

§13.1

Chapter Thirteen

Really we can return to the sines. If it is desired to put together a whole table of sines for thousandths of a degree, [you may proceed as follows:] For a given chord of 60 : 0': degrees the chord for 20:0': is found by trisection, which by five-fold multiplication gives the 100: 0': Chord . Then by bisection, the square of the 50: 0': Chord is found, and subsequently those of | 25: 0': | 12: 30': | 6: 15': | . These Squares give the 50: 0': | 25: 0': | 12: 30': | 6: 15': chords themselves. And these chords give by triplication the chords 18: 45': | 56: 15' | 37: 30': | 75: 0' | 112: 30' | .

And by 5-fold multiplication the chords 31: 15' | 62: 30': | 125: 0': | 93: 45'.

And by 7-fold multiplication the chords 43: 45': | 87: 30': | .

By means of multiplication [of 6: 15':] by 11. 13. 17. 19. the chords of 68: 45': | 81: 15': | 106: 15': | 118: 45': | are found.

The halves of these chords, namely the sines of $3\frac{1}{8}$. $6\frac{2}{8}$. $9\frac{3}{8}$. $12\frac{1}{2}$ degrees, etc., are placed below here.

Deg.	1/1000th parts	Sines of degrees and thousandth parts	Deg.	Min.	Sec.
3	125	545145016380063218757	3	7	30
6	250	1088668748519645759679	6	15	0
9	375	1628954733945887394808	9	22	30
12	500	2164396139381028797595	12	30	0
15	625	2693400539532260178082	15	37	30
18	750	3214394653031615807011	18	45	0
21	875	3725829021441029702515	21	52	30
25	000	4226182617406994361870	25	0	0
28	125	4713967368259976485564	28	7	30
31	250	5187732581605214495200	31	15	0
34	375	5646069259743004405407	34	22	30
37	500	6087614290087206394161	37	30	0
40	625	6511054499119490930562	40	37	30
43	750	6915130557822693777612	43	45	0
46	875	7298640726978356573501	46	52	30
50	000	7660444431189780352024	50	0	0
53	125	7999465651001296901396	53	7	30
56	250	8314696123025452370788	56	15	0
59	375	8605198338560790089653	59	22	30
62	500	8870108331782217010546	62	30	0

[Table 13-1]

From these few given chords, by dividing the quadrant into 144 equal parts, of these I have cared to give their own individual sine. First by the quinquisection rule propounded [on] page 38 [not a valid reference, as this refers to Chapter 12, whereas the rule is in Chapter 5], thus I can fill out all the sines for the first hundred parts, as far as the sine of 62: 30': Degrees. Then the rest of the 44 Sines as far as the end of the Quadrant, being found* by the final section of the following chapter. For given the Sines $37\frac{5}{10}$ and $56\frac{25}{10}$, of these the sum is equal to the sine $63\frac{75}{10}$. , as we see here. By the same method the rest may be computed. [For $\sin(60 + x) - \sin(60 - x) = \sin(x)$]

Sine 3: 45' 654031292301430684
 Sine 56: 15' 8314696123025452371
 Sine 63: 45'8968727415326883055
 [Table 13-2]

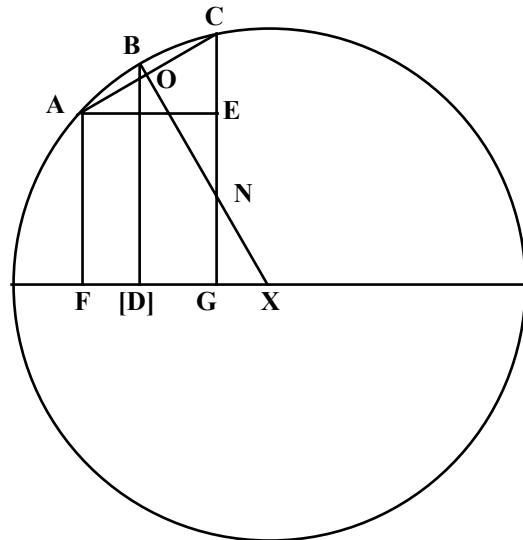
* These few were bound to be present at the end of Chapter 2

6. If the sum of two arcs is equal to 60:0': degrees, the sum of the sines of the given arcs is equal to the sine of the arc composed from 60 Degrees, and from the other given arc.

