§6.1

Chapter Six

Concerning Quinquisection.

Chapter V is followed by one on quinquisection (the opposite of quintuplication) because the fifth part of the subtended chord is sought.

We are able also for any arc with a given chord to find the chord of the fifth part of the arc: but this is found with a little more work, than was the case for finding the third of the subtending chord. In both cases they are found by division, partially ordinary and partially algebraic [involving surds].

Let the chord of 72 degrees be given, or the line of the inscribed pentagon 117557050458.
The places of the figures should be noted of the complete fifth power beyond the four from intermediate places, starting from the first place towards the left, namely from unity. Also to be noted are the places of the cubes, inferred from the same principles, as you will see here.

As this chord really subtends 72 degrees and 288 degrees, the arc gives the subtended fifth part both ways; surely of 14: 24' degrees and 57: 36' degrees. And if we should add on two whole circles to the minor src, they will make 792 degrees, of which the fifth part of this arc is 158: 24' and the chord is arrived at by the same equation: namely 5① -5③ + 1⑤ = 117557050458 . Also, if we add on a single circle to the arcs 72: 0' and 288 :0' the sum of the degrees will be 432 and 648, and the fifth parts of both these will be arrived at, if the signs of the equations are changed thus: 5③-5 ① - 1 ⑤ = 117557050458 . The chords, I say, of 86: 24' and 129: 36' can be found: From the first, the subtended chord of 14: 24': is found. The first figure of the root sought will be 2, the cube of which is 8, and of this the quintuple 40, which is added to the same place, as you see here, and increases the dividend, making this 121557. From which is taken away five times the root,

```
.     .
117557       (20
  40  .    .5③  To be added.
121557
10  .  .   . 5① } To be taken
    32   .  1⑤    away.
  2152505045  The Remaining Dividend
```

[Table 6-1]

and the fifth power of the same root, is located in the correct places[1].

By which the first figure of the root sought is found, it is appropriate to consider correcting the divisor[2]. You see here the divisor with its own places.

2152505045	*To be Divided*	
5	*The Divisor to be taken* 5①	
800000	*Five times the Biquadratic being taken: which by necessity is used in order that we find the Gnomon of five times the power to be taken away.*	
500800000	*[Modified divisor] taken.*	*[By analogy:]*
600	*Five times the triple of the square [taken away].*	*For the cubic gnomon*
440800000	*The corrected divisor obtained which is contained nearly five times in the dividend. And the nearest figure will be 5, because the larger cubic gnomon is the product from the square in the figure found tripled.*	*found by the square of the root found tripled: of which five times will give the principal part five of the cubic gnomon is added to the Dividend [in the quintic].*

[Table 6-2]

			160000	5
			8000	10,4
			400	10,6,3
21525050458	(025066646801	*Subtended chord* 14:24'	20	5,4,3
† 38125 . . .	5③ *to be added* 4			
2533755045	225	*Five times biquadratic* . .	. 800000	5
25	5① *taken away* 625	*Ten times cubic* . .	. .80000	25
†† 65765625	1⑤ *taken away*	*Ten times square* . . .	. . 4000	125
		30036 Five times side . .	. . .100	6125
2718942080000	*Remainder*			
5 . 195 . .	5①+1⑤ *Div. taken*		400 . .	3125
93750	5③ *Div. To be added*		2000 . .	
			5000 . .	
40820 . .	*Corrected Divisor*		6250 .	
2718942080			3125	
*563851080 . . .	5③. *To be added*			
3282793160		††*gnomon* ⑤	6565625	
30	5① *taken*	*Four times cube*	. .32 . .	5
§ 11767935 . .	1⑤	*Six times square*	. . 24 .	25
271025225	*Remainder*	*Four times side*	. . 8	125
56533854	5③ *To be added*		160 .0	625
327559079			120 .	
301183732	5①+1⑤ *taken 16*		48 .	
26375347	*Remainder* *230625		1000	
244663	*Biquadratic* 390625		625	
16311	8	**Gnomon* ④ .	230625	
2779	✠ . . 7625	*Three times square* . . .	. . . 12	5
2447	*Cube* *15625..*	*Three times side* . . .	 6	25
326	4		60 .	125
6	225		150 .	
	Square 625		125	
		✠*Cubic Gnomon* . .	7625	
		† Fifth gnomon	38125	

