# HYDRODYNAMICS SECTION TWELVE.

*Which presents the static properties of moving fluids, what I call static-hydraulics.*

[The theorems presented here are hence of a time independent nature, and relate to the conservation of *vis viva*, or in modern terms, the conservation of the total potential & kinetic energy of water in an experiment. Thus, the rudiments of what is now known as the Bernoulli Principle are presented here; the missing ingredient being the idea of streamlines, later introduced by Euler in his *Neue Gründsatze der Artellerie*, along which lines the sum of these energies is considered to be conserved.]

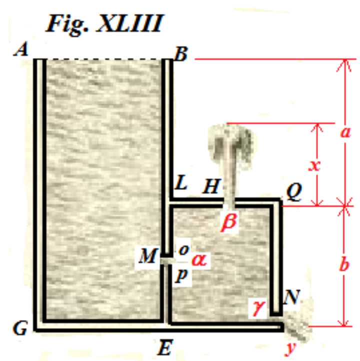
§. 1. Among those, who have given the measures of the pressures of fluids at rest within a vessel, few have traveled beyond the rules of common hydrostatics which we have shown in the Second section: yet there are many others [*i.e.* rules], which thus properly may be said to belong to hydrostatics, just as the centrifugal force is connected with the action of gravity, or the force of inertia, so we have commented on each in the preceding section : and dead forces of this kind may be able to be considered and combined in an infinite number of other ways.
[Recall that Leibnitz originally had the notion of *living* and *dead* forces; essentially the former could make something happen, or as we say now, possess some form of available energy to do useful work, and the latter were associated with action-reaction pairs and the like, which do no work, such as here the normal pressure exerted on the wall of a vessel by the water within, etc.] Truly these are not what may be considered to be greatly desired by me : since it shall not be difficult to give all the general rules concerning that matter. I wish rather for the static rules of a moving fluid, which may be moving within a vessel in a progressive motion, such as the water flowing through tubes to leaping fountains : indeed this has many uses, nor has it been examined by anyone, or if anyone should have made mention of that, then that has been explained by them with minimal correctness: for those who discussed the pressure of water flowing through aquaducts, and who required confirmation for the pressure they were said to be required to support, the laws were handed down for a fluid considered with no motion, and not otherwise.

§. 2. It is peculiar in this *static-hydraulics,* that the pressure of the water cannot be defined in the first place, other than the motion were properly known, which is the reason why this understanding lay concealed for such a long time ; for authors investigating the motion of water were little concerned until now, and almost everywhere the velocities were estimated from the height of the water : but often some motion would approach that velocity so quickly, that the accelerations were unable to be distinguished sensibly, and it may be seen that all motion is generated in an instant, yet it is a concern, that these accelerations may be understood correctly, because otherwise the pressures of the water flowing often cannot be defined; and therefore I have estimated the matter to be of maximum concern from the beginning of the motion as far as to the end, for all these momentary changes are to be considered with care, and to be confirmed by experiments, which I have done everywhere in this treatment, but especially in the third section.

§. 3. If the motion may be defined everywhere, it would be easy to form the most general state of motion of the fluids: for if you may imagine a hole, but that infinitely small, at that place itself for which the pressure of the water may be desired, in the first place you may seek with how great a velocity the water shall erupt through that small opening and to which height that velocity must be due: moreover, you know indeed this height itself to be proportional to the pressure that you seek.

From this principle the pressure is desired which the horizontal plate *LQ* in figure 43 sustains, if there were no perforation : for after it was demonstrated by us in the second corollary of paragraph 31 of the eight section, if the hole *H* were in an infinitely small ratio to the holes *M* and *N*, and the ratio of these holes *M* and *N* [to each other] may be indicated by $\alpha$ & $\gamma$, the height due to the velocity of the water erupting through *H* shall be

$$=\frac{\alpha\alpha\times LB-\gamma\gamma\times NQ}{\alpha\alpha+\gamma\gamma};$$




thence we will judge the pressure of the water on the plate *LQ* without the hole to be proportional to this height itself: which likewise in another way we have given a demonstration in paragraph 19 of the cited section: Hence it follows that it can happen, the plate *LQ* may suffer no pressure, however great the magnitude of the height above that the height of water should be, evidently when $\gamma=\alpha\sqrt{(LB:NQ)}$, indeed the pressure can be changed into a suction.

[Thus the opening at *H* is considered small enough to be ignored as far as the amount of water ejected by it, yet it gives a measure of the pressure at that point. Thus we see a connection arising between the square of the speed of the water and the pressure exerted. This is the main reference point, as it can be determined by experiment; for if $p$ is the pressure immediately under *LQ*, in which case the fountain at *H* rises to the height $\frac{p}{\rho g}$ : hence,

$$\frac{V_\alpha^2}{2g}+\frac{p}{\rho g}=a=LB;\ \text{and}\ \frac{V_\alpha^2}{2g}+\frac{V_\gamma^2}{2g}=a+b=LB+NQ;$$

In addition, we have from the continuity equation:

$$\alpha V_\alpha=\beta V_\beta+\gamma V_\gamma\ \text{ or}: V_\alpha=\frac{\beta}{\alpha}V_\beta+\frac{\gamma}{\alpha}V_\gamma;$$

Hence, if we ignore the cross-section $\beta$ in comparison with $\alpha$ and $\gamma$, the height of the fountain erupting from *LQ* is given by :

$$\frac{p}{\rho g}=a-\frac{V_\alpha^2}{2g};\text{ and } V_\alpha^2=\frac{\gamma^2}{\alpha^2}V_\gamma^2,\text{ giving }\frac{p}{\rho g}=a-\frac{\gamma^2}{2g\alpha^2}V_\gamma^2\text{ and } p=\rho ga-\frac{\gamma^2}{\alpha^2}V_\gamma^2;$$

Again, $\frac{V_\alpha^2}{2g}+\frac{V_\gamma^2}{2g}=a+b;\quad V_\gamma^2=\frac{2g\alpha^2\left(a+b\right)}{\left(\gamma^2+\alpha^2\right)}$; while the height

$$\frac{p}{\rho g}=a-\frac{\gamma^2}{2g\alpha^2}V_\gamma^2=a-\frac{\gamma^2\left(a+b\right)}{\left(\gamma^2+\alpha^2\right)}=\frac{a\left(\gamma^2+\alpha^2\right)-\gamma^2\left(a+b\right)}{\left(\gamma^2+\alpha^2\right)}$$

$$=\frac{a\alpha^2-b\gamma^2}{\left(\gamma^2+\alpha^2\right)}=\frac{\alpha\alpha\times LB-\gamma\gamma\times NQ}{\alpha\alpha+\gamma\gamma}\text{ as above.]}$$

§. 4. Similarly the pressure of the water may be obtained on the plane *LQ*, even if this opening were a hole in a finite ratio of both the remaining magnitudes. For if the plane were perforated by an infinitely small hole besides that at which is at *H*, the water will erupt through each hole with a common velocity: And since this common velocity shall be known (by §. 27 Sect. VIII) for the opening *H*, the velocity also is had, by which the water must erupt through the tiny opening, which truly we conceive, and thus we know the pressure of the water. For example the openings *M*, *H* and *N* were equal to each other, moreover the height *BL* to the height *NQ* had the ratio as 10 to 3, the pressure on the lamina *LQ* will be a tenth part of that, which it is with the openings *H* & *N* stopped up.

[In this case we have the full continuity equation,

$$\alpha V_\alpha=\beta V_\beta+\gamma V_\gamma\text{ or}: V_\alpha=\frac{\beta}{\alpha}V_\beta+\frac{\gamma}{\alpha}V_\gamma;$$

while from the *vis viva*, or from the conservation of energy principle, we have

$$\frac{V_\alpha^2}{2g}+\frac{V_\gamma^2}{2g}=a+b;\text{ giving } V_\alpha^2+V_\gamma^2=2g\left(a+b\right);$$

$$\frac{p}{\rho g}=a-\frac{V_\alpha^2}{2g};\quad\frac{V_\alpha^2}{2g}+\frac{V_\beta^2}{2g}=a,\text{ giving } V_\alpha^2+V_\beta^2=2ga;\text{ also we have :}$$

$$V_\gamma^2=2g\left(a+b\right)-V_\alpha^2\text{ and } V_\beta^2=2ga-V_\alpha^2;$$

$$\text{or } V_\gamma=\sqrt{2g\left(a+b\right)-V_\alpha^2}\text{ and } V_\beta=\sqrt{2ga-V_\alpha^2};$$

Hence, $V_\alpha=\frac{\beta}{\alpha}V_\beta+\frac{\gamma}{\alpha}V_\gamma=\frac{\beta}{\alpha}\sqrt{2ga-V_\alpha^2}+\frac{\gamma}{\alpha}\sqrt{2g\left(a+b\right)-V_\alpha^2}$.

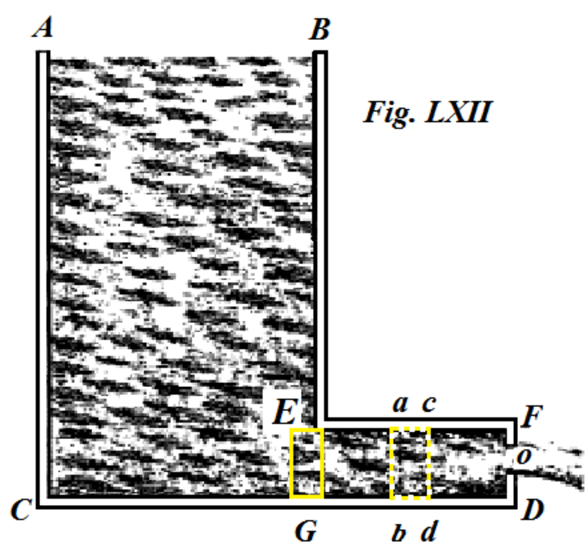
If we have $\alpha=\beta=\gamma=1$, then

$$V_\alpha = V_\beta + V_\gamma = \sqrt{2ga - V_\alpha^2} + \sqrt{2g(a+b) - V_\alpha^2}.$$

That is , if we set $a:b = 10:3$, then ; if the openings *H* and *N* are stopped up, then there is only the hydrostatic pressure, and the pressure on the lamina *a* will be $\frac{10}{13} \times \rho g h$ .]

Finally if you wish to find the pressure of the water at some other place, at least you should add the height, by which the plate *LQ* rises above that place, to the height thrown by the opening *H.* The same method serves to determine the water pressures in the remaining vessels, which we have treated in Section 8. But all these questions are different from these, which are concerned with the motion of fluids through conduits, because the water on account of the infinitely large cross-sections of the vessels put in place by us remain as if at rest in the cavities and nevertheless may exert a far different pressure than they are accustomed to do otherwise. But in water tubes their pressure can change more, when water can flow past with a greater velocity, and it exerts nearly all the customary pressure, if the velocity shall be very small.

Thus these pressures can be determined from the velocities of the fluid by the methods treated by us above. But this matter requires to be treated by an individual method , when water flows through a tubes, and this way of teaching I understand mainly by the name of *static-hydraulics* : Here not only can the pressure be defined from the velocity as inversely, if a small hole may be made in the sides of tubes, the velocity can be defined from the pressure. And in the present section I have decided to act chiefly on that *static-hydraulics*, the use of which is the fullest.




Problem.

§. 5. The widest vessel *ACEB* (Fig. 72) was required to be kept filled constantly with water, with a tube *ED* inserted horizontally into the cylinder*;* and at the end of the tube there shall be a hole *o* discharging water with a uniform velocity; the pressure is sought on the side of the tube *ED.*

Solution.

The height of the surface of the water *AB* above the opening *o* shall be $=a$; the velocity of the water flowing out at *o*, if you should ignore the first instants of the flow, being considered to be uniform and $=\sqrt{a}$; because we have assumed the vessel to be kept full ; and with the ratio of the cross-section of the tube to its hole put $=\frac{n}{1}$, the velocity of the water in the tube $=\frac{\sqrt{a}}{n}$ : Truly if the whole end *FD* were missing, the final velocity of the water in the same tube would be $=\sqrt{a}$, which is greater than $\frac{\sqrt{a}}{n}$ ; therefore the water in the tube approaches the greater motion, but it is impeded by the pressure from its adjacent end *FD*: the water is compressed by this pressure and by the resistance, which compression is itself constrained by the sides of the tube, and these thence likewise sustain a similar pressure. Thus it is apparent the pressure of the sides to be proportional to the acceleration or to the increment of the velocity, which the water shall take, if the whole obstruction to the motion may vanish in an instant, thus so that it may be ejected into the air at once.

The argument now leads to this, so that if during the flow of water through *o* the tube *ED* may be discontinued at *cd* for an instant, it shall be inquired how great an acceleration thence is going to be received by the droplet *abcd*: for the particle *ac* may experience just as much pressure from the sides of the tube from the water flowing past: Hence finally it is required to consider the vessel *ABEcdC,* and from that the acceleration of the particle of water flowing out will be found approximately, if this should have the velocity $\frac{\sqrt{a}}{n}$: This very argument we have dealt with most generally in the third paragraph of Sect. V. But yet as in this case the calculation is particularly brief, so we will undertake this calculation again for the motion in the shortened vessel *ABEcdC.*

The velocity of the water in the tube *Ed,* which now is required to be considered as a variable, shall be $=v$, the cross-section of the tube as before $=n$, with the length $Ec=c$: the length of an infinitely small particle of water *ac* flowing out next by *dx*: An equal droplet will be going to flow into the tube at *E* at the same instant of time when the other is ejected *acdb*: but while the droplet at *E,* the mass of which $=ndx$, enters the tube, it acquires a velocity *v,* and the *living force nvvdx,* which whole *living force* was to be produced anew; for on account of the infinite cross-section of the vessel *AE,* the droplet at *E* had no motion before it entered the tube: to this the increment of the living force *nvvdx* is required to be added to the living force, which the water takes in *Eb*, while the droplet *ad* has flowed out, surely *2ncvdv*: but the sum for the actual descent must be of the drop *ndx* through the height *BE* or *a*: therefore the equation is obtained:

$$nvvdx + 2ncvdv = nadx$$

or

$$\frac{vdv}{dx} = \frac{a - vv}{2c}$$

[Thus, there is a length of water present in the tube initially, which has the *vis viva* $mv^2 \to ncv^2$ which has its value increased by the amount $2ncvdv$ by the inflowing droplet; in addition, the droplet on entering, brings with it the extra vis viva $nvvdx$; the sum of these two increments must be equal the actual descent of the added droplet falling through the height *a*.]

But in the whole motion the increment in the velocity *dv* is proportional to the pressure considered in the time increment [$= dt$], which here is $\frac{dx}{v}$ : therefore in our case the pressure which the droplet *ad* experiences is proportional to the quantity [$= \frac{dv}{dt} =$] $\frac{vdv}{dx}$ [*i.e*. the acceleration of the droplet], that is, to the quantity $\frac{a - vv}{2c}$.

