

CHAPTER III.

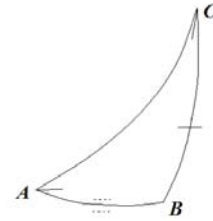
Concerning the measurement of oblique angled triangles.

PROBLEM 1.

With two angles given and with a side opposite to either of the given, to find the side opposite the remaining angle.

In the oblique angled triangle ACB, the side AB is sought.

Given $\left\{ \begin{array}{l} \text{the angles} \left\{ \begin{array}{l} \text{BAC } 103 \underline{999307}; \text{ Compl. } 76 \underline{000693} \\ \text{ACB } 36 \underline{131235} \end{array} \right. \\ \text{with the side CB } 42 \underline{146646}. \end{array} \right.$



The terms of the ratio.

Proportions $\left\{ \begin{array}{l} \text{Sine of the angle of the opposite side given.} \\ \text{Sine of the side given.} \\ \text{Sine angle remaining.} \\ \text{Sine of the side sought, by Cons.3, Ch.2.} \end{array} \right.$

Illustrated numerically.

			Sine		Log.Sin.
Proport.	Sine Angle	CAB	76 <u>000693</u>	97029,86512 . .	9,98690,54277
	Sine side	CB	42 <u>146646</u>	67103,04563 . .	9,82674,22302
	Sine Angle	ACB	36 <u>131235</u>	58963,67471 . .	<u>9,77058,45404</u>
					19,59732,67707
	Sine of the side AB	24 <u>065185</u>	40777,57122 . .		9,61042,13430

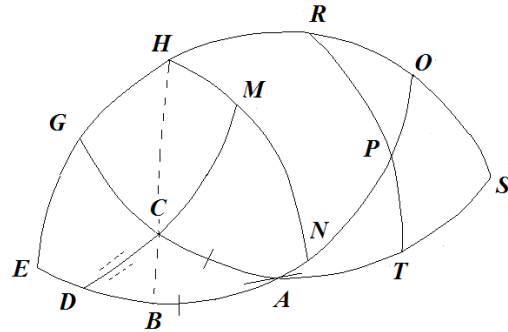
With two of the sides found together with one of the angles opposite to the same, also the remaining side evidently can be found in this manner.

		Sin & Tang.	Log.Sin & Tang.
Proport.	Sine of half the difference of the angles	33 <u>934036</u>	9,74681,94465
	Sine of half the sum of the angles	70 <u>065271</u>	9,97316,55681
	Tangent of half the difference of the sides	9 <u>040731</u>	<u>9,20170,63309</u>
			19,17487,18990
	Tangent of half the remaining side	15 <u>00000</u>	9,42805,24525

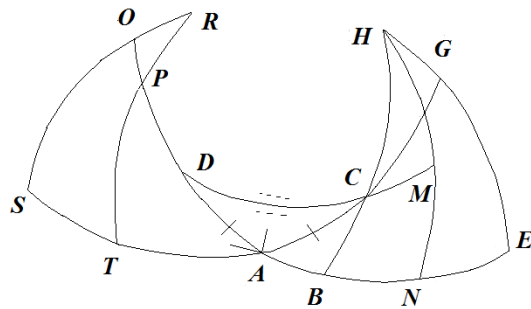
PROBLEM 2.

With two sides given and with the angle determined by the same, the remaining side is sought.

For this and for the solution of almost all the subsequent problems, for which two operations are required, the oblique angled triangle is required to be reduced into two right angled triangles, clearly with the help of a perpendicular drawn from the end of a known side and wherever it shall meet the side sought, or opposite to the angle sought. Thus in the triangles with these right angles the parts of which correspond in similar places, if the individual parts are known,



also the individual parts of the oblique angled triangle will not remain hidden. Truly from these properly understood, which we have set out in the third chapter, the individual parts of these will be able to be sought out without difficulty ; indeed with the perpendicular together with the radius excluded, as we have established in



Consequences 4, 5, 6, & 7 of the second chapter; The sines or tangents of the mean parts of one triangle and the extreme parts of the other, will be the mean proportionals between the sines or tangents of the remaining mean part and the extreme part of the first triangle. So

with our greater understanding, we may proceed to demonstrate the resolution of the following problems one by one. Truly because by sending the perpendicular from the angle to the vertical, whether it falls within or outside the triangle, according as the angles to the base shall be of the same or of different kinds, therefore so that no calculation may be held up by difficulties, we have put in place two diagrams with the same letters assigned, so that the terms of the ratio may be used for each in turn to be illustrated arithmetically.

In the oblique angled ACD the side DC is sought.

		1.Upper diag.	2.Lower diag.
Given	{	the sides	{ AD 42 <u>14646</u>
			{ AC 30 <u>000</u>
		with the angle	DAC 36 <u>131235</u>
		}	}
			{ 24 <u>065185</u>
			{ 30 <u>000</u>
			{ 103 <u>999307</u>

This problem and some of the following as I have said, need a two-fold undertaking, the first of which serves to find the base segments, or the angles to the vertical ; the other truly for finding the angles or the sides required.

Terms of the Ratio.

I. For a segment of the base. *By Prob. 6, Ch. 3.*

Proport. { *As the Radius,*
To the Tangent of the Hypotenuse.
So the Sine of the Complement of the contained angle,
To the Tangent of the segment opposite the contained base angle.
 And thence the remaining segment.

II. For the side sought. *By Conseq. 5.*

Proport. { *As the Sine of the Complement of the Base finally found,*
To the Sine of the Complement of the Hypotenuse;
So the Sine of the Complement of the remaining segment of the base,
To the Sine of the Complement of the side sought.

Illustrated by numbers,
 I.

		<i>Sin. & Tang.</i>	<i>Logs.Sin. & Tang.</i>
Proport. {	<i>As the RadiusRS</i>	90 <u>0000</u> . 100000,00000	. 10,00000,00000
	<i>To the Tan. Hypotenuse TS=AC</i>	30 <u>0000</u> . 57735,02692	. . 9,76143,93726
	<i>So the Sin. compl.OS viz . RO</i>	53 <u>868765</u> . 80766,85560	. . <u>9,90723,31753</u>
	<i>To Tan.segm.Base OP=AB</i>	25 <u>0000</u> . 46630,76582	9,66876,25479
	<i>Of which the compl. is PA=BE</i>	65 <u>0000</u> .	

We are able thus also to establish the terms of the ratio for the segments of the base or of the vertical angles, without the quadrant RST.

Proport. { *As Sine of the compl. of the angle given*
To the Tang. compl. of the Hypot.
So the Radius
To the Tang. of the compl. base segment opp. given angle.

Illustrated by numbers.

I.

			<i>Sin. & Tang.</i>	<i>Logar. Sin. & Tang.</i>
Proport.	As Sin.compl. EG viz . HG	53 <u>868765</u>	. 80766,86698	. . 9,90723,31753
	To Tang. compl. AC viz . GC	60 <u>0000</u>	. 173205,08076	. . 10,23856,06274
	So Radius HE	90 <u>0000</u>	. 100000,00000	. . 10,00000,00000
	ToTang. compl. AB viz . EB	65 <u>0000</u>	. 214450,69205	. . 10,33132,74521
	Therefore segm. AB is	25 <u>0000</u>		

II.

			<i>Sinus.</i>	<i>Logar. Sin.</i>
Proport.	As Sine compl. AB viz . BE	65 <u>0000</u>	. 90630,77870	. . 9,95727,57115
	To Sine compl. AC viz . CG	60 <u>0000</u>	. 86602,54037	. . 9,93753,06317
	So Sine compl. DB viz . BN	72 <u>853354</u>	. 95555,33089	. . <u>9,98025,49202</u>
				19,91778,55519
	To Sine compl. DC viz . CM	65 <u>934815</u>	. 91308,21216.	. 9,96050,98408
	Therefore DC is	24 <u>065185</u>	, the side sought.	

But if the angle assumed shall be obtuse, just as can be seen in the triangle ACD of the lower diagram ACD, the problem is to be worked out in the same manner. The data presented already may be retained.

I.

			<i>Sin. & Tang.</i>	<i>Log. Sin. & Tang.</i>
Proport.	Radius RS	90 <u>0000</u>	. 100000,00000	. . 10,00000,00000
	Tang. of Hypotenuse ST=AC	30 <u>0000</u>	. 57735,02692	. . 9,76143,93726
	Sin.compl. OS viz. RO	13 <u>999307</u>	. 24191,01595	. . <u>9,38365,41009</u>
	Tang.segment.Base OP=AB	7 <u>950891</u>	. 12966,68969	. . <u>9,14509,34735</u>
Of which the Complement is	BE	82 <u>049109</u> .		

II.

		<i>Sine</i>	<i>Log.Sin.</i>	
{	Sine compl. AB viz. BE	82 <u>049109</u> . 99038,69909 . .	9,99580,49270	
	Sine compl. AC viz. CG	60 <u>0000</u> . 86602,54037 . .	9,93753,06317	
	Sines compl. DB viz. BN	57 <u>983930</u> . 84789,94309 . .	<u>9,92834,43146</u>	
				19,86587,49463
	Sine compl. DC viz. CM	47 <u>853354</u> . 74142,97807 . .	9,87007,00193	
	Therefore DC is	42 <u>146646</u> , the side sought.		

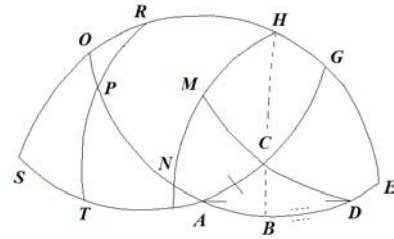
PROBLEM 3.

With two angles given and with a side opposite to either angle given, the side held by the same is sought.

In the oblique angled triangle ADC the side AD is sought

1.Upper diagram. 2.Lower diagram.

{	the angles	CDA	<u>36 131235</u>	{	103 <u>999307</u>
		CAD	<u>46 302020</u>		36 <u>131345</u>
		with the side AC	<u>24 065185</u>		<u>42 146646</u>



Terms of the ratio .

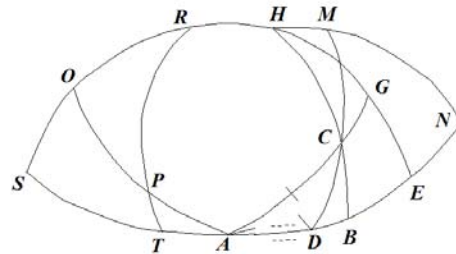
I. For a segment or the base, *by Problem.6. Cap.3.*

{	<i>Proport.</i>	<i>Radius.</i>
		<i>Tangent of the Hypotenuse or of the side.</i>
		<i>Sine of the complement of the angle opposite the Hypotenuse.</i>
		<i>Tangent of the base segment opposite the Hypotenuse.</i>

II. For the remaining segment of the base.

By Conseq.6.

{	<i>Tangent of the angle of the side given opposite.</i>
	<i>Sine of the segment of the base given opposite.</i>
	<i>Tangent of the remaining angle.</i>
	<i>Sine of the remaining segment of the Base.</i>



If the perpendicular falls within, the side sought is the sum of the segments, if it falls without, it is the difference of these.

Illustrated by numbers.

I.

1.

			<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
<i>Proport.</i>	{	Radius . . . RS	90 <u>0000</u>	. 100000,00000 . . 10,00000,00000
		Tang. Hypot. ST=AC	24 <u>065185</u>	. 44659,25969 . . 9,64991,15077
		Sin.compl. OS viz.OR	43 <u>697980</u>	. 69085,69203 . . 9,83938,81107
		Tang.segment. OP=AB	17 <u>146646</u>	. 30853,15726 . . 9 ,48929,96184
		The Complem. is BE	82 <u>049109</u>	

II.

			<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
<i>Proport.</i>	{	Tang. angle at D viz. NM	36 <u>131235</u>	. 73004,79176 . . 9,86335,13634
		Sine of segmen AB	17 <u>146646</u>	. 29481,83629 . . 9,46955,45148
		Tang.angle at D viz. GE	46 <u>302020</u>	. 104651,41007 . . <u>10,01974,50890</u>
				19,48929,9603
		Sine of remaining seg. DB	25 <u>000000</u>	. 42261,82617 . . 9,62594,82404
Segment AB is . . .		17 <u>146646</u>		
Sum of segments is AD		42 <u>146646</u>	, the side sought.	

But if the angles to the base are of diverse kinds, and hence the perpendicular shall fall outside the base, just as in the diagram of the lower triangle, the manner of operation will be similar to the first, and thus the remaining data may be retained.

I.

2.

			<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
<i>Proport.</i>	{	Radius . RS	90 <u>0000</u>	. 100000,00000 . . 10,00000,00000
		Tang. ST=AC	42 <u>146646</u>	. 90504,92470 . . 9,95667,22064
		Sin.compl. SO viz.OR	53 <u>868765</u>	. 80766,85559 . . <u>9,90723,31753</u>
		Tang.segment. OP=AB	36 <u>166052</u>	. 73097,96633 . . 9 ,86390,53817

II.

		<i>Sin. & Tang.</i>	<i>Log. Sin. & Tang.</i>
Proport.	Tang. of ang. at D viz. NM	76 <u>000693</u>	401098,77321 . . 10,60321,13258
	Sine of segment AB	36 <u>166052</u>	59012,73216 . . 9,46955,45148
	Tang. ang. at A viz. GE	36 <u>131235</u>	73004,79176 . . <u>9,86335,13634</u>
			19,63429,71653
	Sine of remaining seg. DB	6 <u>166052</u>	10741,02629 . . 9,03104,58385
	Segment AB is . . .	36 <u>166052</u>	
	Sum of segments is AD	30 <u>000000</u>	, the side sought.

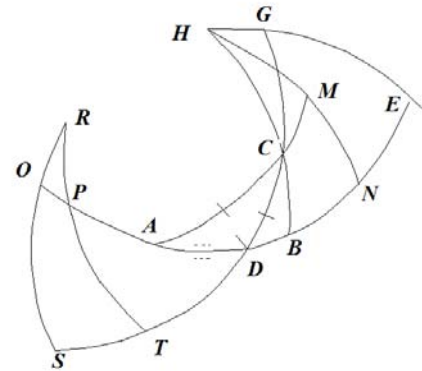
PROBLEM 4.

With two sides given and with one of the opposite angles given, the remaining side is sought.

In the oblique angled triangle ADC the side AD is sought

1.Upper diagram. 2.Lower diagram.

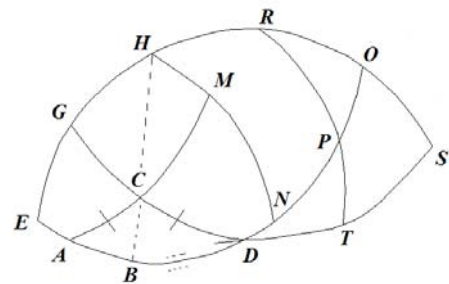
Given	{	The sides	{	DC	30 <u>000000</u>	{	24 <u>065185</u>
		AC	24 <u>065185</u>	137 <u>853354</u>			
		The angle ADC	36 <u>131235</u>				103 <u>999307</u>



Terms of the Ratio.

I. For segment of the base, by *Probl.6, Cap.3*:

Proport.	{	<i>Radius.</i>
		<i>Tangent Hypotenuse to opp.angle given.</i>
		<i>Sine of opp.angle given.</i>
		<i>Tangent of segment of base opp.angle given.</i>



II. For the side sought. By *Conseq.5*:

{	<i>Sine compl.of side opp.angle given.</i>
	<i>Sine compl.segm. base from opp.angle.</i>
	<i>Sine compl.remaining side.</i>
	<i>Sinus complem.remaining segment of base.</i>

If the perpendicular falls within, the sum of segments is the segment sought, but if without, then the difference of the same is the segment sought.

Illustrated by arithmetic.

1.

		I. <i>Sin. & Tang.</i>		<i>Log.Sin. & Tang.</i>				
Proport.	{	Radius . . . RS	90 <u>000000</u>	. 100000,00000	. . 10,00000,00000			
		Tangent. Hypot. DC=ST	30 <u>000000</u>	. 57735,06292	. . 9,76143,93726			
		Sin.compl. OS viz. RO	53 <u>868765</u>	. 80766,85559	. . 9,90723,31753			
		Tang.segment. OP=DB	25 <u>000000</u>	. 46630,76582	. 9 ,66867,25479			
		II.		<i>Sine.</i>		<i>Log.Sin.</i>		
Proport.	{	Sine compl. DC viz. CG	60 <u>000000</u>	. 86602,54037	. . 9,93753,06317			
		Sine compl. DB viz. BE	65 <u>000000</u>	. 90630,77870	. . 9,95727,57115			
		Sine compl. DB viz. BN	65 <u>934815</u>	. 91308,21216	. . 9,96050,98404			
						19,91778,55519		
		Sine compl. AB viz. BN	72 <u>853354</u>	. 95555,33089	. . 9,98025,49202			
		Therefore the seg. AB is	17 <u>146646</u>					
		Truly the seg. DB	25 <u>000000</u>					
	Therefore of the segments, the sum is AD	42 <u>146646</u>	, the side sought.					

The exercise for the side of the AD of the triangle from the lower diagram.

2.

