o* $= p$;

[the letters denoting arc lengths *e, f*, and *g*, and *p* for the weight of water, are not to be confused with the same letters used for points on the circle with the same label; similarly, the point *M* is not to be confused with $\cos\psi = M$.]

the arc $aln = x$; $nm = dx$; $acf = Mx = x\cos\psi$; $fg = Mdx = dx\cos\psi$; $ah = y$; $hi = dy$; $hf = \sqrt{2y - yy}$ [as $hf^2 = y(2-y)$ from elem. geom.]; the weight of the drop in $nm = \frac{pdx}{c}$; truly if the line *hf* may be multiplied by the sine of the angle *aMH,* and divided by the total sine, the lever will be had by which the particle *nm* tries to turn around the spiral: therefore the lever is $= n - \sqrt{2y - yy}$, which multiplied by the stated weight of the drop $\frac{pdx}{c}$ gives the moment of the same $\frac{npdx}{c}\sqrt{2y - yy}$. But from the nature of the circle there is : $Mdx = \frac{dy}{\sqrt{2y - yy}}$: therefore with this value substituted for *dx,* the moment of the drop *nm* likewise becomes $= \frac{npdy}{Mc}$, the integral of which, with the constant subtracted, is $\frac{np(y - f)}{Mc}$ and denotes the moment of the water in the arc *ln;* hence therefore the moment of all the water in *l1o* is $= \frac{np(g - f)}{Mc}$: which divided by the lever of the force applied at *f* or equally by 1, leaves that same force $= \frac{np(g - f)}{Mc}$. Q.E.I

Scholium 1.

(XIII) So that it may be apparent the value of this force does not differ from that, which we found for a ball of the same weight p in *art*. V, surely $\frac{mNp}{M}$, it is required to show the equality between $\frac{np(g-f)}{Mc}$ and $\frac{mNp}{M}$, and between $np(g-f)$ and mNc: truly this equality is required to be deduced from that, because the extremities of the water *l* and *o* shall be placed in the same horizontal height; from thence it follows, as we have shown in *art.* IV, to be the sum from the arc *ac* multiplied by $\frac{mN}{M}$ and from the line *Md* multiplied by $n =$ sum from the equal arc *acMp* multiplied by $\frac{mN}{M}$ and from the line *Mq* multiplied by *n.* And thus from the denominations of the preceding article, there becomes

$$Me\times\frac{mN}{M}+(2-f)\times n=(Me+Mc)\times\frac{mN}{M}+(2-g)\times n,$$

or

$$n(g-f)=mNc;$$

which was the equality required to be shown both of the forces for the ball as well as for the water required to be applied at *f*.

Scholium 2.

(XIV) Because the force $\frac{np(g-f)}{Mc}$ does not differ from $\frac{mNp}{M}$ and the amount $\frac{mN}{M}$ remains the same, whatever amount of water may be raised in one revolution, this force will be proportional to the same amount of water ejected in individual revolutions, or to the weight *p.* It is also easy to show, if the same amount of water will be raised by the same *moving force* with the same velocity to an equal vertical height above to a simple plane, which to this end shall duly be inclined towards the horizontal, to be so that the time of elevation also shall be the same.

Therefore the same work done is required in Archimedes's Screw, as with the above inclined plane, to which all machines are able to be reduced, nor has this screw any preference over the remaining machines with regard to theory. Perhaps in practice fewer inconveniences are in the way indicated in §. 26: by no means do I disagree with its use, but neither do I prefer that before the pumps of Ctesibius.

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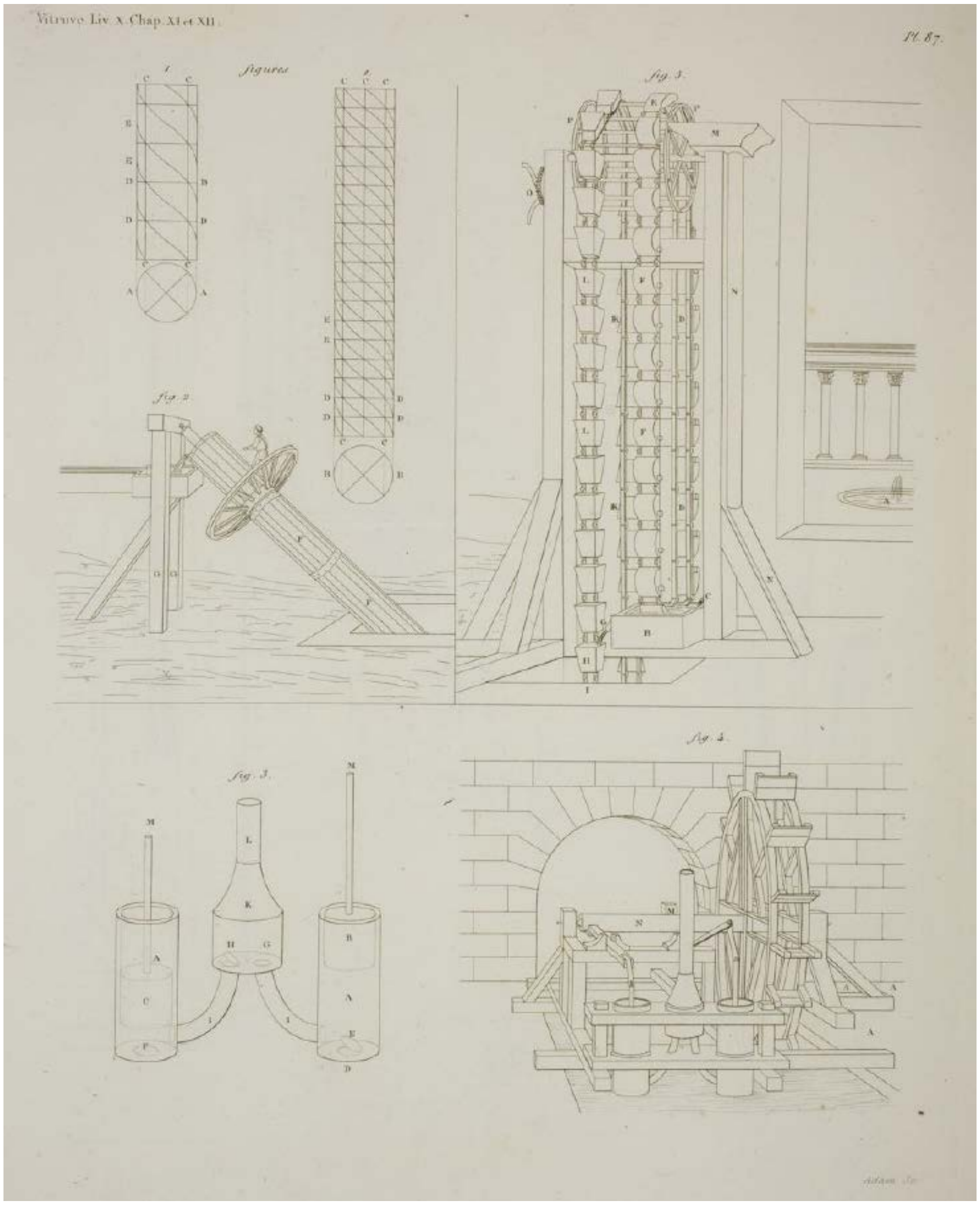
§. 28. It is understood from what has been said to this point, by which title one machine shall be preferred to another, for which machines allow a degree of perfect ; according to which chiefly it shall be required to attend to the construction and usage of these ; how great a part of the *absolute work* may be lost, and other things of a like nature : Indeed we will consider only machines that are said to be moving by *living forces*: moreover it will be readily apparent machines are to be subject to the same laws, which are required to be moved by other principles on account of the force of water, wind, or from the weight of water of some kind; for always the *moving force* multiplied by the time and the velocity of a point to which the force has been applied will give a product from the amount of water and from the height to which that amount in the assumed time shall be raised, with the aid of the proposed machine, with outside hindrances set aside. But I am talking about machines, from which no *work done* is lost ; indeed it can arise, that the maximum part may be lost, which we have shown well enough in the above.

§. 29. It is apparent thence water raised to a certain height again can by its descent perform the same outstanding effect: but the effect will be required to be estimated from the amount of water required to be raised and from the height of the elevation, thus so that for example by 8 cubic feet descending from a height of one foot just as much again shall be able to be raised to the same height or 4 cubic feet to a height of two feet, or one cubic foot to a height of 8 feet and thus whatever it should be desired. A specimen of a machine, which shall be able to raise water to some height with the minimum amount of water descending, is to be seen in the work of Perrault in *Comment. ad Vitruvium lib.* 10, *cap.* 12, which machine he introduces almost as an incredible paradox, and of that he makes the inventor to be the Italian Franchini, by whose industry and careful planning it had constructed with success in the garden of the Royal Library. [See extract from original text below, courtesy *e-rara*, showing also the pump described above supplying water to fountains.] The basis of the machine consists in this, that returning buckets connected together in a circle remove water and transport it to a lower place, where they may discharge, while another series of buckets carry water to a much higher place where they may discharge, yet to be less copious : but it is evident, if all the first series of buckets descending shall be heavier than all the ascending buckets, the other series is going to be moving perpetually in a circle; also there are machines, which perform the same by simple pipes with the aid of bungs requiring to be opened or closed at stated times, in which conversion indeed no work is lost. Charles Fontana has described machines of this kind.



But if anyone should believe it to be possible from the force of water falling from a give height and you may promote impinging on other machines to obtain the same, he has been led astray far. Such a machine belongs to that class, in which the greater part of the *work done* is lost without reward.

It will not be away from the issue to pursue this argument more carefully, and to show how great an effect may be obtained from the force of water or wind, and under what circumstances this effect shall be said to be the greatest of all.

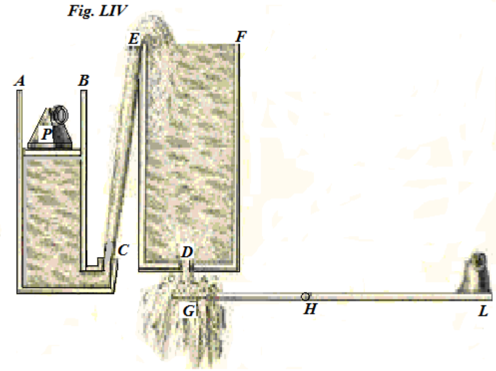
(C) *Concerning machines which may be set in motion by the force of a fluid, such as the strength of the wind.*

§. 30. After water has been raised to a certain height and fallen again through the same height, and impinges continually on the paddles of the wheel requiring to be driven around, it cannot happen otherwise, that the *work available* thus for turning the wheel around shall be much less than that, which was expended in the raising of the water, of which matter the reasoning is in particular, because the water after the impulse by leaping off to the side even now may conserve the velocity, which brought nothing to the rotation of the wheel. Therefore a great part of the work done shall become useless, if the elevation of the water has been effected, so that from the impetus of the same the machine may be turned around and from this finally other water again may be raised to a height ; and indeed a greater or lesser part goes to waste according to the different circumstances ; truly at no time, as I will show, shall less than $\frac{23}{27}$ of the whole be lost, if the calculation shall be made according to the normal common estimate of the impulse of water.

§. 31. Moreover it may be considered generally, if water may flow out from a very wide cylinder through a simple opening with its whole velocity, that is, which shall correspond to the whole height of the water above the opening, and at once the jet may strike a plate directly in front of the opening, so that the force of the fluid shall be against the plate, it shall be in equilibrium with the weight of the cylinder of water, to be erected above the opening to the height of the water. Indeed authors have been lead into fallacy by the experiment and declaring that theory completely false. Yet I have been unwilling here to have departed from that, because at no time have I set out the true theory and then it will be easy from our established theory to correct the calculation. It may be allowed therefore, in its place then we will consider the matter correctly, to adhere to the common opinion, whatever the errors. So that the greater is the force of the fluid, therefore on that account the work done will be required to be increased by a greater ratio, as we will give.

§. 32. Imagine now (Fig. 54) a vessel *ABC* for example a pump which sends out water through the opening *C* only not in a vertical direction: but the water, when it arrives at the peak, to be received by another vessel *EDF*. At the bottom of this other vessel consider an opening *D*, equal to the former opening *C*, and put at the same height, thus so that just as plentiful a supply of water may flow out through *D*, as the amount put in above, and the vessel *EDF* will be itself constantly kept full. Again consider the water flowing out through *D* perpetually to strike the paddles of

Fig. LIV



some wheel, which in this way may raise other water by moving around: In place of this machine a simple lever is described turning around *H*, by putting such a lever continually one after another to be present before the opening *D*, which may accept the water, and by its other extremity may draw up water, and raise it to the same given height.

Thus with these in place I may ask initially about the *work done,* by which the water through the opening *C* may be raised to the height *CE* ; then also I may inquire about the *work done*, which is required at *G* for the lever required to be moving with the same velocity, by which the it may be moved by the force of the water *DG.*

§. 33. Let the cross-section of the opening C or $D = n$, with the cross-section $AB = m$, the velocity of the water at *C* or $D = v$, the weight of the cylinder [of water] above the opening *C* or *D* built up to the height *CE* $, = p$ *;* the time of the flow $= t$ *;* the weight *P* [*i.e.* the pressure $\frac{P}{m} = \frac{p}{n}$] will be $= \frac{m}{n} p$; the velocity, with which the weight descends while the water is being expelled, $= \frac{n}{m} v$ *;* therefore (by §. 3) the *work expended* in the ejection of the water through *C*

$$= \frac{m}{n} p \times \frac{n}{m} v \times t = pvt.$$

§. 34. Now so that the *work available* may be determined [*i.e.* kinetic energy] in the gyration of the lever *GL* expressed about the point *H*, it is required to be noted that by itself to be minimally agreed on; for to be changed by a change in the velocity, however the lever may be going around. Therefore we may make the velocity by which its end may be moving at *G*, $= V$. But here only water may be considered to be striking at *G* with the velocity $v - V$, and thus the pressure to be exercised, which shall be $\left(\frac{v-V}{v}\right)^2 p$ (for pressures are in the square ratio of the velocities of the fluid impinging and for the velocity *v* the pressure is put $= p$). Truly this pressure is in place of the *moving force ;* clearly we may put in place of the fluid pressure a weight to be placed resting on the lever at *G*, which shall be $\left(\frac{v-V}{v}\right)^2 p$. Truly this weight shall be moving with the same velocity by which the point *G* moves, namely with the velocity *V*, and it acts during the time *t:* Therefore the *work done* for the rotation of the lever during the time *t* and with the required velocity *V*

$$= \left(\frac{v-V}{v}\right)^2 p \times V \times t.$$

§. 35. Because therefore if the lever *LG* is not at once turning around, but fluid may be raised to the height *CE*, with that in mind, so that the jet of fluid by its own impulse at *G* by turning the lever around from another part of the fluid raised, the whole work done to the useful work done, will be as pvt to $\left(\frac{v-V}{v}\right)^2 p$, or as v^3 to $(v-V)^2 V$: and the same will be had for its useless part, as v^3 to $v^3 - vvV + 2vVV - V^3$.

§. 36. In nearly all machines, the main motion of which consists in the force of a fluid, it is accustomed to happen, that the velocity of the lever *V*, where it may sustain the force of the fluid, shall be excessively small on account of the velocity of the fluid *v;* moreover with these a great part of the effect is lost, which may be obtained from the same amount of fluid moved with an equal velocity.

§. 37. The maximum effect arises from the motion of a fluid, or what amounts to the same, the *work done* defined by §. 34 shall become a maximum, if there shall be $V = \frac{1}{3}v$; [The work done $= \left(\frac{v-V}{v}\right)^2 p \times V \times t.$ If this is differentiated w.r.t. *V* and put equal to zero, the result $V = \frac{1}{3}v$ emerges. This value can then be put into the first expression to find the maximum work done;] and then that *work done* is $= \frac{4}{27} pvt$, and even now twenty-three from twenty-seven parts is lost by a similar force, which is expended in the raising of water from *C* to *EF*.

