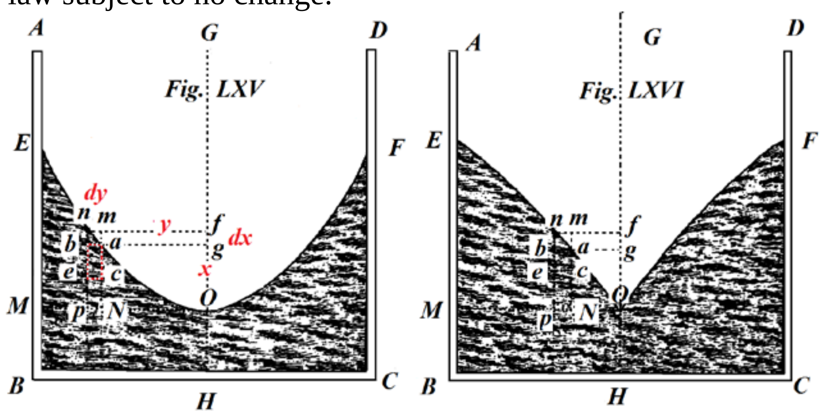
HYDRODYNAMICS: SECTION ELEVEN.

Concerning fluids acting in a vortex, also those which may be contained in moving vessels.

§. 1. From that time when Kepler and Decartes used vortices for explaining various phenomina of nature, many who did not think badly of their work have themselves carefully mulled over that proof: Moreover first, lest I am mistaken, Huygens penetrated its nature correctly in the tract : *Discourse on the Cause of Weight;*[to be found within his treatise on Optics] I shall add certain matters, which pertain to my presentation, perhaps not examined well enough by others.

Moreover, I consider vortices to be reduced usually to a *permanent* state [*i.e.* the steady state solution], or of being permanent, thus so that the fluid may move constantly by the same law subject to no change.



§. 2. Let *ABCD* (Fig. 65 & 66) be a cylinder placed vertically, of which the axis is *GH*, and that it is filled up to a certain height; the water may be considered acting in a vortex and all shall be reduced to a state of permanence: Thus the surface of the water will be depressed towards the axis and raised towards the sides: We will represent a section through the axis terminating on the surface of the water by the curve *EOF*, and now we will give the nature of this curve from the given relation, which the velocities have between themselves at certain distances from the axis.

Now *ga* and *fn* are drawn infinitely close together and horizontal, and *am* is acting along the vertical: Let $Og = x,\ gf$ or $am = dx,\ ga = y,\ mn = dy$: But it is apparent any small droplet situated on the surface with its pressure composed from the centrifugal force horizontally and the force of gravity vertically, to be pressing normal to the surface, because if it may be pressed at an angle, there shall be nothing that may keep the droplet in its place.

Therefore if the centrifugal force of the droplet placed at *a* may be expressed by the horizontal *ba* and the force of gravity by the vertical *ca* and the rectangle *abec* may be completed, the diagonal *ae* will be perpendicular to the curve; from which the triangle *eca* is similar to the triange *amn* and thus

$$dx : dy = ec : ca = ba : ca,$$

or, as the centrifugal force at the point *a* is to the force of gravity.

But Huygens showed the centrifugal force of a body in the act of rotating with the speed, which it would acqure in the free fall through a height of half the radius, to be equal to its force of gravity: because if hence the height corresponding to the velocity of the rotating drop may be called V, the force of gravity g [acting on unit mass; thus here g has its modern interpretation]: the centrifugal force will be $\frac{2gV}{y}$ [*i.e.* the familiar formula $\frac{v^2}{r}$], from which [*i.e.* from the tangent of the elemental triangle]

$$dx : dy = \frac{2gV}{y} : g,$$

or

$$dx = \frac{2Vdy}{y}.$$

§. 3. If one may put $V = \frac{1}{2}y$ there becomes $x = y$ and hence the line *EO* will be right constituting a semi-right angle with the axis *GH* and the cavity will have the shape of a cone: Truly if the velocity may be kept in the same proportion, which everywhere shall be proportional to the roots of the distances from the axis, and the water is driven around either faster of slower, the angle *EOG* therefore becomes more acute as the water moves the faster, thus so that if the velocity were infinitely fast, then the water must be in the form of a wall standing on the base perpendicularly, and to form a cylindrical cavity inside, but only if a cover shall be placed at *AD*, which may impede, so that not all the water may be cast out.

§.4. If there may be put a little more generally $2V = fy^e$, there becomes $dx = fy^{e-1}dy$ or $x = \frac{f}{e}y^e$: Hence it follows the curve always to be concave towards the axis, as in Fig. 65, but if *e* shall be more than one and convex, as in Fig. 66, if it shall be less than one. In the first case the angle *EOG* is right always, in the second always zero: in the single case when $e = 1$ this angle can be of any size.

§. 5. These can serve to distinguish in some way the scale of the velocities produced in a vortex artificially : for if you may see a concave surface, you will judge correctly the velocities to increase in a greater ratio, than the distances from the axis increase, if convex you deduce the opposite. If the curve may not be seen to belong to the family of parabolas, it will be an indication the velocities are not to be prepared from the determination of the distances by some force. So that the further the end of the line *EM* was observed from the horizontal *OM*, or the letter *f*, there the greater the absolute velocity of the particles will be considered.

§. 6. But I consider that it is not possible for a vortex to remain in its state for some perceptible time, if the centrifugal forces of all the equal parts in a homogeneous fluid decrease [the original has 'increase', which must be a typographic error] from the axis

towards the periphery: for if this should come about, since there shall be nothing, which may limit the centrifugal force sufficiently of the parts nearer to the axis, it may happen everywhere, that these closer parts will recede always from the axis and the more remote parts may be propelled towards that, and not in a state of equilibrium at any time or able to maintain a state of permanence. Thence it is apparent this quantity $\frac{2gV}{y}$ (which clearly expresses the centrifugal force of equal parts in homogeneous fluids) is either one to increase with *y* or perhaps not to decrease; and thus again if we may descend to the special hypothesis made before ($2V = fy^e$), *e* cannot become less than one. Therefore in all the vortices which are discussed here, for a state reduced to permanence, the surface at no time will be convex, as in Fig. 66, but either concave, as in Fig. 65, or a conic: and because *e* either is greater than one or equal to the same, it cannot happen otherwise, than that the velocities either to be equal to, or to increase in proportion to, the radii of the distances from the axis. Since I have considered this matter carefully, I do not understand in what manner Newton was able to imagine two vortices of homogeneous fluid reduced everywhere to a state of permanence always, in the first of which *the periodic times shall be as their distances from the axis of the cylinder, but in the other as the square of their distance from the centre of a sphere:* For in the first of these vortices the velocities shall be equal everywhere, and in the other clearly shall decrease from the axis towards the periphery.
[This discovery of an error in Newton's *Principia*, Book II, Prop. 51 and 52, is usually ascribed to Stokes, who came across it in his work on the theory of motion of viscous liquids : *On the theories of the internal friction of fluids in motion* ….. See, e.g. §. 8, p.102, Vol. 1 of Stokes' *Math. and Phys. Papers* ; further remarks by Bernoulli on this topic can be found on pp. 540-541 of Vol. V of the Collected Works of the Bernoulli family: his argument being that Newton consided the equality of tangential forces between contiguous layers of a viscous fluid in a vortex, instead of the equality of the moments of these forces, in describing the motion. The same argument in fact as that advanced 100 years later by G.G. Stokes.]

It is more plausible, in most vortices, which now reach a state of permanence, the periodic times of the individual parts of the fluid, either homogeneous or heterogeneous, to be as if the whole cylinder were solid, but the parts of greater specific gravities are going to be closer to the circumference. In this case *v* [*i.e*. the actual velocity arises here undefined] becomes proportional to *y* itself and *V* proportional to the square of the same, and the curve *EOF* will be the Apollonian parabola, of which the vertex shall be at *O* and its axis *OG.*

I presume this to be approximate thus, if the vortex may be generated from the rotation of a cylindrical vessel about the axis *HG,* or even by the uniform agitation with a stick near the side of the vessel, phenomena of vortices of this kind were published by D. Saulmon in *Comm. Acad. Reg. Sc. Paris. a.* 1716.