		I. <i>Sin. & Tang.</i>		<i>Log.Sin. & Tang.</i>	
Proport.	{	Radius . . . RS	90 <u>000000</u>	. 100000,00000	. . 10,00000,00000
		Tangent. Hypot. DC=ST	24 <u>065185</u>	. 44659,25969	. . 9,64991,15077
		Sin.compl. OS viz. RO	13 <u>999307</u>	. 24191,01595	. . 9,38365,41009
		Tang.segment. OP=DB	6 <u>166050</u>	. 10803,52716	. 9 ,03356,56086

Transl. Ian Bruce.

		II.	Sine.	Log.Sin.
{	Sine compl. DC viz. CG	65 <u>934815</u>	. 91308,21216 . .	9,96050,98404
	Sine compl. DB viz. BE	83 <u>833947</u>	. 99421,47796 . .	9,99748,02351
	Sine compl. DB vid. BN	47 <u>853354</u>	. 74142,97807 . .	<u>9,87007,00249</u>
				19,86755,02600
	Proport. Sine compl. AB viz. BN	53 <u>833956</u>	. 80731,01834 . .	9,90704,04196
	Therefore seg. is AB	36 <u>166050</u>		
and seg. DB is	6 <u>166050</u>			
Therefor of the segments,				
the sum is AD	30 <u>000000</u>	, the side sought.		

PROBLEM 5.

With two angles given with the side included by the same, either side is sought. In the oblique angled triangle ACD, the side DC is sought.

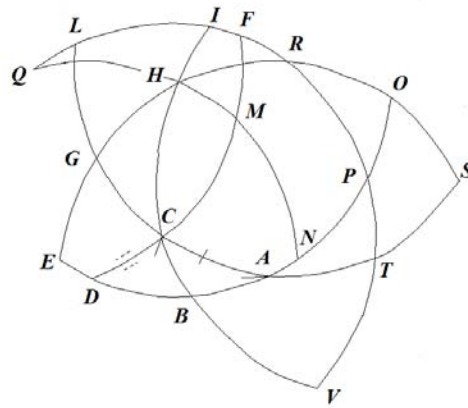
1.Upper diagram. 2.Lower diagram.

{	Given	{	Angles	{	DAC 36 <u>131235</u>	{	36 <u>131235</u>
			ACD 103 <u>999307</u>		46 <u>302020</u>		
			with side AC 30 <u>000</u>		42 <u>146646</u>		

Terms of the ratio.

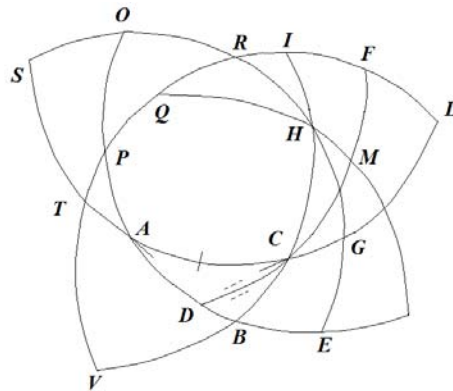
I. For segment of vertical angle, by Prob.16:

{	Proport.	{	Radius.
			Tangent of angle to base, or opp. side sought.
			Sine compl. Hypotenuse; or of side given.
			Tangent compl. segment of angle to vertical.



II. For the side sought, by *Conseq.7.*

Proport. $\left\{ \begin{array}{l} \text{Sine compl. of remaining angle to vertical.} \\ \text{Tangent of side given.} \\ \text{Sine compl. vertical angle last found.} \\ \text{Tangent of side sought.} \end{array} \right.$



Illustrated by numbers.

1.

		I. <i>Sin. & Tang.</i>	<i>Log. Sin. & Tang.</i>
Pro.	{	Radius AS 90 <u>000000</u>	100000,00000 . .10,00000,00000
		Tangent of the angle . CAD viz. SO 36 <u>131235</u>	73004,79176 . . 9,86335,13634
		Sin.compl. of the side . CA viz. AT 60 <u>000000</u>	86602,54037 . . 9,93753,06317
		Tang.compl. seg. angle vert. TP viz. TV 32 <u>302713</u>	63224,00405 . . 9,80088,19951
		Therefore seg. of vert. angle is	57 <u>697287</u> viz. ACB.

		II.	Sinus.	Logar.Sin. & Tang.
{	Proport.	Sine compl. FI viz. QI	43 <u>697980</u>	. 69085,69203 . . 9,83983,81107
		Tangent of side AC viz. LG	30 <u>000000</u>	. 57735,02692 . . 9,76134,93726
		Sine complem. LI viz. RI	32 <u>302713</u>	. 53939,23708 . . <u>9,72786,02462</u>
				19,48929,96188
		Tang.side.sought FM=DC	24 <u>065185</u>	. 44659,25969 . . 9,64991,15081

But if the perpendicular falls without the base, as in the lower diagram ; with the given remaining ; I say

2.

		I.	Sin. & Tang.	Log.Sin. & Tang.
{	Proport.	Radius . . . AS	90 <u>000000</u>	. 100000,00000 . . 10,00000,00000
		Tangent . . . SO	36 <u>131235</u>	. 73004,79176 . . 9,86335,13634
		Sine . . . AT	47 <u>853350</u>	. 74142,97807 . . <u>9,38365,41009</u>
		Tangent . . . TP	28 <u>425765</u>	. 54127,92962 . . <u>9,73342,13883</u>
		Complem. is TV=IL	61 <u>574345</u>	And thence the angle DCB 15 <u>272215</u> .

		II.	Sinus.	Logar.Sin.
{	Proport.	Sine compl. FI viz. QI	74 <u>727785</u>	. 96368,52972 . . 9,98438,56593
		Tang.of side AC viz. LG	42 <u>146646</u>	. 90504,92470 . . 9,95607,22854
		Sine compl. LI viz. RI	28 <u>425765</u>	. 47601,97234 . . <u>9,67762,49419</u>
				19,63429,71473
		Tang.of side FM=DC	24 <u>065185</u>	. 44659,25969 . . 9,64991,14880

Also from the same given, the remaining two sides are able to be obtained likewise clearly by the same labour in this manner.

Transl. Ian Bruce.

	1.	2.
Angle DAC	36 <u>131235</u>	$\left\{ \begin{array}{l} 36 \underline{131235} \\ 46 \underline{303030} \\ \hline 82 \underline{433255} \\ 41 \underline{216627} \\ \hline 10 \underline{170785}^{\frac{1}{2}} \\ 5 \underline{085392}^{\frac{1}{2}} \end{array} \right.$
Angle ADC	103 <u>999307</u>	
<hr/>		
Sum of the angles	140 <u>120542</u>	
half the sum	70 <u>065271</u>	
<hr/>		
Difference of angles	67 <u>868072</u>	
Half difference	33 <u>934036</u>	

1. Operation I.

	Arc	Logs.Sin. & Tang.
$\left\{ \right.$	Sine of half the sum of the angles	70 <u>065271</u> 9,97316,55681
	Sine of half the difference of the angles	33 <u>934036</u> 9,74681,94465
	Tangent of half the included sides	15 <u>000000</u> 9,42805,24525
		<hr/>
	Tan. $\frac{1}{2}$ difference of the remaining sides	9 <u>040731</u> 19,17487,18990 9,20170,63309

Operation II.

	Arc	Logs.Sin. & Tang.
$\left\{ \right.$	Sine compl. half sum of the angles	19 <u>934729</u> 9,53268,99540
	Sine compl. half difference of the angles	56 <u>065964</u> 9,91891,10459
	Tangent of half included side	15 <u>000000</u> 9,42805,24525
		<hr/>
Tangent half sum of sides	33 <u>105916</u> 19,34696,34964 9,81427,35444	
Half sum of sides is	33 <u>105916</u>	
of half the difference of the sides is	9 <u>105916</u>	

Sum of the sum & difference of sides is 42 146647 Side AD sought.

Difference of sum and difference of sides 24 065185 Side DC sought.

Similarly with the given numbers of the lower diagram. I say,

2.

Operation I.

	<i>Arcs</i>	<i>Log.Sin. & Tang.</i>
Sine half sum of angles	41 <u>217728</u>	9,81882,46407
Sine half difference of angles	5 <u>085393</u>	8,94763,13168
Tangent half the side included	21 <u>073323</u>	<u>9,58583,62561</u>
		18,53346,75729
Tang. $\frac{1}{2}$ difference remaining sides	2 <u>967409</u>	8,71464,29322

Operation II.

	<i>Arc</i>	<i>Logs.Sin. & Tang.</i>
Sine compl.half sum angles	48 <u>783372</u>	9,87634,70195
Sine compl.half difference angles	84 <u>904607</u>	9,99828,71096
Tangent half of side included	21 <u>073323</u>	<u>9,58583,62562</u>
		19,58412,33658
Tangent half sum of sides	27 <u>032592</u>	9,70777,63463
Half sum of sides is	27 <u>032592</u>	
half difference of sides is	2 <u>967409</u>	

Sum of sum & difference of sides is 30 000001 Side AD sought.

Difference sum & difference sides is 24 065183 Side DC sought.

PROBLEM 6.

With two sides and with either angle opposite given, th angle opposite the remaining side is sought.

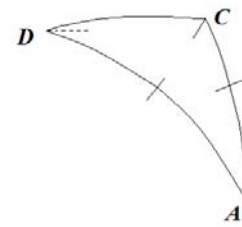
In the oblique angled triangle ACD the angle ADC is sought

$$\text{Given } \left\{ \begin{array}{l} \text{sides } \left\{ \begin{array}{l} \text{AC } 30 \text{ } \underline{000000} \\ \text{AD } 42 \text{ } \underline{144266} \end{array} \right. \\ \text{with the angle ACD } 103 \text{ } \underline{999307} \end{array} \right\}$$

Transl. Ian Bruce.

Terms of the ratio.

- { Sine of the side opposite to the given angle.
- { Sine of the given angle dati.
- { Sine of the remaining side.
- { Sine of the angle sought, per Consec.3, Cap.2.



Illustrated by Arithmetic.

			<i>Sines</i>		<i>Log.Sines.</i>
Proport.	{	Sine of side AD	42 146646	67103,04565 . .	9,82674,22303
		Sine of angle ACD	76 000693	97029,86513 . .	9,98690,57277
		Sine of side AC	30 000000	50000,00000 . .	9,69897,00043
					19,68587,57320
	{	Sine of angle ADC	46 302021	72299,15012 . .	9,85913,35017

With two angles acquired together with the sides opposite the same, we are able also to come upon the remaining angle in the same manner as we have obtained the remaining side by Problem 1 of this chapter.

PROBLEM 7.

With two angles and the side included by the same, the remaining angle is sought.

In the following oblique angled triangle ACD the angle ADC is sought.

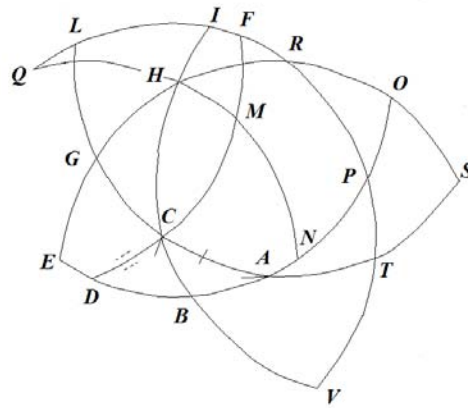
1.Upper diagram. 2.Lower diagram.

Given	{	Angles	{	DAC	103 999307	{	36 131235
				DAC	36 131235		46 302020
		Side	AC	30 000	42 146646		

Terms of the ratio.

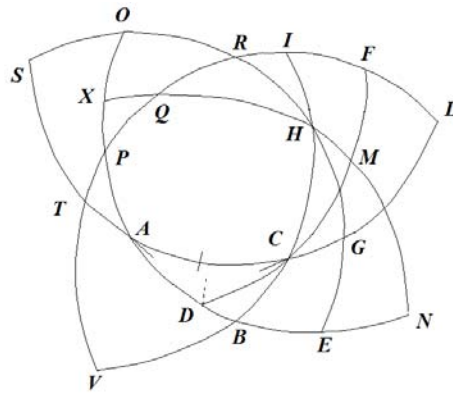
I.For the segment of the vertical angle, Per Probl.6.

- { Radius.
- { Tangent of the angle to the base.
- { Sine comp. of given side.
- { Tangent Comp. of seg. to vertical angle.



II. For the angle sought. *By Conseq.4.*

Proport. $\left\{ \begin{array}{l} \text{Sine of seg. of last found angle to vertical.} \\ \text{Sine of remaining angle to vertical.} \\ \text{Sine compl. of angle to base.} \\ \text{Sine compl. of angle sought.} \end{array} \right.$



Illustrated by arithmetic.

1.

	I. Sin.&Tang.	Log.Sin.&Tang.
Pro. $\left\{ \right.$ Radius AS 90 <u>000000</u>	100000,00000	. 10,00000,00000
Tangent of the angle . CAD viz. SO 36 <u>131235</u>	73004,79176	. . 9,86335,13634
Sin.compl. of side . . . CA viz. AT 60 <u>000000</u>	86602,54037	. . 9,93753,06317
Tang.segment. angle vert.TP viz. TV 32 <u>302713</u>	63224,00405	. $\sqrt{}$ 9,80088,19951
Therefore segment of angle to vert. is 57 <u>697287</u> viz. ACB.		

		II.	<i>Sin.</i>	<i>Logar.Sin.</i>
{	Sine	IL 57 <u>697287</u>	. 84523,65276 . .	9,92697,82551
	Tangent angle . . .	IF 46 <u>302022</u>	. 72299,15012 . .	9,85913,31911
	Sine compl. EG viz. HG	53 <u>808765</u>	. 80766,85559 . .	<u>9,90723,31753</u>
	Sine compl. MN viz. HM	43 <u>697979</u>	. 69085,69203 . .	19,76636,63664 9,83938,81113
Therefore the angle AD is 40 <u>302021</u> the angle sought.				

The same exercise is required in the lower diagram.

2.

		I. <i>Sin.&Tang.</i>	<i>Log.Sin.&Tang.</i>
{	Radius	AS 90 <u>000000</u>	. 100000,00000 . . 10,00000,00000
	Tangent	SO 46 <u>30201</u>	. 104651,41007 . . 10,01974,50864
	Sin.compl. CA viz. AT	47 <u>853354</u>	. 74142,97807 . . 9,87008,00249
	Tangent TP	37 <u>808484</u>	. 77591,67097 . . 9,88981,51113
Complement is TV=IL 52 <u>191526</u>			

		II. <i>Sin.</i>	<i>Log.Sin.</i>
{	Sine	IL 52 <u>191516</u>	. 79006,43467 . . 9,89766,24083
	Sine	IF 16 <u>060281</u>	. 27664,85382 . . 9,44192,83946
	Sine compl. EG viz. HG	43 <u>697979</u>	. 69085,69203 . . <u>9,83938,81107</u>
	Sine compl. MN viz. HM	13 <u>999307</u>	. 24191,01595 . . 19,28131,65053 9,38365,40970
With which added to the quadrant HX 90 <u>000000</u>			
The arc is completed MX 103 <u>999307</u> , the measure of the angle ADC sought.			

PROBLEM 8.

With two sides and one of the opposite angles given, the angle included by the same is sought.

In the oblique angled triangle ADC, the angle ACD is sought.

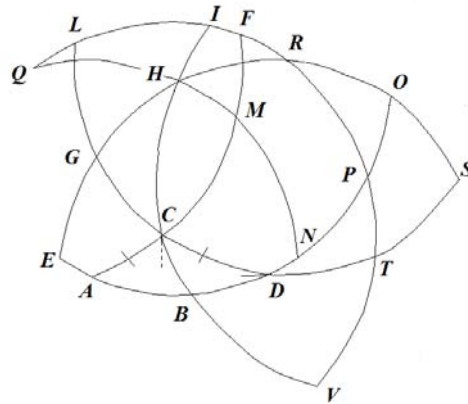
1.Upper diagram. 2.Lower diagram.

$$\text{Given } \left\{ \begin{array}{l} \text{Sides } \left\{ \begin{array}{l} \text{AC } 24 \underline{065185} \\ \text{CD } 30 \underline{000} \end{array} \right. \\ \text{Angle ADC } 36 \underline{131235} \end{array} \right\} \left\{ \begin{array}{l} 30 \underline{000} \\ 42 \underline{146646} \\ 46 \underline{302020} \end{array} \right.$$

Terms of ratio.

I. For segment of vertical angle, by *Problem 6.*

$$\text{Proport. } \left\{ \begin{array}{l} \text{Radius.} \\ \text{Tangent of angle given.} \\ \text{Sine compl. of side opp. to angle given.} \\ \text{Tangent Compl. of segment to vert. angle.} \end{array} \right.$$



II. For angle sought. By *Conseq.7.*

$$\text{Proport. } \left\{ \begin{array}{l} \text{Tangent of side opposite to given angle.} \\ \text{Tangent of remaining side.} \\ \text{Sine Comp. of seg. of vert. angle last found.} \\ \text{Sine Compl. of remaining vertical angle.} \end{array} \right.$$

For the triangle of the lower diagram.

2.