If hence a natural fall of water may be had, and by that it shall be required to raise water or some other similar outstanding task, it is required to be done so that with the machine there in place, when an impulse happens, it may move with a velocity of one third the velocity of the impinging fluid. Truly for this condition to be able to be satisfied always, which is apparent from the example of the lever paddle. For if the point *G* may be moving with a greater velocity, diminish the part *HG* with the rest remaining or to increase that, if the point *G* may be moving with a smaller velocity. Or also with the length *HG* secured, so that a greater or lesser amount of water may be raised at the extremity *L*.

§. 38. From this same account of fluids striking the paddles at right angles: truly another is the computation for fluids incident obliquely on the sails of windmills moving by the force of the wind, and for other similar machines. Concerning these now I may add a little something and with these I may put an end to this section.

With a fluid strikes the surface of a whole sail about to be rotated, placed in some manner in a direction perpendicular to the motion of the fluid, authors teach that the fluid exercises a maximum pressure on the sail towards advancing the rotation, when the sail makes an angle with the direction of the wind, the sine of which shall be to the whole sine as $\sqrt{2}$ to $\sqrt{3}$; truly if the same jet of fluid may be removed wholly by the sail, thus inclined in one way or another to the direction of the fluid, the maximum pressure will be sustained by the sail in the direction of the rotation, which makes an angle of one half right angle with the direction of the fluid.

The first rule applies to machines which are all made to rotate by the wind around them : the second rule for these, which may be moved by a single jet and by a certain determined amount of fluid. Truly each hypothesis depends on, what an excessively small motion of the sail shall be with respect of the motion of the fluid ; for if you look at the motion of the sails, both rules are false ; nor surely can this motion itself be ignored, for I have observed often with windmills, the ends of wings to be carried with a velocity, almost equal to the velocity of the wind itself. Since these shall be thus, now we can put a calculation in place, so that we may have an account of each motion.

§. 39. Therefore let there be a fluid *DEBA* (Fig. 55) which impinges on the whole plate *AB* along the direction *EB :* but the plate is put in place to be moved in a motion parallel to the direction *Bb* and to be perpendicular to *EB* : Again the velocities shall be of this kind, so that while a particle of the fluid traverses the line *EB,* the point *B* of the plane may complete the line *Bb.* With these in place it is allowed for the whole system, evidently the fluid with the plane, to be moving from *b* towards *B* and indeed with the velocity *bB:* Thus truly it arises that the plate *AB* may remain at rest, but the particle of fluid incident at the point *B* may be agreed to have come from the point *e,* by taking $Ee = Bb$, and thus for all the droplets. [*i.e*. we change into the inertial frame of the paddle. Incidentally, in the original diagram the length *Bb* is considerably less than *Ee*.]

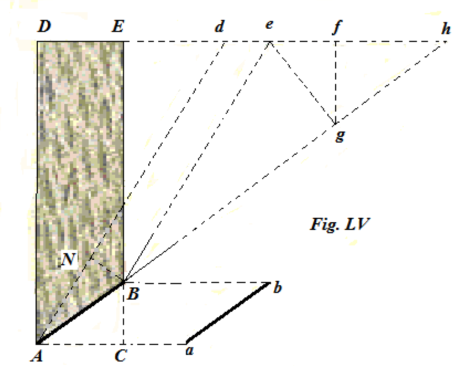


Fig. LV

Therefore in place of the fluid *DEBA* incident vertically downwards with a velocity *v* on the plate *AB* moving with a velocity *EB* or *V*, it will be required to consider the fluid *deBA* being incident on the same plate *AB* at rest, with the velocity *eB:* Now *AB* may be produced as far as *h* and *DEdeh* is acting perpendicular to *EB*, the motion of the particle of fluid represented by *eB* being resolved into *eg* and *gB*, themselves in turn remaining perpendicular, of which the latter has no effect on the plane *AB*, truly the other *eg* again is composed from the two motions *ef* and *fg*, of which the latter *fg* has no effect in trying to propel the plate *AB* in the direction *EB* [as the plate is constrained in this direction] , while the former *ef* alone propels the plate in the direction *Bb* [on reverting to the first inertial frame]. And thus it is shown, however, a particle makes an impulse proportional to the line *ef:* From that it is apparent also, if the line *AB* may represent the whole plate, the number of particles in a given time impinging on the plate to be represented by the line *BN* perpendicular to *Ad* or *Be.* From which finally the pressure of the water for the plate moving in the direction *Bb* is proportional to the line *ef* multiplied by *BN.*

Now so that the inclination of the plate to the fluid may be determined under these most favorable circumstances so that the motion of the plate may be moved forwards in the direction *Bb* : we may put $AB = 1$, *DE* or $AC = x$, $BC = \sqrt{1-xx}$ *;* the line *EB,* which may represent the motion of the fluid, $= v$, and *Bb* as a measure of the motion of the plate $= V$ *;* and thus by putting the calculation in place there is found

$$ef = xv\sqrt{1-xx} - (1-xx)V,$$

and

$$BN = (xv - V\sqrt{1-xx}) : \sqrt{vv+VV};$$

from which

$$ef \times BN = (xv - V\sqrt{1-xx})^2 \times \frac{\sqrt{1-xx}}{\sqrt{vv+VV}},$$

which quantity will be a maximum, when there becomes

$$(9v^4 + 18vvVV + 9V^4)x^6 \;\; -\left(12v^4 + 30vvVV + 18V^4\right)x^4$$
$$+ (4v^4 + 16vvVV + 9V^4)xx - 4vvVV = 0.$$

[Following KF: $ef = Eh - fh = v\cdot\cot\alpha - V - fg\cdot\cot\alpha$, or with $fg = ef\cdot\cot\alpha$ the equation becomes

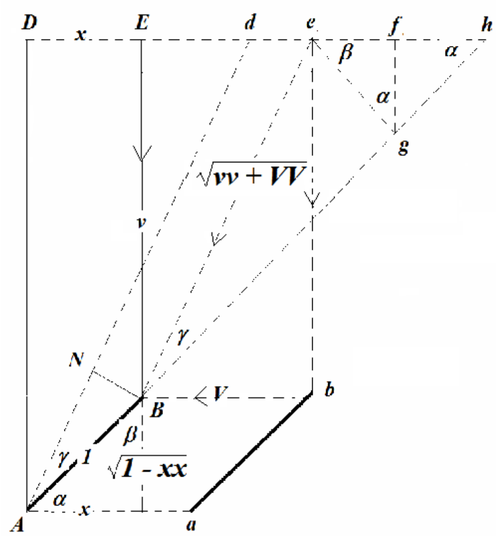
$ef = v\cdot\cot\alpha - V - ef\cdot\cot^2\alpha$; or

$ef\left(1+\cot^2\alpha\right) = v\cdot\cot\alpha - V$; then

$ef\left(1+\dfrac{xx}{1-xx}\right) = ef\,\dfrac{1}{1-xx} = v\dfrac{x}{\sqrt{1-xx}} - V,$

and finally,

$ef = vx\sqrt{1-xx} - V\left(1-xx\right).$



Also,

$NB = 1\cdot\sin\gamma = \dfrac{eg}{\sqrt{vv+VV}} = \dfrac{ef}{\cos\beta\sqrt{vv+VV}}$

$= \dfrac{ef}{\sqrt{1-xx}\sqrt{vv+VV}};$

from that it follows :

$$\sin\gamma = NB = \frac{vx\sqrt{1-xx}}{\sqrt{1-xx}\sqrt{vv+VV}} - \frac{\left(1-xx\right)V}{\sqrt{1-xx}\sqrt{vv+VV}}$$

$$= \left(vx - V\sqrt{1-xx}\right) : \sqrt{vv+VV}.]$$

§. 40. The calculation is different in the account of the inclination of the sails in windmills, because the velocities vary in different places of the sails; for they are proportional to the distances from the centre; but now the calculation for the sails can be put in place easily ; I shall pursue this case no further, it suffices to be observed, because it is not accurate enough for authors to decide $xx = \frac{2}{3}$, and because the true value of x itself shall always be less than $\sqrt{\frac{2}{3}}$. For example if there were $V = v$, and all the points may be agreed to be moving with the same speed, there becomes $x=\sqrt{\frac{1}{2}}$ [as $\alpha=45^{o}$], which shows the sail are required to be inclined at an angle to the direction of the wind of half a right angle. The best construction of the sails would be, if they may be curved, thus, so that the wind may impinge further up at a smaller angle than further down, or if it were made so that the sails everywhere be adjusted to a mean angle of around 50 degrees [to their direction of motion].

§. 41. I go on to another case, where all the fluid by the plate, whatever its inclination may be, is considered to be received by the plate. But here it is apparent, because the number of particles impelling in a given time is the same always, no attention need be paid to the line *BN*, and thus the pressure which the water makes to make the plate *AB* be moving in the direction *Bb* to be represented simply by *ef* or $xv\sqrt{1-xx}-(1-xx)V$. Therefore the maximum pressure itself will be obtained by taking

$$xx=\tfrac{1}{2}+\frac{V}{2\sqrt{(vv+VV)}},$$

and the pressure itself then will be

$$=\tfrac{1}{2}\sqrt{(vv+VV)}-\tfrac{1}{2}V,$$

if by v the direct pressure may be understood, which the jet exerts on the plate, which it meets at right angles.

§. 42. Now we will consider the jet *DEBA* as it emerges immediately from the opening *D* in Figure 54 and again the direction of the pressure p of the jet considered thus, just as in §. 33 ; and the pressure of this water, by which the plate tries in the due manner, so that the pressure becomes a maximum, to propel in a direction perpendicular to the jet

$$=\frac{p}{2v}\times(\sqrt{vv+VV}-V):$$

And if again this pressure may be multiplied by the velocity of the plate V and the time, the *work done* will be obtained, by which a plate with the same velocity in the same interval of time may be able to move through ; therefore the work done will be thus

$$=\frac{pVt}{2v}\times(\sqrt{vv+VV}-V).$$

§. 43. The *work done*, as we have just defined, is to be prepared thus, so that it may increase continually with increasing V, and if the velocity V may be assumed to be infinite, the same *work done* $=\tfrac{1}{4}\times pvt$.

[KF has produced the following justification for this result:

$\frac{pVt}{2v}\times(\sqrt{vv+VV}-V)\sim V^2\left[\sqrt{\frac{v^2}{V^2}+1}-1\right]$. Setting $\frac{v^2}{V^2}=t^2$ and $V^2=\frac{v^2}{t^2}$,

then one obtains the expression $\frac{v^2}{t^2}\left[\sqrt{t^2+1}-1\right]=v^2\varphi(t)\psi(t)$; on writing this expression in the form :

Consider $v^2\frac{\psi(t)}{1/\varphi(t)}$: as $V\to\infty, t\to 0,\ \varphi(t)\to\infty,$ and $\psi(t)\to 0.$

Hence $v^2\frac{\psi(t)}{1/\varphi(t)}\to\frac{0}{0}$ as $t\to 0$; ,]

According to L'Hôpital's Rule, we can differentiate both numerator and denominator and find a finite limit:

Hence $\frac{\psi(t)}{1/\varphi(t)}\to\frac{\psi'(t)}{-\varphi'(t)/\varphi^2(t)}=\frac{2t}{2\sqrt{t^2+1}}:2t\to\frac{1}{2}$ as $t\to 0$.

Hence $\frac{v^2}{t^2}\left[\sqrt{t^2+1}-1\right]=\frac{v^2}{2}$, from which the result follows.]

Therefore since in Figure 54 we may wish to rotate the machine by the jet *DG* used obliquely, at no time can more than a quarter part of this *work done* be able to be obtained, which may be expended in raising water from *C* into *EF*. Truly at no time, as we have shown in §. 37, can more than $\frac{4}{27}$ th of the work done be obtained by a direct force. Therefore the effect shall be nearly twice as great for an oblique impulse either by the movement of the wheel by a direct force, or able to be obtained by a vertical motion to the wheel.

Truly if the impulse of fluids may be estimated other than indicated by §. 31, everywhere the value of the letter *p* would be required to be changed in the same ratio, by which the estimated impulse was changed.

This is the e*xperiment*, about which I made mention in §. 27 Sect. IX. Surely one worker with the help of a pump within seven minutes and a half minutes raised sixteen and a half cubic feet of water to a height of forty feet.

Truly this effect distributed equally shall be equivalent to this action, by which more than around half a cubic foot can be raised in single seconds to a height of one foot: Therefore here the effect is certainly half of that, which a healthy and strong man can be able to do on a strong treadmill from these other principles I have deduced in paragraph seventeen. I would not have believed the difference sought to be all able to fall from the decreases which arise in the *work done* from various causes established in that section, but rather from that, because men may become more tired from the working of pistons in pumps, rather than treading in a treadmill wheel.

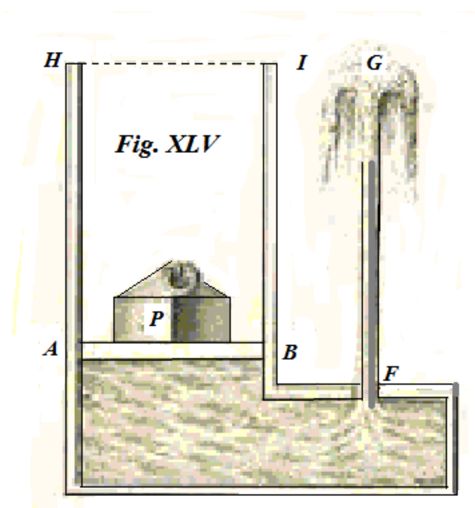
I provided a similar experiment, but clearly with the pump to have been perfected long since and made by the most talented craftsman, several months ago with the most celebrated men present : Professors De la Rive, Calendrin, Cramer & Jalabert of the Geneva Academy; the successes of the experiment was such, that I understood one worker to have lifted four fifths of a cubic foot [of water] to a height of one foot in a time of one second , or rather to have excelled an equal effect. The experiment is notable, and nor do I think a greater effect certainly could have been obtained with any other machine. It is a wonderful thing too, because thus with all kinds of machines, by whatever force they are animated, if you remove obstacles, a not much different effect may appear to be performed. By considering the matter well I can state, a man with the most perfect machine can raise a cubic foot of water to a height of one foot in single seconds, or to produce a similar effect.

Also here the experiments may be relevant, especially in the account of paragraph thirty one, which I have set up most carefully towards estimating the force of a fluid jet impinging on a plane, by which I have been confirmed in the new theory, as by this I have made the matter firm, and likewise been taught well, that a common error had been committed in the time of Mariotte. Because truly here at the end of this section there has been no elegant discussion about the matter, and that has been considered in section thirteen, thus therefore we may delay commenting as far as these inquiries not yet elicited from the observed principles of mechanics are concerned.