§. 7. The pressures which the different parts of the cylinder *ABCD* sustain from the fluid, are proportional to the heights of the vertical columns for the same corresponding parts ; nor indeed is it required, that to this weight of fluid we may add a contribution arising from the centrifugal force, because this effect is now maintained in raising the water: And if the vessel were not cylindrical but some kind of irregular structure, it will be allowed to imagine a cylinder, of which the axis may coincide with the axis of rotation, thus filled with fluid, so that the point *O* in the proposed vessel still may be placed in a fictitious cylinder in the same place : for as great as the pressure will be at some point of the cylinder, it is just as great at the same point, as far as that pertains to the proposed vessel. It is apparent from this itself, it is possible to define the surface of the vortex from a principle another than what we have used before : Namely with the line *OM* drawn to the horizontal and for the vertical *Na* with its infinitely close *pn* it follows the height *Na* or *Og* to be proportional to the centrifugal force of all the particles which are at *ON*, and the difference of the two approximate heights, surely *am* or *gf*, to be proportional to the centrifugal force of the particles *Np:* From which again the last equation may be derived, as we have given in §. 2, surely $dx = \frac{2Vdy}{y}$.

§. 8. We may now considere what must happen to a vortex with floating bodies ; but so that the question there may become more apparent and simple, in place of the body we will consider a globule of the same specific gravity as the fluid vortex.

Such a globule brought together from the fluid is acted on by two forces at the same time, the one tangentially arising from the driving force of the fluid, the other centripetal, which arises from the centrifugal force of the fluid. These forces maintain a constant ratio between each other, namely the square of the velocity of the respective fluid ; either the body may be at rest or it may be carried in a circular motion.

But it deserves to be noted by those, who adhere to Cartesian principles in the explanation of gravitational phenomina, that the tangential force is incomparably greater than the centripetal force: indeed the former is to the latter, as the distance of the body from the axis to the eight thirds parts of the diameter of the globule ; the demonstration is to be seen in *Comment.Acad.Petrop.Tom.* II, *pp.* 318 & 319.

[We should note that Bernoulli acknowledges the Cartesian philosophy of vortices carrying the planets around in their orbits is deficient; his argument with Newton concerns his derivation of vortex motion in Book II, Th. 51 and 52, as we have indicated.]

§. 9. Although I may know many things were alleged by various people, in order that they could show, indeed the subtle matter acting in a vortex can push bodies towards the axis very quickly, nor yet does it follow thence that likewise such bodies would be carried away by a vortex, yet I have been unable to remove this scruple for myself, after I had found the tangential force to be almost infinitely greater than the centripetal force. Nor does it help for this difficulty occuring, if we may put two vortices of equal and opposite strength on the same axis : for it may be seen, most natural phenomena are unable to be in reconciled by the vortex hypothesis, unless we may consider two or more vortices able to move themselves about unrestrained in any direction : or by the force of gravity alone common to all celestial bodies acting towards each other in turn, which cannot be called into doubt; it has been shown clearly enough either to be the farewell of the vortex hypothesis, or several of the freest vortices are to be put in place intersecting in all directions. Therefore if two vortices of equal and opposite strengths may be imagined on the same axis, then the opposite impulses will destroy the forces of each tangential vortex; moreover at the same time each concurrent vortex is required to press a body down towards the common axis.

§. 10. Another difficulty is to be added, the gravity of bodies cannot be demanded from the effect of two contrary vortices moving about the same axis. Thus indeed the bodies would not be attracted towards as if a common point but towards an axis, and by moving they may glide towards the same perpendicular, which disagrees with the vertical fall of bodies and rotundity, or as if with the rotundity of the earth and of celestial bodies [*i.e.* the wrong kind of symmetry is involved].

This other difficulty also may be met, if two axes may be imagined perpendicular to each other in turn or approximately such, around each of which two vortices of equal contrary strength thus may be driven. In as much as the force of all the vortices put together can be understood to be prepared thus, so that a body may be thrust towards the point, where both axes intersect each other ; yet always the earth will be compressed a little amount towards the plane passing through both axes. But it will be possible to contend with this inconvenience, even if it shall be just an inconvenience, by multiplying a large number of vortices together: for even if nearly infinitely many vortices may be put in place, all can cross over each other with the same facility as rays of light, which minimally impede each other.

[Thus Bernoulli ruminates on the possibility of representing the force of gravity acting on a planet such as the earth by some kind of arrangement of vortices; two equal and opposite vortices are needed to cancel the tangential forces acting on the same axis; two such perpendicular axes could define motion towards a point, if such a thing were possible, ete, etc….]

I have wanted to add this here in gratitude of those, who find vorices a source of delight, so that they may see, whether a motion simpler than that devised by Huygens can be considered there: for they are able to explain equally each natural phenomenon. I have expressed this thought a little more accurately in a dissertation which the Paris Royal Academy, with the prize for the year 1734, arranged in effect to have printed.

[Daniel in fact shared the Grand Prize for that year with his father Johann, which led to an enduring split between the two, due to the father's jealosy of his famous son; Johann even went as far as to plagiarise this work, and pretend it was his own, postdating his own efforts published about the same time. See e.g. the MacTutor website for further details.]

§. 11. Because it cannot be doubted that all the planets *gravitate* towards the sun and the satellites of these planets towards their planets according to the reasoning of Newton, and the cause of this gravity shall be similar to that by which terrestrial bodies tend towards the centre of the earth, the hypothesis of vortices will require to be extended to all the systems of the world, if it were to be used for explaining the weight of terrestrial bodies. Thus truly the planets, swimming in a subtle matter, shall be moving in a resisting medium, and little by little losing their motion must approach the centre of the sun in the form of a spiral: truly from the most ancient observations this does not seem to be apparent, the hypothesis of vortices postulates, that the fluid of the vortices may be put beyond a manner of rareness and fineness and that moved with a velocity, that the human mind is scarcely able to follow : where indeed the rarer the fluid there you may imagine it is necessary for the motion to be faster. Perhaps the continuance of the motion will be explained more suitably by the communication of a certain reciprocal motion, thus just as the celestial body shall propel the particles, it may be propelled by these at another time by a similar force.

§. 12. I come now to the remainng properties of gravitating bodies which follow from the hypothesis of vortices. And thus we may put a body at rest into a vortex filled fluid, so that no particles of the fluid may be passed through its pores: thus the body tends towards the centre of the vortex, and the strength of that centripetal force will be exactly equal to the centrifugal force of the fluid vorticity, which shall be located in a similar volume at the same distance from the centre. Therefore whatever bodies placed together in simililar positions in the vortice have the same centripetal force if they may have the same volume likewise, even if the amounts of matter in each body shall be unequal in some manner, and if bodies of this kind shall be able to move freely towards the centre of the vortex, they may be carried with unequal velocities, evidently inversely proportional to the square roots of the quantities of matter, if the measured distances shall be equal.

§. 13. The matters which have been considered in the preceding paragraph are applied easily to the weight of bodies, but only if the source of weight shall be the centrifugal force of some subtle matter driven in the swiftest vortex. Because truly experience teaches that all terrestrial bodies in a vacuum fall with the same velocity and all bodies suspended by a wire make equal tautochonous oscillations, thence we may conclude, *the ultimate heavy particles,* through which surely the gravity-producing fluid shall be unable to penetrate in all terrestial bodies, to be of equal specific density, that is, within equal volumes equal amounts of solid matter are to be contained, and that no less with the *heavy particles,* which compose gold as well as feathers. And truly as I do not want these explained otherwise, it will be required to be said by me, what I understand by the terms themselves : *ultimate heavy particles* and innate *solid matter*.

§. 14. Therefore the *heavy particles* thus are said properly to be these, which are impenetrable by the subtle matter of the vortices: indeed the particles are made of that same kind as the bodies placed in the vortex, that we have discussed in §.12: but although they may be impenetrable by the subtle matter in the manner said, yet I will not believe these to be perfectly solid, such as Huygens seemed to have been assumed in his tract *de Gravitate,* that is, of a kind such that the total volume of which material shall be filled without pores or inflowing fluid : I believe rather on the contrary these heavy particles do have their own pores, and in these there is another far more subtle fluid, which is transferred freely into the same heavy particles, by which the gravitational fluid [also present in the vortex] flows through bodies perceptably: truly the remainer which stick inside the *heavy particles* themselves, I call the *solid matter* pertaining to the same particles.