		I. <i>Sin. & Tang.</i>		<i>Log.Sin. & Tang.</i>	
Proport.	{	Radius DS	90 <u>000000</u>	. 100000,00000	. . 10,00000,00000
		Tangent SO	46 <u>302021</u>	. 104651,41007	. . 10,01974,50864
		Sin.compl. CF viz. DT	47 <u>853354</u>	. 74142,97807	. . 9,87008,00249
		Tangent TP	37 <u>808484</u>	. 77591,67097	. . 9,88981,51113
		Complement is TV=LI	52 <u>191526</u>		

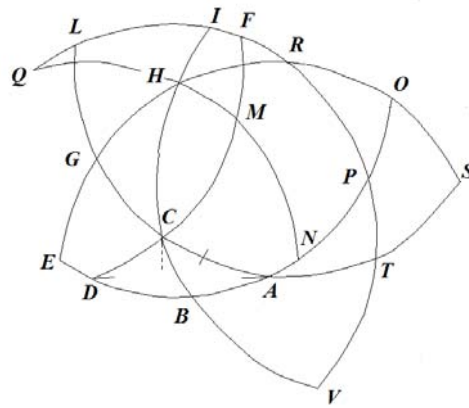
		II.		<i>Sin. & Tang.</i>		<i>Log.Sin. & Tang.</i>	
Proport.	{	Tangent of side AC=FM	30 <u>000000</u>	. 57735,02682	. . 9,76143,93726		
		Tangent of side DC=LG	42 <u>146646</u>	. 90504,92470	. . 9,95667,22064		
		Sine compl. LI viz.LR	37 <u>808484</u>	. 61302,45434	. . <u>9,78747,75192</u>		
		Sine compl. IF viz. IQ	73 <u>939719</u>	. 96097,10775	. . 9,98271,03530	19,74414,97256	
		There the arc is IF	16 <u>020681</u>				
		But the arc found IL	52 <u>191516</u>				
		Difference FL	36 <u>131235</u> , measure of the angle DCA.				

PROBLEM 9

With two angles and with a side opposite one given, the remaining angle is sought.

In the oblique angled triangle ADC, the angle ACD is sought.

		1.Upper diagram.		2.Lower diagram.	
Given	{	Angles	{ DAC	36 <u>131235</u>	} { 46 <u>302020</u>
			{ ADC	46 <u>302021</u>	
		Side AC	30 <u>000000</u>	} { 42 <u>146646</u>	

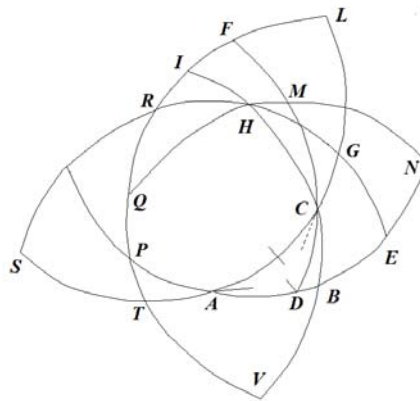


Terms of the ratio.

I. For the segment of the angle to the vertical. *By Problem 6.*

Proport. $\left\{ \begin{array}{l} \text{Radius.} \\ \text{Tangent of the angle opposite to the side given.} \\ \text{Sine comp. of the side opposite to the angle given.} \\ \text{Tangent of the vert. angle of the segment.} \end{array} \right.$

II. For angle sought to the vertical. *By consec. 4.*



Proport. { *Sine compl. of the angle to the opp. side given.*
Sine compl. of the remaining angle given.
Sine of the seg. of the angle to the vertical last found.
Sine of the remaining angle to the vertical.

If the perpendicular falls within, the sum of the angles to the vertical is the angle sought ;
 but if it falls beyond, the difference is the angle sought.

Illustrated by arithmetic.

		I.	<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
Proport. {	Radius AS	90 <u>000000</u>	. 100000,00000	. . 10,00000,00000
	Tangent SO	36 <u>131235</u>	. 73004,79176	. . 9,86335,13634
	Sine compl. AC viz. AT	60 <u>000000</u>	. 86602,54037	. . 9,93753,06317
	Tang. TP	32 <u>302713</u>	. 63224,00405	. . 9,80088,19951
Complement is TV=LI		57 <u>697287</u>	measure of the segment of the angle ACB.	

		II.	<i>Sin.</i>	<i>Logar.Sin.</i>	
Proport. {	Sine compl. EG viz. HG	53 <u>808765</u>	. 80766,85559	. . 9,90723,31753	
	Sine compl. NM viz. HM	43 <u>697979</u>	. 69085,69203	. . 9,83938,81107	
	Sine of vert. angle IL	57 <u>697287</u>	. 84523,65276	. . <u>9,92697,82551</u>	
					19,76636,63658
	Sine vert. angle left IF	46 <u>302020</u>	. 72299,15012	. . 9,85913,31905	
Arc IL is		57 <u>697287</u>			
Sum of arcs is		103 <u>999307</u> ,	measure of angle sought ACD.		

Working for the angle ACD of the lower diagram.

2.

		I.	<i>Sin. & Tang.</i>	<i>Log. Sin. & Tang.</i>
Proport.	Radius AS	90	<u>000000</u>	. 100000,00000 . . . 10,00000,00000
	Tangens SO	46	<u>302021</u>	. 104651,41007 . . . 10,01974,50864
	Sin.compl. AC viz. AT	47	<u>853354</u>	. 74142,97807 . . . 9,87008,00249
	Tangent TP	37	<u>808484</u>	. 77591,67097 . . . 9,88981,51113
	Complement is TV=LI	52	<u>191516</u>	

		II.	<i>Sinus.</i>	<i>Logar. Sin.</i>	
Proport.	Sine compl. EG viz. HG	43	<u>697979</u>	. 69085,69203 . . . 9,83938,81107	
	Sine compl. NM viz. HM	13	<u>99907</u>	. 24191,01595 . . . 9,38365,40970	
	Sine IL	52	<u>191516</u>	. 79006,43467 . . . <u>9,89766,24083</u>	
					19,28131,65053
	Sine remaining angle vert. IF	16	<u>060281</u>	. 27664,85482 . . . 9,44192,83946	

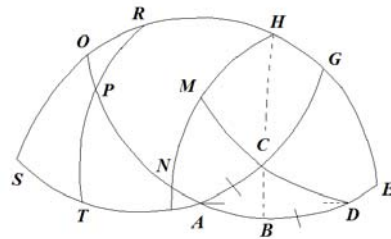
But the arc is IL 52 191516
 Difference is arc LF 36 131245, measure of the angle sought ACD.

PROBLEM 10.

With two sides given with the angle included by the same, either other angle is sought.

In the oblique angled triangle ACD the angle ADC is sought.

		1.Upper diagram. 2.Lower diagram.																
Given	Sides	<table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">AC</td> <td style="padding-right: 5px;">24</td> <td style="padding-right: 5px;"><u>065185</u></td> <td rowspan="3" style="font-size: 2em; vertical-align: middle;">}</td> <td style="padding-left: 5px;">42</td> <td style="padding-left: 5px;"><u>146646</u></td> </tr> <tr> <td style="padding-right: 5px;">AD</td> <td style="padding-right: 5px;">42</td> <td style="padding-right: 5px;"><u>146646</u></td> <td style="padding-left: 5px;">30</td> <td style="padding-left: 5px;"><u>000000</u></td> </tr> <tr> <td style="padding-right: 5px;">Angle CAD</td> <td style="padding-right: 5px;">46</td> <td style="padding-right: 5px;"><u>302020</u></td> <td style="padding-left: 5px;">36</td> <td style="padding-left: 5px;"><u>131235</u></td> </tr> </table>	AC	24	<u>065185</u>	}	42	<u>146646</u>	AD	42	<u>146646</u>	30	<u>000000</u>	Angle CAD	46	<u>302020</u>	36	<u>131235</u>
	AC	24	<u>065185</u>	}	42		<u>146646</u>											
	AD	42	<u>146646</u>		30		<u>000000</u>											
Angle CAD	46	<u>302020</u>	36		<u>131235</u>													



Terms of the ratio.

I. For the segment of the base. by Problem 6, Cap.3.

		I.
Proport.	Radius.	
	Tangent of Hypotenuse.	
	Sine comp. of angle given.	
	Tangent seg. of base opp. given angle.	

The working for the angle ADC of the lower diagram.

2.

		I. Sin. & Tang.		Log. Sin. & Tang.	
{	Proport.	Radius . RS	90 0000	.	100000,00000 . .10,00000,00000
		Tang. AC=ST	42 <u>146646</u>	.	90504,92470 . . 9,95667,22064
		Sin.compl. SO viz.RO	53 <u>868765</u>	.	80766,85559 . . <u>9,90723,31753</u>
		Tang.segm. OP=AB	36 <u>166052</u>	.	73097,96633 . . <u>9,86390,53817</u>
II.					
		Sin. & Tang.		Log. Sin. & Tang.	
{	Proport.	Sine segment of base DB	6 <u>166050</u>	.	10741,02629 . . 9,03104,58364
		Sine of segment AB	36 <u>166052</u>	.	59012,73216 . . 9,77094,58190
		Tang. of ang. given EG	36 <u>131235</u>	.	73004,79176 . . <u>9,86335,13634</u>
		Tang. of angle NM	76 <u>000693</u>	.	401098,77321 . . 10,60325,13460
		Compl. of semicircle is 103 <u>99907</u> , the angle sought ADC.			

The two remaining angles likewise can be found without more difficulty, clearly in the same manner by which we have found the remaining sides for so many problems.

1.			2.	
Side AC	24 <u>065185</u>		Side AC	42 <u>146646</u>
Side AD	42 <u>146646</u>		Side AD	30 <u>000000</u>
Sum of sides	66 <u>211813</u>		Sum of sides	72 <u>146646</u>
Half the sum	33 <u>1059155</u>		Half the sum	36 <u>073323</u>
Diff. of sides	18 <u>081461</u>		Diff. of sides	12 <u>146646</u>
Half the diff.	9 <u>0407305</u>		Half the diff.	6 <u>073325</u>

1. Operation I.

		<i>Logs. Sin. & Tang.</i>	
{	<i>Proport.</i>	Sine of half sum of sides	33 <u>1059155</u> . 9,73734,24996
		Sine of half diff. of sides	9 <u>0407305</u> . 9,19627,71847
		Tang. compl.half included angle	66 <u>848990</u> . <u>10,36897,36696</u>
			19,56525,08543 sum.
		Tang. $\frac{1}{2}$ diff. remaining angles	33 <u>934033</u> . 9,82790,83547 diff.

Operation II.

		<i>Logar. Sin. & Tang.</i>	
{	<i>Proport.</i>	Sine compl.half sum sides	56 <u>894085</u> . 9,92306,89855
		Sine compl.half diff. sides	80 <u>9592695</u> . 9,99457,09124
		Tang. compl. half angle incl.	66 <u>848990</u> . <u>10,36897,36696</u>
			20,36354,45820 sum.
		Tang. half sum angles	70 <u>065270</u> . 10,44047,55965 diff.
		Half diff. remaining angles	<u>33 934033</u>
		Sum	103 <u>999303</u> Angle ACD
		Diff.	36 <u>131237</u> Angle ADC.

2. Operation I.

		<i>Logs Sin. & Tang.</i>	
{	<i>Proport.</i>	Sine half sum sides	36 <u>073323</u> . 9,76998,26167
		Sine half diff. sides	6 <u>073323</u> . 9,02449,00297
		Tang. compl.half incl. angles	71 <u>934825</u> . <u>10,48653,42647</u>
			19,51102,42944
		Tang. $\frac{1}{2}$ diff. remaining angles	28 <u>848643</u> . 9,74104,16777

Operation II.

		<i>Logar. Sin. & Tang.</i>	
{	Sine compl. half sum sides	53 <u>926677</u> . 9,90755,33018	
	Sine compl. half diff. sides	83 <u>926677</u> . 9,99755,55732	
	Tang. compl. half incl. angle	71 <u>9343825</u> . <u>10,78653,42647</u>	
			20,78408,98379
	Tang. half sum remaining angles	75 <u>150664</u> . 10,57653,65361	
	Half diff. angles	<u>28 848643</u>	
Sum		103 <u>999307</u> Angle ADC	
Diff.		46 <u>302021</u> Angle ACD.	

PROBLEM. 11

With the individual sides given, any angle is sought.

In the oblique angled triangle ZPS the angle PZS is sought.

With sides given {
 PS 42 146646
 PZ 30 000000
 ZS 26 065185

Terms of the ratio.

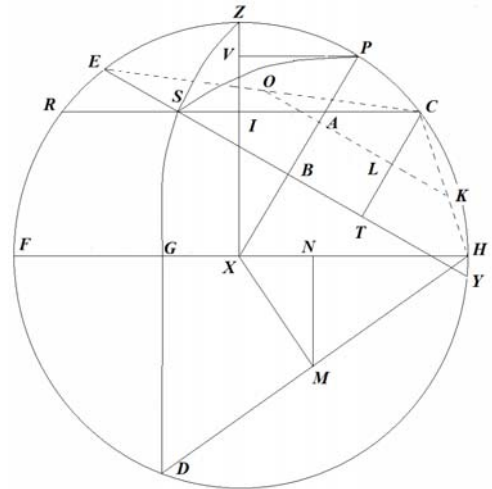
{

Rectangle from Sines of sides including angle sought.

Square of radius.

Rect. from $\frac{1}{2}$ sum of sines & $\frac{1}{2}$ diff. of bases & diff. of same sides.

Square sine $\frac{1}{2}$ angle sought.



Illustrated by arithmetic.

1. The sines of the two sides given is taken that include the angle sought ; and the logarithms of the sines of these taken.
2. The square of the radius is taken; and its logarithm.
3. The base is taken with the difference of the sides or legs, and half the sum and half the difference of these is taken in turn, and of these the sines and the logarithms of the sines.
4. If the first factor shall divide the remaining factor ; the side of the quotient is the sine of half the angle sought ; or if the sum of the first logarithms may be taken from the sum of the rest, half the difference is the logarithm of the sine of half the angle sought.

$$\text{Given sides } \left\{ \begin{array}{l} \text{PZ } 30 \text{ } \underline{000000} \\ \text{ZS } 26 \text{ } \underline{065185} \end{array} \right., \text{ sines } \left\{ \begin{array}{l} 50000,00000 \\ 40777,57121 \end{array} \right\} \& \text{ Logs. are taken } \left\{ \begin{array}{l} 9,69897,00043 \\ 9,61042,13420 \end{array} \right.$$

$$\begin{array}{l} 1. \text{ Rect.of sines of sides } 20388785605000000000 \left\| \begin{array}{l} \text{Log.sum. } 19,30939,13473 \text{ A} \\ \text{Log.sq.rad. } 20,00000,00000 \text{ B} \end{array} \right. \\ 2. \text{ Squared radius } 10000000000000000000 \end{array}$$

$$\begin{array}{ll} \text{Base} & \text{PS } 42 \underline{146646} \\ \text{Diff. legs} & \text{PC } 5 \underline{934815} \\ \text{Sum Base \& diff.} & \text{EPC } 48 \underline{081461} \\ \text{Diff.Base \& diff.sides} & \text{CY } 36 \underline{211831} \end{array}$$

$$\left\{ \begin{array}{l} \text{Half sum . . . CO } 24 \underline{040730} \\ \text{Half diff. . . CK } 18 \underline{105915} \end{array} \right. ; \text{ Sines } \left\{ \begin{array}{l} 40738,59542 \\ 31077,45544 \end{array} \right. \text{ Log. } \left\{ \begin{array}{l} 9,61000,60492 \\ 9,49244,54374 \end{array} \right.$$

$$\begin{array}{l} 3. \text{ Rect. } \frac{1}{2} \text{ Sum. \& } \frac{1}{2} \text{ diff. } 1266051883853238084800000 \text{ etc. } \left\| \begin{array}{l} \text{sum } 19,10245,14866 \text{ C} \\ \text{diff. } 19,79306,01393 \text{ D} \end{array} \right. \\ 4. \text{ Quotient is } 6209502320783665173 \text{ etc.} \end{array}$$

Side of quotient is 7880069944, Sine 51 999650 $\frac{1}{2}$ differ. 9,89653,00696

And thus I say,

		<i>Logar.Sinum.</i>
Proport.	{	<i>Rect.of sines of leg</i> 20388785605000 etc. 19,30939,13473 A
	{	<i>Sq.of radius</i> 10000000000000 etc. 20,00000,00000 B
	{	<i>Rect. $\frac{1}{2}$ sum. \& $\frac{1}{2}$ differ.</i> 12660518838532380 etc. <u>19,10245,14866 C</u>
	{	<i>Sq. sine $\frac{1}{2}$ angle sought</i> 620950232078366 etc. diff. 19,79306,01393 D.

Transl. Ian Bruce.

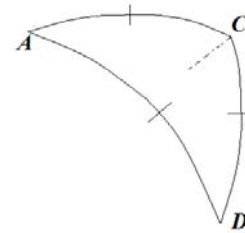
Side of square is 788006944. Sine 51 999650 $\frac{1}{2}$ diff. 9,89653,00696

Of which twice the arc evidently 103 999300 is the angle sought PZS.

And that this compendium of logarithms should be so great a reward, I would rather the practitioner of trigonometry may value from his own senses first rather than from my words.

