HYDRODYNAMICS SECOND SECTION.

Which discusses the equilibrium of fluids at rest, both within themselves, as well as related to other causes.

Theorem 1.

§. 1. The surface of a fluid at rest is parallel to the horizontal.

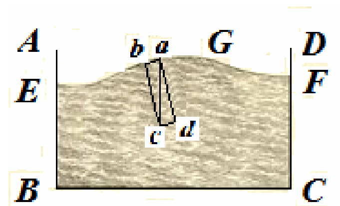


Fig. 1.

Demonstration.

The vessel *ABCD* shall contain the fluid *EBCF* (Fig. 1), of which the surface *EGF*, if it were possible, shall not be parallel to the horizontal : a small drop may be considered at the highest place *a*, which by its gravity is acted on by a force vertically downwards represented by *ac*, this force is resolved into the two collateral forces *ad* & *ab*, the one perpendicular to the surface, and the other which is a tangent to that : But since nothing may be present, which may offer resistance to this latter force, this cannot avoid exerting its influence, and thus the droplet is drawn towards *E*, which shall be contrary to the hypothesis of being at rest, or of an enduring position : Therefore it is necessary, that the force of the tangent *ab* everywhere shall be zero, which cannot happen otherwise, than when the whole surface shall be parallel to the horizontal. Q. E. D.

Corollary.

§. 2. Hence the truth of the general proposition is understood, because evidently the surface of a fluid, of which the parts may be acted on by whatever forces, thus itself is composed always, so that any small drops placed on the surface may be drawn under in a direction perpendicular to the surface.

Theorem 2.

§. 3. A homogeneous fluid enclosed by two pipes of whatever shape are joined together, is composed in equilibrium, when both surfaces are placed on a level, that is, when they maintain an equal vertical distance from the lowest point of the vessel.

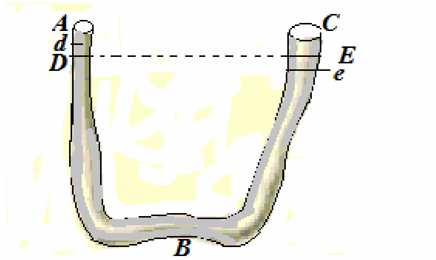


Fig.II

Demonstration.

Let the fluid of the vessel *ABC* (Fig. 2) be enclosed by two legs or composed from pipes joined together, and the fluid may be placed at the same height in each leg : I say that this situation cannot be allowed, whereby the body [*i.e.* of the water enclosed] may receive some weight from a place itself of lower height, because it would be contrary to the nature of gravity : For if the surface *E* may descend to *e,* and from the other side *D* may be raised from *D* to *d,* because the part of the vessel remaining in the same place is filled with fluid remains unchanged before & after the move allowed, the effect in this case of all the change is, that a small amount *Ee* will rise to *Dd.*

Otherwise the same also is likewise apparent from the first theorem, whenever some pipe can be molded into any shape, in which indeed the water shall remain in the place, that it occupied before, either enclosed in the pipe restrained by the sides, or in the surrounding water at rest.

Scholium 1.

§. 4. If in the first demonstration of the preceding paragraph the whole mass may be considered to have changed its position *DBE* into the position *dBe,* it is easily demonstrated that the centre of gravity of the whole mass to have ascended to a higher position, which is no less absurd : Because moreover in our demonstration there is no particle in *Ee,* which will not ascend past the changed position, I have judged the demonstration to be stricter and clearer, if no consideration may be held of the centre of gravity.

Scholium 2.

§. 5. We may consider the phenomena associated with individual capillary tubes ; for water may ascend above the [common] level in a narrower tube, the other extremity being submerged in the water ; indeed mercury does not reach the [common] level. Truly at some time I may consider this matter with the most careful attention, for the meantime I have fallen on the same reasoning, that at one time my uncle Jacob Bernoulli, now blissfully remembered, had given in his treatise *de gravitate aetheris* [*Concerning the weight of the aether*], namely the water in the narrower tube thus ascends above the common level, because the number of particles of the air-aether at the base of the column, which stands above the water in the tube, shall be a smaller number of particles than on a similar base placed just outside the tube ; truly this is understood according to that reason, because with the globules placed adjacent to each other on a horizontal table, if a circular line be drawn, by necessity some of the globules shall be excluded, because they are unable to be divided [*i.e.* they lie either wholly inside or outside the circle drawn]: Truly the downwards forces of the columns of air (of which the base either is either within the tube, or else just outside the tube) are as the [areas of the] bases, that is, as the number of globules within the bases : from which if the number of globules in the former [narrower] base shall be $= a$, in the latter [tube wider base by the diameter of a particle] $= a + b$, then if the downwards force of the inner air-aether column $= g$, then the downwards force of the latter outer column shall be $= \frac{a+b}{a} g$, hence the difference of the downwards forces $= \frac{b}{a} g$, to which the height of the water [in the capillary tube] must be equal above the common level.

So that this may be understood more rightly, it will be required to consider g to be proportional to the square of the diameter, which corresponds to the surface of the fluid enclosed in the tube; and also for the square [of the diameter], on account of the extreme smallness of the globules, to be proportional to a as well, thus so that the ratio g to a shall be agreed to be constant [thus, according to this model, which is of course based on geometrical premises which have little or no bearing on reality, as no account is taken of the force of attraction /repulsion between the liquid and the glass, but which does give rise to a difference in the forces acting on the outside and inside of the tube, the ratio g to a is the weight as it were of a single globule ; thus, we have the simple equation : $\text{downwards force} = \text{wt. of one globule} \times \text{no.of globules in a given area}$; it is convenient to consider unit areas, in which case we can talk about downward pressures or just pressures rather than downward forces, while a and b can be considered as the no. of particles per unit area, and with g defined similarly], and hence the height of the water above the level must follow the proportion of b; this is in fact true, because it is itself apparent that b shall be as the periphery of the surface of the fluid enclosed by the tube,

[This follows from simple geometrical reasoning, for if the inner circle has a radius r, then the no. of particles is proportional to r^2, while the extra particles line within a narrow shell of thickness Δr and of area proportional to $r\Delta r$, which is proportional to b, the 'extra' particles that can be fitted into the shell, and so increasing the force ; hence

$$height \propto \frac{b}{a} \propto \frac{r\Delta r}{r^2} \propto \frac{1}{r}.]$$

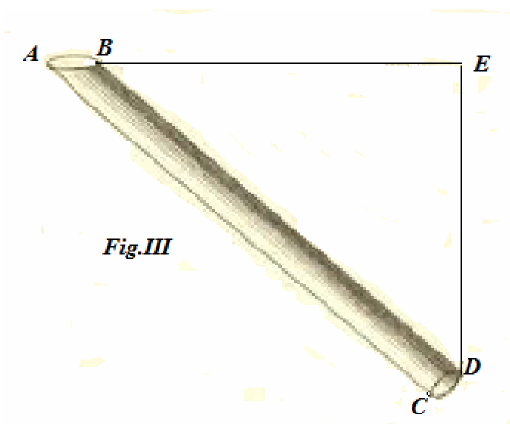
therefore the height above the common level will be as that periphery likewise, as that has been confirmed by experiment for some time now.

[An explanatory argument in German similar to the one presented here can be found in Karl Flierl's work *Anmerkungen zu Daniel Bernoulli's Hydrodynamicae*, p.5; KF in future references; while an exposé in English is presented by G.K.Mikhailov in his introduction in *Die gesammelten Werke der Mathematiker und Physiker der Familie Bernoulli*, *Daniel Bernoulli*, Volume 5, p. 47; edited by P. Radelet-de-Grave and D. Speiser (Birkhäuser). GKM in future references.]

Now if again we may consider different fluids, we will see there the greater the twisting and hence the greater the abovementioned periphery, so that there are more fluid particles, and since the height of the fluid above the common level depends on the magnitude of this periphery, we understand, why this height in the same tube may not follow in the ratio of the inverse specific gravity: thus if the tube may be its end immersed in wine and also in water, the former will ascend less than the latter, yet on account of its smaller specific gravity it ought to ascend more ; truly this shows, if I have followed the matter correctly, the particles of the wine to be smaller than those of water : Yet in no account according to my judgment will I believe that the ascent above the common level can be turned into a descent for any fluid, and all fluids are to be of this same nature in this regard, unless a certain other cause may come about, not yet considered until now, and if from our hypothesis we may argue, it will be required to say, that mercury too should ascend above the common level, but only if its particles may not attracted to each other by a greater force than the particles of water ; for can I attribute everything in fact to this, which make mercury go in the opposite direction ? Experiments, which have guided me to this opinion, I shall append at the end of this section.

Lemma.

