HYDRODYNAMICS

SECTION ONE

Which is the introduction, and contains various matters to be considered initially.

§.1. Since the Theory of Fluids shall be twofold, of which the former shall be Hydrostatics, consisting of the forces on liquids at rest and in various states of equilibrium, and the latter Hydraulics, regarding the motion of fluids, they are accustomed to be treated separately by writers ; truly as I consider both to be closely connected to each other, so that the one may often require the help of the other, without doubt these can be confused, as far as that ordering of matters may be considered, and both are to be included under a common name, and to be called here more generally Hydrodynamics. For although from the most ancient of times the subject was referred to continually under the name Theory of Fluids, yet no very noticeable advances were introduced ; indeed the knowledge of the ancient Mathematicians was limited to that about which Archimedes wrote, which was called the common equilibrium of fluids at rest, and also of the bodies within fluids in which they were present ; & since besides it was apparent by themselves, where no equilibrium was present, a motion occurred towards the part of the lower pressure, various sports and hydraulic machines hence were able to be thought out, partially for amusement and partially serving in an outstanding manner for the common good, where indeed the ancients showed great ingenuity in the matter ; also to be considered, but as if the motion of these were viewed superficially, were those motions which were due to the air pressure: but the true reasons and accurate measurements clearly were ignored in hydraulic matters, and thus were to be stopped almost at the beginning.

[The interested reader is referred to the article on the history of fluid mechanics in *Wikipedia* for details of the personages mentioned below and the books they wrote on the subject, most of which are available in their original language at the internet archive.]

§. 2. It will serve [to introduce] the motion of fluids by determining particularly the efflux of water from a vessel through a very small hole: even if truly one should not depart too quickly from Frontinus [a Roman general put in charge of Rome's water supply around 90 A.D., who wrote a book that has come down to us regarding aqueducts, etc.] and others, or those who believed, regarding the velocity of water from a vessel or reservoir, that the outflow increases [*i.e.* directly] with the increased height above the place of the outflow; even if the same Frontinus, it cannot be denied, put in place unjustified errors in computing the ways of distributing pure water, as well as of expending degraded water. [No mention is made here of subsequent studies by Islamic scholars over a long period in the middle ages.] Benedetto Castelli [a student of Galileo] first considered the problem between velocities and heights, but it was suspected that his law was false, as he

considered both to follow in the same proportion. After this finally Torricelli, [another student of Galileo, perhaps more perceptive of Galilio's work on projectiles] noted that in all motions the velocities increased following in the ratio of the square roots of the heights ; while it was indeed not convenient to measure the absolute velocities, yet experiments were put in place, from which that same measurement was judged to be defined, chiefly among which that was usually alleged, which had been assumed by Guglielmini, and was to be repeated eight times, although that certainly departed from other experiments made at that time: moreover all experiments were accustomed to differ amongst themselves, which were done under differing circumstances, nor always had care been taken, as we shall say several times in the appropriate place, with the time defined by the definite with regard to the efflux, judged to be carried with the speed of the same. [At present this sentence is vague : thus we may presume perhaps either small particles or bubbles are visible, introduced into the flow and traveling along with it, as is indicated in later sections.] Thus since we have recalled the experiment of Guglielmini, of which we have made mention only, it is required to be concluded from the quantity of water, which on consideration flows past [a point on a channel, or through an orifice] in a given time, that its velocity cannot be greater than that, which shall be as the fourth part of the height of the surface of the water above the opening. [Recall the *vis viva* idea of the height being equal to the final speed squared of a falling body in arbitrary units.] And other experiments by the same author, which are examined in *Book.* 2, *prop.1,*of his work *Mensura aquarum fluentium…,* of which the force of the water flowing out may rise to two thirds of its height; but the heights are extended in the works of Mariotte, and in others where they take the half height ; from which with no obstruction of the velocities thus I am persuaded by the diversity of estimates, that the disagreements between the force and the various velocities between themselves, which were considered everywhere to be approximately in proportion to the heads of water, must be as the whole height : but that which was cited finally [at this time], and which was seen to put a half in the foremost position in the works of a great many authors, with the number being moved there beyond doubt by Newton, a man of immortal merit, as he talked about the theory with a little more confidence, where water by the least consideration could be found to be able to rush out upwards from a vessel to ascent to half the height of the water in a stagnant vessel : even if that assertion itself was contradicted at once by all the experiments from which these heights were taken. Newton had established the theory in the first edition of *Princ.Math.Phil.Nat.,* and he had required a pressure, by which the water placed just before the hole would soon to be passed out and set in motion. Truly because the true nature of the matter was hardly ever allowed to be examined, so that a force may be defined *initially* to set the water in motion, rather than *otherwise* by that force arising from the phenomenon of the motion, which I have often found to be possible to judge: there the foundations of the reasoning are to be in doubt. Hence again the man may only be praised for changing his opinion in the second edition of his work [with a little help from Johan. Bernoulli, and at considerable discomfort to himself and the hapless Cotes, as the second edition was in press at the time], and again a little more in the third, affirming indeed that water rises to the whole height, but the jet that it may form in front of the opening contracts or becomes more slender, and thus for each phenomenon to be satisfying the quantity of the efflux in a given time, which may appear to contradict each other. But whatever the true cause may be for that contraction of the jet

of water, on that account, it cannot be denied that the velocity of the efflux of water cannot be estimated from the quantity [flowing out in unit time], I consider that the theory itself may still not be beyond the need of further refinement, because it is by *accidental causes* that the velocity cannot everywhere be constant, unless it may be attributed to unknown causes such as the tenacity of water, and other like causes. Thus, since the water does not flow out through a simple hole but through a cylindrical tube, the jet is not safely constrained by the velocity, with the exception that it may be lost on account of friction on it: but if this may not be considered an obstacle, the flow of water can be deduced from the pressure correctly and with care, I ask this, so that the mind may be turned to more composite cases, e.g. to the flow of water, which Mariotte calls a miracle, from a vessel, which with a diaphragm perforated by some holes allows water to be dispensed filling two cavities, thus so that water can be thought to flow through two holes: Mariotte discusses this motion in his outstanding text *de motu aquarum, part. IV, pag.* 442. [Thus the flow speeds can be compared at different places across a wider initial opening.]

§.3. Thus since occasionally these laws of motion for fluids can be reduced according to the rules of pure geometry, these may be easily judged between themselves while little hope remains [for a more general solution], without some physical hypothesis, since either such laws lie just beyond the insight of the most talented and incomparable of men : nor do I believe that I shall be able to enter into all mathematical rigor in this work which I am about to present : the principal theories of physics are required to be accepted, and not without some largesse, as being approximately true ; moreover with the principles admitted, all will be geometrical, and without tiresome restrictions, and the quantities considered will cohere by the necessary connection between each other. Yet I am unable, how to think best about this same physics which perhaps I have found in place, how should my hand be guided according to the several new properties, since matters are required to be uncovered both about equilibrium as well as the motion of fluids, which unless the love of the labor deceives me, may result finally in a significant improvement in Hydrodynamics, if such properties could be developed more than has been permitted to me so far ; where it will be convenient for me to be reminded, as with many things, whatever is new is customarily to be regarded with suspicion : with the whole theory conceived in the mind, to have been written out as a tract, and generally to have it communicated to friends in private, indeed also to have read the same before our society, before I may put in place any experiment, lest from preconceptions of the measurements under false notions, yet with these satisfied approximately, I might still be open to deception, and even when the most observant men are openly full of praise, such men themselves cannot be persuaded, unless the proof is to be confirmed by experiment; and with all these things accomplished, with the experiments made in the presence of friends, and these in agreement with theory, I was scarcely able to hope for so much. Now truly we may return there, from which we were diverted.

§.4. After the authors were convinced about the differences of the velocities with the change in height, they began to consider more complex vessels, truly equipped with a full variety of pipes inclined diversely and unequally. Moreover Frontinus in his own time

was not ignorant, and he knew in a certain way about the nature of these things : the amount of water to be increased by the inclination or levelness of the pipe, that is, how the designated pipe should be placed, to determine which village or even stream it should supply : thus also the water regulators [see the extract below of his work in translation] ordered to be adjusted according to a line, as he mentions, and at the same depth. And indeed in this respect, justly, whatever Frontinus demanded about certain things, he did not take velocity into account ; truly where he puts in place the calculation of all the water allocations, and he compares that with what is being demanded, I cannot see how it is possible for him to be excused. He had been taught by experience too, what deserved to be noted, in addition for the water requested must be measured according to position by a legal water measuring cup, to which the pipes of the wider water meter should be attached at once, because it shall be thus, and correctly indicated by Fabretti, I will show in due course, although others of the greatest sharpness might have been in agreement, that which is not proven itself to satisfaction or rather concerning which it is to be doubted.

[Here is an extract in the public domain : Clemens Herschel Latin; Tr. Charles E. Bennet, Loeb Library (1925) ; from

Fortinus : *The Aquaducts of Rome* :

36. But the position of the *calix* [a sort of water pipe placed in an aquaduct to draw off a certain amount of water from a stream] is also a factor. Placed at right angles and level, it maintains the normal quantity. Set against the current of the water, and sloping downward, it will take in more. If it slopes to one side, so that the water flows by, and if it is inclined with the current, that is, is less favorably placed for taking in water, it will receive the water slowly and in scant quantity. The *calix*, now, is a bronze ajutage, inserted into a conduit or reservoir, and to it the service pipes are attached. Its length ought not to be less than 12 digits, while its orifice ought to have such capacity as is specified. Bronze seems to have been selected, since, being hard, it is more difficult to bend, and is not easily expanded or contracted.]

