TRIGONOMETRIAE
BRITANNICAE

BOOK TWO PART TWO

[Here divided into quadrantal & non-quadrantal triangle parts.]

Spherical Triangles.

CHAPTER ONE.

With the solutions of plane triangles set out, we move on next to the solutions of spherical triangles. A spherical triangle is one which will be described on the surface of a sphere.

1. The sides of spherical triangles are the arcs of three great circles mutually intersecting each other.
2. The measures of spherical angles are the arcs of the great circles, with the angular points taken as poles, subtending the angles.
3. Great circles are those which bisect the sphere.
4. Of circles which mutually cut each other at right angles, one will pass through the poles of the other, and conversely.
5. The poles of two great circles are separated from each other by the same angle as that taken between the great circles, and conversely.
6. With the three angles of any spherical triangle given, the three sides of another triangle may be given, the angles of which will be equal to the sides of the first triangle.
7. The sides of a spherical triangle taken together will be less than two semicircles. *Reg[-iomontanus] Th.39, Bk 3.*
8. The three angles of a spherical triangle taken together are greater than two right angles, but less than six right angles. *Reg.Th.49, Bk 3.*
9. In any spherical triangle, two angles are greater than the difference between a semicircle and the remaining angle. And thus with a side continued, the exterior angle is less than the sum of the two interior opposite angles.
10. In any spherical triangle, the difference of the sum of two angles and of the whole circle is greater than the difference of the remaining angle and the semicircle.
11. A spherical triangle is either right angled or oblique angled.
12. A right angled spherical triangle is one that has at least one right angle.
13. The legs of right angled spherical triangles are of the same disposition [*i.e.* have similar properties] as are the angles opposite these themselves. *Reg.Th.*3. *Bk* 4.

14. If either leg of a right angled spherical triangle shall be a quadrant, the hypotenuse [*i.e.* the arc subtended by the right angle] will be a quadrant also : But if each leg shall be of the same disposition [*i.e.* both legs are less than a quadrant, or both legs are greater than a quadrant], the hypotenuse will be smaller than a quadrant ; but if they are of different dispositions [*i.e*. one leg is greater than a quadrant, and the other leg less than a quadrant], the hypotenuse will be greater than a quadrant. And conversely. *Reg.Th*.4. *&* 5, *Bk* 4

15. If either of the angles of a right angled spherical triangle shall be at right angles to the hypotenuse, the hypotenuse will be a quadrant ; but if each shall be of the same disposition, then the hypotenuse will be less than a quadrant; while if of differing dispositions, then [the hypotenuse will be] greater than a quadrant. And conversely. *Reg.Th.*6 &7, *Book* 4. [Note the reciprocal nature of this and the previous result, and relating angles to arcs in general.]

16. In a right angled spherical triangle, whichever you please of the oblique angles is greater than the complement of the other, but the difference of the same complement shall be less than a semicircle.

17. An oblique angled spherical triangle is either acute angled or obtuse angled

18. An acute angled spherical triangle is one which has all the angles acute.

19. An obtuse angled spherical triangle is one which has either all the angles obtuse, or has a mixture of obtuse and acute angles.

20. Any side of an acute angled spherical triangle is less than a quadrant. *Reg.Th*.9. *Bk* 4.

21. If two acute angles of an oblique angled triangle were equal, the sides opposite to these will be less than quadrants : but if obtuse, then greater *Reg. Th.*10 &11. *Bk* 4

22. Concerning oblique angled spherical triangles : if two of the acute angles shall be unequal, the side opposite to the smaller of these shall be less than a quadrant ; but if opposite to the obtuse, greater than a quadrant. *Reg. Th.*12 &13. *Bk* 4.

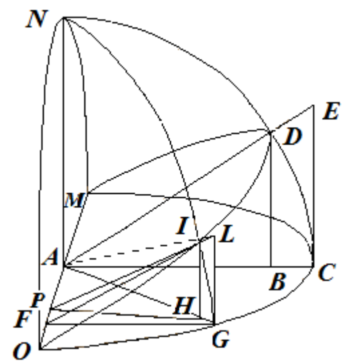
23. In any oblique angled spherical triangle, if the angles to the base shall be of the same disposition, the perpendicular from a vertex falls within the triangle ; but outside if different. *Reg.Th.*8. *Bk* 4.

[The original Latin text by Henry Gellibrand does not have these propositions numbered; the translation of this section by John Newton contained in his book *Trigonometria Britanica* (1658) does do so, and I have followed his example in this instance.]

CHAPTER II.

AXIOM I.

In right angled spherical triangles, if the angle to the base shall be acute also, *the Sines of the hypotenuses shall be in proportion to the sines of their perpendiculars, and conversely.*



Let MCON be the fourth part of the [celestial] sphere ; MCO half of the equinoctial plane, the pole of which is N. OIDM half of the plane of the ecliptic. NDC the quadrant of the intersection of the solstice : MNO half of the plane of the intersection of the equinoxes. NIG the quadrant of some circle of the meridian ; which since by intersecting the equator at right angles, truly cuts the ecliptic obliquely. In this quadrant of the sphere two rectangular spherical triangles DOC & IOG may be put in place, of which the hypotenuses are DO & IO. The perpendiculars are DC, IG. Moreover the bases are OC & OG. The acute angles to the bases are DOC & IOG. Truly the sines of the hypotenuses OD & OI shall be the right lines AD, the whole sine, & PI. The sines of the perpendiculars DC & IG shall be the right lines DB & IH. I say the sines of the hypotenuses AD & PI to be proportional to the sines of the perpendiculars DB & IH. And conversely.

For the triangles BAD & FHI are similar. Because the right lines DB & IH remain perpendicular to the same plane ; and also the parallel lines DA & IP, are in the same plane of the ecliptic, and they have the same inclination ; likewise the angles DBA & DAB, are equal to the angles IHP & IPH, and therefore the remaining ADB & PIH are equal. And thus because the triangles are equal angled, also the sides of these are in homologous proportions. And thus I say :

$$\textit{Prop.}\left\{\begin{array}{ll} \textit{Sines of the } H\textit{'s} & \text{AD . PI.} \\ \textit{Sines..... Perp.} & \text{DB . IH.} \\ \textit{Sines.....}H\textit{'s.} & \text{PI . AD.} \\ \textit{Sines..... Perp.} & \text{IH . DB.} \end{array}\right. \text{And conv. } \textit{Prop.}\left\{\begin{array}{ll} \textit{Sines Perp.} & \text{DB..IH.} \\ \textit{Sines..... } H\textit{'s} & \text{DA..IP.} \\ \textit{Sines..... Perp.} & \text{IH..DB.} \\ \textit{Sines..... } H\textit{'s} & \text{IP..DA.} \end{array}\right.$$

Q.e.d.

AXIOM II.

In spherical right angled triangles, if the angles to the base likewise shall be acute, *the sines of the bases are proportional to the tangents of their perpendiculars. And conversely.*

In the preceding diagram & with the same triangles DOC, IOG, the sines of the bases OC & OG are AC & FG. Moreover the tangents of the perpendiculars DC, IG are CE & GL. I say the sines of the bases, [namely] AC, FG are in proportion to the tangents of the perpendiculars CE, GL; and conversely the tangents of the perpendiculars CE, GL to the sines of the bases AC, FG.

Prop.	*Sines*	AC..FG.	& conv. *Prop.*	*Tang. Perp.*	CE..GL.
	Tang. Perp.	CE..GL.		*Sines*	CA..GF.
	Sines	FG..AC.		*Tang. Perp.*	GL..CE.
	Tang. Perp.	GL..CE.		*Sines*	GF..CA.

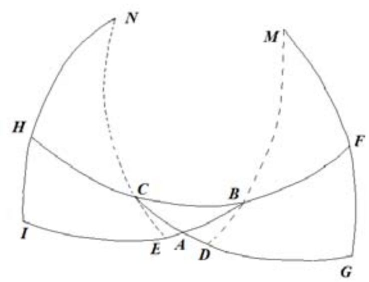
The demonstration of this axiom depends on the same fundamentals by which the above have been shown, and therefore there is no need to repeat the labour.

[In these axioms, we have two similar rt. angled triangles PIH & ADB ; note that the sine is used in its original sense : half the length of the chord of the arc of the great circle subtended at the centre of the sphere. Thus we can talk about the proportions of the sines of the arcs, or of the semi-chords themselves ; the proportionality follows in either direction. The original writer Gellibrand has put the ratio of the proportions into forward and backward forms, that in the first case we might read off as the sines of the hypotenuses to be proportional to the sines of their bases $AD : PI = DB : IH$; and in the contrary case, as the sines of the bases to be proportional to the sines of their perpendiculars $AC : FG = CE : GL$, etc.]

Consequence 1.

If from two angles of a triangle [all triangles henceforth are spherical, unless it is clear that they are planar], the perpendiculars are drawn to the opposite sides, then the sines of the angles and the sines of the perpendiculars are directly proportional. And conversely By Axiom.1.

In Triangle ABC, the perpendiculars CE & BD are drawn from the poles N & M to the opposite sides or to the opposite sides continued, & the arcs HI & FG shall be the measures of the angles CBA & BCA. I say that the sine of the angle CBE (that is the sine of the arc HI) shall be to the sine of the angle BCA (or the arc FG) as the sine CE to the sine BD. For there are



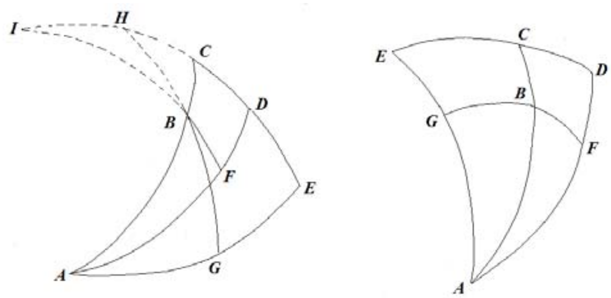
Prop.	*Whole Sines*	BH . . CF.	Therefore *Prop.*	HI.		HI.
	Sines	HI . . FG.		CE.	& alternately	FG.
	Sines	BC . . CB.		FG.		CE.
	Sines	CE . . BD		BD.		BD.

[In the right spherical triangles, the angles at I and G are right, to which there corresponds the whole sine, HB or CF; the two sets of triangles on the left and right give rise to the ratios, following the axioms. Thus we have the ratio of the sines written conveniently as $\sin BH : \sin HI = \sin BC : \sin CE$ & $\sin CF : \sin FG = \sin CB : \sin BD$; and $arc\, BH = arc\, CF$ as both are quadrants, we have $\frac{\sin HI}{\sin CE} = \frac{\sin BH}{\sin BC} = \frac{\sin CF}{\sin BC} = \frac{\sin FG}{\sin BD}$ in the first right columns, etc.]

We can also express this conclusion otherwise, evidently in this manner.

If three arcs of great circles may meet at the same point, and from some point of one perpendiculars are drawn to the rest :

The sines of the angles subtended by the said perpendiculars shall be proportional to the sines of the perpendiculars. And conversely.



Three arcs of great circles AC, AD, AE shall meet at the point A; also from some point B and with the poles I & H the perpendiculars BF & BG are drawn to the arcs AD & AE also with CD & CE taken, the measures of the angles CAD & CAE. I say that the sines CD & CE are proportional to the sines of the perpendiculars BF & BG. For

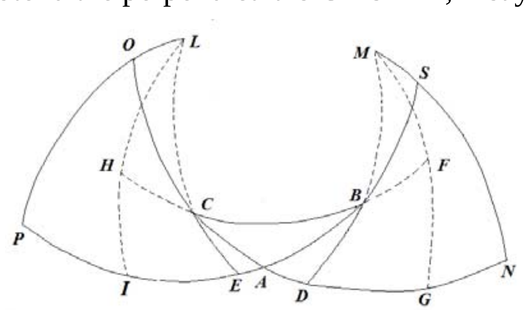
$$\textit{Prop.}\begin{cases}\textit{Whole sine} & AC\ .\ AC.\\ \textit{Sine Hypot.} & AB\ .\ AB.\\ \textit{Sines Ang.} & CD\ .\ CE.\\ \textit{Sines perpen.} & BF\ .\ BG.\end{cases}\ \text{Therefore}\ \textit{Prop.}\begin{cases}CD.\\ BF.\\ CE.\\ BG.\end{cases}\ \text{and conv.}\ \textit{Prop.}\begin{cases}BF.\\ CD.\\ BG.\\ CE.\end{cases}$$

Q.e.d.

Consequence 2.

If two arcs subtend equal angles, then the sines of the perpendiculars and the sines of the hypotenuses are in proportion, and conversely.