§. 15. From these it is evident, different specific gravities of bodies are required to be sought minimely from the different density of the *heavy particles,* but from that, which these particles shall be able to have present, in the same volume in different bodies, but

in unequal numbers, or also from the amount present, thus so that in more compact bodies or with greater specific gravities, the *heavy particles* shall be present either with fewer gaps in place, or they shall be greater in number.
[Thus Bernoulli has in mind that all matter is composed from the same building blocks, and different forms of matter have more or less such fundamental or *heavy particles* in place, or the volumes occupied by such may be different. The difference in specific density thus arises from the distribution of these fundamental particle in some way, and not from differences in the specific density of these particles themselves.]

But if indeed the *heavy particles* should have different specific densities in different bodies, on that account bodies would not able to have different specific gravities, with all else being equal: for such bodies falling from different heights should be going to fall towards the centre of the earth with a velocity between each other: And thus it could happen, that bodies of unequal specific gravity even in a vacuum thus may be said to be descending generally with an unequal velocity, and nevertheless we should consider bodies of different specific gravities to be required to fall with the same velocity: But in bodies of this kind the laws of motion would be far different, and now they are such as where the masses are reckoned only from the weights.

§. 16. Because otherwise all, as far as it agrees with experience, terrestrial bodies have their *heavy particles* of equal specific density, as was advocated in §.13, indeed I may induced to believe the same easily, as it happens in all of the planets considered separately: To me it is certainly probable the particles of planets compared together between themselves have different weights of the specific densities, because I can see no reason, why these particles must be similar in all the planets. But the strength of the centrifugal force on some planet depends on the density of the *heavy particles* or the attempt to recede from the sun. Therefore is it still not allowed to deduce the *centrifugal forces of the planets to be themselves in the inverse square ratio of the distances of the same from the sun, because the ratio of the periodic times may follow the three on two law of the distances*: for such a conclusion presumes a similar density in all of the *heavy particles* in all of the planets.

§. 17. The centrifugal forces of the planets everywhere are equal to the opposite forces by which they are drawn towards the sun: But because, as I have said in the paragraph above, it is not yet certain, in which account with respect to the distances from the sun the centrifugal forces of the sun may be changed, thus nor is it permitted to set up with any certainty the forces of gravity of these towards the sun ; and indeed there are many in the hypothesis of the vortices, which constitute and determine the forces of gravity at different distances.

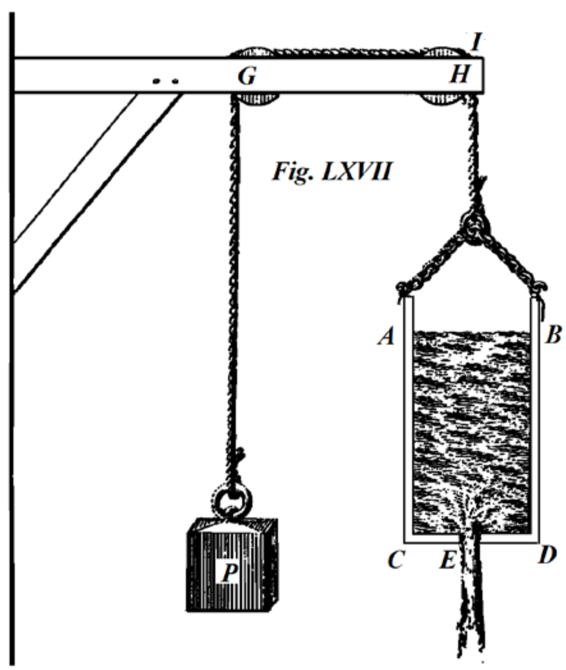
For since the force of gravity shall be equal to the centrifugal force of the subtle matter, which is unable to penetrate the heavy particles of the body, it follows that the greater the force of gravity becomes, by which the passage of a greater amount of subtle matter is denied ; because truly we know the body often to be impenetrable to a single fluid, because it is agreed for another more subtle fluid to be flow freely through it, it can happen, but only if we may consider the vortex matter at unequal distances from the centre of the vortex to be of unequal subtly, so that one and the same planet may be driven at unequal distances from the sun, which likewise can happen in different planets, because a different structure can be added to the weight of the particles.

Besides this there are also the different densities of the matter of the vortex, the velocity and distance from the centre, which agree according to the force of gravity being formed. Truly if the ratio of these may be had, it will be apparent certain forces of gravity decrease with increasing distance from the centre of forces, and nor yet therefore do the centrifugal forces of equal volumes of vortex matter decrease equally, because I believe the latter cannot happen on account of the reason I have set out in §. 6.

Truly they shall suffice, that which generally and along the way we have discussed about the nature of vortices and of their application to the phenomena of gravity : our considerations would not recommend the hypothesis of vortices, but only to draw certain conclusions from that, without which I might have believed that hypothesis itself could not be stopped.

Now I come to the other part of the section, where we will consider briefly the state of fluids, which may be encountered within the motion of vessels: The subject matter is most fertile and variable in an infinite number of ways : But we will mention a few, as examples, to which many others can be recalled.

§. 18. If water may be contained in a vessel with a hole and that vessel may itself fall freely, from itself it is apparent, now water will be going to flow out of the vessel during the fall, because truly the upper particles do not exert their weight on the lower ones : Indeed if the vessel by moving may descent with an acceleration but slower than when bodies may accelerate naturally in a vacuum, water will flow out, but with a smaller velocity as if the vessel were at rest : It will be the opposite, if the vessel by its motion may be drawn upwards : Finally if the vessel may be carried horizontally by an accelerated motion (for now we will not attend to the remaining directions) it can happen, that the velocity of the water flowing out may be greater or smaller than the ordinary velocity on account of the position of the opening: Moreover the velocities of the water thus may be determined.



§. 19. For example, the cylinder *ACDB* (Fig. 67) shall be filled with water as far as to *AB*, the base of which *CD* may have a rather small hole at *E* through which water may flow out, while meanwhile the whole vessel may be drawn upwards by the weight *P* falling by means of the rope running over the two pulleys *H* and *G*. Finally just a much water may be put to be flowing in above constantly as may be flowing out as flows out through the hole *E*: truly the weight of the cylinder and of the water contained in that may be indicated by *p*. Thus it is evident whatever drop of water in the vessel just as at rest in the vessel shall itself have a force desiring it to rise to the natural force of gravity as $\frac{P-p}{P+p}$ to 1: Because truly the reaction of the drop on the base is equal to the force, by which any droplet is forced to rise, besides the other natural pressure evident on the base, which is being expressed by $\frac{P-p}{P+p}$. Truly each pressure taken together will be to the natural pressure taken alone as $\frac{2P}{P+p}$ to 1, thus so that the base may not at all be pressed by the incumbent water, as if the cylinder were at rest and the height of water $=\frac{2P}{P+p}\times AC$, from which the height itself of the corresponding velocity of the water flowing uniformly out follows to be $=\frac{2P}{P+p}\times AC$. Therefore if $P=0$, no water will flow out, with the vessel falling naturally with an acceleration: if $P=p$, the water flows out ordinarily, because then the vessel is at rest; and if $P=\infty$, the velocity of the water flowing out will be to the ordinary velocity as $\sqrt{2}$ to 1.

[Writing this problem in modern terms, with *T* the tension in the rope, *a* the acceleration, *M* the large mass, and *m* the small mass, we have :

$$Mg-T=Ma;\quad T-mg=ma\ \therefore Mg-mg=\left(M+m\right)a$$

$$\therefore a=\frac{M-m}{M+m}g;\quad T=Mg-Ma=Mg-Ma=Mg\left(1-\frac{M-m}{M+m}\right)=\frac{2M}{M+m}mg.$$

Thus the mutual acceleration $a=\frac{M-m}{M+m}g$ is equivalent to $\frac{P-p}{P+p}:1$; while the tension in the rope $\frac{2M}{M+m}mg$ corresponds to the extra pressure force acting on either mass, taken in the ratio to the natural pressure *p* or *mg*, to become as $\frac{2P}{P+p}$ to 1. In addition, the total pressure on the base of the vessel, considered as if at rest, corresponds to a head of water $=\frac{2P}{P+p}\times AC$, from which the rate of flow of the water follows.]