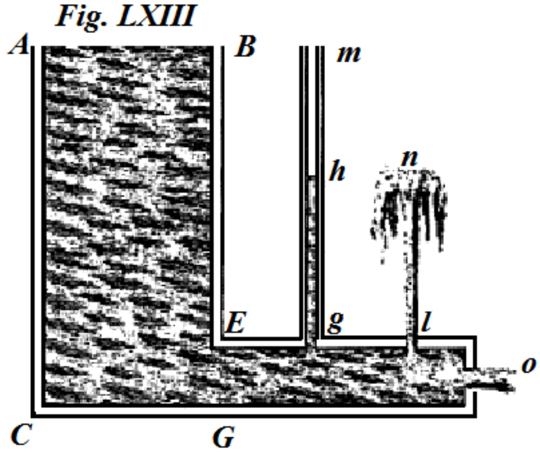
Truly at this instant of time , when the flow in the tube is interrupted, the velocity [as discussed above, for the velocity of the jet in the tube] is $v = \frac{\sqrt{a}}{n}$ or $vv = \frac{a}{nn}$; therefore this value is required to be substituted into the expression $\frac{a - vv}{2c}$, which thus will change into this other form $\frac{nn-1}{2nnc}a$. And this is the quantity, to which the pressure of the water against the particles of the tube *ac* is proportional, whatever the cross-section the tube may have, or by whatever hole its base were perforated. Therefore if the pressure of the water were known in a single case, likewise it would become known in the rest of the cases ; but we have such a case, surely when the opening is infinitely small or *n* is infinitely greater in the ratio of one : for then it is apparent, the water can exercise its own pressure, which agrees with the whole height *a*, and this pressure we will designate by *a:* but when *n* is infinite, the number one vanishes besides the number *nn,* and the quantity to which the pressure shall proportional shall be $= \frac{a}{2c}$ : Therefore if we wish to know generally, how great the pressures shall be when *n* is a number of any kind, such is required to be put in place by proportion. If the quantity $\frac{a}{2c}$ agrees with the pressure *a*, what the pressure shall be for the quantity $\frac{nn-1}{2nnc}a$ : And thus the pressure sought is found $\frac{nn-1}{2nn}a$

Q.E.I.

Corollary 1.

§. 6. Because the letter *c* has gone from the calculation, it follows that all the parts of the tube, both these which are closer to the vessel *AG*, as well as those which are more distant, to be pressed on equally by the water flowing past, and indeed lesser than the parts of the base *CG* : and there the difference is greater, where the opening *o* shall be greater: and the walls of the tube shall sustain no further pressure, if in this the whole obstruction *FD* shall be absent, thus so that the water may flow out through the full opening.

Fig. LXIII




Corollary 2.

§. 7. If the tube may be perforated somewhere by the smallest hole, and indeed with such a ratio of the opening *o*, the water may leap out which may be able to ascent to the height $\frac{nna-a}{nn}$, but only if no other impediments stand in the way: Evidently the height of the throw, will be as in Figure 73, or $ln=\frac{nna-a}{nn}$. Truly if a small tube *gm* shall be present, either vertical or inclined at some angle, communicating with the horizontal tube, but thus still, so that the end of the tubule inserted may not extended into the hollow of the horizontal tube, lest the water flowing past may be pushed against that end, the vertical height *gh* of the water sticking equally in the tube will be of equal height $\frac{nna-a}{nn}$ : and neither is it necessary in this latter case, that the small tube *gm* shall be exceedingly narrow.

[From the continuity equation, if the velocity in the tube is *V*, while in the opening *o* it is *v*, we have $V=\frac{v}{n}$; also we have from the conservation of *vis viva* at the opening : $v=\sqrt{a}$, in which case $V=\frac{v}{n}=\frac{\sqrt{a}}{n}$. It then follows from the conservation of *vis viva* we then have the pressure in the tube $p$ given by : $v^2 = V^2 + p$; or $p = a - \frac{a}{n^2} = a\frac{n^2 - 1}{n^2}$. Here the density of the water is taken as 1, and the usual term $2g = 1$ applies.]

Scholium.

§. 8. Therefore this theory can be confirmed most easily be experiment, with this becoming of greater interest, because nobody at this time has defined equilibriums of this kind, of which the uses appear to be the widest: because by the same method the pressure of the water flowing through the most general channels shall be able to be obtained for water led up whatever inclination, without curvature, and with whatever variation of the cross-section and with the velocity of the water of whatever magnitude; then also, because not only these magnitudes of the pressures, but also the whole of the above theory of motion, that we have deduced above, may be confirmed by experiments of this kind, because they prove correctly from our definitions, to be the accelerations of the water. But it is required to take care in the experiment, that the horizontal tube shall be well polished inside, perfectly cylindrical and horizontal: and it shall be wide enough, so that a notable decrease of the motion cannot arise from the adhesion of the water to the walls of the tube: the vessel itself shall be the widest and shall be kept continually full. It is required to be observed too, by what force still water is raised in the glass tube *gm*, which force is present in all capillary or exceedingly narrow tubes : for this height is required to be taken away from the height *gh*: or rather assuming the tube is of equal thickness and with the opening *o*, it is to be noted the point *m*, and then by the flow of the water the point *h* is required to be noted also : but following the theorem the fall in the pressure shall be

$$mh = \frac{1}{nn} \times a = \frac{1}{nn} \times EB.$$

Finally also it is required to attend to the jet of water flowing out from *o*; for a contraction of this also is made, so that the water in the horizontal tube may flow out with a smaller velocity than $\frac{\sqrt{a}}{n}$. But I have dealt with that contraction in the manner shown in Sect. IV. But although it can occur from these inconveniences, that no noticeable error remains in the experiment, yet if we wish to use greater accuracy, the quantity of water flowing out in a given time is required to be investigated in the experiment, which with the cross-section of the tube prepared correctly will give the velocity of the water flowing within the tube, as we have put in the calculation $= \frac{\sqrt{a}}{n}$ : Truly if from an experiment less were found, surely such as must be due to the height *b*, then the pressure of the water flowing past will be approximately $= a - b$.

Corollary 3.

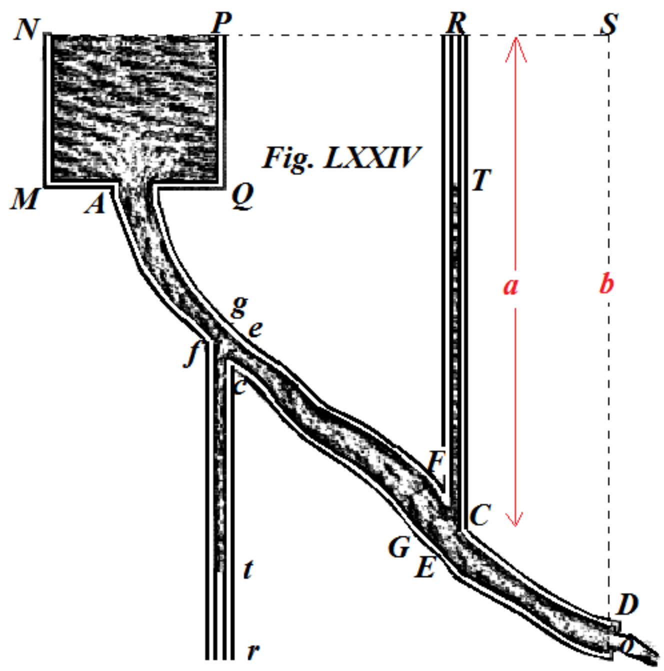
§. 9. If the opening at *o* first may be obstructed with a finger, and later the flow of water may be allowed, the pressure *a* is changed in the first instant of the flow into the pressure $\frac{nna-a}{nn}$ : Truly this change of the pressure does not happen in an instant; indeed if it is required to be said precisely, it happens only after an infinite time, because, as we have observed in section five, the velocity of all the water, of the amount assumed by us corresponding to the whole height *a*, is never present accurately: for it tends towards that velocity with an unbelievable acceleration after the first droplets have been ejected, thus so that the whole amount, as much as can be decided by the senses, may be considered to have been acquired without any sensible delay, unless the water channels shall be very long, for then the accelerations of the water can be distinctly judged by eye, concerning which I gave an example in Sect. V, §.13. Therefore in these water channels leading from a distant reservoir to a leaping fountain, if the pressures may be found there by experiment at various places in the manner I have discussed above, indeed the pressure may be found quickly, yet not in the instant of being diminished, and the pressure will require an interval of time to become known.

Truly so that the pressure of the water may be defined more generally, that velocity is required to be put for *v*, that the water has at that place itself and at a point of time, for which the pressure may be sought, and if that velocity may be understood to be appropriate for the height *b*, the pressure of the water will be $=a-b$.
From which with the present brought together with these which were treated in section five, one will be enabled to define how great the pressure is going to be at individual moments.

From these we are not in the dark to foresee the *static-hydraulic* laws of this, and if both the shape of the vessel and the velocity of the water flowing through the channels may be imagined of whatever kind as it pleases. Evidently the pressure of the water will be constantly $=a-b$, where by *a* the height owed to the velocity is understood, the efflux with which the water shall be emerging from the channel abruptly, after an infinite time and with the vessel kept filled with water constantly, and by *b* the height owed to the velocity, with which the water actually flows out. Certainly this wonderful rule is the simplest to which nature may aspire, and hitherto was able to lie hidden. Now therefore I will show that more expressly.

Problem.

§.10. To find the pressure of the water, along a channel of whatever form and inclination, and with whatever velocity of uniform flow.




Solution.

Let *ACD* (Fig. 74) be the channel through the hole of which *o* the water may be considered to flow with a uniform velocity and of such which may be due to the vertical height *oS: SN* may be drawn and the vessel *NMQP* may be imagined to be infinitely wide and full of water as far as at *NP*, from which channel the water may be drawn off perpetually and uniformly : therefore I imagine these thus, so that a cause of the uniform propelling force may be present, which may propel the water with a given speed or may maintain the uniform flow of the water: And without this hypothesis our problem would become indeterminate, because the same velocity can be generated in the same channel in infinite ways for a point of time, and therefore, so that a measure of the cause propelling the water may be had, uniformity is required to be considered in the motion of the water.

Now the water pressure shall be required to be defined at *CF* (or *cf*)*:* and towards this end we may consider again the channel to be interrupted at the section *CE* (or *ce*) to be considered perpendicular to the channel, whether the droplet *CEGF* (or *cegf*) shall be receiving an acceleration or a retardation after the first instant of the break: for the cause of which generally we need to define the momentary motion through the curtailed vessel *NM*[*AEC*]*QP* (or *NM*[*Ace*]*QP)*. Therefore let the velocity of the infinitely small drop *CEGF* (or *cegf)* from the shortened point $= v$ : its mass $= dx$ : the *living force* [*vis viva*] of the water moving in the shortened vessel shall be proportional to the quantity *vv*, and hence we may make that $= \alpha vv$ , on understanding by the letter $\alpha$ some constant quantity, which depends on the cross-section of the interrupted channel ; but its precise determination is not required here. The *living force* of the water in the devised vessel *NMQP* to be ignored on account of its infinite cross-section: yet no variation should be going to arise in the calculation even if thence it were not of infinite cross-section. We now have the increment of the *living force* of the motion of the water in the shortened vessel $= 2\alpha vdv$ , to which if there may be added the *living force* likewise arising in the ejected droplet, there arises $2\alpha vdv + vvdx$ , which is the whole increment of the *living force,* due to the *actual descent* of the droplet *dx* through the vertical height of the water above the point *C* (or *c*), which we will designate by *a:* hence therefore that increment of the total *living force* required to be made is equal to *adx,* thus so that there shall be

$$2\alpha vdv + vvdx = adx$$

or

$$\frac{vdv}{dx} = \frac{a - vv}{2\alpha}.$$

[The common term for the cross-sectional area has been taken as 1, as has the density of the water.]

If the remainder may be done as in paragraph five, and the velocity *v* such as what may be due to the height *b,* the pressure of the water at *CF* (or *cf* ) may be found to be as great as the amount in still water for the height $a - b$. Where it can be noted the height *b* to be to the height *oS,* if there shall be no other impediments to the motion, and the out flowing jet at *o* shall not be contracted, in the square ratio of the opening *o* and of the section *CE* (or *ce*).

Corollary.

§. 11. When *b* is greater than *a*, the quantity $a - b$ becomes negative and thus the pressure is changed into a suction, that is, the sides of the channel are pressed inwards : but then the matter is required to be considered thus, as if in place of the overlying column of water *CT* placed in equilibrium with water flowing past, the water column *et* will be hung on *et*, the pressure of which requiring to fall may be impeded by the attraction of the water flowing past : even as if for example the cross-section of the channel *ce* shall be equal to the opening *o,* then there will be $b = oS$ , with no account taken of accidental impediments to the motion*:* hence if the tube may descend from the channel *cr,* and this shall be full of water from its origin *c* as far as to the point *t* with the opening *o* according to the position labeled, the water *et* will remain suspended without motion: truly if the point *t* shall be placed beyond *o*, water will fall through the tube

*cr,* and will flow out perpetually at *r,* nor yet, as which cannot yet be estimated easily from this theory, the velocity of the water flowing out at *r* will be such, as must be due to the height *NP* above *r,* even if all the impediments may be taken away; this velocity will correspond rather to the height *tr* , but only if the tube shall be excessively narrow in proportion to the channel. If the point *t* may be put higher from the point *o*, water at once rises by itself, and when every channel will have been filled, air is drawn up through the tubule, and soon the jet of water flowing out at *o* is disturbed by mixing with air, deprived from its clearness and uniformity. Therefore it is apparent, when the pressure is going to become positive and when it shall be negative: evidently the pressure is greater in that tube, when it is wider and placed lower: indeed the height *b* in that theory is $=\frac{1}{nn}\times oS,$ if $\frac{1}{n}$ denotes the ratio between the cross-section of the opening and of the cross-section of its tube, for which the pressure is required to be defined. Truly when obstacles diminish the motion notably, it will be agreed rather in estimating the pressures, that the velocity of the water, such as it actually becomes, may be known from experiment and the height due to that velocity may be substituted for *b* :
similarly the pressure will be estimated more accurately, if for *a* not the height of the surface of the water *NP* may be put above the place of efflux, but rather the height corresponding to the velocity, by which the water may actually flow from the same channel at an interrupted place: Yet these corrections are not always useful. Truly I will now illustrate that general theory by certain examples.

Example 1.

§. 12. *ABFG* (Fig. 75) shall be a vessel from the middle of the base of which a tube *DE* a tube falls having the form of a truncated cone diverging more lower down: It is continually filled with water at *AG*, thus so that the vessel may be kept full.

Moreover let the height of the surface of the water above the opening $E=a$, and above *D* (which place is for where the water pressure may be sought) $=c$ *;* the cross-section of the opening at $E=m$ *;* and the cross-section or the horizontal section at $D=n$. Therefore the pressure of the water at $D=c-\frac{mm}{nn}a,$ which amount is negative by virtue of the hypothesis, thus so that the walls of the channel are pressed inwards by a column of water of height $\frac{mm}{nn}a-c.$

Therefore if the curved tube *DLN* may be considered to be inserted into the other *DE*, the water flowing past

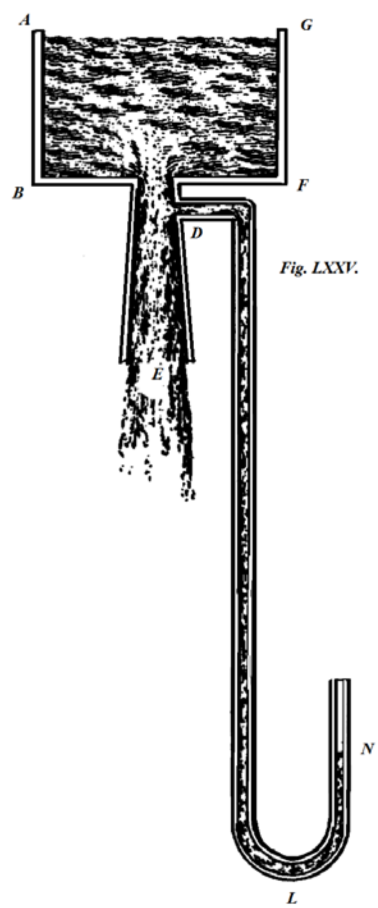


Fig. LXXV.

at *D* will be in equilibrium with the water in *DLN*, when the height *D* above *N* is
$\frac{mm}{nn}a-c.$
If the height shall be less with this, by itself the water will ascend nor cease to ascend, as long as the opening *N* is submerged in water, thus so that thus the water may be able to be raised from a lower place into a higher one without any external force, if a sufficiently copious amount of water may flow at *AG*. But truly when the height of the vertical *D* above *N* is greater than $\frac{mm}{nn}a-c$, the water will rise into the leg *IN*, until it should be equal to that.