[Table 6-3]

1875 . .	6
75	36
11250 . .	216
450 .	
225216	
112770216	(3)
563851080	

[Table 6-5]

390625		5		
13625 . . .		10	4	
625		10	6	3
25		5	4	3
1953125 . . .		6		
136250		36		
6250		216		
125		1296		
11718750		7776		
817	500			
4087	50			
	3750	0		
	625			
12	50			
§1176793	5			

[Table 6-4]

We have in the right margin a way of finding Gnomons of fifth powers as of third powers, also have been added fourth powers, or biquadratics, and second powers, or Quadratics: because these lead together to the finding of fifth and third: which we apply only in the solution of equations.

§6.2 Notes on Chapter Six :Section 1.

[1] The first task is to find an approximate value of the smallest positive root, essentially by trial and error, of the 5[th] power polynomial $f(x) = -x^5 + 5x^3 - 5x + A$, where A = 1.17557050458 = 2sin 36. Briggs first evaluates $f(x_1)$, where $x_1 = 0.2$, to find 0.2152505045, according to Table 6-1. ***Briggs always arranges things so that he subtracts positive quantities*** : he uses -f(x)/f'(x) for the smallest +ve root, and f(x)/ -f'(x) for the next one. See Figure 6-1 later in the text.

If x_1 is the first approximation to the required smallest +ve root, then a better approximation, according to Newton's method - though the method predates Newton a little as previously discussed in Note 4 of Chapter 3 - is furnished by

$$x_2 = x_1 - f(x_1)/f'(x_1): \; x_2 = x_1 + \frac{-x_1^5 + 5x_1^3 - 5x_1 + A}{5 - 15x_1^2 + 5x_1^4}, \text{and}$$

$$x_3 = x_2 + \frac{-x_2^5 + 5x_2^3 - 5x_1 + A}{5 - 15x_2^2 + 5x^4} = x_2 + \frac{-(x_1+\Delta)^5 + 5(x_1+\Delta)^3 - 5(x_1+\Delta) + A}{5 - 15(x_1+\Delta)^2 + 5(x_1+\Delta)^4}$$

$$= x_2 + \frac{f(x_1) - g_5(x_1,\Delta) + 5g_3(x_1,\Delta) - 5\Delta}{f'(x_1) + 5g_4(x_1,\Delta) - 5(6x_1\Delta + 3\Delta^2)}$$

where $g_5(x_1,\Delta) = 5x_1^4\Delta + 10x_1^3\Delta^2 + 10x_1^2\Delta^3 + 5x_1\Delta^4 + \Delta^5$; $g_4(x_1,\Delta) = 4x_1^3\Delta + 6x_1^2\Delta^2 + 4x_1\Delta^3 + \Delta^4$; and $g_3(x_1,\Delta) = 3x_1^2\Delta + 3x_1\Delta^2 + \Delta^3$; while $f'(x_1) = 5 - 15x_1^2 + 5x_1^4$.

Thus, an 'ordinary' long division is performed, but only after the numerator or dividend has been adjusted by the correcting terms of the two gnomons called here g_5 and g_3 and the 5Δ terms, while the denominator or divisor is corrected by the g_4 term, and the last two terms in the divisor, which Briggs does not bother to write as the gnomon g_2. These operations constitute the algebraic part of the solution.

[2] This is an extended note on Table 6-3.