Otherwise 2.

The logarithms of the legs of the angle sought may be subtracted from the logarithms of the square of the radius and of half the sum of the three sides and of half the difference of the base and the sum of the legs: The remainder will be the logarithm of the square of the sine of the complement of half the angle sought to the quadrant.



Precepts Illustrated by Arithmetic.

In the oblique angled triangle ADC the angle ACD is sought.

Given	{	Base AD 42 <u>146646</u>	<i>Logar.Sin.</i>	
	{	Legs { AC 30 <u>000000</u>	9,69897,00043	
		CD 26 <u>065185</u>	9,61042,13430	
				19,30939,13473 B Sum Logs.
Sum sides		96 <u>211831</u>	. 20,00000,00000 E log.sq.rad.	}
Half sum		48 <u>105915</u>	. 9,87179,49769	
Diff.legs & Base		11 <u>918539</u>		
$\frac{1}{2}$ Diff.legs & Base		5 <u>959269</u>	. 9,01628,70114	
				38,88808,19883 F sum.
				<u>19,30939,13473 B</u>
				19,57869,06410 Diff. B & F.
Logarithm Sine Deg.38 <u>000344</u>		9,78934,53205	Half diff.	
Compl. of this arc		51 <u>999656</u> .		
Compl. double arc		103 <u>999312</u> is the angle sought ACD.		

Transl. Ian Bruce.

Otherwise 3.

From half the sum of the sides, the sides of the triangle are taken away individually, and the sum of the logarithms of the sine of half the sum of the sides and of the difference of the side subtending the given angle, is taken from the sum of the logarithms of the remaining difference of the sines and twice the logarithm of the radius : half of the remaining will be the Logarithm of the tangent of half the angle sought.

Arithmetical illustration of the precept.

The data from the above question may be retained.

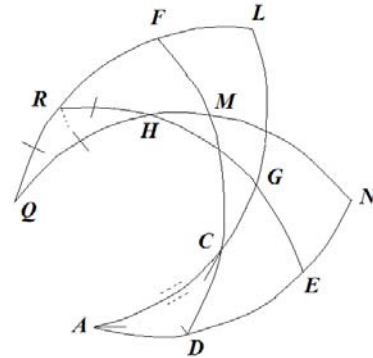
Sides	{	AD . 42 <u>146646</u>		
		AC . 30 <u>000000</u>		
		CD . 26 <u>065185</u>		
Sum of sides . . .		96 <u>211831</u>	<i>Log.Sin.</i>	
Half the sum		48 <u>105915</u>	. 9,87179,49769	}
Diff.1 of Base AD & $\frac{1}{2}$ sum		5 <u>959269</u>	. 9,01628,70114	
Diff.2 of side AC & $\frac{1}{2}$ sum		18 <u>105915</u>	. 9,49244,54374	}
Diff.3 of side CD & $\frac{1}{2}$ sum		24 <u>040730</u>	. 9,61000,60492	
		Twice log radius	<u>20,00000,00000</u>	
			39,10245,14866	}
			<u>18,88808,19883</u>	
			Log. difference	
			20,21436,94983	
			Half difference . . .	
			10,10718,47491	
Log. Tangent is		51 <u>999653</u> .		
Of which twice the arc		103 <u>999306</u>		is the angle sought ACD.

PROBLEM 12

With the angles given, any side is sought.

In the oblique angled triangle ADC the side AC is sought

$$\text{Angles given } \begin{cases} \text{ADC} . 103 \underline{999307} \\ \text{DAC} . 36 \underline{131235} \\ \text{ACD} . 46 \underline{302020} \end{cases}$$



This problem itself is the converse of problem eleven, and may be resolved in the same manner, provided that the angles and sides may be interchanged. For the two smaller angles can always be equated to the two sides of the triangle included by the major arcs of the circles drawn from the same pole. But the third angle can be greater than a quadrant, and thus its complement to a semicircle can be taken conveniently for the third side. Moreover the angle where it shall have been found, will be one of the three sides sought ; but with one found from the same, the remainder will be found from the rule of proportion beside *Problem 1*.

As in triangle ACD, of which the arcs of the pole are HRQ, and its sides are equal to the angles of this ; AD is equal to the angle at H or to the arc EN. DC to the angle Q or to the arc MF. And AC to the angle HRF or to the arc GL. Therefore if the angles A.D.C shall be given, the sides QR.QH.RH are given. Moreover the angle CDE or MN may be equal to the side QH. And thus the triangle QRH may be resolved, following the precepts of problem 11.

	<i>Log.Sin.</i>	
Legs $\begin{cases} \text{RQ } 46 \underline{302020} \\ \text{RH } 36 \underline{131235} \end{cases}$	9,85913,31911 9,77058,45404	
Difference of legs	10 <u>170785</u>	+ 19,62971,77315
Base	QH 76 <u>000693</u>	Sum Logs.
Sum Diff. of legs & base	86 <u>171478</u>	
Diff.Base & diff. of legs	65 <u>829908</u>	
Half sum diff. of legs & Base	43 <u>085739</u>	9,83447,92046
Half differ. Base & diff. of legs	32 <u>914954</u>	<u>9,73511,44888</u>

Transl. Ian Bruce.

		19,56959,36934 sum Logs.
Twice Log radius	<u>20,00000,00000</u>	
	+	<u>19,62971,77315</u>
		19,93987,59619 Diff.Logs.
Log. Sine $\frac{1}{2}$ Angle HRQ	68 <u>926676</u>	9,96993,79809 Half diff.
Of which twice the angle is	137 <u>853352</u>	
And twice the angle compl.to semicircle	42 <u>146648</u>	is the angle GRL, of which
the measure is GL=AC, the side sought.		

Otherwise 2.

With the given and the sought as above. I say,

Proport. { *Rectangle from sines of the legs of the angle sought.*
Square of the radius.
Rectangle from $\frac{1}{2}$ sines of the sum of the sides
& diff. of the Base & half the sum.
Square sine of compl. of half angle sought.

		<i>Logs.Sin.</i>
Legs {	RQ 46 <u>302020</u>	9,85913,31911
	RH 36 <u>131235</u>	9,77058,45404
		<i>Logs.Sin.</i>
Legs {	RQ 46 <u>302020</u>	9,85913,31911
	RH 36 <u>131235</u>	9,77058,45404
Base	QH 76 <u>000693</u> +	19,62971,77315 Sum Logs.
Sum legs & base	<u>158 433948</u>	
Half sum legs & base	79 <u>216974</u>	9,99226,30294
Differ. base & of half sum	3 <u>226281</u>	<u>8,74900,27530</u>
		8,74126,57824 sum.Logs.
Twice log. radius		<u>20,00000,00000</u>
		+ 19,62871,77315
Log. Sine angle	21 <u>073312</u>	9,55577,40254
Compl. of this angle	68 <u>926688</u>	
Twice which is QRH	137 <u>853376</u>	
And its compl.to semicircle is	42 <u>146624</u> .	Side sought AC.

Sides $\left\{ \begin{array}{l} \text{QH} . 76 \underline{000693} \\ \text{RQ} . 46 \underline{302020} \\ \text{RQ} . 36 \underline{131235} \end{array} \right.$

Sum of sides	158 <u>433948</u>	<i>Log.Sines.</i>	
Half sum	79 <u>216974</u>	. 9,99226,30294	} 18,74126,57824 sum Log.
Diff.1 of side QH	3 <u>216281</u>	. 8,74900,27530	
Diff.2 of side RQ	32 <u>914954</u>	. 9,83447,92046	} 19,56957,36934 Sum Logs.
Diff.3 of side RH	43 <u>085739</u>	. 9,73511,44888	
			20,82832,79110 Diff.
			10,41416,39555 Half Diff. is
Log. Tangent $\frac{1}{2}$ angle HRQ	68 <u>926686</u>		
double is the whole angle	137 <u>853372</u>		
And its Compl.to semicircle	42 <u>146628</u>	of which the measure is LG=AC.	

CHAPTER V.

1. So that all can be done with the minimum of labour.

If the first of the three given proportionals shall be the whole sine or radius, the middle two logarithms are added with the first place to the left removed ; the remainder will be the Logarithm sought.

The illustration of Problem 3, Ch.4. of plane triangles may be recalled, & Problem1, Ch.3 of spherical triangles :

		<i>Logs.</i>	$\left\ \right.$		<i>Logs.</i>
<i>Prop.</i>	$\left\{ \begin{array}{l} \text{Radius} \ 90 \underline{0000} \\ \text{Tangent} \ 283300 \\ \text{Leg} \ 11237943 \\ \text{Leg} \ 6058601 \end{array} \right.$	10,00000,0000	$\left\ \right.$	$\left\{ \begin{array}{l} \text{Whole Sine} \ 90 \underline{000} \\ \text{Sine} \ 30 \underline{000} \\ \text{Sine Hyp.} \ 51 \underline{076287} \\ \text{Sine} \ 22 \underline{891768} \end{array} \right.$. 10,00000,00000
		9,73168,5536	$\left\ \right.$. 9,69897,00043
		<u>3,05068,6815</u>	$\left\ \right.$. <u>9,89097,02062</u>
		$\cancel{2},78237,2351$	$\left\ \right.$. $\cancel{2},78237,2351$

2. If the first of the given amounts shall be some sine or side ; For the Logarithm of the first its arithmetical complement may be taken, just as the most learned *Master Briggs* taught in Ch. 15 of the *Arithmetica Logarithmica* ; and with the addition of three made, the one emerging to the left may be removed.

Examples may be borrowed from Problem 1, Ch.4 on planar triangles, & Problem 2.Ch.3. on spherical triangles.

Prop.	{	Leg	11237943	{	3,05068,6815	Log.		{	Sine	67	108232	{	9,96437,34001	Log.			
		Leg	6058601	2,78237,2352							Sin.Tot.	90	000000	10,00000,00000			
		Radius	90	00000	10,00000,0000						Sine	38	923713	9,79815,67532			
		Tangent	28	33	9,73168,5537			Sine	43	00000	9,83378,33531						

Or if the logarithm of the secant of the complement of the arc is put in the first place, the same fourth member comes about : Because the radius is the mean proportion between the sines and the secant of the complement : and therefore if from twice the logarithm of the radius the logarithm of the given arc may be taken, it will produce the logarithm of the secant of the complement ; which agrees with the arithmetical complement, if you remove the one placed at the left. Therefore if we use the logarithm of the secant, after performing the addition two is taken away, so that there appears

$$\begin{array}{r}
 10,03562,65999 \text{ Log. secant} \quad 22 \ 891768 \\
 0,03562,65999 \text{ Compl.Arith.Log.} \ 67 \ 108232
 \end{array}$$

3. If the first of the three given shall be the tangent, as above its arithmetical complement may be substituted, just as you see here from example 5 of the problem, Ch.3 on spherical triangles.

Prop.	{	Tangent	30	000000	{	9,76143,93726	Log.Tang.		
								0,23856,06274	Compl.Arith.Log.Tang.
		Radius	90	000000	10,00000,00000				
		Tangent	22	891768	9,62556,69364				
		Sine	47	000000	9,86412,74638				

Or the logarithm of the complement of the tangent may be taken, which as before is almost the same as with the arithmetic complement, so that

$$\begin{array}{r}
 \text{Tangent Complement of the arc} \ 60 \ 000000 \quad 10,23856,66274 \\
 \text{Arithmetical Complement first put in place} \quad 0,23856,66274
 \end{array}$$

Transl. Ian Bruce.

Since the mean proportional radius gives the ratio between the tangent of the arc and the complement. And therefore if the logarithm of the tangent may be taken from twice the logarithm of the radius, the logarithm of the complement of the tangent will remain.

4. But if the four proportional numbers do not express lines but rectangles, as when some angle is sought with the three sides given, or conversely ; the arithmetical complements will be able to be taken for each logarithm of the first rectangle ; if which is done, everything can be performed by addition only. We can make use of Problem 11, Ch.4.

Legs	{	PZ 30 000000	Log.Sin 9,69897,00043	Compl.Arith.	0,30120,99957
		ZS 26 <u>067185</u>	Log.Sin 9,61042,13430	Compl.Arith.	0,38957,86570
		{ Half sum		24 <u>040730</u>	9,61000,60492
		{ Half diff.		18 <u>105915</u>	<u>9,49244,54374</u>
					19,79306,01393
		Half sum, as above			9,89653,00696

Log. of sines.

Half sum of sides	48 <u>105915</u>	9,87179,49769	Compl.Arith.	0,12820,50341
First Difference	5 <u>959269</u>	9,01628,70114	Compl.Arith.	0,98371,29886
		Second Diff.		9,49244,54274
		Third Diff.		<u>9,61000,60492</u>
		Sum		20,21436,94983
		Half sum, as above		10,10718,47491

Here the arithmetical complements are taken for the first two logarithms ; because these two do account for the first rectangle from the four proportions, and twice the logarithm of the radius is left out for the subtraction of two from the sum that comes about from the addition of the five logarithms : just as Mr. *Briggs* has demonstrated, and we ourselves have shown many times in our public lectures on Astronomy.

The whole doctrine of ellipses that I have in preparation, with curved lines in place that I intended to attach, whereby I show the use of planer as well as spherical triangles, but because of the swiftness of printing and the short time available finally I was unable to put in hand. But with this present in the following (God willing).

End.

CAPUT III.

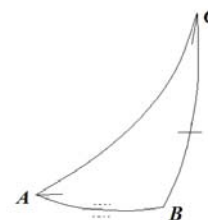
De Triangulorum Obliquangulorum diminsione.

PROBLEMA 1.

Datis Angulis duabus & Latere alteri datorum opposito, Quaeritur LATUS reliquo angulo oppositum.

In Triangulo Obliquangulo ACB quaeritur Latus AB

$$\text{Datis } \left\{ \begin{array}{l} \text{Angulis } \left\{ \begin{array}{l} \text{BAC } 103 \text{ } \underline{999307} \\ \text{ACB } 36 \text{ } \underline{131235} \end{array} \right. \text{ Compl. } 76 \text{ } \underline{000693} \\ \text{Latere } \text{CB } 42 \text{ } \underline{146646} \end{array} \right.$$



Termini Rationis.

$$\text{Proport. } \left\{ \begin{array}{l} \text{Sinus anguli lateri dato oppositi.} \\ \text{Sinus lateris dati.} \\ \text{Sinus anguli reliqui.} \\ \text{Sinus lateris quaesiti. per Cons.3.Cap.2.} \end{array} \right.$$

Illustrato per numeros.

			<i>Sinus</i>		<i>Logar.Sin.</i>
<i>Proport.</i>	Sinus Anguli	CAB	76 <u>000693</u>	97029,86512 . .	9,98690,54277
	Sinus Lateris	CB	42 <u>146646</u>	67103,04563 . .	9,82674,22302
	Sinus Anguli	ACB	36 <u>131235</u>	58963,67471 . .	<u>9,77058,45404</u>
	Sinus Lateris	AB	24 <u>065185</u>	40777,57122 . .	19,59732,67707 9,61042,13430

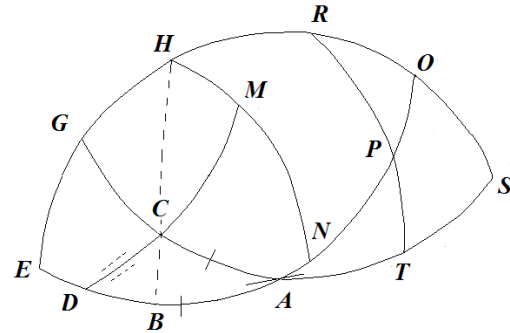
Acquisitis Lateribus duobus una cum angulis iidem oppositis, potest etiam reliquum latus hunc scilicet in modum obtineri.

		<i>Sin & Tang.</i>	<i>Log.Sin & Tang.</i>
<i>Proport.</i>	Sinus semisiis differentiae angulorum	33 <u>934036</u>	9,74681,94465
	Sinus semisiis summae angulorum	70 <u>065271</u>	9,97316,55681
	Tangens semisiis differentiae laterum	9 <u>040731</u>	<u>9,20170,63309</u>
	Tangens semisiis lateris reliqui	15 <u>00000</u>	19,17487,18990 9,42805,24525

PROBL. 2.

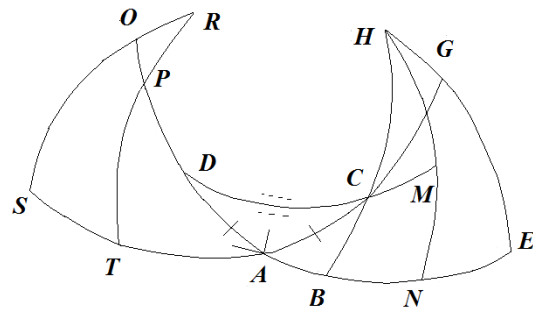
Datis Lateribus duabus & Angulo ab iisdem comprehenso, Quaeritur LATUS reliquum.