§. 6. *ABDC* shall be a cylindrical tube (Fig. 3) inclined to the horizontal in some manner, of which the base *CD* shall be perpendicular to the side of the pipe, and it is understood to be full of water as far as to *AB;* I say that the pressing down force of all the water on the base *CD* to be equal to the weight of a cylinder of water, of which the base is *CD*, and the altitude of which is the vertical line *DE* terminated by the horizontal line *BE.*

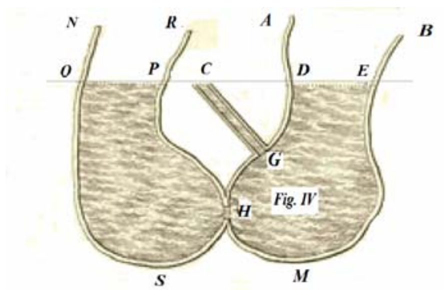


Demonstration.

Since the shape of the pipe shall be cylindrical, and the base in addition shall be perpendicular to the sides of the pipe, in any case it is seen, that the action of the fluid on the base shall be the same as a solid cylinder of the same weight placed on the inclined plane ; but it is agreed from mechanics, that the pressure of the solid cylinder on the base to be that, which is defined in the proposition ; therefore and of such a kind will be the action of the fluid, but only if the adhesion of the fluid to the sides of the pipe may not be considered, and to be of the same nature as in the account of capillary tubes, which we have refrained from considering. Q. E. D.

Theorem 3.

§. 7. Now generally a vessel shall be formed in whatever manner *AHMB* (Fig. 4) and filled with water as far as *DE,* the pressure of the water on the individual particles of the vessel, such as at *G* or *H,* always is equal to the weight of the aqueous cylinder, of which the base is the surface of these particles, and of which the height is equal to the vertical distance of the same particles from the surface of the water.



Demonstration.

In the first place a cylindrical pipe *CG* shall be present standing at right angles to the vessel at *G,* and with *ED* produced, this pipe may be understood to be full of liquid as far as to *C*. If now the vessel may have a hole formed at *G;* each fluid will be in equilibrium (per §.3); therefore the fluid of the pipe *CG* presses just as much towards the interior, as the fluid press from the vessel to the exterior. But the first pressure agrees with the proposed pressure, (by §.6), and therefore likewise with the other.

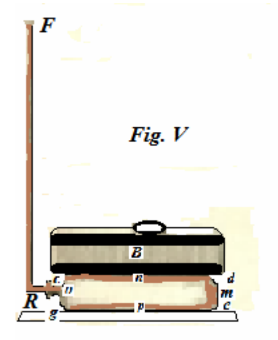
II. Truly if in place of the point *G* some such point *H* may be taken, so that a line, which stands in place perpendicularly in that place of the vessel, lies within a vessel ; then the whole vessel *RHSON* to be considered, in the first place united at *H*, and filled with water as far as *PO.* Thus indeed it is evident, if the particle *H*, which is common to each vessel, may be perforated, the fluid thus shall be in equilibrium (§. 3) and thus the pressure of each on *H* is equal. But the pressure of the fluid in *RSN* is that, which is indicated in the proposition (by the first part of this demonstration), and therefore is the pressure of the fluid, which is in the vessel *AMB.* Q. E. D.

Scholium.

§. 8. From these propositions the equilibriums of fluids at rest are deduced easily in more composite cases. But I am unwilling to pursue everything, not even the account demanded of our investigations, content with the demonstrations of the *fundamental propositions* in hydrostatics, which I have given only. What truly pertains to the pressures of fluids which are not at rest, surely these are [worthy] of deeper investigation. The pressure of a fluid flowing through channels or pipes with a given change of the velocity would still not be determined correctly anywhere [from hydrostatic considerations], although that kind of argument would be of the greatest use both in matters concerning water as well as in many other situations. Truly it is not advisable to act on these situations before we shall have discussed the motion of the fluids.

§. 9. An account of the forces of small bladders is apparent from the preceding, by which they are able to support huge weights : thence also the force is known, that the sides of the pipe sustain, in which the water is at rest ; which argument we will now run through, as it is usually treated by writers on hydrostatics, especially since many other things depend on that, concerning which it will be necessary for us to discuss.

In the first place the bladder *onmp* (Fig. 5) shall be inserted between a hard slab and the weight *B*, into which water is poured through the pipe *FRo*, of which the vertical leg *FR* for the sake of brevity we may put to be of incomparable greater length than the diameter of the bladder. The weight *B* will not be lifted at once, but if the water may be poured in further as far e.g. to *F*, finally the weight is raised ; moreover it will be in equilibrium, since the contact place *cd* is itself in the ratio to the opening *o*, as the weight *B* shall be to the weight of the water cylinder of height *FR* pressing on the base *o*. And thus the determination of the absolute height will depend on the structure of the bladder, which for example if it were composed from perfectly flexible filaments, and with no extension allowed, and if likewise it should have the natural shape of a sphere, it is readily apparent, the contact lengths *cnd* and *gpe* to be equal and & wrinkled, and the remaining expanded part having the form of a spherical zone. And hence the size of the elevation *np* is deduced from geometry, which will be zero, as long as the greatest circle of the bladder shall have a smaller ratio to that orifice *o*, than that which there is between the weight *B* and the weight of the previously mentioned cylinder of water, nor from the beginning will the whole vesicle be unfolded until the height became infinite, that is, never. Truly if the fibers were of a different nature, matters would be otherwise, because many things have not considered well enough, in which the discussion would be about the shape of the inflated bladder, and that they would wish to connect that to the arrangement of small muscular cavities for the animal, concerning which I may respond a little further.



§.10. *DC* shall be the bladder (Fig. 6) and the same weight *P* appended, and likewise with the pipe *DA* attached, the length of which hanging together we may imagine again therefore to be much greater than the length *DC*. With these in place indeed anything you please can be examined easily, with the bladder and the pipe to be filled, so that the former may swell up, and the appended weight *P* may rise: but no one will understand the state of equilibrium, and the belly like figure, unless clearly the structure of the same fibres of the bladder may be understood ; which since thus they may be which can occur more frequently, we will examine some individual cases.

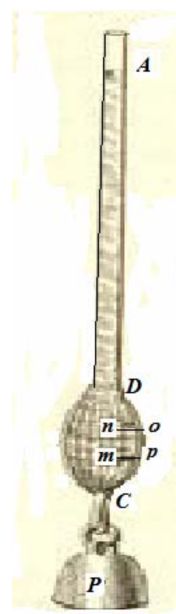


Fig. VI

Case I.

§. 11. If the bladder were composed from longitudinal fibres *DpC*, *DmC* &c. in the image of meridians concurring with the poles as it were at the points *D* & *C*, with perfect flexibility and & uniformity, the individual fibres of which shall be connected with the smallest transverse fibres, and with both thus loose, so that they allow a sufficient extension by the smallest or as if

zero force. Thus any fiber *DpC* will be curved into an elastic shape, and the whole bladder will assume the form of a solid, which is generated from the revolution of this curve about the axis *DC.* Again if the height *AD* is infinite, *DpC* shall be an elastic rectangle and then the maximum thickness of the bladder shall be to the length of the axis *DC* as 25 to 11 very nearly and the length of the arc *DpC* is to the same axis as 5 to 2 approximately, thus so that the maximum elevation of the weight the bladder may be shortened by three fifth parts.
[These ratios come from earlier papers by the author on muscular fibres : see GKM, note 10, p.121 , vol.5 *Werkes der D.B.*]

Case II.

§. 12. If, with the rest put in place as above, the small transverse filaments *no, mp* &c. which are perpendicular to the longitudinal fibres, are resistant to extension, it is apparent that the shape of the fiber *DopC* is unable to be determined, unless the two kinds of general forces may be considered at some point, the first of which shall be acting perpendicular to the curve, and press the filament outwards, the other is perpendicular to the axis of the curve *DC* and pulls inwards : also it is understood easily that an infinite number of laws of these pressures can be devised, so that for some given curve the fiber *DopC* may compose itself, and thus also for example into a circle, and which shape is attributed by most physiologists to the fibrils, which pertain to the small mechanisms of muscles: But there is also another way, by which a fiber of longitude *DopC* is able to acquire the shape of circular arc, truly when all the transverse fibrils *no, mp* &c. are missing. Since thus while the vesicle is being inflated, there may be a gap between the two nearby longitudinal fibres *DopC* & *DnmC*, through which the fluid bursts out, but likewise, since it may not flow out with enough speed, it extends the fibres, and these it composes into a circular figure : and in this case the greatest part of the bladder cut off, which in the former case was $\frac{3}{5}$ of the whole length of the uninflated bladder, now is only approximately $\frac{4}{11}$.