§. 5. But what the ancients saw to be obscure and without making true measurements, that at last in the treatise concerned with the *Measurement of Flowing Water*, the distinguished Guglielmini says in a more accurate and general proposition of such a kind, that the velocity to be the same of water flowing along an inclined channel and if it were flowing from a vessel in the same manner, *and for equal section, as far away from the surface of the water as the section from the horizontal surface through the starting container*, which proposition opposes the proposition of Dionysius Papin, itself straying far from the truth. But because there, as we have discussed, we may examine in particular both the Hydrostatics as well as the Hydraulics, here also that is required to be enumerated, with the pressure of the fluids to be known from the force of the motion, namely *the force of a fluid jet rushing with a given velocity towards a plane at right angles, to be equal to the weight of a cylinder of fluid erected on that plane, the height of which shall be such, as from which a moving body falling freely from fluid at rest may acquire that velocity.* With the aid of this most useful problem it is allowed to estimate the force of fluids exciting machines, or as the wind is of such a kind, the propelling of

ships, the motion of bodies in resisting mediums, and many others. But about Hydrostatics, which is concerned in particular with the finest of tubes or capillaries, I have nothing to say, because at this point it cannot be reduced to the common general laws of all fluids: Besides the author is unsure, who first had observed the nature of these tubes ; yet it can be agreed the observation to be recent, because in other books some 70 or 80 years before these nothing can be seen.

§. 6. Besides authors cited from the times of Galileo, in matters concerned with water the more celebrated are Torricelli, Borelli, Vivian, Pascal, Boyle, and in more recent times there are Varignon, Newton, Poleni, Hermann, Jacob & Johanne Bemoulli, the discoveries of which are set out in *Comment.Acad.Reg.Sc.Paris., Princ.math.phil.nat., Tractatu de castellis notisque ad Frontinum, Phoronomia, Actis Lips.,* and with various works by others. Which truly were about showing the curvatures of the fluid arising from the pressure and other matters of this kind found by geometers, which were readily reduced to pure geometry, however for the rest I pass over silently with all praise. From these expositions, which pertain to others, I think it reasonable that an account of my own be submitted, and I may say with sincerity, from these they may be able to hope for some or be indebted for a great increase in their knowledge of hydrodynamics. Therefore briefly, as far as I am able, I will indicate a brief description of the work to be undertaken.

§. 7. In the first place particular theorems will be shown, which pertain to the equilibrium of fluids at rest [in Section II]: it seemed to me the plan would be to put in place what is required, although I admit freely that no new propositions would be added by me: Indeed the way requiring to be shown, as far as I know is peculiar to myself, but since it shall be easy to devise innumerable demonstrations for these propositions, what I myself arrogate is not sufficient here in this part. In addition the phenomena of capillary tubes are examined along the way, and finally with the occasion arising, of the pressure that fluids exercise on the sides of vessels; several theorems including some new ones are added about the figures of bladders filled with liquids, about the forces of the same required for lifting loads, and about the construction and strength of aquaducts, and other related matters.

§. 8. Afterwards it is a question of determining the motion of fluids flowing from a vessel, and since all who have been involved with this until now in their theories have considered only the most obvious case, in which the opening is agreed to be infinitely small in comparison with the size of the vessel, our theory is not limited by its width and indeed extends to the case of holes of any size, and likewise to vessels of any shape. For although the inner shape of the vessel is required minimally, since the hole can be considered as indefinitely small, yet without that the motion of the water cannot be defined, since it is of a perceptible size. Corollaries may be deduced from the general theory, which illustrate the variable motion of water and the unusual characteristics of the same, and they confirm either whatever may be taught from experiments or they indicate the attributes of the theory made evident by these. Indeed the theory teaches us, when the internal amplitudes either barely or do overcome the amplitude brought to light, that the

error is insensible, which arises from the consideration of the hole arising as infinitely small, and thus perhaps with several of our additions seen to be quite useless.
Truly these, if only what shall be in the future, that I may wish to consider, besides what I write is not only for people concerned with water, but for the geometers, who are delighted with the naked truth, the greatest use of our considerations may be by others in other matters, which they understand more, since they consider carefully the motion to begin from rest, and by passing through an infinite number of steps, before a certain speed may be obtained ; the greatest changes often indeed made in the shortest instants of time, clearly as they shall not be perceived by the senses in any manner, still are required to be determined at individual points, so that both the motion can be correctly perceived by the mind, as well as because from that, various theorems can be deduced. Thus bringing to mind (because an example concerning the nature of the thing shall be the same for all), it cannot be possible, that the pressure of the water flowing along a channel with a given velocity, may be defined on the side of the same, unless these changes, which I may call momentary, may be understood correctly in the mind in some way imperceptible to the senses. I had first thought concerning these, thus that the part to be added to a new theory of water to succeed with the most optimism, which, because both the motion as well as the pressure of fluids could be considered at the same time, most appropriately would be seen to be called *hydraulic-static*. After these examples of the general theory are shown, both from simple cylindrical vessels as well as from those equipped with pipes, and in accordance with these latter ones especially the changes are determined, which arise from the beginning of the flow, as well as reached by steps in the velocity, and that indeed in the case of the hypothesis of the largest vessels; but it should be observed, that these are certainly observable changes, even if the vessels are of infinite size, and ones that can be shown from experiments, while all the water flowing from the largest vessel through a simple hole, certainly can have the velocity [measured] at once, from the first point of time. The aforementioned changes depend both on the length as well as the shape of the pipe. Finally also analytical calculations for the times found for the various kinds can be added on, together with physical annotations pertaining to that [section IV]. Finally from the indicated theory, it will not be possible, that waters may not ascent much higher than the surface of a gushing spring, is shown at the end of this section, individual phenomena which often themselves are observed, do not pertain to our hypothesis, and for pleasure I try to imitate, mention of which is added to *Hist. Reg. Acad.Sc.Paris ad ann.* 1702, where it is said, seeing that it can happen, that water leaping in fountains rises to a height three or four times the height corresponding to the maximum height of the uppermost water surface, yet soon the enormous jet of water sinks to the accustomed height, and afterwards a true account of this phenomena is advanced with the desired true measures from our theory, and the manner may be indicated required to produce these unaccustomed leaps, and indeed to be increased at will.

§. 9. Again the theory may be extended to the examination of the motions from vessels constantly being filled, from which indeed water continually flows in at the same rate as it flows out from these : the kinds of which consist chiefly in that, as the fluids runs out it approaches more and more quickly to that sudden increase [*i.e.* step] in the velocity, which is due to the whole height of the surface of the fluid above the aperture, that truly generally they never attain, unless after an infinite time: yet water to be on a slope is

shown to increase in speed so quickly, that only for an imperceptibly short time has it not acquired the whole speed, unless it is carried by the longest aquaducts or rivers, and to be thrust forwards with great liveliness; for then the accelerations as well as the speeds are not, as they cannot be perceived, what may be confirmed from the single example chosen from the book *de motu aquarum* by the celebrated Mariotte. Truly because the motion begins from rest and increases continually, formulas are given, with the aid of which either from the flow with time or from the quantity of water ejected the velocity can be defined for individual points of time and vice versa.

§.10. In the following sections fluids are considered, which are moving within vessels, where especially the motion of the fluids reciprocate or oscillate about mean positions, and careful studies of the same are indicated. Moreover Newton gave a similar theorem for the oscillations of a fluid [*Principia,* Bk. II, Prop. XLIV.], in a tube with oscillations of the same size (the two legs of which were vertical at the greatest separation, with the intermediate part horizontal), which theorem my father rendered in *Comm. Acad. Imp. Sc. Petrop. tom.* 2, *p.* 201, more generally with the inclination of the legs placed at any angle to the horizontal. Our theory includes the whole matter without any restriction, considering the tubes at individual places with the direction or position and amplitude variable as it pleases : whereupon it is shown, in which cases it comes about, that oscillations of different amplitudes shall be isochronous, with which in place the length of the most general simple isochronous pendulum is determined. And moreover besides this kind of oscillation indeed others will be subjected to examination in the subsequent section, such as these, which happen in infinite tubes of water, or also with the ends immersed, in which there is a need for individual investigation, because all the phenomena used correspond to a rule [Section VII], truly that same shall be ignored only between these in disagreement, as far as between the laws of motion which prevail between perfectly elastic bodies and for those which are softer.

§. 11. After this we progress to others consisting of several parts, certainly considering the motion of either homogeneous or heterogeneous fluids, which are forced to flow through one or several apertures, and at first being ejected into air, where that rule commonly understood about the springing of water to the highest level fails completely, with the ordinary laws of pressure ceasing. But of all these indeed no vestige is found in the works of the authors, except what Mariotte has in place in the above citation : *part.* IV, *p.m.* 442, *de motu aquar.,* where indeed the flow of water is retarded, to have been itself testified, being informed from experiments, but likewise it is evident, that it departs far from the true theory of these motions, and indeed this theory seems to be nearly all about principles, at this point by making use of the similar things, but with the force missing, thus so that there shall be nothing, which confirms the excellence of our understanding more: indeed from the truth of these, the experiments set up permit me no further doubt. But its usefulness is not lacking from these considerations, whenever great ideas can be put in place in perfecting hydraulic machines.

§.12. Dissertations concerning hydraulic machines are following, in which chiefly it is shown, some have reached a certain limit of perfection, beyond which it may not be possible to go ; but failure from this ultimate degree of perfection in many machines is

revealed mainly from the numerical calculation received, with the added rules or precepts, to which the mind shall pay close attention in the construction of new machines: [Io. Friderici Weidleri : *Tractatus de Machinis Hydraulicis…. Marlyensi et Londinensi.*…etc. (1733)] in the place of an example the most noteworthy is brought forwards by all the [water-] wheels of the *machine de Marley* [This was the largest pumping system in existence in the late 17th century, and was used to transport water from the Seine to the fountains in the gardens of Versailles, it consisted of 14 water-wheels each 38 ft. in diameter, and needed a small army of attendants], from which it may be shown, if the manner of description may be trusted, that not more than nearly a 56th part of its quantity of water supplied, as with the remaining parts the machine may be supposed to function perfectly according to theory. Also special attention is put in place about the most useful machines from the most ancient of times up to the present, especially the spiral of Archimedes, not unworthy for the attention of the Geometers, as an account of these extends not only to pure geometry but also to hydraulics.