§. 20. Now it is sought what must happen to the fluid contained in a vessel, to which the motion of a uniform acceleration is impressed. That truly is easiest to see from this alone, because now the inertia of the particles or the direction, along which the vessel may move, shall be horizontal contrariwise [to the impressed force], while the weight of the same is vertical: truly each remains the same constantly.

Therefore after the fluid has arrived at a state of permanence or of permanency, its surface will be a plane, but inclined towards the direction of motion. Moreover the angle of inclination may be determined as follows.

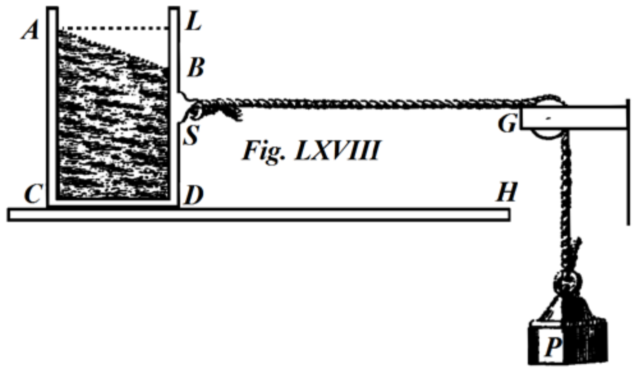


Fig. LXVIII

ACDL shall be a cylindrical vessel (Fig. 68) placed vertically, which on a horizontal plane *CDH*, by means of the weight *P* with the help of the pulley *G* joined to the vessel at *S*, is moved in a uniformly accelerated motion, and the weight of the vessel and of the water contained in that shall be to the weight *P* as *p* to P: with ordinary gravity $=1$; and the pressure of any droplet in the direction *GS* on account of its weight shall be $\frac{P}{P+p}$: Therefore if *AB* shall be in the same plane with *SG* and with the surface of the water, and *AL* may be drawn, it is apparent the action of natural gravity to be to the reaction arising from the force *P* to be, as *BL* to *AL*, or as 1 to $\frac{P}{P+p}$: and by calling the whole sine equal to 1, the sine of the angle *LAB* to be

$$=\frac{P}{\sqrt{(2PP+2Pp+pp)}}.$$

[We can solve this problem by considering the horizontal motion of the mass m : If the tension in the rope is *T*, and the mass falling is *M*, then

$Ma = Mg - T$; while $ma = T$; $\therefore a = \frac{Mg}{M+m} \rightarrow a : g = \frac{P}{P+p} : 1.$ Therefore the horizontal force acting on a small mass δm on the inclined plane, into the plane, shall be $\frac{M\delta m}{M+m} g$, and hence the component of this force up the plane, inclined at an angle θ to the horizontal shall be $\frac{M\delta m}{M+m} g\cos\theta$, while the component of its weight acting down the

plane shall be $\delta m \cdot g \sin\theta$. Hence from the equality of these forces, we have
$$\frac{M}{M+m} = \tan\theta = \frac{P}{P+p}; \text{ or } \sin\theta = \frac{P}{\sqrt{P^2+2Pp+p^2}}.]$$

Hence also it is understood the base *CD* undergoes a greater pressure from the incumbent water at *C* than at *D*, and that in the ratio of the heights AC and *BD:* and if likewise the same base may be perforated by the smallest opening, water is going to be ejected with a velocity, which shall correspond to the height of the overlying vertical column of water. Truly thus now it will be, after all will have reached a state of *permanency* ; if the weight *P* shall be variable, on no account in that situation will the surface *AB* remain permanent : but the velocity depends on that weight, by which the vessel is moved from place to place. Therefore if the whole weight may be removed, now after the vessel will have acquired a motion, the vessel will go on to move with its velocity, but the slope of the surfaces of the water will decrease, and again will be composed according to the horizontal position, just as if the vessel were at rest; therefore in these cases it is not the motion of the vessel, which will change the state of the fluids, but rather the variation of the motion.

§. 21. What we have taught in the preceding paragraph about a cylindrical vessel placed vertically is extended easily to a vessel of any form: indeed just as the inclination of the surface of the water *AB* to the horizontal in a cylindrical vessel, is will be such in all the remaining vessels : but the pressure of the water at the sides of the vessel will be defined everywhere, if a column may be considered vertical from that point, for which the

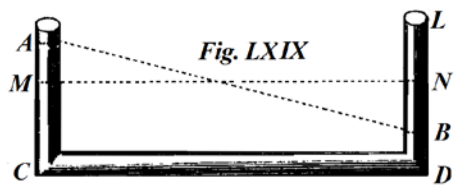


Fig. LXIX

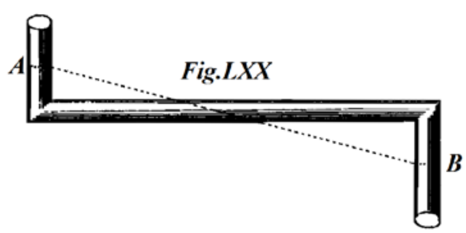


Fig.LXX

pressure of the water is required to be defined, as far as to the surface of the water, by which reason it will be produced, if there were a need for that. For example if in place of the vessel we may take a tube with each part bent, just as *ACDL* (Fig. 69), and just as that may be moved in the direction *CD*, while each surface *M, N* will change position into *A, B*, then the right line *AB* must maintain the inclination defined before ; also it can happen that a part of the water may flow through *A*, before equilibrium shall be present: if the leg *DL* may be considered downward, as in Fig. 70, the water will remain as if suspended: for in each case the inclination of the line *AB* will be the same with all else equal.

But in Fig. 69 that line *MA* will be greater, where the horizontal leg *CD* is longer : thus so that the smallest accelerations or also retardations shall be able to be observed, which often can be of use for other things, such as for determining the accelerations of ships, and the forces the oarsmen may exert with the individual immersions of the oars ; but still in these cases, because it cannot support a state of continuance or permanency, all the motion of the fluid which may be replicated in turns, is required to be investigated.

This same account furnishes that at no time shall it be allowed generally to determine from these premises, what may happen when vessels containing fluid may be beaten.

But the rules of percussion are able to be deduced from the ordinary rules of pressure, since the percussion shall be none other, except for a huge pressures lasting for a short time.

Fig. LXXI

§. 22. For example let the cylindrical tube *ABCD* (Fig. 71) be placed horizontally full of water, and the globe *P* on the projection *AP* to the tube may be struck *:* then the water suddenly presses the base *BA* strongly towards *P:* so that we may understand this pressure correctly, initially we may consider no weight to be present for the tube : thus it is apparent from the equality between the action and reaction from the enduring impulse of the globe to be driven out somewhat from the water, if this may strike the base immediately. Truly if the weight of the water and of the tube have a ratio that may be put as p to π, the impulse of the water at the base may be diminished, and the total impulse will be, to the impulse remaining, as $p+\pi$ to p; for the impulse is distributes equally both in all the water as well as in the material of the tube, and only the fluid at the base reacts.

But now we may imagine a little hole *m* in the base *BA*, but yet through that water may be considered to flow freely ; thus we understand, a small amount of water to be ejected by an enduring impulse through the little opening *m* ; nor yet will this amount of water be able to be determined ; for it depends on the rigidity of the material *AP* receiving the impulse : surely if the material shall be made from that most rigid, a greater force is required to be substituted for the attempt, but not lasting so long; likewise for example the impulse may be considered to be divided into two different cases: moreover in the first case the force shall be quadrupled, in the other the duration of the force quadrupled, as [the ejection of the water] can happen in the first case rather than in the second, when the material is more rigid: thus by the impulse of the force, in going from the lesser to the greater duration, around twice the quantity thus may flow out, than in the other case. In this manner the rigidities of material can be investigated: but they can also be investigated with sound.