§. 13. From this it follow to be with difficulty, that the shape of the inflated bladder, to which the weight has been attached, may be determined correctly, since there shall be nobody, who shall be able to understand perfectly the nature of the smallest fibrils : yet I will transcribe here certain examples, which may be considered with the greatest plausibility from my leaves without demonstration, as if from which a demonstration may be desired, it will be found in *Book* 3, *Comm. Acad. Sc. Petrop* ; [*CP*1728]. But before everything I will give the equation for the curve, which is formed from two kinds of forces, as I have said in the preceding paragraph, and with these observing some law or other.

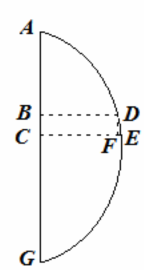


Fig. VII

[A translation of Daniel Bernoulli's *CP* 1728 paper : *A General Method for finding the curvature of a string…..* is provided by me in the contents page for this work, in which these equations are derived.]

§. 14. Therefore the filament *AEG* (Fig. 7) shall be fixed from the two points *A* and *G*; the right line *AG* may be drawn*:* and *D* & *E* shall be two points infinitely close on the filament, from which *DB* & *EC* may be acting perpendicular to *AG ;* moreover the small line *DF* shall be drawn parallel to the line *AG*. It is understood from the individual points *D* or *E* some variable forces any two forces are to be acting, of which the one shall be everywhere perpendicular to the curve and the other perpendicular to *AG* : the former we may put equal to *A* at the point *D*, and at the point *E* equal to $A+dA$, the latter at the point $D=C$, and at the point $E=C+dC$. Again there shall be $AB=x,\ BD=y,\ AD=s,\ BC=dx,\ \ FE=dy,\ DE=ds$, which element of the curve may be considered as of constant magnitude; the radius of osculation at the point $D=R$, and at the point $E=R+dR$. I say that this shall be the equation for the curve :

$$-A\,dR-R\,dA=\left(RdC\,dx+2C\,dyds+C\,dxdR\right):ds,$$

or on putting *CR ddx* for *Cdyds* (indeed there is $R=\frac{dyds}{ddx}$), there will be had :

$$-AdR-RdA=(RdCdx+CRddx+Cdyds+CdxdR):ds,$$

or

$$\frac{-ARds-2RCdx}{ds}=\int Cdy.$$

§. 15. It is understood from the preceding equation, that since the forces which are normal to the curve act alone, there shall be $AR=$constant quantity, truly because thus there shall be $C=0$: therefore then the radius of curvature everywhere follows the ratio of the inverse corresponding potential. But if the forces perpendicular to the axis [abscissa] shall act alone, then the letter *A* shall vanish, and

$$-\frac{2RCdx}{ds}=\int Cdy.$$

But this equation can be integrated and reduced to this form $RCdx^2=$ constant quantity; from which it is apparent the force drawn from the radius of osculation is everywhere in the inverse ratio of the square of the sine, which the applied line makes with the curve.

[See *Prob*. 4, *Coroll*. 2 of the above *CP* 1728 paper:

For the constant of integration, Bernoulli takes gds^2, from which it follows, that

$RCdx^2=gds^2=const.$, as required. In addition, the angle made by an element of the curve to the *x*-axis (i.e. $\frac{dx}{ds}$) is the same the angle between the *y*-axis and the radius of curvature of the element in question.]

Similarly the canonical equation admits to integration, when the forces, which are perpendicular to the axis, are all equal to each other or proportional to the element of the curve *ds*. For thus on putting $dC=0$, there is obtained :

$$-AdR - RdA = (2n\,dyds + n\,dxdR) : ds\ ,$$

on being understood to be multiplied by a constant quantity *n*, from which the equation treated correctly shall become :

$$nydy + mmdy - nsds = ds\int Adx\ ,$$

where *m* is a constant coming from the integration.

If besides the forces normal to the curve there are put forces proportional to the applied lines y, our last equation can be further reduced to this :

$$-dx = \left(2\,ff - \frac{gyy}{h}\right)dy : \sqrt{(2ny + 2mm)^2 - \left(2\,ff - \frac{gyy}{h}\right)^2}\ ,$$

for which the constants *f* and *m* are to be applied in particular cases, while *n* and *g* depend on a relation of the forces at some point: from which if $g = 0$, catenaries arise, and if $n = 0$, the relation will produce elastic curves: indeed generally the equation will be used in the determination of the curvature of a sail of uniform weight, to which fluid is superimposed. The most simple case of this description is, if one puts $f = m = 0$, then indeed there becomes :

$$-dx = \frac{-gydy}{\sqrt{(4nnhh - ggyy)}},$$

or with the integration made with the addition of the due constant,

$$x = -\sqrt{\frac{4nnhh}{gg} - yy} + \frac{2nh}{g},$$

which is the equation for a semicircle, to which the equation will be adapted to the sail according to the following hypothesis [See Prob. 7 of *CP* 1728] :

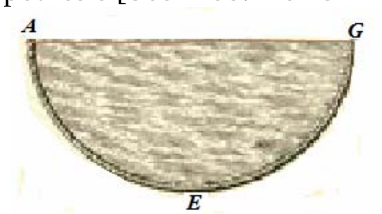


Fig. VIII

The filament of a heavy sail *AEG* (Fig. 8) shall be curved into a semicircle, of which the diameter *AG* shall be placed on the horizontal ; the fluid shall be resting above the filament as far as to *AG*, I say that if the weight of the fluid shall be equal to the weight of the filament, as the filament shall be perfectly flexible and of uniform thickness it will maintain the figure of a semicircle. But whatever the weights of the filament and fluid, so as they shall be equal, it shall be agreed from the elements of geometry that this figure shall be accomplished. And then if it may be put in place, that both the force *A* as well as the force C to be applied everywhere to be proportional to the corresponding *y* (which hypothesis certainly may to be agreed on to the greatest extent when the true figure of the bladder is seen in figure six) again the canonical equation, which contains differentials of the third order, will be able to be reduced to a simpler differential equation and that can be constructed by quadrature. Surely there shall be $A = my$ and $C = ny$, I say the nature of the curve ADG in Fig. 7 to be expressed by this equation :

$$dx = \left(g^3 + \tfrac{1}{2}myy\right)dy : \sqrt{\left(f^3 + \tfrac{1}{2}nyy\right)^2 - \left(g^3 + \tfrac{1}{2}myy\right)^2},$$

in which the letters of constant magnitude f and g again will be produced from integrations : moreover the value of the letter *n* shall become negative, since the equation for the figure of the inflated bladder used is required to be determined.

§. 16. I have no wish to dwell on these matters, which are not concerned closely to Hydrodynamics: also I add nothing about elastic fluids, because I have arranged for the theory of these to be dealt with separately; yet because it touches on the pressures of elastic fluids, these will be able to be deduced and demonstrated easily from the nature of the simpler heavy fluids set out above, by requiring the fluid to be devoid of elasticity, and a cylinder of the same fluid of infinite altitude, or as if of infinite altitude, to be superimposed ; moreover we will say how these shall be required to be understood in its place: Now indeed I will go to that, which it is customary to ask chiefly with regard to matters concerned with water, namely how great must the strength of channels be, so that they shall resist the pressure of water, where especially channels are considered, which bear water to fountains, about which I will give a little advice also.

§. 17. The pressure of water at rest in channels must be distinguished properly from the pressure of flowing water, because I know that until now nobody has paid any attention to that; hence it is the case, that the rules shown by various authors prevail only for fluids at rest, even if from the words they use, they may be able persuade [the reader] that these likewise pertain to flowing water. Truly so that each theory may appear to be distinguished within its own limit, I shall present a certain example, the demonstration of which will be apparent from what follows.

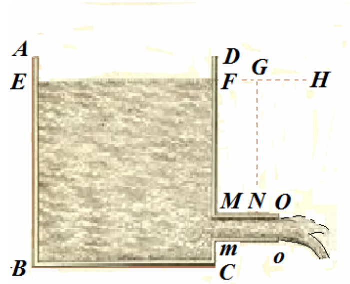


Fig. IX

In place of a reservoir there shall be the largest vessel *ABCD* (Fig. 9) filled with water as far as to *EF*, and constructed in the lower part with a horizontal cylindrical pipe *MOmo*, through which water is understood to be able to flow without impediment ; the limited vertical line *NG* is drawn from the horizontal *EH.* Thus from these preparations, I say that if the whole opening *Oo* may be obstructed by a finger, the point *N* to be pressed outwards according to the whole height *NG;* if half the orifice may be obstructed, that pressure to be diminished to a quarter of its value, and if then with the finger be removed the water may flow freely, all the pressure vanishes, thus so that with authors, the whole may be accustomed to be confused with the part or even with nothing. But I will show that is possible even for the pressure to be negative, and thus to be changed into suction. Truly since I am unable to act on that before I have put the whole theory of flowing water in place, now I will consider water at rest only, as if the whole orifice *Oo* were obstructed.