As Sine 13:45'... 2376858923
 Sine 46:15'..... 7223639620
 Sine 73:45'.... 9600498543
 106:15.....

[Table 13-3]

For the triangles ACE, NCO, NXG, are right angled and similar: and the angles CAE, CNO of 30:0': and therefore OC, EC equal and EC the difference of the sines AF, CG. [For, if the angle BXA is x , then the angle AXF = $60 - x$; while the angle CXF is $60 + x$. $OC = R \sin x$, while $EC = 2.OC.\sin 30 = OC$.]



[Figure 13-1]

Hence : if two arcs keep the same [angular] distance from 60 degrees, and the third is equal to the same distance: by giving two of the sines, it is permitted to find the sine of the third. For if the sines of the smaller arcs are given, the sum of these is the sine of the largest arc. And by taking away the sine of the other small arc from the sine of the third there will remain the sine of the remaining small [Arc].

Section 2 of this Chapter

The canon of sines for dividing the quadrant into 144 equal parts.

$$\left. \begin{array}{l} 625 \\ 1250 \\ 1875 \end{array} \right\} \text{ have value } \left\{ \begin{array}{l} 5/8 \\ 1^2/8 \\ 1^7/8 \end{array} \right.$$

Degrees 1/10000	Sines	Degrees 1/10000	Sines	Degrees 1/10000	Sines
0 000	00000000000000000000	15 625	2693400539532260178	31 250	5187732581605214495
0 625	109080914941823718	16 250	104889600777660376	31 875	92945924898465490
1 250	109080914941823719	16 875	2798290140309920552	32 500	5280678506503679978
1 875	109067935403787470	17 500	104556632234703108	33 125	92317576964558380
2 500	218148850345611192	18 125	2902846772544623659	33 750	5372996083468238351
	109041977872150200		104211222498107546		91678244162195902
	327190828217761393		3007057995042731203		5464674327630434254
	109003045435598586		103853412668178146		91028002565588017
	436193873653359981		3110911407710909348		5555702330196022275
	108951142726703236		103483245320706461		90366929546982126
3 125	545145016380063219	18 750	3214394653031615807	34 375	5646069259743004405
3 750	108886275921367464	19 375	103100764501905066	35 000	89695103767456522
4 375	654031292301430684	20 000	3317495417533520905	35 625	5735764363510460936
5 000	108808452738092418	20 625	102706015723166453	36 250	89012605167560502
5 625	762839745039523102	21 250	3420201433256687390	36 875	5824776968678021447
	108717682437058656		102299045955647701		88319514957802264
	871557427476581758		3522500479212335123		5913096483635823720
	108613975819024275		101879903624681330		87615915608985666
	980171403295606033		36243803828370016485		6000712399244809395
	108497345224039726		101448638604013186		86901890842396990
6 250	1088668748519645760	21 875	3725829021441029703	37 500	6087614290087206394
6 875	108367804529979473	22 00	101005302209867949	38 125	86177525619842988
7 500	1197036553049625241	22 625	3826834323650897675	38 750	6173791815707049392
8 125	108225369150890661	23 125	100549947194843039	39 375	85442906133541286
8 750	1305261922200515910	23 750	3927384270845740738	40 000	6259234721840590688
	108070056035159003	24 375	100082627741631545	40 625	84698119795864344
	1413331978235674921		4027466898587372306		6343932841636455043
	107901883663492086		99603399456575022		83943255228938249
	1521233861899167015		4127070298043947352		6427876096865393302
	107720872046720371		99112319363046987		83178402254097618
9 375	1628954733945887395	25 000	4226182617406994362	41 250	6511054499119490931
10 000	107527042723416086	25 625	98609445894667609	41 875	82403651881197704
10 625	1736481776669303482	26 250	4324792063301661979	42 500	6593458151000688642
11 25	107320418757330351	26 875	98094838888350801	43 125	81619096297785147
11 875	1843802195426633834	27 500	4422886902190012788	43 750	6675077247298473797
	107101024734648817		97568559577184055		80824828858128577
	1950903220161282665		4520455461767196851		6755902076156602381
	106868886761066215		97030670583142433		80020944072110379
	2057772106922348884		4617486132350339292		6835923020228712767
	106624032458679893		96481235909637185		79207537593981005
12 500	2164396139381028798	28 125	4713967368259976486	44 375	6915130557822693778
13 125	106366490962703260	28 750	95920320933899944	44 000	78384706210977038
13 750	2270762630343732077	29 375	4809887689193876422	45 625	6993515264033670796
14 375	106096292917998843	30 000	95347992399203513	46 250	77552547831804495
15	2376858923261730939	30 625	4905235681593079926		7071067811865475271
	105813470475431895		94764318406920063		76711161474988665
	2482672393737162852		4999999999999999980		7147778973340463916
	105518057288044755		94169368408417770		75860647257091921
	2588190451025207627		5094169368408417741		7223639620597555817
	105210088507052530		93563213196796763		75001106380800776