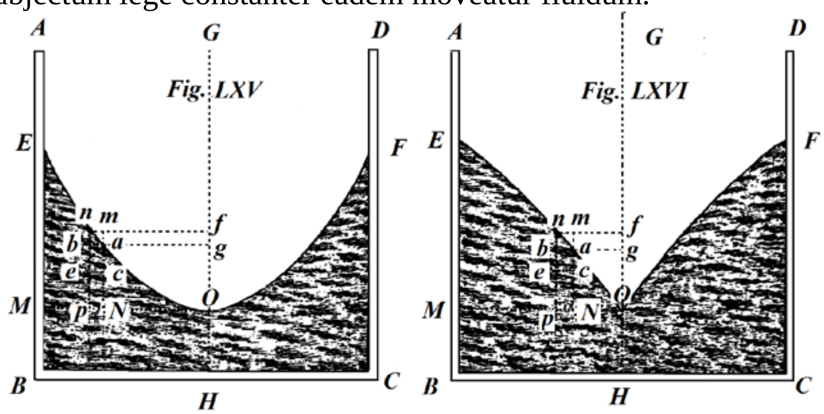
[*i.e*. the impulse is defined as av. force by the time acting, being equal to the change of momentum, which is presumably what Bernoulli means by impetus. It appears that the amount of water expelled is proportional to the square root of the impulse.]

HYDRODYNAMICAE SECTIO UNDECIMA.

De fluidis in vorticem actis,tum etiam de iis, quae in vasis motis continentur.

§. 1. Ex eo tempore quo Keplerus & Cartesius vortices adhibuere pro variis naturae phaenomenis explicandis, multi operam suam haud male se collocaturos rati sollicite istud argumentum ruminati sunt: primus autem, ni fallor, naturam ejus recte penetravit Hugenius in *Tract. sur la pesanteur;* superaddam quaedam, quae ad institutum meum pertinent, ab aliis fortasse non satis examinata.

Poni autem solent vortices ad statum *permanentiae* seu durationis reducti, ita ut nulli mutationi subjectum lege constanter eadem moveatur fluidum.



§. 2. Sit cylindrus *ABCD* (Fig. 65 & 66) verticaliter positus, cujus axis *GH*, isque ad certam altitudinem plenus sit; concipiatur aqua in vorticem acta sintque omnia jam ad statum durationis reducta: Ita superficies aquae deprimetur versus axem & elevabitur versus latera: Sectionem per axem terminatam a superficie aquae repraesentabimus curva *EOF*, hujusque curvae nunc indolem dabimus ex data relatione, quam inter se habent velocitates sub certis ab axe distantiis.

Ducantur *ga* & *fn* infinite propinquae & horizontales, agaturque *am* verticalis:Sit $Og = x,\ gf$ seu $am = dx,\ ga = y,\ mn = dy$: Patet autem quamlibet guttulamin superficie positam nisu suo, ex vi centrifuga horizontali & vi gravitatis verticali composito, perpendiculariter superficiei insistere, quia si oblique contranitatur nihil sit, quod guttulam in loco suo conservet.

Igitur si vis centrifuga guttulae in *a* positae exprimatur per horizontalem *ba* & vis gravitatis per verticalem *ca* compleaturque rectangulum *abec*, erit diagonalis *ae* ad curvam perpendicularis; unde triangulum *eca* simile est triangulo *amn* & sic

$$dx : dy = ec : ca = ba : ca,$$

vel ut vis centrifuga in puncto *a* ad vim gravitatis.

Demonstravit autem Hugenius vim centrifugam corporis in gyrum acti celeritate, quam lapsu libero per altitudinem dimidii radii acquirere possit, aequalem esse vi suae gravitates : quod si proinde altitudo respondens guttulae velocitati gyratoriae dicatur V, vis gravitas g: erit vis centrifga $\frac{2gV}{y}$, unde

$$dx : dy = \frac{2gV}{y} : g,$$

vel

$$dx = \frac{2Vdy}{y}.$$

§. 3. Si ponatur $V = \frac{1}{2}y$ fiet $x = y$ & proinde linea *EO* erit recta constituens cum axe *GH* angulum semirectum habebitque cavitas formam coni: Si vero servata eadem proportione velocitatum, quae nempe sint ubique radicibus distantiarum ab axe proportionales, aquae celerius tardiusve circumagantur, fiet angulus *EOG* eo acutior, quo celerius moventur, ita ut si infinita fuerit velocitas, tunc aquae perpendiculariter fundo insistere debeant instar muri, cavitatemque cylindricam interius formare, si modo operculum sit in *AD*, quod impediat, quominus aquae omnes ejiciantur.

§.4. Si ponatur paullo generalius $2V = fy^e$, fiet $dx = fy^{e-1}dy$ vel $x = \frac{f}{e}y^e$: Hince sequitur curvam semper fore versus axem concavam, ut in Figura 65, si sit *e* major unitate atque convexam, ut in Fig. 66, si sit minor. In priori casu est angulus *EOG* semper rectus, in altero semper nullus: in solo casu quo $e = 1$ potest angulus iste esse qualiscunque.

§. 5. Inservire possunt haec ad dignoscendam quodammodo scalam velocitatum in vortice artificiose producto: si enim superficiem videas concavam, recte judicabis velocitates majori crescere ratione, quam distantiae ab axe crescant, si convexam contrarium deduces. Si curva non videatur ad parabolicum genus pertinere, indicium erit velocitates non posse comparari cum distantiarum determinata aliqua potentia. Quo major observata fuerit linea *EM* terminata ab horizontali *OM*, eo major putabitur velocitas particularum absoluta seu littera *f*.

§. 6. Existimo autem non posse vorticem in statu suo per tempus aliquod notabile permanere, si vires centrifugae partium aequalium in fluido homogeneo decrescent ab axe versus peripheriam: hoc enim si esset, cum nihil sit, quod partium axi viciniorum vim centrifugam sufficienter coerceat, fieret utique, ut partes illae viciniores perpetuo ab axe recederent, remotioresque ad illum propellerent, neque unquam in hoc statu aequilibrium aut status durationis obtineri posset. Apparet inde quantitatem hanc $\frac{2gV}{y}$ (quae nempe in fluidis homogeneis vim centrifugam partium aequalium exprimit) aut una crescere cum y aut saltem non decrescere; atque sic si rursus ad specialem hypothesin antea factam ($2V = fy^e$) descendamus, nonpoterit *e* esse minor unitate. Igitur in omnibus vorticibus, de quibus hic sermo est,ad statum durationis reductis, superficies nunquam convexa erit, ut in Figura 66, sed semper aut concava, ut in Figura 65, aut conica: & quia e vel major est unitate vel eidem aequalis, aliter fieri non potest, quin velocitates aut aequali aut majori ratione crescant cum radicibus distantiarum ab axe. Haec cum ita mecum perpendo, nonintelligo quemadmodum Newtonus fingere sibi potuerit duos vortices fluidi ubique homogenei ad statum perpetuae durationis reductos, in quorum altero *tempora periodica partium sint ut earum distantiae ab axe cylindri, in altero autem ut quadrata distantiarum a centro sphaerae:* Nam in horum vorticum altero velocitates ubique essent aequales, & in altero plane decrescerent ab axe versus peripheriam.

Magis verosimile est, in plerisque vorticibus, qui statum per durationis jam attigerint, fluidi sive homogenei sive heterogenei partium singularum tempora periodica eadem fore, quasi totus cylindrus solidus fuerit, partes autem quae sint specifice graviores circumferentiae viciniores futuras esse. In hoc casu fit v proportionale ipsi y & V proportionale ejusdem quadrato, curvaque *EOF* erit parabola Apolloniana, cujus vertex in *O* & cujus axis sit *OG*.

Praesertim haec ita proxime fore praesumo, si vortex generetur a rotatione vasis cylindrici circa axem *HG*, vel etiam ab agitatione uniformi baculi juxta latera vasis, cujusmodi vorticum phaenomena exposuit D. Saulmon in *Comm. Acad. Reg. Sc. Paris. a.* 1716.