The angles CAE & BAD shall be equal, and because the verticals drawn from the poles, clearly L & M, subtend the perpendiculars CE & BD, I say that the sines of the



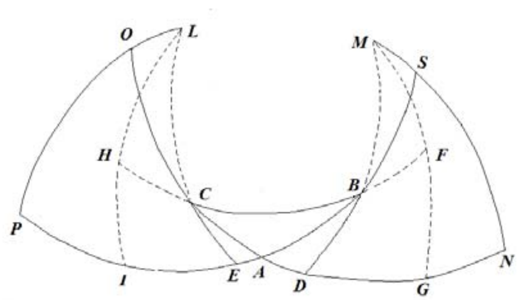
perpendiculars CE & BD are in proportion to the sines of the hypotenuses CA & BA; and conversely. Indeed the sine of the angle SAN, that is of the arc SN, is equal to the sine OP or to the angle OAP. And thus I say :

$$\textit{Prop.}\left\{\begin{array}{lll} \textit{Sine} & \text{SN} & \text{. OP.} \\ \textit{Whole Sine} & \text{SA} & \text{. OA.} \\ \textit{Sine perp.} & \text{BD} & \text{. CE.} \\ \textit{Sine Hypo.} & \text{BA} & \text{. CA.} \end{array}\right. \text{Therefore } \textit{Prop.}\left\{\begin{array}{ll} \textit{Sine perp.} & \text{BD.} \\ \textit{Sine Hypo.} & \text{BA.} \\ \textit{Sine perp.} & \text{CE.} \\ \textit{Sine Hypo.} & \text{CA.} \end{array}\right. \text{\& conv. } \textit{Prop.}\left\{\begin{array}{ll} \textit{Sine Hypo.} & \text{BA.} \\ \textit{Sine perp.} & \text{BD.} \\ \textit{Sine Hypo.} & \text{CA.} \\ \textit{Sine perp.} & \text{CE.} \end{array}\right.$$

Q.e.d.

Consequence 3.

The sines of the angles are in proportion to the sines of the opposite sides, & conversely.



Let CAB be the oblique angled triangle of the adjoining diagram. I say that the sines of the angles CBA and BCA to be proportional to the sines of the sides CA and BA. For the sines of the angles HI and FG are proportional to the sines of the perpendiculars CE and BD *by Consec.* 1. Likewise the sines of the hypotenuses CA and BA are proportional to the same sines of the perpendiculars CE and BD *by Conseq.*2. And thus I say :

$$\textit{Prop.}\left\{\begin{array}{ll} \textit{Sine of the angle} & \text{HI.} \\ \textit{Sine of the Hypo.} & \text{CA.} \\ \textit{Sine of the angle} & \text{FG.} \\ \textit{Sine of the Hypot.} & \text{BA.} \end{array}\right. \text{\& conv.} \textit{Prop.}\left\{\begin{array}{l} \text{CA.} \\ \text{HI.} \\ \text{BA.} \\ \text{FG.} \end{array}\right.$$

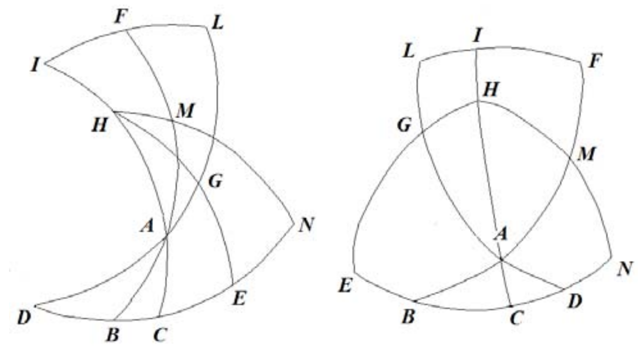
Q.e.d.

[This consequence is the sine rule for spherical triangles.]

Consequence 4.

In oblique angled triangles : *If the perpendicular may be drawn from the vertical angle to opposite side (if need be it may be continued) then the sines of the complements of the angles to the base are directly proportional to the sines of the angles to the vertical. And conversely.*

In the oblique angled triangle ABD, the perpendicular AC may be drawn from the vertex A. And with the sides BA & DA continued together with the perpendicular AC as far as to the quadrants, clearly the arcs AL, AF, AI ; from the vertex A the periphery IFL is described, the measure of the vertical angles IAF, IAL, which are equal to the angles



BAC & DAC. And the quadrantal arcs HGE & HMN are described from the angular points B & D.

I say that the sines HG & HM (evidently the complements of the angles at the base ADB & ABC) are proportional to the sines of the angles to the vertical LI & FI. For by *Conseq.*1. there are :

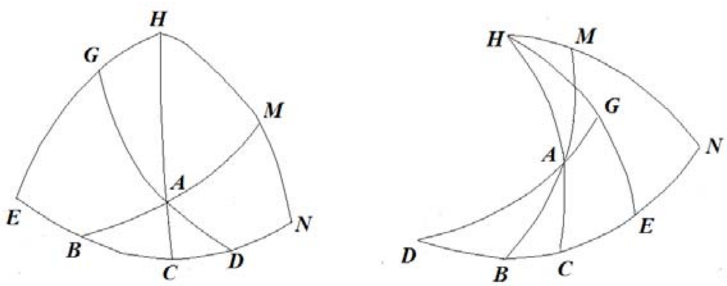
$$\textit{Prop.}\begin{cases} \textit{Sine} & \text{AH . AH.} \\ \textit{Whole Sine} & \text{AI . AI.} \\ \textit{Sine} & \text{HG . HM.} \\ \textit{Sine} & \text{IL . IF.} \end{cases} \text{Therefore } \textit{Prop.}\begin{cases} \textit{Sine} & \text{HG.} \\ \textit{Sine} & \text{HM.} \\ \textit{Sine} & \text{IL.} \\ \textit{Sine} & \text{IF.} \end{cases} \text{\& conversly.}$$

Q.e.d.

Consequence 5.

In oblique angled triangles : *If a perpendicular may be drawn from the angle to the vertical to the opposite side (it may be continued if there is a need ;) then the sines of the complements of the base segments are directly proportional to the sines of the complements of the sides for the angle to the vertical from which the perpendicular is drawn.*

In the oblique angled triangle ABD, the perpendicular AC may fall to the base DB, which may be continued from the sides as far as to a quadrant. I say that the sines of the



complements of the segments to the bases CN & CE, are proportional to the sines of the complements of the sides, AM & AG. For *by Consec*.1., there are [the ratios :]

$$\textit{Prop.}\left\{\begin{array}{lcl} \textit{Whole Sine} & HC & . \; HC. \\ \textit{Sine} & HA & . \; HA. \\ \textit{Sine} & CN & . \; CE. \\ \textit{Sine} & AM & . \; AG. \end{array}\right. \text{Therefore } \textit{Prop.}\left\{\begin{array}{ll} \textit{sine} & CN. \\ \textit{sine} & AM. \\ \textit{sine} & CE. \\ \textit{sine} & AG. \end{array}\right. \text{\& conv. } \textit{Prop.}\left\{\begin{array}{l} AM. \\ CN. \\ AG. \\ CE. \end{array}\right.$$

Q.e.d.

Consequence 6.

In oblique angled triangles : *If a perpendicular may be drawn from the angle to the vertical to the side opposite (it may be continued if there is a need) ; then the sines of the complements of the segments to the base are inversely proportional to the tangents of the opposite angles to the base. And conversely.*

In the oblique angled triangle of the above diagram ABD, the perpendicular AC may be drawn from the vertex A to the base DB ; which may be continued from the sides as far as a quadrant. Also from some points with the angles *D* & B & from the pole H, the quadrantal arcs HE & HN may be described. I say that the sines of the segments of the base CD & BC, are inversely proportional to the tangents of the opposite angles NM & EG. Indeed by *Axiom 2*, the ratios are :

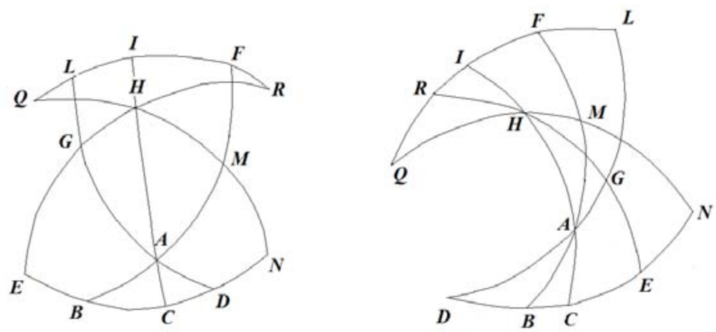
$$Prop.\left\{\begin{array}{lll} sine & DC\,.\;BC & \textit{Hence with} \\ Tang. & CA\,.\;CA & \textit{the intermediate} \\ Radius & DE\,.\;BN & \textit{ratios excluded,} \\ Tang. & EG\,.\;NM & \textit{there will be} \end{array}\right. \quad Prop.\left\{\begin{array}{ll} sine & DC. \\ sine & BC. \\ Tang. & NM. \\ Tang. & EG. \end{array}\right. \text{\& likewise } Prop.\left\{\begin{array}{l} BC. \\ DC. \\ EG. \\ NM. \end{array}\right. \text{\& conv.}$$

Q.e.d.

Consequence 7.

In oblique angled triangles : *If a perpendicular may be drawn from the angle to the vertical to the side opposite (it may be continued if there is a need ;), then the sines of the complements of the vertical angles are inversely proportional to the tangents of the sides. And conversely.*

In the oblique angled triangle ABD, the periphery QIR may be described from the vertex A, also the peripheries NMQ & EGR from the vertices of the angles B & D. And with these sides AB & AD continued together with the perpendicular AC as far as to F, L & I ; the tangents LG & FM will be equal to the sides AD & AB. Also the arcs IR & IQ will be the sines of the complements of the angles to the vertical CAD & CAB, the measures of which are LI & IF. And thus I say that the sines IR & IQ of the complements of the vertical angles to be inversely proportional to the tangents of the sides FM, LG.



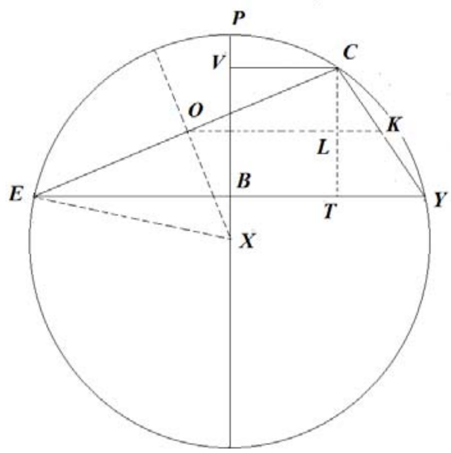
For by *Axiom* 2, the ratios are

$$Prop.\left\{\begin{array}{lll} sine & QR\,.\;RI & \textit{Hence with the} \\ Tang. & IH\,.\;HI & \textit{intermediate} \\ Radius & QF\,.\;RL & \textit{ratios excluded,} \\ Tang. & FM\,.\;LG & \textit{there will be} \end{array}\right. \quad Prop.\left\{\begin{array}{ll} sine & QI. \\ sine & RI. \\ Tang. & LG. \\ Tang. & FM. \end{array}\right. \text{\& inversely } \left\{\begin{array}{l} RI. \\ QI. \\ FM. \\ LG. \end{array}\right. \text{Et conv.}$$

Q.e.d.

LEMMA.

With the versed sines of two arcs given, the right sines of half the sum and of half the difference of the same arcs will be the mean proportionals between the whole sine and half of the difference of the versed sines.



Let the arcs PE & PC be given, and the versed sines of these PB & PV, of which the difference is BV. The sum of the given arcs shall be EPC ; and the difference of the same CY. The sine of the half sum shall be EO or OC ; the sine of the half difference shall be CK. I say that EO & CK shall be the mean proportional between XE & CL.
For the angles EXO, CYT & CKL are equal by 2, Prop.3. And the angles at O & L shall be right. Therefore the triangles XEO & KCL are similar, & XE, EO ; KC, CL are in proportional. Q.e.d.

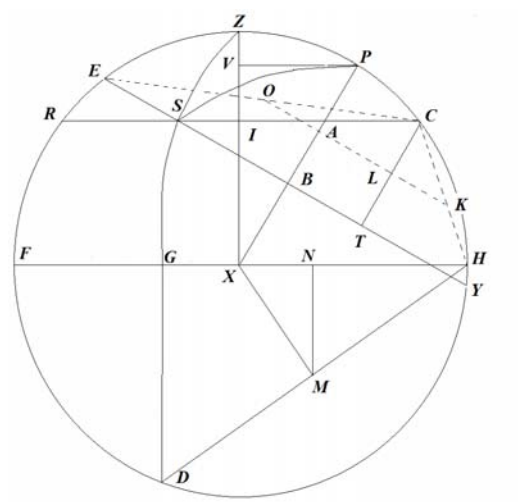
A Consequence.

If the triangle ECY may be inscribed in the circle, & from any angle ECY the perpendicular CT may be drawn to the opposite side EY : half the legs of the said angle OC, & CK are the mean proportionals between half the perpendicular CL & the semidiameter XE. Therefore the rectangles OC, CK : XE, CL are equal.

PROPOSITION.