HYDRODYNAMICAE SECTIO NONA

De motu fluidorum, quae non proprio pondere, sed potentia aliena ejiciuntur, ubi praesertim de Machinis Hydraulicis earundemque ultimo qui dari potest perfectionis gradu, & *quomodo mechanica tam solidorum quam fluidorum ulterius perfici possit.*

§. 1. In hac Sectione, qua Machinas examinare hydraulicas, usumque earum, quantum fieri potest, perficere potissimum constitui, animum abstrahemus a variationibus motus, quae originem ducunt a potentia vel inertia fluidi interni, quia ut vidimus motus aquae internae tantum non aequabilis est a primo fere fluxus initio, si orificium exile sit, uti est in Machinis hydraulicis plerisque ratione amplitudinum internarum. Res enim foret ridicula in rebus practicis sollicitos esse de mutationibus, quae primis fluxus momentis fiunt, quasque jam determinavimus in sectione quarta, quod ibi operae pretium esse poterat ut omnis theoriae vis inde elucesceret. Igitur durante toto motu, brevitatis gratia, ponemus aquam constanter velocitate expelli, quae se habeat ut radix potentiae internae prementis, postquam haec potentia ad pondus cylindri aquei foramini superincumbentis reducta fuerit: nam quaecunque fuerit ista potentia, considerandum erit pondus cylindri verticalis aquei superficiei aqueae internae superincumbentis, atque altitudo istius cylindri dabit altitudinem velocitati aquae exilientis debitam, si modo nulla adsint obstacula extrinseca, & aqua ex vase amplissimo ejiciatur. Hoc ita intelligendum est, ut si operculum *AB* pondere *P* oneratum (Fig. 45) aquam per orificium *F* expellat, pondus autem *P* aequale sit ponderi cylindri aquei *HABI,* tunc vena aquea *FG* altitudinem *HI* attingere debeat.



Definitiones.

§. 2. Per *potentiam moventem* deinceps intelligam principium illud agens, quod consistit in pondere, pressione animata aliisve hujuscemodi viribus, uti dicuntur, mortuis.

Productum autem quod oritur a multiplicatione *potentiae* istius *moventis* per ejusdem velocitatem aeque ac tempus durante quo pressionem suam exerit, designabo per *potentiam absolutam.* Vel quia productum ex velocitate & tempore proportionale est simpliciter spatio percurso, licebit etiam *potentiam absolutam* colligere ex *potentia movente* multiplicata per spatium, quod eadem percurrit. Id vero productum ideo voco *potentiam absolutam,* quia ex illo demum aestimandi sunt labores hominum operariorum in elevandis aquis exantlati, quod mox demonstratum dabo in regulis, quae mihi in hanc rem observatae fuerunt. Interim visae mihi fuerunt machinae hydraulicae commode se reduci pati ad duo genera, quorum alterum aquas cum impetu ejicit, alterum de loco in locum placide veluti transportat. Utrumque ordine suo pertractabo genus & denique sub finem quaedam addam de diversis potentiis moventibus.

(A) *De machinis aquas cum impetu in altum projicientibus.*

Regula 1.

§. 3. labores hominum operariorum, qui machinis hydraulicis pro aquis elevandis apponuntur, aestimandi sunt ex *potentia absoluta*, id est, ex *potentia movente* seu pressione quam exerunt, ex tempore & ex velocitate puncti, cui *potentia movens* applicator.

Demonstratio.

(α) De *potentia movente* res est perspicua: labores enim caeteris omnibus paribus sunt utique proportionales numero operariorum seu *potentiae moventi.* (β) Ratione temporis res est non minus manifesta ex omnium circumstantiarum replicatione, quae ex duplicatione temporis oritur. (γ) Denique quod ad velocitatem attinet res ex eo est deducenda, quod sive *potentiam moventem* duplices, sive ejus velocitatem, non diversus oriatur effectus, nempe duplus ab utraque parte. Finge pondus *P* descensu suo aquam per orificium *F* ejicere ad altitudinem *FG:* deinde manentibus reliquis duplicatum puta orificium *F,* & vides ad eandem altitudinem *FG* eodemque tempore duplam aquae quantitatem ejectum iri ab eadem *potentia movente P,* sed ea duplo celerius descendente. Pariter quantitas aquae manentibus reliquis duplicabitur, si & orificium *F* & amplitudinem *AB* & pondus seu *potentia movente. P* duplices, tunc vero velocitas hujus potentiae duplicatae invariata manet. Igitur utroque modo effectus geminatur. Q. E. D.

Scholium.

§. 4. Propositio praecedens non sensu physiologico sed morali est interpretanda: moraliter neque plus neque minus aestimo laborem hominis, qui eadem celeritate conatum duplum exercet, quam ejus qui eodem conatu celeritatem duplicat, quia nempe uterque eundem edit effectum; fieri tamen potest, ut alterius labor, quam vis altero non minus robusti, sensu physiologico sit admodum major. Si quis conatu 20 librarum singulis minutis primis spatium 200 *ped.* faciat, is facile conatum geminabit, difficillime vero velocitatem. Ex hoc consequens est in omni machinarum genere dispiciendum praesertim esse, quomodo debeant esse constitutae, ut pro eodem tempore minima hominum defatigatione productum ex conatu eorum in velocitatem omnium maximum sit: atque exinde patebit, quaenam in ergatis longitudo vectibus sit tribuenda, quantus in rotis seu tympanis calcatoriis radius sit faciendus, quanta remis longitudo sit concillanda, & sic de aliis machinis.

Fig. XLVI

Ratione usus autem tympanorum calcatoriorum, quae frequentissime adhibentur, ut momentum nostrae animadversionis eo magis fiat perspicuum, hoc experimentum intelligatur:

Ponamus in Fig. 46 altitudinem verticalem multorum milliarum, ad quam homo dato tempore ascendere debeat: tempus autem sumemus decem horarum, quia talis laboribus diumis terminus esse solet, dein fingamus plures vias, *AC*, *AD* &c. diverse ad horizontalem *BD* inclinatas: His positis intelligimus eo celerius viatori progrediendum esse, quo viam selegerit minus inclinatam, ut eodem tempore culmen montis *A* attingat, & patet viam aliquam fore veluti *AC*, super qua minima defatigatione iter absolvet, quandoquidem nemo nec super plano verticali incedere nec dato tempore viam infinitam absolvere potest; statuamus viam hanc minimae defatigationis cum horizontali angulum facere *ACB* 30 graduum.

Quod si ita sit, erit tympanum calcatorium ita fabricandum, ut pondus desiderata velocitate superetur, cum calcator perpetuo triginta gradibus a puncto tympani infimo distat.

Ex eodem principio etiam inter machinas diversi generis selectus est faciendus: ita v. gr. si in ergatis vectiarius potentiam exerat, seu pressionem horizontalem, quae efficiat quartam sui proprii ponderis partem, hocque nisu singulis minutis primis spatium 200 *ped.* absolvat, is fere ut puto eodem defatigabitur modo, ac si eadem velocitate tympanum rotatorium ad angulum 30 *grad.* calcet; interim tamen pondus duplum eodem tempore ad eandem altitudinem hoc modo feret calcator, quia caeteris paribus pressionem duplam exerit.

Regula 2.

§. 5. Existente eadem *potentia absoluta* dico omnes machinas, quae nullas patiuntur frictiones & quae nullos motus ad propositum finem inutiles generant, eundem effectum praestare neque adeo unam alteri praeferendam esse.

Demonstratio.

Ex mechanicis constat machinam utcunque compositam reduci posse ad vectem simplicem: igitur omnem machinationem hydraulicam repraesentare licebit simplici antlia vecte instructa Fig. 47, ubi nempe embolus ope vectis *MN* mobilis circa punctum *M* detruditur, atque sic aqua per orificium *F* expellitur. At vero si potentia movens *P* vecti applicata intelligatur in *N*, videmus ex praecedente propositione nihil lucri accedere *potentiae absolutae* ab aucta vel diminuta longitudine vectis *MN:* & certe quaecunque sit ista longitudo fieri potest, ut *potentia movens* eadem atque invariata velocitate mota eandem aquae quantitatem eodem impetu expeltat, si modo amplitudo antliae *AB* rationem habeat constantem ad longitudinem vectis *MN.* Ex quibus perspicuum est, omnes machinas eadem *potentia absoluta* eundem effectum praestare, si modo a frictionibus motibusque ad destinatum finem inutilibus animus abstrahatur.

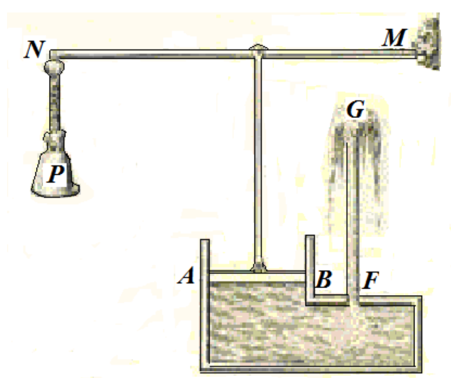


Fig. XLVII

Scholium.

§. 6. Non desunt qui putent machinam excogitari posse, cujus ope minimo labore maxima aquae quantitas ad quamcunque altitudinem elevari possit, animumque excrucient in anquirendis rotis, vectibus, ponderibus appendendis: sed operam perdunt, neque audiendi sunt hujusmodi promissores, cum magni quid sibi videntur invenisse: Optima machina est, si solum ejus effectum respiciamus, quae minimas patitur frictiones, nullosque generat motus inutiles, de quo utroque evitando praecepta trademus infra.

Regula 3.

§. 7. In antliis, quales Figuris 45 & 47 repraesentantur, in quibus superficies aquae interna *AB* in eadem propemodum altitudine est cum foramine *F*, sunt *potentiae absolutae* pro iisdem temporibus in triplicata ratione velocitatum aquarum exilientium.

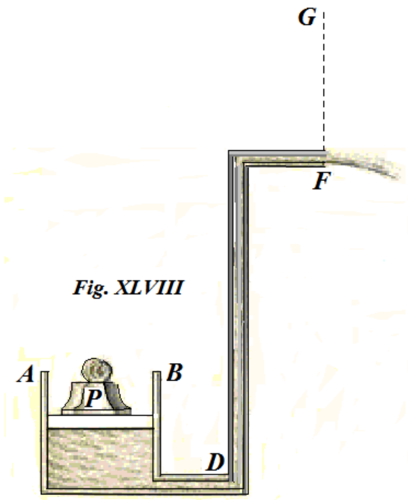
Demonstratio.

Sunt enim *potentiae moventes* in duplicata ratione velocitatum, quibus aquae per foramen *F* erumpunt, & velocitates *potentiarum moventium* sequuntur ipsam rationem velocitatum aquarum exilientium: Sed pro iisdem temporibus sunt *potentiae absolutae* ut potentiae moventes multiplicatae per suas velocitates, ergo patet propositio.

Scholium.

§. 8. Sequitur ex ista regula, si animus sit aquam per foramen F ad altitudinem *FG* elevare, magnam *potentiae absolutae* partem sine fructu perdi, cum aquae majori impetu erumpunt, quam quae altitudini *FG* respondeat; fac enim aquas dupla velocitate expelli, requiretur *potentia absoluta* octupla, neque tamen ratione finis propositi effectus plus quam duplus est censendus, quia nempe eodem tempore dupla aquarum quantitas elavatur: potuissetque iste effectus obtineri *potentia absoluta* subquadrupla exprimendo aquas simplici velocitate per foramen duplum; hoc igitur nomine tres quartae partes istius potentiae inutiliter impensae dicendae sunt. Originem hujus detrimenti indicavi §. 5, eaque consistit in motu qui generatur ad propositum finem inutili: nempe omnis motus qui aquis residuus est postquam altitudinem *G* attigerunt in nostro casu superfluus est dicendus.

Regula 4.



§. 9. Cum aquae expelluntur per canalem *D F* (Fig. 48) habentque in orificio *F* velocitatem quae debeatur altitudini verticali *GF*, est *potentia absoluta* eadem tempore impensa proportionalis velocitati aquae in *F* ductae in altitudinem *G* supra *AB.*

Demonstratio.

Est enim potentia movens *P* proportionalis praefatae altitudini & velocitas istius potentiae est ut velocitas aquae in *F*.

Scholium.

§. 10. *Potentiae absolutae* majori ratione crescunt quam velocitates aquarum effluentium, id est, quam quantitates eodem tempore ejectae: attamen differentia rationum fere insensibilis est, cum altitudo *FG* parva admodum est ratione altitudinis canalis *FD:* Sit ex. gr. *FG* aequalis $\frac{1}{4}FD$ (negligendo altitudinem *BD):* mox vero ejiciantur aquae velocitate dupla, ita, ut nunc sit $FD = FG$ *;* sic erunt *potentiae absolutae* ut $1\times\frac{5}{4}$ ad 2×2 seu ut 5 ad 16, sic ut ad ejiciendam duplam aquae quantitatem *potentia absoluta* requiratur plusquam tripla. Si vero *FG* statuatur prius $=\frac{1}{100}FD$, & deinde aquae rursus dupla velocitate exprimi ponantur, erunt nunc *potentiae absolutae* ut 1×101 ad 2×104 seu ut 101 ad 208, quae ratio a subdupla parum deficit. Sequitur inde, quo minori velocitate aquae hauriantur, eo majori cum fructu *potentiam absolutam* impendi, & tunc demum eam propemodum omnem utiliter impendi, cum fere insensibili velocitate aquae per orificium *F* effluunt: poterit autem magnitudo orificii compensare velocitatis exiguitatem, ut dato tempore notabilis aquarum quantitas hauriri possit. Dispendium *potentiae absolutae* sic definietur.

Regula 5.

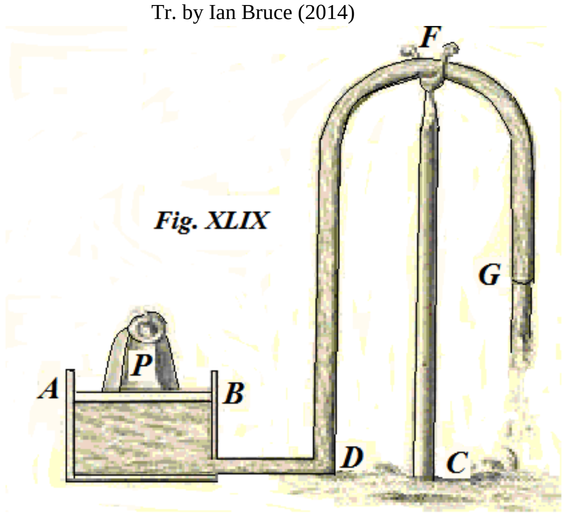
§.11. Constitutum fuerit ope antliae *ABDF*, valvula in fundo instructae & aquae impositae, aquas ex loco humiliori *AD* in altiorem *F* transfundere, fueritque velocitas media aquae in *F* effluentis debita altitudini *FG*, erit dispendium *potentiae absolutae* ad integram hanc potentiam ut *FG* ad altitudinem *G* supra *AB*.

Demonstratio.

Fingamus augeri admodum orificium *F* diminuta in eadem ratione velocitate aquarum effluentium in *F;* sic non mutabitur quantitas aquae dato tempore effluentis, si velocitas *potentiae moventis* eadem sit, atque proinde idem erit effectus. Sed si velocitas ita diminuatur, ut altitudo ipsi debita sit insensibilis, exprimetur *potentia movens* per altitudinem *F* supra *AB*, cum antea *potentia movens* erat aequalis altitudini *G* supra *AB;* & cum in utroque casu eadem sit velocitas *potentiae moventis,* erunt *potentiae absolutae* pro iisdem temporibus ut altitudo *G* ad altitudinem *F* supra communem *AB*. Igitur differentia altitudinum *G* & *F* exprimet dispendium, cum integra altitudo *G* supra *AB* repraesentat totam *potentiam absolutam.*

§.12. Idem ratiocinium valet pro omni machinationum genere: Quoties nempe aquae in locum, ad quem elevandae sunt, evectae notabilem habent velocitatem, magnum fit *potentiae absolutae* dispendium: posita enim altitudine elevationis $= A$, altitudine debita velocitati aquarum in loco quo effunduntur $= B$, integra potentia *B* absoluta $= P$, perdetur $\frac{B}{A+B} \times P$.

Notari etiam potest, cum aquae trans altitudinem aliquam, cujus culmen in *F* positum sit, fundi debent ope antliae tubo instructae, continuandum esse tubum *DF* inferiora versus quantum id liceat, nec abrumpendum in *F*, prouti id apparet ex Fig. 49. Nam si v. gr. punctum *F* duplo altius positum sit quam extremitas tubi G, duplo major *potentia absoluta* requiritur pro transfundendis aquis per canalem abruptum in *F*, quam per continuatum usque in G; si parvula utrobique velocitate effluant, cujus nempe altitudo genitrix parva sit ratione altitudinum *FD* vel *GD.*



Regula 6.