Table 6-2 is concerned with evaluating $f'(x_1)$ as defined immediately above in Note 1, when $x_1 = 0.2$. The dividend is the first number in the top row, not used immediately in the calculation that follows, and the places are not aligned vertically with the numbers in the subsequent rows. These subsidiary calculations, of the nature of subroutines in modern computing, are designed to limit the arithmetic to the simplest operations possible. Note that a considerable calculation will be needed to evaluate the root to the 22 places required to obtain a reference sine in the table of sines of adjacent angles used for interpolation to give the final tables of sines. It is appropriate in the present case to present Briggs' work in some detail, in order that the reader is left in no doubt as to what is going on. Later calculations will not be given in such detail, but the present example can be used to expand on them as required. We add the decimal point for our convenience.

2152505045	0.2152505045	$f(x_1)$; $x_1 = 0.2$
5	5.000000000	5
800000	0.008	$5(x_1)^4$
500800000	5.008000000	$5 + 5(x_1)^4$
600	0.600000000	$15(x_1)^2$
440800000	4.408000000	$5 + 5(x_1)^4 - 15(x_1)^2 = -f'(x_1)$.

[Table 6-2A]

			Types of terms found in gnomons
160000	$(x_1)^4$	5	$5(x_1)^4\Delta$
8000	$(x_1)^3$	10,4	$10(x_1)^3\Delta^2$, or $4(x_1)^3\Delta$
400	$(x_1)^2$	10,6,3	$10(x_1)^2\Delta^3$, $6(x_1)^2\Delta^2$, $3(x_1)^2\Delta$
20	$x_1 = 0.2$	5,4,3	$5(x_1)\Delta^4$, $4(x_1)\Delta^3$, $3(x_1)\Delta^2$

[Table 6-3A]

Table [6-3A] appears to be a reference table of the different kinds of terms to be met with in evaluating the three kinds of gnomons, which share common factors, which need only be evaluated once.

We first enlarge on the gnomon of the 5th power, g_5, that occupies the top right hand corner of Table 6-3:

. 800000	$5(x_1)^4$	0.008	5	Δ	.05
. .80000	$10(x_1)^3$	0.08	25	Δ^2	.0025
. . 4000	$10(x_1)^2$	0.4	125	Δ^3	.000125
. . .100	$5(x_1)$	1	625	Δ^4	.00000625
			3125	Δ^5	0.0000003125
400 . .	$5(x_1)^4\Delta$	0.0004			
2000 . .	$10(x_1)^3\Delta^2$	0.0002			
5000 . .	$10(x_1)^2\Delta^3$	0.00005			
6250 .	$5(x_1)\Delta^4$	0.00000625			
3125	Δ^5	0.0000003125			
6565625	g_5	0.0006565625			

[Table 6-3B]

Note that Briggs is only interested in the relative place of the contributions to his scheme at this stage, which he combines in an efficient way by considering all the terms as whole numbers [as he uses a radius of 10^{10} or larger]: thus, $5(x_1)^4\Delta$ gives $5 \times 800000 = 4000000$, etc. The connection between Δ, set as 5, and $5(x_1)$, set as 100, gives internal consistency. There still remains the task of placing the leading number into the correct position in the decimal expansion. We next enlarge on the gnomon of the 4th power, g_4,

. .32 . .	$4(x_1)^3$		5	Δ	.05
. . 24 .	$6(x_1)^2$		25	Δ^2	.025
. . 8	$4(x_1)$		125	Δ^3	.000125
160. . .	$4(x_1)^3\Delta$	.0016	625	Δ^4	.00000625
120. .	$6(x_1)^2\Delta^2$	.0006			
48. . .					
1000.	$4(x_1)\,\Delta^3$	.0001			
625	Δ^4	.00000625			
230625	g_4	.00230625			

[Table 6-3C]

that occupies the middle right hand side of Table 6-3:

Note that the $6(x_1)^2\Delta^2$ term has been arrived at by adding two parts.