In general spherical triangles for which the sides are known, there will be :

Proport.
- *The rectangle formed from the sines of the sides of the vertical angle.*
- *The square of the radius.*
- *The Rectangle formed from the sines of half the sum & of half the difference of the base & the legs.*
- *The square of the sines of half the vertical angle.*



Let PZS be the triangle of the given sides and the vertical angle shall be PZS. Either leg ZS and ZC will be equal, and PC the difference of the legs : Also the base PS of the vertical angle, and also PY and PE are equal, & CE the sum of the base and of the difference of the legs, and CY the difference of the same. All these are known. PV the sine of the leg PZ, & CI the sine of the remaining leg ZS, which is continued to S, are drawn. Likewise CT the perpendicular of the right line EY, and OLK bisecting the right lines EC, SC, CT, and CY, are drawn. Also let the arc HD be a measure of the vertical angle PZS, and the right line MN perpendicular to the right line HG bisects the right lines HG, Hd ; I say that the following are in proportion :

- The Rectangle from the sines of the given sides PV & CI.
- The square of the radius PX.
- The Rectangle from the sines of the sum & difference of the base and the difference of the sides OC & OK.
- The square of the sine of half angle HM to the vertical.

For the triangles XPV, XAI, SAB, & SCT are equal angled, and therefore

Prop.	CT	Likewise in prop.	CS.	and these rectangles will be proportional:	Rect: CT, CS.
	CS		HG.		Rect: CS, HG.
	PV		CI.		Rect: PV, CI.
	PX		HX.		Rect: PX, HX.

Also there are :

Prop. $\begin{cases} \text{CT.} \\ \text{HG.} \\ \text{PV, CI rect.} \\ \text{PX square} \end{cases}$ or in prop. $\begin{cases} \text{CL. . . (for they are halves} \\ \text{HN. . . of the rect. CT,HG.)} \\ \text{PV,CI rect.} \\ \text{PX square} \end{cases}$ Likewise in prop. $\begin{cases} \text{CL.} \\ \text{HN.} \\ \text{CL, HX rect:} \\ \text{HN, HX rect:} \end{cases}$

But truly, the rectangles $\begin{cases} \text{CL, HX} \\ \text{OC, CK.} \end{cases}$ are equal by the Lemma;

also $\begin{cases} \text{rect.HN, HX. which are as the altitudes CL, HN.} \\ \text{The square HM. by } \textit{Consec.} \text{ 2.\& 4 } \textit{Book}\text{.8.}\textit{Ramus}. \end{cases}$

Therefore there are the proportions :

$\begin{cases} \text{Rect. PV, CI.} \\ \text{Square PX.} \\ \text{Rect. OC, CK.} \\ \text{Square HM.} \end{cases}$

Q.e.d.

CHAPTER III.

Concerning the measurement of right angled spherical triangles.

Equipped with these aids, it will not be difficult to penetrate into the innermost parts of spherical triangles : and thus in the first place we approach the solutions of right angled spherical triangles. Since in every triangle, there shall be three sides and just as many angles, if in a spherical triangle some one of the six parts shall be 90 degrees, of the remaining parts the two which are adjacent to the part of 90 degrees together with the complements of the three remaining parts, are the five circular parts drawn by the most illustrious man, the Baron of Merchiston , of which if two were given, any from the three remaining can be found readily. Indeed:

I. *The Radius & the sine of any assumed part are the mean proportionals between the tangents of parts assumed to be the nearest on both sides or situated around*.

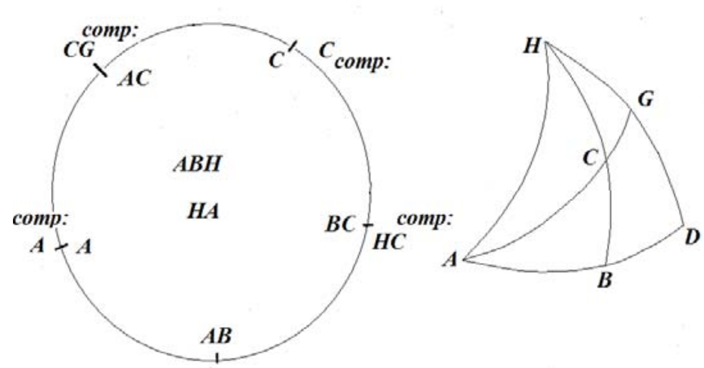
[Thus,
$$\text{tan(close ass.part on one side):sin(any ass. part)} = \text{Radius} : \text{tan(close ass. part on other side)}.$$
]

Also if the part assumed shall be more distant from the two remaining.

2. *The Radius & the sine of the assumed parts, are the mean proportionals between the sines of the complements of the opposite parts.*
[Thus, likewise,
$$\text{Radius} \times \text{sin(any ass.part)} = \text{cos(further ass.part on one side)} \times \text{cos(otherfurther ass. part)}.]$$

And if the four parts are taken

3. *The sines if the complements of the middle parts are the mean proportionals between the tangents of the extremes*.



This third proposition evidently shall be true, yet it has no convenient use, since the two given parts are sufficient to elicit the finding of any of the third parts.

So that all these shall be made more clear, the circular parts of the right angled triangle ABC have been placed within the circle, and the individual sides are less than quadrants, and the two angles acute :

moreover the parts of the oblique angled triangle ACH are placed outside the circle, and the two sides are less than quadrants, and the angles opposite to these sides are acute, the third angle opposite to the quadrant is obtuse. And so that the truth of the proposition may be seen more easily, I have recommended putting the logarithms of these parts nearby, so that we shall be able to indicate the equality of these sines and tangents by proportions.

		Logs. of Sines	*Logs. of Tangents.*
Side AB or angle AHB	gr. 47 <u>000</u> .	9,86412,74638 .	10,03034,41335
The complement BD	43 <u>000</u> .	9,83378,33303 .	9,96965,58665
Side BC, or compl. of side CH	22 <u>891768</u> .	9,58994,02104 .	9,62556,68364
The compl. CH	67 <u>108232</u> .	9,96437,34001 .	10,37443,31636
Complement of Angle ACB	70 <u>062238</u> .	9,97315,72316 .	10,44040,38970
	19 <u>937762</u> .	9,53275,33346 .	9,55959,61030
Complement of side AC,	38 <u>923713</u> .	9,79815,67304 .	9,997I8,64982
The side AC	51 <u>076187</u> .	9,89097,02061 .	10,09281,35018
Complement Angle BAC	60 <u>00</u> .	9,93753,06317 .	10,23856,06274
	30 <u>00</u> .	9,69897,00043 .	9,76143,93726

And so that we may see each example of the proposition, let the parts given be AB; BC, and the nearest part may be sought, clearly the complement of the angle ACB.

Proport.

As the Tangent AB gr. 47<u>00</u>		10,03034,41335
To the Radius		10,00000,00000
So the Sine BC.	22 <u>891768</u> .	<u>9,58994,02104</u>
To the Tangent Compl. ACB	19 <u>937762</u> .	9,55959,60769

But if the side AC may be sought by the second proposition :

Proport.

As the Whole Sine		. 10,00000,00000
To the Sine Compl. BC gr.	67 <u>108232</u>	. 9,96437,34001
So the Sine Compl. AB .	43 <u>000</u>	. <u>9,83378,33303</u>
To the Sine Compl. AC .	38 <u>923713</u>	. ~~1~~9,79815,67304

If the angle may be sought BAC;

Proport.	*As the Tangent* BC	gr.22 891768	.	9,62556,68364
	To the Radius			10,00000,00000
	So the Sine AB . .	47 000 .	. .	9,86412,74638
	To the Tangent Compl. BAC	60 000 .	. .	10,23856,06274

Thus in a rectangular spherical triangle with two parts given from the five circular parts, the three remaining will be able to be found.

Because if two circular parts shall be given in the quadrantil triangle ACH, the three remaining parts are found in almost the same manner.

Let the angles AHC gr.47000 & HAB gr. 60000 be given, and the angle ACH is sought.

Proport.	*As the Whole Sine*		10,00000,00000
	To the Sine Compl. AHC gr.	43 000 .	9,83378,33303
	So the Sine Compl. HAC .	30 000 .	9,69897,00043
	To the Sine Compl. ACH .	19 937762	~~1~~9,53275,33346

The excess of the obtuse angle ACH above a right angle, or the complement of the angle ACB.

If AC may be sought. The given parts : HAC & AHC; the part sought the complement of AC. These three are nearby or contiguous parts, and the intermediate part is HAC.

Proport.	*As the Tangent Angle* AHC	gr.47 000 . .	10,03034,41335
	To the Radius		10,00000,00000
	So the Sine HAC 47 000 . . .	60 000 . .	9,93753,66317
	To the Tangent Compl. AC . . .	38 923713 .	9,90718,64982
	Therefore AC is	51 070287	

If HC is sought, the manner does not differ from that which is set out for finding the side AC. The angles HAC, AHC and the side HC are neighbouring circular parts, and therefore :

Proport.	*As the Tangent Angle* HAC	gr.60 000 . .	10,23856,06274
	To the Radius		10,00000,00000
	So the Sine CHA . .	47 000 . .	9,86412,74638
	To the Tangent Complem.	HC 22 891768.	9,62556,68364
	Therefore HC is .	67 108232	

Therefore in triangles of this kind we will be able to use either this general rule although allowed to be more obscure, or with these particular instances of the given rules and which are of service in the cases provided, and which are strengthened significantly by demonstrations. But this general rule will not afford more difficulty than these particular demonstrations, provided in the particular cases arising it will be seen to change, so that the same quantities assuredly may be retained on both sides. And thus there shall be:

PROBLEM I.

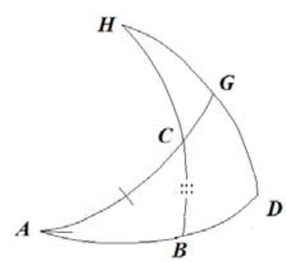
With the hypotenuse & and the angle opposite to that given, the leg opposite a given angle is sought.

In the right angled triangle ABC the leg BC is sought :

From the given { Hypotenuse AC 51 076287
with the angle BAC 30 000 }

Terms of the ratio.

Proport. { *As the Whole Sine*
To the Sine of the angle given.
So the Sine of the Hypotenuse.
To the Sine of the leg sought. }



Illustrated by numbers.

				Sines	*Logs of Sines.*
Proport.	Whole sine	AG	90 000	1000000,00000	10,00000,00000
	Sine angle.	GD.	30 000	500000,00000	9,69897,00043
	Sine Hypot.	AC.	51 076287	777983,18409	9,89097,02062
	Sine of leg	CB.	22 891768	388991,59204	~~1~~9,58994,02504

PROBLEM. 2.

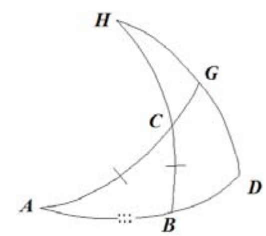
With the hypotenuse and a leg given, the remaining leg is sought.

In the right angled triangle ABC the leg AB is sought.

Given { hypotenuse. AC. 51 076287
leg BC. 22 891768 }

Terms of the ratio.

Proport.
- *As the Sine compl. of given leg.*
- *To the Whole Sine.*
- *So the Sine of the compl. of the hypotenuse*
- *To the Sine compl. of leg sought.*



Illustrated by numbers.

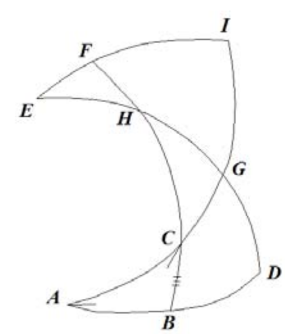
				Sine	*Logs of Sines.*
Proport.	Sine	HC	67 108232	92124,13016	9,96437,34001
	Whole sine.	HB	90 000	1,00000,00000	10,00000,00000
	Sine	CG	38 923713	62828,50929	9,79815,67532
	Sine	BD	43 000	68199,83600	9,83378,33531
	Compl.	AB	47 000 Leg sought.		

PROBLEM 3.

With the oblique angles given, either leg is sought.

In the right angled triangle ABC the leg BC is sought.

With the angles { ACB . 70 062238; BAC . 30 000 }



Terms of the ratio.

Proport.
- *As the Sine of the angle of the opposite side sought*
- *To the Sine compl. of the opposite side sought.*
- *So the Whole Sine to the*
- *Sine compl. of the side sought.*

Illustrated by Arithmetic.

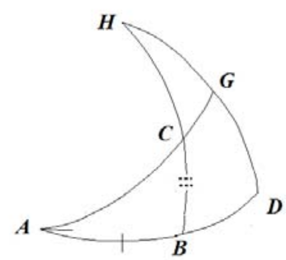
				Sines	*Logs of the Sines.*
Proport.	Sine	FI	70 062238	94006,35859	9,97315,72287
	Sine	HG	60 000	86602,54037	9,93753,06317
	Whole Sine	FC	90 000	1,00000,00000	10,00000,00000
	Sine	HC	67 108232	92124,13016	9,96437,34030
	Compl.	CB	22 891768	Leg sought.	

PROBLEM 4.

With a leg given and with the angle given opposite the other leg, the leg is sought.

In the right angled triangle ABC the leg BC is sought.

With given { the leg AB 47 000; with the angle BAC 30 000



Terms of the ratio.

Proport. { *As the Radius to the*
Tangent of the oblique angle given.
So the Sine of the leg given to the
Tangent of the leg sought.

Illustrated by arithmetic.