§. 13. Certain kinds of motion of elastic fluids, such as of air and of ignited gunpowder, follow from these established matters, which pertain to the nature of fluids ; which truly I cannot consider otherwise than from the physical hypothesis themselves, about which I cannot confirm with confidence. The propositions and problems of this section are new, and chosen with that in mind, so that with many physical questions requiring to be shown, they may also present the occasion of being solved. Certain are added concerning the estimation of the innate living forces for the elastic fluids, which perhaps will be of some use in practical machines; for example, the effect of a fifth of a pound of ignited gunpowder in raising heavy weights to greater than a hundred of the fittest men laboring continually would be able to achieve in the space of a whole day
[Section X : the contemporary concept of kinetic energy and its relation to mechanical work].

§. 14. Again it is a question of the forces acting on and produced by fluids involved in circular motion, which remain unchanged in rotating vessels ; and with various others interspersed. Moreover which are carried forwards in a circular motion, how they are able to be maintained and how the phenomena of gravity can be explained by vortices ; to what extent they will be able, as much as they can be [Thus, Decarte's vortices lived on].

§. 15. Having dealt with the theory of the motions of fluids, the work descends again to the equilibrium of fluids, but now of moving fluids, the laws of which have not yet been shown. The wonder is, since the former motion may be defined from the pressure, here by the inverse method the pressure is sought from the motion, first requiring to be defined from the circumstances ; nor would I believe that another way can be entered upon without risk besides that, which I have followed : moreover I have considered a channel, through which the water flowing passes through a point in place and at a moment of time convenient to the question be cut off ; and afterwards by our rules set out I have investigated the acceleration of the particles of water projecting forwards, and of the efflux approximately. From this acceleration it may be possible to deduce the compression of that particle of water, which compression from the nature of the fluid is equal to the pressure on the sides of the channel. From this known pressure it will appear,

what must occur if the channel were perforated at the same place, and a tube be placed corresponding to the hole [*i.e.* Mariotte's technique] ; to be sure, so that water in that may rise until standing still at some level in the tube, and besides being sustained by the water flowing along the channel below, thus, so that this equilibrium exists between the flowing and standing water: I have considered that this theorem be called conveniently by this name *hydraulico-stat* [, but not now used]. Again it deserves to be noted, this fundamental theorem itself again shall be the source of other motions unknown before. The theorems which are expounded are not only new, but also there are several which are unexpected, of which nor was the truth of all sufficient to persuade me completely, before I had set up experiments, which removed all doubts for me. Moreover they have a conspicuous use, since the true valuation of the pressure of the water flowing along aquaducts or rivers depends on them, and hence the required strengths of the pipes can be determined. On this too depends the accurate measurements of water requiring to be distributed by water meters inserted laterally into the stream : in physiology now the pressures which pertain to the motions of humors in the bodies of an animal are understood more correctly, and which can be different.

§. 16. Finally I progress certain other ways in which water can make progress, requiring to be explained : thus certainly water, while it escapes through the opening, presses against the vessel in no other way than the cannon ball presses against the cannon, from which it explodes [Section XIII]: many new properties of this backwards repelling pressure are discovered, which show the uncommon nature of the pressure, and the general laws of these which are assumed, show the same nature of the thing in mechanics on serious contemplation. These investigations I have done, because it appeared to me, to be able to sail a little without oars, or to grasp the aid of a little unknown wind ; where I will bring a little something to the thing itself in place, even if I am not ignorant of all the first beginnings of things of this kind, and by most people considered laughable by themselves. Finally also some theorems are added concerning the force of the water arising from the impulse and hence from the strain, that moving bodies encounter in fluids.

§. 17. And these theorems indeed have been shown, which have allowed a geometric deduction from permitted principles to be made by me. Because truly there is nothing in the theory so firmly demonstrated, that does not demand some restriction in its application to bodies; thus it is readily apparent, that no theory of fluids is going to be anticipated, which shall not give the fullest satisfaction to all the experimental measures known ; I would wish to remind anyone about this, who would wish to confirm our theorems by experiment. And indeed generally they will find some agreement, but not perfect, and that in a more strict or relaxed way, according to the circumstances of the phenomenon. But as often as some experiment is itself conducted, for me to bear in mind before all, to what extent the principles of the theory agree with the case proposed ; and thus never or on the rarest occasion has it failed me. For not only indeed was I accustomed to foresee, in what manner an outcome would be different in the future, if something were to be of note, but also to what extent ; which is evident enough itself if I judge rightly the certain laws that influence fluids, which we have considered to be prescribed, but which themselves encounter certain obstacles everywhere, sometimes

greater or less. Quite a few other experiments the individual ones of which I have located at the end of the section to which they pertain : indeed I have been concerned especially, with propositions unknown before and with several questions resolved well enough. It is not about trust in the experiments that there may be doubt, since I shall have performed these in the presence of friends and that after publishing the theory ; yet the great part of the experiments, which I have in mind considered, when it is not allowed to go through one by one, with others remaining to be set up. From reading through our propositions, and which themselves touch on many things, I have judged that thus there is no need to expound on everything to the extent that I had wished ; for I have set out everything to some extent.

§. 18. Truly now at last an account has been rendered of the principles of which at some point we have made mention. In particular is the *conservation of living forces,* or, as I may say, *the equality between the active descending and ascending powers*
[The Latin *potentialem* or power means '*having the ability or strength to get something done or effected*', originally meant by the Romans in a political or military sense ; thus, it is in this sense that the word *potential* is used in this translation]*:* I will use this latter manner of speaking, because the former moreover signifies the same kind as found in the works of the philosophers, who are moved to use the name *vis vivae* only, which perhaps will be proven to be more suitable. I think that here our way of speaking about this is a little more eloquent.

§. 19. After Galileo had shown that a body descending either vertically or on some curved plane, acquires the same velocity, provided the height of the fall shall be the same, which also can be shown from the nature of the force pressing down ; Huygens by the same proposition, made use of for a more general hypothesis, in successfully eliciting the laws of motion for the collisions of elastic bodies, and also in establishing the centre of oscillation of a compound pendulum; moreover advancing this axiom in his own words :

If as many weights as you please may begin to move in whatever manner by their force of gravity, and every one again may be returned to rest at once of its own accord, the centre of gravity of the composite body shall be returned to the starting height, where by saying *'in whatever manner'* it is understood, *'if they may strike against each other or press against each other descending, or the bodies may interact amongst themselves in some other way.'*

The principle of the conservation of the living forces follows at once from this axiom, which Huygens himself has shown also, and in which it is assumed:

If some number of weights [*i.e.* masses] *may begin to move in some manner under the force of their gravity, the velocities of each everywhere shall be such that the products gathered together of the squares of the velocities multiplied by their masses, shall be proportional to the vertical height, through which the centre of gravity of the composite body has fallen multiplied by the total mass.*

The astonishing thing is, how great a use this hypothesis may have in the philosophy of mechanics, because, if compared with other things, without my father observing correctly here and there, but initially shown in the Paris publication of his *Dissertation Concerning the Laws of Motion,* and it is likewise what I have used for fluids in vol. 2 *Comment.*

Acad. Imp. Sc. Petrop., for investigating the proper laws of motion in fluids arising from gravity ; indeed I ha considered the velocities of particles constantly to be such that, with the individual particles moving vertically upwards as far as to a state of rest, the common centre of gravity of those may ascend to the initial height: but I have preferred on account of what I have said above to use this hypothesis as said by Huygens rather than by my father, and to designate by that name of the *equality of the ascending and descending potential*, rather than by the other way, by the *conservation of the living forces*, which even if some people disdain, and I know not why they speak so, especially in England. Indeed for me in the whole Leibnitzian teaching about the *living forces* there appears to be nothing, about which not everything may be in agreement, being discussed in its own way, which unless I am mistaken, to be shown in *Comm. Acad. Sc. Imp. Petrop. book.* I, *p.* 131 *& seq.* 47 , here I would choose this place, lest by which words it may give offence to some of the readers, and nothing may be known to be accepted by me, which shall not be accepted in mechanics by everyone, and which may not by necessity be in agreement with that which Galileo has now put in place as stated: the increments of the velocities to follow a proportion composed from the pressing down forces [*i.e.* the force of gravity] and the instants of time.

§. 20. Concerning the other approach, although the proposed principle shall be general, it should not be used without a detailed inspection, because often it happens that the motion passes into unknown matters. Thus for example the position prevails for the rules of motion elicited from collisions, only if the bodies shall be completely elastic; but when such bodies are not, it is easy to see, part of the living forces or of the ascending potential in the immoderate compression of bodies is not to be restored to bodies, but an impression to adhere to some subtle matter, through which it has passed through : yet if the matter may be considered correctly, an account of which is known, which is between between the residual part in the bodies, and that which is transferred to the subtle matter; it will be apparent, this inconvenience can easily occur, and thus for the laws of motion for soft bodies to be defined correctly. A comparison which follows in the computation of the motion of water, where whenever it is evident, a part of the ascending potential to be lost continually; certainly an account of this matter is required to be had on being removed from the calculation: with which attended to properly, it has pertained for me to uncover many new theorems concerning the flow of water, which is seen in Sections VIII & IX, and from which I do not see yet, whether they may be able to be demonstrated much more by any other method to be thought out.