We next enlarge on the gnomon of the 3rd power, g_3, that occupies the bottom right hand side of Table 6-3:

We are now in the position of tackling the main division algorithm, with the correcting factors being added to the numerator and denominator:

2152505045: f(0.2)		(025066646801	f(0.2) = 0.2152505045
† 38125 . . .	5③ *to be added*	4	$5g_3(0.2,0.05) = 0.038125$
2533755045		225	$f(0.2) + 5g_3(0.2,0.05)$
25	5① *taken away*	625	$5\Delta = 5 \times 0.05 = 0.25$
†† 65765625	1⑤ *taken away*		$g_5(0.2, 0.05) = 0.0006565625$
2718942080000	*Remainder*		$f(.2)+5g_3(.2,.05)- g_5(.2, .05) -.25=$**.00 271894208**
5 .195 . .	5 + 5× *Biquadratic***		$5+5(g_4(.2,.05) + (.2)^4)= 5.019531$
93750	15× *Quadratic***		$15(g_2(.2,.05) + (.2)^2)= 0.9375$
40820 . .	*Corrected Divisor*		$5+5(g_4(.2,.05)+(.2)^4)-(15(g_2(.2,.05)+(.2)^2)=$**4.0820**
2718942080	*Remainder*		As above $f(x_2)$
*563851080 . . .	5③. *To be added*		$5g_3 = .000563851080$
3282793160			$f(x_2) + 5g_3$
30	5① *taken*		$5\Delta = 5\times0.0005 = 0.0030$
§ 11767935 . .	1⑤		$g_5(0.25,0.0006) = 0.\ 000011767935$
271025225	*Remainder*		*Etc, etc.*
56533854	5③ *To be added*		
327559079			
301183732	5①+1⑤ *taken 16*		
26375347	*Remainder*	*230625	
244663	*Biquadratic*	390625	
16311		8	
2779	✠ . .	7625	
2447	*Cube*	*15625..*	
326		4	
6		225	
	Square	625	

[Table 6-3E]

16	$(x_1)^4 = .2^4 = 0.0016$	
Remainder *230625	$g_4(.2,.05)$ = 0.00230625	See Table 6-3C
Biquadratic 390625	*Biquad.* = 0.00390625	*5+5×Biquad.* = 5.019531
8	$(x_1)^3 = .2^3 = 0.008$	
✠ . . 7625	$g_3(.2,.05)$ = 0.007625	See Table 6-3D
Cube 15625..	*Cube* = 0.015625	
4	$(x_1)^2 = .2^2 = 0.04$	$(x_1+\Delta)^2 = (x_1)^2 + 2x_1\Delta + \Delta^2$
225	$2x_1\Delta + \Delta^2 = 0.0225$	
Square 625	*Square* = 0.0625	*15× Square* = 0.9375

[Table 6-3F]

** The instruction 5①+1⑤ seems to have been placed inadvertently into this slot: the aim is to evaluate the gradient here, which is a slowly varying function .
We note that the ordinary division of the corrected dividend and divisor give the correction : 0.00 271894208/4.0820 = .000666. Hence, the new value of Δ is 0.0006, while x_2 now has the value 0.250. Table 6-5 indicates how the new gnomon $g_3(0.25,0.0006)$ = .000112770216.

1875 .	$3(x_1)^2$	.1875	6e-4	Δ
75	$3(x_1)$	.7500	36e-8	Δ^2
11250 .	$3(x_1)^2\Delta$	.0001125	216e-12	Δ^3
450	$3(x_1)\Delta^2 +$			
225216	Δ^3			
112770216	g_3	.000112770216		
	$5g_3$	.000563851080		

[Table 6-4A]

In the same way, Table 6-5 gives the powers of 25 corresponding to the approximate root x_2 = 0.25, together with the powers of Δ = 0.0006, and their various products, leading to $g_5(0.25,0.0006)$ = 0. 000011767935. Note that the same divisor may be used without further change here, as only the first sig. fig. of the quotient is required: 0.0000271025225/4.0820 = 0.000066….The dividend is further corrected, and the same divisor then gives: 0.0000026375347/4.0820 = 0.00000646…as the correction. The same divisor will fail eventually in supplying several correct consecutive figures and needs to be corrected again: so the process continues….