Degrees	1/10000	Sines	Degrees	1/10000	Sines	Degrees	1/10000	Sines
46	875	7298640726978356574 74132641122883848	62	500	8870108331782217011 49840234110924893	78	125	9785988655009383145 21864149022921314
47	500	7372773368101240435 73255354822021906	63	125	8919948565893141887 48778849433741140	78	750	9807852804032304449 20697112620707362
48	125	7446028722923262354 72369351866511633	63	750	8968727415326883055 47711660561277872	79	375	9828549916653011836 19527613469068835
48	750	7518398074789774000 71474737681844434	64	375	9016439075888160925 46638794478338713	80	000	9848077530122080692 18355790726709452
49	375	7589872812471618449 70571618718161891	65	000	9063077870366499635 45560378845258550	80	625	9866433320848790166 17181783828816986
50	000	7660444431189780352 69660102437589223	65	625	9108638249211758183 44476541982713234	81	250	9883615104677607173 16005732470471900
50	625	7730104533627369591 68740297301448380	66	250	9153114791194471415 43387412856450721	81	875	9899620837148079095 14827776590024961
51	250	7798844830928817987 67812312757352231	66	875	9196502204050922142 42293121061944413	82	500	9914448613738104069 13648056352446049
51	875	7866657143686170234 66876259226181387	67	500	9238795325112867547 41193796808977616	83	125	9928096670090550133 12466712132645879
52	500	7933533402912351637 65932248088945248	68	125	9279989121921845155 40089570906139855	83	750	9940563382223196026 11283884498772758
53	125	7999465651001296901 64980391673528752	68	750	9320078692827985002 38980574745271991	84	375	9951847266721968799 10099714195486485
53	750	8064446042674825655 64020803241326492	69	375	9359059267573256986 37866940285826863	85	000	9961946980917455298 8914342127211087
54	375	8128466845916152150 63053596973765725	70	000	9396926207859083834 36748800039168460	85	625	9970861323044666384 7727909341368675
55	000	8191520442889917877 62078887958719943	70	625	9433675007898252281 35626287052804413	86	250	9978589232386035063 6540557011596038
55	625	8253599330848637823 61096792176814546	71	250	9469301294951056665 34499534894554585	86	875	9985129789397631105 5352426420946531
56	250	8314696123025452371 60107426487626324	71	875	9503800829845611238 33368677636657966	87	500	9990482215818577643 4163658945078805
56	875	8374803549513078678 59110908615778343	72	500	9537169507482269233 32233849839819412	88	125	9994645874763656464 2974396035434454
57	500	8433914458128857005 58107357136931872	73	125	9569403357322088651 31095186537198067	88	75	9997620270799090917 1784779202406750
58	125	8492021815265788860 57096891463677101	73	75	9600498543859286756 29952823218339974	89	375	9999405050001497667 594949998502293
58	750	8549118706729465045 56079631831324162	74	375	9630451367077626768 28806895813056090	90		1000000000000000000
59	375	8605198338560790090 55055699283596404	75	000	9659258262890682898 27657540675248035			
60	000	8660254037844386494 54025215658227372	75	625	9686915803565930975 26504894566683338			
60	625	8714279253502613866 52988303572463346	76	250	9713420698132614330 25349094640722103			
61	250	8767267557075077212 51945086408473107	76	875	973876979277336426 24190278425997167			
61	875	8819212643483550319 50895688298666691	77	500	9762960071199333584 230285838100497569			

If we have divided the quadrant into 144 equal parts, of these individual [angles] set out, these sines above are described, from which the whole canon of sines is found for the smallest part, resolved by quinquisection alone, of which examples follow nearby.