§. 13. Cum in antliis quas hucusque consideravimus opercula *AB* seu potius emboli non bene lateribus machinarum respondent, hiatus relinquitur, & ab hoc aliud dispendii genus in potentiis absolutis oritur, quod in antliis, in quibus altitudo orificii supra embolum negligi potest, sic determinatur. Ut aggregatum ex foramine effluxus & praedicto hiatu, ad eundem hiatum, ita *potentia absoluta*, quae impenditur, ad partem illius quae inutilis est, seu ad ejusdem dispendium.

Demonstratio.

Nam aquae per foramen & hiatum aequaliter premuntur, & aequali velocitate fluunt; perditur autem omnis *potentia absoluta*, quae aquas per hiatum cogit, & haec se habet ad integram *potentiam absolutam*, ut hiatus ad summam foraminis & hiatus.

Scholium.

§. 14. Convenit utique embolis uti bene formatis & politis; necesse quoque est ut cavitas antliae sit plane cylindrica, ejusdemque latera pariter perpolita. Vix autem crediderim, nisi id fiat alio fine, e re esse, ut emboli cavitates ultima accuratione expleant, quia fortasse sic majus oritur virium dispendium a frictionibus, quam si circumcirca parvulus relictus fuisset hiatus: Si enim hiatus ille centesimam v. gr. partem foraminis effluxus efficiat, vix amplius locus erit frictionibus & non nisi centesima praeterpropter *potentiae absolutae* pars inde perditur, & fortasse a frictione emboli cavitatem antliae exacte occupantis majus dispendium oritur. Igitur hoc respectu non est quod nimis sollicite evitemus transitum aquae per hiatum ab embolo relictum. Non respicit autem haec animadversio illas machinas, in quibus emboli retractione aquae in antliam attrahendae sunt. Hic enim justa & plena emboli magnitudo omnino est necessaria.

Regula 7.

§. 15. In machinis quae plura habent foramina aquas transmittentia ex una cavitate in alteram, aliquid de *potentia absoluta* perditur, cujus rei rationem in praecedente Sectione esse diximus, quod singularum guttularum ex una cavitate in alteram per foramen commune fluentium *ascensus potentialis* perit.

Quo plura sunt & quo minora hujusmodi foramina, eo majus oritur *potentiae absolutae* dispendium, quod magni momenti esse solet, idque fortasse praeter communem opinionem, in machinis, quas Vitruvius ab inventore vocat, Ctesibianis. Loquor autem de foraminibus ita dispositis, ut omnis aqua effluxura per illa transire debeat. Istud jam detrimenti genus tali definietur calculo.

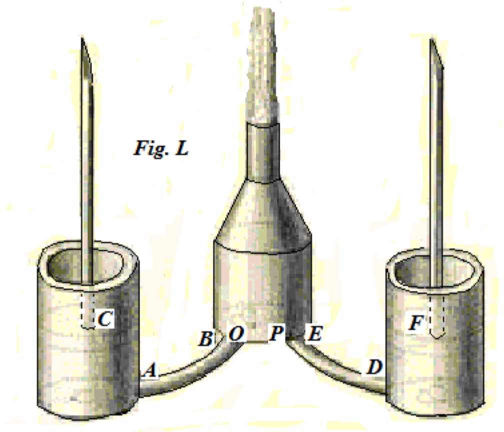
Sit amplitudo foraminis ultimi aquas in aërem emittentis $= n$, amplitudines autem reliquorum foraminum, per quae aquae trajiciuntur intra machinam, designentur litteris α, β, γ &c. & erit, posita utrobique eadem *potentia movente*, altitudo debita velocitati aquae effluentis ad similem altitudinem nullis obstantibus foraminibus internis, ut 1 ad $1+\frac{nn}{\alpha\alpha}+\frac{nn}{\beta\beta}+\frac{nn}{\gamma\gamma}+\&c.$ (per §.11 Sect. VIII); sequitur inde factis istis altitudinibus inter se aequalibus, fore *potentias moventes* ut $1+\frac{nn}{\alpha\alpha}+\frac{nn}{\beta\beta}+\frac{nn}{\gamma\gamma}+\&c.$ ad 1, & quia utrobique velocitates potentiarum moventium eaedem sunt, similem quoque pro iisdem temporibus rationem habebunt *potentiae absulutae.* Superflua igitur est pars ejus $\frac{nn}{\alpha\alpha}+\frac{nn}{\beta\beta}+\frac{nn}{\gamma\gamma}+\&c.$ unde dispendium *potentiae absolutae* erit ad totam hanc potentiam ut $\frac{nn}{\alpha\alpha}+\frac{nn}{\beta\beta}+\frac{nn}{\gamma\gamma}+\&c.$ ad $1+\frac{nn}{\alpha\alpha}+\frac{nn}{\beta\beta}+\frac{nn}{\gamma\gamma}+\&c.$

Scholium.

§.16. Quoties idea machinae foramina postulat, per quae aquae ex uno modiolo in alium transfluant (quod fit in omni antliarum genere; veluti aspirantium, *aspirantes* gallice aut prementium, *foulantes* &c.), sunt illa foramina, quantum id reliquae circumstantiae permittunt, amplissima facienda, ita ut amplitudo orificii effluxus parva admodum sit respectu illorum foraminum internorum: Ut vero usus regulae clarius pateat, exempla considerabimus machinarum aliarum non minus usitatarum.

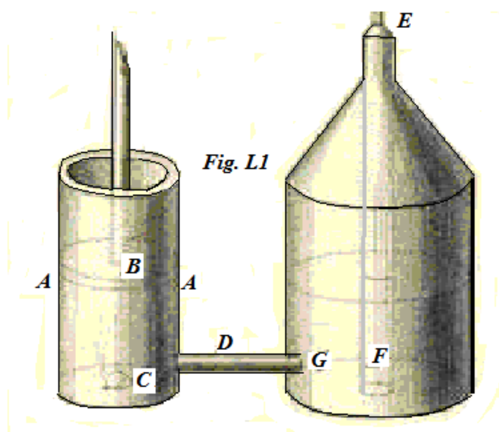
Exemplum 1.

Proposita sit machina (quam repraesentat Figura 50) in qua emboli C & *F* alternatim deprimuntur, atque per diabetes *AB*, *DE* aquae in *modiolum BEH* intruduntur , ut sic jactus fiat continuus per orificium *H.* Cum hic emboli alternatim agant, alterutrum considerabimus quasi solum sed continue agentem; ita vero considerandum est foramen effluxus *H*, amplitudinis *n,* & alterutrum foraminum *o*, *p,* quibus singulis sit amplitudo *a;* ita erit dispendium *potentiae absolutae* $\frac{nn}{\alpha\alpha}$, posita potentia integra $=1+\frac{nn}{\alpha\alpha}$, quae



quantitates sunt ut *nn* ad $nn + \alpha\alpha$. Considerabile certe est hoc dispendium, si iconibus harum machinarum fidere licet, in quibus saepe orificia *o* & *p* minora sunt orificio effluxus *H*, quod si foret plus quam dimidium perderetur *potentiae absolutae*. Erunt autem canales *AB* & *DE* per totum tractum, quantum id licet, amplificandi, ut machina parum de sua praestantia perdat.

Ceterum fuit haec machina excogitata, ut jactus fieret continuus per *H*. Quia tamen fieri non potest, quin aliquod temporis intervallum intercedat inter ultimum emboli elevationis punctum, instantisque ejusdem depressionis initium, non poterit jactus omnino esse continuus & aequabilis. Huic vero incommodo optimum remedium attulit auctor machinae illius, cujus mentionem facit D. Perrault in *Comment. ad Vitruvium, pag.* 318 edit. 2 Paris, quamque in Bibliotheca Regia Paris. asservari dicit; inserviet nobis haec machina alterius exempli loco: figuram autem desumam una cum ejusdem descriptione ex ipso Perraultio.



Exemplum 2.

«Machina est» referente praefato Perraultio, «in qua aqua expellitur ex modiolo *A* (Fig. 51) mediante embolo *B* in catinum *FG*, ex quo aër, si modo aliquid aquae jam adsit, egredi non valet; quia tubus *EF* usque ad fundum fere descendit: sic enim fit, ut aqua propulsa ex modiolo *A* per diabeten *D* imumque catini occupans claudat orificium tubae in *F*, aërique transitum neget. Igitur cum embolus novas intrudit aquas in modiolum, partim aëre partim aqua repletum, hae aquae de novo affusae vim exerunt in utrumque fluidum, & cum aqua non possit exilire per tubum *FE* eadem velocitate qua irruit ex antlia per diabeten *D*, quia scilicet (sunt verba Perraultii) tubus *FE* in extremitate sua *E* orificio perforata est multo minori, quam est orificium tubi *D*, aqua in catino accumulata aërem comprimit, ab eodemque reciproce pressa, etiam dum embolus elavatur, per tubam *FE* exilit.»

Perditur in hac machina magna *potentiae absolutae* pars a transitu aquae per diabeten *D*, hocque dispendium eo majus erit, quo angustior est iste tubulus: fiat igitur amplus aut etiam plures tubi construantur aquas transmittentes: majoris est momenti haec annotatio in praesenti casu, quod multo majus dispendium ab angustia diabetes *D* oritur, quam in aliis machinis; fac enim amplitudinem hujus diabetes eandem, quae est orificio *E*, & pone insuper aequalibus temporis intervallis embolum deprimi retrahique: non perdetur jam solum dimidia *potentiae absolutae* pars, ut alias, sed plane quatuor quintae partes inutiles fient. Quia vero multa sunt in hac machina, quae singularem postulant calculum, placet illam seorsim perlustrare.

Digressus continens aliquas commentationes in Machinam Hydraulicam quam repraesentat Figura 51.

(α) Non potest jactus aqueus per *E* esse omnino aequabilis, durante tota emboli agitatione: Dum enim embolus elevatur, novae aquae non accedunt, atque sic diminuitur quantitas aquae in catino *GE* contentae, aërque eidem superincumbens dilatatur ac denique elater ipsius diminuitur: hinc quoque velocitate continue minori aqua erumpit donec rursus ab embolo intruso acceleretur.

Verum si ponatur spatium, quod aër in catino occupat, longe majus spatio illo ab aqua, quae durante una emboli elevatione ejicitur, occupato, cessat fere tota haec inaequalitas, posito embolum uniformiter agitari & diu ante fuisse agitatum, quae posterior hypothesis ideo necessaria est, quod primae agitationes valde differant a sequentibus. Igitur brevitatis ergo omnibus hisce hypothesibus satisfaciemus, id est, ubique *statum*, qui dicitur, *permanentiae* ponemus.

(β) Cum igitur primis emboli agitationibus sensim augeatur velocitas aquae per *E* effluentis, mox fit ut jactus aqueus velocitatem tantum non integram attingat; quo rei statu posito, patet tantum aquae depressione emboli impelli in catinum, quantum ex eodem tota emboli agitatione ejicitur. Primis autem agitationibus plus intruditur, quam ejicitur, idque non ideo, ut putavit Dn. Perrault, quod orificium in *E* altero in *G* minus sit (idemque enim succederet si vel majus esset), sed quod causa efficiens non possit statim omnem suum exerere effectum in ejiciendis aquis.

(γ) Videbitur fortasse rem non satis perlustrantibus fore, ut omnibus in statu permanente jam positis, nullisque praesentibus obstaculis alienis, aqua per foramen *E* velocitate exiliat, qua ascendere possit ad altitudinem columnae aqueae in aequilibrio positam cum pressione emboli: atque ita sane foret, si pressio emboli sine interruptione adesset, nullusque in aqua *ascensus potentialis* perderetur: quia vero in utroque res aliter se habet, non potest non alia oriri in jactu aqueo velocitatis aestimatio: Hinc quisque non obscure videt animum advertendum esse ad temporum rationem, quibus embolus deprimitur, retrahiturque, tum etiam ad rationem amplitudinum in canaliculo *D* & orificio *E*.

(δ) Ponamus igitur tempus quo embolus deprimitur $= \theta$ *;* tempus unius integrae agitationis $= t$, amplitudinem orificii $E = \mu$, & diabetes $D = m$ *:* deinde comparata potentia embolum detrudente cum superincumbente columna aquea, faciamus hujus columnae altitudinem $= a$, altitudinem vero aquae exilientis velocitati debitam $= x$. His ita ad calculum praeparatis licebit duobus indagare modis rationem quae futura sit inter velocitates aquarum in orificio *E* & diabete *D*, atque hinc valorem incognitae *x* elicere.

Primo enim patet tempore θ (quo scilicet embolus detruditur) tantum aquae fluere per diabeten *D*, quantum tempore *t* (quo embolus deprimitur retrahiturque) effluit per *E*. Est igitur velocitas in *D* ad velocitatem in *E* ut $\frac{1}{m\theta}$ ad $\frac{1}{\mu t}$:& quum posterior haec velocitas sit $= \sqrt{x}$, erit altera $= \frac{\mu t}{m\theta}\sqrt{x}$.

Secundo quia velocitas aquae effluentis debetur pressioni aëris in catino, sequitur hanc pressionem aequivalere ponderi columnae aqueae altitudinis *x;* sed si a pressione emboli auferas pressionem aëris, habebis pressionem, quae velocitatem aquae in *D* generet; hinc quia differentia pressionum exprimitur per $a - x$, repraesentabitur velocitas aquae in *D* per $\sqrt{a-x}$; igitur nunc est velocitas aquae in *D* ad velocitatem aquae in orificio *E* ut $\sqrt{a-x}$ ad $\sqrt{x}$. Combinatis rationibus utroque modo inventis, fit

$$\sqrt{a-x} : \sqrt{x} = \frac{1}{m\theta} : \frac{1}{\mu t},$$

sive

$$x = \frac{mm\theta\theta}{mm\theta\theta + \mu\mu tt} \times a.$$

Patet ex ista aequatione altitudinem jactus duplici titulo deficere ab altitudine columnae prementis *a;* magis nempe deficit, cum celerius deprimitur tardiusve elavatur embolus, tum etiam cum orificium *E* ratione canaliculi *D* amplitudine crescit. Fuerit v. gr. amplitudo istius orificii aequalis amplitudini tubuli *D* atque pari celeritate embolus deprimatur eleveturque & prodibit $x = \frac{1}{5}a$, sic ut ad quintam partem tantum assurgat vena effluens altitudinis *a*.

(ε) Dispendium *potentiae absolutae* jam hoc modo eruetur, posito prius nullum laborem in elevandum embolum impendi. Sit velocitas qua embolus deprimitur $= v$, & erit *potentia absoluta* tempore unius agitationis integrae impensa $= av\theta$ (per paragraphum tertium); quia vero effectus in eo consistit, ut effluxus fiat per *E* durante tempore *t* ipsaque aqua ad altitudinem $x = \frac{mm\theta\theta}{mm\theta\theta + \mu\mu tt} \times a$ elevetur, potuisset id antlia simplex Figurae quadragesimae quintae efficere, si pro *potentia premente* in illa sumtus fuisset cylindrus aqueus altitudinis $\frac{mm\theta\theta}{mm\theta\theta + \mu\mu tt} \times a$, atque haec potentia durante tempore *t* velocitate $\frac{\theta}{t}v$ egisset, unde *potentia absoluta* in hac machina simplici, qua nihil de illa perditur, requisita futura fuisset

$$=\frac{mm\theta\theta}{mm\theta\theta+\mu\mu tt}\times a\times\frac{\theta}{t}v\times t=\frac{mm\theta\theta}{mm\theta\theta+\mu\mu tt}\times av\theta.$$

Est igitur tota *potentia absoluta* ad partem ejus inutiliter perditam ut $av\theta$ ad $av\theta-\frac{mm\theta\theta}{mm\theta\theta+\mu\mu tt}\times av\theta$ seu ut $mm\theta\theta+\mu\mu tt$ ad $\mu\mu tt$. Igitur si integra *potentia absoluta* designetur per *P*, erit ejus dispendium $=\frac{\mu\mu tt}{mm\theta\theta+\mu\mu tt}\times P$.

