§2.1 Chapter Two

Concerning How the Subtended Chords Are Sought in the Writings of the Ancients.

1. *Prop. Ptol.*

Ptolemy[1] showed, First, following Euclid, the way the [lengths of the] sides of particular figure inscribed in a circle could be found, namely the equilateral triangle by *Prop. 12. Book 13*. The square by *Prop. 6. Book 4.* The Pentagon, Hexagon, and Decagon by *Prop 9. & 10. Book 13.*

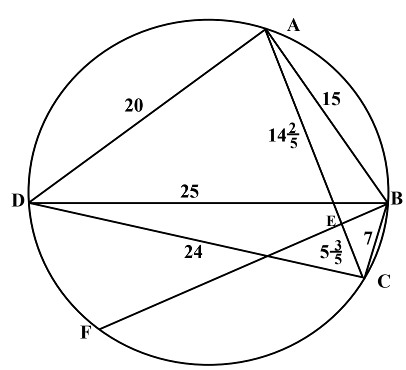
	Let the Diameter be	200000.
The Sides Will be	*of the Hexagon*	100000
	of the Square	141213562373095
	of the Triangle	1732050807568877
	of the Decagon	0618033988749895
	of the Pentagon	1175570504584946

2. *Prop. Ptol.*

Secondly, *by Prop. 30, Book 3: from Euclid,* he teaches*, the Square of the Given Subtended Chord, taken from the Square of the given Diameter, leaves the Square of the Complement of the Subtended [Chord] to the Semi-circle.* And according to this method the Subtended Chords of 60, 90,120, 36, 72 & of the Complements 144 & 108 degrees can be found.

3. Thirdly, this *Lemma* is proposed as it is required in subsequent proofs. *If a quadrilateral [lit.* quadrangle*] is inscribed in the Circle, the Rectangle comprising the Diagonals is equal to the [the sum of the] Rectangles comprising the opposite sides.*

As in circle ABCD, let the diagonals be DB, 25; AC, 20. The Rectangle from the Diagonals 500 is equal to [the sum of] the Rectangles AB, DC, 360. And AD, BC, 140. Indeed, the line BE is drawn such that the angles ABE, DBC should be equal. The Triangles DBC, ABE are similar; because the Angles CDB, CAB are equal, when they are in the same section, by *Prop. 21, Book 3*. & CBD, EBA shall be equal from the construction; the remaining AEB, DCB therefore are equal by *Prop. 31, Book 1.*



[Figure 2-1]

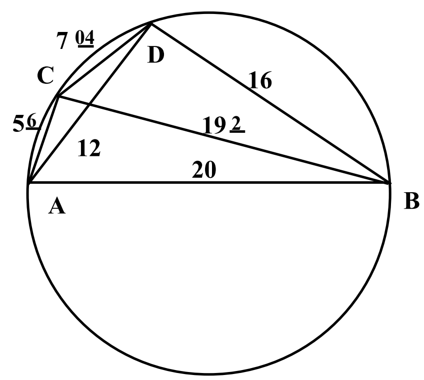
BD, DC : BA, AE are therefore in proportion: & the Rectangles BD, AE; DC, BA are equal from *Prop.16, Book 6.* Likewise, the Triangles BDA, BCE are similar, because the Angles BCE, BDA are on the same section, & therefore equal. And as ABE, CBD shall be equal from the construction, with the common part DBE taken away, the remaining [angles] CBE, DBA are equal; & therefore CB, CE : BD, DA, are proportionals, & the rectangles CB, DA; CE, BD are equal; but [the sum of] the rectangles DB, EA; DB, CE, is equal to the rectangle DB, CA by *Prop. 1, Book 2, Euclid:* & therefore the rectangle with the Diagonals DB, CA is equal to [the sum of] the Rectangles DC, BA, & BC, DA, which had to be shown[2].

DB 25		DC 24	
	Rect.: DB, AE. 360		Rect.: DC,AB.: 360
AE $14^2/_5$		AB 15	
DB 25		DA 20	
	Rect.: DB, CE. 140		Rect.: DA,BC.140
CE $5^3/_5$		BC 7	
DB 25			
	Rect.: DB, AC. 500		
AC 20			

[Table 2-2]

4. Fourthly, as permitted by the above *Lemma*, from the two given arcs with unequal Subtended Chords, the Subtended Chord of the difference can be found.