Moreover this is required to be brought to mind, that from time to time I have learned from experience, evidently much to be lacking, by which less water may flow out from the vessel through the tubes, to which they have been attached, with the whole velocity being less than ought to be determined from the strength of the theory; the reasons for which I have indicated in paragraph 25 Sect. III. Thus it happens thence that the height *D* above *N* shall be definitely less, than it should be according to the theory put in place: The error is corrected, if in place of $\frac{mm}{nn}a$, the height corresponding to the velocity that the water has at *D* may be put in place; which height may be found by experiment from the amount of water supposed to be flowing out in a given time.

## Example 2.

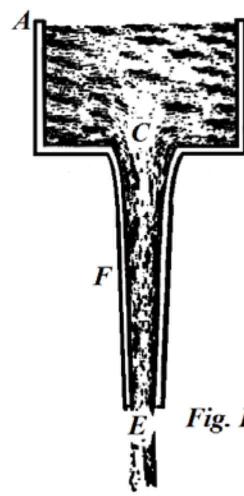


Fig. LXXVI

§.13. If a vertical tube shall be appended to a similar vessel, such as may be shown in Fig.76 by *CE*, in which the cross-section amplitudes everywhere may have the inverse square root ratio of the height of the incumbent water above, this tube is never affected by the water flowing past, nor is any pressure or suction sustained.

Thence it follows the natural shape of the vertical thread of water, while this is held together, to be the same as of the tube *CFE*, which both reasoning and experiments confirmed : but the thread will be attenuated quicker there where the height of the water above the surface of the water above the opening *C* is smaller, or where the flow of the water is slower : it is apparent the thread of water to be of this nature, so that the same amount of water may flow across the same sections, nor will the velocity be changed anywhere, wherever the thread may be cut off, which same properties also occur in the tube *CFE*, thus so that they may agree amongst themselves most correctly.
[This being of course the natural way for a jet of water to fall under gravity.]

## Example 3.

[Following GHM, all the derivations of the following section follow from the equation $\frac{P}{\gamma} = x - ay^2$, where P is the pressure of the water at $x$, $\gamma$ is the specific weight, and $y = \frac{\omega}{\omega_d}$ is the ratio of the tube cross-section to the area of the final orifice, etc.]

§.14. Water may be carried away downstream from a reservoir through a channel, in the base of which channel there shall be a hole, through which the water may burst out vertically, as if in a leaping fountain; I say the pressure of the water at the individual points of the channel to be everywhere equal, if the cross-sections of this shall be as $\sqrt{\frac{a}{x-b}}$, where $a$ expresses the height of the water in the channel above the opening of the efflux, $x$ the height of the same water above a place taken freely in the channel, and $b$ an arbitrary constant height ; and then the pressure of the water flowing everywhere to be to the pressure of still water as $b$ to $a$. [The pressure in the tube is constant only if $\omega$ is equal to $\omega_d \sqrt{\frac{a}{x-b}}$.] Because truly with all else being equal the wider channels resist bursting less than narrower ones, and that indeed in the ratio of the radii or because the attempt of the water to burst the channel, with all else equal, follows the square root ratio of the cross-sections, it is apparent the same channel to be in danger of rupturing at the following individual points, if the cross-section ($y$) to the ratio of the opening ejecting the water (1) everywhere may follow the law of this equation

$$\left(x - \frac{a}{yy}\right)\sqrt{y} = b$$

or

$$xxy^4 - bby^3 - 2axyy + aa = 0.$$

In the channel through all its treatment with the equality of the cross-section, the pressure of the water towards rupturing the channel everywhere will be proportional to the strength of the channel, if the thickness of the sides of the channel may follow as $x - \frac{a}{mm}$, with the cross-section of the channel understood to be in the ratio $m$ to the opening (1).

## Example 4.

§. 15. It can happen, that the height of a surface of water in the ratio of a place, for which the pressure is sought, shall be negative, just as with curved siphons by leading water from one vessel into another of lower height: And then the pressure shall be twice negative in the account, evidently $= -a - b$, with $a$ denoting the height of the place above the surface of the water and *b* the height corresponding to the velocity of the water at that place.

Truly these may suffice, as I think, towards correctly understanding the statics of fluid motions: I come now to certain other phenomena, the solution of which will depend on these rules which we have just given.

§. 16. In the third section, §. 25, I made mention of the cohesion of the water flowing through the tube: but truly the measure of this cohesion is a matter to be defined everywhere, which cannot be put in order without the above mentioned *static-hydraulics* : and nor indeed will the vertical heights above the opening of the efflux suffice, as is commonly thought, for also it is required to know the agreeing velocities of the water, and these may be known from the cross-sections. So that the general law may be apparent at once in being defined by the force of cohesion or by knowing, so that the fluid may be acted upon towards mutual separation, I say that force of cohesion to be equal to that force, by which the walls of the channel are pressed inwards, this Proposition as we have defined in §.11 does not seem to need another demonstration by me ; for as the compression of the water, or the force by which the parts are compressed towards each other, is equal to the overlying column of still water, thus in turn the attempt to separate the fluid of is agreed to be equal to the appended column of still water, which shall be in equilibrium with the water flowing past. In place of examples we may assume the same, with which above we have used for indicating the negative pressures of water.

(I) In §.12 Figure seventy-five it was explained, if in the tubule *DIN* the height *D* above *N* shall be such, that the water in that being still shall be in equilibrium with the water flowing past at *D*, the force of cohesion at *D* must be just as great, that the water in the same place may not be torn apart, as the weight of the column of water may have with a similar base and vertical height *DN.* Thence it may be understood what I said in §. 25 Sect. III, *the length of the tube thus can be increased, so that finally the water shall cease to be continuous in the tube, otherwise it would rather be divided into columns, and that happens with cylindrical tubes when they descend beyond thirty-two feet; but in diverging tubes it is required to descend less: thus if, for example, the lower opening were twice as great as the upper one above in the open reservoir, the lower tube cannot fall beyond eight feet, without the danger being present of the water column separating.* [*i.e.* the phenomenon of cavitation.] Yet in these examples considered theoretically the water was considered with all its velocity to flow out without diminution.

(II) From the same reasoning it is apparent, if the tubes converge towards the lower parts, then these allow a descent of more than 32 *ft.*: indeed a tube can be continued without end in the case of Figure 76 set out in §.13, and in innumerable other ways.

(III) Truly if the surface of the water in the reservoir on account of the place proposed were negative, such as happens, when waters are to be carried across a mountain, at no time shall it be able, in whatever manner the matter may be instituted, for the height to exceed thirty two feet, which is apparent from §.15. For if water were flowing across even with an infinitely small velocity, a force of cohesion now is required, which shall be equal to the whole height of the column of water, and a greater force is required, if the water shall be flowing across with an observable velocity. Hence I think the remedies brought forth by other writers are empty words ; indeed I know water often remains stuck beyond a height of 32 feet without another artifice, and of mercury beyond 30 inches ; but this effect is uncertain and nor itself constant. Also indeed they confirm the flow of water to happen through curved siphons in a vacuum : but truly the vacuum were to be such, that indeed not a sixtieth part of the air would have remained in retention, and I do not know whether the height of the tube would have departed by more than half a foot of water being drawn above the surface. Therefore thus, concerning what I have said in the subsequent freeing of the water, these things should not be considered other than what I wished to say hypothetically. It shall suffice that I have determined accurately by how much force waters may be urged into a mutual separation.

§. 17. Again there are other natural phenomena, the explanation of which truly depends on this *static-hydraulic* theory: such as how smoke rising through a chimney may draw air through an opening made in the chimney after itself with great impetus ; how a wind blowing from a more confined place into one more open may lose some of its elasticity, just as may be gathered from that, as open windows may be closed by the air, by that escaping from the room on account of retaining its greater elasticity ; and others of this kind, which one cannot examine individually.

The pressures of moving fluids indeed are able to be varied in innumerable ways ; yet I think all can be reduced to our general principle : we will examine two examples of this theory ; the first I have deduced from the known motion, which a fluid is going to have, if in the place of determining the pressure the vessel may be perforated with an infinitely small hole: the other, as they say, to be deduced *a priori* from our general theory ; and often each may be obtained in the same place, as one may require the help of the other, and then for the other an estimation of the pressure arises, as I will show be a single example.

§. 18. We may consider in the vessel, as set up in Figure 72, the horizontal tube to have a hole not only at the extremity, but also by inserting a lamina *EG* which has a hole in the middle in the vertical plane at its place of insertion, with all else indicated remaining in the same positions, as in §. 5: another pressure will be experienced by the walls of the tube *ED* from the water flowing past, and indeed smaller than with no lamina *EG* in place, the water flowing past with a somewhat smaller velocity. So that this pressure may be defined accurately, the path being followed is the same as what was cited in paragraph five: evidently before all the velocity is sought, by which the water flows through the tube *ED*, now after this has been made uniform. Then also it is required to find the value $\frac{vdv}{dx}$, if the water flow in the tube may be broken somewhere.

But how this shall be able to come about, the matter is chiefly what belongs to section eight, with similar cautions used as in §.14 of Section seven: In Section eight the motion of fluids flowing through several holes is shown generally, and in Sect. 7, §. 14 shows in a kind, how the *ascent potential* shall be estimated, which is generated by droplets, when these flow in through a side, not as in still water but in moving water, which cannot be ignored.

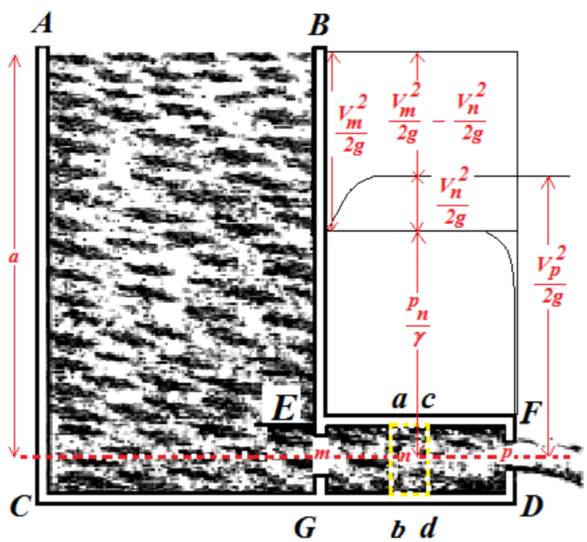



***Fig. LXII a***

If you follow the method with these correctly indicated, you will find the velocity, with which the water flows uniformly through the tube *ED*, to agree with this height :

$$\frac{mmppa}{mmnn+nnpp-mmpp},$$

where the respective cross-sections of the holes made in the plates *EG* and *FD*, and of the tube *ED*, are indicated by *m*, *p*, and *n* : but by *a* is understood the height of the water above the tube *ED* placed horizontally.
[Following KF, p.125, the amended Fig. 72*a* is made into a kind of 'energy-level' diagram, where the heights corresponding to the kinetic energies for the assumed rate of water flow through the openings are put in place, and the height corresponding to the water pressure is put in place, where the specific gravity of water is taken as $\gamma$. In addition, using the continuity equation as previously, we have :

$$\frac{V_p^2}{2g} = a - \left(\frac{V_m^2}{2g} - \frac{V_n^2}{2g}\right)$$ or, since $mV_m = nV_n = pV_p$,

$$\frac{V_p^2}{2g} = a - \frac{pp}{mm}\cdot\frac{V_p^2}{2g} + \frac{pp}{nn}\cdot\frac{V_p^2}{2g},$$ from which it follows :

$$\frac{V_p^2}{2g}\left(1+\frac{pp}{mm}-\frac{pp}{nn}\right) = a; \frac{V_p^2}{2g} = \frac{a}{1+\frac{pp}{mm}-\frac{pp}{nn}} = \frac{mmnna}{mmnn+nnpp-mmpp};$$

$\therefore$

$$\frac{V_n^2}{2g} = \frac{pp}{nn}\frac{V_p^2}{2g} = \frac{mmpp}{mmnn+nnpp-mmpp}a.]$$

If again you consider the tube broken at *cd*, and the droplet *ad* to be moving with the velocity *v* or the height due to this velocity $= vv$, and likewise you indicate the length *Ec* by *c*, and the minimum length *ac* by *dx*: you arrive at this equation :

$$2cvdv + \frac{nn}{mm}vvdx = adx,$$

or

$$\frac{vdv}{dx} = \frac{mma - nnvv}{2mmc};$$

now for *vv* the value just indicated may be substituted : $\frac{mmppa}{mmnn+nnpp-mmpp}$, and there will be

$$\frac{vdv}{dx} = \frac{mmnn - mmpp}{2c(mmnn+nnpp-mmpp)}a,$$

to which the pressure sought is proportional. But just as if the cross-section of the extreme opening indicated by *p* is infinitely small, the pressure becomes $= a$ [*i.e.* as a limiting value]; therefore generally the pressure sought by virtue of paragraph five equals

$$\frac{mmnn - mmpp}{mmnn+nnpp-mmpp}a.$$

[From the diagram and above, on taking $\gamma = 1$:

$$\frac{p_n}{\gamma} = \frac{V_p^2}{2g} - \frac{V_n^2}{2g} = \frac{V_p^2}{2g}\left(1-\frac{pp}{nn}\right) = \frac{(nn-pp)mma}{mmnn+nnpp-mmpp}. ]$$

§.19. If the cross-section of the tube *n* is as if in an infinite ratio of the cross-sections of the holes in the plates, the pressure becomes $=\frac{mma}{mm+pp}$: and so great also is the height, to which the water flowing out at *o* can rise by its velocity: therefore that agrees with paragraph four of Section eight, because the shape of the vessel as it were everywhere of infinite cross-section makes no difference to the speed of the out flowing water.

When there is no plate at *F*, there can be put $p=n$, and the whole pressure vanishes [*i.e.* above atmospheric pressure]. That deserves to be noted, because it shows the reason, why the suction shall not be as great in diverging tubes, as the amount to be due by virtue of the hypothesis, by which all *living force* is considered to be conserved : for in the present case we have had the reason of that *vis vivae*, which is used up continually. Thus also no pressure was being experienced by the walls of the tube, when the plate *EG* has a hole which is as if indefinitely smaller than that which is in *FD*. Finally it deserves also to be noted, because whatever fluids moved generally through channels with no plates constructed furnish a pressure, which may correspond to the difference of the heights owed by these velocities, by which a fluid may flow after an infinite time along a broken channel and by which actually it flows along an unbroken channel, yet this law in the present case barely prevails, which I wish those to think about, who wish to demonstrate generally [the laws] synthetically from our *static- hydraulic* theory seen in §. 10. Indeed perhaps there are [those], for whom this matter by itself will be seen to be obvious, so that it shall scarcely require to be demonstrated : but those, if they shall be present, who show particular laws of this kind which occur in *static-hydraulics*, impose on themselves a certain false likeness of the truth.