§. 21. Therefore thus not without caution I have been using our principle, and in this way not only concerning the motion of water but also its pressure, and what is amazing to be seen, is that many previously unknown features present themselves, which are not yet established in the analysis, and which no one could easily have foreseen or expected. Truly when it happens, that the *ascending potential* from the nature of things shall be unable neither to be conserved, nor to be foreseen, how great a part may be absorbed, the motion of the flows cannot be determined with enough accuracy, nor do I think, can be determined by any other way. Therefore I wish the reader to proceed with caution in the corollaries deduced from our theorems, which often on account of changed circumstances cannot agree accurately with the experiments.

§. 22. It is clear enough from the aforementioned, to be required from our method, that the velocities of the individual particles of the fluid to be determined from an assumed velocity which is present everywhere, just as in the outflow. Hence it has been necessary to add another hypothesis, which is this: after we have considered the fluid to be divided into layers, perpendicular to the direction of the motion, we may consider the particles of the same layer to be moving with the same speed, thus, so that everywhere the velocity of the fluid shall be inversely proportional to the size [*i.e.* normal cross-section] of the corresponding vessel. This is the usual hypothesis, although it remains to be noted, the fluid at the sides of the vessel to be moving a little slower but in the middle faster, which shall be from friction, and also other exceptions being required to be made in addition; yet a notable error arises most rarely from deficiencies of this kind.

§. 23. I shall finish with this preview concerning our hypothesis of the phenomena which are able to illustrate and confirm the conservation of the living forces in the motion of fluids to some extent : certain of these occur many times in the work itself, but which on account of the calculation that they demand, I will not present. But the commonplace and obvious nature concerned with a drop falling into water at rest is observed : certainly it excites circles on the surface of the stationary water, the more of these are present there, in which the drop was larger or dropped from higher, and there is doubt why these should not propagate without end, except for the stickiness of the fluid and other similar obstructions. And when it is possible to observe also another effect of this kind with water dripping, especially with water leaking through a large hole, while many smaller drops may be projected upwards from the surface of the water below, and then it will be apparent constantly, and what shall be especially pertinent here, that there the higher the drops rise, so that the fewer and smaller in volume they will become, and when the height fallen shall be two feet, often the smaller drops will rise beyond the height fallen by the large drops.

[Thus, a large drop falling from a height into an open circular container filled with water, will send out circular ripples, and occasionally the reflected circular waves will converge to the centre of the circle again, and send up a smaller drop that may rise higher; or a single large drop produces a 'ring' of rising small drops; thus apparently defying the rule that a falling body on suffering an elastic collision will rebound to its own height. Incidentally, I wrote a short paper published in the July 1990 issue of the *American Journal of Physics* involving springs and masses that illustrate the same apparent increase in potential energy of a weight falling and rebounding to a greater height.]

With the former noted, the latter too is worthy of note, which concerns the particles of water held in a channel and with that horizontal for example, and at that extremity, towards which the water flows, a lid with a small hole may be observed. Clearly from that point of time, at which the water may come as far as the lid, a few small drops may burst forth with great impetus, and soon the motion of all the remaining water is brought to rest ; moreover, from which it may be easy to suppose, the water with its velocity to be on the point of moving through the hole, the rest to be stopped, which truly will correspond minimally to the conservation of the *living forces*; but this corresponds

unusually to the momentary strong efflux of the water, or as if by exploding : more about this elsewhere.

§. 24. These are [the explanations of the phenomena] which follow from our hypothesis, and I have wished to warn in advance not only about the outstanding successes but also about the deficient properties [of my theory]. I will say that it remains as certain regarding the nature of fluids, evidently around which our lucubrations will be turning, not what I may consider to have been seen by me more than by others, but what shame I may experience [from the unresolved matters], and to solemnly retire from this habit with all the other writers. And indeed in the first place they are all accustomed to agree that the motion for any kind of a fluid to belong inside the fluid, without which surely nobody would rightly pursue the [properties of] fluidity to such an extent, the effervescence of different fluids, the dissolving of submerged solids in fluids, evaporation, and indefinitely many other phenomena; and hence many of the most solid things with sufficient heat turn into liquids, which sets everything in motion : but the internal motion itself enables, that the particles themselves shall not be too close to each other, but as if they may fly apart, so that it may happen, that without friction they may depart from the place by the smallest impulse, which may not succeed at all, with the placing of nearby particles between themselves, as in a pile of sand. Thus it is easy to understand as follows, because it may be said to imitate milk boiling in a pan over a fire with a fine powder from [ground up] eggshells. Because the more intense is the heat, there the motion of the particles is stronger everywhere, and these disperse from each other to greater separations; which agrees with the expansion of all the fluid from the increased heat, and likewise by contraction from cold, to which law the freezing of water is still not made subject : but because, while it may freeze, it shall be of a different nature, that fortuitously intervenes from another cause, it may be considered to be deduced, surely from that reason because water enfolds air in the spaces between its particles, which thus does not increase the volume of water, just as sugar dissolved in water does not increase the volume of the same ; because at the instant of time of freezing the motion of the particles of water may be diminished ; thus because the same particles approach more towards each other, and thus the particles of air are expelled from its interstices, which placed elsewhere less conveniently are able to increase the volume, just as sugar in solution does not yet increase the volume of the water, in which it is mixed. Hence the reason is deduced conveniently, why freezing water well purged of air before freezing is not specifically lighter but becomes rather heavier. Moreover Mariotte conducted the outstanding experiments about the true solution of air in water as far as to the point of saturation, and that has been examined in his *Traité du mouvement des eaux et des autres corps fluides* [*Treatise on the motion of water and of other fluids*]. Therefore there is room to remain in doubt about fluids solidifying (as I have said), because the inner motion may cease or be diminished, while indeed the particles in turn will fall together amongst themselves, and become contiguous, and likewise heterogeneous particles are expelled from the interstices, if which may have lingered there ; nor yet hence is the hardness of solidifying bodies more clearly understood, rather a body may be considered, with that intermediate nature of the motion between being a solid and a liquid ceasing, unless which may added to another body, to become and to be compared with, a heap of sand : but what the nature of the matter shall be, lest indeed I follow a conjecture, meanwhile will be allowed to

adopt the form in whatever manner the particles attract each other, or, as I may say in the costomary words of the English, to attract each other in turn, and the designated attraction to increase with the particles approaching each other ; to be of different strengths in diverse bodies, e.g. lesser in oil than in water, the solid [ice] of which is harder; fluids to solidify more quickly and more easily, the particles of which either attract each other more strongly or they may be disturbed by a slower motion. From which one may conclude, water impregnated with sugar or salt freezes more slowly, because the particles of sugar or salt are interposed between the particles of water, the attraction of which may be diminished, nor shall these be able to be joined together, to be dried out and solidified, unless the heterogeneous particles be expelled from the place : and surely for all fluids, which have been impregnated with heterogeneous particles, a certain part of the time of solidification shall be in the expulsion from the pores, or the secretion and precipitation of these particles. There are infinitely many other phenomena regarding bodies both of solids and fluids, which certainly agree wonderfully well with the principle of mutual gravitation [presumably Bernoulli has in mind just attraction, not necessarily gravitational; but that is an open question], thus, so that it shall be a source of pain, the principle itself put in place to be so far above human understanding, that I do not consider there to be anyone who may be able to understand it in any way.

§. 25. Finally here it will be convenient to be reminded, that this treatise has been considered by me as being about physics rather than mathematics, lest the learned be led astray by me, nevertheless the geometrical method to be effected in the hypothesis, definitions, and with the remaining parts to be established, and everywhere I have followed the order and language of the Geometers, who are accustomed to begin from the egg, with propositions completed and there with the whole order treated, so that from the first principles the individual parts can be duly deduce, and nothing shall be left out not demonstrated afterwards, whatever that might be from everything now demonstrated. This concern by me was not on account of these things, which have been drawn from other sources, whether they should be definitions or axioms, or also theorems, nor yet have I left out the definitions of these, which are new, certainly also in the second section demonstrations of the theorems are put in place, of demonstrations from others here and there; and since certain terms occur from the others not to be explained or to be used, I will show the definitions of these in the text itself. In the same way I will propose the rest according to the manner of the Geometricians in the form of propositions, theorems, problems, corollaries and scholia, but also I will give explanations continually in the discourse

One thing remains, about which chiefly I wish to forewarn the reader: it has not been possible for me to have lavished the care and attention on this work that I should have done, and that I had wished for that. And thus I do not doubt, why some errors will have crept in, while I considered the calculations, which I hope, no one will explain in the wrong manner: others, which have met the eye at once while re-reading the text, I have corrected myself ; yet others even now I am convinced still survive.

HYDRODYNAMICAE SECTIO PRIMA.1

Quae introitus est, variaque continet praenotanda.

§.1.

Duplex cum sit Theoria Fluidorum, quarum altera Hydrostatica, liquorum stagnantium pressiones & aequilibria varia, altera Hydraulica, fluidorum motum spectans, seorsum pertractari a scriptoribus consueverunt, utramque vero tam arcto nexu inter se cohaerere perciperem, ut altera alterius ope plurimum egeat, haud dubitavi eas confundere, quantum id ordo rerum postulare videbatur, ambasque nomine communi & generaliori Hydrodynamicae complecti. Quamvis autem ab antiquissimis temporibus fuerit continuo exculta Theoria fluidorum, incrementa tamen non admodum notabilla cepit; veterum quidem Mathematicorum cognitio eo terminabatur, quod aequilibrium commune fluidorum stagnantium, aut etiam corporum cum fluidis, quibus insident, de quibus Archimedes scripsit, intelligebant; & cum praeterea per se pateat, ubi aequilibrium non est, motum versus partem minoris pressionis fieri, varios lusus, machinasque hydraulicas hinc excogitare potuerunt, partim oblectationi, partim publicis commodis egregie inservientes, qua quidem in re peringeniosos se monstrarunt; videbant etiam, sed quasi per transennam motus illos, qui pressioni aeris debentur: Veras autem rationes accuratasque mensuras in Hydraulicis rebus plane ignorabant, atque sic fere in limine subsistebant.