Necesse igitur est in hac prae aliis antliis, ut diabetes amplitudine admodum superet orificium *E*, vel ut multiplex adsit. Si enim unicus adesset, isque amplitudine orificio *E* aequalis, simulque uniformi velocitate sursum deorsumque agitari ponatur embolus, dispendium orietur quatuor quintarum totius partium: atque si vel duplo amplior fiat, etiamnum perdetur dimidium *potentiae absolutae.*

(ζ) Denique perspicuum est minorem pressionem sustinere latera catini GE, quam modioli *AA*, quippe pressiones istae sint ut *x* ad *a*, id est, ut $mm\theta\theta+\mu\mu tt$ ad $mm\theta\theta$, ex qua ratione artifices judicabunt de firmitate laterum, quae pro utroque requiritur.

Regula 8.

§. 17. Quando embolus in antliis retrahitur & aqua in modiolum influit, non solum proprio pondere solicitata sed maximam partem ab embolo attracta, tunc omnis *potentia absoluta* in hanc attractionem impensa casu supervenit, quia antlia, sub aquis, ut fit, posita, sua sponte impleretur si sufficiens huic impletioni tempus concederetur; nec adeoque attractio illa ita pertinet ad ejiciendas aquas certa cum velocitate, quin tota vitari possit, hocque nomine labor in illam impensus mihi inutilis dicitur.

Quia vero influxus aquarum partim proprio pondere fit, partim etiam elevatione emboli, non potest *dispendium potentiae absolutae* ab effectu aestimari: Quin potius calculus ita est ponendus, ut positis potentia embolum in certo situ elevante $=\pi$, velocitate emboli $=v$, tempusculoque quantitatibus π & v respondente *dt*, dicatur omnis *potentia absoluta* in elevationem emboli impensa $=\int\pi v dt$ vel $=\int\pi dx$, si per *dx* intelligatur elementum spatioli tempusculo *dt* percursi. Sequitur inde, si constantis magnitudinis sit, uti fere est, conatus, quo embolus elevatur, fore *potentiam absolutam* aequalem *potentiae moventi* ductae in spatium percursum: simile autem ratiocinium cum valeat etiam pro depressione emboli simulque tantum elevetur embolus quantum deprimitur, apparet *potentias absolutas*, quae in attrahendas expellendasque altematim aquas impenduntur, proxime esse ut *potentiae* utrobique *moventes;* unde dispendium oritur quod est $=\frac{\pi}{\pi+p}\times P$, factis scilicet potentia elevante $=\pi$, potentia deprimente $=p$ & *potentia absoluta* in elevationem depressionemque emboli impensa $=P$.

Potest aliter dispendium *potentiae absolutae* proxime aestimari ex eo, quod omnis *ascensus potentialis* aquae in antliam influentis inutiliter generatus censeri debeat. Sed si iisdem temporibus, sive eadem velocitate embolus sursum deorsumque movetur, erit velocitas qua aquae admittuntur ad velocitatem qua ejiciuntur reciproce ut foramina respondentia, ipsique *ascensus potentiales* utrobique erunt in ratione quadrata inversa foraminum respondentium. Si deinde diversis temporibus fiant emboli elevatio & depressio, sunt velocitates reciproce ut tempora & *ascensus potentiales* reciproce ut quadrata temporum. Est igitur *ascensus potentialis* aquae influxu generatus ad *ascensum potent.* qui ab effluxu oritur solusque intenditur, in ratione reciproca quadrata composita ex ratione foraminis influxus ad foramen effluxus & temporis, quo hauriuntur aquae, ad tempus, quo expelluntur.

Scholium.

§. 18. Ex utraque aestimandi ratione sequitur lente embolum esse elevandum: ita enim parva fit *potentia movens* ratione primae methodi aut magnum fit tempus elevationis ratione secundae, atque sic operarii singulis elevationis emboli intervallis a conatu praecedentis depressionis exantlato reficientur. Posterior porro methodus indicat foramina, per quae aquae attrahuntur, amplianda & multiplicanda esse; id vero etiam priori conforme est methodo, quia sic sufficiens fere aquae quantitas sua sponte influit, minorique adeo *potentia movente* opus est.

Regula 9.

§. 19. Denique jactum aqueum verticaliter assurgentem nunquam eam attingere altitudinem observandum est, quae debeatur aquae velocitati initiali, id est, si vena fluidi verticaliter assurgere incipiat a sua origine velocitate tali, quam grave libere cadendo ex altitudine a acquirat, non poterit fluidum ascendere ad totam altitudinem a, etiamsi aëris resistentiam removeas, aut quicquid excogitare velis, quod casu motum retardare queat. Ipsa enim rei natura defectum aliquem exigit necessario, cujus rei ratio physica haec est: Nempe quaelibet guttula etiamsi ascensum incipiens verticalem, non potest tamen, quin sensim ad latera deftectatur & tandem, cum ad summum pervenit, motu feratur horizontali, qui notabilis esse debet, quia per supremum limbum vel sectionem venae aqueae omnis aqua transit, quae per foramen effluxit: fac igitur unicuique guttulae eo temporis puncto quo horizontaliter movetur velocitatem inesse, quam grave lapsu libero per altitudinem b acquirit: ita vides non posse venam ultra altitudinem $a-b$ assurgere: Atque hoc titulo dispendium oritur ratione *potentiae absolutae* totius ut b ad a.

Scholium.

§. 20. Observatum fuit inter aquas communi velocitate ex tubulis diversimode formatis ejectas alias aliis altius assurgere: Ergo hic attendendum est ad ultimorum tubulorum aquas emittentium *(des ajutages)* conformationem aptissimam. Hac de re experimenta instituit D. Mariotte in *Tract. de mot. aquar.*

Scholium Generale.

§. 21. Examinavimus adhuc impedimenta, quae casu superveniunt in machinis hydraulicis aquas cum impetu ejicientibus: Praecipua illa esse puto, quae exposui; poterunt tamen alia insuper excogitari, sed, ut credo, minoris admodum momenti. Ubique fere mensuras dedimus omnino geometricas simulque modum indicavimus, quo iisdem impedimentis maxima ex parte obviam iri possit. Qui majoribus intendit, putans posse minimo labore seu (quod eodem recidere demonstravi §. 3) minima *potentia absoluta* quemvis effectum in elevandis aquis desideratum praestari, opinione fallitur, atque oleum & operam perdet. Si enim ab impedimentis istis expositis aliisve similibus fortasse excogitandis animum abstrahas, machina in rerum natura perfectissima erit simplex antlia Figurae quadragesimae quintae, atque si aquae ejus ope in altum projectae colligantur in *G*, dico fieri non potuisse ut minori labore eadem aquarum quantitas ad eandem altitudinem *FG* elevaretur.

Est deinde aliud machinarum genus, quod a machinationibus adhuc pertractatis differt in eo, quod hae aquas cum impetu ejiciant, illae placide sine motu notabili transferant. Sed & in his ultimus perfectionis qui dari potest gradus eodem recidit. Sunt autem pleraeque multis obstaculis iisque maximi momenti obnoxiae. De his igitur nunc directe nobis erit agendum.

(B) *De machinis hydraulicis aquas sine notabili impetu ex loco humiliori in altiorem transportantibus.*

Regula 10.

§. 22. Si pondus aliquod per datam altitudinem verticalem *(a) potentia movente* utcunque variabili sed directe applicata elevetur nullusque motus in summitate altitudinis propositae corpori supersit, constanter erit eadem *potentia absoluta* in elevationem ponderis impensa, nempe aequalis producto ex pondere corporis elevati & altitudine elevationis *a*.

Demonstratio.

Nam si pondus, quod vocabo *A*, ascenderit per altitudinem *y*, eoque in loco animari ponatur potentia movente variabili *P* directe applicata, moverique velocitate *v*, erit tempusculum, quo pondus per elementum *dy* elevatur, $=\frac{dy}{v}$, quod ductum in *potentiam moventem P* ejusdemque velocitatem *v* dat elementum *potentiae absolutae* (per defin. §. 2) $=Pdy$, ergo $\int Pdy$ dabit totam *potentiam absolutam,* si post integrationem fiat $y=a$; in omni vero motu incrementum velocitatis *dv* est aequale potentae animanti seu moventi, $\frac{P-A}{A}$, quae hic est ductae in tempusculum quod nunc est $\frac{dy}{v}$; habemus igitur

$$dv = \left(\frac{P-A}{A}\right) \times \frac{dy}{v}$$

vel $$Avdv = Pdy - Ady\,,$$

id est $$\tfrac{1}{2}Avv = \int Pdy - Ay\,,$$

sive $$\int Pdy = \tfrac{1}{2}Avv + Ay\,,$$

ubi faciendum est $y = a$ & $v = 0$ (per hypoth.) ita ut sit $\int Pdy = Aa$.

Quia autem, ut vidimus, $\int Pdy$ exprimit integram *potentiam absolutam* in elevandum pondus impensam, erit eadem haec potentia constanter eadem, nominatimque aequalis producto ex pondere *A* & altitudine *a*, ut habet propositio. Q. E. D.

Corollarium.

§. 23. Ex demonstratione nostra apparet, esse quoque *potentiam absolutam* eandem, quoties velocitas in summitate est eadem, id est, quoties altitudo ad quam corpus velocitate sua residua ascendere potest, nempe $\frac{1}{2}vv$, est constans: atque si altitudo ista dicatur *b*, erit *potentia absoluta* $= A(a+b)$. Igitur patet nunc, quanta pars *potentiae absolutae* perdatur, cum animus sit pondus *A* ad altitudinem *a* elevare, idemque in summitate velocitatem residuam habeat debitam altitudini *b;* erit nempe dispendium potentiae ad integram potentiam ut *b* ad $b + a$.

Scholium l.

§. 24. Cavendum itaque est, ne machinae ita sint constructae, ut vehementi motu aquae ad locum destinatum transportentur. Parvum autem esse solet istud dispendii genus in plerisque machinis.

Scholium 2.

§. 25. Omnia similiter se habent si corpus non verticaliter, sed super plano utcunque inclinato, aut etiam curva qualicunque elevetur: semper enim tota *potentia absoluta* erit aequalis $A(a+b)$, id est, producto ex pondere in altitudinem elevationis auctam altitudine velocitati corporis in summitate residuae debita, cujus rei demonstratione supersedeo, quod parum differt a praecedente demonstratione.

Scholium Generale.

§. 26. Quia omnium machinarum utcunque compositarum effectus reduci possunt ad naturam plani inclinati, perspicuum est omnes machinas, si a frictionibus iisque *potentiarum absolutarum* dispendiis, quae hactenus recensuimus, animum removeamus, eodem recidere, quia *potentia absoluta* simpliciter pendet ab altitudine ad quam corpus est elevandum ejusdemque pondere. Habet hoc commune *potentia absoluta* cum *vi viva* seu cum *ascensu descensuve actuali.* Isque ultimus est perfectionis machinarum gradus, quem transgredi non licet, imo nec attingere quidem; semper enim remotis omnibus frictionibus dispendiisque potuisset eadem *potentia absoluta* majus pondus ad eandem altitudinem elevari. Ut jam comparatio institui possit quaedam circa machinarum defectum, tam illarum quae aquas ad desideratam altitudinem veluti projiciunt, quam quae easdem transportant, nunc harum posteriorum defectus maxime notabiles quoque indicabimus.
(I) Frictiones tanto obstaculo sunt in plerisque hujusmodi machinis, ut solae maximam *potentiae* partem absorbeant, praesertim autem cum asserculi quadrati aut globi ovales, catena in circulum redeunte connexi, per canalem, cui sunt accommodati, transeuntes aquas elevant.
(II) Pleraeque machinae, praesertim vero rursus quas modo indicavimus, rosariorum nomine designari solitae , ita sunt comparatae, ut aqua dum elevatur continue pars ejus destillet, sive plane decidat in locum ex quo hausta fuit sive saltem ex loco superiori in inferiorem, uti in rosariis; si in his globuli aut asserculi canali sunt bene adaptati, frictio fit fere insuperabilis, sin minus, maxima aquae quantitas per hiatus relictos destillat ex superioribus divisionibus in inferiores, ita ut minima aquae pars in illis supersit, cum culmen attigerunt, ejus quantitatis quam in toto itinere receperunt. Videntur itaque vel solo hoc nomine istae machinae admodum improbandae, praesertim vero si aquae limpidae sint elevandae, quae antliis hauriri possint.

(III) Solent quoque machinae ejus esse indolis, ut aquam ultra altitudinem propositam attollant: Perditur autem potentia quae excessui respondet, atque si aquae trans molem sunt evehendae, difficulter id obtinetur, quod indicavi §.12.

(IV) Sunt & machinae, quae directam potentiae moventis applicationem non admittunt, ex qua obliquitate rursus dispendium aliquod oritur.

§. 27. Istaque fere sunt, quae notabilis momenti mihi visa fuerunt, obstacula; nescio autem an illis in tantum obviam iri possit, quantum de prima machinarum genere demonstravimus: frictionum diminuendarum artificia quaedam norunt mechanici: machinas quae situlis aquas hauriunt atque elevant praetulerim rosariis: situlae autem ita sint fabricatae, si modo id fieri possit, ut in situ infimo statim impleantur nihilque emittant priusquam ad situm supremum pervenerint. Cum aqua transfundenda est trans locum altiorem in alium minus altum, opera danda est, ut impetus aquae labentis promoveat motum tympani seu rotae in gyrum agendae, quamvis multum absit ut sic omnis *potentia absoluta* utiliter impendatur, prouti fieri antlia Figurae 49 indicavimus

(§. 12). Principium actionis consistet, si recte judico, aptissime in calcatura: homines enim isti labori maxime sunt assueti; pertinet huc, quod monui §. 4 occasione regulae primae de angulo acclivitatis, sub quo viator dato tempore minima defatigatione certam attingere possit altitudinem verticalem. Crediderim hominem mediocris staturae, sanum & robustum super via ad 30 gradus acclivi incedentem non difficulter singulis horis 3600 pedes confecturum, atque proinde ad altitudinem verticalem 1800 pedum pondus corporis sui, quod ponam 144 librarum seu duorum pedum cubicorum aquae, elevaturum. Talis igitur homo poterit ope machinae calcatura circumagendae & perfectissimae (in qua scilicet nihil de *potentia absoluta* perdatur) singulis horis duos pedes cubicos aquae ad altitudinem verticalem 1800 pedum elevare, seu quod idem est, singulis minutis secundis unum ped. cub, ad alt. unius pedis: machinas quae multo minoris sunt effectus, officium facientibus operariis, parum puto commendabiles: Interim instituto experimento in aedibus Ill. D. *General* de Coulon cum antlia, quod in fine sectionis apponam, effectum haud parum minorem expertus sum, quo confirmatus sum in sententia mea operarios calcatura plurimum praestare: facile autem praevideo in machinis admodum compositis longe minorem effectum prodire, quia in his *maxima potentiae absolutae* pars inutilis impenditur. Notabile istius rei nunc afferam exemplum a notissima machina Marlyensi, ostensurus quam incredibile fere *potentiae absolutae* dispendium ab omnibus impedimentis collectis oriatur.