§. 7. Pressiones quas diversae cylindri *ABCD* partes a fluido sustinent, proportionales sunt altitudinibus columnarum verticalium iisdem partibus respondentium;neque enim requiritur, ut huic ponderi conatum fluidi a vi centrifuga oriundum addamus, quia conatus iste effectum jam obtinuit in elevandis aquis: Atque si vas non fuerit cylindricum sed irregularis utcunque structurae, licebit cylindrum fingere,cujus axis coincidat cum axe rotationis, fluido ita plenum, ut punctum *O* tamen vase proposito quam in cylindro fictitio in eodem loco positum sit: tanta enim in quovis cylindri puncto pressio erit, quanta est in eodem puncto, quatenus id ad vas propositum pertinet. Apparet ex hoc ipso, posse superficies vorticum ex alio principio quam quo ante usi sumus definiri: Ducta nempe linea horizontali *OM* &verticali *Na* cum sua infinite propinqua *pn* sequitur altitudinem *Na* seu *Og* proportionalem esse vi centrifugae omnium particularum quae sunt in *ON*, & differentiam altitudinum duarum proximarum, nempe *am* seu *gf*, proportionalem vi centrifugae particulae *Np:* Unde rursus derivatur aequatio finalis, quam§. 2 dedimus, nempe $dx = \frac{2Vdy}{y}$.

§. 8. Videamus nunc quid accidere debeat corporibus vortici innatantibus; ut autem quaestio eo distinctior atque simplicior fiat, corporis loco considerabimus globulum parvum ejusdem cum fluido vorticoso gravitatis specificae.

Globulus talis fluido commissus duabus statim potentiis sollicitatur, altera tangentiali ab impetu fluidi ortum trahente, altera centripeta, quae a vi fluidi centrifuga nascitur. Istae vires constantem servant inter se rationem, quadratam nempe velocitatis fluidi *respectivae;* sive quiescat corpus sive motu circulari feratur.

Notari autem meretur ab iis, qui in explicandis gravitatis phaenomenis adhaerent principiis Cartesianis, vim tangentialem esse incomparabiliter majorem vi centripeta:est enim illa ad hanc, ut distantia corporis ab axe vorticis ad octo tertias partes diametri globi; demonstrationem videre est in *Comment. Acad. Petrop. tom.* II,*p.* 318 & 319.

§. 9. Quamvis sciam multa a variis allegata fuisse, ut ostenderent, materiam subtilem celerrime in vorticem actam corpora quidem versus axem detrudere posse neque tamen inde sequi, ut simul a vortice deferantur ista corpora, non potui tamen hunc mihi scrupulum eximere, postquam cognovi vim tangentialem vi centripetal esse pene infinite majorem. An non melius huic difficultati occurritur, si duos super eodem axe vortices statuamus contrarios & aequalis virtutis: Videtur enim, phaenomena naturae plurima conciliari non posse cum vorticum hypothesi, nisi ponamus duos pluresve vortices liberrime sub qualicunque directione se invicem trajicere posse: vel sola gravitatio communis omnium corporum caelestium versus se invicem, quae in dubium vocari nequit, satis ostendit aut valedicendum esse hypothesi vorticum, aut liberrimam vorticum plurium in omnes plagas decussationem statuendam esse. Si igitur duo vortices aequalis virtutis contrarii super eodemque axe fingerentur, tunc impetus contrarii destruerent vires utriusque vorticis tangentiales;simul autem uterque vortex concurreret ad corpus versus axem communem deprimendum.

§. 10. Altera accedit difficultas, quominus possit corporum gravitas peti ex effectu duorum vorticum contrariorum super eodem axe motorum. Ita enim corpora non versus punctum commune aut quasi punctum sed versus axem gravitarent, motuque ad eundem perpendiculari laberentur, quod cum descensu corporum verticali & rotunditate vel quasi rotunditate terrae corporumque coelestium pugnat.

Huic alteri quoque difficultati occurretur, si fingantur duo axes ad se invicem perpendiculares aut proxime tales, circa quorum utrumque duo vortices contrarii aequalis virtutis circumagantur. Namque vis composita omnium vorticum ita intelligi potest comparata, ut corpus detrudat proxime versus punctum, quo ambo axes se invicem intersecant; semper tamen foret terra aliquantum compressa versus planum per ambos axes transiens. Poterit autem vel huic incommodo, si modo incommodum sit, obviam iri, multiplicando admodum vorticum numerum: nam si vel infiniti fere statuantur vortices, poterunt omnes eadem facilitate se trajicere, ac radii luminis, qui se minime impediunt.

Volui ista hic adjicere in gratiam eorum, qui vorticibus delectantur, ut videant,an motus iste facilius concipi possit eo, quem Hugenius finxit: utroque enim phaenomena naturae aequaliter explicari possunt. Hanc sententiam paullo accuratius exposui in dissertatione, quam Academia Reg. Sc. Paris. praemio a. 1734 affectam imprimi curavit .

§. 11. Quia dubitari nequit, quin omnes planetae versus solem & satellites versus suos planetas ad mentem Newtoni *gravitent,* hujusque gravitatis causa affinis sit cum illa qua corpora terrestria versus centrum terrae tendunt, erit vorticum hypothesis ad totum systema mundi extendenda, si pro gravitate corporum terrestrium explicanda adhibeatur. Ita vero planetae, materiae subtili innatantes, moverentur in medio resistente, paulatimque de motu suo aliquid perdentes ad centrum solis accedere sub forma spiralis deberent: hoc vero cum ex antiquissimis observationibus non appareat, postulat vorticum hypothesis, ut fluidum vorticosum ponatur supra modum rarum atque subtile idque velocitate, quam mens humana vix assequi possit, motum: quo enim rarius fluidum, eo celerius motum fingas necesse est. Fortasse opportunius motuum perennitas explicabitur

a communicatione quadam motus reciproca, ita ut quas modo corpus coeleste propulsit particulas, ab his alio tempore vi simili propellatur.
§. 12. Venio jam ad reliquas corporum gravitantium proprietates, quae ex hypothesi vorticum sequuntur. Ponamus itaque corpus in fluido vorticoso quiescens, quod nullas fluidi particulas per poros suos transmittat: ita tendet corpus versus centrum vorticis, eritque vis ejus centripeta praecise aequalis vi centrifugae fluidi vorticosi, quod sub simili volumine in eadem a centro distantia positum sit. Ergo corpora quaecunque in simili vorticis loco constituta eandem habent vim centripetam si idem habeant volumen, etiamsi quantitates materiae in unoquoque corpore sint utcunque inaequales, & si hujusmodi corpora libere versus centrum vorticis moveri possint, ferentur velocitatibus inaequalibus, reciproce scilicet proportionalibus quantitatum materiae radicibus quadratis, si spatia emensa sint aequalia.

§. 13. Quae in praecedente paragrapho monita sunt, facile applicantur gravitati corporum, si modo principium gravitatis sit vis centrifuga alicujus materiae subtilis celerrime in vorticem actae. Quia vero experientia docet omnia corpora terrestria in vacuo simili descendere velocitate omniaque corpora e filo suspensa aequali vibrationes facere tautochronas, inde concludemus, *particulas ultimas graves*, per quas nempe fluidum gravificum penetrare nequeat, in omnibus corporibus terrestribus esse aequalis densitatis specificae, id est, sub aequalibus voluminibus aequales *materiae solidae* quantitates continere, idque non minus in *particulis gravibus*, quae aurum quam quae plumas componunt. Ne vero haec secus ac volo explicentur dicendum mihi erit, quid intelligam per *ultimas particulas graves* & per *materiam solidam* ipsis insitam.

§. 14. Sunt igitur *particulae graves* proprie sic dictae illae, quae impenetrabiles sunt materiae subtili vorticosae: hujusmodi enim particulae idem faciunt, quod corpora in vortice posita, de quibus §. 12 diximus: quamvis autem impenetrabiles sint materiae subtili modo dictae, non crediderim tamen illas perfecte solidas, quales Hugenius praesumsisse videtur in *Tract.* suo *de gravitate,* id est tales quorum spatium totum materia repletum sit sine poris aut fluido interfluo: existimo potius has *particulas graves* suos rursus habere poros, atque in illis fluidum aliud esse longum subtilius, quod particulas graves eadem libertate trajicit, qua fluidum gravificum fluit per corpora sensibilia: residuum vero quod in *particulis gravibus* sibi cohaeret voco *materiam solidam* ad particulas easdem pertinentem.