§. 2.

Motui fluidorum determinando inservit praecipue effluxus aquae ex vase per foramen valde parvum: tametsi vero non omnino fugeret Frontinum aliosque, uti aliqui credunt, velocitatem aquarum ex vase vel castello effluentium crescere ab aucta altitudine aquae supra effluxus locum, negari tamen non potest, quin idem Frontinus in computandis aquarum modulis, seu erogandis aquis turpes & injustos commiserit errores Benedictus Castellius primus de nexu velocitates inter & altitudines cogitare, falsam autem legem suspicatus est, putans, ambas eandem rationem sequi. Post hunc demum Torricellius observavit, velocitates crescere in subduplicata ratione altitudinum, quem secuti sunt omnes; nec dum vero conveniebant de absoluta velocitatis mensura, experimenta tamen instituerunt, qua istam mensuram definiri existimarunt, inter quae potissimum allegari solet illud, quod a Guglielmino sumtum, octiesque repetitum fuit, quamvis id ab aliis

experimentis ex illo tempore factis admodum recedat: solent autem omnia inter se differre, quae sub diversis fiunt circumstantiis, nec semper tutum est, uti suo loco dicemus pluribus, ex quantitate aquae, definito tempore per definitum lumen effluentis, judicium ferre de ejusdem velocitate. Sic cum ad calculum revocamus experimentum Guglielminianum, cujus modo mentionem fecimus, concludendum esset ex quantitate aquae, quae per lumen datum tempore dato effluxit, velocitatem ejus non majorem fuisse illa, quae debetur quartae parti altitudinis superficiei aqueae supra foramen. Et alia sunt eodem Auctore experimenta, quae recensentur *lib.* 2, *prop.1, Mens. aquarum fluent.,* vi quorum aqua effluens velocitate sua ascendere possit ad duas tertias istius altitudinis; apud Mariottum aliosque extant, quae pro dimidia altitudine faciunt; qua non obstante velocitatum ita aestimatarum diversitate, mihi persuadeo, vix a se invicem veras velocitates discrepasse, ratione habita ad altitudines aquae & ubique tales proxime fuisse, quae integrae altitudini debeantur: illa autem, quae loco ultimo fuere citata, quaeque pro dimidia altitudine prima fronte videntur stare, numero apud Authores plurima, movebant procul dubio Newtonum, Virum meritis suis immortalem, ut paulo confidentius loqueretur de Theoria, qua aquam per lumen minimum ex vase verticaliter sursum exilientem ad dimidiam altitudinem aquae in vase stagnantis ascendere posse invenerat, etsi assertum istud omnibus experimentis, quae de his altitudinibus immediate sumta fuere, contradicat: Theoriam exposuit in edit. prima *Princ. math. phil. nat.,* eamque petiit ex pressione, qua aqua prae foramina posita moxque egressura ad motum cietur. Quoniam vero natura rei haud semper permittere videtur, ut *a priori* definiatur vis aquam ad effluxum animans, atque potius de ea vix aliter, quam ex phaenomenis motus, id est, *a posteriori,* quod saepe expertus sum, judicare licet, suspectum esse debet ratiocinium isti principio innixum. Hinc etiam Vir modo laudatus sententiam suam mutavit in secunda Operis sui editione, rursusque aliquantum in tertia, affirmans aquam ad totam quidem altitudinem ascendere, venam autem, quam efformat, prae foramine contrahi seu gracilescere, atque sic utrique phaenomeno velocitatis quantitatisque dato tempore effluentis, quae sibi contradicere videbantur, satisfaciens. Quamvis autem contractionem istam fili aquei veram esse causam, ob quam,velocitas aquae effluentis non possit aestimari ex quantitate, negandum non sit, puto tamen, Theoriam ipsi non esse superinstruendam, quia *accidentalis* est, nec sibimet ubique constans, dum velocitas non variat nisi a causis alienis veluti attritu, tenacitate aquae, aliisque similibus. Sic cum aqua non per simplex foramen, sed per tubulum cylindricum effluit, vena notabiliter non contrahitur salva velocitate, excepto eo, quod propter attritum ei demitur: si quis autem hoc non obstante putet, ex pressione posse recte & tuto aquarum fluxum deduci, hunc rogarim, ut ad casus magis compositos animum advertat, v. gr. ad fluxum aquae, quem mirabilem vocat Mariottus, ex vase, quod diaphragma aliquod foramine perforatum in duas cavitates aqua implendas dispescit, sic ut aqua per duo foramina transfluere cogatur: de hoc motu loquitur Mariottus in *Tractatu* suo egregio *de motu aquarum12, part. IV, pag. m.* 442.

§. 3.

Haec cum ita sint, facile quisque secumjudicabit, quam parum spei supersit, aliquando Leges motuum pro fluidis ad regulas Geometriae purae reductum iri, sine ulla hypothesi physica, cum vel in ipso limine effugerint perspicaciam Viri ingenio praepotentis & incomparabilis: neque ego credo posse ea, quae in hoc opere expositurus

sum, omnem rigorem mathematicum subire: Principia Theoriae physica sunt & non sine largitione acceptanda ut proxime vera; admissis autem principiis, omnia erunt Geometrica, & nullis obnoxia restrictionibus, necessario nexu inter se cohaerebunt. Non possum tamen, quam bene sentire de physicis istis positionibus, in quas forte incidi, quandoquidem me manuduxerunt ad plurimas novas proprietates, cum de aequilibrio tum de motu fluidorum detegendas, quae, nisi me amor suscepti laboris fallit, aliquando Hydrodynamicam insigniter promovebunt, si magis excolantur, quam mihi licuit; ubi monuisse conveniet, quando multis, quicquid novum est, suspectum esse solet, totam me Theoriam animo concepisse, tractatum conscripsisse, pleraque cum amicis privatim communicasse, quaedam etiam coram Societate nostra praelegisse, priusquam ullum experimentum instituerim, ne ex praeconceptis mensuris opinione falsa, proxime tamen illis satisfaciente, me falli paterer, quandoque etiam Viros perspicacissimos intellectis theorematis aperte fassos esse, se sibi talia persuadere non posse, nec experimentis confirmatum iri existimare; hisque omnibus gestis, facta demum fuisse experimenta coram Amicis, haecque ita convenisse cum Theoria, quantum ipse vix sperare poteram. Nunc vero redeamus illuc, unde divertimus.

§. 4.

Postquam certi fuerunt Authores de diversitate velocitatum a mutatis altitudinibus, vasa considerare coeperunt magis composita, fistulis nempe varie inclinatis atque inaequaliter amplis instructa. Harum autem indolem jam suo tempore quodammodo cognovit Frontiuus, non ignarus, modulum augeri a deClivitate vel humilitate calicis, id est, fistulae signatae, quae castello, aut aliquando etiam rivo induebatur: unde etiam calices ad lineam, uti loquitur, ordinari & in eadem altitudine poni jussit. Et hoc quidem respectu in juste postulatur Frontinus a quibusdam, velocitatis nullam habuisse rationem; ubi vero calculum ponit omnis aquae acceptae, illamque comparat cum eroganda, non video, quomodo excusari possit. Experientia quoque edoctus fuerat, quod notari meretur, plus debito aquae erogari per calicem legitimae tum mensurae, tum positionis, cui statim fistulae amplioris moduli subjectae sint, quod ita esse, recteque a Fabretto indicatum fuisse, suo loco monstrabo, quamvis Vrri alias acutissimi, id non satis sibi liquere vel potius de eo se dubitare, innuerint.

§. 5.

Quod autem veteres obscure & sine veris mensuris viderunt. id demum Cl. Guglielminus in *Tract. de aquarum fluentium mensura* propositione accuratiori & generaliori complexus est tali, *eandem velocitatem,* inquiens, *esse aquae fluentis per canalem inclinatum, ac si fluxerit e vase per lumen simile, & aequale sectioni, tantundem a superficie aquae remotum, quantum sectio ab horizontali per initium alvei,* quam propositionem impugnavit Dionysius Papinus, ipse multum a veritate aberrans. Quoniam autem in eo sumus, ut commenta, tum Hydrostatica, tum Hydraulica praecipua recenseamus, hoc loco etiam numerandum est illud, de pressione fluidorum ex impetu cognoscenda, nempe *vim fluidi, in planum ad angulum rectum irruentis data velocitate, aequalem esse ponderi cylindrici fluidi super illo plano exstructi, cujus altitudo talis sit, ex qua mobile libere cadendo a quiete fluidi velocitatem*

acquirat. Problematis hujus utilissimi ope aestimare licet vim fluidorum machinas agitantium, aut, quale est ventus, naves propellentium, motus corporum in mediis resistentibus plurimaque alia. De Hydrostatica autem, quae tubulis tenuissimis seu capillaribus particularis est, nihil dico, quia hactenus ad Leges generales omnibus fluidis communes reduci non potuit: Incertus praeterea est Author, qui primus horum tubulorum indolem observaverit; constat tamen recentem esse observationem, quia de illa in libris ante hos 70 vel 80 annos editis nihil videre est.