Ad hujus & subsequentium fere omnium solutionem, pro quibus duae requiruntur operationes, Tringulum Obliquangulum daturum in duo Rectangula est reducendum, ope scilicet perpendicularis ab extremitate lateris noti ductae & quoties fieri possit in latus quaesitum, aut angulo quaesito oppositae. In Triangulis itaque his Rectangulis quorum partes simili situ respondent, si singulae innotescunt, non latebunt etiam & Obliquanguli singulae. Ex



iis vero, quae Capiti tertio praefiximus, probe intellectis, singulas horum partes exquirere non erit difficile; Exclusa enim perpendiculari una cum Radio, quemadmodum *ad Consectaria 4.5.6.7 Capitis secundi*

praecipimus; Erunt Sinus vel Tangentes partis mediae alterius Trianguli & extremae reliqui, medii proportionales inter Sinus vel Tangentes partis mediae reliqui & extremae alterius Trianguli. Nos autem majoris perspicuitatis ergo, demonstrative Problematum



subsequentium resolutiones sigillatim aggredimur. Quoniam vero perpendicularis ab angulo verticali demittenda, aliquando intra, quandoque extra Triangulum cadat, prout anguli ad Basin eiusdem vel diversae fuerint affectionis, idcirco ne ulla calculo ad haereat remora, duo opposuimus schemata iisdem consignata litteris, ut Rationis Termini, nec non Illustratio Arithmetica, utrisque inserviant.

In Triangulo Obliquangulo ACD quaeritur Latus DC

		1. <i>Super.schem.</i>	2. <i>Infer.schem.</i>	
Datis	{ Lateribus	{ AD 42 14646	{ 24 065185	
		{ AC 30 000		{ 30 000
		{ Angulo DAC 36 131235		{ 103 999307

Hoc & nonnulla ut dixi subsequencia Problemata duplici indigent praxi, quarum prior inventioni segmentorum Basis, aut Angulorum verticalium subservit; Altera vero Angulorum aut Laterum quaesitorum.

Termini Rationis.

I. Pro Segmento Basis. *Per Probl.6.Cap.3.*

Proport. { *Radius.*
Tangens Hypotenusae.
Sinus Complementi anguli comprehensi.
Tangens segmenti comprehens Basis angulo contermini.
Atque inde reliquum segmentum,

II. Pro Latere quaesito. *Per Consec.5.*

Proport. { *Sinus Complementi Basis ultimo inventi.*
Sinus Complementi Hypotenusae.
Sinus Complementi reliqui segmenti Basis.
Sinus Complementi lateris quaesiti.

Illustratio per numeros,

I.

			<i>Sin. & Tang.</i>	<i>Logar. Sin. & Tang.</i>
Proport. {	Radius RS	90 <u>0000</u>	. 100000,00000	. . 10,00000,00000
	Tang. Hypotenusae TS=AC	30 <u>0000</u>	. 57735,02692	. . 9,76143,93726
	Sin.compl. OS videlicet RO	53 <u>868765</u>	. 80766,85560	. . <u>9,90723,31753</u>
	Tang.segm.Basis OP=AB	25 <u>0000</u>	. 46630,76582	9,66876,25479
	Cuius compl. est PA=BE	65 <u>0000</u>		

Possumus etiam absque quadrantali RST terminos Rationis pro segmentis Basis aut angulorum verticalium ita constituere.

Proport. { *Sinus complementi anguli comprehensi.*
Tangens complimentae Hypotenusae.
Radius.
Tangens compl. segmenti Basis angulo comprehenso & contermini,

Illustratio per numeros.

I.

Transl. Ian Bruce.

			<i>Sin. & Tang.</i>	<i>Logar. Sin. & Tang.</i>
<i>Proport.</i>	Sin. compl. EG vid. HG	53 <u>868765</u>	. 80766,86698 . .	9,90723,31753
	Tang. compl. AC vid. GC	60 <u>0000</u>	. 173205,08076 . .	10,23856,06274
	Radius HE	90 <u>0000</u>	. 100000,00000 . .	10,00000,00000
	Tang. compl. AB vid. EB	65 <u>0000</u>	. 214450,69205 . .	10,33132,74521
	Est ergo segm. AB	25 <u>0000</u>		

II.

			<i>Sinus.</i>	<i>Logar. Sin.</i>
<i>Proport.</i>	Sinus compl. AB vid. BE	65 <u>0000</u>	. 90630,77870 . .	9,95727,57115
	Sinus compl. AC vid. CG	60 <u>0000</u>	. 86602,54037 . .	9,93753,06317
	Sinus compl. DB vid. BN	72 <u>853354</u>	. 95555,33089 . .	<u>9,98025,49202</u>
				19,91778,55519
	Sinus compl. DC vid. CM	65 <u>934815</u>	. 91308,21216 . .	9,96050,98408
	Est ergo DC	24 <u>065185</u>	Latus quaesitum.	

Quod si Angulus comprehesus sit obtusus, quemadmodum videre est in Triangulo obliquangulo ACD inferioris Diagraphae, eodem operandum est modo. Retineantur reliqua data praemissa.

I.

			<i>Sin. & Tang.</i>	<i>Logar. Sin. & Tang.</i>
<i>Proport.</i>	Radius RS	90 <u>0000</u>	. 100000,00000 . .	10,00000,00000
	Tang. Hypotenusae ST=AC	30 <u>0000</u>	. 57735,02692 . .	9,76143,93726
	Sin. compl. OS vid. RO	13 <u>999307</u>	. 24191,01595 . .	<u>9,38365,41009</u>
	Tang. segment. Basis OP=AB	7 <u>950891</u>	. 12966,68969 . .	9,14509,34735
	Cuius Complem. est BE	82 <u>049109</u>		

II.

			<i>Sinus.</i>	<i>Logar.Sin.</i>
{	Proport.	Sinus compl. AB vid. BE	82 <u>049109</u> . 99038,69909 . .	9,99580,49270
		Sinus compl. AC vid. CG	60 <u>0000</u> . 86602,54037 . .	9,93753,06317
		Sinus compl. DB vid. BN	57 <u>983930</u> . 84789,94309 . .	<u>9,92834,43146</u>
				19,86587,49463
		Sinus compl. DC vid. CM	47 <u>853354</u> . 74142,97807 . .	9,87007,00193
	Est ergo DC	42 <u>146646</u> Latus quaesitum.		

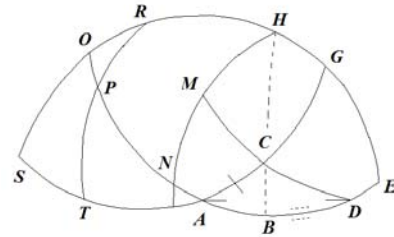
PROBL. 3.

Datis Angulis duabus & Latere alteri datorum opposito, Quaeritur LATUS ab iisdem comprehensum.

In Triangulo Obliquangulo ADC quaeritur latus AD

1.Superior.scheme. 2.Infer.scheme.

{	Datis	{	Angulis	{	CDA	36 <u>131235</u>	{	103 <u>999307</u>
			CAD		46 <u>302020</u>	36 <u>131345</u>		
			Latere AC		24 <u>065185</u>	42 <u>146646</u>		



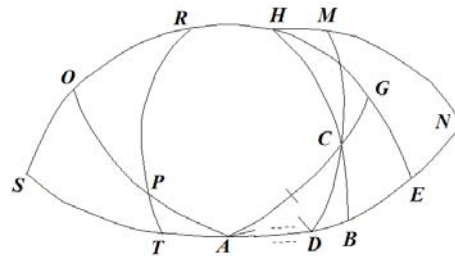
Termini Rationum.

I. Pro segmento Basis. *per Probl.6.Cap.3.*

{	Proport.	<i>Radius.</i>
		<i>Tangens Hypotenusae sive lateris dati.</i>
		<i>Sinus compleminti anguli Hypotenusae contermini.</i>
		<i>Tangens segmenti Basis Hypotenusae contermini.</i>

II. Pro reliquo Basis segmento. *Per Consect.6.*

{	<i>Tangens anguli lateri data oppositi.</i>
	<i>Sinus segmenti Basis lateri data contermini.</i>
	<i>Tangens reliqui anguli.</i>
	<i>Sinus reliqui segmenti Basis.</i>



Si perpendicularis cadit intra, segmentorum summa; Sin extra, differentia eorundem est latus quaesitum.

Illustratio per numeros.

Transl. Ian Bruce.

I.

1.

			<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
<i>Proport.</i>	Radius . . . RS	90 <u>0000</u>	. 100000,00000	. . 10,00000,00000
	Tang. Hypot. ST=AC	24 <u>065185</u>	. 44659,25969	. . 9,64991,15077
	Sin.compl. OS vid.OR	43 <u>697980</u>	. 69085,69203	. . 9,83938,81107
	Tang.segment. OP=AB	17 <u>146646</u>	. 30853,15726	. . 9,48929,96184
	Cuius Complem. est BE	82 <u>049109</u>		

II.

			<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
<i>Proport.</i>	Tang. anguli ad D vid. NM	36 <u>131235</u>	. 73004,79176	. . 9,86335,13634
	Sinus segmenti AB	17 <u>146646</u>	. 29481,83629	. . 9,46955,45148
	Tang.anguli ad D vid. GE	46 <u>302020</u>	. 104651,41007	. . <u>10,01974,50890</u>
				. . 19,48929,9603
	Sinus reliqui segmenti. DB	25 <u>000000</u>	. 42261,82617	. . 9,62594,82404
	Segmentum AB est . . .	17 <u>146646</u>		
	Summa segmentorum est AD	42 <u>146646</u>	Latus quaesitum.	

Quod si Anguli ad Basim sint diversae affectionis, ac proinde Perpendicularis cadat extra,quemadmodum in inferioris schematis triangulis, operandi modus priori consimilis erit, Retineantur itaque reliqua data.

I.

2.

			<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
<i>Proport.</i>	Radius . RS	90 <u>0000</u>	. 100000,00000	. 10,00000,00000
	Tangs. ST=AC	42 <u>146646</u>	. 90504,92470	. . 9,95667,22064
	Sin.compl. SO vid.OR	53 <u>868765</u>	. 80766,85559	. . <u>9,90723,31753</u>
	Tang.segment. OP=AB	36 <u>166052</u>	. 73097,96633	. . 9,86390,53817

II.

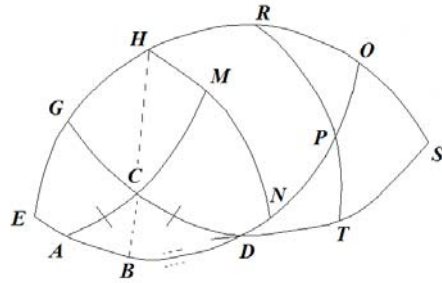
			<i>Sin. & Tang.</i>	<i>Log. Sin. & Tang.</i>
Proport.	{	Tang. anguli ad D vid. NM 76 <u>000693</u>	401098,77321	10,60321,13258
		Sinus segmenti AB 36 <u>166052</u>	59012,73216	9,46955,45148
		Tang. anguli ad A vid. GE 36 <u>131235</u>	73004,79176	9,86335,13634
				19,63429,71653
		Sinus reliqui segmenti. DB 6 <u>166052</u>	10741,02629	9,03104,58385
	Segmentum AB est . . .	36 <u>166052</u>		
	Summa segmentorum est AD	30 <u>000000</u>	Latus quaesitum.	

PROBL. 4.

Datis Lateribus duobus & Angulo altero datorum opposito, Quaeritur LATUS reliquum.

In Trangulo Obliquangulo ADC quaeritur latus AD

		1. <i>Super. scheme.</i> 2. <i>Infer. schem.</i>		
Datis	{	Lateribus { DC 30 <u>000000</u>	{	24 <u>065185</u>
		AC 24 <u>065185</u>		137 <u>853354</u>
		Angulo ADC 36 <u>131235</u>		103 <u>999307</u>



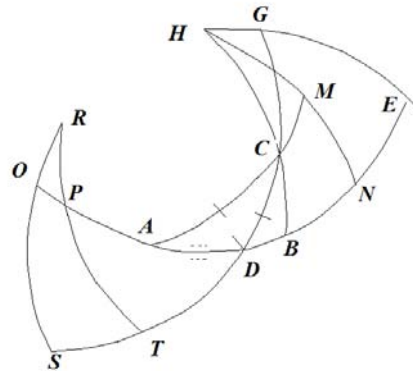
Termini Rationum.

I. Pro segmento Basis. *per Probl.6.Cap.3.*

Proport.	{	<i>Radius.</i>
		<i>Tangens Hypotenusae angulo dato conterminae.</i>
		<i>Sinus compleminti anguli dati.</i>
		<i>Tangens segmenti Basis angulo dato contermini.</i>

II. Pro Latere quaesito. *Per Consect.5.*

{	<i>Sinus compl. Lateris angulo dato contermini.</i>
	<i>Sinus compl. segm. Basis etde angulo contermini.</i>
	<i>Sinus compl. reliqui Lateris.</i>
	<i>Sinus complem. reliqui segmenti Basis.</i>



Si perpendicularis cadit intra, segmentorum summa; Sin extra, differentia eorundem est
latus quaesicum.

Illustratio Arithmetica.

1.

		I. <i>Sin. & Tang.</i>		<i>Log. Sin. & Tang.</i>		
Proport.	Radius . . . RS	90 <u>000000</u>	. 100000,00000	. .	10,00000,00000	
	Tangens. Hypot. DC=ST	30 <u>000000</u>	. 57735,06292	. .	9,76143,93726	
	Sin.compl. OS vid. RO	53 <u>868765</u>	. 80766,85559	. .	9,90723,31753	
	Tang.segment. OP=DB	25 <u>000000</u>	. 46630,76582	. .	9,66867,25479	
II. <i>Sinus.</i> <i>Logar. Sin.</i>						
Proport.	Sinus compl. DC vid. CG	60 <u>000000</u>	. 86602,54037	. .	9,93753,06317	
	Sinus compl. DB vid. BE	65 <u>000000</u>	. 90630,77870	. .	9,95727,57115	
	Sinus compl. DB vid. BN	65 <u>934815</u>	. 91308,21216	. .	9,96050,98404	
						19,91778,55519
	Sinus compl. AB vid. BN	72 <u>853354</u>	. 95555,33089	. .	9,98025,49202	
	Est ergo segmentum AB	17 <u>146646</u>				
	Segmentum vero DB	25 <u>000000</u>				
Ergo segmentorum summa AD		42 <u>146646</u>	Latus quaesitum.			

Praxis pro Latere AD Trianguli inferioris schematis.

2.

		I. <i>Sin. & Tang.</i>		<i>Log. Sin. & Tang.</i>	
Proport.	Radius . . . RS	90 <u>000000</u>	. 100000,00000	. .	10,00000,00000
	Tangens. Hypot. DC=ST	24 <u>065185</u>	. 44659,25969	. .	9,64991,15077
	Sin.compl. OS vid. RO	13 <u>999307</u>	. 24191,01595	. .	9,38365,41009
	Tang.segment. OP=DB	6 <u>166050</u>	. 10803,52716	. .	9,03356,56086

		II.	Sinus.	Logar.Sin.
Proport.	Sinus compl. DC vid. CG	65 <u>934815</u>	. 91308,21216 .	. 9,96050,98404
	Sinus compl. DB vid. BE	83 <u>833947</u>	. 99421,47796 .	. 9,99748,02351
	Sinus compl. DB vid. BN	47 <u>853354</u>	. 74142,97807 .	. <u>9,87007,00249</u>
				19,86755,02600
	Sinus compl. AB vid. BN	53 <u>833956</u>	. 80731,01834 .	. 9,90704,04196
	Est ergo segmentum AB	36 <u>166050</u>		
	Segmentum vero DB	6 <u>166050</u>		
Ergo segmentumorum				
	summa AD	30 <u>000000</u>	Latus quaesitum.	

PROBLEMA 5.

Datis Angulis duobus cum Latere ab iisdem comprehenso, Quaeritur LATUS alterutrum. In Triangulo Obliquangulo ACD quaeritur Latus DC.

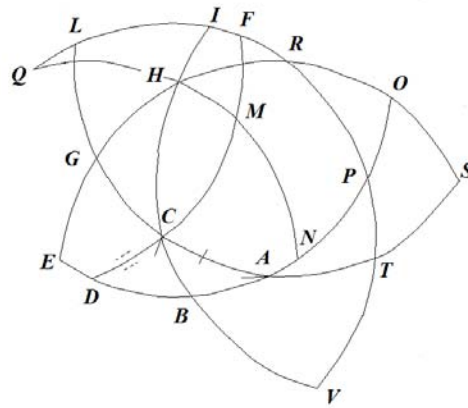
1.Super.schem. 2.Infer.schem.

Datis	Angulis	DAC	36 <u>131235</u>	36 <u>131235</u>	
		ACD	103 <u>999307</u>		46 <u>302020</u>
		Latere AC	30 <u>000</u>		42 <u>146646</u>

Termini rationum.

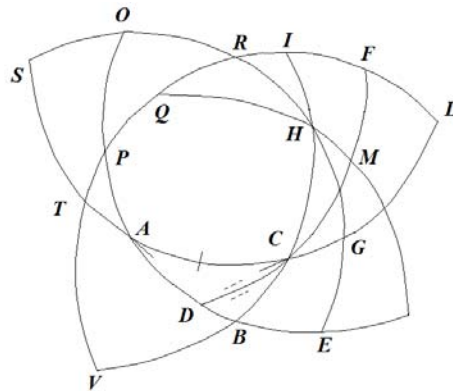
I.Pro segmento Anguli verticalis. Per Probl.16.