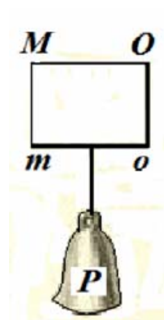


Fig. X

§. 18. Moreover it is agreed from mechanics the sides of the pipe *MOmo* (the diameter of which we consider to be incomparably smaller than the height *NG*) is not to be extended otherwise, than if the sides shall be unfolded into the rectilinear figure *MOmo* (Fig. 10) and they may have the weight *P* hung on, which shall be equal to the weight of a prism of water, of which the three sides shall be : 1st the radius of the pipe, 2nd the length of the same, and 3rd the height of water above the pipe. From this proposition not only an account of the tensions is understood, if the heights of water or the diameters of the pipe were different, but also the measurement of the tensions : Because in the first place if the strength of the pipe shall be greater than that tension, there will be no danger of the pipe rupturing ; if otherwise certainly the pipe will be burst. Besides experiments concerning the strength of pipes have been set up by various authors; but experiments of this kind are difficult and expensive ; therefore it will be easier to determine the strength of pipes of either lead or iron, if by experiment it may be known, how great a weight a lead or iron fiber of a given thickness shall be able to sustain without the danger of splitting. A similar experiment conducted by me placed at the end of the section shows how the strength of a pipe of given thickness and diameter shall be deduced.

Experiments follow pertaining to the second Section.

To §. 5. *About capillary tubules* : Innumerable experiments concerning the nature of these tubules were selected from various sources, among whom George Bernard

Bulfinger is eminent, who not only gathered particular ones together, but also added a lot of his own, see. *Comm. Acad. Sc. Petrop. book.2*,(1729) *pp. 233-287.*

I. So that it may be clearly observed by eye, of how contrary a nature there shall be here on the part of mercury with the remaining fluids, I have carefully looked after the construction of a glass vessel *ABC* (Fig. 11) composed of two vertical legs, of which the one *AB* had a diameter of three or four lines [a line was the twelfth part of an inch], while the other *BC* was scarcely the third part of a line. When the vessel was being filled with some liquid, the height of the surface in the narrower leg was higher than in the wider leg, just as in *D* and *G,* but in the case of mercury alone it was lower in the narrower tube than in the wider one, or as in *F* and *G.*

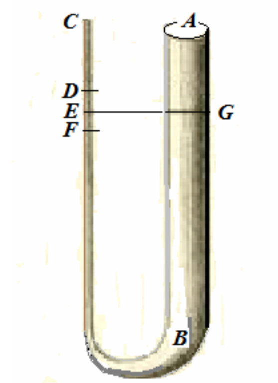


Fig. XI

II. I have considered from these experiments that mercury is not going to be shown to depart from the nature of other fluids on that account, but rather because of the stronger mutual attraction of its particles: certainly I filled up the more slender tube by suction more slowly and with that placed horizontally, and erected it. Thus the mercury ran out, yet at no time all of it and the vertical height of the mercury remaining in the tube to be itself agreed upon in every situation. But which, with the mercury thus suspended in the tube, the ends of the tube may be moved into a vessel with still mercury, all the mercury flowed out at once. The former phenomena, unless I am mistaken, indicate that with mercury and other fluids in contact with the same, there is no place for a force of attraction ; but the final phenomenon shows the strongest attraction between the particles of mercury themselves.

Fig. XII

III. A cylindrical glass tube of 3 or 4 lines diameter, with the bottom constructed from the most fine paper, or prepared from the thinnest iron and perforated with the smallest hole in the middle, as Fig. 12 shows. The tube *ACDB* may be inclined and the whole filled with mercury, then slowly erected ; it becomes as before, and although the tube shall be most full, yet not all of the mercury will flow out, but a part of this will remain suspended, even as *MCDN,* and this thus will be the greater the smaller the small hole *o.* Then when the base is submerged in mercury to a small extend, provided in another vessel, thus so that the submerged part of the tube shall be $C\alpha$, the mercury does not ascend in the tube as far as β (evidently taking $C\alpha = M\beta$) but with almost all flowing out, until the surface *MN* arrives at α . Again the tube *ACDB* was submersed deeply enough in the mercury, which was in the other vessel, but still none began to flow from the vessel into the tube, which was to be submerged to the depth *CM* ; and then thus suddenly it flowed in until each part was placed on the level, evidently as far as *MN,* if it were submerged as far as that place. All these may be deduced from the mutual attraction of the mercury particles. Besides I performed the task in order to investigate the relation which exists between the height *MC* and the size of the small hole *o;* certainly it is plausible that the height be in the inverse ratio to the diameter pertaining to the small hole; yet I was not yet able to confirm the conjecture well enough by the experiment performed, on account of the impure mercury I was using at the time, so that there was no general agreement among themselves with the variation in the hole with the height of the suspended mercury in repeated experiments, then also, because the small holes can be measured with the least accuracy; indeed the small holes must be as small as possible, whenever the height of the suspended mercury is scarcely six or eight lines, since the diameter of the hole is equal to the sixth part of a line. Yet I may say something about the method I used. Namely with brass wires of different thickness, which are used in musical instruments, of which the small diameters are correctly known from the length and the weight of those, I perforated the small paper hole *CD* ; but thus fringes are accustomed to arise around the sides of the small hole which impede the outflow, and it follows readily that the hole shall be greater than the thickness of the wire.

To §.18. *Concerning the strength of the pipes.* Round brass wires, the diameter of which was $\frac{2}{11}^{\text{th}}$ of a Parisian line, to which successively greater weights were appended, were not broken at first until the weight was in excess of 18 *Nuremberg lb.* Then the thinnest lead plate, for which the figure was rectangular, $\frac{5}{4}$ *lin.* wide, $\frac{1}{131}$ *lin.* thick was observed to be broken when a weight was appended to the same of the and a half ounces. And from these two observations it follows for the other parts that the wire made from brass to be more than around 28 times stronger than the wire made from lead. It is deduced from the first experiment also, if the brass tube had a diameter of one foot, and the thickness of the sides were $\frac{2}{11}^{\text{th}}$ *lin.,* that it could sustain a height of 518 feet of water before it might be ruptured. In this calculation I have given a weight of 70 pounds to a cubic foot of water. Truly if the same pipe were made of lead, it would support water to a height of 18 *ft.* for the force of the other observation, and the height of the water will be around 99 *ft.,* if the sides of the pipe should have a whole line in thickness. This agrees with what Mariotte has in his *Tract. de motu aquarum, p.* 472, where indeed he says a lead pipe, the diameter of which was one foot, and the thickness of the side of two and a half lines water could be raised to a height of one hundred feet without bursting ; because it was observed when he abraded the sides of the pipe slowly, then finally when they were diminished to the thickness of a line, so that only then the force of the water ruptured the pipe.

From the observed strength of brass wires also the strength of cannons can be deduced: for example a cannon of which the bore may have a diameter of three inches ; but the thickness of the sides not far from the ignition hole, where the force of the powder is greatest, are accustomed to be approximately equal to the diameter of the bore, thus so that the whole diameter shall be three times the diameter of the bore. Therefore because this thickness is not to be ignored in comparison with the diameter of the bore, we will consider all the material to be concentrated in the middle of the side and thus to be three inches distant from the axis of the bore. With this in place the maximum height of water that the cannon can bear near the ignition hole shall be almost

$$=\frac{11}{2}\times 12\times 3\times 2\times 518=205128,$$

[This follows from the simple proportion of the heads of water H_1 and H_2 to the thickness and inversely as the radius of the bore, where H_1 is sufficient to rupture a cannon of wall

thickness D_1 and radius of bore R_1, and where D_2 and R_2 correspond to H_2, according to $H_2 : H_1 = D_2 R_1 : D_1 R_2$, where $D_2 = 36\, lines; D_1 = \frac{2}{11} lines;$ and $H_1 = 518\, ft.$]
which force is around seven thousand times the natural elasticity of the air [*i.e.* the standard atmosphere. See GKM, note 32].
But in what follows it is shown, gunpowder ignited can exert a force indeed a little greater than what has been said, able to burst cannon, but yet not much greater. But the rest of the strength, which cannon require, is had from belts or bands, which are called *plattes bandes & moulures*, besides that which arose from the first casting of the cannon (*a l'endroit de la culasse*) the thickness shall be greater than what has been assumed by us. Meanwhile we are not surprised that quite a few cannon do rupture.