				Sin.&Tang.	*Log. Sin.&Tang.*
Proport.	Radius	AD	90 000	1,00000,00000	10,00000,00000
	Tangent	DG	30 000	57735,02692	9,76143,93726
	Sine	AB	47 000	73135,37016	9,86412,74638
	Tangent	CB	22 891768	42224,72564	9,62556,69364

PROBLEM 5.

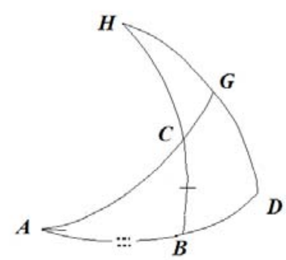
With a leg and with the angle opposite to it given, the remaining leg is sought.

In the right angled triangle ABC the leg AB is sought.

Given { the leg BC 22 891768
with the angle BAC 30 000

Terms of the ratio.

Proport. { *As the Tangent of the oblique angle given to the Radius.*
So the Tangent of the leg given
To the Sine of the leg sought.



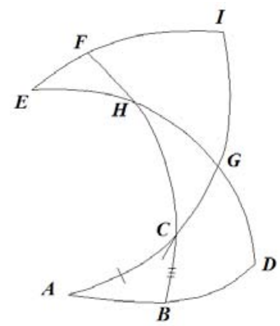
Illustrated by arithmetic.

					Sin.&Tang.		*Log. Sin.&Tang.*
Proport.	Tangent	DG	30 000	. .	57735,02692	. .	9,76143,93726
	Radius	AD	90 000	. .	1,00000,00000	. .	10,00000,00000
	Tangent	BC	22 891768	. .	42224,72564	. .	9,62556,69364
	Sine	AB	47 000	.	73135,37016	. .	9,86412,74638

PROBLEM 6.

With the hypotenuse and an angle given, the leg is sought for the opposite angle.

In the right angled triangle ABC the leg BC is sought.



Given	Hypotenuse	AC	51 076287
	Angle	BCA	70 062238

Terms of the ratio.

Proport.
- *As the Radius to the*
- *Tangent of the Hypotenuse.*
- *So the Sine of the compliment of the angle given*
- *To the Tangent of the leg sought.*

Illustrated by numbers.

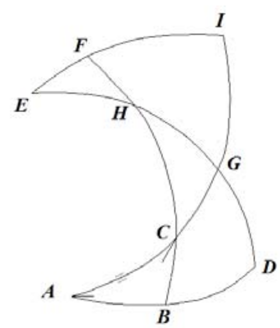
				Tangent. & Sine	*Logs.Tang.& Sin.*
Proport.	Radius	EI	90 000	100000,00000	10,00000,00000
	Tangent	IG	51 076287	123826,46138	10,09281,35061
	Sine	EF	19 937762	34099,91921	9,53275,33303
	Tangent	FH	22 891768	42224,71767	19,62556,68364

PROBLEM 7.

With the oblique angles given, the hypotenuse is sought.

In the right angled triangle ABC the hypotenuse AC is sought.

With the angles given { BAC 30 000; ACB 70 062238 }



Terms of the ratio.

Proport. { *As the Tangent of either angle to the Radius. So the Tangent of the compliment of the remaining angle To the Sine of the compliment of the hypotenuse.* }

Illustrated by arithmetic.

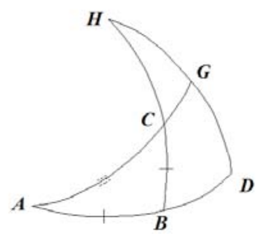
				Tangent & Sinus	*Logs.Tang.& Sin.*
Proport.	Tangent	IG	70 062238	275679,13657	10,44040,38970
	Radius	IC		100000,00000	10,00000,00000
	Tangent	GH	60 000	173205,08076	10,23856,06274
	Sine	GC	38 923713	62828,50929	9,79825,67304

PROBLEM 8.

With the legs given, the hypotenuse is sought.

In the right angled triangle ABC, the hypotenuse AC is sought.

With the legs given { AB 47 000
BC 22 901768 }



Terms of the ratio.

Proport. {
As the Whole Sine to the
Sine complement of the one leg.
So the Sine compliment of the other leg required
to the Sine compliment of the hypotenuse sought. }

Illustrated by numbers.

				Sines	*Logs. of Sines.*
Proport.	Whole Sine	HB		. . 1,00000,00000 . .	10,00000,00000
	Sine	BD	43 000	. . 68199,83600 . .	9,83378,33303
	Sine	CH	67 108232	. . 92124,13016 . .	9,96437,34030
	Sine	CG	38 923713	. . 62828,50929 . .	~~1~~9,79815,67304
	Compl. is	CB	51 076287	Hypotenuse sought.	

PROBLEM 9

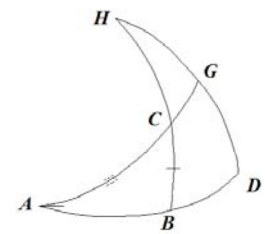
With a leg and the angle opposite given, the hypotenuse is sought.

In the right angled triangle ABC the hypotenuse AC is sought.

Given: { the leg BC 22 891768
with the angle BAC 30 000

Terms of the ratio.

Proport. {
As the Sine of the oblique angle given
To the whole Sine.
So the Sinus of the side given
To the Sinus of the Hypotenuse sought.

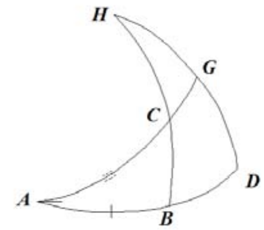


				Sines	*Logs.Sines.*
Proport.	Sinus	GD	30 000	50000,00000	9,69897,00043
	Radius	AG	90 000	100000,00000	10,00000,00000
	Sinus	CB	22 891768	38899,15834	9,58994,02104
	Sinus	AC	51 076287	77798,31669	9,89097,02061

PROBLEM 10

With a leg and the opposite angle given, the hypotenuse is sought.

In the right angled triangle ABC the hypotenuse AC is sought.



Given { the leg AB 47 000
the angle BAC 30 000 }

Terms of the ratio.

Proport. {
As the Radius
To the Tangent of the complement of the leg given.
So the Sine complement of the angle given
To the Tangent of the complement of the Hypotenuse sought.
}

Illustrated by numbers.

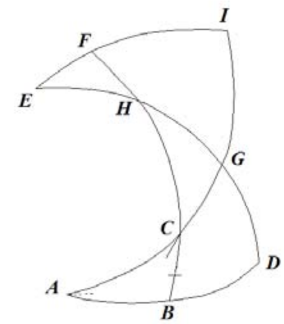
				Sines	*Log.Sin.&Tangs.*
Proport.	Radius	HD	90 000 .	100000,00000 . .	10,00000,00000
	Tangens	DB	43 000 .	93251,50861 . .	9,96965,58665
	Sinus	HG	60 000 .	86602,54037 . .	9,73753,06317
	Tangens	GC	38 923713 .	80758,18517 . .	19,90718,64982
	Compl. is	AC	51 076287 Hypotenuse sought.		

PROBLEM 11.

With an angle and the leg opposite to it given, the remaining angle is sought.

In the right angled triangle ABC the angle BAC is sought.

Given	the angle	ACB	70 062238
	the side	BC	22 891768



Terms of the ratio.

Proport.
- *As the whole Sine*
- *To the Sine of the oblique angle given.*
- *So the Sine of the compliment of the leg given*
- *To the Sine of the compliment of the angle sought.*

Illustrated by arithmetic.

				Sines	*Logs.Sines.*
Proport.	Whole Sine	FC	90 000	100000,00000	10,00000,00000
	Sine	FI	70 062238	94006,35859	9,97315,72316
	Sine	HC	67 108232	92124,13016	9,96437,34001
	Sine	HG	60 000	96602,54037	~~1~~9,93753,06317

Compl. is GD 30 000 The measure of the angle sought.

PROBLEM 12.

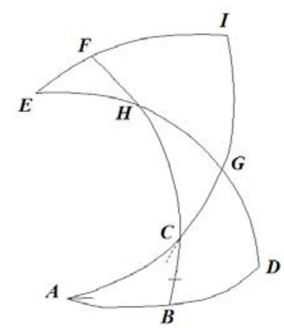
With an angle and the opposite side given, the remaining angle is sought.

In the right angled triangle ABC the angle ACB is sought.

Given	the angle	BAC	30 000
	the leg	BC	22 891768

Terms of the ratio.

Proport.
As the Sine of the compliment of the leg given.
To the Sine of the compliment of the Angle given.
So the whole Sine
To the Sine of the angle sought.



Illustrated by numbers.

				Sine	*Logs.Sines.*
Proport.	Sine	HC	67 108232	92124,13016	9,96437,34001
	Sine	HG	60 000	96602,54037	9,93753,06317
	Whole Sine	FC	90 000	100000,00000	10,00000,00000
	Sine	FI	70 062238	94006,35859	9,97315,72287

PROBLEM 13

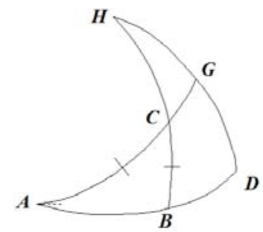
With the hypotenuse and leg given, the angle opposite the given leg is sought.

In the right angled triangle ABC the angle BAC is sought.

Given	Hypotenuse	AB	51 <u>072687</u>
	Leg	BC	22 <u>891768</u>

Terms of the ratio.

Proport.
- *As the Sine of the Hypotenuse*
- *to the Sinus of the leg.*
- *So the whole Sine*
- *To the Sine of the angle sought.*



Illustrated by numbers.

					Sines		*Logs.Sines.*
Proport.	Sine	AC	51 <u>076287</u>	.	77798,31814	. .	9,89097,02061
	Sine	CB	22 <u>891768</u>	.	38899,15834	. .	9,58994,02104
	Whole Sine	AG	90 <u>000</u>	.	100000,00000	. .	10,00000,00000
	Sine	GD	30 <u>000</u>	.	50000,18517	. .	9,69897,00043

PROBLEM 14

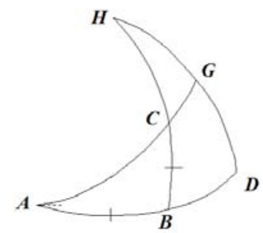
With the legs given, either angle is sought.

In the right angled triangle ABC the angle BAC is sought.

With the legs given { AB 47 000; BC 22 891768 }

Terms of the ratio.

Proport. { *As the Sine of the leg opposite the angle sought*
To the Tangent of the other leg.
So the Radius
To the Tangent of the angle sought. }



Illustrated by numbers.

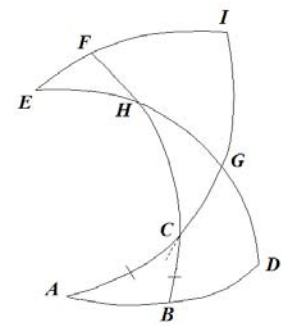
				Sines	*Logs of Sines.*
Proport.	Sine	AB	47 000	73135,37016	9,86412,74638
	Tangent	BC	22 891768	42224,72384	9,62556,78364
	Radius	AD	90 000	100000,00000	10,00000,00000
	Tangent	DG	30 000	57735,02692	9,76143,93726

PROBLEM 15.

With the hypotenuse and a leg given, the angle is sought enclosed by the same.

In the right angled triangle ABC the angle ACB is sought.

Given { Hypotenuse AC 51 076287
LEG BC 22 891768



Terms of the ratio.

Proport. { *As the Tangent of the Hypotenuse*
To the Radius.
So the Tangent of the side
To the Sine of the compliment
of the angle sought.

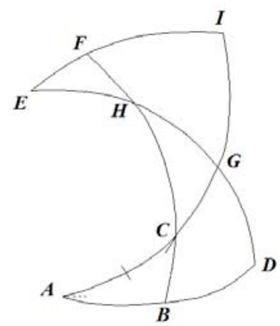
Illustrated by arithmetic.

				Sines	Logs.Sines.
Proport.	Tangent	IG	51 076287 . .	123826,46138 . .	10,09281,35061
	Radius	IE	90 000 . .	100000,00000 . .	10,00000,00000
	Tangent	FH	22 89768 . .	4,2224,72384 . .	9,62556,68364
	Sine	FE	19 93776 . .	34099,91921 . .	9,53275,33303
	Compl. is	FI	70062238 Measure of the angle sought ACB.		

PROBLEM 16.

With the hypotenuse and an angle given, the remaining angle is sought.

In the right angled triangle ABC the angle BAC is sought.



Given { Hypotenuse AC 51 <u>076287</u>; Angle ABC 70 <u>062238</u>

Terms of the ratio.

Proport.
- *As the Radius*
- *To the Tangent of the oblique angle given.*
- *So the Sine of the compliment of the Hypotenuse*
- *To the Tangent of the compliment of the angle sought.*

Illustrated by numbers.

				Sin.&Tang.	*Log.Sin.&Tang.*
Proport.	Radius	CI	90 <u>000</u>	100000,00000	10,00000,00000
	Tangent	FI	70 <u>062238</u>	275679,13657	10,44040,38970
	Sine	CG	38 <u>923713</u>	62828,50920	9,79815,63704
	Tangent	GH	60 <u>000</u>	173205,08076	10,23856,06274

Compl. is GD 30 <u>000</u> Measure of the angle sought CAB.