Let the given Diameter [Figure 2-2] be AB, 20. Of the Inscribed [Chords] AD, 12. AC $5^3/_5$. CD is sought. Firstly, DB & CB can be found by the *2nd Prop.* Since the angles ADB, ACB are in the semi-Circle, they are right, by *Prop. 30, Book 3*. & therefore if the Square AD, 144, is taken from the Square AB, 400, there remains the Square DB, 256. *By Prop. 47, Book 1*. DB is therefore 16. By the same method, by taking the Square AC, $31^9/_{25}$ from the Square AB, 400, there remains the square CB, $368^{16}/_{25}$, or $368\underline{64}$. And CB will be $19\underline{2}$.

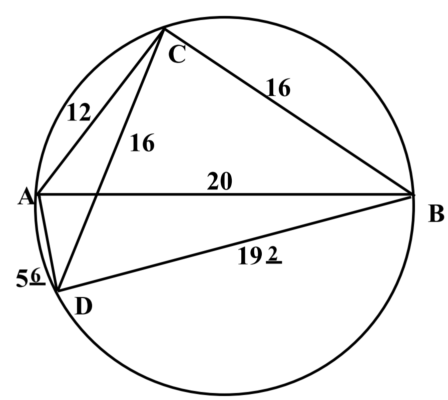


[Figure 2-2]

Therefore given AB, 20; AD, 12; AC, $5\underline{6}$; & by finding DB, 16. CB, $19\underline{2}$. CD is sought by the preceding Lemma. The rectangle[3] AC, DB, $89\underline{6}$, is taken from the rectangle AD, CB, from the diagonals, $230\underline{4}$, will leave the rectangle $140\underline{8}$, comprising the diameter AB, 20 and CD. Therefore with $140\underline{8}$ divided by 20, the Quotient $7\underline{04}$ will be the length of the straight line CD, which is required.

5. By the same Lemma: From two given arcs with Subtended Chords, the Subtended Chord of the sum is found[4].

Let the given diameter [Figure 2-3] be AB, 20. AC, 12. AD, $5\underline{6}$. DC is sought. In the first place CB, DB are to be found, as before[5]: and this gives CB as 16, DB as $19\underline{2}$ We have therefore AB, AC, and AD given; & CB and DB have been found. The rectangles AD, CB, $89\underline{6}$, & AC, BD, $230\underline{4}$ are taken, the sum of which is $320.\underline{0}$, which is equal to the rectangle



[Figure 2-3]

comprising the Diameter AB & the chord sought DC, *by the preceding Lemma, Prop. 3*. Therefore given the Diameter AB, 20; let it divide 320.0 - the rectangle taken from the diameter and DC together, then the quotient 16 is the Subtended Chord DC sought[6].

§2.2 Notes On Chapter Two

1 The First Lemma looks at the problem of finding the lengths of the sides of some regular figures inscribed in a circle of unit radius. The Second Lemma applies the Theorem of Pythagoras to the sides of the appropriate right angled triangle to determine the ratios for the specified angles.

2 In more modern terminology, we have AE.BD = AB.CD, and CE.BD = CB.DA; hence, (AE + EC).BD = AB.CD + CB.DA, or AC.BD = AB.CD + CB.DA, as required. See the *CRC Handbook of Modern Mathematics*, p.84, for the 'whole story' on this theorem from a modern perspective. This theorem is the main 'workhorse' used by Ptolemy in the construction of his Table of Sines alluded to in the first chapter.

3 These are referred as 'oblongs' in the text always.

4 This should be called the 5^{th} Lemma, or the converse of Lemma 4.

5 That is, we find the remaining sides of the cyclic quadrilateral first.

6 This is a rather perfunctory and incomplete look at Ptolemy's method. Ptolemy uses these results to find the lengths of half chords in terms of the known chords of larger angles: in this way he subdivides 12^0 formed from the difference of 72^0 from the regular pentagon and 60^0 from the regular hexagon to find the half chords corresponding to 6^0, 3^0, $1{}^1/_2{}^0$, ${}^3/_4{}^0$ and eventually for ${}^1/_2{}^0$, which he uses as the unit to build up his table. See his *Almagest I* for details. Briggs, however, wishes to move on to his own methods…..