HYDRODYNAMICAE SECTIO SECUNDA.

*Quae agit de fluidis stagnantibus eorundemque aequilibrio
tum inter se, tum ad alias potentias relato.*

Theorema 1.

§. 1. Superficies fluidi stagnantis horizonti est parallela.

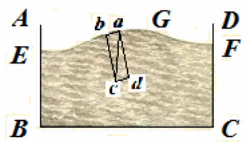


Fig. 1.

Demonstratio.

Contineat vas *ABCD* (Fig. 1) fluidum *EBCF*, cujus superficies *EGF*, si fieri possit, horizonti non sit parallela: consideretur guttula in loco eminentiori *a*, quae gravitate sua verticaliter deorsum sollicitatur vi repraesentata per *ac*, resolvatur haec vis in duas collaterales *ad* & *ab* alteram perpendicularem ad superficiem, alteram quae tangat illam: Cum autem nihil adsit, quod huic vi posteriori resistat, haec non potest non effectum suum exerere, ipsamque adeo guttulam versus *E* trahere, quod esset contra hypothesin stagnationis, seu status permanentis: Igitur necesse est, ut vis tangentialis *ab* ubique nulla sit, quod non aliter contingit, quam cum superficies tota horizonti est parallela. Q. E. D.

Corollarium.

§. 2. Hinc intelligitur veritas propositionis generalis, quod nempe superficies fluidi, cujus partes viribus quibuscunque sollicitantur, se ita semper componat, ut quaelibet guttula, in superficie posita, trahatur sub directione ad superficiem perpendiculari.

Theorema 2.

§. 3. Fluidum homogeneum, duobus tubis communicantibus utcunque formatis inclusum, ad aequilibrium est compositum, quando ambae superficies ad libellam positae sunt, id est, aequalem a puncto vasis infimo distantiam verticalem servant.

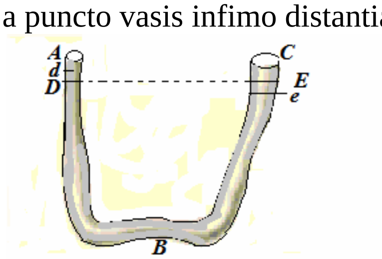


Fig.II

Demonstratio.

Sit fluidum vasi *ABC* (Fig. 2) ex duobus cruribus seu tubis communicantibus composito inclusum, ponaturque in utroque crure ad eandem altitudinem positum: dico non posse situm hunc mutari, quin corpus aliquod grave ex situ inferiori in altiorem se recipiat, quod esset contra naturam gravium : Nam si superficies *E* descendat in *e,* & ab altera parte D ex *D* elevetur in *d,* quoniam pars vasis reliqua eodem fluido ante & post situm mutatum plenum est, omnis mutationis effectus in hoc situs est, quod particula *Ee* ascenderit in *Dd.* Caeterum idem quoque liquet ex Theoremate primo, quandoquidem in aqua stagnante tubus utcunque formatus fingi potest, in quo utique aqua situm servabit, quem antea habuit, cum perinde sit, sive aqua tubo inclusa coerceatur lateribus tubi, sive circumstagnante aqua.

Scholium 1.

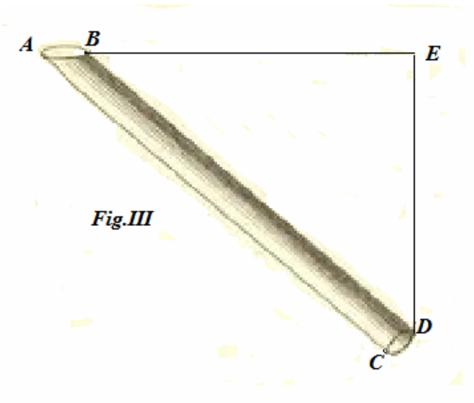
§. 4. Si in demonstratione prima praecedentis paragraphi tota massa *DBE* situm suum commutasse concipiatur cum situ *dBe,* facile demonstrator centrum gravitatis totius massae in situm altiorem ascendisse, quod non minus absurdum est: Quoniam autem in nostra demonstratione nulla est particula in *Ee,* quae non ascenderit post mutatum situm, existimavi strictiorem & clariorem fore demonstrationem, si centri gravitatis nulla consideratio habeatur.

Scholium 2.

§. 5. De tubis capillaribus phaenomena habemus singularia; aqua enim ascendit supra libellam in tubo strictiori, cujus altera extremitas aquae submergitur; mercurius vero libellam non attingit. Haec vero cum aliquando attente perpenderem, in eandem praeter propter incidi causam, quam olim Patruus meus Jacobus Bernoulli beate defunctus dederat in *Tractatu* suo *de gravitate aetheris,* nempe aquam in tubo strictiori ideo ultra libellam ascendere, quod numerus particularum aereoaetherearum in basi columnae, quae aquae in tubo supereminet, minor sit numero particularum in simili basi extra tubum; hoc vero intelligitur ex eo, quod positis juxta se globulis in tabula horizontali, si circino circulus fiat, globulorum aliquot necessario excludantur, quia dividi nequeunt: Sunt vero pressiones columnarum aereo-aetherearum (quarum basis altera est in tubo, altera extra tubum) ut bases, id est, ut numeri globulorum in basibus: unde si numerus globulorum in prima basi sit $=a$, in altera $=a+b$, pressio columnae prioris $=g$, erit pressio alterius columnae $=\frac{a+b}{a}g$, hinc differentia pressionum $=\frac{b}{a}g$, cui aequari debet altitudo aquae supra libellam. Haec ut rectius intelligantur, considerandum erit esse *g* proportionalem quadrato diametri, quae respondet superficiei fluidi tubo inclusi, & eidem quadrato ob extremam globulorum parvitatem proportionalem quoque esse *a,* sic ut ratio *g* ad *a* censenda sit constans, atque proin altitudo aquae supra libellam proportionem sequi debeat ipsius *b;* est vero, quod per se patet, *b* ut peripheria superficiei fluidi tubo inclusi, erit igitur altitudo supra libellam, ut eadem illa peripheria, id quod experientia jam diu confirmavit. Si porro nunc diversa consideremus fluida, videbimus eo tortuosiorem atque proin majorem esse praememoratam peripheriam, quo majores sunt fluidi particulae, & cum a magnitudine hujus peripheriae pendeat altitudo fluidi supra libellam, percipimus, cur haec altitudo in eodem tubo non sequatur rationem gravitatis specificae inversam: ita si idem tubulus immergatur spiritui vini & aquae, ille minus ascendit, quam haec, cum tamen ob minorem suam gravitatem spiritus ascendere deberet magis; hoc vero indicat, si recte rem assecutus sum, minores esse particulas spiritus vini, quam aquae: Nunquam tamen meo judicio ascensus supra libellam in ullo fluido mutari potest in descensum, & omnia fluida ejusdem esse hac in re indolis, crediderim, nisi alia quaedam causa, nondum hactenus considerata, superveniat, & si ex nostra hypothesi argumentamur, dicendum erit, mercurium quoque supra libellam fuisse ascensurum, si modo particulae ejus non majori vi se invicem attraherent, quam particulae aquae; huic enim attractioni omnia tribuo, quae mercurium in diversa ire faciunt? Experimenta, quae ad hanc sententiam me manuduxerunt, apponam in fine hujus Sectionis .

Lemma.

§. 6. Sit tubus cylindricus *ABDC* (Fig. 3) utcunque versus horizontem inclinatus, cujus fundum *CD* ad latera tubi sit perpendiculare, plenusque intelligatur aqua usque in *AB;* dico pressionem omnis aquae in fundum *CD* esse aequalem ponderi cylindri aquei, cujus basis est *CD,* & cujus altitudo est verticalis *DE* terminata ab horizontali *BE.*

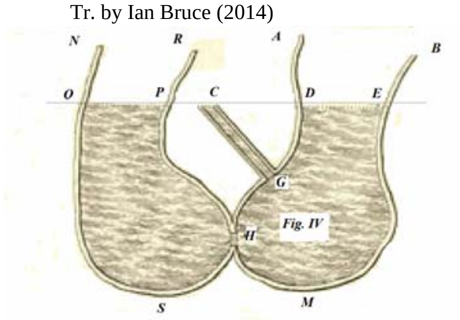


Demonstratio.

Cum forma tubi sit cylindrica, & fundum insuper ad latera tubi perpendiculare, quilibet videt, quod actio fluidi in fundum eadem sit, quam haberet cylindrus solidus ejusdem ponderis super plano inclinato; constat autem ex mechanicis, pressionem cylindri solidi in fundum eam esse, quae in propositione definitur; ergo & talis erit actio fluidi, si modo non respiciatur adhaesio fluidi in lateribus tubi, ejusdemque indoles ratione tubulorum capillarium, a quibus animum abstrahimus. Q. E. D.