Tractatum edidit Weidlerus *de machinis hydraulicis* in quo plenam descriptionem facit machinae Marlyensis, atque refert omnes aquas elevari a motu 14 rotarum, quarum alae ab impetu sequanae propellantur: hunc impetum facit pro omnibus rotis aequalem ponderi 1000594 librarum, isque est quem nos designavimus nomine *potentiae moventis.* Praeterea alas motu ferri ex aliquibus circumstantiis colligere potui, quo conficiant $3\frac{3}{4}$ pedes singulis minutis secundis, atque haec velocitas habenda est pro velocitate *potentiae moventis;* deinde addit singulis diebus elevari vi illius machinae 11 700 000 libras aquae ad altit. 500 *ped.* His ita positis videamus nunc in machina simplicissima Fig. 45, qua nihil de *potentia absoluta* perdi intelligatur, quanta ad istum effectum potentia *P* pariter velocitate ut $3\frac{3}{4}$ mota requiratur. Erit autem altitudo $FG = 500$ *ped.* & quoniam tempore 24 horarum ejici debeant per lumen *F* 11700 000 librae, id est, 162 500 *ped. cub.,* magnitudo istius luminis ponenda erit $= 0,0108$ partium pedis unius quadrati: Velocitas aquae in *F* tanta est, ut absolvat singulis minutis secundis 173 *ped.* Igitur continet velocitatem $3\frac{3}{4}$, quam pondus *P* habere ponitur, 46 vicibus & toties superare debet amplitudo antliae *AB* amplitudinem luminis *F:* Erit proinde amplitudo *AB* fingenda 0, 4968 *part. ped. quadrat.*, ex quo consequens est, pondus *P* aequale futurum ponderi cylindri aquei super basi *AB* ad altitudinem 500 *ped.* constructi seu ponderi 248,4 *pedum cub.* aquae, id est, ponderi 17 885 librarum, quae tantum quinquagesimam sextam partem efficit *potentiae moventis* quam eadem velocitate motam applicari ostendit Weidlerus. Sic igitur in tota machina dispendium fit quod $\frac{55}{56}$ integrae *potentiae absolutae* exaequat.

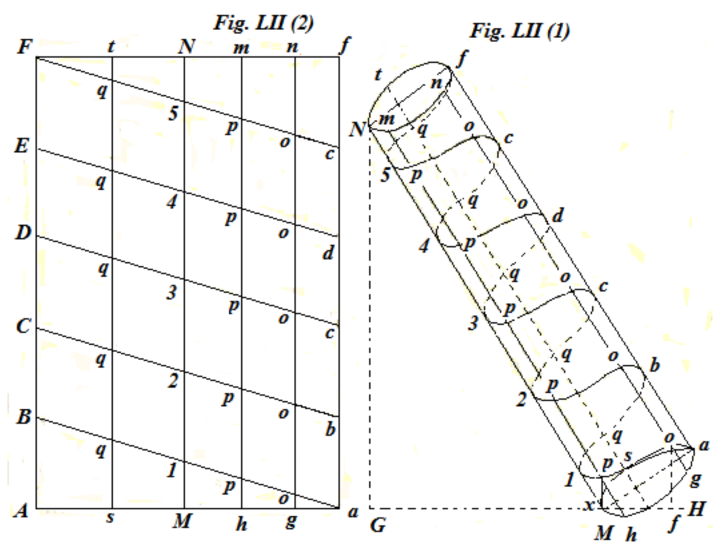
Postquam ita naturam machinarum hydraulicarum, quantum illud in generalibus fieri potest, examinavimus, haud abs re erit exemplum aliquod speciale accuratius pertractare, & quia cochlea Archimedis multis gaudet egregiis proprietatibus, quas nemo satis, quantum scio, aperuit, ab hac exemplum desumam idque eo libentius, quod multi sint, qui contra nostras regulas putant singularem huic cochleae virtutem inesse pro elevanda magna aquae quantitate brevi tempore parvaque vi: falluntur autem qui ita cogitant: nam si obstaculorum accidentalium nulla habeatur ratio, idem praestat eadem *potentia absoluta*, quod reliquae machinae omnes.

Commentationes speciales de Cochlea Archimedis.

(I) Varii sunt auctores, qui modum docuerunt construendi hanc cochleam: summa huc redit, ut canalis quidam aut plures superficiei cylindricae circumflectantur, & ita quidem ut canalis ubique eandem habeat inclinationem ratione axis cylindri, quam Vitruvius praeter necessitatem in omnibus cochleis fieri jubet ad angulum semirectum. Requiritur ergo ante omnia, ut in superficie cylindri linea spiralis ducatur ad cujus normam canalis sit ponendus, id quod facillime meo judicio in superficie admodum polita fieri poterit (praesertim cum helices non parum a se distare debent) circumvolvendo eidem aliquoties funiculum: hic enim tensus sua sponte desideratam lineam faciet, neque enim spiralis sibi similis ubique esse potest, aut constantem habere ad axem cylindri inclinationem, quin arcus inter duo puncta interceptus sit omnium arcuum eosdem terminos habentium minimus, quam indolem funiculo extenso competere palam est: si vero frictiones impedimento sint, filum ad minora intervalla extendi poterit. Sed non est, cur in re per se pluribus modis facillima scrupulosi simus.

Lex spiralis primaria est, ut ubique aequaliter ad axem cylindri inclinet, cui legi sequens innititur constructio, quam in gratiam infra dicendorum apponam.

Finge cylindrum rectum *MafN* (Fig. 52, (1)), cujus superficiei sit inscribenda



desiderata spiralis *a1b2c3d* &c.; eandemque superficiem puta explicatam in planam figura praeditam parallelogrammi rectanguli *AafF* (Fig. 52, (2)); sumantur hic ab una parte *AB, BC, CD, DE* & *EF,* ab altera *ab, bc, cd, de* & *ef,* singulae singulis aequales; jungantur lineis rectis puncta *B, C, D, E* & *F* cum punctis *a, b, c, d* & *e:* his ita factis, si superficies plana rursus in cylindricam convolvatur, junctis lineis *AF* & *af,* coincidentibusque punctis *A* & *a, B* & *b* &c., fiet ut lineae *aB, bC, cD* &c. in superficie cylindrica lineam continuam forment, quae ipsa erit spiralis desiderata. Ad faciliorem intellectum in utraque figura puncta homologa communibus litteris distinxi.

(II) Propositus jam fuerit cylindrus *MafN* (Fig. 52, (1)), habens ad ductum spiralis modo descriptae circumflexum canalem, cujus diametrum veluti infinite parvum censebimus ratione diametri ad cylindrum pertinentis: atque sic habebitur cochlea Archimedis, quasi uti velimus ad elevandas aquas ex *M* in *N*, cylindrus erit horizontern versus inclinandus, & ita quidem ut angulus *aMH* (interceptus inter diametrum baseos *Ma,* quae est in plano verticali, & horizontalem *MH*) sit major quam angulus *sao,* quem faciunt tangentes circuli & spiralis in communi puncto *a.* Deinde converso cylindro circa axem suum in directione *aghMs* aquae influent per inferius canalis circumducti orificium effluentque per superius.

(III) Ut naturam hujus elevationis recte intelligamus, tria se nobis offerunt puncta in qualibet spiralis helice examinanda, nempe puncta *o, p* & *q*, quorum primum *o* maxime distat ab horizonte, alterum *p* eidem proximum est, & *q* in eadem altitudine positum est cum puncto *o* in helice proxime inferiori sumto: per singula puncta *o* ducta est recta *gn*, per puncta *p* recta *hm* & per puncta *q* recta *st.* Situs vero harum linearum determinabuntur in sequentibus.

(IV) Sit radius, qui pertinet ad basin cylindri, $=1$ sumaturque pro sinu toto; sinus anguli $sao=m$, ejusdemque cosinus $=M$, sinus anguli $aMH=n$, ejusdemque cosinus $=N$; arcus $ag=X$; cosinus illius arcus $=x$, erit perpendiculum ex *o* in horizontem demissum, nempe $or=\frac{mNX}{M}+n(1+x)$. Quia vero *or* maxima est, fit

$$\frac{mNdX}{M}+ndx=0,$$

& cum ex natura circuli sit $dX=\frac{-dx}{\sqrt{1-xx}}$, erit

$$\frac{-mNdx}{M\sqrt{1-xx}}+ndx=0,$$

ergo

$$\sqrt{1-xx}=\frac{mN}{Mn}.$$

Est igitur sinus arcus quaesiti $ag=\frac{mN}{Mn}$ aut cosinus $x=\pm\frac{\sqrt{nn-mm}}{Mn}$: signum superius dat arcum *ag,* inferius arcum *ah* determinantem puncta infima *p.*

Atque sic determinavimus tum puncta suprema *o,* tum ima *p,* patetque arcus *Mh* & *ag* esse inter se aequales, simul autem ex quantitate irrationali $\sqrt{nn-mm}$ valorem litterae *x* afficiente colligitur fieri non posse, ut *m* sit major quam *n*: neque enim in hoc casu punctum datur infimum, quod tota spiralis ubique ascendit continue: Neque etiam inserviet sic cochlea ad elevandas aquas; unde jam patet ratio ejus, quod monui in articulo hujus digressionis secundo, de requisito excessu anguli *aMH* supra angulum *sao.*

(V) Ponamus nunc globum alicubi esse in cavitate canalis, cochleamque in situ suo firmari: sic minime quiescet globus, quin existat in puncto aliquo *p*. Quod si vero cochlea non retineri ponatur, globus descendet, descensuque cochleam circumaget, atque si praeterea fingatur, nullius esse ponderis cochleam motumque globi liberrime fieri nihil obstantibus frictionibus, descendet globus super recta *mh* non alia lege, quam globus libere super plano inclinato descendens. Apparet itaque potentiam requiri ad impediendum globi descensum, firmandamque cochleam. Istam potentiam applicatam ponemus in puncto *f* in plano circuli & perpendiculariter ad radium inquisituri in rationem, quam habeat ad pondus globi in puncto aliquo *p* quiescentis.

Sit pondus globi $= p$: quia vero actio globi est verticalis, resolvenda erit in duas alias ad perpendiculum sibi insistentes, quarum una communem habeat cum axe cochleae directionem, altera eidem perpendicularis sit; prior cum nihil ad circumagendam cochleam conferat rejicienda, posteriorque sola consideranda erit; est vero actio illa residua $= np$ & agit in vectem, qui est $=$ sinui arcus seu arcus *ag*, hicque sinus (per *art.* IV) est $= \frac{mN}{Mn}$. Est igitur momentum actionis $= \frac{mN}{Mn} \times np = \frac{mNp}{M}$; hoc si dividas per radium baseos, qui est vectis pertinens ad potentiam applicatam in *f* in aequilibrio positam cum actione globi, habebis istam potentiam quaesitam $= \frac{mNp}{M}$. Sic igitur directe ex natura vectis deducere licet, quod alii ex principio alieno petere solent. Praemissis istis praemittendis usum machinae considerare nunc incipiemus, quem habet pro elevandis aquis.

Problema.

(VI) Quaeritur quaenam maxima sit aquae quantitas quam cochlea quavis revolutione ejicere potest.

Solutio.

Consideremus helicem integram *alb*, sitque quantitas aquae quam plena continet, $= q$*:* Notandum autem est non posse helicem esse totam aqua repletam, si enim totus canalis plenus esset, effluerent aquae per orificium inferius, igitur quivis ramus, qualis est *alb*, partim aëre partim aqua occupatur; erit autem altera aquae extremitas in *o* ceu puncto supremo, altera in *q*, ceu puncto ad libellam cum priori composito: pars igitur aqua plena est *opq*, atque si haec pars ponatur ad longitudinem totius helicis *alb* ut *g* ad *h*, erit maxima aquae quantitas una revolutione ejicienda $= \frac{gq}{h}$. Q. E. I.

Scholium 1.

(VII) Quoniam, ut diximus, fieri non potest ut aqua per totum canalis tractum sit contigua, cavendum est, ne separatio aquae impediatur, quod facile fieri potest cum totum cylindri fundum aquae immergitur, quia sic aëri prohibetur ingressus per orificium inferius canalis: Neque faciendum est, ut nimia fundi pars extra aquam promineat, quia sic cochlea non omnem, quam una revolutione alias posset, aquam haurit; imo nihil hauriet, si immersio punctum *h* non attingat: Debita autem fiet immersio usque ad punctum *g*, quia sic arcus helicis *opq*, qui aquam retinere valet, maximus fit. Etsi enim nunquam rei periculum fecerim, & plerique auctores aliter de illa loqui videantur, malim tamen rationi, quam auctoritati illorum, qui ad immersionem hanc animum non adverterunt, credere.

Regula igitur *ratione immersionis* haec observabitur, fundum nempe submergetur, donec chorda arcus extra aquam emmentis sit $=\frac{2mN}{Mn}$,ubi litterae *m, N, M, & n* idem significant, quod in articulo quarto.

Scholium 2.

(VIII) Apparet quidem post levem rei contemplationem eo majorem esse rationem inter arcum helicis *opq* & integram helicem *alb,* id est, inter *g* & *h,* atque proinde eo majorem ceteris paribus aquae quantitatem singulis revolutionibus ejici, quo minor est angulus *sao* & quo major angulus *aMH,* seu quo minor est distantia inter duas proximas helices & quo magis cochlea versus horizontem inclinat: Veram autem illam rationem algebraice exprimere non licet: In omni tamen casu particulari id facili appropinquatione obtinetur.

Exemplum praecedentis regulae desumam a cochlea, qualem Vitruvius adhibere & construere docet. Facit autem angulum *sao* semirectum & sic $m = M\sqrt{\frac{1}{2}} = 0,70710$: deinde inter *NG* & *MG* rationem statuit, quae est ut 3 ad 4; inde deducitur angulus *GNM* vel $aMH = 53°,\ 8'$, ejusque sinus $n = 0,80000$ atque cosinus $N = 0,60000$: ergo (per *art.* III) est sinus arcus *ag* altissimum punctum *o* definientis $\frac{mN}{Mn} = \frac{3}{4}$, ipseque arcus $ag = 48°,35'$. Debet adeoque vi regulae *art.* VII arcus extra aquam eminens in fundo esse $97°,10'$; immergeturque arcus $262°,50'$.

Ut jam praeterea definiamus rationem inter arcum helicis *opq* & helicem integram *alb,* notandum est, eandem esse illam rationem, quae intercedit inter arcum circularem *ghMs* & circumferentiam circuli, quod ex Figura socia manifestum est. Determinatur autem arcus *ghMs* hunc in modum. Est nempe $\text{arc.}ghMs = \text{arc.}aghMs - \text{arc.}ag$. Sed vidimus in articulo tertio, si ex quocunque puncto spiralis, veluti *o* & *q,* perpendicula ad horizontem punctum *M* radentem demittantur, qualia sunt *or* & *qx,* fore istud perpendiculum $=\frac{mN}{M}+\ n(1+x)$: seu in nostro casu $= 0,60000X + 0,80000(1+x)$, denotante *X* arcum circularem, puncto in spirali assumto respondentem, nempe arcum *ag* aut arc. *aghMs,* & *x* significante ejusdem arcus cosinum. Est vero arc.

$ag = 48°,35'$ = (quia radius exprimitur unitate) 0,84797, ejusque cosinus $= 0,66153$: Igitur in nostro casu fit $or = 0,50878 + 1,32922 = 1,83800$. Quia porro puncta *o* & *q* sunt in eadem altitudine posita, atque lineae *or* & *qx* inter se aequales, apparet quaestionem nunc eo esse reductam, ut alius arcus *aghMs* inveniatur puncto *q* respondens, qui si vocetur *X*, ejusque cosinus *x*, sit $0,60000X + 0,80000(1+x) = or = 1,83800$: pro ista conditione invenitur arcus *aghMs* proxime $175\frac{1}{2}$ *grad.* incidente puncto *s* in plagam *agM:* Et cum arcus *ag* fuerit $48°,35'$, erit tandem arcus *ghMs* $126°,55'$, qui proinde erit ad circumferentiam circuli praeterpropter ut 10 ad 29: similisque ratio intercedit inter arcum helicis *opq* integramque helicem.