Proport.	R	Radius.
		Tangens anguli ad Basim, sive lateri quaesito oppositi.
		Sinus complem ; Hypotenusae; sive lateris dati.
		Tangens complem. segmenti anguli verticalis.



II. Pro Latere quaesito. *Per Consect.7.*

Proport. { *Sinus compl.reliqui segmenti anguli verticalis.*
Tangens Lateris dati.
Sinus compl.anguli verticalis ultimo inventi.
Tangens Lateris quaesiti.



Illustratio per numeros.

1.

		I. Sin.&Tang.	Log.Sin.&Tang.
Pro.	{ Radius AS 90 <u>000000</u>	. 100000,00000	. . 10,00000,00000
	{ Tangens anguli . . . CAD vid. SO 36 <u>131235</u>	. 73004,79176	. . 9,86335,13634
	{ Sin.compl. lateris . . . CA vid. AT 60 <u>000000</u>	. 86602,54037	. . 9,93753,06317
	{ Tang.segment. angulus vert.TP vid TV 32 <u>302713</u>	. 63224,00405	. . 9,80088,19951
Est ergo anguli vertical.segmentum		57 <u>697287</u>	vid. ACB.

		II.	Sinus.	Logar.Sin. & Tang.
Proport.	Sinus compl. FI vid. QI	43 <u>697980</u>	. 69085,69203	. . 9,83983,81107
	Tangens lateris AC vid. LG	30 <u>000000</u>	. 57735,02692	. . 9,76134,93726
	Sinus complem. LI vid. RI	32 <u>302713</u>	. 53939,23708	. . <u>9,72786,02462</u>
				19,48929,96188
	Tang.later.quaesiti FM=DC	24 <u>065185</u>	. 44659,25969	. 9,64991,15081

Sin extra cadit Perpendicularum, ut in inferiore schemate; Retentis reliquis datis ; Aio

2.

		I.	Sin. & Tang.	Log.Sin. & Tang.
Proport.	Radius . . . AS	90 <u>000000</u>	. 100000,00000	. . 10,00000,00000
	Tangens . . . SO	36 <u>131235</u>	. 73004,79176	. . 9,86335,13634
	Sinus . . . AT	47 <u>853350</u>	. 74142,97807	. . <u>9,38365,41009</u>
	Tangens . . . TP	28 <u>425765</u>	. 54127,92962	. . <u>9,73342,13883</u>
	Complem.est TV=IL	61 <u>574345</u>	Atque inde Angulus DCB	15 <u>272215</u> .

		II.	Sinus.	Logar.Sin.
Proport.	Sinus compl. FI vid. QI	74 <u>727785</u>	. 96368,52972	. . 9,98438,56593
	Tang.lateris AC vid. LG	42 <u>146646</u>	. 90504,92470	. . 9,95607,22854
	Sinus compl. LI vid. RI	28 <u>425765</u>	. 47601,97234	. . <u>9,67762,49419</u>
				19,63429,71473
	Tang.lateris FM=DC	24 <u>065185</u>	. 44659,25969	. . 9,64991,14880

Poterunt ctiam ex iisdem datis reliqua duo latera eodem labore simul obtineri hunc scilicet in modum.

Transl. Ian Bruce.

	1.	2.	
<i>Angulus DAC</i>	36 <u>131235</u>	}	36 <u>131235</u>
<i>Angulus ADC</i>	103 <u>999307</u>		46 <u>303030</u>

<i>Summa angulorum</i>	140 <u>120542</u>	}	82 <u>433255</u>
<i>Semissis summae</i>	70 <u>065271</u>		41 <u>216627</u>

<i>Differentia angulorum</i>	67 <u>868072</u>	}	10 <u>170785</u> ^{$\frac{1}{2}$}
<i>Semissis differentiae</i>	33 <u>934036</u>		5 <u>085392</u> ^{$\frac{1}{2}$}

1. Operatio I.

	<i>Arcus</i>	<i>Logar.Sin. & Tang.</i>
{	<i>Sinus semissis summae angulorum</i>	70 <u>065271</u> 9,97316,55681
	<i>Sinus semissis differentiae angulorum</i>	33 <u>934036</u> 9,74681,94465
	<i>Tangens semisiis lateris comprehensi</i>	15 <u>000000</u> <u>9,42805,24525</u>
		19,17487,18990
{	<i>Tang. $\frac{1}{2}$ differentiae reliquorum laterum</i>	9 <u>040731</u> 9,20170,63309

Operatio II.

	<i>Arcus</i>	<i>Logar.Sin. & Tang.</i>
{	<i>Sinus compl.semmissis summae angulorum</i>	19 <u>934729</u> 9,53268,99540
	<i>Sinus compl.semmissis differentiae angulorum</i>	56 <u>065964</u> 9,91891,10459
	<i>Tangens semisiis lateris comprehensi</i>	15 <u>000000</u> <u>9,42805,24525</u>
		19,34696,34964
{	<i>Tangens semissis summae laterum</i>	33 <u>105916</u> 9,81427,35444
	<i>Semissis summae laterum est</i>	33 <u>105916</u>
	<i>Semissis differentiae laterum est</i>	9 <u>105916</u>

Summa summae & differentiae laterum est 42 146647 Latus AD quaesitum.

Differentia summae & differentiae laterum est 24 065185 Latus DC quaesitum.

Retentis similiter datis Trianguli inferioris schematis. Aio

2. Operatio I.

	<i>Arcus</i>	<i>Logar.Sin.&Tang.</i>
Sinus semissis summae angulorum	41 <u>217728</u>	9,81882,46407
Sinus semissis differentiae angulorum	5 <u>085393</u>	8,94763,13168
Tangens semisiis lateris comprehensi	21 <u>073323</u>	9,58583,62561
		18,53346,75729
Tang. $\frac{1}{2}$ differentiae reliquorum laterum	2 <u>967409</u>	8,71464,29322

Operatio II.

	<i>Arcus</i>	<i>Logar.Sin.&Tang.</i>
Sinus compl.semmissis summae angulorum	48 <u>783372</u>	9,87634,70195
Sinus compl.semmissis differentiae angulorum	84 <u>904607</u>	9,99828,71096
Tangens semisiis lateris comprehensi	21 <u>073323</u>	9,58583,62562
		19,58412,33658
Tangens semissis summae laterum	27 <u>032592</u>	9,70777,63463
Semissis summae laterum est	27 <u>032592</u>	
Semissis differentiae laterum est	2 <u>967409</u>	

Summa summae & differentiae laterum est 30 000001 Latus AD quaesitum.

Differentia summae & differentiae laterum est 24 065183 Latus DC quaesitum.

PROBL. 6.

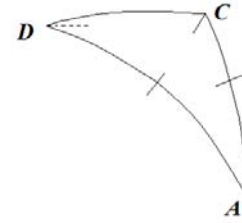
Datis Lateribus duobus & Angulo alteri datorum opposito, Quaeritur ANGULUS reliquo lateri oppositus.

In Triangulo Obliquangulo ACD quaeritur Angulus ADC

$$\text{Datis } \left\{ \begin{array}{l} \text{Lateribus } \left\{ \begin{array}{l} \text{AC } 30 \text{ } \underline{000000} \\ \text{AD } 42 \text{ } \underline{144266} \end{array} \right. \\ \text{Angulo } \text{ACD } 103 \text{ } \underline{999307} \end{array} \right\}$$

Termini Rationis.

- Sinus lateris angulo dato oppositi.*
- Sinus anguli dati.*
- Sinus lateris reliqui.*
- Sinus anguli quaesiti. per Cons.3.Cap.2.*



Illustratio Arithmetica.

			<i>Sinus</i>	<i>Logar.Sin.</i>
<i>Proport.</i>	{	Sinus Lateris	AD 42 <u>146646</u>	67103,04565 . . . 9,82674,22303
		Sinus Anguli	ACD 76 <u>000693</u>	97029,86513 . . . 9,98690,57277
		Sinus Lateris	AC 30 <u>000000</u>	50000,00000 . . . <u>9,69897,00043</u>
		Sinus Anguli	ADC 46 302021	72299,15012 . . . 9,85913,35017

Angulis acquisitis duobus una cum lateribus iisdem oppositis, possumus etiam reliquum angulum eundem in modum deprehendere quo reliquum latus obtinuimus ad Probl.1. huius Capituli.

PROBL. 7.

Datis duobus Angulis & Latere ab iisdem comprehenso, Quaeritur ANGULUS reliquus.

In sequente Triangulo Obliquangulo ACD quaeritur Angulus ADC.

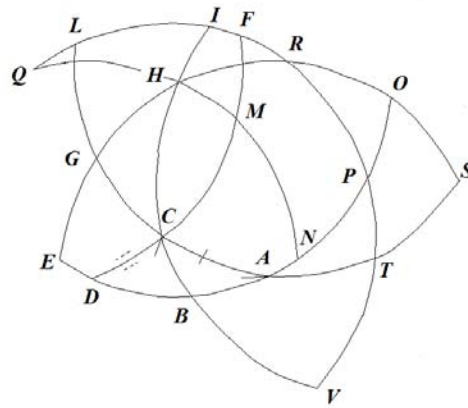
1.*Super.scheme.* 2.*Infer.schem.*

Datis {	{	Angulis {	DAC 103 <u>999307</u>	{	36 <u>131235</u>
			DAC 36 <u>131235</u>		46 <u>302020</u>
		Latere	AC 30 <u>000</u>		42 <u>146646</u>

Termini rationum.

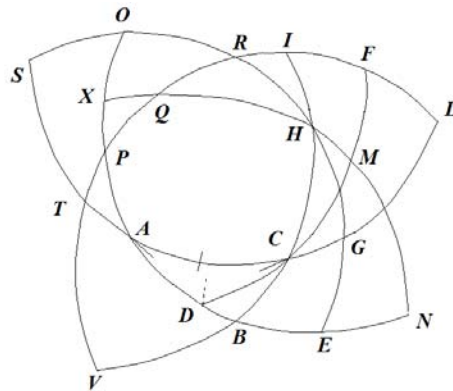
I.Pro segmento Anguli verticalis. *Per Probl.6.*

- Radius.*
- Tangens anguli ad Basim.*
- Sinus complem lateris dati.*
- Tangens Complem. segmenti anguli verticalis.*



II. Pro Angulo quaesito. *Per Consect.4.*

Proport. { *Sinus segmenti anguli verticalis ultimo inventi.*
Sinus reliqui anguli verticalis.
Sinus complementi anguli ad Basim.
Sinus complementi anguli quaesiti.



Illustratio Arithmetico.

1.

		I. Sin.&Tang.	Log.Sin.&Tang.
Proport.	Radius AS	90 <u>000000</u>	. 100000,00000 . . 10,00000,00000
	Tangens anguli . . . CAD vid. SO	36 <u>131235</u>	. 73004,79176 . . 9,86335,13634
	Sin.compl. lateris . . . CA vid. AT	60 <u>000000</u>	. 86602,54037 . . 9,93753,06317
	Tang.segment. angulus vert.TP vid TV	32 <u>302713</u>	. 63224,00405 . . 19,80088,19951
	Est ergo anguli vertical.segmentum	57 <u>697287</u> vid. ACB.	

	II.	Sin.	Logar.Sin.
Proport.	Sinus IL 57 <u>697287</u>	. 84523,65276 . .	9,92697,82551
	Tangens anguli . . . IF 46 <u>302022</u>	. 72299,15012 . .	9,85913,31911
	Sinus compl. EG vid. HG 53 <u>808765</u>	. 80766,85559 . .	<u>9,90723,31753</u>
			19,76636,63664
	Sinus compl. MN vid HM 43 <u>697979</u>	. 69085,69203 . .	9,83938,81113
Est ergo angulus ADC	40 <u>302021</u>	angulus quaesitus.	

Eadem requiritur praxis in Triangulo inferioris schematis.

2.

	I.	Sin. & Tang.	Log.Sin. & Tang.
Proport.	Radius AS 90 <u>000000</u>	. 100000,00000 . .	10,00000,00000
	Tangens SO 46 <u>30201</u>	. 104651,41007 . .	10,01974,50864
	Sin.compl. CA vid. AT 47 <u>853354</u>	. 74142,97807 . .	9,87008,00249
	Tangens TP 37 <u>808484</u>	. 77591,67097 . .	1 9,88981,51113
	Complement est TV=IL 52 <u>191526</u>		

	II.	Sin.	Logar.Sin.
Proport.	Sinus IL 52 <u>191516</u>	. 79006,43467 . .	9,89766,24083
	Sinus IF 16 <u>060281</u>	. 27664,85382 . .	9,44192,83946
	Sinus compl. EG vid. HG 43 <u>697979</u>	. 69085,69203 . .	<u>9,83938,81107</u>
			19,28131,65053
	Sinus compl. MN vid HM 13 <u>999307</u>	. 24191,01595 . .	9,38365,40970
Quo addito Quadranti HX	90 <u>000000</u>		
Conflatur arcus MX	103 <u>999307</u>	mensura Anguli ADC quaesiti.	

PROBL. 8.

*Datis Lateribus duobus & Angulo alteri datorum opposito, Quaeritur ANGULUS ab
 iisdem comprehensus.*

In Triangulo Obliquangulo ADC quaeritur Angulus ACD.

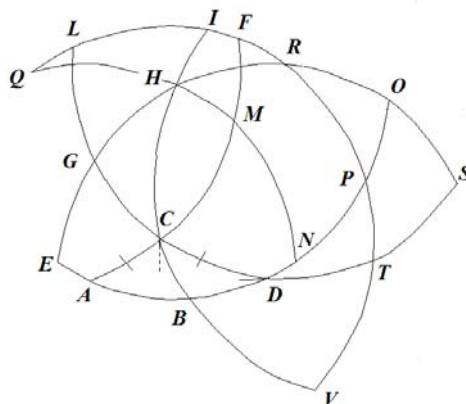
1. *Super.scheme.* 2. *Infer.schem.*

$$\text{Datis} \left\{ \begin{array}{l} \text{Lateribus} \left\{ \begin{array}{l} \text{AC} \quad 24 \quad \underline{065185} \\ \text{CD} \quad 30 \quad \underline{000} \end{array} \right. \\ \text{Angulo} \quad \text{ADC} \quad 36 \quad \underline{131235} \end{array} \right\} \left\{ \begin{array}{l} 30 \quad \underline{000} \\ 42 \quad \underline{146646} \\ 46 \quad \underline{302020} \end{array} \right.$$

Termini rationum.

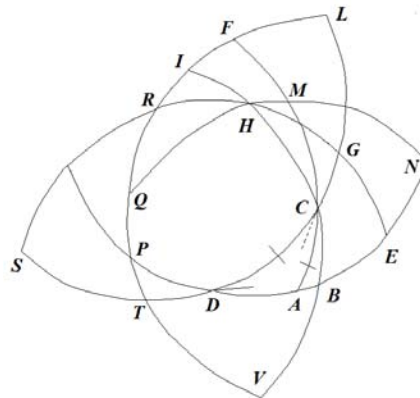
I. Pro segmento Anguli verticalis. *Per Probl.6.*

$$\text{Proport.} \left\{ \begin{array}{l} \text{Radius.} \\ \text{Tangens anguli dati.} \\ \text{Sinus complem. lateris dato contermini.} \\ \text{Tangens Complem. segmenti anguli verticalis.} \end{array} \right.$$



II. Pro Angulo quaesito. *Per Consect.7.*

$$\text{Proport.} \left\{ \begin{array}{l} \text{Tangens lateris angulo dato oppositi.} \\ \text{Tangens reliqui lateris.} \\ \text{Sinus Complementi segmenti anguli verticalis ultimo inventi.} \\ \text{Sinus Compl. reliqui anguli verticalis.} \end{array} \right.$$



Si Perpendicularis cadit intra, angulorum verticalium summa; Sin extra, differentia eorundem est angulus quaesitus.

Illustratio Arithmetica.

		I.	<i>Sin. & Tang.</i>	<i>Log. Sin. & Tang.</i>
Proport.	Radius DS	90 <u>000000</u>	. 100000,00000	. . 10,00000,00000
	Tangens SO	36 <u>131235</u>	. 73004,79176	. . 9,86335,13634
	Sinus compl. CD vid. DT	60 <u>000000</u>	. 86602,54037	. . 9,93753,06317
	Tang. TP	32 <u>302713</u>	. 63224,00405	. . 9,80088,19951
		Complement est TV=LI	57 <u>697287</u>	mensura segmenti Anguli DCB.

		II.	<i>Sinus.</i>	<i>Logar. Sin. & Tang.</i>
Proport.	Tang. lateris AC=FM	24 <u>065185</u>	. 44659,25969	. . 9,64991,15077
	Tangens lateris DC= LG	30 <u>000000</u>	. 57735,02692	. . 9,76134,93726
	Sinus compl. LI vid. IR	32 <u>302713</u>	. 53939,23708	. . <u>9,72786,02462</u>
	Sinus compl. IF vid. IQ	43 <u>697980</u>	. 69085,69203	. . 19,48929,96188
Est ergo arcus IF		46 <u>302021</u>		
Et arcus primo inventus LI		57 <u>697287</u>		
Summa angulorum verticalium		103 <u>999308</u>	Angulus quaesitus ACD.	