TRIGONOMETRIAE
BRITANNICAE

LIBRI SECUNDI PARS SECUNDA

De Trianguls Sphaericis.

CAPUT PRIMUM.

Expeditis Triangulorum Planorum solutionibus, Sphaericorum proxime aggredimur.
Triangulum Sphaericum est quod in superficie Sphaerica describitur.
Trianguli Sphaerica Latera sunt arcus trium circulorum maximorum Sphaerae sese mutuo intersecantium.
Angulorum Sphaericorum mensurae sunt arcus maximorum circulorum ex punctis angularibus tanquam e polis descripti angulosque subtendentes.
Circuli maximi sunt qui Sphaericam bisecant.
Circulorum qui se mutuo recte secant unus transit per alterius polos. Et contra.
Poli duorum Circulorum maximorum habent distantiam angulo ab iisdem comprehenso aequalem. Et contra. Itaque
Datis tribus Angulis cuiuslibet Trianguli Sphaerici, dantur tria latera alterius Triangula, cuius anguli aequabuntur lateribus prioris Trianguli.
Trianguli Sphaerici latera simul sumpta minora duobus semicirclis. *Reg.39.3.*
Trianguli Sphaerici tres anguli simul sumpti, duobus rectis sunt maiores, vel sex rectis minores. *Reg.49.3*.
In quolibet Triangulo Sphaerico duo anguli majores sunt differentia inter reliquum & semicirculum. Itaque
Continuato latere angulus exterior duobus interioribus oppositis minor est.
In quolibet Triangulo Sphaerico Differentia summae duorum angulorum & integri Circuli major est differentia reliqui anguli & semicirculi.
Triangulum Sphaericum aut est Rectangulum aut Obliquangulum.
Rectangulum est quod saltem unicum habet angulum rectum.
Trianguli Sphaerici Rectanguli crura sunt eiusdem affectionis cuius sunt anguli ipsis oppositi. *Reg*.3.3.
Trianguli Sphaerici Rectanguli, si Crus alterum sit quadrans,erit etiam & Hypotenesa

quadrans: Sin utrumque sit affectionis eiusdem, Hypotenesa erit quadranta minor ; sin diversae,major. Et contra. *Reg*.4. *&* 5.4

Trianguli Sphaerica Rectanguli, si alteruter angulorum ad Hypotenesam rectus sit, Hypotenusa erit quadrans, Sin uterque affectionis eiusdem,minor: sin diversae maior. Et contra. *Reg*.6·7·4.

In Triangulo Sphaerico Rectangulo, uterlibet obliquorum maior est complemento alterius minor, autem differentia eiusdem complemti ad semicirculum.

Obliquangulum Triangulum Sphaericum aut est Actutangulum aut Obtuseangulum

Acutangulum est quod omnes angulos habet acutos.

Obtusanguli est quod vel omnes obtusos, vel obtusos & acutos mixtim habet.

Trianguli Sphaerici Acutanguli, laterum quodvis quadrante minus est. *Reg*.9.4

Trianguli Sphaerici Obliquanguli; Si duo anguli acuti aequales sint, erunt latera iis oppositi quadrantibus minora : sin obtusi, maiora *Reg*.10.11.4

Trianguli Sphaerica obliquanguli; Si duo anguli acuti inaequales sint, erit Latus minori eorum oppositum quadrante minus; sin obtusi maiori, maius. *Reg*.12.13.4

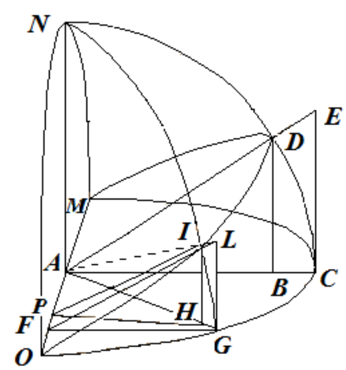
In quolibet Triangulo Sphaerico obliquangulo, si anguli ad Basim sind eiusdem affectionis, perpendicularis a vertice cadit Intra ; at Extra si diversae. *Reg*.8.4.

CAPUT II.

AXIOMA I.

In Triangulis Sphaericis Rectangulis, si idem sit angulus acutus ad Basim, *Sinus* Hypotenesarum,Sinubus *suorum perpendiculorum sunt proportionales* ; *& contra.*

Sit MCON quaerta Sphaerae pars; MCO semissis plani Aequinoctialis, cuius polus N. OIDM semissis plani Ecliptice. NDC quadrans Coluri Solstitiorum : MNO semissis plani Coluri Equinoctiorum. NIG quadrans Circuli alicuis Meridiani ; qui cum Coluris, Aequatorem ad angulos rectos, Eclipticam vero ad obliquos secat. In hac Sphaere quadrante duo constitutuuntur Triangula Sphaerica Rectangula DOC & IOG quorum Hypotenusae sunt DO & IO. Perpendiculares DC, IG. Bases autem OC & OG. Angulus ad bases acutus DOC, IOG. Sinus vero Hypotenusarii OD, OI sint rectae AD, Sinus Totus, & PI. Sinus perpendicularium DC, IG sint rectae DB, IH. Aio Sinus Hypotenusarum AD, PI Sinubus perpendicularum DB, IH proportionales esse. Et contra. Sunt enim Triangula BAD, FHI, similia. Quia rectae DB, IH perpendiculares insistent eidem plano; Et parallelae etiam DA, IP, sunt in eodem Eclipticae plano, eandemque habent inclinationem ; Anguli item DBA, DAB, angulis IHP, IPH aequantur, ergo & reliqui ADB, PIH aequantur. Quoniam itaque triangulia sunt aequiangula, sunt etiam & latera eorum homologa proportionalia. Aio itaque :



Proport. { *Sinus Hypotenusae* AD.PI. / *Sinus Perpendic.* DB.IH. / *Sinus Hypotenusae.* PI.AD. / *Sinus Perpendic.* IH.DB. }
et Contra *Proport*. { *Sinus Perpendic.* DB..IH. / *Sinus Hypotenusae* DA..IP. / *Sinus Perpendic.* IH..DB. / *Sinus Hypotenusae.* IP..DA. }

Quod erat demonstrandum.

AXIOMA II.

In Triangulis Sphaericis Rectangulis, si idem sit angulus acutus ad Basim,
Sinus Basium, Tangentibus suorum Perpendiculorum sunt proportionales. Et contra.

In praecedente schemate & iisdem Triangulis DOC, IOG, Sinus Basium OC & OG sunt AC & FG. Tangentes autem Perpendiculorum DC, IG sunt CE & GL. Aio Sinus Basium AC, FG Tangentibus Perpendiculorum CE, GL. Item Tangentes Perpendiculorum CE, GL Sinubus Basium AC, FG proportionales esse.

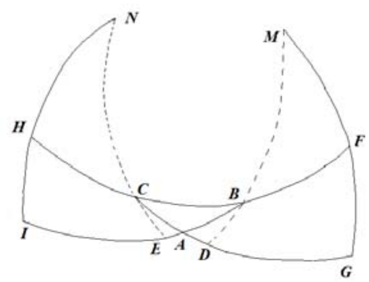
$$Proport.\begin{cases} Sinus & AC..FG. \\ Tang.\ Perpendic. & CE..GL. \\ Sinus & FG..AC. \\ Tang.\ Perpendic. & GL..CE \end{cases} \text{et Contra}\ \ Proport.\begin{cases} Tang.\ Perpendic. & CE..GL. \\ Sinus & CA..GF. \\ Tang.\ Perpendic. & GL..CE. \\ Sinus & GF..CA \end{cases}$$

Axiomatis huius demonstratio iisdem nititur fundamentis quibus & superioris, ideoque repetito labore non opus est.

Consectarium 1.

Si a duabus anguli Trianguli, Perpendiculares ducantur in latera opposita;Sinus angulomun, Sinubus Perpendicularium sunt directe proportionales. Et contra. per Axiom.1.

In Triangulo ABC, a polis M & N ducantur perpendiculares CE, BD in latera opposita & continuata, & sint arcus HI, FG mensurae angulorum CBA, BCA. Aio Sinum anguli CBE (id est Sinum arcus HI) esse ad Sinum anguli BCA (vel arcus FG) ut Sinus CE ad Sinum BD. Sunt enim



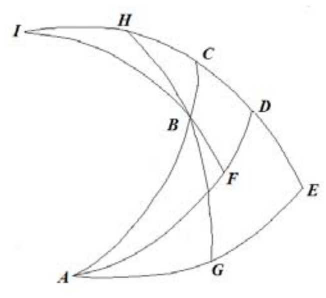
$$Proport.\begin{cases} Sinus\ \ Totus & BH\ .\ .\ CF. \\ Sinus & HI\ \ .\ .\ FG. \\ Sinus & BC\ .\ .\ CB. \\ Sinus & CE\ .\ .\ BD \end{cases} \text{Ergo}\ \ Proport.\begin{cases} \text{HI.} & \text{HI.} \\ \text{CE. et alterne} & \text{FG.} \\ \text{FG.} & \text{CE.} \\ \text{BD.} & \text{BD.} \end{cases}$$

Poterimus etiam Consectarium istud aliter exprimere, hunc scilicet in modum.

Si tres Arcus majorum Circulorum in codem puncto concurrant,& a quolibet puncto unius ducantur perpendiculares ad reliquos,

Sinus Angulorum a dictis perpendicularibus subtensorum, Sinubus Perpendicularium sunt proportionales. Et contra.

Sint tres Arcus maiorum Circulorum AC, AD, AE, concurrentes in puncto A; a puncto ctiam B & e polis I & H ducantur BF,BG, perpendiculares arcubus AD, AE sunto etiam



CD & CE, E mensurae angulorum CAD, CAE. Aio Sinus CD,CE Sinubus Perpendicularium BF,BG, proportionales esse. Nam

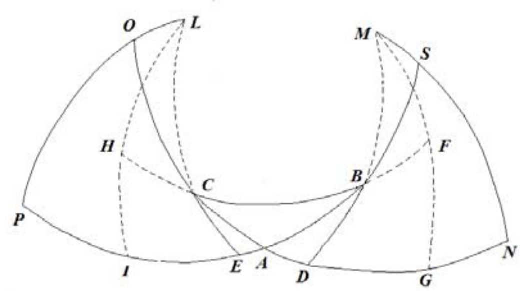
$$Prop.\left\{\begin{array}{lll} \textit{Sinus Totus} & AC & . \ AC. \\ \textit{Sinus Hypot}. & AB & . \ AB. \\ \textit{Sinus Ang}. & CD & . \ CE. \\ \textit{Sinus perpen.} & BF & . \ BG \end{array}\right. \text{Ergo } Prop.\left\{\begin{array}{l} CD. \\ BF. \\ CE. \\ BG. \end{array}\right. \text{et contra } Prop.\left\{\begin{array}{l} BF. \\ CD. \\ BG. \\ CE. \end{array}\right.$$

Quod erat demonstrandum.

Consectarium 2.

Si duo arcus Perpendiculares subtendant angulos aequales ; *Sinus Perpendicularium, Sinibus Hypotenusarum sunt proptionales. Et contra.*

Sint Anguli CAE, BAD aequales,quia verticales, quos subtendent Perpendiculares CE,



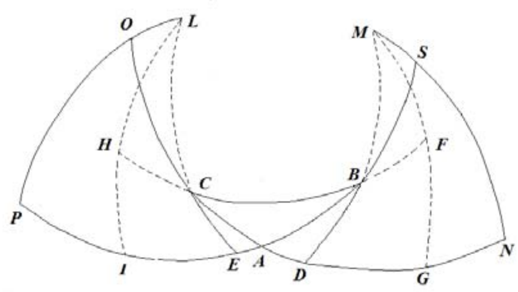
BD a polis scil: L & M ducti. Aio Sinus perpendicularium CE, BD Sinubus Hypotenusarum CA, BA proportionales esse; Et contra. Est enim Sinus anguli SAN id est arcus SN aequalis Sinui OP vel Anguli OAP. Aio itaque

$$Prop.\left\{\begin{array}{lll} \textit{Sinus} & SN & . \ OP. \\ \textit{Sinus Totus} & SA & . \ OA. \\ \textit{Sinus perp.} & BD & . \ CE. \\ \textit{Sinus Hypo.} & BA & . \ CA. \end{array}\right. \text{Ergo } Prop.\left\{\begin{array}{ll} \textit{Sinus perp.} & BD. \\ \textit{Sinus Hypo.} & BA. \\ \textit{Sinus perp.} & CE. \\ \textit{Sinus Hypo.} & CA. \end{array}\right. \text{et contra } Prop.\left\{\begin{array}{ll} \textit{Sinus Hypo.} & BA. \\ \textit{Sinus perp.} & BD. \\ \textit{Sinus Hypo.} & CA. \\ \textit{Sinus perp.} & CE. \end{array}\right.$$

Quod erat demonstrandum.

Consectarium 3.