§. 20. From that matter also there will be concerning these, which were discussed in §. 18, experiments being made, both for the velocity of the water before flowing out at *o*, as well as for the pressure; thence indeed as well as the pressure the laws will be confirmed also by that theory of the acceleration, which it may be obtained, when a certain part of the *vis vivae* is wasted continually, which discussion we have handled especially in Section eight ; but obstacles shall be avoided as much as possible in an experiment being done, of which now we have made mention often.

[Following GKM, the reasoning in the following paragraphs §. 21 & §. 22 is entirely erroneous, and therefore no corrections have been made for these sections. ]

§. 21. I may add here a question which certainly does not pertain to the statics of fluids, but to hydraulics or to the motion of fluids, which truly cannot be solved without these premised rules of. In Fig. seventy two (here I consider no longer a plate at *EG* ) if the tube may be perforated by a hole at *ac* by having a finite ratio both to the cross-section of the tube as well as to the hole at *o*, and the motion of the water now were made uniform, I say, it is sought, with what velocity the water shall be going to erupt from each opening.

Now again the height shall be $BE=a$, the cross-section of the tube $=n$, the cross-section of the opening at $o=p$, the cross-section of the hole $ac=m$, the velocity of the water flowing out through $o=v$ :

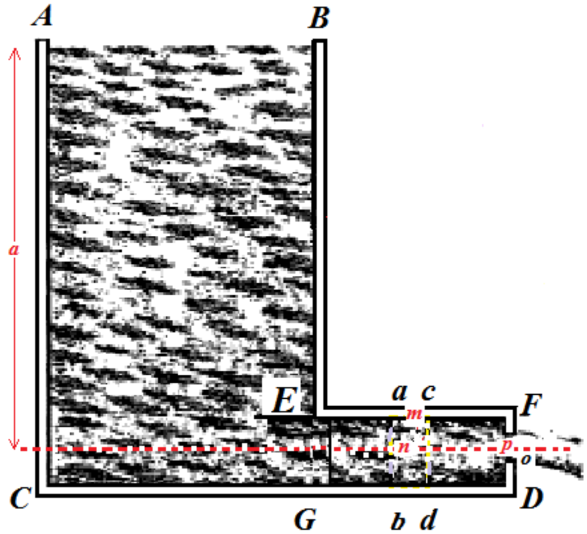



***Fig. LXII a***

The velocity of the water which flows past the opening *ac* will be $=\frac{p}{n}v$. Therefore at the same time it exerts a pressure on the sides of the tube, which is $=a-\frac{ppvv}{nn}$ (by §. 5), and therefore I suppose to be approximately also as great as the height, by which the water leaps out through the opening *ac*: truly the velocity of the same will be $=\sqrt{\left(a-\frac{ppvv}{nn}\right)}$.

[This is the flaw in Bernoulli's argument; the pressures everywhere at the bottom can be taken as *a*, wherever the holes are placed, assuming the tube is quite wide.]

With these in place the velocities at the openings *o* and *ac* will be as *v* to $\sqrt{\left(a-\frac{ppvv}{nn}\right)}$ : and thus any droplet entering the tube at *GE*, when it arrives at the region of the first hole, is separated into two parts, of which the one flows out through *ac*, the other through *o* : and these parts shall be respectively as the velocities, by which the efflux is made, and with each multiplied by the cross-sections of the holes. Therefore if the mass of the whole droplet *GE* may be called *g*, the part of it flowing out through *ac* equals

$$gm\sqrt{\left(a-\frac{ppvv}{nn}\right)}:\left(pv+m\sqrt{\left(a-\frac{ppvv}{nn}\right)}\right)$$

and the part flowing out by the other opening *o*

$$=gpv:\left(pv+m\sqrt{\left(a-\frac{ppvv}{nn}\right)}\right).$$

If these parts may be multiplied *respectively* by the squares of their velocities, the *living forces* of these will be found, the sum of which is required to be equal to $g \times a$, that is, with the *actual descent* of the droplet *g* through the height *a*. Thus such an equation may be obtained, if it may be reduced,

$$n^3vv - n^3a = mpv\sqrt{(nna - ppvv)}$$

or

$$vv = \frac{2n^6 + mmnnpp + nnmp\sqrt{(4n^4 + mmpp - 4nnpp)}}{2n^6 + 2mmp^4}a,$$

and this quantity expresses the height for the water flowing out at *o*, from which also the height may be known similarly for the other opening *ac*, which clearly is

$= a - \frac{ppvv}{nn}$.

[In these formulas for $v^2$ in this and the next paragraph, the negative root must be taken, otherwise it leads to the impossible solution $v^2 > a$ (GHM.), which Bernoulli finds hard to explain.]

§. 22. If $p = n$, there becomes $vv = a$; therefore now all the water leaps out with a single velocity through the opening *o*, and nothing flows out through the other opening *ac*. Again in each opening the velocity corresponds to the whole height *a*, if *p* is as if infinitely small : Truly if *m* is infinitely small, indeed there becomes $vv = a$, but the height corresponding to the velocity for the small opening *ac* is $= a - \frac{ppvv}{nn}$, as §. 7 now had indicated. If $m = p$, there becomes

$$vv = \frac{n^4a}{n^4 - nnpp + p^4}$$

and

$$a - \frac{ppvv}{nn} = \frac{(nn - pp)^2 a}{n^4 - nnpp + p^4}.$$

Finally it can be observed, the water to be ejected always from the opening *o* with a greater velocity, that may correspond to the height *a*, because certainly it happens, because the water in *Ed* exerts as if an impale on the water *dF*.

Meanwhile whenever all these outstanding Corollaries are in agreement with the nature of the argument, yet a solution of this problem cannot be considered truly other than approximately.

*Static-Hydraulic Experiments for Section XII.*

For §§. 3 & 4. The pressures, which were set out in the said paragraphs, are easily confirmed by experiment, if a vessel, such as set up in Figure forty-three, each is described in §. 26 of Section VIII, to be constructed with care, and a glass tube may be put in place vertically through the plate *LQ* of the same figure, each orifice of which shall be open : thus it will be observed with the holes *H* and *N* stopped up and with the whole system filled with water, water will rise in the glass tube to the level *AB*, or on account of that nature of capillary tubes to go beyond. But then if the finger may be removed from the opening *N*, it will be observed, the water in the glass tube falls and with the measures taken it will be found, unless I am mistaken, the height of the water in the glass tube (with the height that ought to be taken away because of the capillary tube) to be

$$=\frac{\alpha\alpha\times LB-\gamma\gamma\times NQ}{\alpha\alpha+\gamma\gamma},$$

as said in §. 3, where the names of these letters are explained.

If again a finger may be removed from each orifice *H* and *N*, then the same height of water will be left in the glass tube such as §. 4 indicates. Similarly a glass tube can be inserted into the plate *QN*, and it then bent, so that it can be known, whether the pressures on the plate *QN* were defined correctly also.

Truly the experiments which pertain to the pressures of water carried along tubes, I have carried out myself carried out for our Society, and they have been described in *Vol.* IV *Commentarii* [*Academiae scientiarum imperialis Petropolitanae*], *pag.* 194. Therefore these I will insert here, as they have been described there.

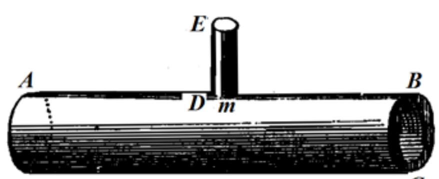
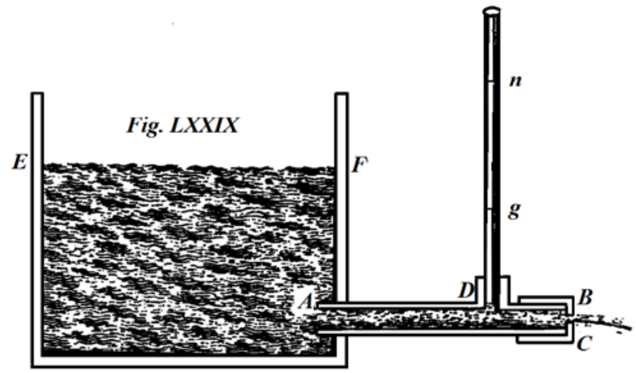


Fig. LXXVII


Fig. LXXVIII.

«I used a wooden box, the width of which was of one foot, the length of three feet, and the height of 14 inches. This I filled with water and to the lower part of that I put in place a tube made from iron accurately placed horizontally. But thus this same tube was made from iron : surely had a length *AB* (Fig. 77) of 4 *English in.,* 2 *lin.,* the diameter *BC* 7 *lin.;* in the middle the tube was perforated by a small hole *m*, and at the same place a small tube *DE* equally of iron six lines long and one and a half lines in diameter having been welded, thus so that the small hole *m* may be placed carefully in the middle : To this afterwards was put in place a glass tube of equal cross-section, as appears in Figure 79, which indicates the measures of the whole experiment. Again I attended to making three covers fitted to the iron tube, with a hole of a different size bored through:

such a cover is shown in Figure 78.




And with all these joined together in that manner, which Figure 79 shows, and on making sure, lest water might flow out by other cracks, than through the opening in *BC*, I bored a hole in *BC*, and then I observed the point *n* on the glass tube placed vertically, to which the water was rising, I noted that with a silken thread wound around : but first I investigated the capillary strength of this glass tube, and I found that to be five lines [i.e. $\frac{5}{12}^{th}$ of an inch], thus so that with the tube of water set in vertically the difference between each surface of the water should be of five lines: therefore the point *n* was raised above the surface *EF* by just as many lines, and hence in any calculation the height *Dn, Dg* is agreed to be reduced by five lines.

In the individual experiments the box thus was kept filled with water, thus so that the height *AF* was to be 9 *in.* 7 *lin.*, but the height *Dn* to be 10 *poll.* Thus with all these prepared for the experiment, then with the orifice open at *BC* the efflux of the water may be conceded, and immediately the water falls in the glass tube, such as from *n* to *g*, which place *g* I noted with another silken thread wound around the glass tube before. And thus finally we began such experiments which correspond to §. 5 and what follows.

Experiment 1.

With the diameter of the hole in the cover *BC* to be $2\frac{1}{5}$ *lin.*, the descent *ng* was a little more than one line, thus so that no difference was able to be observed between the theory and the outcome of the experiment.

Experiment 2.

With another cover taken, in which the diameter of the opening was $3\frac{2}{5}$ *lin.* or a little greater, the observed descent *ng* was of six and two third lines, clearly again as the theory indicates.

## Experiment 3.

With the third cover used, in which the diameter of the opening was *5 lin.* or a little amount less, we observed the descent *ng* to be 28 *lin.* By the strength of the theory it would be around 29 *lin.;* indeed nor was there seen to have an opening of five lines exactly. The little difference is to be attributed to impediments, which the water suffers in flowing through the tube, with a greater amount than in the preceding experiments, on account of the greater motion within the pipe.

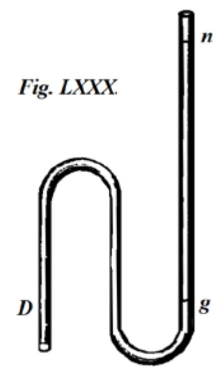
## Experiment 4.

Finally with no cover attached we have allowed to flow out through the whole opening, and then nearly all the water had left the glass tube : yet a little part remained, that we took to be eight lines high: but of these lines five are to be attributed to the strength of the capillary tube, the three remaining are due to impediments, which stand in the way of the water flowing across from *D* as far as to *B*.

Therefore thus the experiments agree according to the rule : But thence it is not hard to see, it can happen, that the sides of the tube not only are not pressed towards the exterior, but also so that they may be compressed inwards towards the axis (see. §. 11 ). But that I have deduced from another experiment.

## Experiment *5.*

Fig. LXXX.




In place of the cylindrical tube *AB* I have used a conical one, of which the outer opening was greater than the internal opening, and likewise I used a curved glass tube, such as Figure 80 shows. And when before the flow the water was raised in the glass tube to *n*, the water fell in the same tube as far as *g*, when water flowed through the conical tube: and the point *g* was below *D*, indicating a compression was endured by the flow of water through the tube. But in these cases the impediments of the motion are significant, which act so that the velocities of the water in the outer opening were exceedingly smaller, than what correspond to the height of the water: and because of this reason the height of the point *D* above *g* was not so great, as otherwise would be present, although there was a little height. A similar effect has been obtained by others in another way, but much more remarkable (see. §.12). This other experiment I presented myself to the Academics in the following year, in the presence of the most Serene Prince Emanuel of Portugal. [ Who visited the St. Petersburg Academy in 1730.]

Experiment 6.

In Figure 81 *ACFB* shall represent a cylinder, in the base of which the conical tube *DGHE* was established *;* and this had a small tube *l* at the side, which received the end of the curved glass tube *lmn;* the height *CA* was 3 *in.* 10 *lin.,* *El 4 lin., lH* 2 *in.* $9\frac{1}{2}$ *lin.;* the cross-section of the conical tube at *l* was to the cross-section of the opening *GH* as 10 to 16; *ln* was 5 *in.* 6 *lin.,* and its opening *n* was submerged in water in the little vessel *M*.

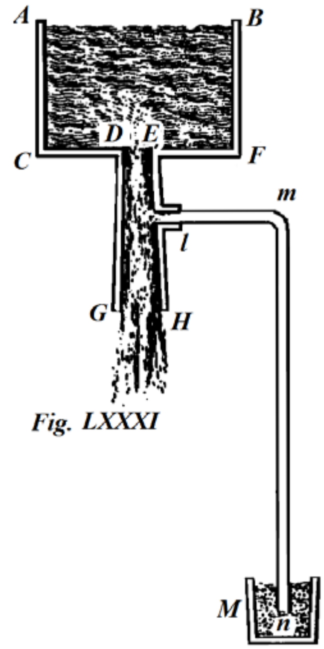


*Fig. LXXXI*

With a finger placed over the opening *GH* and with water dripped from the vessel through the glass tube *lmn* into the vessel *M:* but with the finger removed and now with the water flowing out through *GH*, by a reciprocal motion water at once rises from the small vessel *M* along the tube *nml,* and together with the rest flows out through *GH,* then the whole vessel *M* shall be empty. But the upper is filled with water continually, so that the vessel may be kept full. If a finger may obstruct part of the opening *GH* , it will be effected readily that as it pleases water may be moved up or down in the glass tube *lmn*.»

If anyone should want to investigate by experiments, whether the theory may agree with the problem in §. 18, he will not have arranged his work badly, since not only thus will he have illustrated easily this, our new *static- hydraulics*, but also equally the new theory of Sect. VIII, treated by no-one, and by an outstanding example.