We can appreciate fully the nature of Briggs' work by resorting to a spreadsheet (which has the obvious advantage of setting out all the numbers in tables). For the present case, this has been done for 7 iterations, where the beautiful nature of the convergence becomes apparent, and one can appreciate Briggs' label of gnomon or pointer. For each correction has more correct figures, and after 7 iterations, the next 6 places are guaranteed correct: this was a powerful tool indeed for someone working by hand. However, rather than present the whole spreadsheet, only a few salient numbers are extracted from it:

$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$
2.15250504580E-01	2.71894208000E-03	2.71018024838E-04	2.63681500579E-05
x_1	x_2	x_3	x_4
2.00000000000E-01	2.50000000000E-01	2.50600000000E-01	2.50660000000E-01
Δ_1	Δ_2	Δ_3	Δ_4
5.00000000000E-02	6.00000000000E-04	6.00000000000E-05	6.00000000000E-06
$gnomon_3$	$gnomon_3$	$gnomon_3$	$gnomon_3$

7.62500000000E-03	1.12770216000E-04	1.13067714960E-05	1.13097491230E-06
gnomon$_4$	gnomon$_4$	gnomon$_4$	gnomon$_4$
2.30625000000E-03	3.76352161296E-05	3.77842155615E-06	3.77991419291E-07
gnomon$_5$	gnomon$_5$	gnomon$_5$	gnomon$_5$
6.56562500000E-04	1.17751351621E-05	1.18373226023E-06	1.18435578934E-07
Numerator 1	Numerator 2	Numerator3	Numerator 4
2.71894208000E-03	2.71018024838E-04	2.63681500577E-05	1.90458904040E-06
$f'(x_1)$	$f'(x_2)$	$f'(x_3)$	$f'(x_4)$
4.08203125000E+00	4.07771402608E+00	4.07728178419E+00	4.07723855481E+00
Divisor 1	Divisor 2	Divisor3	Divisor 4
4.08203125000E+00	4.07771402608E+00	4.07728178419E+00	4.07723855481E+00
Quotient 1	Quotient 2	Quotient 3	Quotient 4
6.66075763139E-04	6.64632250090E-05	6.46709044246E-06	4.67127202590E-07

$f(x_5)$	$f(x_6)$	$f(x_7)$
1.90458904026E-06	2.73694194641E-07	2.90600674635E-08
x_5	x_6	x_7
2.50666000000E-01	2.50666400000E-01	2.50666460000E-01
Δ_5	Δ_6	Δ_7
4.00000000000E-07	6.00000000000E-08	7.00000000000E-09
gnomon$_3$	Gnomon$_3$	gnomon$_3$
7.54002525869E-08	1.13100586432E-08	1.31950719440E-09
gnomon$_4$	Gnomon$_4$	gnomon$_4$
2.52003930600E-08	3.78006936425E-09	4.41008269309E-10
gnomon$_5$	Gnomon$_5$	gnomon$_5$
7.89610845858E-09	1.18442061586E-09	1.38182479053E-10
Numerator 5	Numerator 6	Numerator 7
2.73694194734E-07	2.90600672415E-08	5.19420956430E-10
$f'(x_5)$	$f'(x_6)$	$f'(x_7)$
4.07723567281E+00	4.07723524051E+00	4.07723519008E+00
Divisor 5	Divisor 6	Divisor 7
4.07723567281E+00	4.07723524051E+00	4.07723519008E+00
Quotient 5	Quotient 6	Quotient 7
6.71273913743E-08	7.12739528804E-09	1.27395387368E-10

Thus, the required root has the value 0.2506664671273953..

[*End of Notes for Section 1.*]

§6.1(cont'd)

2. If the fifth part of the subtended chord is sought from the same given chord of 288 degrees: with the given points noted above and below as before,
[i.e. to the right of], 5 ①-5③ + 1⑤, the chord of 57: 36' is sought, 117557050458^3.