1. Page 45 [here the next 4 page for all of these], with the individual Degrees divided into eight parts. As the total fourth part has 720 equal small parts.

2. *Page 46. The degree cut in 40 parts. The fourth part in 3600.*
3. *Page 47. The degree cut in 200 parts. The fourth part in 18000.*
4. *Page 48. The degree cut in 40 parts. The fourth part in 90000.*

Section 3.

The Sines being given

Page 43.

40.625
41.250
41.875
42.500
43.125
43.750

The remaining Sines are placed between those you see here following the method expounded on page 38 in the previous chapter.

7 th	6 th Diff.	5 th Diff.	4 th Diff.	3 rd Diff.	2 nd Diff.	Sine of eighth Degree with 1 st Difference	Degrees	1/1000 th Degrees
	702.04	374770	147502882.77	78738933546	30990309920778.48	6511054499119490931 16543025507733584	40	625
	704	374066	147877653	7859105	31069048854324	6527597524627224516 16511956458879260	40	750
1.777	706	373359.92	148251719	78442804174.52	31147639910217	6544109481086103778 16480808818969043.3	40	875
	707	372652	148625078	78294179096	31226082714391	6560590289905072822 16449582736254652	41	000
	709	371943	148997730	78145181366	31304376893487	6577039872641327476 16418278359361165	41	125
	710.90	371233	149369673.39	77995811693	31382522074853.08	6593458151000688642 16386895837286312	41	250
	713	370520	149740906	77846070787	31460517886547	6609845046837974957 16355435319399764	41	375
1.760	715	369805.20	150111426	77695959361.36	31538363957334	6626200482157374724 16323896955442429.7	41	500
	716	369088	150481232	77545478129	31616059916696	6642524379112817156 16292280895525731	41	625
	718	368370	150850320	77394627809	31693605394825	6658816660008342890 16260587290130904	41	750
	719.73	367650	151218690.73	77243409118	31771000022635.88	6675077247298473797 16228816290108268	41	875
	721`	366929	151586341	77091822777	31848243431754	6691306063588582.68 16196968046676513	42	000
1.740	722	366206.47	151953270	76939869506.07	319253352554531	6707503031635258583 16165042711421980.2	42	125
	724	365482	152319477	76787550028	32002275124038	6723668074346680566 16133040436297939	42	250
	726	364755	152684959	76634865068	32079062674066	6739801114782978507 16100961373623871	42	375
	728.44	364026	153049714.42	76481815353	32155697539135.64	6755902076156602381 16068805676084733	42	500
	730	363296	153413740	76328401612	32232179354489	6771970881832687115 16036573496730342	42	625
1.726	731	362564.20	153777036	76174624575.79	32308507756101	6788007455329417359 16004264988974138.7	42	750
	733	361803	154139600	76020484975	32384682380676	6804011720318391498 15971880306593460	42	875
	735	361095	154501431	75865983544	32460702865651	6819983600624984960 15939419603727806	43	000
	737.07	360358	154862526.78	75711121017	32536568849195.26	6835923020228712767 15906883034878609	43	125
	739	359619	155222884	75555898132	32612279970210	6851829903263591375 15874270754908397	43	250
1.708	740	358879.76	155582503	75400315629.89	32687835868340	6867704174018499771 15841582919040054.9	43	375
	742	158137	155941382	75244374247	32763236183967	6883545756937539824 15808819682856085	43	500
	744	357393	156299519	75088074728	32838480558212	699354576620395908 15775981202297871	43	625
	745.61		156656912.03		32913568632937.53	6915130557822693778	44	750

[Table 13-5]

Section 4.

The Sines being given [on]

Page 45

41.500
41.625
41.750
41.875
42.000
42 125

As before the rest which here have been computed being placed between.