§. 6. Authores praeter citatos a Galilaei temporibus, in rebus aquariis celebriores sunt Torricellius, Borellus, Vivianus, Pascalius, Boilius, recentioris aetatis sunt Varignonius, Newtonus, Polenus, Hermannus, Jacobus & Johannes Bemoulli, quorum inventaextantin *Comment. Acad. Reg. Sc. Paris., Princ. math. phil. nat., Tractatu de castellis notisque ad Frontinum, Phoronomia, Actis Lips.,* aliisque operibus variis. Quae vero circa curvaturas ex pressione fluidi genitas aliaque hujusmodi inventa a Geometris exhibita fuerunt, quia facile ad Geometriam puram reducantur, utut de reliquo omni laude digne silento praetereo.

Expositis his, quae ad alios pertinent, aequum esse sentio, ut meorum quoque ratione subducta, dicam sincere, an aliqua & quanta Hydrodynamicae incrementa ab illis sperari possint aut debeant. Breviter igitur, quantum potero, momenta operis suscepti indicabo.

§. 7. Exhibentur primo loco Theoremata praecipua, quae ad aequilibrium fluidorum stagnantium pertinent: visa mihi fuit instituti ratio id postulare, quamvis libenter fatear, nullas a me novas adjectas fuisse propositiones: Demonstrandi quidem modus, quantum scio, mihi proprius est, sed cum facile sit, innumeras sibi fingere demonstrationes, parum est, hac quoque in parte, quod mihi arrogo. Phaenomena praeterea aliqua tubulorum capillarium obiter recensentur, & denique occasione pressionis, quam fluida in latera vasis exercent, Theoremata varia & nonnulla nova adduntur, circa figuram vesicarum liquore impletarum, circa earundem potentias ad onera elevanda, circa constructionem & firmitatem aquaeductuum, aliaque affinia.

§. 8. Agitur postea de motu fluidorum ex vase effluentium, & cum omnes, qui hactenus de hac re egerunt, casum unicum maxime obvium, quo foramen ratione amplitudinis vasis internae infinite parvum censetur, in Theoria sua consideraverint, nostra non parum commendatur sua latitudine; extendit enim se ad positionem foraminis cujuscunque magnitudinis, imo & vasis cujuscunque figurae. Quamvis enim figurae vasis internae consideratio minime requiritur, cum foramen ut infinite parvum considerari potest, attamen sine illa motus aquae definiri nequit, cum est notabilis magnitudinis. Ex Theoria generali corollaria deducuntur, quae motum aquarum variabilem ejusdemque affectiones egregie illustrant, confirmantque, quicquid aut experientia docuit, aut rei attributiones per se manifeste indicant. Docet quidem Theoria, quando amplitudines internae vel mediocriter superant amplitudinem luminis, errorem esse insensibilem, qui ex consideratione foraminis ut infinite parvi nascitur, atque sic nostrae additiones nonnullis fortasse videbuntur satis inutiles. Hos vero, si modo qui futuri sint, secum cogitare velim, praeter quod non solum aquariis scribo, sed & Geometris, qui veritatibus nudis etiam delectantur,

usum nostrarum meditationum aliis in rebus maximum esse, quod magis intelligent, cum perpenderint, motum incipere a quiete, & per infinitos transire gradus, priusquam certam celeritatem obtineat; maximas mutationes saepe quidem tam brevi fieri temporis momento, ut sensibus nullo plane modo percipi possint, determinandas tamen esse ad singula puncta, tum ut motus animo recte percipiatur, tum quia exinde varia deduci possunt Theoremata. Ita animadverti (quod exemplum ob rei momentum sit instar omnium), fieri non posse, ut pressio aquae, per canalem data velocitate ftuentis, in ejusdem latera definiatur, nisi mutationes istae, quas *momentaneas* dicam, utcunque sensibus inperceptibiles recte animo intelligantur. De his ego, ut primus cogitavi, ita optatissimo cum successu novam Theoriae aquarum partem addidi, quae, quia fluidorum tum motum tum pressionem simul respicit, *hydraulico-statica* aptissime vocari visa fuit. Post haec Theoriae generalis specimina, de vasis cylindricis tam simplicibus, quam iis, quae tubis instructa sunt, exhibentur, & in his posterioribus praesertim determinantur mutationes, quae ab initio fluxus oriuntur, dum datus velocitatis gradus attingitur, & id quidem in hypothesi vasorum amplissimorum; notandum autem est, has mutationes sensibiles admodum esse, etiamsi vasa sunt infinitae amplitudinis, posseque illas experimentis demonstrari, dum aquae ex vase amplissimo per foramen simplex effiuentes prima statim temporis puncta totam, quantam possunt, velocitatem habent. Pendent praedictae mutationes tum a longitudine tum a figura tubi. Denique etiam calculi analytici pro varii generis temporibus inveniendis una cum annotationibus physicis eo pertinentibus adjiciuntur. Indicante denique Theoria, fieri non posse, ut aquae multum ultra supremam scaturiginis superficiem ascendant, monstratur sub fine sectionis , non pertinere ad hypotheses nostras phaenomenon singulare, quod ipse saepius observavi, & pro lubitu imitari possum, cujusque mentio injicitur in *Hist. Reg. Acad. Sc.. Paris ad ann.* 1702, ubi dicitur, accidere quandoque, ut aquae in fontibus salientibus assurgant ad altitudinem triplam, aut quadruplam ejus, quae respondet aquae superficiei supremae, mox tamen enormem aquae jactum ad consuetam altitudinem deprimi, posteaque genuina istius phaenomeni ratio cum veris mensuris ex Theoria nostra petitis affertur, modusque indicatur saltum insolitum producendi, imo & ad lubitum augendi.

§. 9. Porro Theoria extenditur ad examen motuum ex vasis constanter plenis, quibus nempe tantum aquae continue affunditur, quantum ex illis effluit : horum indoles in eo potissimum consistit, ut fluida emanantia magis magisque accedant ad illum velocitatis gradum, qui toti altitudini superficiei fluidi supra foramen debetur, eum vera nunquam omnino attingant, nisi post tempus infinitum: vergere tamen demonstrantur aquae tam cito ad velocitatem istam, ut post tempusculum insensibile tantum non totam acquirant, nisi cum per longissimos rivos aut aquaeductus feruntur, magnoque lumine ejiciuntur; tunc enim accelerationes tam celeres non sunt, quin percipi possint, quod singulari exemplo ex Cl Mariotti Libro *de motu aquarum* desumto confirmatur. Quoniam vero motus a quiete incipit & perpetuo crescit, formulae dantur, quarum ope vel ex fluxus tempore vel ex quantitate aquarum ejectarum velocitas singulis temporis punctis definiri possit & vicissim.

§.10. In sequentibus fluida considerantur, quae intra vasa moventur, ubi praesertim motus fluidorum reciproci seu oscillatorii ad mensuras revocantur, earumque affectiones

indicantur. Dedit autem Newtonus Theorema simile pro oscillationibus fluidi, in tubo uniformis amplitudinis (cujus crura duo extrema verticalia, intermedia pars horizontalis) oscillantis, quod Theorema Pater meus in *Comm. Acad. Imp. Sc.Petrop. tom.* 2, *p.* 201, generalius reddidit posita inclinatione qualicunque crurum extremorum versus horizontem. Nostra Theoria totam rem sine ulla restrictione complectitur, tubos considerans in singulis locis directione seu positione atque amplitudine pro lubitu variabiles: ostenditur dein, quibus in casibus fiat, ut oscillationes diversae excursionis sint isochronae, quibus stantibus longitudo penduli simplicis isochroni generalissime determinatur. Sed & praeter hoc oscillationum genus in subsequente sectione quaedam aliae examini subjiciuntur, veluti illae, quae fiunt in tubis aquae infinitae vel etiam terminatae immersis, in quibus singulari circumspectione opus est, qua adhibita omnia phaenomena calculo ad amussim respondent, eadem vero neglecta tantus fit inter ea dissensus, quantus est inter leges motus, quae pro corporibus perfecte elasticis, iisque quae pro mollibus valent.

§. 11. Post haec ad alia magis composita progredior, motum nempe fluidorum considerans sive homogeneorum sive heterogeneorum, quae per unum aut plura foramina transfluere coguntur, priusquam ejiciantur in aërem, ubi regula illa communiter recepta de saltu aquae ad supremam aquae libellam vehementer fallit, cessantibus etiam legibus pressionis ordinariis. Horum autem omnium apud Authores ne vestigium quidem reperitur, nisi quod Mariottus habet, loco supra citato *part.* IV, *p. m.* 442, *de motu aquar.,* ubi quidem fluxum aquarum retardari, fuisse se experientia edoctum, testatur, simul autem manifestat, quam procul abfuerit a vera horum motuum Theoria, & videtur sane haec Theoria omnium fere principiorum, adhuc in rebus similibus adhiberi solitorum, vim eludere, ita ut nihil sit, quod nostrorum praestantiam magis confirmet: de eorum veritate enim experimenta instituta me amplius dubitare non sinunt. Non deest autem hisce meditatis sua utilitas, quandoquidem magni momenti esse possint in perficiendis machinis hydraulicis.

§.12. Sequuntur commentationes de machinis hydraulicis, quibus potissimum monstratur, certum quendam perfectionis terminum esse, ultra quem ire non liceat; defectus autem ab ultimo hoc perfectionis gradu in multis machinis maxime receptis calculo numerico subjiciuntur, additis regulis seu praeceptis, ad quae in construendis novis machinamentis animus sit advertendus: exempli loco affertur notissima per totum orbem machina Marlyensis, de qua monstratur, si modo descriptionibus fidendum sit, quod non ultra quinquagesimam sextam prope partem suppeditet ejus aquae quantitatis, quam caeteris paribus machina perfectissima theoretice subministrare queat. Speciale etiam examen instituitur de machina ab antiquissimis temporibus ad nostram usque aetatem usitatissima, Cochlea nimirum Archimedis, attentione Geometrarum non indigna, tam ratione eorum, quae ad Geometriam puram, quam quae ad Hydraulicam pertinent.