Theorema 3.

§. 7. Sit jam generaliter vas utcunque formatum *AHMB* (Fig. 4) & aqua repletum usque in *DE,* erit pressio aquae in singulas vasis particulas, veluti in *G* aut *H,* semper aequalis ponderi cylindri aquei, cujus basis est superficies illius particulae, & cujus altitudo aequalis est distantiae verticali ejusdem particulae a superficie aquea.



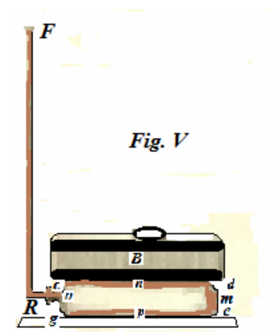
Demonstratio.

Primo concipiatur in *G* tubulus cylindricus *CG* perpendiculariter vasi insistens, productaque *ED,* intelligatur hic tubus simili liquore plenus usque in C. Si nunc fingatur vas perforatum in *G;* erit utrumque fluidum in aequilibrio (per §.3) ; tantum ergo premit fluidum tubuli *CG* versus interiora, quantum premit fluidum vasis versus exteriora. Sed prior pressio convenit propositioni (per §. 6), ergo & altera.

II. Si vero loco puncti *G* sumatur aliud *H* tale, ut linea, quae eo in loco vasi perpendiculariter insistit, cadat intra vas; tunc potest vas integrum concipi *RHSON,* priori unitum in *H,* & aqua repletum usque in *PO.* Sic enim apparet, si particula *H,* quae utrique vasi communis est, perforetur, fluida sic fore in aequilibrio (§. 3) adeoque utriusque pressionem in *H* esse aequalem. Pressio autem fluidi in *RSN* ea est, quae indicatur in propositione (per partem primam hujus demonstrationis), ergo & pressio fluidi, quod est in vase *AMB.* Q. E. D.

Scholion.

§. 8. Ex his propositionibus facile deducuntur aequilibria fluidorum stagnantium in casibus magis compositis. Nolo autem omnia prosequi, neque enim instituti nostri ratio id postulat, contentus demonstrationibus, quas modo dedi, propositionum *fundamentalium* in hydrostatica. Quod vero attinet ad pressiones fluidorum non stagnantium, sunt certe hae altioris indaginis. Necdum a quoquam recte determinata fuit pressio fluidorum, per canales seu tubos dato velocitatis gradu fluentium, quamvis id argumenti genus tam in rebus aquariis quam multis aliis sit utilissimum. De his vero prius agere non licet, quam de motu fluidorum commentati simus.



§. 9. Patet ex praecedentibus ratio potentiarum vesicularium, quibus ingentia pondera superari possunt: Inde etiam noscitur vis, quam sustinent latera tubi, in quo aquae stagnant; quod argumentum, quoniam pertractari solet ab hydrostaticae scriptoribus, nunc percurremus, praesertim cum multa alia eo innitantur, de quibus nobis dicendum erit.

Sit primo vesica *onmp* (Fig. 5) pavimento & ponderi *B* interposita, in quam aqua infundatur per tubum *FRo,* cujus crus verticale *FR* brevitatis gratia incomparabiliter longius ponemus, quam diametrum vesicae: Non elevabitur statim pondus *B;* at si aqua porro infundatur usque v. gr. in *F,* demum attolletur pondus; erit autem aequilibrium, cum locus contactus *cd* se habet ad orificium *o,* ut pondus *B* ad pondus cylindri aquei altitudinis *FR* super basi *o* insistentis. Pendet itaque absoluta elevationis determinatio a structura vesicae, quae si exempli gratia composita fuerit ex filamentis perfecte flexibilibus, extensionemque nullam admittentibus, simulque figuram naturalem habuerit sphaericam, facile apparet, fore spatia contactus *cnd* & *gpe* aequalia & corrugata, partemque reliquam expansam habituram esse formam zonae sphaericae; Atque hinc per Geometriam deducitur quantitas elevationis *np*, quae nulla erit, quamdiu circulus maximus vesicae minorem habuerit rationem ad orificium *o* illa, quae est inter pondus *B* & pondus praefati cylindri aquei, nec prius tota explicabitur vesica quam altitudo fuerit infinita, id est, nunquam. Si vero fibrae alius sunt indolis, aliter se res habet, quod multi non satis considerarunt, quibus de figura vesicae inflatae sermo fuit, eamque cavernulis muscularibus in oeconomia animali applicare voluerunt, qua de re nunc paullo fusius agam.

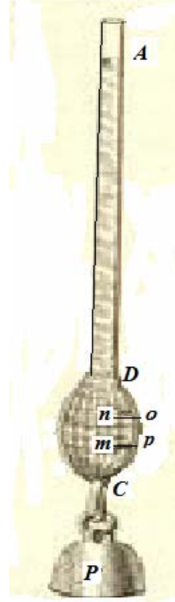


Fig. VI

§.10. Fuerit vesica *DC* (Fig. 6) eidemque appensum pondus *P*, simulque alligata tubulo *DA*, cujus rursus longitudinem compendii ergo incomparabiliter majorem longitudine *DC* fingemus. His positis facile quidem quivis perspicit, repletis vesica & tubulo fore, ut illa intumescat, pondusque appensum *P* elevet: nemo autem intelliget statum aequilibrii, figuramque ventricosam, nisi plane intelligatur structura vesicae ejusdemque fibrarum; quae cum ita sint, casus aliquot singulares examinabimus, qui frequentius occurrere possunt.

Casus I.

§. 11. Si vesica composita fuerit ex fibris longitudinalibus *DpC, DmC* &c. instar meridianorum in punctis *D* & *C*, ceu Polis concurrentibus aequalibus, perfecte flexibilibus & uniformibus, quarum singulae inter se proximae minimis connectantur fibrillis transversalibus, bisque ita laxis, ut minima vel quasi nulla vi sufficientem extensionem admittant. Sic quaelibet fibra *DpC* incurvabitur in figuram elasticae, totaque vesica formam assumet solidi, quod generatur ex revolutione istius curvae circa axem *DC.* Si porro altitudo *AD* est infinita, fit elastica *DpC* rectangula & tunc est crassities maxima vesicae ad longitudinem axis *DC* ut 25 ad 11 praeter propter atque longitudo arcus *DpC* est ad eundem axem proxime ut 5 ad 2, ita ut maxima elevatione ponderis vesica tribus quintis partibus decurtetur.

Casus 11.

§. 12. Si positis caeteris, ut antea, minima filamenta transversalia *no, mp* &c. quae sunt perpendicular[ia] ad fibras longitudinales, extensioni resistant, apparet non posse figuram fibrae *DopC* determinari, quin duo potentiarum genera unicuique puncto applicata considerentur, quorum alterum curvae perpendiculariter insistit, & filum extrorsum premit, alterum ad axem curvae *DC* est perpendiculare & introrsum trahit: facile etiam intelligitur infinitas posse harum pressionum excogitari leges, ut ad curvam quamvis datam fibra *DopC* se componat, atque adeo etiam v. gr. ad circularem, quae figura a plerisque Physiologis tribuitur fibrillis, quae pertinent ad machinulas musculares: Sed est alius etiam modus, quo fibra longitudinalis *DopC* acquirere potest figuram arcus circularis, nempe cum omnino absunt fibrillae transversales *no, mp* &c. Sic enim dum inflatur vesica, hiatus fit inter duas fibras longitudinales proximas *DopC* & *DnmC,* per quem fluidum erumpit, simul autem, cum non satis cito effluere possit. fibras extendit.

easque ad figuram circularem componit: atque hoc in casu maxima vesicae decurtatio, quae in priori casu fuit $\frac{3}{5}$ totius longitudinis vesicae non inflatae, nunc tantum est proxime $\frac{4}{11}$.

§. 13. Sequitur ex hisce, difficile esse, ut figura vesicae inflatae, cui pondus appensum est, recte determinetur, quandoquidem nemo sit, qui indolem minimarum fibrillarum perfecte cognoscere possit: transcribam tamen huc exempla quaedam, quae maxime videntur probabilia, ex schedis meis sine demonstratione, quam si quis desideret, reperiet in *tom.* 3 *Comm. Acad. Sc. Petrop*. Ante omnia autem aequationem dabo ad curvam, quae ex duobus potentiarum generibus, ut dixi in praecedente paragrapho, iisque quamcunque legem observantibus formatur.