The first figure will be 9, because by adding 5③ the dividend is 4820 [$5(0.9)^3$ + 1.1755 = 4.82], whence the divisor 5 can be taken away nine times. But for a lesser amount, five of the quotient cannot be taken away from the given number with the [lesser] increase of the five cubes. [f(.9) = 0.23, while f(0.8) = 0.59, and f(1) = -0.1755].

But the investigation itself is not different in any way from the preceding. For this given chord and 5③ [on addition] (as they shall be the smaller in this operation with the main part as 5① and 1⑤) ought to be taken from five times the root and the fifth power of the root: as thus,

117557050458	*Given subt'd ch.*[A]	(09635073477	*Subt'g* 57:36'	
3645	5③ *Added* [$5\times.9^2$}			
482057050458	*Sum taken from*			
509049	5① + 1⑤ [$5\times.9+.9^5$]			
26991949542	*Remaining dividend* [f(.9)]	[f(.9)/-f '(.9)=0.069]		
30	5① [5×.06 and]			
† 2248826976	1⑤ *Added* [g_5(.9,.06)]		5 . .	5① [*and*]
79480219302			42467	*5 biquad. sub.*
† 778680 . . .	5③ *Taken* [$5g_3$(.9,06)]		13824	*3 of 5 of Sq.*
¢ 161221930200	[f(.96) = .016122]	[f(.96)/ f '(.96)= 3.5e-3]	¢4557	*Corrected*
15	5① [5×.003 and]		[f '(.96)= 4.557	*divisor*
* 1282007394 . .	1⑤ *Added* [g_5(.96,.003)]		5 . . .	5①
4394226696			4300	*5 biquad..*
§ 41601735 . .	5③ *Taken* [$5g_3$(.96,003)]		1391 . .	15 sq.
γ 234053196	[f(.963) = .00234053196]	[f(.963)/ f'(.96)=.5.07e-4]	γ 461	*Corrected*
25	5① [5×.0005 and]			*divisor*
4 215226171	1⑤ *Added* [g_5(.963,.0005)]			
699279367				
♂ 695887937	5③ *Taken* [$5g_3$(.963,0005)]			
3391430	[f(.9635) = .00003391430]	[f/f ' =7.34e-6]		
35	5① [5×.00007 and]			
♁ 3016339	1⑤ *Added* [g_5(.9635,.00007)]			
9907769				
☿ 9747559	5③ *Taken* [$5g_3$(.9635,00007)]			
160210	[f(.963507) = .00000160210	[f/f ' =3.48e-7]		

	②	③	④
13821			
1843	81 . . [$(0.9)^2$]	729 [$(0.9)^3$]	6561 [$(0.9)^4$]
357	1116	155736 ♋	19324656 ♌
322	9216 *V*[$(0.96)^3$]	884736 [$(0.96)^3$]	84934656 [$(0.96)^4$]
	5769	8320347	10666402161 F
	927369 I[$(0.963)^3$]	893056347 [$(0.963)^3$]	860011162161 [$(0.963)^4$]
	96325	1391776	1787504229 ↣
	92833225 [$(0.9635)^3$]	894448123 [$(0.9635)^3$]	861798666390 [$(0.9635)^4$]

[Table 6-5]

6561 . . 729 . . 81 . 9	5 10 | 4 | 10 | 6 | 3 5 | 4 | 3	
32805 . . . 7290 . . 810 . 45	6 36 216 1296	
 196830 . . . 43740 2187 2160 1728 6480 5184 7776	7776	[Δ^5 $5\times(9)^4\times 6$ $10\times(9)^3\times 6$ $10\times(9)^3\times(30)$ 10×216 800×216 5×1296 40×1296]
† 2248826976	⑤	[$g_5(0.9,.06)$]
2916 . . . 486 . . 36 .	6 36 216	
17496 . . . 2916 . . 1458 . . . 1296 . 648 . . 1296	1296 [$=6^4$]	$4 \times 9^3 \times 6$ $6 \times 9^2 \times 6$ $6 \times 9^2 \times 30$ 6×216 30×216 6^4
♌ 19324656	④	[$g_4(0.9,.06)$]
243 . . 27 . 1458 . . 162 . 81 . . 216	6 36 216	 6×243 6×27 30×27 6^3
♋ 155736	③	[$g_3(0.9,.06)$]
† 778680	5 ③	
6561 [$(0.9)^4$]		
19324656 ♌		
84934656		[$(0.96)^4$]