§. 15. Perspicuum ex his est, diversas corporum gravitates specificas minime petendas esse ex diversa densitate *particularum gravium,* sed ex eo, quod hae particulae possint esse in diversis corporibus sub eodem volumine numero inaequales, aut etiam magnitudine, sic ut in corporibus compactioribus majorisve gravitatis specificae *particulae graves* vel minoribus interstitiis positae vel volumine majores sint.
Etsi vero diversas densitates specificas habuissent *particulae graves* in diversis corporibus, non propterea diversas habitura fuissent gravitates specificas corpora caeteris positis paribus: talia autem corpora ex alto delapsa diversa inter se velocitate fuissent descensura versus centrum terrae: Fieri itaque potuisset, ut corpora aequalis gravitatis specificae vel in vacuo communiter ita dicto inaequali velocitate descendissent, non minus atque corpora videmus diversae gravitatis specificae aequali velocitate

descendentia: In hujusmodi autem corporibus leges motuum longe aliae forent, atque nunc sunt, ubi massae ex solis ponderibus aestimantur.

§. 16. Caeterum quia omnia, quantum experientia constat, corpora terrestria habent suas *particulas graves* aequalis densitatis specificae, ut§. 13 monitum fuit, facile quidem inducar, ut credam idem in omnibus planetis fieri seorsim consideratis: Planetas vero inter se comparatos *particulas* suas *graves* diversae habere densitatis specificae mihi admodum est probabile, quia nullam video rationem, cur in omnibus planetis similes esse debeant istae particulae. Sed a *particularum gravium* densitate in quolibet planeta pendet hujus vis centrifuga seu conatus recedendi a sole. Igitur nondum licet colligere *planetarum vires centrifugas se habere in ratione quadrata reciproca eorundem distantiarum a sole ex eo, quod tempora periodica rationem sequantur sesquiplicatam distantiarum:* talis enim conclusio supponit similem in omnibus planetis *particularum gravium* densitatem.

§. 17. Planetarum vires centrifugae aequales utique sunt viribus contrariis quibus versus solem trahuntur: Quia autem, ut dixi in superiori paragrapho, nondum certum est, in quanam ratione respectu distantiarum a sole vires planetarum centrifugae mutentur, ideo neque de eorum viribus gravitatis versus solem aliquid certi statuere licet; et plurima quidem sunt in vorticum hypothesi, quae vires gravitatis in diversis distantiis constituunt & determinant.

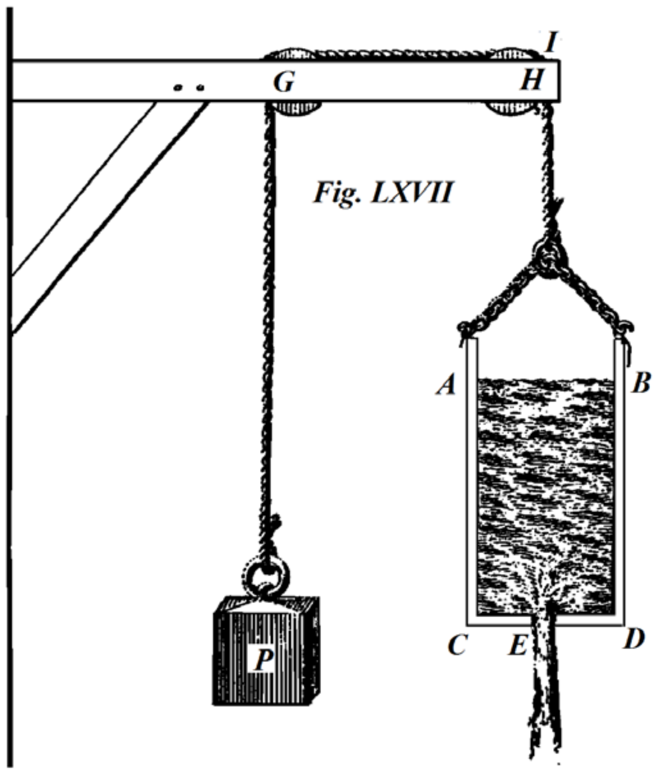
Cum enim vis gravitatis sit aequalis vi centrifugae materiae subtilis, quae particulas corporis graves penetrare nequit, sequitur eo majorem esse vim gravitatis, quo majori materiae subtilis quantitati transitus negatur; quia vero scimus corpus saepe fluido uni impenetrabile esse, quod alii fluido subtiliori liberrimum concedit transfluxum, fieri potest, si modo materiam vorticosam in diversis a centro vorticis distantiis inaequaliter subtilem putemus, ut unus idemque planeta in inaequalibus a sole distantiis inaequaliter ad solem pellatur, quod idem facilius contingere potest in diversis planetis, quia accedit diversa quae esse potest particularum gravium structura.

Praeter haec sunt etiam diversa materiae vorticosae densitas, velocitas distantiaque a centro, quae concurrunt ad vim gravitatis formandam. Si vero eorum ratio habeatur, apparebit posse quidem vires gravitatis decrescere crescentibus distantiis a centro virium, neque tamen propterea vires centrifugas aequalium materiae vorticosae voluminum pariter decrescere, quod posterius ob rationem §. 6 expositam fieri non posse existimo.

Ista vero quae generaliter & obiter disputavimus de natura vorticum eorumque ad phaenomena gravitatis applicatione, sufficiant: animus non fuit vorticum commendare hypothesin, sed quasdam tantum inde conclusiones facere, sine quibus ipsam hypothesin subsistere non posse crediderim.

Venio jam ad alteram sectionis partem, qua breviter considerabimus statum fluidorum, quae intra vasa mota continentur: Argumentum est fertilissimum infinitisque modis variabile: Sed pauca attingemus, ceu exempla, ad quae multa alia revocari poterunt.

§. 18. Si aqua in vase perforato contineatur ipsumque vas libere cadat, ex se patet, nihil aquae durante vasis lapsu esse effluxurum, quia nempe particulae superiores non gravitant in inferiores: Si vas motu quidem accelerato descendat sed tardiore quam quo corpora naturaliter in vacuo accelerantur, effluet aqua, sed minori velocitate ac si vas quiescat: Contrarium erit, si vas motu accelerato sursum trahatur: Denique si vas horizontaliter accelerato motu feratur (jam enim ad reliquas non attendemus directiones) fieri potest, ut velocitas aquae effluentis major sit vel minor velocitate ordinaria pro ratione situs foraminis: Velocitates autem aquae sic determinabuntur.



§. 19. Sit v. gr. cylindrus *ACDB* (Fig. 67) aqua plenus usque in *AB*, cujus fundum *CD* foramen habeat in *E* valde parvum per quod aquae effluant, dum interea totum vas sursum trahatur a pondere *P* descendente mediante funiculo super duabus trochleis *H* & *G* excurrente. Denique constanter tantum aquae superius affundi ponatur, quantum effluit per foramen E: pondus vero cylindri & aquae in eo contentae indicetur per *p*. Ita apparet quamlibet guttam aquae in vase veluti stagnantis vi animari ad ascensum quae se habeat ad vim gravitatis naturalem ut $\frac{P-p}{P+p}$ ad 1: Quia vero reactio guttulae in fundum aequalis est vi, qua ad ascensum animatur quaevis guttula, praeter pressionem naturalem aliam exeret in fundum, quae exprimenda erit per $\frac{P-p}{P+p}$. Utraque vero pressio simul sumta ad pressionem solam naturalem ut $\frac{2P}{P+p}$ ad 1, adeo ut fundum haud secus ab incumbente aqua prematur, quam si cylindrus quiesceret essetque altitudo aquae $=\frac{2P}{P+p}\times AC$, ex quo ipso sequitur altitudinem velocitati aquae uniformiter effluentis debitam esse $=\frac{2P}{P+p}\times AC$. Igitur si $P=0$, nulla effluet aqua, cadente vase motu naturaliter accelerato: si $P=p$, effluet aqua velocitate ordinaria, quia tunc vas quiescit; atque si $P=\infty$, erit velocitas aquae effluentis ad velocitatem ordinariam ut $\sqrt{2}$ ad 1.