Now with these experiments themselves put together in a table, just as I have made mention of which: I have used the same apparatus for that, as described in the manner show in Figure 79: but in addition, as the nature of the matter requires, I have put another cover on the tube at *A* : and the height of the water *AF* was 8 *London in.,* the diameter of the iron tube *AC* again 7 *lin.* Also I used the same covers as before: But in whatever experiment I observed the descent *n* which the surface made, when the finger was removed from the cover *BC*: but at the same time with the measure taken of the vertical height of the opening *C* above the floor, I observed the distance of this vertical line from the place, at which the jet of water was striking. I will call this distance the amplitude of the throw: moreover this vertical height was 19 *inches* in individual experiments. Thus with these preparations I did experiments of such a kind.

Experiment 7.

When the diameter of the inner opening was $2\frac{1}{5}$ *lin.* and the diameter of the external opening $3\frac{2}{5}$ *lin.*, the descent *ng* was a little less than 7 *in.*, the amplitude of the throw 9 *in.* But in the theory established in §.18 the descent *ng* was indicated to be 6 *in.* 10 *lin.* and the amplitude of the throw $9\frac{1}{2}$ *in.*

Experiment 8.

Then the diameter of the internal opening was *5 lin.* and the diameter of the other opening $3\frac{2}{5}$ *lin.;* the fall *ng* was almost 17 *lin.* and the amplitude of the throw 24 *in.* In theory *ng* $17\frac{3}{4}$ *lin.* and the amplitude of the throw 23 *in.*

Experiment 9.

Again when the diameter of the internal opening was $3\frac{2}{5}$ *lin.* and the diameter of the exterior opening 5 *lin.,* the fall *ng* was almost the same as it was in experiment 7, namely around 7 *in.* Truly the amplitude of the throw was greater, clearly 11 *in.* In theory *ng* is 6 *in.* 11 *lin.* and the amplitude of the throw nearly 11 *in.*

Experiment 10.

Finally with the diameter of the interior opening proving to be $3\frac{2}{5}$ *lin.* and with the diameter of the exterior opening $2\frac{1}{5}$ *lin.,* the descent *ng* was around one inch and the amplitude of the throw 23 *in.* In theory $ng = 14$ *lin.* and the amplitude of the throw $= 22\frac{1}{2}$ *in.*

All these experiments actually agree with theory; perhaps a greater agreement would arise, if it were allowed to take more accurate measurements of the openings ; yet no one I think, will be offended by these small differences of the numbers. But mainly they arise from the compression of the water at *AC*, which is produced, while the droplets entering through the interior opening lose part of the motion ; hence the amplitude of the throw are a little greater and the fall *ng* a little less in theory than in the experiments ; I have been unwilling to add a measure of this effect, although that was possible, lest the calculation became more involved.

# HYDRODYNAMICAE SECTIO DUODECIMA.

*Quae staticam fluidorum motorum, quam hydraulico-staticam voco, exhibet.*

§. 1. Inter eos, qui pressionis fluidorum intra vasa subsistentium mensuras dederunt, pauci regulas Hydrostaticae vulgares, quas in Sectione secunda demonstravimus, transgressi sunt: multa tamen alia sunt, quae ad Hydrostaticam proprie sic dictam pertinent, veluti cum actioni gravitatis vis centrifuga conjuncta est, aut vis inertiae, quod utrumque in praecedente sectione commentati sumus: possentque hujusmodi vires mortuae excogitari & combinari infinitis aliis modis. Non vero haec sunt, quae maxime desideranda mihi videntur: cum difficile non sit regulas ad id negotium dare generales. Desidero potius fluidorum staticam, quae intra vasa moventur motu progressivo, veluti aquarum per canales ad fontes salientes fluentium: multiplicis enim usus est, nec ab ullo tractata, aut si qui mentionem de illa fecisse dici possunt, ab his minime recte fuit explicata: qui enim de pressione aquarum per aquae ductus fluentium horumque requisita firmitate ad pressionem illam sustinendam dixerunt, non alias, quam pro fluidis nullo motu latis leges tradiderunt.

§. 2. Singulare est in ista *hydraulico-statica,* quod nisus aquarum prius definiri non possit, quam motus recte fuerit cognitus, quae ratio est, quod tam diu latuit haec doctrina; parum enim solliciti hactenus fuerunt Auctores in motu aquarum disquirendo, & velocitates ubique fere ex sola aquae altitudine aestimarunt: quamvis autem saepe motus tam cito ad hanc velocitatem tendat, ut accelerationes sensibus plane distingui nequeant, & in instanti omnis motus generari videatur, interest tamen, ut hae accelerationes recte intelligantur, quia aliter pressiones aquarum fluentium definiri saepe non possunt; proptereaque existimavi, rem esse maximi momenti a motus principio usque ad datum terminum mutationes illas utcunque *momentaneas* omni cura perpendere, experimentisque confirmare, quod passim in hoc tractatu, praesertim autem in sectione tertia, feci.

§. 3. Si ubique motus definiri posset, facile foret staticam in fluidis motis generalissimam formare: si enim foramen, sed id infinite parvum fingas, eo ipso in loco pro quo pressio aquarum desideratur, quaeres primo quanta velocitate aquae per illud foraminulum sint erupturae & cui altitudini illa velocitas debeatur: intelligis autem huic ipsi altitudini proportionalem esse pressionem, quam quaeris.

Ex hoc principio petenda est pressio quam sustinet lamina horizontalis *LQ* in Figura quadragesima tertia, si perforata non fuerit: postquam enim demonstratum a nobis fuit in corollario secundo paragraphi vicesimi septimi Sectionis octavae, si foraminulum *H* infinite parvum fuerit ratione foraminum *M* & *N*, ratioque horum foraminum *M* & *N* indicetur per $\alpha$ & $\gamma$ , fore altitudinem velocitati aquae per *H* erumpentis debitam

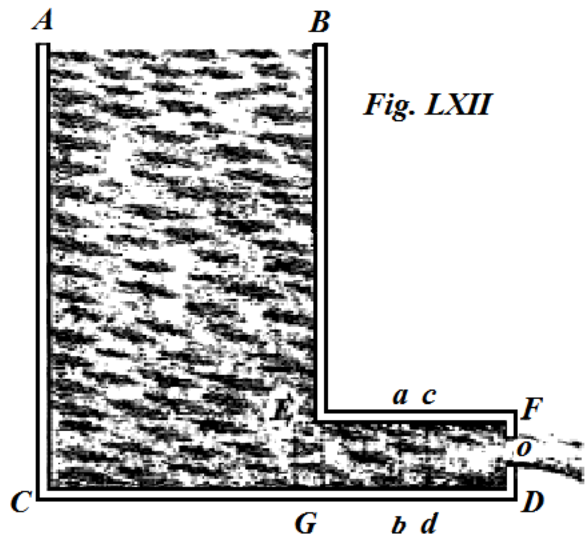
$$=\frac{\alpha\alpha\times LB-\gamma\gamma\times NQ}{\alpha\alpha+\gamma\gamma};$$

inde judicabimus nisum aquae in laminam *LQ* non perforatam huic ipsi altitudini proportionalem esse: quod idem alio modo demonstratum dedimus in paragrapho decimo nono citatae Sectionis: Hinc sequitur fieri posse, ut lamina *LQ* nullam pressionem patiatur, quantumvis magna supra eam fuerit altitudo aquae, scilicet quando $\gamma = \alpha\sqrt{(LB : NQ)}$, imo pressionem in suctionem mutari posse.

§. 4. Similiter obtinetur pressio aquae in laminam *LQ*, si vel haec perforata fuerit foramine finitae ratione amborum reliquorum magnitudinis. Si enim foraminulo infinite parvo lamina praeter illud, quod est in *H*, perforata fuerit, non potest non velocitate communi aqua per utrumque erumpere: Et cum haec velocitas cognita sit (per §. 27 Sect. VIII) pro foramine *H*, habetur quoque velocitas, qua aqua per foraminulum, quod nempe concipimus, erumpere debeat, atque sic pressionem aquae cognoscimus. Fuerint v. gr. foramina *M*, *H* & *N* inter se aequalia, altitudo autem *BL* habuerit ad altitudinem *NQ* rationem ut 10 ad 3, erit pressio in laminam *LQ* subdecupla illius, quae est obturatis foraminibus *H* & *N*.

Denique si in alio loco pressionem aquae desideres, addes saltem altitudinem, qua lamina *LQ* supra illum locum eminet, altitudini jactus per orificium *H*. Eadem methodus inservit ad pressiones aquarum in reliquis vasis, quae in Sectione octava tractavimus, determinandas. Differunt autem omnes hae quaestiones ab iis, quae ad motum fluidorum per canales pertinent, quod aquae ob infinitam vasorum a nobis positam amplitudinem veluti quiescant in cavitatibus & nihilominus pressionem longe aliam exerceant, quam aliter solent. In canalibus autem aquae pressionem suam eo magis mutant, quo majori velocitate praeterfluunt, & omnem fere consuetam pressionem exerunt, si velocitas ista sit valde parva.

Haec ita, cum velocitates fluidorum determinari possunt per methodos jam supra a nobis traditas. Singulari autem methodo res pertractanda est, cum aquae per canales fluunt, hancque doctrinam potissimum titulo hydraulico-staticae intelligo: Hic non tam pressio ex velocitate quam reciproce velocitas, si foraminulum in lateribus canalis fiat, ex pressione definiri potest. Et de ista hydraulico-statica, cujus usus amplissimus est, in praesenti sectione potissimum agere constitui.




Problema.

§. *5.* Fuerit vas amplissimum *ACEB* (Fig. 72) aqua constanter plenum conservandum, tubo instructum cylindrico & horizontali *ED;* sitque in extremitate tubi foramen *o* aquas velocitate uniformi emittens; quaeritur pressio aquae in latera tubi *ED.*

Solutio.

Sit altitudo superficiei aqueae *AB* supra orificium $o = a$; erit velocitas aquae in *o* effluentis, si prima fluxus momenta excipias, uniformis censenda & $= \sqrt{a}$, quia vas constanter plenum conservari assumimus; positaque ratione amplitudinum tubi ejusque foraminis $= \frac{n}{1}$, erit velocitas aquae in tubo $= \frac{\sqrt{a}}{n}$ : Si vero omne fundum *FD* abesset, foret velocitas ultima aquae in eodem tubo $= \sqrt{a}$, quae major est quam $\frac{\sqrt{a}}{n}$ ; igitur aqua in tubo tendit ad majorem motum, nisus autem ejus ab apposito fundo *FD* impeditur: Ab hoc nisu & renisu comprimitur aqua, quae ipsa compressio coercetur a lateribus tubi, haecque proinde similem pressionem sustinent. Apparet sic pressionem laterum proportionalem esse accelerationi seu incremento velocitatis, quod aqua sit acceptura, si in instanti omne obstaculum motus evanescat, sic ut immediate in aerem ejiciatur.

Res igitur jam eo perducta est, ut si durante fluxu aquae per *o* tubus *ED* in temporis puncto abrumpatur in *cd*, quaeratur quantam accelerationem guttula *abcd* inde sit perceptura: tantam enim pressionem sentiet particula *ac* in lateribus tubi sumta a praeterfluente aqua: Hunc in finem considerandum est vas *ABEcdC*, atque pro eo invenienda acceleratio particulae aqueae effluxui proximae, si haec habuerit velocitatem $\frac{\sqrt{a}}{n}$ : Istud negotium fecimus generalissime in paragrapho tertio Sect. V. Attamen quia in hoc casu particulari brevis est calculus, motum in vase decurtato *ABEcdC* hic iterum calculo subducemus.

Sit velocitas aquae in tubo *Ed*, quae nunc ut variabilis consideranda est, $= v$, amplitudo tubi ut antea $= n$, longitude $Ec = c$ : indicetur longitudo *ac* particulae aqueae infinite parvae & effluxui proxime per *dx*: Erit guttula aequalis in *E* tubum ingressura eodem temporis puncto quo altera *acdb* ejicitur: dum autem guttula in *E*, cujus massa $= ndx$, tubum ingreditur, acquirit velocitatem *v*, atque *vim vivam nvvdx*, quae *vis viva* tota fuit de novo generata; nullum enim, ob amplitudinem vasis *AE* infinitam, motum guttula in *E* habuit tubum nondum ingressa: huic *vi vivae nvvdx* addendum est incrementum *vis vivae*, quod aqua in *Eb* accipit, dum guttula *ad* effluit, nempe *2ncvdv*: aggregatum debetur *descensui actuali* guttulae *ndx* per altitudinem *BE* seu *a*: habetur igitur

$$nvvdx + 2ncvdv = nadx$$

sive

$$\frac{vdv}{dx} = \frac{a - vv}{2c}$$

In omni autem motu est incrementum velocitatis *dv* proportionale pressioni ductae in tempusculum quod hic est $\frac{dx}{v}$ : igitur in nostro casu est pressio, quam guttula *ad* patitur, proportionalis quantitati $\frac{vdv}{dx}$ , id est, quantitati $\frac{a - vv}{2c}$.

Est vero in eo temporis puncto, quo tubus abrumpitur, $v = \frac{\sqrt{a}}{n}$ vel $vv = \frac{a}{nn}$;

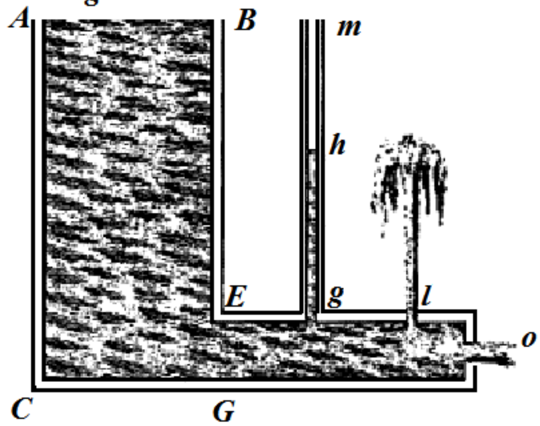
hic igitur valor substituendus est in expressione $\frac{a - vv}{2c}$, quae sic abit in hanc alteram $\frac{nn-1}{2nnc}a$. Et haec est quantitas, cui pressio aquae contra particulam tubi *ac* proportionalis est, quamcunque amplitudinem tubus habuerit, aut quocunque foramine ipsius fundum perforatum fuerit. Igitur si in unico casu pressio aquae cognita fuerit, innotescet simul in omnibus reliquis; talem autem habemus, nempe cum foramen est infinite parvum aut *n* infinite magna ratione unitatis: tunc enim ex se patet, aquam exercere integram suam pressionem, quae toti altitudini *a* convenit, hancque pressionem designabimus per *a:* sed quando n est infinita, evanescit unitas prae numero *nn*, fitque quantitas cui pressio est proportionalis $= \frac{a}{2c}$ : Ergo si generaliter scire velimus, quanta sit pressio cum *n* est numerus qualiscunque, talis instituenda est analogia. Si quantitati $\frac{a}{2c}$ convenit pressio *a*, quaenam erit pressio pro quantitate $\frac{nn-1}{2nnc}a$ : Et sic invenitur pressio quaesita $\frac{nn-1}{2nn}a$

Q.E.I.

Corollarium 1.

§. 6. Quia litera *c* ex calculo abiit, sequitur omnes partes tubi, tam eas quae sunt vasi *AG* propiores, quam quae remotiores, aequaliter ab aqua praeterfluente premi, & quidem minus quam partes fundi *CG* : differentiamque eo majorem esse, quo majus sit foramen *o:* nullamque amplius pressionem sustinere latera tubi, si in hoc omnis obex *FD* absit, sic ut pleno orificio aquae effluant.