Sinus Angulorum Sinubus oppositorum Laterum sunt Proportionales, & contra.
Esto Triangulum Obliqangulum adjuncti schematis CAB. Aio Sinus angulorum CBA BCA esse Sinubus laterum CA, BA proportionales. Sunt enim Sinus angulorum HI,



FG proportionales Sinubus perpendicularium CE, BD *per Consect.* 1. Item Sinus Hypotenusarum CA, BA proportionales iisdem Sinubus perpendicularium CE, BD *per Consectar*.2. Aio itaque

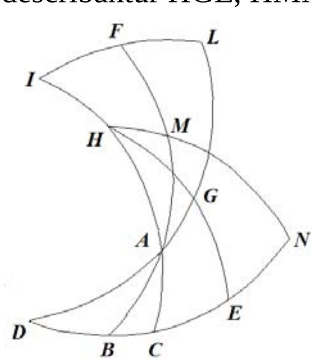
$$Prop.\begin{cases} \textit{Sinus anguli}. \ \ \mathrm{HI}. \\ \textit{Sinus Hypot}. \ \ \mathrm{CA}. \\ \textit{Sinus Anguli} \ \ \mathrm{FG}. \\ \textit{Sinus Hypot}. \ \ \mathrm{BA}. \end{cases} \text{Et contra } Prop.\begin{cases} \mathrm{CA}. \\ \mathrm{HI}. \\ \mathrm{BA}. \\ \mathrm{FG}. \end{cases}$$

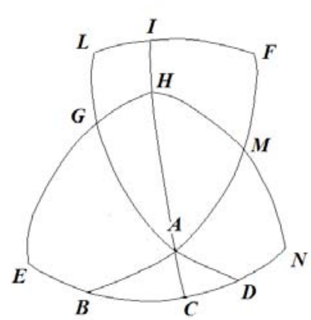
Quod erat demonsitandum.

Consectarium 4.

In Triangulis Obliquangulis: *Si Perpendicularis ducatur ab angulo verticali in latus oppositum (si opus sit continuatum;) Sinus complementorum angulorum ad Basim, Sinubus angulorum ad verticem sunt directe proportionales. Et contra.*

In Triangulo Obliquangulo ABD, a vertice A ducatur Perpendicularis AC.
Et continuatis lateribus BA , DA una cum perpendiculo AC usque ad quadrantes, videlicet AL, AF, AI ; ab A vertice describatur peripheria IFL, mensura angulorum verticalium IAF, IAL, qui aequantur angulis BAC, DAC. Et a punctis angularibus B & D arcus quadrantales describantur HGE, HMN.





Aio Sinus HG, HM (complementorum scilicet angulorum ad Basim ADB, ABC) Sinubus angulorum ad verticem LI, FI proportionales esse. Sunt enim *per Consect.*1.

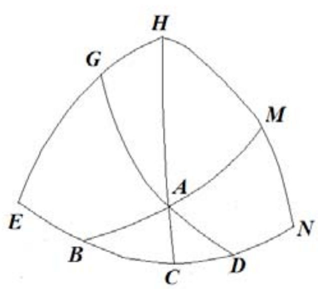
$$\textit{Prop.}\begin{cases}\textit{Sinus} & AH \,.\, AH.\\ \textit{Sinus Totus} & AI \,.\, AI.\\ \textit{Sinus} & HG \,.\, HM.\\ \textit{Sinus} & IL \,.\, IF\end{cases}\text{Ergo }\textit{Prop.}\begin{cases}\textit{Sinus} & HG.\\ \textit{Sinus} & HM.\\ \textit{Sinus} & IL.\\ \textit{Sinus} & IF.\end{cases}\text{et contra}$$

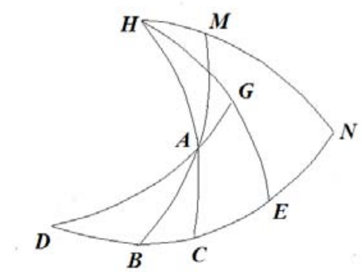
Quod erat demonstrandum.

Consectarium 5.

In Triangulis Obliquangulis: *Si Perpendicularis ducatur ab angulo verticali in latus oppositum (si opus sit continuatum ;) Sinus complementorum segmentorum Basis, Sinubus complementorum laterum anguli verticalis a quo perpendicularis ducitur sunt directe proportionales.*

In Triangulo Obliquangulo ABD , a vertice A cadat Perpendicularis AC in Basim DB, quae cum lateribus continuetur ad quadrantes usque. Aio Sinus complementorum segmentorum Basis CN, CE, Sinubus complementorum laterum AM, AG, proportionales esse. Sunt enim *per Consec*.1.





$$\textit{Prop.}\begin{cases}\textit{Sinus Totus} & HC \,.\, HC.\\ \textit{Sinus} & HA \,.\, HA.\\ \textit{Sinus} & CN \,.\, CE.\\ \textit{Sinus} & AM \,.\, AG\end{cases}\text{Ergo }\textit{Prop.}\begin{cases}\textit{sinus} & CN.\\ \textit{sinus} & AM.\\ \textit{sinus} & CE.\\ \textit{sinus} & AG.\end{cases}\text{et contra }\textit{Prop.}\begin{cases}AM.\\ CN.\\ AG.\\ CE.\end{cases}$$

Quod erat demonstrandum.

Consectarium 6.

In Triangulis Obliquangulis: *Si Perpendicularis ducatur ab angulo verticali in latus oppositum (si opus sit continuatum;) Sinus complementorum segmentorum Basis, Tangentibus angulorum Basi counterminorum, sunt reciproce proportionales. Et contra.*

In Triangulo Obliqangulo superioris schematis ABD, a vertice A ducatur perpendicularis AC in Basim DB ; quae cum lateribus continuetur usque ad quadrantes. A punctis quoque angularibus *D* & B & e polo H describantur arcus quadrantales

HE, HN. Aio Sinus segmentorum Basis CD, BC,Tangentibus angulorum conterminorum NM, EG,esse reciproce proportionales. Sunt enim *per Axioma 2*.

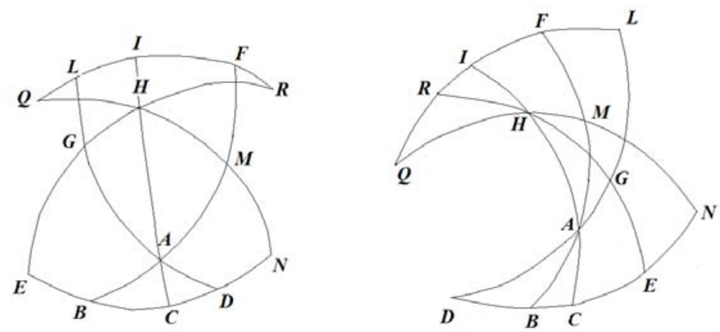
$$Prop.\left\{\begin{array}{lll} sinus & DC\,.\,BC & \\ Tang. & CA\,.\,CA & \textit{Ergo exclusis} \\ Radius & DE\,.\,BN & \textit{intermediis} \\ Tang. & EG\,.\,NM & \textit{erunt} \end{array}\right. Prop.\left\{\begin{array}{l} sinus\ DC. \\ sinus\ BC. \\ Tang.\ NM. \\ Tang.\ EG. \end{array}\right. \text{Et alterne } Prop.\left\{\begin{array}{l} BC. \\ DC. \\ EG. \\ NM. \end{array}\right. \text{Et contra.}$$

Quod erat demonstrandum.

Consectarium 7.

In Triangulis Obliquangulis: *Si Perpendicularis ducatur ab angulo verticali in latus oppositum (si opus sit continuatum;) Sinus complementorum angulorum verticalium,Tanentibus laterum sunt reciproce proportionales. Et contra.*

In Triangulo Obliquangulo ABD, a vertice A peripheria describantur QIR, nec non a punctus angularum B & D peripheriae NMQ, EGR. Continuatisque lateribus AB, AD una cum perpendiculari AC ad F, L & I usque ; erunt Tangentes LG, FM aequales laterius AD, AB. Erunt etiam arcus IR, IQ_Sinus complementorum angulorum verticalium CAD, CAB, quorum mensurae sunt LI, IF. Aio itaque Sinus IR, lQ complementorum angulorum verticalium Tangentiubus laterum FM, LG reciproce proportionales esse. Sunt enim *per Axioma* 2.

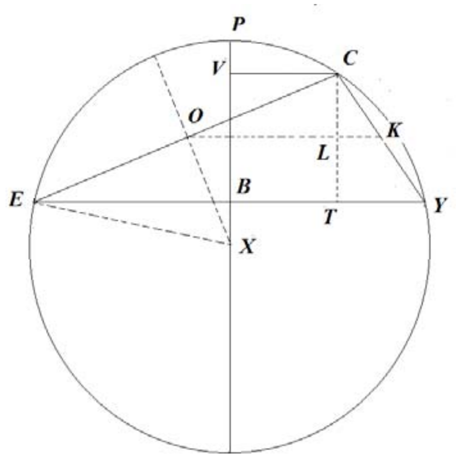


$$Prop.\left\{\begin{array}{lll} sinus & QR\,.\,RI & \\ Tang. & IH\,.\,HI & \textit{Ergo exclusis} \\ Radius & QF\,.\,RL & \textit{intermediis,} \\ Tang. & FM\,.\,LG & \textit{erunt} \end{array}\right. Prop.\left\{\begin{array}{l} sinus\ QI. \\ sinus\ RI. \\ Tang.\ LG. \\ Tang.\ FM. \end{array}\right. \text{Et inverse } \left\{\begin{array}{l} RI. \\ QI. \\ FM. \\ LG. \end{array}\right. \text{Et contra.}$$

Quod erat demonstrandum.

LEMMA.

Datis duarum arcuum Sinibus versis , Erunt Sinus recti semisummae & semidifferentiae eorundem arcuum medii proportionales inter Sinum Totum & semissem differentiae Sinuum versorum.



Sunto arcus dati PE, PC & eorum Sinus versis PB, PV, quorum differentia est BV. Summa datorum arcuum sit EPC ; Differentia eorundem CY. Esto Sinus semissummae EO vel OC ; Sinus semidifferentiae CK. Aio EO, CK, medios proportionales esse inter XE, CL.
Sunt enim anguli EXO, CYT, CKL aequales per 2, Prop.3. Et anguli ad O & L recti. Sunt igitur Triangula XEO, KCL similia, & XE, EO ; KC, CL proportionales. Quod erat demonstrandum.

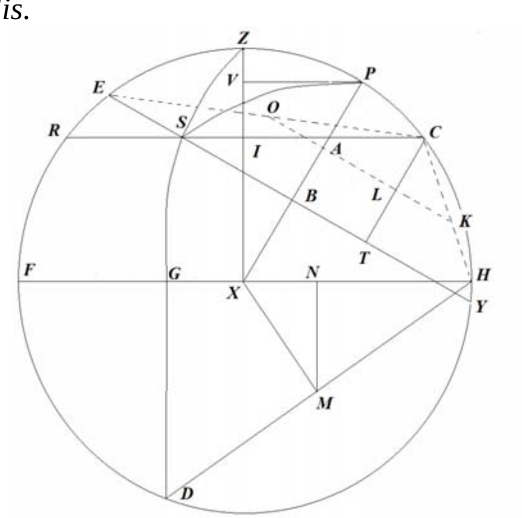
Consectarium.

Si Triangulum ECY Circulo inscribatur, & ab angulo quolibet ECY ducatur perpendicularis CT in latus oppositum EY : Semissis crurum dicti anguli OC, CK sunt medii proportionalis inter semissem perpendicularis CL, & semidiametrum XE. Rectangula ergo OC, CK : XE, CL aequantur.

PROPOSITIO.

In trianglis Sphaericis universis quibus nota sunt latera, erunt

Prop. {
Rectangulum e Sinibus laterum angulum verticalem comprehendentium.
Quadratum Radii.
Rectangulum e Sinibus semisummae & semidifferentiae basis & differentiae crurum
Quadratum Sinus semissisanguli verticalis.