§. 13. Succedunt specimina quaedam de motu fluidorum elasticorum, veluti aeris & pulveris pyrii accensi, praemissis iis, quae ad naturam horum fluidorum pertinent ; quae vero ipse non aliter, quam ut hypotheses physicas considero, de quibus nihil confidenter affirmabo. Propositiones & Problemata hujus sectionis nova sunt, &

eo selecta animo, ut multis quaestionibus physicis illustrandis, aut etiam solvendis occasionem praebere possint. Adjiciuntur quaedam de aestimatione virium vivarum fluidis elasticis insitarum, quae aliquando fortasse in praxi mechanica nonnullius usus erunt: monstratur enim, unius v. gr. librae pulveris pyrii accensi effectum in elevandis ponderibus majorem esse posse, quam vel centum homines robustissimi labore continuo intra unius diei spatium efficere possint.

§. 14. Agitur porro de fluidorum motu circulari, ut & de fluidis, quae in vasis motis stagnant; variaque alia intermiscentur. Quae autem de motu circulari proferuntur,

inservire quodammodo possunt ad phaenomena gravitatis per vortices explicanda; caetera valeant, quantum poterunt.

§. 15. Praemissa Theoria motuum, rursus ad aequilibria fluidorum descenditur, sed fluidorum motorum, quorum leges exhibitae nondum fuerunt.• Mirum est, cum alias motus ex pressione definiatur, hic inversa methodo pressionem ex motu peti, prius ex circumstantiis definiendo; nec crediderim aliam viam tuto iniri posse praeter eam, quam ego secutus sum: consideravi autem canalem, per quem aquae fluunt eo in loco eoque temporis puncto, quae quaestioni conveniunt, amputatum; posteaque per regulas nostras praemissas accelerationem indagavi particulae aquae imminentis, proximeque effluxurae. Ex ista acceleratione colligere licebat compressionem
illius particulae aqueae, quae compressio per naturam fluidorum aequalis est pressioni in latera canalis. Cognita hac pressione apparet, quid fieri debeat, si canalis eodem in loco perforatus fuerit, tubulusque foramini respondeat; fore nempe, ut aqua in eo ascendat ad certum usque gradum stagnans in tubulo, & ab aqua inferius per canalem praeter fluente sustenta, sic, ut hic aequilibrium adsit inter aquas fluentes & stagnantes: hoc vero nomine Theoriam istam *hydraulico-staticam* commode vocari posse existimavi. Notari porro meretur, ipsam hanc Theoriam fundamentum rursus esse & fontem aliorum motuum antehac incognitorum. Theoremata, quae exponuntur, non nova solum, sed & pleraque inexpectata sunt, quorum omnium veritatem nec ipse plane mihi persuadere potui, priusquam experimenta instituissem, quae mihi omnem scrupulum demebant. Habent autem insignem usum, quandoquidem iis innititur vera pressionis aquarum, per aquaeductus seu rivos fluentium, aestimatio, hincque deducendae tuborum firmitates requisitae. Inde quoque pendent accuratae mensurae aquarum per modulos, rivo lateraliter insertos, erogatarum: in Physiologia rectius jam intelligentur, quae pertinent ad motum humorum in corpore animali, & quae sunt alia.

§. 16. Denique progredior ad alios quosdam modos, quibus aqua nisum facere pot est, explicandos: ita nempe aqua, dum per foramen effluit, in contrarium premit vas non aliter, atque globus retropellit tormentum, ex quo exploditur: istius retropulsionis plures proprietates deteguntur novae, quae pressionum naturam egregie illustrant, earumque leges, quas affectant, generales in mechanicis rem istam serio meditantibus indicabunt. Has disquisitiones feci, quia mihi visum est, posse ea novae aliquando navigationi sine remorum, aut venti adminiculo excogitandae occasionem praebere ; qua de re suo loco pauca quaedam afferam, etsi non ignoro,

omnium hujusmodi rerum primordia per se plerisque videri ridicula. Tandem etiam de vi aquarum ex impulsu hincque nato renixu, quam corpora in fluidis mota offendunt, Theoremata quaedam adjiciuntur.

§. 17. Et haec quidem sunt, quae mihi ex admissis principiis geometricam deductionem pati visa sunt. Quoniam vero nihil est in Theoria tam rigorose demonstratum, quod non in applicatione ad corpora restrictionem aliquam postulet, ideo facile apparet, nec ullam Theoriam de fluidis expectandam esse, quae omnibus mensuris experientia cognitis plenissime satisfaciat; cujus rei memores esse velim, qui Theoremata nostra experimentis confirmare voluerint. Ubique invenient quidem aliquem consensum, sed non perfectum, eumque modo strictiorem, modo laxiorem, pro rerum circumstantiis. Quoties autem ipse aliquod experimentum effeci, ante omnia mecum perpendi, quousque principia Theoriae cum casu proposito congruerent; atque sic me nunquam aut rarissime eventus fefellit. Non solum enim praevidere solebam, in quam partem futura esset differentia, si quae notabilis esse debebat, sed & quanta; quod ipsum, si recte judico, satis manifestat, fluida affectare quidem leges, quas ipsis praescriptas esse ponimus, obstacula autem ubique offendere nunc majora, nunc minora. Caeterum experimenta institui non pauca, quorum singula in fine sectionis, ad quam pertinent, locavi: praesertim vero sollicitus fui, in propositionibus antea incognitis & plerisque sat paradoxis confirmandis. De experimentorum fide non est, quod quis dubitet, cum praecipua coram Amicis eaque post publicatam Theoriam fecerim; magnam tamen experimentorum, quae animo concepi, partem, quando per singula ire non licet, aliis relinquens instituendam. Perlectis nostris propositionibus quisque sibi finget innumera, neque proin opus esse judicavi, omnia, qualia sunt a me desiderata, exponere; exposui tamen aliqua.

§. 18. Jam vero tandem principiorum, quorum toties mentionem fecimus, ratio reddenda est. Praecipuum est *conservatio virium vivarum*, seu, ut ego loquor, *aequalitas inter descensum actualem ascensumque potentialem:* Utar hac posteriore voce, quia idem quod altera significat, sortem autem apud nonnullos Philosophos, qui vel ad solum *vis vivae* nomen moventur, magis benignam fortasse experietur. Puto, hic e re nostra fore, hac de re paulo copiosius dicere.

§. 19. Postquam Galilaeus docuisset, corpus, sive verticaliter, sive super plano utcunque incurvato, descendens eandem velocitatem acquirere, modo altitudo lapsus sit eadem, quod ex natura pressionum demonstrari potest, Hugenius eadem hac propositione, sed generaliori pro hypothesi feliciter usus est in eruendis legibus motuum corporum elasticorum ex percussione, nec non in stabiliendo centro oscillationis penduli compositi; protulit autem axioma hoc suum talibus verbis : *Si pondera quotlibet vi gravitatis suae moveri incipiant utcunque, singulaque rursus ad quietem sponte reducantur, centrum gravitatis ex ipsis compositae ad pristinam altitudinem rediturum esse,* ubi per vocem *utcunque* intelligit, *sive se percutiant inter descensum, sive premant, aliove modo in se invicem agant corpora.* Ex isto axiomate statim sequitur principium conservationis *virium vivarum,* quod ipse etiam Hugenius demonstravit, & quo assumitur: *Si pondera quotlibet vi gravitatis suae moveri incipiant utcunque, singulorum velocitates ubique tales fore, ut producta,*

ex earum quadratis in suas massas collecta, sint proportionalia altitudini verticali, per quam centrum gravitatis ex corporibus compositae descendit, multiplicatae per massas omnium. Mirum est, quantam habeat haec hypothesis in Philosophia mechanica utilitatem, quod, si quis alios, sane Pater meus recte animadvertit, qui id sparsim, imprimis autem in *Dissertatione* Parisiis edita *de legibus motuum* & in *tom.* 2 *Comment. Acad. Imp. Sc. Petrop.* ostendit , idemque est, quod pro investigandis Legibus motuum, ex propria gravitate ortorum, in fluidis adhibui; posui enim velocitates particularum constanter tales esse, ut, singulis verticaliter sursum motis ad statum quietis usque, centrum earum gravitatis commune ad pristinam altitudinem ascendat: malui autem ob rationem supra dictam hanc hypothesin verbis Hugenianis quam Paternis accommodare, eamque nomine *aequalitatis inter descensum actualem ascensumque potentialem* insignire, quam altero *conservationis virium vivarum,* quod etiamnum aliqui, praesertim in Anglia, nescio quo fato, fastidiunt. Mihi quidem in tota doctrina Leibnitiana de *viribus vivis* nihil esse videtur, de quo non omnes, suo tamen quivis loquendi modo, conveniant, quod, ni fallor, dare ostendi in *Comm. Acad. Sc. Imp. Petrop. tom.* I, *p.* 131 *& seq.* 47 , quem locum hic allegare volui, ne quis Lectorum se verbis offendi patiatur, sciatque nihil a me accipi, quod in Mechanica receptum non sit ab omnibus, & quod non necessario nexu cohaereat cum eo, quod jam Galilaeus posuit, cum statueret, incrementa velocitatum proportionem sequi compositam ex pressionibus & momentis temporum.