[Table 6-6A]

84934656 . . 884736 . . 9216 96	5 10 | 4 | 10 | 6 | 3 5 | 4 | 3	
424673280..... 8847360. ... 92190... 480.	3 9 27 81	
1274019840 . . . 79626240 645330 18438 480 384243	243	
*1282007394 | 2043	⑤	[$g_5(0.96,.003)$]
3538244. . . . 55296 . . 384 .		
F 10614732000.	[④]	[$g_4(0.96,.003)$]
27648. . 288 82944. . 2592. 27	3 9 27	
§ 8320347	[③]	$g_3(0.96,.003)$]
729. .. ♋ 15763	[$(.9)^2$ + $g_3(0.9,.06)$ =	
884736	$(.96)^2$]	

[Table 6-6B]

860011162161 . .	5			
883056347 . .	10	4		
927369 . .	10	6	3	
963 .	5	4	3	
4300055810805…..	[5			
8930563470 ...	25			
9273690 .	125			
4815	625]			
215002790|54025				
44652|817350	5			
178611|2694	[20			
4|63845 .	5			
18|54738.	20			
92|7369	100]			
♃ 215226170|5	⑤			[g_5(0.963,.0005)]
3572225388. . . .				
5564214 . .				
3852 .				
17861126940				
27821070 …				
11128428.				
19260 . .				
7704 .				
3852 . . .				
➳ 8320347	[④]			[g_4(0.963,.0005)]
2782107..				
2889 . . .				
13910535. . . .				
14445. . .				
5778. . . .				
125. .				
1391775875. .	[③]			g_3(0.963,.0005)]
♂ 6958879375 ..				
86179866634				
89444				
43089933317				
89444				
301629533219				
804996				
357776				
♁ 30163391. .	⑤			[g_5(0.9635,.00007)]
92833225. . .				
9635..				
278499675…..				
28905..				
1949497725 ….				
2601. . .				
1156. . . .				
19491188	[③]			g_3(0.9635,.00007)]
☿ 97475594				

[Table 6-6C]

§6.2 (cont'd) Notes on Chapter Six :Section 2.

3 We now look for a larger root of the 5th power polynomial: for reference, the 5 real roots, and a graph of the function are included:

x_1 = -1.8096541049325596908121166953066;
x_2 = -1.3690942118564859775239691956940;
x_3 = .25066646712739535044210045155490;
x_4 = .96350734820450208227900164702114;
x_5 = 1.9645745014571482356149837924245.

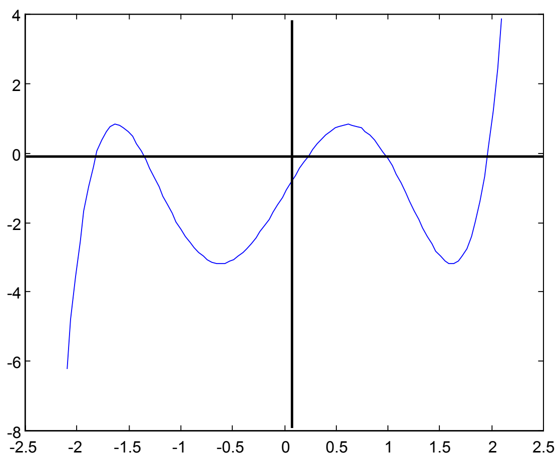

Figure 6 - 1: Graph of the quintic $f(x) = x^5 - 5x^3 + 5x - A$ with the 5 real roots above, where A = 1.17557050458.