Consequens inde est, singulis revolutionibus cochlea a Vitruvio descripta proxime ejici $\frac{10}{29}$ illius quantitatis, quam helix integra & plena continet, seu paullulum ultra trientem.

Scholium 3.

(IX) Notandum tamen est, quaecunque sit aquae quantitas, quae qualibet cochleae revolutione canalem inferius ingreditur, superiusque ex eodem effluit, nullum nec detrimentum nec lucrum propterea cadere in *potentiam absolutam* si nulla habeatur frictionum ratio, quia *potentia movens* caeteris paribus illi quantitati proportionalis est. At vera quia frictiones semper obstant, eaedemque fere sunt ob pondus machinae proprium, sive major sive minor quantitas aquae hauriatur, opera utique danda est, ut ista quantitas caeteris paribus fiat maxima: Hac de re nunc agam paullo disertius.

Scholium 4.

(X) Jam innui supra, crescere rationem arcus *ghMs* ad circumferentiam circuli decrescentibus angulis *sao* & *NMG:* uterque igitur minimus esset construendus, nisi alia obstarent incommoda, praesertim ratione anguli *NMG.* Quod ad angulum *sao* attinet, potest is fere ad lubitum diminui, neque aliud inde incommodum resultat, nisi quod latera canalis circumflectendi nimis ad se invicem accedere possunt: E contrario a diminutione istius anguli aliud obtinetur compendium, nempe quod tunc eo verticalius possit erigi machina ipsaque aqua eo altius elevari, etenim angulus *aMH* semper major esse debet angulo *sao:* a verticaliori autem cochleae positione simul obtinetur, ut minori incommodo sit machinae proprium pondus eaque facilius sustineatur.

Haec ita perpendens crediderim fere sufficere posse angulum 5 graduum, quem faciat canalis cum base nuclei. Cardanus quoque minorem istum fecit angulum quam Vitruvius, & cum eo pauciores super eodem nucleo circumflecti possint canales, quo obliquius sunt inserti, Vitruvius octo, Cardanus tres tantum ponendos statuit : sunt autem canales longiores in cochlea Cardani, ita ut longitudinibus accedat, quod numero canalium decedit. Ratione alterius anguli *NMG* observari meretur, aquam altius elevari posse, quo major iste fiat angulus, sed e contrario minorem aquae quantitatem singulis ejici revolutionibus. Justum fortasse tenebunt medium, qui angulum istum 60 facient graduum.

(XI) Subducemus nunc hujus nostrae quoque ad normam praecedentis articuli constructae cochleae calculum, prouti fecimus de cochlea ad Vitruvii praeceptum constructa, *art.* VIII. Quia vero per hypothesin angulus *sao* est 5^{o} & angulus $NMG = 60°$,reperietur per art. IV arcus *ag* $8^{o},43'$,& linea verticalis $or = 1,00574$, cui aequalis erit altera verticalis *qx,* si dentur arcui $aghMs$ $284°,57'$, a quo si subtrahatur arcus *ag,* remanet arcus $ghMs$ $276°,14'$: qui respondet arcui helicis aquam retinere valenti: est igitur haec pars ad totam helicem ut 16574 ad 21600 vel ut 8287 ad 10800, sic ut singulis revolutionibus ejici possint plus quam quatuor quintae partes integrae helicis capacitatis, duplumque cum triente praeterpropter hac machina efficiatur, quam obtinetur simili machinatione ad mentem Vitruvii fabricata: altius quoque eodem nucleo elevantur aquae in ratione ut $\sqrt{3}$ ad $\sqrt{2}$. Venio jam ad *potentiam* tum *moventem* tum *absolutam,* quae in elevandis aquis impenditur.

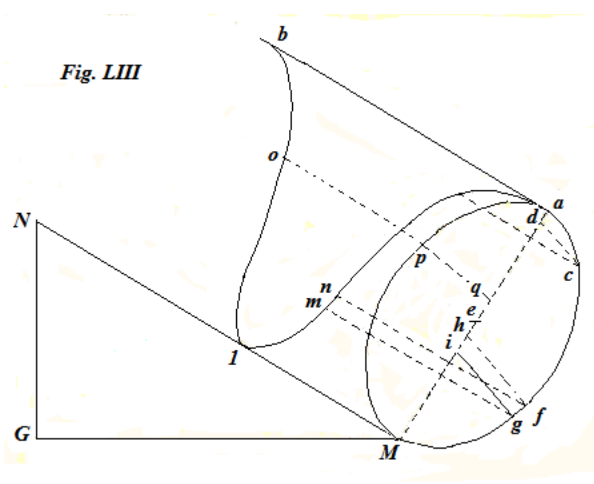
Problema.

(XII) Dato pondere aquae in helice quiescentis, invenire potentiam tangentialem in f in aequilibrio cum illo pondere positam.

Solutio.

Vidimus quomodo problema hoc geometrice solvendum sit ratione globi in puncto infimo *p* quiescentis. In praesenti vero casu paullo aliter se res habet, quod pondus aquae per magnum helicis arcum est distributum, neque in puncto aliquo dato concentratum. Facile quidem est in antecessum praevidere, in utroque casu easdem fore potentias ex regulis mechanicae indirectis; placet tamen hujus rei demonstrationem dare ex natura vectis petitam, quia mechanici eo omnia reducere amant.

Helicem considerabimus *alb* ex Figura quinquagesima secunda seorsim desumtam, ad



evitandam linearum confusionem, conservatis denominationibus *art.* IV adhibitis.

Sic igitur in Figura 53 erit rursus angulus *NMG* angulus quem facit nucleus cum horizonte, cujus sinus $= N$, sinusque anguli $aMH = n$; *alb* est una spiralis circumvolutio: basis nuclei est circulus *acMpa;* sinus anguli *pal* est ut ante $= m$, ejusque cosinus *M;* puncta vero *l* & *o* sunt extremitates aquae in spirali quiescentis & in eadem altitudine ab horizonte posita; ex istis punctis ductae sunt ad peripheriam basis rectae *le*

& *op* ad basin perpendiculares. In parte helicis quam aqua occupat sumta sunt duo puncta infinite propinqua *m* & *n* & per haec ductae sunt rectae *nf* & *mg* rursus ad basin perpendiculares. Denique ex punctis *c, f, g, p* ductae sunt ad diametrum *aM* perpendiculares *cd, fh, gi* & *pq;* atque centrum basis ponitur in *e,* radiusque $ea = 1$. Sit jam arcus spiralis *l1o* aqua plenus $= c$ & consequenter arcus circularis eidem respondens $cMp = Mc$; $al = e$; $ac = Me$; *ad* (seu sinus versus arcus *ac*) $= f$; $aq = g$; pondus aquae in $l1o = p$; $arcus\ aln\ x$; $nm = dx$; $acf = Mx$; $fg = M\ dx$; $ah = y$; $hi = dy$; $hf = \sqrt{2y - yy}$; erit pondus guttulae in $nm = \frac{pdx}{c}$; si vero linea *hf* multiplicetur per sinum anguli *aMH,* dividaturque per sinum totum, habetur vectis quo particula *nm* cochleam circumagere tentat: est igitur vectis iste $= n - \sqrt{2y - yy}$ qui multiplicatus per praefatum guttulae pondus $\frac{pdx}{c}$ dat ejusdem momentum $\frac{npdx}{c}\sqrt{2y - yy}$. Sed ex natura circuli est $Mdx = \frac{dy}{\sqrt{2y - yy}}$: hoc igitur valore substituto pro *dx*, fit idem guttulae *nm* momentum $= \frac{npdy}{Mc}$, cujus integralis, subtractacta debita constante, est $\frac{np(y - f)}{Mc}$ denotatque momentum aquae in arcu *ln;* hinc igitur momentum omnis aquae in *l1o* est $= \frac{np(g - f)}{Mc}$: quod divisum per vectem potentiae in *f* applicatae seu per 1 relinquit potentiam istam quaesitam pariter $= \frac{np(g - f)}{Mc}$. Q.E.I

Scholium 1.

(XIII) Ut appareat, non differre valorem istius potentiae ab illa, quam pro globo ejusdem ponderis *p* invenimus articulo V, nempe $\frac{mNp}{M}$, demonstranda est aequalitas inter $\frac{np(g - f)}{Mc}$ & $\frac{mNp}{M}$ & inter $np(g - f)$ & mNc: ista vero aequalitas deducenda est ex eo, quod extremitates aquae *l* & *o* in eadem ab horizonte altitudine positae sint; inde enim sequitur, ut demonstravimus *art.* IV, esse aggregatum ex arcu *ac* multiplicato per $\frac{mN}{M}$ & ex linea *Md* multiplicata per $n =$ aggregato ex arcu *acMp* pariter multiplicato per $\frac{mN}{M}$ & ex linea *Mq* multiplicata per *n.* Adhibitis itaque denominationibus praecedentis articuli, fit

$$Me \times \frac{mN}{M} + (2 - f) \times n = (Me + Mc) \times \frac{mN}{M} + (2 - g) \times n,$$

vel $$n(g-f)=mNc\,;$$

quae aequalitas demonstranda erat ad demonstrandam aequalitatem potentiarum tum pro globo tum pro aqua in *f* applicandarum.

Scholium 2.

(XIV) Quia potentia $\frac{np(g-f)}{Mc}$ non differt ab $\frac{mNp}{M}$ & quantitas $\frac{mN}{M}$ eadem manet, quaecunque aquae quantitas una revolutione hauriatur aut ejiciatur, erit potentia ista proportionalis eidem quantitati aquae singulis revolutionibus ejectae seu ponderi *p*. Facile quoque demonstratu est, si eadem aquarum quantitas eadem *potentia movente* eademque velocitate ad parem altitudinem verticalem elevetur super simplici plano, quod ad hunc finem debite versus horizontem inclinatum sit, fore ut tempus elevationis quoque idem sit.

Igitur eadem *potentia absoluta* requiritur in cochlea Archimedis, quam super plano inclinato, ad quod omnes machinae reduci possunt, nec ullam habet ista cochlea praërogativam prae reliquis machinis in theoria spectatis. Fortasse in praxi minus est obnoxia incommodis §. 26 indicatis: nequaquam improbo ejus usum, sed nec eam praefero prae antliis Ctesibianis.

§. 28. Intelligitur ex hactenus dictis, quibus titulis una machina alteri praeferenda sit, quemnam machinae perfectionis gradum admittant; ad quid potissimum attendendum sit in illarum constructione & usu; quanta *potentiae absolutae* pars perdatur, aliaque similla: Equidem machinas tantum consideravimus *potentiis* ut dicuntur *animatis* motas: facile autem apparet iisdem legibus subjectas esse machinas, quae ab impetu aquarum, venti, aut ab aquarum gravitatione hujusmodique aliis principiis sunt movendae; semper enim *potentia movens* ducta in tempus & velocitatem puncti cui potentia est applicata, dabit productum ex quantitate aquae & altitudine ad quam ista quantitas assumto tempore elevari possit ope machinae propositae, sepositis impedimentis alienis. Loquor autem de machinis, quibus nihil de *potentia absoluta* perditur; fieri enim potest, ut maxima pars pereat, quod satis ostendimus in superioribus.

§. 29. Apparet exinde aquam ad certam altitudinem elevatam posse rursus suo descensu eundem praestare effectum: effectus autem erit aestimandus ex quantitate aquarum elevandarum & ex altitudine elevationis, sic ut v. gr. descensu 8 pedum cubicorum ex altitudine unius pedis possint totidem rursus elevari pedes cubici ad similem altitudinem aut 4 pedes cubici ad altitudinem duorum pedum, aut unus pes cubicus ad altitudinem 8 pedum & sic utcunque libuerit. Specimen machinae, quae possit aquam ad quamcunque altitudinem elevare minimo aquarum descensu, videre est apud D. Perrault in *Comment. ad Vitruvium lib.* 10, *cap.* 12, quam machinam ut incredibile fere paradoxon affert ejusque inventorem facit D. Franchini ltalum, cujus industria & consiliis in horto Bibliothecae Regiae cum successu constructa fuit. Fundamentum machinae in eo consistit, ut situlae concatenatae & in circulum redeuntes aquam excipiant eamque in locum transportent infimum, ibique effundant, dum alia sitularum series aquas hauriunt & ad locum longe altiorem, minori tamen copia ferunt atque effundunt: perspicuum autem est, seriem priorem si omnes situlae descendentes graviores sint omnibus situlis ascenderitibus, alteram perpetuo in gyrum acturam esse; machinae etiam sunt, quae idem praestant per simplices tubos ope epistomiorum statis temporibus convertendorum, in quam quidem conversionem nulla potentia impenditur. Hujusmodi machinationes describit Carolus Fontana.

At si quis credat posse ex impetu aquarum ex certa altitudine delapsarum & in machinae alas impingentium idem obtineri, is longe aberrabit. Talis machinatio pertineret ad illarum classem, quibus maxima *potentiae absolutae* pars evanescit sine fructu.

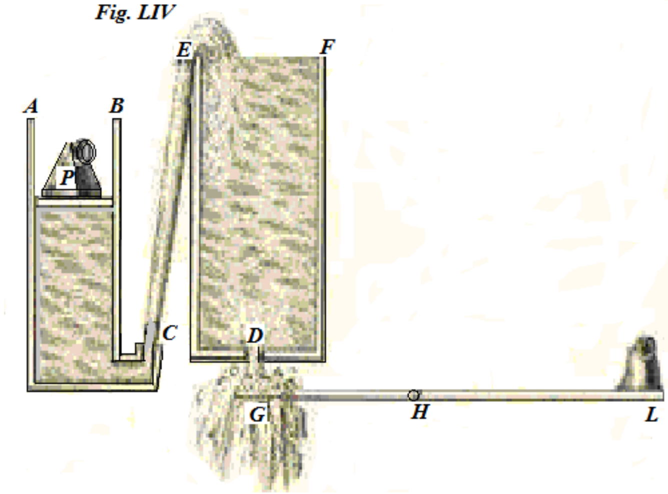
Non abs re erit istud argumentum accuratius prosequi, & ostendere quantus effectus ab impetu aquarum aut venti obtineri possit & sub quibus circumstantiis effectus iste sit omnium maximus dicendus.

(C) *De machinis, quae ab impetus fluidi, veluti vi venti moventur*.

§. 30. Postquam aquae ad certam altitudinem elevatae ex eadem rursus decidunt, continueque in alas rotae circumagendae impingunt, fieri aliter non potest, quin *potentia absoluta* ad rotam sic circumagendam requisita multo minor sit illa, quae in elevationem aquarum impensa fuit, cujus rei praecipua ratio est, quod aquae post impulsum ad latera desilientes velocitatem etiamnum conservent, quae ad rotae rotationem nihil confert. Igitur magna *potentiae absolutae* pars inutilis fieret, si elevatione aquarum efficiendum esset, ut ab impetu earundem machina circumagatur & ab hac denique aquae rursus aliae ad certam altitudinem eleventur; & quidem major minorve pars perit pro diversis circumstantiis, nunquam vero, ut monstrabo, minus quam $\frac{23}{27}$ totius perdetur, si ad normam vulgaris impulsus aquarum aestimationis computus fiat.

§. 31. Statuitur autem communiter si aquae ex cylindro valde amplo per simplex foramen tota sua velocitate, id est, quae toti altitudini aquae supra foramen debeatur, fluant, atque vena statim prae foramine directe impingat in planum, fore ut impetus fluidi contra planum in aequilibrio sit cum pondere cylindri aquei, super foramine ad altitudinem aquae erecti. Experimento quidem fallaci auctores seducti hanc stabiliverunt theoriam omnino falsam. Nolui tamen hic ab illa recedere, quia veram theoriam nondum exposui atque deinceps facile erit exposita nostra theoria calculum corrigere. Liceat igitur, donec suo loco rem rectius perpenderimus, vulgari sententiae, quamvis erroneae, adhaerere. Quo major est impetus fluidi, eo majori ratione erit *potentia absoluta,* quam dabimus, augenda.