6 th Diff.	5 th Diff.	4 th Diff.	3 rd Diff.	2 nd Diff.	Sine of 1/40 th Degree with 1 st Difference	Degrees	1/1000 th Degrees
0.045	118.4	240178.46	622048742.0	1261535038650.2	6626200482157374724 3267304948639721.7	41	500
	118.3	240296.8	621808445.2	1262157087392.2	6629467787106014446 3266042791552329.5	41	525
	118.34	240415.2	621568029.9	1262778895837.4	6632733829897566776 3264780012656492.1	41	550
	118.3	240533.5	621327496.4	1263400463867.3	6635998609910223269 3263516612192624.9	41	575
	118.2	240651.8	621086844.8	1264021791363.7	6639262126522415894 3262252590401261.3	41	600
0.045	118.2	240770.15	620846074.2	1264642878208.5	6642524379112817156 3260987947523052.9	41	625
	118.1	240888.1	620605185.9	1265263724282.7	6645785367060340210 3259722683798770.3	41	650
	118.10	241006.5	620364179.3	1265884329468.6	6649045089744138981 3258456799469301.9	41	675
	118.0	241124.6	620123054.5	1266504693647.9	6652303546543608284 3257190294775653.9	41	700
	118.0	241242.6	619881811.9	1267124816702.4	6655560736838383938 3255923169958951.5	41	725
0.045	117.9	241360.69	619640451.2	1267744698514.3	6658816660008342890 3254655425260437.1	41	750
	117.9	214478.6	619398972.7	1268364338965.5	6662071315433603328 3253387060921471.6	41	775
	117.88	241596.5	619157376.1	1268983737938.2	6665324702494524800 3252118077183533.3	41	800
	117.8	241714.3	618915661.7	1269602895314.3	6668576820571708334 3250848474288219.0	41	825
	117.8	241832.2	618673829.5	1270221810976.0	6671827669045996554 3249578252477242.9	41	850
0.045	117.7	241950.08	618431879.3	1270840484805.5	6675077247298473797 3248307411992437.3	41	875
	117.7	242067.8	618189811.5	1271458916684.8	6678325554710466235 3247035953075752.4	41	900
	117.65	242185.6	617947625.9	1272077106496.3	6681572590663541988 32457638759689256.0	41	925
	117.6	242303.2	617705322.6	1272695054122.2	6684818354539511245 3244491180915133.7	41	950
	117.5	242420.8	617462901.9	1273312759444.8	6688062845720416379 3243217868155688.8	41	975
0.045	117.5	242538.32	617220363.5	1273930222346.7	6691306063588582068 3241943937933342.0	42	000
	117.4	242655.8	616977707.8	1274547442710.2	6694548007526515410 3240669390490631.7	42	025
	117.42	242773.3	616734934.47	1275164420418.0	6697788676917006042 3239394226070213.6	42	050
	117.3	242890.7	616492043.7	1275781155352.5	6701028071143076256 3238118444914861.1	42	075
	117.3	243008.1	616249035.6	1276397647396.3	6704266189587991118 3236842047267464.8	42	100
0.045		243125.41		1277013896432.0	6707503031635258583	42	125

[Table 13-6]

Section 5.

The Sines are given

Page 46

As before the rest are placed between.

42.000
42.025
42.050
42.075
42.100
42.125

5 th	4 th Diff.	3 rd Diff.	2 nd Diff.	1 st Diff.	Sine of 1/200 th Degree	Degrees	1/1000 th Degrees
	388.06		50957209670.0		6691306063588582068	42	000
		4938539.3		6484907217579001.1			
	388.10		50962148209.3		6691954554310341069	42	005
		4938151.1		648439759610791.8			
0.04	388.14		50967086360.4		6692602994069951861	42	010
		4937763.0		648388792524431.4			
	388.18		50972024123.4		6693251382862476292	42	015
		4937374.8		648337820500307.9			
	388.22		50976961498.2		6693899720682976600	42	020
		4936986.6		648286843538808.7			
	388.25		50981898484.9		6694548007526515410	42	025
		4936598.4		648235861640324.8			
	388.29		50986835083.3		6695196243388155735	42	030
		4936210.1		648184874805241.5			
0.04	388.33		50991771293.4		6695844428262960977	42	035
		4935821.8		648133883033948.1			
	388.37		50996707115.2		6696492562145994925	42	040
		4935433.4		648082886326832.9			
	388.41		51001642548.6		6697140645032321758	42	045
		4935045.0		648031884684284.3			
	388.44		51006577593.6		6697788676917006042	42	050
		4934656.6		647980878106690.8			
	388.48		51011512250.2		6698436657795112732	42	055
		4934268.1		647929866594440.6			
0.04	388.52		5101644518.3		6699084587661707172	42	060
		4933879.6		647878850147922.3			
	388.56		51021380397.9		6699732466511855093	42	065
		4933491.1		647827828767524.5			
	388.60		51026313889.0		6700380294340622617	42	070
		4933102.5		647776802453635.5			
	388.63		51031246991.4		6701028071143076253	42	075
		4932713.9		6477257712066644.2			
	388.67		51041112030.4		6701675796914282898	42	080
		4932325.2		647674735026939.0			
0.04	388.71		51036179705.2		6702323471649309838	42	085
		4931936.5		647623693914908.7			
	388.75		51046043966.8		6702971095343224747	42	090
		4931547.8		647572647870941.9			
	388.79		51050975514.6		6703618667991095690	42	095
		4931158.9		647521596895427.3			
	388.82		51055906673.5		6704266189587991118	42	100
		4930770.1		647470540988753.8			
	388.86		51060837443.6		6704913660128979872	42	105
		4930381.3		647419480151310.2			
0.04	388.90		51065767824.9		6705561079609131182	42	110
		4929992.4		647368414383485.3			
	388.94		51070697817.3		6706208448023514668	42	115
		4929603.5		647317343685668.0			
	388.97		51075627420.9		6706855765367200336	42	120
		4929214.5		647266268058247.1			
	389.00		510805586635.3		6707503031635258583	42	125