In Triangulo inferioris schematis.

2.

		I. <i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
Proport.	Radius DS 90 <u>000000</u>	. 100000,00000 . .	10,00000,00000
	Tangens SO 46 <u>302021</u>	. 104651,41007 . .	10,01974,50864
	Sin.compl. CF vid. DT 47 <u>853354</u>	. 74142,97807 . .	9,87008,00249
	Tangens TP 37 <u>808484</u>	. 77591,67097 . .	9,88981,51113
	Complement est TV=LI 52 <u>191526</u>		

		II.	<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
Proport.	Tangens lateris AC=FM 30 <u>000000</u>	. 57735,02682 . .	9,76143,93726	
	Tangens lateris DC=LG 42 <u>146646</u>	. 90504,92470 . .	9,95667,22064	
	Sinus complem. LI vid.LR 37 <u>808484</u>	. 61302,45434 . .	<u>9,78747,75192</u>	
	Sinus complem. IF vid. IQ 73 <u>939719</u>	. 96097,10775 . .	19,74414,97256	
	Ergo est arcus IF 16 <u>020681</u>		9,98271,03530	
At inventus Arcus IL 52 <u>191516</u>				
Differentia FL 36 <u>131235</u>	mensura Anguli DCA.			

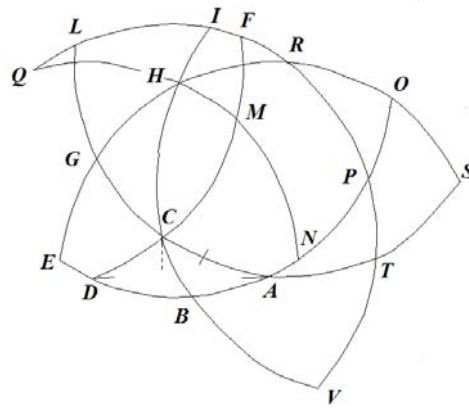
PROBL. 9

Datis Angulis duobus & Latere alteri datorum opposito, Quaeritur ANGULUS reliquus.

In Triangulo Obliquangulo ADC quaeritur Angulus ACD

1.*Super.scheme.* 2.*Infer.schem.*

Datis	{	Angulis	{ DAC 36 <u>131235</u> }	{	46 <u>302020</u>
			{ ADC 46 <u>302021</u> }		103 <u>999307</u>
			{ Latere AC 30 <u>000000</u> }		42 <u>146646</u>

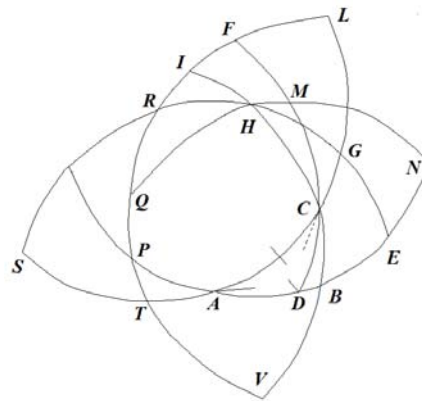


Termini Rationum.

I. Pro segmento angulo verticalis. *Per Probl.6.*

$$\text{Proport.} \left\{ \begin{array}{l} \text{Radius.} \\ \text{Tangens anguli lateri dato contermini.} \\ \text{Sinus complementi lateris angulo dato contermini.} \\ \text{Tangens segmenti anguli verticalis.} \end{array} \right.$$

II. Pro Angulo verticali quaesico. *Per Consect. 4.*



Proport. { *Sinus complementi anguli lateri dato contermini.*
Sinus complementi reliqui anguli dati.
Sinus segmenti anguli verticalis ultimo inventi.
Sinus reliqui anguli verticalis.

Si perpendicularis cadit intra, angulorum verticalium summa; Sin extra, differentia est Angulus quaesitus.

Illustratio Arithmetica.

		I.	<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
Proport. {	Radius AS	90 <u>000000</u>	. 100000,00000 . .	10,00000,00000
	Tangens SO	36 <u>131235</u>	. 73004,79176 . .	9,86335,13634
	Sinus compl. AC vid. AT	60 <u>000000</u>	. 86602,54037 . .	9,93753,06317
	Tang. TP	32 <u>302713</u>	. 63224,00405 . .	9,80088,19951
Complement est TV=LI 57 <u>697287</u> mensura segmenti Anguli ACB.				

		II.	<i>Sin.</i>	<i>Logar.Sin.</i>
Proport. {	Sinus compl. EG vid. HG	53 <u>808765</u>	. 80766,85559 . .	9,90723,31753
	Sinus compl. NM vid. HM	43 <u>697979</u>	. 69085,69203 . .	9,83938,81107
	Sinus angulus verticalis IL	57 <u>697287</u>	. 84523,65276 . .	<u>9,92697,82551</u>
				19,76636,63658
	Sinus reliqui anguli vert. IF	46 <u>302020</u>	. 72299,15012 . .	9,85913,31905
Arcus IL est		57 <u>697287</u>		
Arcuum summa est		103 <u>999307</u>	mensura anguli quaesiti ACD.	

Praxis pro Angulo ACD Trianguli inferioris schematis.

2.

		I.	<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
Proport. {	Radius AS	90 <u>000000</u>	. 100000,00000 . .	10,00000,00000
	Tangens SO	46 <u>302021</u>	. 104651,41007 . .	10,01974,50864
	Sin.compl. AC vid. AT	47 <u>853354</u>	. 74142,97807 . .	9,87008,00249
	Tangens TP	37 <u>808484</u>	. 77591,67097 . .	9,88981,51113
Complement est TV=LI 52 <u>191516</u>				

Transl. Ian Bruce.

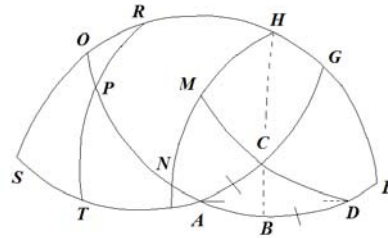
		II.	Sinus.	Logar.Sin.
Proport.	{	Sinus compl. EG vid. HG 43 <u>697979</u>	. 69085,69203 . .	9,83938,81107
		Sinus compl. NM vid. HM 13 <u>99907</u>	. 24191,01595 . .	9,38365,40970
		Sinus IL 52 <u>191516</u>	. 79006,43467 . .	<u>9,89766,24083</u>
				19,28131,65053
	{	Sinus reliqui anguli vert. IF 16 <u>060281</u>	. 27664,85482 . .	9,44192,83946
Est autem arcus		IL 52 <u>191516</u>		
Differentia est arcus		LF 36 <u>131245</u>	mensura Anguli quaesiti ACD.	

PROBLEMA 10.

Datis Lateribus duobus cum Angulo ab iisdem comprehenso, Quaeritur ANGULUS alteruter.

In Triangulo Obliquangulo ACD quaeritur Angulus ADC.

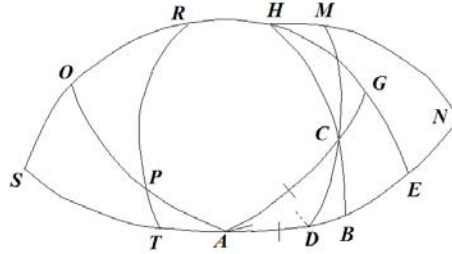
		1.Superior.scheme.	2.Infer.schem.
Datis	{	Lateribus { AC 24 <u>065185</u>	{ 42 <u>146646</u>
		AD 42 <u>146646</u>	{ 30 <u>000000</u>
		Angulo CAD 46 <u>302020</u>	{ 36 <u>131235</u>



Termini Rationum.

I. Pro segmento Basis. per *Probl.6.Cap.3.*

Proport.	{	<i>Radius.</i>
		<i>Tangens Hypotenusae.</i>
		<i>Sinus compleminti anguli dati.</i>
		<i>Tangens segmenti Basis angulo dato contermini.</i>



II. Pro angulo quaesito. *Per Consect.6.*

- Sinus segmenti Basis angulo quaesito contermini.*
- Sinus segm. Basis angulo comprehenso contermini.*
- Tangens anguli comprehensi.*
- Tangens anguli quaesiti.*

Illustratio per numeros.

I.

1.

		<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
<i>Proport.</i>	Radius . . . RS	90 <u>0000</u>	100000,00000 . . . 10,00000,00000
	Tang. Hypot. AC=ST	24 <u>065185</u>	. 44659,25969 . . . 9,64991,15077
	Sin.compl. SO vid.RO	43 <u>697980</u>	. 69085,69203 . . . 9,83938,81107
	Tang.segment. OP=AB	17 <u>146646</u>	. 30853,15726 . . . 9,48929,96184
Ergo reliquum segment. DB		25 <u>000000</u>	

II.

		<i>Sin. & Tang.</i>	<i>Log.Sin. & Tang.</i>
<i>Proport.</i>	Sinus segmenti Basis DB	25 <u>000000</u>	. 42261,82617 . . . 9,62594,82594
	Sinus reliqui segmenti AB	17 <u>146646</u>	. 29481,83629 . . . 9,46955,45148
	Tang.anguli EG	46 <u>302020</u>	. 104651,41007 . . . <u>10,01974,50890</u>
	Tangens anguli quaesiti.NM	36 <u>131235</u>	. 73004,79176 . . . 9,86335,13418

Praxis pro Angulo ADC Triangulo inferioris schematis.

2.

		I. <i>Sin. & Tang.</i>		<i>Log.Sin. & Tang.</i>	
{	<i>Proport.</i> Radius .	RS	90 <u>0000</u>	. 100000,00000	. .10,00000,00000
	Tangs.	AC=ST	42 <u>146646</u>	. 90504,92470	. . 9,95667,22064
	Sin.compl.	SO vid.RO	53 <u>868765</u>	. 80766,85559	. . <u>9,90723,31753</u>
	Tang.segm.	OP=AB	36 <u>166052</u>	. 73097,96633	. . 9 ,86390,53817

II.

		<i>Sin. & Tang.</i>		<i>Log.Sin. & Tang.</i>	
{	<i>Proport.</i> Sinus segmenti	Basis DB	6 <u>166050</u>	. 10741,02629	. . 9,03104,58364
	Sinus segmenti	AB	36 <u>166052</u>	. 59012,73216	. . 9,77094,58190
	Tang.anguli dati	EG	36 <u>131235</u>	. 73004,79176	. . <u>9,86335,13634</u>
	Tang.anguli	NM	76 <u>000693</u>	. 401098,77321	. . 10,60325,13460

Compl. ad semicirculum est 103 99907 Angulus quaesitus ADC.

Non minore molestia reliqui duo Anguli simul investigari poterunt, eodem prorsus modo quo ad Problema quantum reliqua latera venati sumus.

1.			2.	
Latus AC	24 <u>065185</u>		Latus AC	42 <u>146646</u>
Latus <u>AD</u>	42 <u>146646</u>		Latus <u>AD</u>	30 <u>000000</u>
Summa Laterum	66 <u>211813</u>		Summa Laterum	72 <u>146646</u>
Semmissis summae	33 <u>1059155</u>		Semmissis summae	36 <u>073323</u>
Differentia Laterum	18 <u>081461</u>		Differentia Laterum	12 <u>146646</u>
Semmissis differentiae	9 <u>0407305</u>		Semmissis differentiae	6 <u>073325</u>

1. Operatio I.

Logar. Sin. & Tang.

<i>Proport.</i> {	Sinus semissis summae laterum	33 <u>1059155</u>	. 9,73734,24996
	Sinus semissis differentiae laterum	9 <u>0407305</u>	. 9,19627,71847
	Tangens compl.semmissis anguli comprehensi	66 <u>848990</u>	. <u>10,36897,36696</u>
			19,56525,08543 sum.
	Tang. $\frac{1}{2}$ differentiae reliquorum angulorum	33 <u>934033</u>	. 9,82790,83547 differ.

Operatio II.

Logar. Sin. & Tang.

<i>Proport.</i> {	Sinus compl.semmissis summae laterum	56 <u>894085</u>	. 9,92306,89855
	Sinus compl.semmissis differentiae laterum	80 <u>9592695</u>	. 9,99457,09124
	Tangens compl. simissis anguli comprehensi	66 <u>848990</u>	. <u>10,36897,36696</u>
			20,36354,45820 sum.
	Tangens simissis summae angulorum	70 <u>065270</u>	. 10,44047,55965 differ.
	Semissis differentiae reliquorum angulorum	33 <u>934033</u>	
	Summa	103 <u>999303</u>	Angulus ACD
	Differentia	36 <u>131237</u>	Angulus ADC.

2. Operatio I.

Logar. Sin. & Tang.

<i>Proport.</i> {	Sinus semissis summae laterum	36 <u>073323</u>	. 9,76998,26167
	Sinus semissis differentiae laterum	6 <u>073323</u>	. 9,02449,00297
	Tangens compl.semmissis anguli comprehensi	71 <u>934825</u>	. <u>10,48653,42647</u>
			19,51102,42944
	Tang. $\frac{1}{2}$ differentiae reliquorum angulorum	28 <u>848643</u>	. 9,74104,16777

Operatio II.

Logar. Sin. & Tang.

{	Proport.	Sinus compl.semmissis summae laterum	53 <u>926677</u>	.	9,90755,33018
		Sinus compl.semmissis differentiae laterum	83 <u>926677</u>	.	9,99755,55732
		Tangens compl. simissis anguli comprehensi	71 <u>9343825</u>	.	<u>10,78653,42647</u>
					20,78408,98379
		Tangens simissis summae reliquorum angulorum	75 <u>150664</u>	.	10,57653,65361
		Semmissis differentiae <u>angulorum</u>	28 <u>848643</u>		
		Summa	103 <u>999307</u>	Angulus ADC	
		Differentia	46 <u>302021</u>	Angulus ACD.	

PROBL. 11

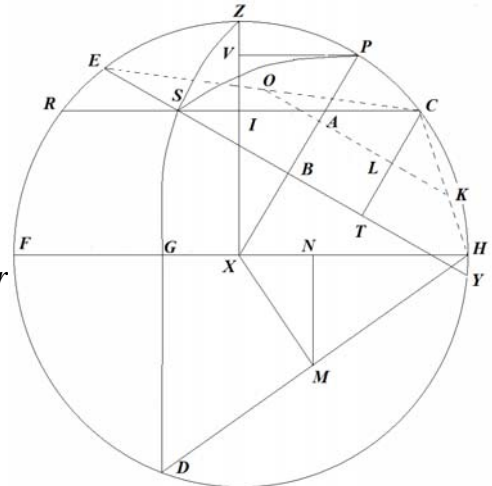
Datis singulis Lateribus, Quaeritur ANGULUS quilibet.

In Triangulo Obliquangulo ZPS quaeritur Angulus PZS

Datis Lateribus { PS 42 146646
 PZ 30 000000
 ZS 26 065185

Termini Rationis.

Proport. { *Rectangulum e Sinibus laterum angulum
 quaesitum comprehenduntium.
 Quadratum Radii.
 Rectangulum e $\frac{1}{2}$ Sinibus summae & $\frac{1}{2}$ differ
 Basis & differentiae laterum eorundem.
 Quadratum Sinus $\frac{1}{2}$ anguli quaesiti.*



Illustratio Arithmetica.

1. Sumantur Sinus datorum laterum angulum quaesitum comprehendentium; eorumque Sinuum Logarithmi.
2. Sumantur quadratum Radii; eiusque Logarithmus.
3. Conferatur Basis, cum differentia laterum sive crurum, & sumatur semissis summae & semissis differentiae, earumque Sinus; & Sinuum Logarithmi.
4. Si factus primi, reliquorum factum diviserit ; quoti latus est Sinus semissis anguli quaesiti; vel si summa Logarithmorum primi, e reliquis auferatur; differentiae semissis est Logarithmus Sinus semissis anguli quaesiti.

$$\text{Sint datorum Laterum } \left\{ \begin{array}{l} \text{PZ 30 } \underline{000000} \\ \text{ZS 26 } \underline{065185} \end{array} \right. \text{ Sinus } \left\{ \begin{array}{l} 50000,00000 \\ 40777,57121 \end{array} \right. \Big| \text{Logar. } \left\{ \begin{array}{l} 9,69897,00043 \\ 9,61042,13420 \end{array} \right.$$

$$\begin{array}{l} 1. \text{ Rectangulum Sin.Lat. } 20388785605000000000 \Big| \text{Logar.summa . } 19,30939,13473 \text{ A} \\ 2. \text{ Quadratum Radii } 10000000000000000000 \Big| \text{Log.Quadr.Radii } 20,00000,00000 \text{ B} \end{array}$$

$$\begin{array}{l} \text{Basis PS } 42 \underline{146646} \\ \text{Differentia Crurum . PC } 5 \underline{934815} \\ \text{Summa Basis \& differ. EPC } 48 \underline{081461} \\ \text{Diff.Basis \& diff, Laterum CY } 36 \underline{211831} \end{array}$$

$$\left\{ \begin{array}{l} \text{Semissis summae . . CO } 24 \underline{040730} \\ \text{Semissis differentiae . CK } 18 \underline{105915} \end{array} \right. \text{ Sinus } \left\{ \begin{array}{l} 40738,59542 \\ 31077,45544 \end{array} \right. \Big| \text{Log. } \left\{ \begin{array}{l} 9,61000,60492 \\ 9,49244,54374 \end{array} \right.$$

$$\begin{array}{l} 3. \text{ Rectangulum } \frac{1}{2} \text{ Sum. \& } \frac{1}{2} \text{ diff. } 1266051883853238084800000 \text{ \& c. } \Big| \text{summa } 19,10245,14866 \text{ C} \\ 4. \text{ Quotus est } 6209502320783665173 \text{ \& c. } \Big| \text{diff. } 19,79306,01393 \text{ D} \\ \text{Quoti Latus est } 7880069944 \text{ Sinus } 51 \underline{999650} \quad \frac{1}{2} \text{ differ. } 9,89653,00696 \end{array}$$

Aio itaque

		<i>Logar.Sinum.</i>	
<i>Proport.</i> {	<i>Rectangulum Sinuum Crurum</i>	20388785605000 & c.	19,30939,13473 A
	<i>Quadratum Radii.</i>	10000000000000 & c.	20,00000,00000 B
	<i>Rectangulum $\frac{1}{2}$ sum. \& $\frac{1}{2}$ differ.</i>	12660518838532380 & c.	<u>19,10245,14866 C</u>
	<i>Quadratum Sinus $\frac{1}{2}$ anguli quaesiti</i>	620950232078366 & c. \& diff.	19,79306,01393 D.