***Fig. LXIII***




Corollarium 2.

§. 7. Si alicubi foraminulo minimo, & quidem tali ratione foraminis *o*, perforetur tubus, exiliet aqua velocitate, qua ad altitudinem $\frac{nna-a}{nn}$ ascendere possit, si modo impedimenta aliena nihil obstent: Erit nempe altitudo jactus, in Figura 73, seu $ln=\frac{nna-a}{nn}$ . Si vero tubulus adsit verticalis aut etiam utcunque inclinatus *gm*, communicans cum tubo horizontali, sed ita tamen, ut extremitas tubuli inserti non promineat intra cavitatem tubi horizontalis, ne aqua praeterfluens illidat in illam extremitatem, erit altitudo aquae verticalis *gh* in tubo inserto haerentis pariter aequalis $\frac{nna-a}{nn}$ : neque necesse est in hoc posteriori casu, ut tubulus *gm* sit admodum strictus.

## Scholium.

§. 8. Poterit ergo haec theoria experimento confirmari facillimo, eo majoris futuro momenti, quod nemo adhuc hujusmodi aequilibria, quorum usus latissime patet, definiverit: quod eadem methodo nisus aquarum per canales fluentium generalissime obtineri possit pro aquae ductibus utcunque inclinatis, incurvatis, amplitudinisque variatae ac velocitate aquarum qualicunque; tum etiam, quod nonsolum haecce pressionum, sed tota insuper motuum theoria, quam supra dedimus, hujusmodi experimentis confirmetur, quia arguunt, recte a nobis definitas fuisse accelerationes aquarum. Curandum autem est in experimento, ut tubus horizontalis sit interius bene politus, perfecte cylindricus atque horizontalis: sitque satis amplus, ut ab adhaesione aquae ad latera tubi notabile motus decrementum oriri non possit: vas ipsum sit amplissimum atque continue plenum conservetur. Observandum quoque est, quanta sit virtus tubulo vitreo *gm* aquas stagnantes elevandi, quae virtus omnibus tubis capillaribus aut admodum strictis in est: haec enim elevatio ab altitudine *gh* est subtrahenda: aut potius assumendus est tubus aequalis crassitiei & obturato orificio *o*, notandum est punctum *m*, tumque fluxu aquis concesso notandum quoque est punctum h: erit autem secundum theoriam descensus

$$mh=\frac{1}{nn}\times a=\frac{1}{nn}\times EB.$$

Tandem etiam attendendum est ad venam aquae in *o* effluentis; hujus enim contractio etiam facit, ut aqua in tubo horizontali minori transfluat velocitate, quam $\frac{\sqrt{a}}{n}$. De ista contractione eamque praeveniendi modo egi in Sect. IV. His autem quamvis ita occurri possit incommodis, ut error sensibilis in experimento superesse nequeat, tamen si majorem adhibere velimus accurationem, experimento indaganda erit quantitas aquae dato tempore effluentis, quae cum amplitudine tubi comparata rectissime dabit velocitatem aquae intra tubum fluentis, quam in calculo posuimus $=\frac{\sqrt{a}}{n}$ : Si vero experimento minor inventa fuerit, talis nempe, quae debeatur altitudini *b*, tunc erit proxime pressio aquae praeterfluentis $=a-b$.

## Corollarium 3.

§. 9. Si orificium in *o* prius digito obturetur, posteaque fluxus aquis concedatur, mutatur a primo fluxus momento pressio *a* in pressionem $\frac{nna-a}{nn}$ : Ista vero pressionum mutatio non fit in instanti; imo si accurate loquendum est, fit demum post tempus infinitum, quia, ut vidimus in sectione quinta, onmis aquarum velocitas, quanta in calculo a nobis posita fuit integrae altitudini *a* respondens, nunquam accurate adest: attamen incredibili acceleratione statim post primas ejectas guttulas ad hanc velocitatem tendunt, ita ut totam, quantum sensibus dijudicari potest, sine mora ulla sensibili acquisivisse videantur, nisi praelongi sint aquae ductus, tum enim aquarum accelerationes oculis distincte dijudicari possunt, cujus rei exemplum dedi in Sect. V §.13. In his igitur canalibus aquas ex castello longissime sito ad fontem salientem ducentibus, si pressiones alicubi experimento explorentur eo quo supra dixi modo, invenietur pressionem celeriter quidem, nec tamen in instanti diminui, pressionumque intervalla dignoscere licebit.

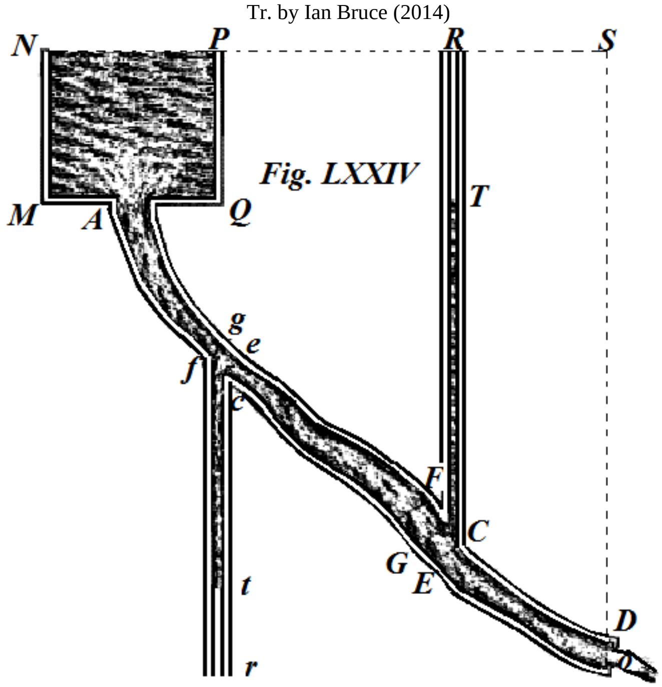
Ut vero generaliter nisus aquarum definiatur, ponenda est pro $v$ ea velocitas, quam aqua eo ipso in loco temporisque puncto, quibus nisus desideratur, habet, sique ea velocitas convenire intelligatur altitudini $b$, erit nisus aquarum $= a - b$. Unde collatis cum praesentibus his quae in sectione quinta tradita fuerunt, definire licebit quanta singulis momentis pressio futura sit,

Ex his non obscurum est praevidere leges hujusce *hydraulico-staticae,* si & figura vasis & aquarum per canales transfluentium velocitas pro lubitu fingantur qualescunque. Erit nempe pressio aquarum constanter $= a - b$, ubi per $a$ intelligitur altitudo debita velocitati, quacum aqua abrupto canali vaseque constanter pleno conservato post tempus infinitum effluxura sit, & per $b$ altitudo debita velocitati, qua cum aqua actu transfluit. Mirum sane est simplicissimam hanc regulam, quam natura affectat, adhuc latere potuisse. Jam igitur illam demonstrabo ex pressius.

## Problema.

§.10. Invenire pressionem aquae, per canalem utcunque formatum atque inclinatum, velocitate quacunque fluentis uniformi.

## Solutio.




Sit canalis *ACD* (Fig. 74) per cujus foramen *o* transfluere ponantur aquae velocitate uniformi & tali quae debeatur altitudini verticali *oS:* ducatur *SN* & fingatur vas infinite amplum *NMQP* aquis plenum usque in *NP,* ex quo canalis aquas suas perpetuo & aequabiliter hauriat: haec ideo sic fingo, ut causa adsit seu vis propellens uniformis, quae aquas data velocitate propellat seu fluxum aquarum conservet aequabilem: Et sine hac hypothesi problema nostrum foret indetermininatum, quia velocitas eadem in eodem canali infinitis modis ad temporis punctum generari potest & propterea, ut habeatur mensura causae aquas propellentis, fingenda est uniformitas in motu aquarum.

Fuerit nunc aquarum pressio definienda in *CF* (aut *cf):* huncque in finem putabimus rursus abrumpi canalem in *CE* (aut *ce)* sectione ad canalem perpendiculari examinaturi, quamnam accelerationem retardationemve guttula *CEGF* (vel *cegf)* post primum rupturae momentum receptura sit: qua de causa generaliter motum *momentaneum* per vas decurtatum *NM*[*AEC*]*QP* (vel *NM*[*Ace*]*QP)* definiendum habemus. Sit igitur velocitas guttulae infinite parvae *CEGF* (seu *cegf)* ipso decurtationis puncto $=v$ : massa ejus $=dx$ : erit *vis viva* aquae in vase decurtato motae proportionalis quantitati *vv,* eamque proinde faciemus $=\alpha vv$, intelligendo per litteram $\alpha$ quantitatem quamcunque constantem, quae pendet ab amplitudinibus canalis abrupti; praecisa autem ejus determinatio hic non requiritur. Notetur *vim vivam* aquae in vase ficto *NMQP* negligi ob infinitam ejus amplitudinem: nulla tamen si vel infinitae non esset amplitudinis inde in calculo oritura fuisset variatio. Habemus jam incrementum *vis vivae* aquae in vase decurtato motae $=2\alpha vdv$, cui si addatur *vis viva* simul genita in guttula ejecta, oritur $2\alpha vdv+vvdx$, quod est incrementum *vis vivae* totale, debitum *descensui actuali* guttulae *dx* per altitudinem verticalem aquae supra punctum *C* (vel *c*), quam designabimus per *a:* hinc igitur istud incrementum *vis vivae* totale faciendum est aequale *adx*, sic ut sit

$$2\alpha vdv+vvdx=adx$$

vel

$$\frac{vdv}{dx} = \frac{a - vv}{2\alpha}.$$

Reliqua si fiant ut in paragrapho quinto, & ponatur velocitas *v* talis quae debeatur altitudini *b*, invenietur pressionem aquae in *CF* (aut *cf*) tantam esse, quanta in aqua stagnante ad altitudinem $a - b$. Ubi notari potest esse altitudinem *b* ad altitudinem *oS*, si nulla motus impedimenta aliena sint, venaque effluens in *o* non contrahatur, in ratione quadrata foraminis *o* & sectionis *CE* (aut *ce*).

Corollarium.

§. 11. Cum *b* major est quam *a*, fit quantitas $a - b$ negativa atque sic pressio in suctionem mutatur, id est, latera canalis introrsum premuntur: tunc autem res ita consideranda est, ac si loco columnae aqueae *CT* superincumbentis & in aequilibrio positae cum aqua praeterfluente, sit columna aquea appensa *et*, cujus nisus descendendi impediatur ab attractione aquae praeterfluentis: veluti si v. gr. amplitudo canalis *ce* aequalis sit orificio *o*, tunc erit $b = oS$, nulla habita ratione motus impedimentorum *accidentalium:* hinc si tubulus ex canali descendat *cr*, hicque sit aqua plenus a sua origine *c* usque in punctum *t* cum orificio *o* ad libellam positum, manebit aqua *et* suspensa sine motu: si vero punctum *t* infra *o* positum sit, descendet aqua per tubulum *cr*, & effluet perpetuo in *r*, neque tamen, ut facile quis existimare potuisset nondum hac visa theoria, velocitas aquae in *r* effluentis talis erit, quae debeatur altitudini *NP* supra *r*, etiamsi omnia impedimenta auferantur; respondebit potius haec velocitas, si modo tubulus admodum strictus sit ratione canalis, altitudini *tr*. Si punctum *t* altius positum sit puncto *o*, aqua sua sponte ascendet & cum omnis canalem ingressa erit, aer per tubulum attrahetur, moxque vena aquea in *o* effluens ab admixto aere turbabitur pelluciditate atque soliditate orbata. Apparet igitur, quando pressio futura sit affirmativa & quando negativa: nempe eo major est in tubo pressio, quo amplior est & quo humilius positus: Altitudo *b* est quidem in theoria $= \frac{1}{nn} \times oS$, si $\frac{1}{n}$ denotet rationem inter amplitudinem orificii & ejus tubi sectionis, pro qua pressio est definienda. Cum vero obstacula notabiliter diminuunt motum, conveniet potius in aestimandis pressionibus, ut velocitas aquae, qualis actu est, experimento cognoscatur & altitudo illi velocitati debita pro *b* substituatur: similiter accuratius aestimabitur pressio, si pro *a* non tam ponatur altitudo superficiei aqueae *NP* supra effluxus locum, quam altitudo velocitatis, quacum aquae actu effluant ex canali eodem in loco abrupto: Hae tamen correctiones non semper locum habent. Istam vero theoriam generalem jam exemplis quibusdam illustrabo.

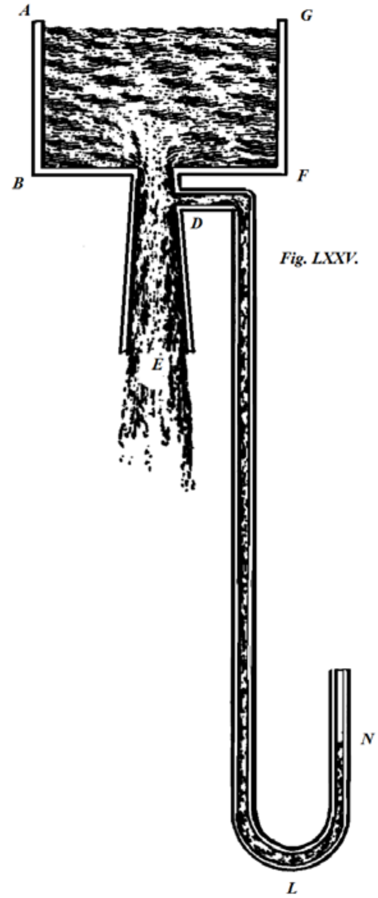


*Fig. LXXV.*

## Exemplum 1.

§. 12. Sit vas *ABFG* (Fig. 75) ex cujus fundi medio descendit tubus *DE* formam habens coni truncati inferiora versus divergentis: Affundantur perpetuo aquae in *AG*, ita ut sic vas plenum conservetur. Sit autem altitudo superficiei aqueae supra orificium $E = a$, & supra *D* (qui locus est pro quo pressio aquae quaeritur) $= c$; amplitudo orificii in $E = m$; & amplitudo seu sectio horizontalis in $D = n$. Erit pressio aquae in $D = c - \frac{mm}{nn}a$, quae quantitas vi hypothesium est negativa, sic ut latera canalis introrsum premantur a columna aquea altitudinis $\frac{mm}{nn}a - c$.

Igitur si concipiatur tubus incurvus *DLN* alteri *DE* insertus, erit aqua praeterfluens in *D* in aequilibrio cum aqua *DLN*, quando altitudo *D* supra *N* est $\frac{mm}{nn}a - c$. Si altitudo haec minor est, sua sponte aqua ascendet nec ascendere desinet, quamdiu aquis orificium *N* submersum est, ita ut sic aquae ex loco humiliori in sublimiorem sine ulla vi externa elevari possint, si in *AG* aquae sufficiente copia affluant. At vero cum altitudo verticalis *D* supra *N* major est quam $\frac{mm}{nn}a - c$, ascendet aqua in crure *IN*, donec illi fuerit aequalis.

Caeterum hic in memoriam revocandum est, quod passim monui experientiam docere, nempe multum abesse quo minus aquae per tubos a vase, cui implantati sunt, divergentes tota sua velocitate, quam vi theoriae obtinere deberent, effluant; cujus rei rationes indicavi paragrapho 25 Sect. III.