Fig. LIV



§. 32. Finge nunc (Fig. 54) vas *ABC* ceu antliam quae aquas per foramen *C* in directione tantum non verticali expellat: aquas autem, cum ad summum pervenerint, ab alio vase *EDF* excipi. In alterius hujus vasis fundo concipe foramen *D*, priori *C* aequale, & in eadem altitudine positum, ita ut tanta aquarum copia effluat per *D*, quanta superius injicitur, ipsumque vas *EDF* constanter plenum servetur. Porro puta aquas per *D* effluentes perpetuo impingere in alas alicujus rotae, quae hoc modo circumacta aquas alias elevet: Loco istius machinae describitur in figura simplex vectis volubilis circa *H*, ponendo talem vectem continue alium atque alium adesse prae foramine *D*, qui aquas excipiat, atque altera sua extremitate aquas hauriat, easdemque ad datam altitudinem elevet. His ita positis inquiram primo in *potentiam absolutam*, quae aquas per foramen *C* ad altitudinem *CE* elevat; deinde quoque in *potentiam absolutam*, quae requiritur in G ad vectem eadem velocitate movendum, qua movetur ab impulsu aquarum *DG*.

§. 33. Sit amplitude foraminis *C* vel $D = n$, amplitude $AB = m$, velocitas aquarum in *C* vel $D = v$, pondus cylindri super foramine *C* aut *D* ad altitudinem *CE* extructi $= p$; tempus fluxus $= t$; erit pondus $P = \frac{m}{n} p$; velocitas, qua pondus dum aquae expelluntur descendit, $= \frac{n}{m} v$; est igitur (per§. 3) *potentia absoluta* in aquas *m* per *C* ejectas impensa

$$= \frac{m}{n} p \times \frac{n}{m} v \times t = pvt.$$

§. 34. Ut jam *potentia absoluta* in gyrationem vectis *GL* circa punctum *H* impensa determinetur, notandum est illam minime sibimet constare; mutari enim a mutata velocitate, quacum vectis circumagitur. Igitur faciemus velocitatem qua extremitas ejus in *G* movetur, $= V$. Hoc autem modo aquae impingere censendae sunt in *G* velocitate $v - V$, atque sic pressionem exercere, quae sit $\left(\frac{v-V}{v}\right)^2 p$ (sunt enim pressiones in ratione quadrata velocitatum fluidi impingentis atque pro velocitate *v* ponitur pressio

$= p$). Ista vero pressio est loco *potentiae moventis;* possumus nempe loco pressionis fluidi ponere pondus vecti superincumbens in *G*, quod sit $\left(\frac{v-V}{v}\right)^2 p$. Istud vero pondus eadem velocitate movebitur qua punctum *G*, nempe velocitate *V*, agitque durante tempore *t:* Est igitur *potentia absoluta* ad rotationem vectis durante tempore *t* & velocitate *V* requisita

$$=\left(\frac{v-V}{v}\right)^2 p\times V\times t.$$

§. 35. Quod si igitur vectis *LG* non immediate circumagitur, sed fluidum ad altitudinem *CE* elevatur, eo animo, ut vena fluidi suo impulsu in *G* vectem circumagendo ab altera parte aquam elevet, erit *potentia absoluta* integra ad *potentiam absolutam* utilem, ut *pvt* ad $\left(\frac{v-V}{v}\right)^2 p$, seu ut v^3 ad $(v-V)^2 V$: eademque se habebit ad partem sui inutilem ut v^3 ad $v^3 - vvV + 2vVV - V^3$.

§. 36. In omnibus fere machinis, quarum principium motus consistit in impulsu fluidi, fieri solet, ut velocitas vectis, ubi fluidi impetum sustinet, seu *V* sit admodum parva ratione velocitatis fluidi *v;* in his autem maxima pars effectus, qui ab eadem fluidi quantitate pari velocitate moti obtineri posset, perditur.

§. 37. Maximus oritur ab impulsu fluidi effectus, sive, quod idem est, maxima fit *potentia absoluta* §. 34 definita, si sit $V = \frac{1}{3}v$; & tunc est ista *potentia absoluta* $= \frac{4}{27}pvt$, atque etiamnum viginti tribus vigesimis septimis partibus deficit a potentia simili, quae in elevandas aquas ex *C* in *EF* impenditur.

Si proinde naturalis habeatur aquarum descensus, atque illo utendum sit ad elevandas aquas aliudve simile quid praestandum, faciendum est ut machina eo in loco, quo fit impulsus, velocitate moveatur subtripla velocitatis fluidi impingentis. Huic vero conditioni semper satisfieri potest, quod ex allato vectis exemplo patet. Si enim majori velocitate moveatur punctum *G*, diminue partem *HG* manentibus reliquis aut eam auge, si minori moveatur velocitate punctum G. Vel etiam salva longitudine *HG* fac, ut aquae in extremitate *L* majori minorive quantitate hauriantur.

§. 38. Ista vero ratione fluidorum ad perpendiculum in alas impingentium: alius est computus pro fluidis oblique incidentibus in alas moletrinarum vi venti agitandarum aliarumque similium machinarum. De his nunc pauca quaedam superaddam atque iis sectioni huic finem imponam.

Quum fluidum in superficiem totius alae utcunque positae & in directione ad motum fluidi perpendiculari rotaturae impingit, docent auctores, fluidum maximum in alam exercere nisum ad promovendam rotationem, quando ala cum directione venti angulum facit, cujus sinus sit ad sinum totum ut $\sqrt{2}$ ad $\sqrt{3}$; si vero vena fluidi eadem atque tota excipiatur ab ala, modo sic modo aliter ad directionem fluidi inclinata, maximam pressionem sustinebit in directione rotationis ala, quae facit angulum semirectum cum directione fluidi.

Prima Regula pertinet ad machinas quae a vento omnia ambiente circumaguntur: altera ad illas, quae a vena solitaria & a certa determinataque fluidi quantitate moventur. Utraque vero hypothesi innititur, quod motus alarum admodum parvus sit respectu motus fluidi; si enim ad motum alarum respicias, ambae regulae falsae sunt; neque profecto iste motus negligendus est, in moletrinis enim saepe observavi, extremitates alarum velocitate ferri, ipsam fere venti velocitatem exaequante. Haec cum ita sint, calculum nunc ita ponemus, ut utriusque motus rationem habeamus.

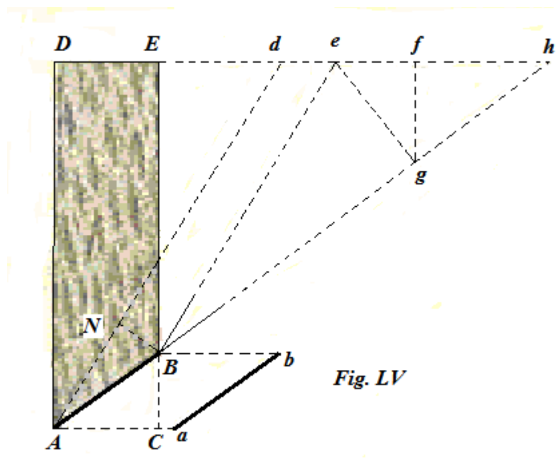


Fig. LV

§. 39. Sit igitur fluidum *DEBA* (Fig. 55) quod sub directione *EB* impingit in totum planum *AB:* moveri autem ponitur planum motu parallelo in directione *Bb* ad *EB* perpendiculari: Sint porro velocitates ejusmodi, ut dum particula fluidi percurrit lineam *EB*, punctum plani *B* absolvat lineam *Bb.* His positis fingere licet totum systema, fluidum nempe cum plano, moveri a *b* versus *B* & quidem velocitate *bB:* Ita vero fiet, ut planum *AB* quiescat, particula autem fluidi in punctum *B* incidens censenda sit venisse ex puncto *e,* sumta $Ee = Bb$, & sic de omnibus guttulis. Igitur loco fluidi *DEBA* in planum motum *AB* incidentis velocitate *EB* concipiendum erit fluidum *deBA* in idem planum *AB* sed immotum incidens velocitate *eB:* Producatur jam *AB* usque in *h* agaturque *DEdeh* perpendicularis ad *EB,* erit motus particulae fluidi repraesentatus per *eB* resolvendus in *eg* & *gB,* sibi invicem perpendiculariter insistentes, quorum posterior nihil in planum *AB* agit, alter vero *eg* rursus ex duobus compositus est motibus *ef &fg,* quorum posterior fg planum *AB* inutiliter in directione *EB* propellere tentat, dum prior *ef* solus idem planum in directione *Bb* propellit. Demonstratum itaque est, quamlibet particulam facere impulsum proportionalem lineae *ef:* Dein patet quoque, si linea *AB* repraesentet totum planum, fore numerum particularum dato tempore in planum impingentium repraesentandum per lineam *BN* perpendicularem ad *Ad* seu *Be.* Unde tandem nisus aquarum ad movendum planum in directione *Bb* est proportionalis lineae *ef* ductae in *BN.*

Ut jam determinetur inclinatio plani ad fluidum sub his circumstantiis maxime favorabilis ut motus plani in directione *Bb* promoveatur: ponemus $AB = 1$, *DE* seu $AC = x$, $BC = \sqrt{1-xx}$; lineam *EB*, quae repraesentat motum fluidi, $= v$, & *Bb* ceu mensuram motus plani $= V$; atque sic instituto calculo invenitur

$$ef = xv\sqrt{1-xx} - (1-xx)V,$$

atque

$$BN = (xv - V\sqrt{1-xx}) : \sqrt{vv+VV};$$

unde

$$ef \times BN = (xv - V\sqrt{1-xx})^2 \times \frac{\sqrt{1-xx}}{\sqrt{vv+VV}},$$

quae quantitas maxima erit, cum fit

$$(9v^4 + 18vvVV + 9V^4)x^6 - \left(12v^4 + 30vvVV + 18V^4\right)x^4 + (4v^4 + 16vvVV + 9V^4)xx - 4vvVV = 0 .$$

§. 40. Calculus ratione inclinationis alarum in moletrinis alius est, quia velocitates in diversis alarum locis variae sunt; sunt enim proportionales distantiis a centro; facile autem nunc cuivis erit computum pro moletrinis instituere; huic casui non ulterius insistam, sufficiat id notasse, quod non satis accurate statuatur ab auctoribus $xx = \frac{2}{3}$. & quod verus valor ipsius x semper minor sit quam $\sqrt{\frac{2}{3}}$. Si fuerit v.gr. $V = v$, & omnia alae puncta simili velocitate moveri censeantur, fiet $x = \sqrt{\frac{1}{2}}$, quod indicat inclinandam esse alam ad directionem venti sub angulo semirecto. Optima alarum constructio foret, si incurvarentur, ita, ut sub angulo minori ventus in illas impingat superius quam inferius, aut si fiat ut alae ubique ventum sub angulo medio quinquaginta praeterpropter graduum excipiant.

§. 41. Pergo ad alterum casum, quo omne fluidum a plano, utcunque id inclinatum sit, excipi ponitur. Hic autem patet, quia numerus particularum dato tempore impellentium semper idem est, nullam esse attentionem faciendam ad lineam *BN*, atque sic nisum quem aquae faciunt ad movendum planum *AB* in directione *Bb* simpliciter repraesentari per *ef* seu $xv\sqrt{1-xx} - \left(1-xx\right)V$. Igitur nisus iste maximus obtinebitur sumendo

$$xx = \frac{1}{2} + \frac{V}{2\sqrt{(vv+VV)}},$$

atque erit ipse nisus tunc

$$= \frac{1}{2}\sqrt{(vv+VV)} - \frac{1}{2}V,$$

si per v intelligatur pressio directa, quam vena exerit in planum cui perpendiculariter occurrit.

§. 42. Consideremus nunc venam *DEBA* tanquam immediate ex orificio *D* in Figura 54 egressam & vocemus rursus directam pressionem venae ita consideratae p, sicut §. 33 ;

atque erit nisus istius aquae, quo conatur planum debito modo, ut nisus maximus fiat, inclinatum propellere in directione ad venam perpendiculari

$$=\frac{p}{2v}\times(\sqrt{vv+VV}-V):$$

Et si porro iste nisus multiplicatur per velocitatem plani V atque tempus, obtinetur *potentia absoluta*, qua planum eadem velocitate per idem temporis spatium moveri queat; sic igitur *potentia absoluta* erit

$$=\frac{pVt}{2v}\times(\sqrt{vv+VV}-V).$$

§. 43. *Potentia absoluta*, quam modo definivimus, ita est comparata, ut continue crescat crescente V, atque si velocitas V infinita sumatur, fit eadem potentia $=\frac{1}{4}\times pvt$. Igitur cum in Figura 54 vena DG uti volumus ad rotandam machinam per impulsum obliquum, nunquam plusquam quarta pars obtineri potest illius *potentiae absolutae*, quae in elevationem aquarum ex C in EF impenditur. Impulsu vero directo nunquam plus quam $\frac{4}{27}$, obtineri vidimus §. 37. Ergo effectus fere duplo major impulsu obliquo seu motu rotae horizontali quam impulsu directo, seu motu rotae verticali obtineri potest.

Si vero impulsus fluidorum aliter aestimetur quam §. 31 indicatum fuit, erit ubique in eadem ratione mutandus valor litterae p, qua impulsus aestimatio fuit mutata.

Experimentum, de quo §. 27 Sect. IX mentionem feci, hoc est. Nempe unus operarius ope antliae intra septem minuta prima cum dimidio pedes cubicos sedecim cum dimidio ad altitudinem quatuordecim pedum everit.

Iste vero effectus aequaliter distributus aequivalet huic actioni, qua dimidius praeter propter pes cubicus singulis minutis secundis elevatur ad altitudinem unius pedis: Hic igitur effectus dimidius admodum est illius, quem hominem sanum & robustum calcatura dare posse ex aliis deduxi principiis in paragrapho decimo septimo. Non crediderim defectum petendum esse omnem a decrementis, quae in *potentiam absolutam* ex variis causis in ista sectione expositis cadere possunt, sed potius ab eo, quod plus defatigentur homines ab agitatione emboli in antlia, quam a calcatura in rota calcatoria.

Experimentum plane simile, sed cum antlia longe perfection artificioque singulari fabricata, ante aliquot demum menses Genevae sumsi praesentibus Viris Clarissimis D. D. De la Rive, Calendrin, Cramer & Jalabert Acad. Genev. Profess.; successus experimenti talis fuit, ut intellexerim operarium unum singulis minutis secundis quatuor quintas partes unius pedis cubici ad altitudinem unius pedis elevasse vel potius effectum aequalem praestitisse. Notabile est experimentum, nec puto ulla alia machina effectum obtineri posse admodum majorem. Mirabile quoque id est, quod sic omnis generis machinas, quacunque potentia animatas, si obstacula demas, effectum haud multo dissimilem praestare appareat. Re bene perpensa statuo, hominem machina perfectissima singulis minutis secundis pedem cubicum aquae ad altitudinem unius pedis elevare posse aut effectum similem producere.

Huc etiam pertinerent, praesertim ratione paragraphi trigesimi primi, experimenta quae accuratissime institui ad aestimandum impetum venae fluidae in planum impingentis, quibus confirmatus fui in theoria nova, quam hac de re stabiliveram, simulque edoctus, errorem e Mariotti temporibus communem fuisse commissum. Quia vero in fine hujus sectionis hac de re non diserte sermo fuit, atque in sectione decima tertia expresse eam pertractare animus est, ideo eo usque disquisitiones hasce, ex principiis mechanicis nondum observatis, erutas differemus.