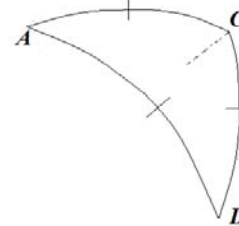
Transl. Ian Bruce.

Quadrati latus est 788006944. Sinus 51 999650; $\frac{1}{2}$ diff. 9,89653,00696
 Huius arcus duplum videlicet 103 999300 est Angulum quaesitus PZS.

Quale ac quanti pretii sit compendium istud Logarithmicum, malim ut ex proprio sensu quam meis verbis aestimet Trigonomet.

Aliter 2.

Subtrahantur logarithmi Crurum anguli quaesiti, e Logarithmi Quadrati Radii & semissis summae trium laterum & semissis differentiae Basis & summae Crurum : Reliquus erit Logarithmus Quadrati Sinus complimenti semissis anguli quaesiti ad Quadrantem.



Praecepti Illustratio Arithmetica.

In Triangulo Obliquangulo ADC quaeritur Angulus ACD

Datis	{	Basi	{	AD 42 <u>146646</u>	Logar.Sin.	
		Cruribus	{	AC 30 <u>000000</u>	9,69897,00043	
			}	CD 24 <u>065185</u>	9,61042,13430	
				19,30939,13473 B	Summa Logar.	
					Radii dupl.Logar.	
Summa Laterum	96	<u>211831</u>	.	20,00000,00000 E	}	
Semissis summae	48	<u>105915</u>	.	9,87179,49769		
Diff.Crurum & Basis	11	<u>918539</u>				
$\frac{1}{2}$ Diff.Crurum & Basis	5	<u>959269</u>	.	9,01628,70114		
				38,88808,19883 F	Summa.	
				19,30939,13473 B		
				19,57869,06410	Differentia B & F.	
Logarithmus Sinus Gra.38	<u>000344</u>	.		9,78934,53205	Semissis differentiae.	
Huius arcus Complem.	51	<u>999656</u>				
Complem.arcus duplus	103	<u>999312</u>				

est Angulus quaesitus ACD.

Transl. Ian Bruce.

Aliter 3.

De dimidio collectorum laterum, latera Trianguli sigillatim subducantur, summaque Logarithmorum Sinuum semissis summae laterum & differentiae lateris angulum quaesitum subtendentis, auferatur e summa Logarithmorum Sinuum reliquarum differentiarum & duplicato Radii Logarithmo : semissis reliqui erit Logarithmus Tangentis semissis anguli quasiti.

Illustratio Praecepti Arithmetica.

Retineantur data cum quaesito superioris schematis.

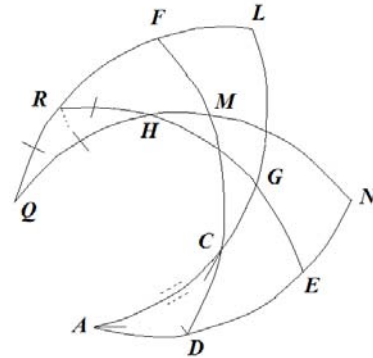
Latera	{	AD . 42 <u>146646</u>		
		AC . 30 <u>000000</u>		
		CD . 26 <u>065185</u>		
<hr style="width: 100%;"/>				
Laterum summa		96 <u>211831</u>	<i>Logar.Sin.</i>	
Semissis summae		48 <u>105915</u>	. 9,87179,49769	} 18,88808,19883 summa.
Diff.1Basis AD & $\frac{1}{2}$ summae		5 <u>959269</u>	. 9,01628,70114	
Diff.2lateris AC & $\frac{1}{2}$ summae		18 <u>105915</u>	. 9,49244,54374	} 19,10245,14866
Diff.3lateris CD & $\frac{1}{2}$ summae		24 <u>040730</u>	. 9,61000,60492	
		Radii duplicatus <u>Logar.</u>	20,00000,00000	
				} summa.
			39,10245,14866	
			<u>18,88808,19883</u>	
		Logarithmorum differentia	20,21436,94983	
		Semissis differentiae	10,10718,47491	
est Logarithmus Tangentis 51 <u>999653</u> .				
Huius arcum duplum 103 <u>999306</u> est angulus quaesitus ACD.				

PROBL. 12

Datis Angulis, Quaritur LATUS quodlibet.

In Triangulo Obliquangulo ADC quaeritur Latus AC

$$\text{Datis Angulis } \begin{cases} \text{ADC} . 103 \underline{999307} \\ \text{DAC} . 36 \underline{131235} \\ \text{ACD} . 46 \underline{302020} \end{cases}$$



Problema istud conversum est undecimi, eundemque in modum solvitur, si modo anguli in latera permutentur. Duo enim Anguli minores semper aequantur duobus lateribus Trianguli comprehensi ab arcibus maiorum circularum a polis eorundem ductorum. Tertius autem angulus esse potest maior quadrante, & idcirco complementum eius ad semicirculum sumi potest commodissime pro tertio latere.

Angulus autem ubi inventus fuerit, erit unus e tribus quaesitis lateribus ; Inventis autem ex iisdem uno, reliqua per proportionis regulam *juxta Probl. 1.* invenientur.

Ut in triangulo ACD, horum arcuum poli sunt HRQ, & illius latera aequantur huius Angulis; AD aequalis angulo ad H vel arcui EN. DC angulo Q vel arcui MF. Et AC angulo HRF vel arcui GL. Idcirco si dati sint anguli ADC dantur latera QR, QH, RH. Angulus autem CDE vel MN aequatur lateri QH. Resolvatur itaque triangulum QRH, secundum praeceptum Problematis 11.

Logar.Sin.

$$\text{Crura } \begin{cases} \text{RQ } 46 \underline{302020} & 9,85913,31911 \\ \text{RH } 36 \underline{131235} & 9,77058,45404 \end{cases}$$

Differentia Crurum . . . $10 \underline{170785} + 19,62971,77315$ Summa Logar.

Basis $\underline{\text{QH } 76 \underline{000693}}$

Summa Differentiae Crurum & Basis $86 \underline{171478}$

Diff.Basis & differentiae Crurum $\underline{65 \underline{829908}}$

Semissis summae diff. Crurum & Basis $43 \underline{085739}$ $9,83447,92046$

Semissis differ. Basis & differ. Crurum $32 \underline{914954}$ $\underline{9,73511,44888}$

19,56959,36934 sum.Logar.

Logarithmus duplicatus Radii 20,00000,00000
 + 19,62971,77315

19,93987,59619 Diff.Logar.

Logarithmus Sinus $\frac{1}{2}$ Anguli HRQ 68 926676 9,96993,79809 Demiss diff.

Huius anguli duplum est 137 853352

Et anguli duplicati compl.ad semicirculii 42 146648 est Angulus GRL, cuius
 mensura est GL=AC Lateri quaesito.

Aliter 2.

Sunto data & quaesita ut supra. Aio

Proport. $\left\{ \begin{array}{l} \text{Rectangulum e Sinibus crurum anguli quaesiti.} \\ \text{Quadratum Radii.} \\ \text{Rectangulum e } \frac{1}{2} \text{ Sinibus summae laterum \& differentiae Basis \& semisummae.} \\ \text{Quadratum Sinus complementi semissis anguli quaesiti.} \end{array} \right.$

Logar.Sin.

Crura $\left\{ \begin{array}{l} \text{RQ 46 } \underline{302020} \quad 9,85913,31911 \\ \text{RH 36 } \underline{131235} \quad 9,77058,45404 \end{array} \right.$

Logar.Sin.

Crura $\left\{ \begin{array}{l} \text{RQ 46 } \underline{302020} \quad 9,85913,31911 \\ \text{RH 36 } \underline{131235} \quad 9,77058,45404 \end{array} \right.$

Basis QH 76 000693 + 19,62971,77315 Summa Logar.

Summa Crurum & Basis 158 433948

Semissis summae crurum & Basis 79 216974 9,99226,30294

Differ. Basis & semissis summa 3 226281 8,74900,27530

8,74126,57824 sum.Logar.

Duplicatus Radii Logarithmus 20,00000,00000
 + 19,62871,77315

Logarithmus Sinus anguli 21 073312 9,55577,40254

Complementum huius anguli 68 926688

Cuius duplum est QRH 137 853376

Et huius compl.ad semicirculum est 42 146624. Latus quaesitum AC.

Latera { QH . 76 000693
 RQ . 46 302020
 RQ . 36 131235

Summa Laterum 158 433948 *Log.Sinuum.*
 Semissis summae 79 216974 . 9,99226,30294 } 18,74126,57824 summa Log. }
 Diff.1lateris QH 3 216281 . 8,74900,27530 } 20,00000,00000 Radii dup.Log. }
 Diff.2lateris RQ 32 914954 . 9,83447,92046 }
 Diff.3lateris RH 43 085739 . 9,73511,44888 } 19,56957,36934 Summa Logarith.
 20,82832,79110 Differentia
 10,41416,39555 Differ. semissis est

Logarithmus Tangens $\frac{1}{2}$ anguli HRQ 68 926686
 Cuius duplum est totus angulus 137 853372
 Et huius Compl. ad semicirculum est 42 146628 cuius mensura LG=AC.

CAPUT V.

1. Ut omnia minimo cum labore fiant.
 Si primus trium datorum proportionalium sit Sinus Totus vel Radius, addantur medii duo Logarithmi amputata prima versus sinistram nota; reliquus erit Logarithmus quaesitus.
 Revocetur illustratio Probl.3.Cap.4.Planorum: & Probl.1.Cap.3 Sphaericorum.

	<i>Logarithmi.</i>		<i>Logarithmi.</i>	
Prop.	<i>Radius</i> 90 <u>0000</u>	10,00000,0000	<i>Sinus Totus</i> 90 <u>000</u>	. 10,00000,00000
	<i>Tangens</i> 28 <u>3300</u>	9,73168,5536	<i>Sinus</i> 30 <u>000</u>	. 9,69897,00043
	<i>Crus</i> 1123 <u>7943</u>	<u>3,05068,6815</u>	<i>Sinus Hyp.</i> 51 <u>076287</u>	. <u>9,89097,02062</u>
	<i>Crus</i> 605 <u>8601</u>	$\cancel{2}$,78237,2351	<i>Sinus</i> 22 <u>891768</u>	. $\cancel{9}$,58994,02104

2. Si primus datorum sit Sinus alius quilibet vel latus; Pro Logarithmo primi sumatur eius complementum Arithmeticum, quemadmodum praecepit doctissimus *D. Briggsius* ad Cap. 15 Arithmeticae Logarithmicae ; & facta trium additone, auferatur unitas ad sinistram emergens.

Exempla mutantur Prob I.Cap.4 Planorum, & Prob.2.Cap.3.Sphaericorum.

$$\text{Prop.} \left\{ \begin{array}{l} \text{Crus} \quad 11237943 \\ \text{Crus} \quad 6058601 \\ \text{Radius} \quad 90 \ 0000 \\ \text{Tangens} \quad 28 \ 33 \end{array} \right. \left\{ \begin{array}{l} \frac{3,05068,6815 \text{Log.}}{6,94931,3185 \text{c.Ar.}} \\ 2,78237,2352 \\ \frac{10,00000,0000}{\cancel{9,73168,5537}} \end{array} \right. \left\| \left\{ \begin{array}{l} \text{Sinus} \quad 67 \ 108232 \\ \text{Sin.Tot.} \quad 90 \ 000000 \\ \text{Sinus} \quad 38 \ 923713 \\ \text{Sinus} \quad 43 \ 00000 \end{array} \right. \left\{ \begin{array}{l} \frac{9,96437,34001 \text{Log.}}{0,03562,65999 \text{c.Ar.}} \\ 10,00000,00000 \\ \frac{9,79815,67532}{\cancel{9,83378,33531}} \end{array} \right.$$

Vel si primo loco ponatur Logarithmus Secants complementi arcus quartus eveniet : Quia Radius est medius proportionalis inter Sinuum & Secantem complementi : Et idcirco si e duplicato Radii Logarithmo auferatur Logarithmus dati arcus, prodibit Logarithmus Secantis complementi ; qui cum complemento Arithmetico congruit, si unitatem ad sinistram sitam excipias. Idcirco si Secantis Logarithmo utamur, post peractam additionem auferendus est binarius, ut patet

$$10,03562,65999 \text{ Logar.Secantis} \quad 22 \ 891768$$

$$0,03562,65999 \text{ Compl.Arith.Log.} \quad 67 \ 108232$$

3. Si primus trium datorum sit Tangens, substituatur eius complementum Arithmeticum ut supra, quemadmodum hic vides Exemplo Problematis. Cap.3 Sphaericorum.

$$\text{Propert.} \left\{ \begin{array}{l} \text{Tangens} \quad 30 \ 000000 \\ \text{Radius} \quad 90 \ 000000 \\ \text{Tangens} \quad 22 \ 891768 \\ \text{Sinus} \quad 47 \ 000000 \end{array} \right. \left\{ \begin{array}{l} \frac{9,76143,93726 \text{ Log.Tang.}}{0,23856,06274 \text{ Compl.Arith.Log.Tang.}} \\ 10,00000,00000 \\ \frac{9,62556,69364}{\cancel{9,86412,74638}} \end{array} \right.$$

Vel sumatur Logarithmus Tangentis complemeti, qui ut ante fere idem est cum complemento Arithmetico, ut

$$\text{Tangens Complementi arcus} \quad 60 \ 000000 \quad 10,23856,66274$$

$$\text{Complementum Arithmeticum prius positum} \quad 0,23856,66274$$

Quia Radius mediam obtinet rationem inter Tangens arcus & complementi. Et idcirco si Logarithmus Tangentis auferatur e duplicato Radii Logarithmo, restabit Logarithmus Tangentis complementi.

4. Quod si numeri quatuor proportionales non exprimant lineas sed rectangular, ut cum datis tribus lateribus quaeritur angulus aliquis, vel contra ; sumi poterunt complementia Arithmetica pro utroque Logarithmo primi rectanguli ; quod si fiat, omnia per solam additionem perficiuntur. Exampelis utamur Problematis 11. Cap.4.

Transl. Ian Bruce.

Crura	{	PZ 30 000000	Log.Sin	9,69897,00043	Compl.Arith.	0,30120,99957
		ZS 26 <u>067185</u>	Log.Sin	9,61042,13430	Compl.Arith.	0,38957,86570
			{	Semissis summae	24 <u>040730</u>	9,61000,60492
				Semissus differentiae	18 <u>105915</u>	<u>9,49244,54374</u>
						19,79306,01393
				Semissis summae, ut supra		9,89653,00696

Log.Sinuum.

Semissis summae laterum	48 <u>105915</u>	9,87179,49769	Compl.Arith.	0,12820,50341
Differentia prima	5 <u>959269</u>	9,01628,70114	Compl.Arith.	0,98371,29886
		Differentia secunda		9,49244,54274
		Differentia tertia		<u>9,61000,60492</u>
		Summa		20,21436,94983
		Semissis summae, ut supra		10,10718,47491

Hic sumantur Complementa Arithmetica pro duobus primis Logarithmis;
 Quia hi duo inserviunt primo Rectangulo e quatuor proportionalis, atque omittitur
 Logarithmus duplicati Radii pro subtractione binarii e summa quae
 proveniret ex additione quinque Logarithmorum : quemadmodum admonuit
 D.Briggus, nosque in publicis nostris praelectionibus Astronomicis saepissimè
 ostendimus.

Integram Eclipsium doctrinam quam in apparatu habeo, coronidis loco subnectere
 statuissem, qui usum Triangulorum tam Planorum quam Sphaericorum copiose exhibeo,
 si modo per Typographi festinationem ac temporis angustias ultimam manum imponere
 licuisset. De istis autem (si DEUS volet) in sequentibus.

FINIS.