Esto Triangulum datorum laterum PZS & sit angulus verticalis PZS. Erunt ZS alterum Crus & ZC aequalia, & PC differentia Crurum : Erunt etiam PS Basis anguli verticalis & PY, PE aequales, & CE summa Basis & differentiae Crurum, & CY differentia earundem. Haec omnia cognita sunt. Ducantur PV Sinus cruris PZ, & CI Sinus reliqui Cruris ZS qui continuetur in S. Ducantur item CT perpendicularis rectae EY, & OLK bisecans rectas EC, SC, CT, CY. Sit etiam arcus HD mensura anguli verticalis PZS, & recta MN perpendicularis rectae HG bisecet rectas HG,Hd ; Aio

$$\begin{cases}\text{Rectangulum e Sinubus datorum laterum PV, CI.}\\ \text{Quadratum Radii PX.}\\ \text{Rectangulum e Sinubus ; summae \& differentiae Basis \& differentia Crurum , OC, OK.}\\ \text{Quadratum Sinus semissis anguli verticali HM.}\end{cases}$$

Sunt enim Triangula XPV, XAI, SAB, SCT aequiangula, ac propterea

$$\textit{Prop.}\begin{cases}\text{CT}\\ \text{CS}\\ \text{PV}\\ \text{PX}\end{cases} \quad \textit{Item Prop.}\begin{cases}\text{CS.}\\ \text{HG.}\\ \text{CI.}\\ \text{HX.}\end{cases} \text{Et oblongata ista proportionalia erunt} \begin{cases}\text{Oblong: CT, CS.}\\ \text{Oblong: CS, HG.}\\ \text{Oblong: PV, CI.}\\ \text{Oblong: PX, HX.}\end{cases}$$

Sunt etiam

$$\textit{Prop.}\begin{cases}\text{CT.}\\ \text{HG.}\\ \text{PV, CI obl:}\\ \text{PX quadrat.}\end{cases} \text{vel } \textit{Prop.}\begin{cases}\text{CL. . .}\\ \text{HN. . .}\\ \text{PV,CI obl.}\\ \text{PX Quadrat :}\end{cases} \text{(sunt enim semisses rectarum CT,HG.)} \quad \text{Item } \textit{Prop.}\begin{cases}\text{CL.}\\ \text{HN.}\\ \text{CL, HX obl:}\\ \text{HN, HX obl:}\end{cases}$$

$$\text{At vero aequantur per Lemma, oblonga} \begin{cases}\text{CL, HX.}\\ \text{OC, CK.}\end{cases} \text{Item} \begin{cases}\text{Obl.HN, HX. quia sunt ut altitudines CL, HN.}\\ \text{Quadratum HM. } \textit{per Consec.}\ 2.el.4l.8.\textit{Rami.}\end{cases}$$

Sunt igitur

$$\textit{Prop.}\begin{cases}\text{Oblongum PV, CI.}\\ \text{Quadratum PX.}\\ \text{Oblongum OC, CK.}\\ \text{Quadratum HM.}\end{cases}$$

Quod erat demonstrandum.

CAPUT III.

De Triangulorum Sphaericorum Rectangulorum Dimensione.

Adminiculis hisce instructi, intima Triangulorum Sphaericorum adita penetrare non erit difficile : Primo itaque Rectangulorum solutiones aggredimur.
Cum in omni Triangulo, tria sint latera & totidem anguli, si inTriangulo Sphaerico harum six partium aliqua sit gradus 90, reliquaerum partium duae quae proxime adjacent parti graduum 90 una cum complementis trium reliquaerum, ducuntur a viro Clarissimo *Barone Marchistonii* quinque partes circulares, quaerum si duae datae fuerint, quaelibct e tribus reliquis facile invenitur. Sunt enim

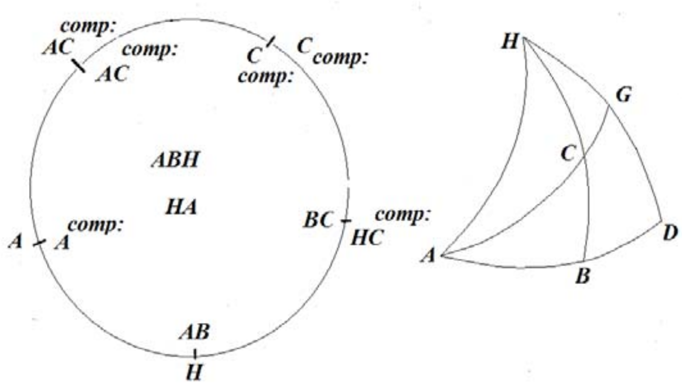
I. *Radius & Sinus cuiuslibet partes assumptae, medii proportionales, inter Tangentes partium utrinque assumptae parti proximarum vel circumpositarum*.

Sunt etiam (si pars assumta sit remotior a duabus reliquis)

2. *Radius & Sinus partes assumptae, medii proportionales inter Sinus complementorum partium oppositarum*

Et si partes sumantur quatuor

3. *Sinus complementorum partium mediarum sunt medii proportionales inter Tangentes extremarum*.



Tertia haec propositio licet vera sit, nullum tamen commodum habet usum, cum duae datae partes sufficiant ad expeditam inventionem tertiae partis cuiuscunque.

Quo haec omnia sint magis manifesta, partes circulares Trianguli Rectanguli ABC sitae sunt intra circulom, suntque singula latera quadrantis minora, & anguli duo acuti :
Partes autem Trianguli Obliqanguli ACH sunt positae extra Circulum, suntque latera duo minora quadrantibus, & anguli his lateribus oppositi sunt acuti, tertius angulus quadranti oppositus est obtusus. Atque ut Propositionum istarum veritas facilius percipiatur, harum partium Logarithmos apponendos censui, ut eorum aequalitatae possimus de Sinuum & Tangentium proportionibus indicare.

		Logar. Sinuum	*Logar. Tangentium.*
Latus AB vel angulus AHB	gr. 47 000	9,86412,74638	10,03034,41335
BD complementum	43 000	9,83378,33303	9,96965,58665
Latus BC vel compl. lateris CH	22 891768	9,58994,02104	9,62556,68364
CH compl.	67 108232	9,96437,34001	10,37443,31636
Complementum Anguli ACB	70 062238	9,97315,72316	10,44040,38970
	19 937762	9,53275,33346	9,55959,61030
Complementum Lateris AC	38 923713	9,79815,67304	9,997I8,64982
Latus AC	51 076187	9,89097,02061	10,09281,35018
Complementum Anguli BAC	60 00	9,93753,06317	10,23856,06274
Complem.	30 00	9,69897,00043	9,76143,93726

Atque ut utriusque Propositionis exemplum videamus, sunto datae partes AB; BC, & quaeratur pars proxima, videlicet complementum Anguli ACB.

Proport.	*Tangens* AB gr. 4700		10,03034,41335
	Radius		10,00000,00000
	Sinus BC.	22 891768	9,58994,02104
	Tangens Compl. ACB	19 937762	9,55959,60769

Sin quaeratur latus AC *per Propositionem secundam*;

Proport.	*Sinus Totus*		10,00000,00000
	Sinus Compl. BC gr.	67 108232	9,96437,34001
	Sinus Compl. AB .	43 000	9,83378,33303
	Sinus Compl. AC .	38 923713	~~1~~9,79815,67304

Si quaeratur angulus BAC;

Proport.	*Tangens* BC	gr.22 891768	9,62556,68364
	Radius		10,00000,00000
	Sinus AB	47 000	9,86412,74638
	Tangens Compl. BAC	60 000	10,23856,06274

Idcirco in Triangulo Rectangulo datis duabus e quinque partibus circularibus, inveniri poterunt tres reliquae.
Quod si datae sint duae partes circulares in Triangulo Quadrantali ACH, inveniuntur tres reliquae eodem fere modo.

Sunto dati anguli AHC gr.47000. HAB gr. 60000, & quaeratur Angulus ACH.

Proport.	*Sinus Totus*	. . . .		10,00000,00000
	Sinus Compl. AHC gr.	43 000 .		9,83378,33303
	Sinus Compl. HAC .	30 000 .		9,69897,00043
	Sinus Compl. ACH .	19 937762		19,53275,33346

Excessus anguli obtusi ACH supra rectum, vel complementum anguli ACB.

Si quaeratur AC. Partes datae, HAC, HCA; pars quaesita complementum AC. Hae tres sunt partes proximae vel contiguae, intermedia est HAC.

Proport.	*Tangens Anguli* AHC	gr.47 000 . .		10,03034,41335
	Radius	. . . .		10,00000,00000
	Sinus HAC 47 000 . . .		60 000 . .	9,93753,66317
	Tangens Compl. AC . . .		38 923713 .	9,90718,64982
	Ergo AC est		51 070287	

Si quaeratur HC, non differt investigationis medus ab eo qui traditur pro inventione lateris AC. Anguli HAC & AHC & latus HC sunt partes circulares proximae, & idcirco

Proport.	*Tangens Anguli* HAC	gr.60 000 . .	10,23856,06274
	Radius	. . . .	10,00000,00000
	Sinus CHA . .	47 000 . .	9,86412,74638
	Tangens Complem.	HC 22 891768.	9,62556,68364
	Ergo HC est . .	67 108232	

Poterimus igitur in huiusmodi Triangulis, uti vel hac generali regula licet magis obscura, vel particularibus illis quae singulis datorum & quae sitorum casibus inserviunt, & sigillatim demonstrationibus firmantur. Generalis autem illa regula poterit non difficulter ab hisce particularibus demonstrationes, mutare prout particulares casus exigere videbuntur, ut eadem utrobique certitudo retineatur. Sit itaque.

PROBL. I.

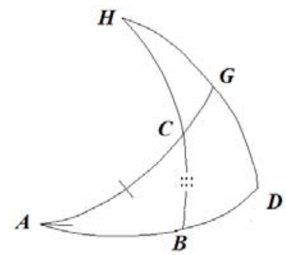
Datis Hypotenusa & Angulo ipsi contermino, Quaeritur CRUS *angulo dato oppositum*.

In Triangulo Rectangulo ABC quaeritur Crus BC

E datis { Hypotenusa AC 51076287
Angulo BAC 30000

Termini Rationis.

Proport. { *Sinus Totus.*
Sinus anguli dati.
Sinus Hypotenusae.
Sinus Cruris quaesiti.



Illustratio per numeros.

				Sinus	*Logarithmi Sinuum.*
Proport.	Sinus Totus	AG	90 000	1000000,00000	10,00000,00000
	Sinus anguli.	GD.	30 000	500000,00000	9,69897,00043
	Sinus Hypot.	AC.	51 076287	777983,18409	9,89097,02062
	Sinus cruris	CB.	22 891768	388991,59204	1 9,58994,02504

PROBL. 2.

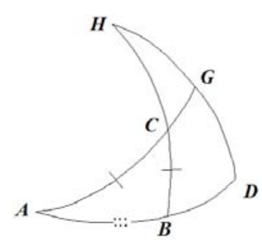
Datis Hypotenusa & Crure, Quaeritur CRUS *reliquum.*

In Triangulo Rectangulo ABC quaeritur Crus AB

Datis { Hypotenusa. AC. 51 076287
Crure BC. 22 891768

Termini Rationis.

Proport. { *Sinus complementi Cruris dati.*
Sinus Totus.
Sinus complementi Hypotenusae
Sinus complementi Cruris quaesiti.



Illustratio per numeros.

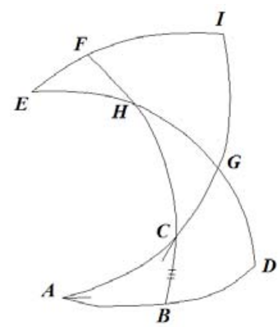
				Sinus	*Logarithmi Sinuum.*
Proport.	Sinus	HC	67 108232	92124,13016	9,96437,34001
	Sin. Totus	HB	90 000	1,00000,00000	10,00000,00000
	Sinus	CG	38 923713	62828,50929	9,79815,67532
	Sinus	BD	43 000	68199,83600	9,83378,33531
	Compl.	AB	47 000 Crus quaesitum.		

PROBL. 3.

Datis Angulis obliquis, Quaeritur CRUS *utrumlibet*.

In Triangulo Rectangulo ABC quaeritur Crus BC

Datis Angulis { ACB . 70 062238
BAC . 30 000 }



Termini Rationis.

Proport. {
Sinus anguli Cruri quaesito contermini.
Sinus compl. Cruris quaesiti oppositi.
Sinus Totus.
Sinus complimenti Cruris quaesiti. }

Illustratio Arithmetica.

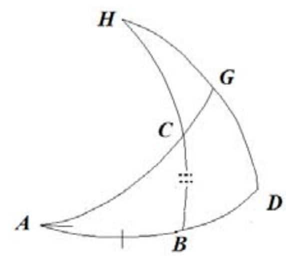
				Sinus	*Logarithmi Sinuum.*
Proport.	Sinus	FI	70 062238	94006,35859	9,97315,72287
	Sinus	HG	60 000	86602,54037	9,93753,06317
	Sin. Totus	FC	90 000	1,00000,00000	10,00000,00000
	Sinus	HC	67 108232	92124,13016	9,96437,34030
	Compl.	CB	22 891768 Crus quaesitum.		

PROBL. 4.

Datis Crure & Angulo Cruri dato contermino, Quaeritur CRUS *reliquum*.

In Triangulo Rectangulo ABC quaeritur Crus BC.

Datis { Crure AB 47 000
Angulo BAC 30 000



Termini Rationis.

Proport. { *Radius.*
Tangens anguli obliqui dati.
Sinus cruris dati
Tangens cruris quaesiti.

Illustratio Arithmetica.

				Sin. & Tang.	Log. Sin. & Tang.
Proport.	Radius	AD	90 000	1,00000,00000	10,00000,00000
	Tangens	DG	30 000	57735,02692	9,76143,93726
	Sinus	AB	47 000	73135,37016	9,86412,74638
	Tangens	CB	22 891768	42224,72564	9,62556,69364

PROBL. 5.

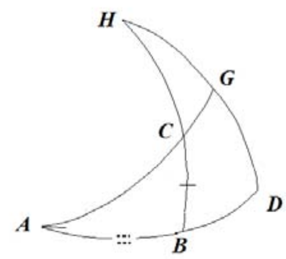
Datis Crure & Angulo ipsi opposito, Quaeritur CRUS *reliquum*.

In Triangulo Rectangulo ABC quaeritur Crus AB.