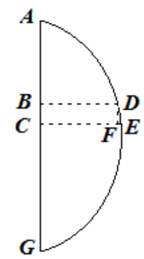


Fig. VII

§. 14. Sit igitur filum *AEG* (Fig. 7) duobus punctis *A* & *G* affixum; ducatur recta *AG:* sintque duo puncta in filo infinite propinqua *D* & *E*, ex quibus agantur ad *AG* perpendiculares *DB* & *EC;* lineola autem *DF* sit lineae *AG* parallela. Intelligatur singulis punctis *D* vel *E* applicatas esse duas potentias utcunque variabiles, quarum altera sit ubique ad curvam, altera ad *AG* perpendicularis: priorem ponemus in puncto *D* aequalem *A*, in puncto *E* aequalem $A+dA$, alteram in puncto $D=C$,
in puncto $E=C+dC$. Sit porro $AB=x,\ BD=y,\ AD=s,\ BC=dx,$ $FE=dy,\ DE=ds$, quod elementum curvae constantis magnitudinis ponatur; Radius Osculi in puncto $D=R$, in puncto $E=R+dR$. Dico aequationem ad curvam fore hanc

$$-A\,dR-R\,dA=\left(RdC\,dx+2C\,dyds+C\,dxdR\right):ds,$$

vel posito *CR ddx* pro *Cdyds* (est enim $R=\dfrac{dyds}{ddx}$), habebitur

$$-AdR-RdA=(RdCdx+CRddx+Cdyds+CdxdR):ds,$$

sive

$$\frac{-ARds-RCdx}{ds}=\int Cdy.$$

§. 15. lntelligitur ex praecedente aequatione, quod cum potentiae, quae sunt ad curvam perpendiculares, solae agunt, fiat $AR=$ constanti quantitati, quia nempe sic fit $C=0$: tunc igitur radius osculi ubique sequitur rationem inversam potentiae respondentis. At si potentiae ad axem perpendiculares solae adsunt, tunc evanescente littera *A* fit

$$-\frac{RCdx}{ds}=\int Cdy.$$

Potest autem haec aequatio integrari & ad hanc reduci formam $RCdx^2=$ constanti quantitati; ex qua apparet potentiam ductam in radium osculi ubique esse in ratione reciproca quadrati sinus, quem applicata facit cum curva. Similiter aequatio canonica integrationem admittit, cum potentiae, quae ad axem perpendiculares sunt, omnes inter se sunt aequales seu proportionales elemento curvae *ds*. Ita enim posito $dC = 0$, obtinetur

$$-AdR - RdA = (2n\,dyds + n\,dxdR) : ds,$$

intelligendo per *n* constantem quantitatem, qua aequatione recte tractata fit

$$nydy + mmdy - nsds = ds\int Adx,$$

ubi *m* constans est ab integratione proveniens.

Si praeterea potentiae ad curvam normales ponantur applicatis *y* proportionales, poterit ulterius reduci postrema aequatio ad hanc

$$-dx = \left(2\,ff - \frac{gyy}{h}\right)dy : \sqrt{(2ny + 2mm)^2 - \left(2\,ff - \frac{gyy}{h}\right)^2},$$

cujus constantes *f* & *m* casibus particularibus erunt applicandae, dum *n* & *g* pendent a relatione potentiarum in puncto aliquo: unde si $g = 0$, oritur catenaria, & si $n = 0$ prodit elastica: generaliter vero inservit aequatio ad curvaturam lintei uniformiter gravis, cui fluidum superincumbit, determinandam. Casus simplicissimus hujus rei est, cum supponitur $f = m = 0$, tunc enim fit

$$-dx = \frac{-gydy}{\sqrt{(4nnhh - ggyy)}},$$

seu facta integratione cum additione debitae constantis,

$$x = -\sqrt{\frac{4nnhh}{gg} - yy} + \frac{2nh}{g},$$

quae est aequatio ad semicirculum, ad quem nempe se linteum accommodabit in sequenti hypothesi:

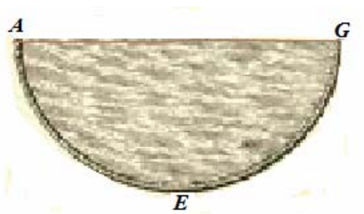


Fig. VIII

Sit filum lintei gravis *AEG* (Fig. 8) in semicirculum incurvatum, cujus diameter *AG* ad libellam posita sit; superincumbat filo fluidum usque ad AG, dico si fluidi pondus sit aequale ponderi fili , fore ut filum perfecte flexile & uniformis crassitiei figuram semicircularem conservet. Quomodo autem pondera fili & fluidi, ut aequalia fiant, efflciendum sit, ex elementis Geometriae constat. Denique si statuatur tam potentias *A* quam C esse ubique applicatae respondenti *y* proportionales (quae hypothesis sane maxime convenire videtur cum vera figura vesicae in figura sexta) poterit rursus aequatio canonica, quae continet differentialia tertii Ordinis, reduci ad aequationem simpliciter differentialem eamque per quadraturas facile construendam. Sit nempe $A = my$ & $C = ny$, dico naturam curvae *ADG* in Fig. 7 exprimi hac aequatione

$$dx = \left(g^3 + \tfrac{1}{2}myy\right)dy : \sqrt{\left(f^3 + \tfrac{1}{2}nyy\right)^2 - \left(g^3 + \tfrac{1}{2}myy\right)^2},$$

in qua literae constantis magnitudinis *f* & *g* rursus ab integrationibus prodierunt: fit autem valor literae *n* negativus, cum aequatio ad vesicae inflatae figuram determinandam adhibetur.

§. 16. Nolui his nimis insistere, quod non proxime pertinent ad Hydrodynamicam: Nihil etiam addo de fluidis elasticis, quia horum theoriam seorsim tradere constitui; attamen quod ad pressiones fluidorum elasticorum attinet, poterunt illae ex natura fluidorum simpliciter gravium supra exposita facile deduci & demonstrari, fingendo fluidum elasticitate esse destitutum, cylindrumque fluidi similis altitudinis infinitae vel quasi infinitae superincumbere; haec autem quomodo intelligenda sint suo loco dicemus: Nunc quidem pergo ad id, quod in rebus aquariis potissimum quaeri solet, quanta nempe debeat esse firmitas canalium, ut pressioni aquae resistere possint, ubi praesertim considerantur canales, qui aquas ad fontes vehunt. de quibus ego quoque pauca monebo.

§. 17. Probe distinguendae sunt pressiones aquarum in canalibus stagnantium a pressionibus fluentium, quamvis id nemo adhuc animadverterit, quod sciam; hinc est, quod regulae a variis exhibitae valeant tantum pro aquis stagnantibus, tametsi verbis utantur, quae perinde eas pertinere ad aquas fluentes persuadere possint. Ut vero discrimen utriusque Theoriae appareat in ipso limine, exemplum quoddam afferam, cujus demonstratio ex inferioribus patebit. Sit loco castelli vas amplissimum

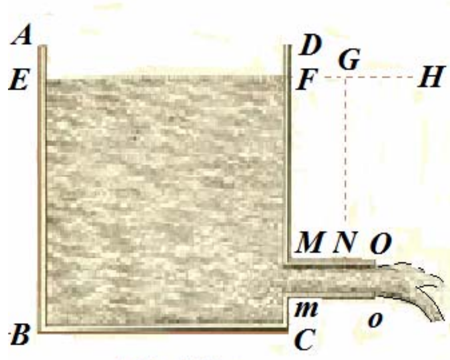


Fig. IX

ABCD (Fig. 9) aqua repletum usque in *EF*, & in parte inferiori tubulo cylindrico horizontali *MOmo* instructum, per quem aquae sine impedimento transfluere posse intelligantur; ducatur verticalis *NG* terminata ab horizontali *EH*. His ita praeparatis, dico si orificium *Oo* totum digito obstruatur, punctum *N* premi extrorsum secundum totam altitudinem *NG;* si dimidium orificium obturetur, hanc pressionem quarta sui parte diminui, & si denique remoto digito aquae liberrime effluant, omnem pressionem evanescere, sic ut totum cum parte aut etiam cum nihilo confundi ab Authoribus soleat. Sed demonstrabo posse pressionem vel negativam fieri, atque ita in suctionem mutari. Quoniam vero id agere non possum priusquam integram theoriam de aquis fluentibus praemiserim, nunc aquas considerabo saltem stagnantes, veluti si orificium *Oo* totum fuerit obstructum.