Fit inde ut altitudo *D* supra *N* admodum minor sit, quam vi theoriae exposita esse deberet: Error corrigetur si loco $\frac{mm}{nn}a$ ponatur altitudo velocitatis, quam aqua in *D* habet; quae altitudo per experimentum de quantitate aquae dato tempore effluentis sumtum obtinetur.

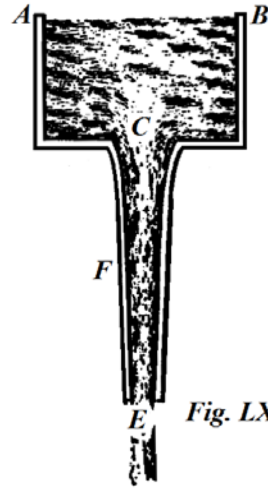


Fig. LXXVI

Exemplum 2.

§.13. Si simili vasi appensus sit tubus verticalis, qualis repraesentatur in Fig. 76 per *CE*, in quo amplitudines ubique rationem habeant inversam subduplicatam altitudinum aquae superincumbentis, tubus iste nihil afficitur ab aqua praeterfluente, neque ullibi vel pressionem sive suctionem sustinet.

Sequitur inde figuram naturalem fili aquei verticalis, quamdiu hoc contiguum est, eandem esse, quae tubi *CFE*, quod & ratio & experientia confirmat: filum autem eo citius attenuabitur quo minor est altitudo superficiei aqueae supra orificium *C*, seu quo tardius effluunt aquae: apparet filum aqueum ejus esse indolis, ut eadem aquae quantitas per singulas sectiones transfluat, nec velocitas ullibi mutetur, ubicunque filum abrumpatur, quae eadem proprietas etiam in tubum *CFE* cadit, adeo ut rectissime haec inter se conveniant.

Exemplum 3.

§.14. Devehantur aquae e castello per canalem, in cujus fundo foramen sit per quod aquae veluti in fonte saliente verticaliter exiliant; dico pressionem aquae in singula canalis puncta ubique aequalem fore, si amplitudines ejus sint respective ut $\sqrt{\frac{a}{x-b}}$, ubi $a$ exprimit altitudinem aquae in castello supra orificium effluxus, $x$ altitudinem ejusdem aquae supra locum ad libitum in canali sumtum, & $b$ altitudinem arbitrariam constantem; & tunc fore ubique pressionem aquae fluentis ad pressionem aquae stagnantis ut $b$ ad $a$. Quia vero caeteris paribus canales ampliores minus rupturae resistunt quam strictiores, & id quidem in ratione radiorum seu quia conatus aquae ad canalem rumpendum caeteris paribus rationem sequitur subduplicatam amplitudinum, patet canalem idem rupturae periculum in singulis locis subiturum esse, si amplitudo ($y$) ratione orificii aquas ejicientis (1) ubique sequatur legem hujus aequationis

$$\left(x-\frac{a}{yy}\right)\sqrt{y}=b$$

vel

$$xxy^4-bby^3-2axyy+aa=0.$$

In canali per totum suum tractum aequabilis amplitudinis aquarum nisus ad rumpendum canalem ubique proportionalis erit firmitati canalis, si crassities laterum canalis rationem sequatur ut $x-\frac{a}{mm}$, intellecta per $m$ amplitudine canalis ratione orificii (1).

Exemplum 4.

§. 15. Fieri potest, ut altitudo superficiei aqueae ratione loci, pro quo pressio indaganda est, sit negativa, veluti in siphonibus recurvis aquas ex vase uno in aliud humilius positum ducentibus: Tuncque pressio fit duplici titulo negativa, nempe $= -a - b$, denotante $a$ altitudinem loci supra superficiem aquae & $b$ altitudinem velocitati aquae in illo loco debitam.

Ista vero sufficient, ut puto, ad recte intelligendam fluidorum motorum staticam: Venio jam ad alia quaedam phaenomena, quorum solutio ab istis, quas dedimus modo, regulis pendet.

§. 16. In Sectione tertia, §. 25, mentionem feci cohaesionis aquae per tubos fluentis: veras autem istius cohaesionis mensuras ubique definire res est, quae sine ista praemissa *hydraulico-statica* expediri nequit: neque enim altitudines considerasse verticales supra orificium effluxus sufficit, ut vulgo putatur, sed oportet etiam nosse velocitates aquis convenientes, haeque cognoscuntur ex amplitudinibus. Ut vero statim appareat lex generalis in definienda vi cohaesionis seu conatu, quo fluida ad mutuam separationem sollicitantur, dico illam vim cohaesionis aequalem esse vi, qua latera canalis introrsum premuntur, quam definivimus §. 11. Propositio haec alia demonstratione egere mihi non videtur; prouti enim compressio aquae, seu vis qua ejus partes ad se invicem comprimuntur, aequalis est superincumbenti columnae aqueae stagnanti, ita vicissim conatus fluida separandi aequalis censendus est appensae columnae verticali aqueae stagnanti, quae cum aquis praeterfluentibus in aequilibrio sit. Exemplorum loco eadem accipiemus, quibus supra pro indicandis aquarum pressionibus negativis usi sumus.

(I) In Figura septuagesima quinta §. 12 explicata, si in tubulo *DIN* altitudo *D* supra *N* talis sit, ut aqua in eo stagnans cum aquis in *D* praeterfluentibus in aequilibrio sit, tanta debet esse vis cohaesionis in *D*, ne aqua ibidem discerpatur, quantam habet pondus columnae aqueae similis basis & altitudinis verticalis *DN*. Inde intelligitur quod dixi §. 25 Sect. III *posse longitudinem tubi ita augeri, ut tandem aquae desinant esse continuae in tubo, quin potius in columnas dividantur, idque fieri in tubis cylindricis cum infra triginta duos pedes descendant; in tubis divergentibus autem minorem descensum requiri: ita si v. gr. orificium inferius duplo majus fuerit orificio superiori in castellum hiante, non posse tubos infra octo pedes descendere, quin periculum adsit aquarum dissolutionis.* In his tamen exemplis theoretice consideratis aquae omni sua velocitate sine diminutione motus effluere ponuntur.

(II) Ex eadem ratione patet, si tubi inferiora versus convergant, tunc illos majorem quam 32 *ped.* admittere descensum: imo sine fine tubum continuari posse in casu Figurae 76 §. 13 explicatae, ut & infinitis aliis modis.

(III) Si vero altitudo superficiei aqueae in castello ratione loci propositi negativa fuerit, veluti fit, cum aquae trans montem vehendae sunt, nunquam poterit, quomodocunque res instituatur, altitudo excedere triginta duos pedes, quod patet ex §. 15. Si enim aquae vel plane infinite parva transfluant velocitate, vis cohaesionis jam requiritur, quae sit aequalis toti columnae aqueae, atque major vis requiritur, si notabili velocitate transfluxerint. Hinc remedia ab aliquibus Scriptoribus allata vana puto; scio quidem sine alio artificio aquas saepe suspensas haerere ultra altitudinem 32 pedum, & mercurium ultra 30 pollices; sed is effectus incertus est nec sibi constans. Quidam etiam affirmant fluxum aquarum per siphones recurvos fieri in vacuo: an vero vacuum tale fuerit, ut ne sexagesima quidem aeris pars in recipiente remanserit, & num altitudo tubi plus quam dimidio pede superficiem aquae hauriendae excesserit ignoro. Sic igitur, quae de subsecutura aquarum solutione dixi, non aliter quam hypothetice dicta velim considerentur. Sufficiet quod accurate determinaverim quanta vi aquae ad separationem mutuam urgeantur.

§. 17. Sunt porro alia naturae phaenomena, quorum vera explicatio ab ista theoria *hydraulico-statica* pendet: veluti quod fumus per caminum ascendens aerem per foramen in camino factum magno post se trahat impetu ; quod ventus ex loco angustiori in apertiorem flans aliquid de sua elasticitate perdat, prouti id colligitur ex eo, quod fenestrae apertae ab aere, e camera egressum ob majorem suam elasticitatem tentante, claudantur; & hujusmodi alia, quae examinare singula non licet.

Possunt fluidorum motorum pressiones quidem infinitis variari modis; puto tamen omnia ad principia nostra reduci posse: duas istius theoriae examinavimus species; primam deduxi ex cognito motu, quem fluidum habiturum sit, si in loco determinandae pressionis foraminulo infinite parvo vas perforetur: alteram *a priori*, ut dicunt, ex theoria nostra generali deduxi; saepe utraque simul locum obtinet, ut altera alterius opem requirat, & tunc alia oritur pressionum aestimatio, quam unico indicabo exemplo.

§. 18. Putemus in vase, quod Figura 72 sistit, tubum horizontalem nonsolum in extremitate, sed & in sua insertione *EG* laminam habere in plano verticali in medio perforatam, manentibus caeteris positionibus §. 5 indicatis: aliam patientur pressionem latera tubi *ED* a transfluente aqua, quam nulla apposita lamina *EG* & quidem minorem, quamvis minori velocitate transfluant. Ut pressio haec accurate definiatur, via calcanda est eadem, quae in citato paragrapho quinto: nempe ante omnia quaerenda est velocitas, qua aquae in tubo *ED* transfluunt, postquam haec jam uniformis facta est. Deinde etiam inquirendum est in valorem $\frac{vdv}{dx}$, si tubus alicubi abrumpi ponatur.

Quomodo autem hoc inveniri possit, res est quae potissimum pertinet ad Sectionem octavam, adhibitis simul cautelis §.14 Sectionis septimae: In Sectione octava generaliter ostenditur motus fluidorum per plura foramina transfluentium & in §. 14 Sect. VII in specie monstratur, quomodo aestimandus sit *ascensus potentia/is*, qui in guttulis generatur, quando hae per foramen, non in aquam veluti stagnantem, sed in aquam motu, qui negligi nequit, latam influit.

Si recte indicatis hisce insistas vestigiis, reperies velocitatem, quacum aqua uniformiter per tubum *ED* transfluit, convenire huic altitudini

$$\frac{mmppa}{mmnn+nnpp-mmpp},$$

ubi per *m, p,* & *n* indicantur *respective* amplitudines foraminum in laminis *EG* & *FD* factorum ut & tubi *ED:* per *a* autem intelligitur altitudo aquae supra tubum *ED* horizontaliter positum.

Si porro tubum abrumpi ponas in *cd*, guttulamque *ad* velocitate moveri *v* seu altitudinem huic velocitati debitam $=vv$, simulque longitudinem *Ec* indices per *c*, longitudinem minimam *ac* per *dx*: aequationem invenies hanc

$$2cvdv+\frac{nn}{mm}vvdx=adx,$$

sive

$$\frac{vdv}{dx}=\frac{mma-nnvv}{2mmc};$$

substituatur nunc pro *vv* valor modo indicatus $\frac{mmppa}{mmnn+nnpp-mmpp}$, & erit

$$\frac{vdv}{dx}=\frac{mmnn-mmpp}{2c\left(mmnn+nnpp-mmpp\right)}a,$$

cui pressio quaesita est proportionalis. Sed si amplitudo orificii extremi indicata per *p* est veluti infinite parva, pressio fit $=a$ *;* igitur est generaliter pressio quaesita vi paragraphi quinti aequalis

$$\frac{mmnn-mmpp}{mmnn+nnpp-mmpp}a.$$

§.19. Si amplitudo tubi *n* est veluti infinita ratione amplitudinum in laminarum foraminibus, fit pressio $=\frac{mma}{mm+pp}$: & tanta etiam est altitudo, ad quam aqua in *o* effluens velocitate sua ascendere potest: id igitur conforme cum paragrapho quarto Sectionis octavae, quia figura vasis ceu ubique infinitae amplitudinis non differre facit velocitatem aquae exilientis.

Cum nulla est lamina in *F*, fit $p=n$, totaque pressio evanescit. Notari id meretur, quia rationem ostendit, cur in tubis divergentibus suctio tanta non sit, quanta vi hypotheseos, qua omnis *vis viva* conservari ponitur, esse deberet: in praesenti enim casu rationem habuimus illius *vis vivae,* quae continue absumitur. Ita quoque nullam pressionem patiuntur latera tubi, cum lamina quae est in *EG* foramen veluti infinite minus illo, quod est in *FD,* habet. Denique notari id quoque meretur, quod quamvis fluida per canales nullis laminis instructos mota generaliter affectent pressionem, quae respondeat differentiae altitudinum illis velocitatibus debitarum, qua fluidum effluat post tempus infinitum per canalem abruptum & qua actu transfluit per canalem non abruptum, hanc legem tamen in praesenti casu minime valere, ad quod animum attendere velim hos, qui visa theoria nostra *hydraulico-statica,* propositionem generalem §. 10 synthetice demonstrare volent. Erunt enim fortasse, quibus res haec ita per se obvia videbitur, ut vix demonstranda sit: hos autem, si qui futuri sint, ex falsa quadam verisimilitudine sibimet imponere, ostendunt hujus modi leges particulares, quae in *hydraulico-statica* occurrunt.

§. 20. E re erit de his quoque, quae §. 18 dicta sunt, experimenta sumere, tum provelocitate aquarum in *o* effluentium, tum pro pressione; inde enim praeter pressionum leges confirmabitur etiam illa accelerationum theoria, quae obtinet, cum continue pars quaedam *vis vivae* inutiliter absumitur, quod argumentum in sectione octava praesertim pertractavimus; in experimento autem sumendo evitentur, quantum fieri potest, impedimenta, quorum jam saepe mentionem fecimus.

§. 21.Adjiciam hic quaestionem quae quidem non ad staticam fluidorum pertinet, sed ad hydraulicam seu motum fluidorum, quae vero sine istis praemissis regulis *hydraulico-staticis* solvi nequit. Quaeritur in Figura septuagesima secunda (nullam hic amplius in *EG* laminam considero) si tubus foramine in *ac* perforetur finitam rationem habente tum ad amplitudinem tubi tum ad amplitudinem foraminis *o*, motusque aquarum jam uniformis factus fuerit, quaeritur, inquam, quanta velocitate aquae per utramque aperturam erupturae sint.

Sit jam rursus altitudo $BE = a$, amplitudo tubi $= n$, amplitudo orificii in $o = p$, amplitudo foraminis $ac = m$, velocitas aquae per *o* effluentis $= v$: Erit velocitas aquae quae foramen *ac* praeterfluit $= \frac{p}{n}v$. Igitur ibidem in latera tubi exercet pressionem, quae est $= a - \frac{ppvv}{nn}$ (per §. 5), & propterea suppono proxime fore tantam quoque altitudinem, quae generare possit velocitatem, qua aqua per foramen *ac* exilit: ipsam vero hanc velocitatem esse $= \sqrt{\left(a - \frac{ppvv}{nn}\right)}$. Hoc posito erunt velocitates in foraminibus *o* & *ac* ut *v* ad $\sqrt{\left(a - \frac{ppvv}{nn}\right)}$ : sicque quaelibet guttula tubum in *GE* ingrediens, cum pervenit ad regionem primi foraminis, in duas dispescitur partes, quarum altera per *ac*, altera per *o* effluit: suntque hae partes respective ut velocitates, quibus fit effluxus, utrobique ductae in amplitudines foraminum. Igitur si massa guttulae integrae *GE* dicatur *g*, erit pars ejus per *ac* effluens aequalis

$$gm\sqrt{\left(a - \frac{ppvv}{nn}\right)} : \left(pv + m\sqrt{\left(a - \frac{ppvv}{nn}\right)}\right)$$

& pars altera per *o* effluens

$$= gpv : \left(pv + m\sqrt{\left(a - \frac{ppvv}{nn}\right)}\right).$$

Si hae partes multiplicentur *respective* per quadrata suarum velocitatum, habebuntur earundem *vires vivae*, quarum aggregatum aequandum est cum $g \times a$, id est, cum *descensu actuali* guttulae *g* per altitudinem *a*. Sic obtinetur talis aequatio, si reducatur,

$$n^3vv - n^3a = mpv\sqrt{(nna - ppvv)}$$

sive

$$vv = \frac{2n^6 + mmnnpp + nnmp\sqrt{(4n^4 + mmpp - 4nnpp)}}{2n^6 + 2mmp^4}a,$$

haecque quantitas exprimit altitudinem pro velocitate aquae in *o* effluentis, qua cognita habetur quoque altitudo similis pro altero foramine *ac*, quae nempe est $= a - \frac{ppvv}{nn}$.