Datis { Crure BC 22 891768
Angulo BAC 30 000

Termini Rationis.

Proport. { *Tangens anguli obliqui dati.* / *Radius.* / *Tangens cruris dati.* / *Sinus cruris quaesiti.* }



Illustratio Arithmetica.

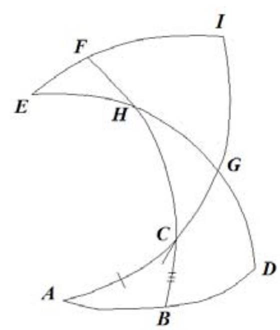
					Sin.&Tang.	*Log. Sin.&Tang.*
Proport.	Tangens	DG	30 000	. .	57735,02692 . .	9,76143,93726
	Radius	AD	90 000	. .	1,00000,00000 . .	10,00000,00000
	Tangens	BC	22 891768	. .	42224,72564 . .	9,62556,69364
	Sinus	AB	47 000	.	73135,37016 . .	9,86412,74638

PROBL. 6.

Datis Hypotenusa & Angulo, Quaeritur CRUS *angulo dato conterminum.*

In Triangulo Rectangulo ABC quaeritur Crus BC

Datis	Hypotenusa	AC	51 076287
	Angulo	BCA	70 062238



Termini Rationis.

Proport. { *Radius.* / *Tangens Hypotenusae.* / *Sinus complimenti anguli dati.* / *Tangens Cruris quaesiti.* }

Illustratio per numeros.

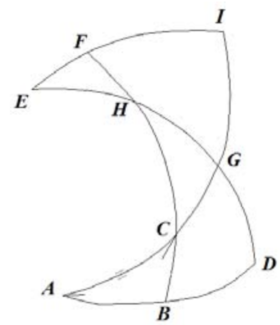
				Tangent. & Sinus	Logar.Tang.& Sin.
Proport.	Radius	EI	90 000	100000,00000	10,00000,00000
	Tangens	IG	51 076287	123826,46138	10,09281,35061
	Sinus	EF	19 937762	34099,91921	9,53275,33303
	Tangens	FH	22 891768	42224,71767	19,62556,68364

PROBL. 7.

Datis Angulis obiquis, Quaeritur HYPOTENUSA.

In Triangulo Rectangulo ABC quaeritur Hypotenusa AC

Datis anglis { BAC 30 000; ACB 70 062238 }



Termini Rationis.

Proport.
- *Tangens anguli alterutri.*
- *Radius.*
- *Tangens complimenti reliqui anguli.*
- *Sinus complimenti Hypotenusae.*

Illustratio Arithmetica.

				Tangent. & Sinus	Logar.Tang.& Sin.
Proport.	Tangens	IG	70 062238	275679,13657	10,44040,38970
	Radius	IC		100000,00000	10,00000,00000
	Tangens	GH	60 000	173205,08076	10,23856,06274
	Sinus	GC	38 923713	62828,50929	9,79825,67304

PROBL. 8.

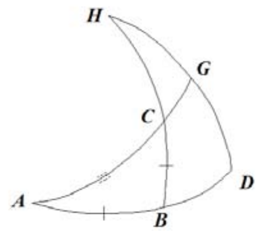
Datis Cruribus, Quaeritur HYPOTENUSA.

In Triangulo Rectangulo ABC, quaeritur Hypotenusa AC

Datis Cruribus { AB 47 000; BC 22 901768 }

Termini Rationis.

Proport. {
Sinus Totus.
Sinus complementi Cruris alterius.
Sinus complimenti Cruris reliquli.
Sinus complimenti Hypotenusae quaesitae.
}



Illustratio per numeros.

				Sinus	*Logarithmi Sinuum.*
Proport.	Sin. Totus	HB		. . 1,00000,00000	. . 10,00000,00000
	Sinus	BD	43 000	. . 68199,83600	. . 9,83378,33303
	Sinus	CH	67 108232	. . 92124,13016	. . 9,96437,34030
	Sinus	CG	38 923713	. 62828,50929	. . 19,79815,67304
Compl. est		CB	51 076287	Hypotenusa quaesita.	

PROBL. 9

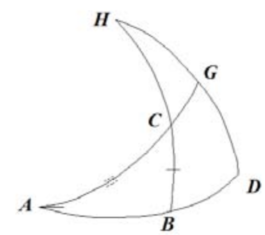
Datis Crure & Angulo ipsi oppositi, Quaeritur HYPOTENUSA.

In Triangulo Rectangulo ABC quaeritur Hypotenusa AC

Datis { Crure . BC 22 891768
Angulo BAC 30 000

Termini Rationis.

Proport. {
Sinus anguli oblique dati.
Sinus Totus.
Sinus Cruri dati.
Sinus Hypotenusae quaesitae.



				Sinus	*Logar.Sin.*
Proport.	Sinus	IG	30 000	50000,00000	9,69897,00043
	Radius	EI	90 000	100000,00000	10,00000,00000
	Sinus	CB	22 891768	38899,15834	9,58994,02104
	Sinus	AC	51 076287	77798,31669	9,89097,02061

PROBL. 10

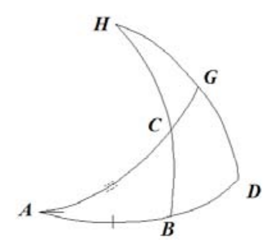
Datis Crure & Angulo ipsi contermino, Quaeritur HYPOTENUSA.

In Triangulo Rectangulo ABC quaeritur Hypotenusa AC

Datis { Crure AB 47 000
Angulo BAC 30 000

Termini Rationis.

Proport. {
Radius.
Tangens complementi Cruris dati.
Sinus complementi Anguli dati.
Tangens complementi Hypotenusae quaesitae.



Illustratio per numeros.

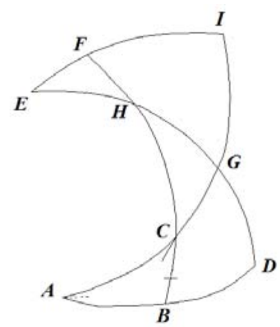
					Sinus	*Logar.Sin.*
Proport.	Radius	HD	90 000	.	100000,00000 . .	10,00000,00000
	Tangens	DB	43 000	.	93251,50861 . .	9,96965,58665
	Sinus	HG	60 000	.	86602,54037 . .	9,73753,06317
	Tangens	GC	38 923713	.	80758,18517 . .	~~1~~9,90718,64982
	Compl. est	AC	51 076287	Hypotenusa quaesita.		

PROBL. 11.

Datis Angulo & Crure ipsi contermino, Quaeritur ANGULUS *reliquus*.

In Triangulo Rectangulo ABC quaeritur Angulus BAC

Datis	Angulo	ACB	70 062238
	Crure	BC	22 891768



Termini Rationis.

Proport.
- *Sinus Totus.*
- *Sinus anguli obliqui dati.*
- *Sinus complimenti Cruris dati.*
- *Sinus complimenti anguli quaesiti.*

Illustratio Arithmetica.

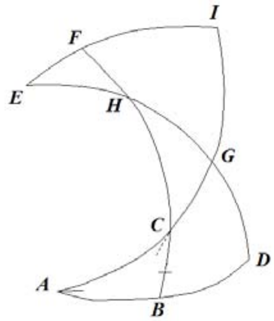
					Sinus	*Logar.Sinuum.*
Proport.	Sinus Totus	FC	90 000	. .	100000,00000 . .	10,00000,00000
	Sinus	FI	70 062238	. .	94006,35859 . .	9,97315,72316
	Sinus	HC	67 108232	. .	92124,13016 . .	9,96437,34001
	Sinus	HG	60 000	.	96602,54037 . .	~~1~~9,93753,06317
	Compl. est	GD	30 000	Anguli quaesiti mensura.		

PROBL. 12.

Datis Angulo & Crure ipsi opposito, Quaeritur ANGULUS *reliquus*.

In Triangulo Rectangulo ABC quaeritur Angulus ACB

Datis	Angulo	BAC	30 <u>000</u>
	Crure	BC	22 <u>891768</u>



Termini Rationis.

Proport.
- *Sinus complimenti Cruris dati.*
- *Sinus complimenti Anguli dati.*
- *Sinus Totus.*
- *Sinus anguli quaesiti.*

Illustratio per numeros.

				Sinus	*Logar.Sinuum.*
Proport.	Sinus	HC	67 <u>108232</u>	92124,13016	9,96437,34001
	Sinus	HG	60 <u>000</u>	96602,54037	9,93753,06317
	Sinus Totus	FC	90 <u>000</u>	100000,00000	10,00000,00000
	Sinus	FI	70 <u>062238</u>	94006,35859	9,97315,72287

PROBL. 13

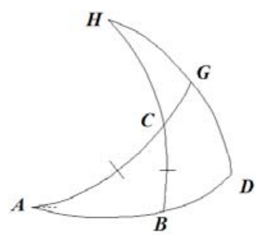
Datis Hypotenusa &Crure, Quaeritur ANGULUS *Cruri dato oppositus*.

In Triangulo Rectangulo ABC quaeritur Angulus BAC

Datis	Hypotenuse	AB	51 <u>072687</u>
	Crure	BC	22 <u>891768</u>

Termini Rationis.

Proport.
- *Sinus Hypotenusae.*
- *Sinus Cruris.*
- *Sinus Totus.*
- *Sinus anguli quaesiti.*



Illustratio per numeros.

				Sinus	*Logar.Sin.*
Proport.	Sinus	AC	51 076287	77798,31814	9,89097,02061
	Sinus	CB	22 891768	38899,15834	9,58994,02104
	Sin. Totus	AG	90 000	100000,00000	10,00000,00000
	Sinus	GD	30 000	50000,18517	9,69897,00043

PROBL. 14

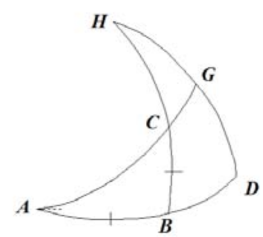
Datis Cruribus, Quaeritur ANGULUS *alteruter*.

In Triangulo Rectangulo ABC quaeritur Angulus BAC

Datis Cruribus { AB 47 000; BC 22 891768 }

Termini Rationis.

Proport. {
Sinus Cruris Angulo quaesito contermini.
Tangens Cruris reliqui.
Radius.
Tangens anguli quaesiti.
}



Illustratio per numeros.

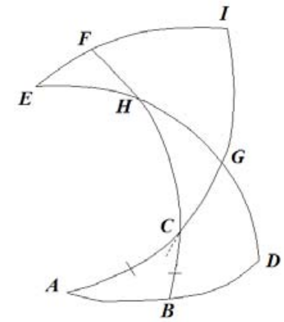
				Sinus	*Logar.Sin.*
Proport.	Sinus	AB	47 000	73135,37016	9,86412,74638
	Tangens	BC	22 891768	42224,72384	9,62556,78364
	Radius	AD	90 000	100000,00000	10,00000,00000
	Tangens	DG	30 000	57735,02692	9,76143,93726

PROBL. 15.

Datis Hypotenusa & Crure, Quaeritur ANGULUS *ab iisdem comprehensus*.

In Triangulo Rectangulo ABC quaeritur Angulus ACB

Datis	Hypotenusa	AC	51 076287
	Crure	BC	22 891768



Termini Rationis.

Proport.
- *Tangens Hypotenusae.*
- *Radius.*
- *Tangens Cruris.*
- *Sinus complimenti anguli quaesiti.*

Illustratio Arithmetica.

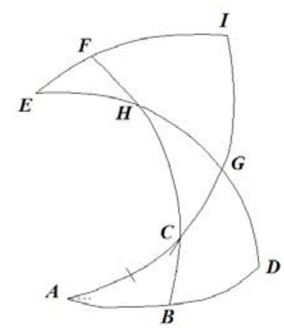
					Sinus	*Logar.Sinuum.*
Proport.	Tangens	IG	51 076287	. .	123826,46138 . .	10,09281,35061
	Radius	IE	90 000	. .	100000,00000 . .	10,00000,00000
	Tangens	FH	22 89768	. .	4,2224,72384 . .	9,62556,68364
	Sinus	FE	19 937762	. .	34099,91921 . .	9,53275,33303
	Compl. est	FI	70062238	Mensura anguli quaesita ACB.		

PROBL. 16.

Datis Hypotenusa & Angulo, Quaeritur ANGULUS *reliquus*.

In Triangulo Rectangulo ABC quaeritur Angulus BAC

Datis	Hypotenusa	AC	51 076287
	Angulo	ABC	70 062238



Termini Rationis.

Proport.
- *Radius.*
- *Tangens anguli obliqui dati.*
- *Sinus complimenti Hypotenusae.*
- *Tangens complimenti Anguli quaesiti.*

Illustratio numeros.

					Sin. & Tang.	Log. Sin. & Tang.
Proport.	Radius	CI	90 000	. .	100000,00000	. . 10,00000,00000
	Tangens	FI	70 062238	. .	275679,13657	. . 10,44040,38970
	Sinus	CG	38 923713	. .	62828,50920	. . 9,79815,63704
	Tangens	GH	60 000	. .	173205,08076	. . 10,23856,06274
	Compl. est	GD	30 000	Mensura anguli quaesita CAB.		