§. 18. Constat autem ex Mechanicis latera tubi *MOmo* (cujus diametrum incomparabiliter censebimus minorem altitudine *NG)* non aliter tendi, quam si explicata essent in figuram rectangularem *MOmo* (Fig. 10) appensumque haberent pondus *P,* quod sit aequale ponderi prismatis aquei, cujus tria latera sint 1°. radius tubuli, 2°. longitudo ejusdem & 3°. altitudo aquae supra tubum. Ex hac propositione intelligitur nonsolum ratio tensionum, si diversae fuerint altitudines aquae aut diametri tuborum, sed & ipsa tensionum mensura: Quod si proin firmitas tuborum major sit ista tensione, nullum erit rupturae periculum; si secus certo rumpetur tubus. Caeterum de firmitate tuborum experimenta instituta fuerunt a variis; sunt autem ejusmodi experimenta difficilla & sumtuosa; poterit igitur facilius firmitas tuborum sive plumbeorum sive ferreorum cognosci, si experimento innotescat, quantum pondus filum plumbeum aut ferreum datae crassitiei sustinere possit sine rupturae periculo. Experimentum simile a me institutum apponam in fine sectionis ostensurus quomodo inde firmitas tubi datae crassitiei & diametri deduci possit.

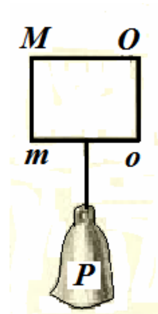


Fig. X

Sequuntur experimenta quae ad Sectionem pertinent secundam.

Ad §. 5. *De tubules capillaribus* : Experimenta innumera de horum tubulorum indole a variis sumta fuerunt, quos inter eminet Georgius Bernhardus Bulfingerus, qui non solum praecipua collegit, sed & plurima de suis addidit, vid. *Comm. Acad. Sc. Petrop. tom.2, pag. 233 & seqq.*

I. Ut oculis recte appareret, quam contrariae sint indolis hac in parte mercurius & reliqua fluida, confici curavi vas vitreum *ABC* (Fig. 11) ex duobus cruribus verticalibus compositum, quorum alterum *AB* diametrum habebat trium linearum vel quatuor, alterum *BC* vix tertiae partis lineae. Cum vas liquore quocunque implebatur, superficies altius erat in crure strictiore quam ampliore, veluti in *D* & *G,* mercurius autem solus depressior est in strictiore quam ampliore, vel uti in *F* & *G.*

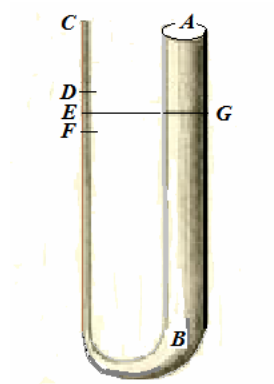


Fig. XI

II. Ostensurus mercurium non aliam ob rationem a natura aliorum fluidorum recedere, quam ob fortiorem particularum suarum mutuam attractionem, cogitavi de his experimentis: tubulum nempe gracilem mercurio suctione implevi eumque horizontaliter positum sensim erexi:

Sic effluxit mercurius, nunquam tamen omnis & altitudo verticalis mercurii in tubulo residui in omni situ sibi constabat. Quod si autem, cum mercurius in tubulo sic suspenditur, extremitas tubi mercurio in vase stagnanti admovetur, protinus omnis effluit. Priora Phoenomena, ni fallor, indicant mercurio & aliis fluidis idem contingere, cum vi attractrici nullus est locus; mercurium autem fortissime se attrahere docet phoenomenon ultimum.

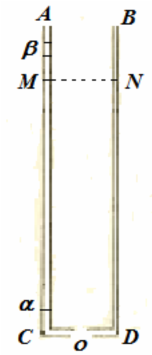


Fig. XII

III. Sumatur tubus cylindricus vitreus diametri 3 aut 4 linearum, fundo instructus ex charta subtili, aut tenuissima lamina ferrea parato & in medio minimo foraminulo perforato, ut ostendit Figura 12. Inclinetur tubus *ACDB* & impleatur totus mercurio, dein sensim erigatur; fiet quod antea, & quamvis tubus sit amplissimus, non tamen effluet omnis mercurius, sed suspensa haerebit ejus pars, vel uti *MCDN*, haecque eo major erit quo minus est ejus foraminulum *o.* Dein cum fundum submergitur mercurio, in vase alio servato, tantillum, sic ut pars submersa tubi sit $C\alpha$, non solum non ascendit mercurius in tubo usque in β (sumta scilicet $C\alpha = M\beta$) sed & omnis fere effluit, donec superficies *MN* pervenit in α . Porro tubum *ACDB* vacuum sat profunde mercurio, qui erat in vase alio, submersi, nec tamen prius quicquam influere coepit ex vase in tubum, quam ad altitudinem *CM* esset submersus; & tunc statim eo usque influit donec ab utraque parte ad libellam sit constitutus, nempe usque in *MN*, si ad eum locum usque erat submersus. Omnia haec ex mutua particularum mercurialium attractione facile deducuntur. Caeterum dedi operam ut investigarem relationem, quae est inter altitudinem *MC* & amplitudinem foraminuli *o;* verisimile utique est altitudinem illam esse in ratione reciproca diametri ad foraminulum pertinentis; nec tamen experimento conjecturam satis confirmare potui, tum ob impuritatem mercurii quo utebar, quae faciebat, ut non variato foramine in iteratis experimentis altitudo suspensi mercurii sibimet ipsi non omnino constaret, tum etiam, quod difficile est foraminula minima accurate metiri; debent enim foraminula esse minima, quandoquidem altitudo mercurii suspensi vix est sex octove linearum, cum diameter foraminis sextam partem lineae aequat, dicam tamen qua methodo usus fuerim. Filis nempe aeneis, quibus in instrumentis musicis utuntur, diversae crassitiei, quorum diametros minimas ex longitudine & pondere eorum rectissime cognovi, chartulam *CD* perforavi; sed sic solent oriri circa latera foraminis fimbriae quae effluxum impediunt, & facile succedit ut foramen majus sit quam est crassities fili

Ad §.18. *De firmitate tuborum.* Filum aeneum rotundum, cujus diameter erat $\frac{2}{11}$ *lin. Paris.,* cui successive pondera continue majora appendebantur, prius non disruptum fuit, quam ad 18 *lib. Norimb.* pondus excrevisset. Dein tenuissimam lamellam plumbeam, cui rectangularis figura erat, $\frac{5}{4}$ *lin.* latam, $\frac{1}{131}$ *lin.* crassam rumpi observavi cum eidem appensum esset pondus trium unciarum cum dimidia. Ex hisce observationibus duabus sequitur caeteris paribus filum ex aere plus quam 28 vicibus fortius esse, quam filum ex plumbo. Ex priori experimento quoque deducitur, si tubus aereus diametrum habuerit unius pedis, & crassities laterum fuerit $\frac{2}{11}$ *lin.,* posse eum aquam sustinere ad altitudinem 518 pedum priusquam rumpatur. In hoc calculo dedi pedi cubico aquae pondus 70 librarum. Si vero idem tubus fuerit plumbeus, sustinebit aquam ad altitudinem 18 *ped.* vi alterius observationis, poteritque altitudinem aquae ferre 99 *ped.,* si latera tubi habeant in crassitie lineam integram. Convenit hoc cum eo quod Mariottus in *Tract. de motu aquarum, p.* 472, habet,ubi nempe dicit tubum plumbeum, cujus diameter unius erat pedis, & laterum crassities duarum linearum cum dimidia sine ruptura aquam tulisse ad altitudinem centum pedum; quod cum observaret abrasit sensim latera, donec tandem ad lineae crassitiem essent diminuta, & tum denique vim aquae tubum disrupisse.

Ex observata fili aenei firmitate colligitur etiam firmitas tormentorum bellicorum: fuerit v. gr. tormentum bellicum cujus animae diameter habeat tres poll.; solent autem haud procul a lumine accensorio, ubi maxima est vis pulveris, crassities laterum esse praeterpropter aequales diametro animae, ita ut diameter tota sit tripla diametri animae. Quia igitur crassities haec non est negligenda prae diametro animae, censebimus materiam omnem concentratam in medio atque sic ab axe animae distantem tribus pollicibus. Hoc posito erit altitude maxima aquae quam tormentum haud procul a lumine accensorio ferre potest $=\frac{11}{2}\times 12\times 3\times 2\times 518=205128$, quae vis fere septies millies superat elasticitatem aeris naturalis. Ostendam autem in sequentibus, pulverem pyrium accensum vim exercere posse ad rumpendum tormentum aliquantum quidem majorem, quam quae dicta fuit, sed non multum tamen excedentem. Reliquum autem firmitatis, quod requirunt tormenta, habent a cingulis seu fasciis, quae dicuntur *plattes bandes & moulures*, praeter id quod in primo ortu tormenti (*a l'endroit de la culasse*) crassities major sit quam quae a nobis assumta fuit. Interim non pauca tormenta disrumpi, sic non mirabimur.