§. 22. Si $p = n$, fit $vv = a$; ergo tunc aquae tota velocitate exiliunt solita per foramen *o*, & per alterum foramen *ac* nihil effluit. In utroque porro foramine velocitas respondet integrae altitudini *a*, si *p* est veluti infinite parva: Si vero *m* est infinite parva, fit quidem $vv = a$, sed altitudo velocitatis pro foraminulo *ac* est $= a - \frac{ppvv}{nn}$, ut§. 7 jam indicatum fuit. Si $m = p$, fit

$$vv = \frac{n^4a}{n^4 - nnpp + p^4}$$

&

$$a - \frac{ppvv}{nn} = \frac{(nn - pp)^2 a}{n^4 - nnpp + p^4}.$$

Denique observari potest, aquas per foramen *o* semper majori velocitate ejici, quam quae altitudini *a* respondet, quod utique fit, quia aquae in *Ed* veluti impetum faciunt in aquas *dF*.

Interim quamvis omnia haec Corollaria egregie cum indole argumenti consentiunt, non potest tamen solutio istius problematis aliter quam proxime vera censeri.

*Experimenta hydraulico-statica pro Sectione XII.*

Ad §§. 3 & 4. Pressiones, quae dictis expositae fuerunt paragraphis, facili experimento confirmari poterunt, si vas, quale Figura quadragesima tertia sistit, quodque §. 26 Sect. VIII describitur, confici curetur, ejusdemque laminae *LQ* tubus vitreus verticaliter implantetur, cujus orificium utrumque apertum sit: observabitur sic obturatis foraminibus *H* & *N* totoque systemate aquis repleto, aquam in tubo vitreo ad libellam *AB* ascendere, aut illam propter naturam tubulorum capillarium transcendere. Dein autem si digitus ab orificio *N* removeatur, observabitur, aquam in tubo vitreo descendere & captis mensuris invenietur, ni fallor, altitudinem aquae in tubo vitreo residuam (detracta altitudine virtuti tuborum capillarium debita) esse

$$=\frac{\alpha\alpha\times LB-\gamma\gamma\times NQ}{\alpha\alpha+\gamma\gamma},$$

uti dictum est§. 3, ubi denominationes harum litterarum explicantur.

Si porro ab utroque orificio *H* & *N* digitus removeatur, tunc erit eadem altitudo aquae in tubo vitreo residua talis, quae§. 4 indicatur. Similiter potest tubus vitreus laminae *QN* inseri, isque deinde inflecti, ut cognosci possit, an pressiones quoque in lamina *QN* recte definitae fuerint.

Experimenta vero quae ad pressiones aquarum per tubos latarum pertinent ipsemet coram Societate nostra institui & descripta sunt in *tom.* IV *Commentariorum, pag.* 194. Illa igitur, ut ibi descripta sunt, hic allegabo.

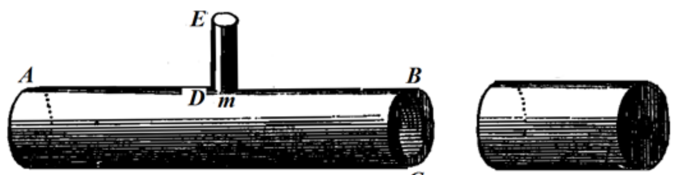
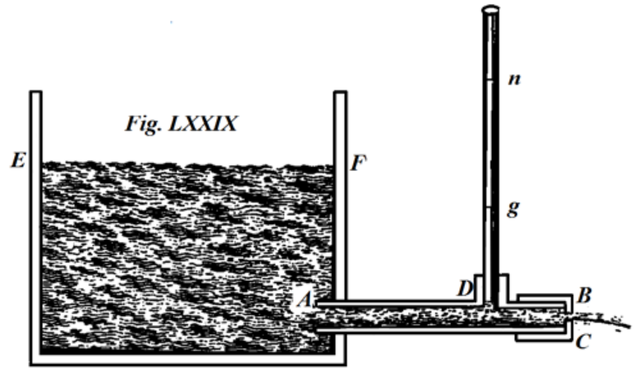


Fig. LXXVII Fig. LXXVIII.

«Usus sum arca lignea, cujus latitudo erat unius pedis, longitudo trium pedum, altitudo 14 pollicum. Hanc aqua implevi ejusque parti infimae fistulam accurate cylindricam ex ferro fabricatam infixi horizontaliter. Ita autem factus .erat tubus iste ferreus: longitudinem nempe habuit *AB* (Fig. 77) 4 *in. 2lin. Angl.,* diametrum *BC* 7 *lin.;* in medio tubus foraminulo *m* erat perforatus, ibidemque tubulus *DE* pariter ferreus sex lineas longus ac sesquilineam in diametro habens afferruminatus erat, ita ut foraminulum *m* in medio basis foveret: Huic postmodum tubulo imposui tubum vitreum aequabilis amplitudinis, ut apparet in Figura 79, quae modum totius experimenti indicat. Porro tria opercula confieri curavi tubo ferreo adaptata, foramine diversae magnitudinis pertusa: tale operculum repraesentatur Figura 78.




Hisce omnibus conjunctis eum in modum, quem ostendit Figura 79, factoque, ne aqua per alias rimas, quam per aperturam in *BC* efflueret, obturavi orificium in *BC*, tumque observavi in tubo vitreo verticaliter posito punctum *n*, ad quod aquae ascendebant, idque filo sericeo circumvoluto notavi: prius autem exploraveram virtutem capillarem istius tubi vitrei, hancque inveneram quinque linearum, ita ut tubo aquae verticaliter immisso differentia inter utramque aquae superficiem esset quinque linearum: propterea punctum *n* supra superficiem *EF* elevatum fuit totidem lineis, hincque in calculo quaevis altitudo *Dn*, *Dg* quinque lineis diminuta censenda est.

In singulis experimentis arca aquis ita plena conservata fuit, ut altitudo *AF* esset 9 *in.* 7 *lin.*, altitudo autem *Dn* 10 *in.* His omnibus ita ad experimentum praeparatis, tunc aperto orificio in *BC* aquis effluxus concedebatur & protinus descendit aqua in tubo vitreo, veluti ex *n* in *g*, quem locum *g* rursus alio filo sericeo antea tubo circumvoluto notavi. Et sic denique talia cepimus experimenta quae respondent§. 5 & seqq.

Experimentum 1.

Cum diameter foraminis in operculo *BC* esset $2\frac{1}{5}$ *lin.*, fuit descensus *ng* tantillo major una linea, ita ut nulla differentia inter theoriam & successum experimenti observari potuerit.

Experimentum 2.

Assumto alio operculo, in quo diameter foraminis erat $3\frac{2}{5}$ *lin.* aut paululum major, descensus *ng* observatus fuit sex linearum cum duabus tertiis, plane rursus, ut theoria indicat.

Experimentum 3.

Adhibito tertio operculo, in quo diameter foraminis erat *5 lin.* aut aliquantulum minor, descensum *ng* observavimus 28 *lin.* Vi theoriae debebat esse circiter *29 lin.;* nec enim foramen omnino quinque lineas in diametro habere visum fuit. Differentia parvula tribuenda est impedimentis, quae aqua in transfluxu per fistulam patitur, majoribus quam in praecedentibus experimentis, ob auctum motum intra fistulam.

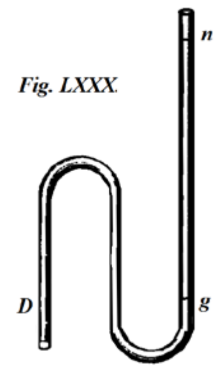
Experimentum 4.

Denique nullo apposito operculo aquas pleno orificio effluere sivimus, tuncque omnis fere aqua e tubo vitreo egressa fuit: pars tamen aliqua remansit, quam deprehendimus octo lineas altam: Earum autem quinque tribuendae sunt virtuti tubi capillaris, tres reliquae debentur impedimentis, quae aqua in transfluxu a *D* usque ad *B* offendit.

Sic igitur experimenta ad amussim cum theoria conveniunt: Inde autem non difficile est praevidere, fieri posse, ut latera fistulae non solum non premantur versus exteriora, sed & ut versus axem fistulae introrsum comprimantur (confer. §. 11 ). Id autem edoctus sum hoc alio experimento.

Experimentum *5.*

Fig. LXXX




Loco tubi cylindrici *AB* adhibui conicum, cujus orificium extemum erat majus orificio interno, simulque usus sum tubo vitreo incurvato, qualem ostendit Figura 80. Et cum ante fluxum aqua haesit in tubo vitreo in n, descendit in eodem tubo aqua usque in *g*, cum aquae effluerent per tubum conicum: fuitque punctum *g* infra *D,* indicio compressum fuisse durante fluxu tubum conicum.
In his autem casibus impedimenta motus sunt insignia, quae faciunt ut velocitates aquae in orificio externo admodum minores sint, quam quae respondent altitudini aquae: hancque ob rationem altitudo puncti *D* supra *g* tanta non fuit, quanta alias futura fuisset, fuit tamen aliqua. Similem effectum alio obtinui modo, sed admodum notabiliorem (confer. §.12). Experimentum hoc alterum subsequente anno coram Academicis institui, praesente Serenissimo Portugaliae Principe Emanuele .

Experimentum 6.

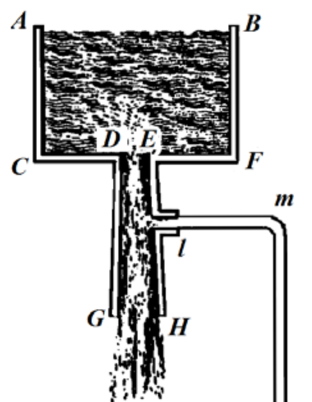



Fig. LXXXI

In Figura 81 repraesentat *ACFB* cylindrum, in cujus fundo implantatus erat tubus conicus *DGHE;* hicque ad latus habuit parvulum tubulum in *l,* qui reciperet extremitatem tubi vitrei incurvati *lmn;* altitudo *CA* erat 3 *in.* 10 *lin.*, *El 4 lin.*, *lH* 2 *in.* $9\frac{1}{2}$ *lin.;* amplitudo tubi conici in *l* erat ad amplitudinem orificii *GH* ut 10 ad 16; *ln* erat 5 *in.* 6 *lin.* ejusque orificium *n* erat aquae in vasculo *M* submersum.

Apposito digito orificio *GH* impletoque vase stillabat aqua per tubum vitreum *lmn* in vas *M:* remoto autem digito & effluentibus jam aquis per *GH,* motu reciproco aqua sponte ex vasculo *M* ascendit per tubum *nml,* & una cum reliquis effluxit per *GH,* donec totum vasculum *M* evacuatum esset. Affundebantur autem superius continue aquae, ut vas plenum conservaretur. Si digito pars orificii *GH* obtegebatur, facile erat efficere ut pro lubitu aquae in tubo vitreo *lmn* sursum deorsumve moverentur.»

Si quis etiam experimentis explorare voluerit, num theoria cum problemate §. 18

conveniat, non male operam suam collocaverit, quandoquidem non solum sic novam hanc nostram *hydraulico-staticam*, sed & theoriam Sect. VIII novam pariter & a nemine tractatam egregio exemplo eoque facillimo illustraverit.

Hisce jam in chartam conjectis ipse experimenta sumsi, quorum modo mentionem feci: Machina ad id usus sum eadem, quam modo descripsi, quaeque Figura 79 repraesentatur: sed insuper, ut natura rei postulat, in *A* tubo aliud operculum imposui: eratque altitudo aquae *AF* 8 *in. Lond.*, diameter tubi ferrei *AC* rursus 7 *lin.* Operculis quoque iisdem usus sum, quibus ante: In quovis autem experimento descensum observavi, quem superficies *n* fecit, cum digitus ab operculo *BC* removeretur: simul autem mensura capta altitudinis verticalis orificii *C* supra pavimentum observavi distantiam istius lineae verticalis a loco, in quem vena aquea incidebat. Hanc distantiam vocabo *amplitudinem jactus:* altitudo autem haec verticalis erat in singulis experimentis 19 *in.* His ita praeparatis experimenta feci talia.

## Experimentum 7.

Cum diameter orificii interioris operculi esset $2\frac{1}{5}$ *lin.* & diameter orificii exterioris $3\frac{2}{5}$ *lin.*, fuit descensus *ng* paullo minor, quam 7 *poll.*, amplitudo jactus 9 *poll.* In theoria autem §. 18 exposita indicatur descensus *ng* 6 *poll.* 10 *lin.* & amplitudo jactus $9\frac{1}{2}$ *poll.*

## Experimentum 8.

Deinde fuit diameter orificii interni *5 lin.* & diameter alterius orificii $3\frac{2}{5}$ *lin.;* fuit descensus *ng* fere 17 *lin.* & amplitudo jactus 24 *poll.* In theoria est *ng* $17\frac{3}{4}$ *lin.* & amplitudo jactus 23 *poll.*

## Experimentum 9.

Porro cum esset diameter orificii interni $3\frac{2}{5}$ *lin.* & diameter orificii exterioris 5 *lin.*, fuit descensus *ng* fere idem, qui in experimento 7, nempe circiter 7 *poll.* Verum amplitudo jactus fuit major, scilicet 11 *poll.* In theoria est *ng* 6 *poll.* 11 *lin.* & amplitudo jactus fere 11 *poll.*

## Experimentum 10.

Denique existente diametro orificii interioris $3\frac{2}{5}$ *lin.* & diametro orificii exterioris $2\frac{1}{5}$ *lin.*, fuit descensus *ng* circiter unius pollicis atque amplitudo jactus 23 *poll.* In theoria est *ng = 14 lin.* & amplitudo jactus $= 22\frac{1}{2}$ *poll.*

Omnia profecto haec experimenta egregie cum theoria conveniunt; fortasse major consensus futurus fuisset, si majori accuratione foraminum mensuras accipere licuisset;

nemo tamen, ut puto, minimis istis numerorum differentiis offendetur. Oriuntur autem maxime a compressione aquae *AC*, quae producitur, dum guttulae per orificium interius canalem ingredientes partem motus amittunt; hinc amplitudo jactus tantillo major & descensus *ng* minor sunt in theoria quam in experimentis; nolui hujus rei mensuram adjicere, quamvis id in potestate fuisset, ne calculus fierit intricatior.