§. 20. Quaeritur nunc quid accidere debeat fluido, quod in vase continetur, cui motus horizontalis uniformiter acceleratus imprimitur. Id vero facillimum est videre ex hoc solo, quod nunc inertia particularum ceu directioni, sub qua vas movetur, contraria sit horizontalis, dum gravitatis earundem est verticalis: Utraque vero manet constanter eadem.

Igitur postquam fluidum ad statum durationis seu *permanentiae* pervenit, superficies ejus plana erit, sed inclinata versus plagam motus. Angulus autem inclinationis determinabitur ut sequitur.

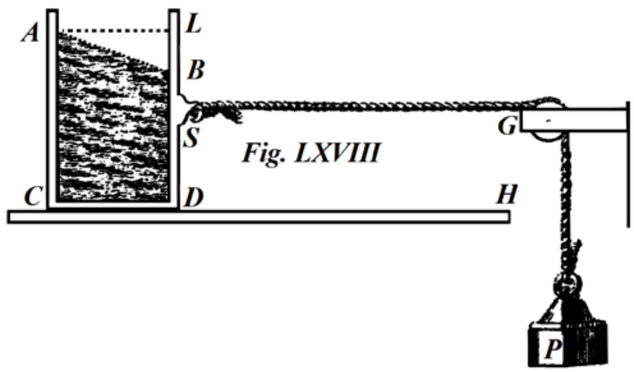


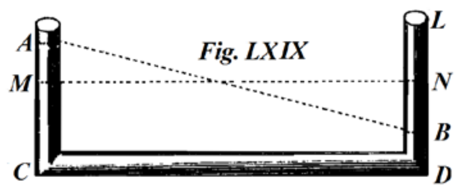
Fig. LXVIII

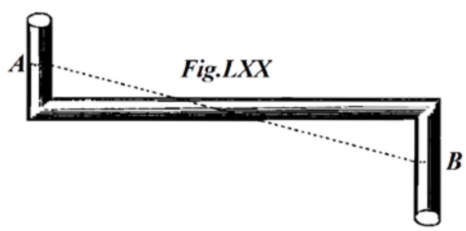
Sit vas cylindricum ACDL (Fig. 68) verticaliter positum, quod super plano horizontali *CDH*, mediante pondere *P* ope trochleae *G* vasi annexo in *S*, movetur motu uniformiter accelerato, sitque pondus vasis & aquae in illo contentae ad pondus *P* ut *p* ad P: gravitatio naturalis $=1$; eritque nisus cujuslibet guttulae in directione *GS* ratione suae gravitationis $\frac{P}{P+p}$: Igitur si *AB* sit in eodem plano cum *SG* & cum superficie aquae, ducaturque *AL*, patet actionem gravitatis naturalis fore ad reactionem a pondere *P* oriundam, ut *BL* ad *AL* seu ut 1 ad $\frac{P}{P+p}$: vocatoque sinu toto 1, fore sinum anguli *LAB*

$$=\frac{P}{\sqrt{(2PP+2Pp+pp)}}.$$

Hinc etiam intelligitur fundum *CD* majorem ab incumbente aqua pressionem pati in *C* quam in *D*, idque in ratione altitudinum AC & *BD:* sique idem fundum perforetur minimo foraminulo, aquam ejectum iri velocitate, quae respondeat altitudini columnae verticalis superincumbentis. Ita vero erit, postquam omnia jam ad statum *permanentiae* pervenerint; si pondus *P* variabile sit, nunquam in eodem situ permanebit superficies *AB:* a pondere autem isto pendet velocitas, qua vas movetur insingulis locis. Igitur si totum pondus auferatur, postquam vas jam motum acquisiverit, perget vas sua velocitate moveri, superficies autem aquae declivitatem deponet, rursusque ad situm horizontalem componetur, veluti si quiescat vas; in his adeoque casibus non est vasis motus, qui fluidorum statum permutet, sed motus variatio.

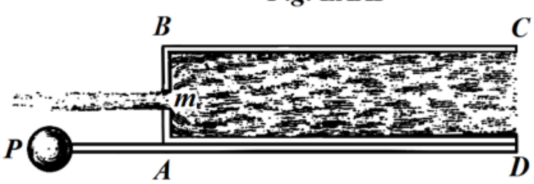
§. 21. Quod in praecedente paragrapho monuimus de vase cylindrico verticaliter posito facile extenditur ad vas cujuscunque figurae: qualis enim est inclinatio superficiei aqueae *AB* ad horizontem in vase cylindrico, talis erit in omnibus reliquis vasis: pressio autem aquae in latera vasis ubique definietur, si columna concipiatur verticalis ab eo puncto, pro quo pressio aquae definienda est, usque ad superficiem aquae, quae cogitatione producenda erit, si id opus fuerit. Si loco vasis sumatur v. gr. tubus ab utraque parte inflexus, veluti *ACDL* (Fig. 69) isque moveatur in directione *CD*, tum utraque superficies *M*, *N* situm mutabit in *A*, *B*, donec recta *AB* debitam obtineat inclinationem antea definitam; fieri etiam potest ut pars aquae effluat per *A*, priusquam aequilibrium adsit: si crus *DL* deorsum spectet, ut in Figura 70, aqua manebit veluti suspensa: in utroque enim casu inclinatio lineae *AB* caeteris paribus eadem erit. In Figura autem 69 erit linea *MA* eo major, quo longius est crus horizontale *CD:*sic ut minimae accelerationes aut etiam retardationes observari possint, quod saepe aliis rebus inservire potest, veluti dignoscendis accelerationibus navium, nisibusque quos exercent singulis remorum





submersionibus remiges; in his tamen casibus,quia non potest status supponi durationis seu *permanentiae*, omnis fluidi motus,qui singulis vicibus replicatur, esset inquirendos. Facit eadem haec ratio, ut nondum liceat omnino ex praemissis determinare, quid fieri debeat cum vasa fluidum continentia percutiuntur. Possunt autem regulae percussionum ex ordinariis legibus pressionum deduci, quandoquidem percussio nihil aliud sit, nisi ingens pressio parum durans.

Fig. LXXI



§. 22. Sit v. gr. tubus cylindricus horizontaliter situs *ABCD* (Fig. 71) aqua plenus, impingatque globus *P* in tubi prominentiam *AP:* tunc aqua subito premet vehementer fundum *BA* versus *P:* ut hanc pressionem recte intelligamus, ponemus primo nullum inesse pondus tubo: ita apparet ex aequalitate inter actionem & reactionem fundum durante globi impulsu non aliter impelli ab aqua, quam pelleretur in contra clampartem a globo, si hic immediate in fundum impingat. Si vero pondera aquae & tubi rationem habere ponantur ut *p* ad π, diminuetur impulsus aquae in fundum, eritque impulsus totus ad impulsum residuum ut $p+\pi$ ad *p;* distribuitur enim impulsus aequaliter in omnem tum aquae tum tubi materiam, solumque fluidum in fundum reagit.

Nunc autem in fundo *BA* parvulum fingamus foramen *m*, sed per id tamen aqua liberrime fluere putetur; ita intelligimus, particulam aquae per foraminulum *m* ejectum iri durante impulsu; neque tamen quantitas istius aquae determinari poterit;pendet enim a rigiditate materiae *AP* impulsum recipientis: si nempe materia ista rigidissima sit, fortior pressio substituenda est impetui, sed minus durans; consideretur v. gr. idem impetus in duobus diversis casibus: sit autem in uno pressio quadrupla, in altero duratio pressionis quadrupla, quod fieri potest cum materia rigidior est in casu priori quam posteriori: ita effluet in impulsu pressionis minoris magisque durantis dupla circiter quantitas quam in altero. Possunt hoc modo rigiditates materiarum explorari: sed possunt etiam ex sono.