§. 20. De caetero, quamvis principium praedictum universale sit, non tamen est sine circumspectione adhibendum, quia saepe contingit, ut motus transeat in materiam alienam. Ita verbi gratia positio illius valet pro regulis motuum ex percussione eruendis, si modo corpora sint perfecte elastica; sed cum talia non sunt, facile est videre, partem *virium vivarum* sive *ascensus potentialis* in compressionem corporum impensam corporibus non restitui, sed materiae cuidam subtili, ad quam transiit, impressam haerere: si tamen res recte consideretur, quum ratio cognoscitur, quae est inter partem corporibus residuam, eamque quae ad materiam subtilem transiit; apparebit, facile occurri posse isti incommodo, sicque recte definiri leges motuum pro corporibus mollibus. Simile quid succedit in motu aquarum computando, ubi quandoque manifestum est, partem *ascensus potentialis* continue perdi; cujus utique rei in subducendo calculo ratio habenda est: ad quod probe attento multa de aquarum fluxu Theoremata nova mihi contigit detegere, quae videre est in Sect. [Oct. & Non.] & de quibus nondum video, an ulla alia methodo demonstrari nedum excogitari possint.

§. 21. Sic igitur non incautus principio nostro usus sum, hocque modo non solum de motu aquarum, sed & de earum pressione, quod mirum videri potest, multa antea incognita se offerunt, quae nondum instituta Analysi nemo facile praeviderit nec expectarit. Quum vero fit, ut *ascensus potentialis* nec omnis conservari possit ex rei natura, nec praevideri, quanta pars absorbeatur, non satis accurate motus fluidorurn determinari potest, nec puto, ulla alia methodo posse. Igitur Lectorem cautum esse velim in corollariis ex Theoria nostra deducendis, quae saepe propter mutatas circumstantias non accurate cum experimentis convenire poterunt.

§. 22. Ex praememoratis jam satis liquet, ex nostra methodo requiri, ut singularum

particularum fluidi definiatur velocitas ex assumta velocitate, quae est aliquo in loco, veluti in loco effluxus. Necesse proin fuit, aliam superaddere hypothesin, quae haec est: postquam scilicet mente concepimus divisum fluidum in strata, ad directionem motus perpendicularia, ponemus fluidi particulas ejusdem strati eadem velocitate moveri, ita, ut ubique velocitas fluidi reciproce proportionalis sit amplitudini vasis respondenti. Usitata est haec hypothesis, quamvis caeterum notum sit, fluidum ad latera vasis paullo tardius, in medio autem velocius moveri, quod ab attritu fit, aliasque etiam exceptiones subinde esse faciendas; error tamen notabilis ab hujusmodi defectibus rarissime potest oriri.

§. 23. Finiam haecce de hypothesibus nostris praemonita recensione phaenomenorum, quae conservationem *virium vivarum* in motu fluidorum aliquantum & illustrare & confirmare poterunt: eorum quidem in ipso opere plurima occurrent, quae autem ob calculum, quem postulant, non allegabo. Triviale autem & obvium est, quod de gutta, in aquam stagnantem delapsa, observatur: orbes nempe excitat in superficie aquae stagnantis, horumque eo plures, quo vel major fuerit gutta, vel altius delapsa, nec dubium est, quin isti orbes sine fine se propagaturi essent, nisi tenacitas fluidi, aliaque similia obstaculo essent. Quandoque etiam alium effectum ab hujusmodi stillis observare licet, dum plures guttulae minores a superficie aquae inferioris in altum projiciuntur, tuncque constanter apparet, quod praesertim huc pertinet, eo altius assurgere guttulas, quo pauciores numero atque minores volumine fuerint, & cum altitudo lapsus esset duorum pedum, saepius guttulae minores ultra altitudinem lapsus ascendebant, stillante praesertim aqua per foramen magnum. Hic notatu quoque dignum est illud, quod de particula aquae per canalem tenuem, eumque v. gr. horizontalem, atque in ea extremitate, versus quam aqua fluit, operculo perforato opertum observatur. Scilicet eo temporis puncto, quo aqua ad operculum usque pervenit, magno impetu paucae guttulae exiliunt, moxque omnis aquae motus sistitur; facile autem quis suspicari posset, aquam foramini imminentem sua velocitate moveri pergere, reliquam sisti, id vero conservationi *virium vivarum* minime responderet; respondet autem egregie vehemens iste aquae effluxus *momentaneus*, vel quasi explosio: de his alibi plura.

§. 24. Haec sunt, quae de hypothesibus nostris, earumque tum praestantia tum defectu volui in antecessum monere. Superest ut quaedam dicam de indole fluidorum, quippe circa quae lucubrationes nostrae versabuntur, non quod eam me aliis magis perspectam habere putem, sed quod nefas existimem, a more hoc scriptoribus omnibus solenni recedere . Et primo quidem hoc omnes convenire solent, motum fluidis quibusvis inesse intestinum, sine quo nemo profecto tantam fluiditatem, effervescentias diversorum fluidorum, dissolutiones solidorum fluidis submersorum, evaporationes, aliaque phaenomena infinita recte assequetur; hinc pleraeque res solidissimae a sufficiente calore, qui omnia in motum rapit, liquescunt: facit autem motus iste intestinus, ut particulae sibi non sint contiguae, sed quasi volitent, quo fit, ut sine frictione a minimo impulsu loco cedant, quod minime succederet, positis iisdem particulis inter se, sicut in acervo arenae, contiguis. Ita facile intellectu est, pollinem ex putaminibus ovorum in patella igni superimpositum lac bulliens, quod dicitur, mentiri. Quo intensior autem est calor, eo vehementior utique est motus

particularum, haeque majori intervallo a se invicem dispersae; quod convenit cum dilatatione omnium fluidorum ab aucto calore, eorundemque contractione ex frigore, cui legi ipsa etiam aqua nondum congelata subjicitur: quod autem, dum congelatur, contrariae sit indolis, id ex alia causa, fortuito superveniente, deducendum videtur, nempe ex eo, quod aqua in interstitiis suis particulas foveat aëreas, quae sic volumen aquae non augent, prouti saccharum in aqua solutum non auget ejusdem volumen; quod tempore instantis congelationis particularum aquearum motus minuatur; quod sic eaedem particulae magis ad se invicem accedant, adeoque ex interstitiis suis particulas aëreas pellant, quae alibi minus commode locatae volumen augere possunt, prouti saccharum nondum solutum volumen auget aquae, cui permistum est. Hinc commode ratio deducitur, cur glacies aquae ab aere ante congelationem bene purgatae non specifice levior, quin potius gravior fiat. Egregia autem experimenta circa solutionem veram aeris in aqua ad punctum saturationis usque instituit Mariottus, eaque in *Tractatu* suo *de motu aquarum* recensuit. Suspicioni igitur locus est, fluida (ut dixi) congelari, cum cessat vel valde diminuitur motus intestinus, tum enim particulae in se invicem collabuntur, fiuntque contiguae, simulque ex interstitiis particulas heterogeneas, si quae ibi commorentur, expellunt; nec tamen clarius hinc intelligitur durities corporum congelatorum, quinimo videtur, cessante isto motu corpus mediae naturae inter fluidum & solidum, nisi aliud quid accedat, fieri, & comparandum cum acervo arenae: quid autem id rei sit, ne conjectura quidem assequor, licebit interim fingere quaslibet particulas ad se gravitare, vel, ut voce Anglis usitata utar, se invicem attrahere, attractionemque insigniter crescere, accedentibus ad se invicem particulis; diversae esse virtutis in diversis corporibus, minoris v. g. in oleis quam in aquis, quarum glacies durior est; fluida citius & facilius congelari, quorum particulae vel fortius se attrabunt vel motu lentiori agitantur. Exinde conjicere liceret, aquam saccharo vel sale impraegnatam tardius congelari, quod particulae sacchari vel salis, particulis aqueis interpositae, harum attractionem diminuant, neque hae conjungi possint, siccidumque congelari, quin particulae heterogeneae loco pellantur: & certe omnibus in fluidis, quae particulis heterogeneis sunt impraegnata, tempore congelationis fit quaedam partium ex poris expulsio, seu secretio atque praecipitatio. Infinita sunt alia corporum tum solidorum tum fluidorum phaenomena, quae mire admodum cum principio mutuae gravitationis conveniunt, ita, ut dolendum sit, principium ipsum tam alte supra mentem humanam positum esse, ut neminem esse putem, qui id ullo modo intelligere possit.

§. 25. Denique hic monuisse conveniet, tractatum hunc ut Physicum potius quam Mathematicum mihi considerari, nec proin consultum me duxisse, methodum Geometricam in hypothesibus, definitionibus caeterisque apparatibus praemittendis nimium affectare, & ubique ordinem sermonemque Geometrarum sequi, qui solent ab ovo ordiri, propositionibus complecti, & eo ordine omnia pertractare, ut ex primis praemissis singula rite deducantur, nihilque indemonstratum post se relinquant, quamvis id a tot aliis jam demonstratum fuerit. Non mihi haec cura fuit ratione eorum, quae ab aliis tradita sunt, sive definitiones fuerint & axiomata, sive etiam theoremata, non omisi tamen demonstrationes eorum, quae nova sunt, imo etiam in [secunda] Sectione apponuntur demonstrationes Theorematum, ab aliis passim demonstratorum; & cum quidam occurrant termini, ab aliis non explicati nec

usitati, horum definitiones in ipso textu exhibebo. Caetera modo sub forma Propositionum, Theorematum, Problematum, Corollariorum, Scholiorumque pro more Geometrarum proponam, modo etiam sermone continuo explicata dabo.

Unum superest, de quo Lectorem praemonitum potissimum volo: non potuisse me huic operi eam adhibere sive diligentiam sive attentionem, quam debuissem, & quam ipse desideravi. Nullus adeoque dubito, quin nonnulli irrepserint errores, dum calculos ponerem, quos, spero, nemo sinistre explicabit: aliquos, qui in oculos incurrerunt, dum tractatum leviter relegerem, ipse correxi; alios tamen etiamnum superesse mihi persuadeo.