[Table 13-7]

Section 6.

The sines are given

Page 47

As before the rest is placed between.

42.000
42.005
42.010
42.015
42.020
42.025

4 th Diff.	3 rd Diff.	2 nd Diff.	1 st Diff.	Sine of 1/1000 th Degree	Degrees	1/1000 th Degrees
0.62	39509.5	2038288388.0	129702221086612.6	6691306063588582068	42	000
	39508.9	2038327897.6	129700182758715.0	6691435765809668681	42	001
	39508.3	2038367406.5	129698144391308.5	6691565465992427396	42	002
	39507.6	2038406914.9	129696105984393.6	6691695164136818704	42	003
	39507.0	2038446442.5	129694067537971.1	6691824860242803098	42	004
0.62	39506.4	2038485929.6	129692029052041.5	6691954554310341069	42	005
	39505.8	2038525436.0	129689990526605.5	66920842463393111	42	006
	39505.2	2038564941.9	129687951961663.6	6692213936329919716	42	007
	39504.5	2038604447.1	129685913357216.5	6692343624281881380	42	008
	39503.9	2038643951.7	129683874713264.7	66924733101952 38596	42	009
0.62	39503.3	2038683445.6	129681836029809.1	6692602994069951861	42	010
	39502.7	2038722958.9	129679797306850.1	6692732675905981670	42	011
	39502.1	2038762461.7	129677758544388.4	6692862355703288520	42	012
	39501.4	2038801963.8	129675719742424.5	6692992033461832908	42	013
	39500.8	2038841465.3	129673680900959.2	6693121709181575333	42	014
0.62	39500.2	2038880966.1	129671642019993.0	6693251382862476292	42	015
	39499.6	2038920466.3	129669603099526.7	6693381054504496285	42	016
	39499.0	2038959966.0	129667564139560.6	6693510724107595812	42	017
	39498.3	2038999465.0	129665525140095.5	6693640391671735373	42	018
	39497.7	2039038963.4	129663486101132.0	6693770057196875468	42	019
0.62	39497.1	2039078461.1	129661447022670.8	6693899720682976600	42	020
	39496.5	2039117958.2	129659407904712.6	6694029382129999271	42	021
	39495.9	2039157454.7	129657368747257.8	6694159041537903983	42	022
	39495.3	2039196950.6	129655329550307.2	6694288698906651241	42	023
	39494.7	2039263445.9	129653290313861.3	6694418354236201548	42	024
		2039275940.6		6694548007526515410	42	025

[Table 13-8]

And by this most convenient means the Canon of Sines can be prepared to one thousandth of a degree. Because if we are content to make hundredths of degrees, other sines can be taken from the canon which gives the sines for 144 parts of the quadrant. And so with the quadrant divided into 72 equal parts, we are able by the same method of quinquisection to increase the number of sines first to 360 parts, then for 1800, finally for 9000. For the individual degrees should have 100 sines computed. I need to show the sines, tangents, and secants with logarithms.