Note that Briggs avoids approaching a root from negative values; hence the first root found used the inverted function -f(x) as the numerator, but kept f'(x) as the denominator; while in the second case, f(x) is used, while -f'(x) is employed.

A spreadsheet analysis can be performed similar to that for the first root. Here we merely indicate the form of the correction factors as the iteration converges:

quot1	quot2	quot3	quot4
3.52223112953E-03	5.07655954053E-04	7.34826893268E-06	3.48204646819E-07
quot5	quot6	quot7	quot8
4.82045048800E-08	8.20450238659E-09	2.04501910565E-10	4.50211616788E-12

End of Notes on Section Two.

§6.1(cont'd)

3. If we add two whole circles to 72 Degrees, the sum of the Degrees will be 792, the fifth part of which [will be] 158:24': Of which the Chord also should be given from the side [root] of the Pentagon: 5 ① -5③ + 1⑤ ⹀ 117557050458

b

117557050458 (1964517451. Subtended by 158: 24'

5	5 ③	1….	5		
617557		1…	10	4	
5	5①	1..	10	6	3
1	1 ⑤	1.	5	4	3
17557050458		5…..	9		
† 29295	5 ③	10…..	81		
2947057		10…	729		
45.	5①	5.	6561		
‡ 2376099		45…...	59049		
12095805045		810….			
3352680	5 ③	7290…			
45622605045		32805.			
30	5①	59049			
41644754976		2376099	⑤‡		
9778500698		4000	9		
* 230966720	5 ③	6. ..	81		
32875172698		4.	729		
20	5①	36. . . .	6561		
* 29636499809		486. . .			
Prob. 1238672889		2916 ..			
4 2893708562	5 ③	6561			
413281451		120321	④		
25	1 ⑤	3 . .	9		
♂ 3721577522		3 .	81		
160803929	≈	27. . .	729		
II 5760 . . . 15 *Sq.*		243 . .			
5 . . .	5①	729 . .			
♋ 7378 *5Biquad.*		5859	③		
211 *Divisor corrected*		29295	†		
≈ b		130321	5		
Corr. 2158)16080(7451		6859	10	4	
Divisor. If this 15106		361	10	6	3
remainder has 863		19 . .	5	4	3
been divided		651605	6		
in the common 111		68590	36		
way. 108		3610	216		
		95	1296		
39		3909630 . . .	7776		
19645		411540 . .			
1 . .		20577			
261 . *Sq..*		21660.			
2316		361			
15696 .		722			
19645		6480			
		11664			
41644754976 ⑤ ← [sum]		7776			

2475789056	5			
7529536 . . .	10	4		
38416 . .	10	6	3	
196 .	5	4	3	
♋ 778945280	4			
75295360	16			
II 384160 . . .	64			
980 . .	528			
29515781120	1024			
451772160				
75295360				
1536640 . .				
230496				
20480 .				
23040 . .				
1024				
*296364998093824	⑤			
30118144 . . .	4			
230496 . .	16			
784 .	64			
120472576 . . .	256			
1382976 . .				
230496 . . .				
3136 .				
4704 . .				
556				
120841871916	④			
115248 . .				
588				
460992 . .				
3528.				
588 .				
64				
46193344	③			
*230966720				
1487873243 | 1916	5			
75757 | 29344 . . .	10	4		
3 | 857296 . .	10	6	3	
1964 .	5	4	3	
7439366215 | 9580	5			
757572 | 93440 . . .	25			
38 | 572960 . .	125			
9820 .	625			
3719683107 | 9 | 7900	31625			

[Table 6-7]

1	
5859 *Cubes*	
670536	
46193344	
5787417125	
1	
120321 *Biquadratics*	
172579056	
120841871916	
15157245614	
1	
2376099 ⑤ *powers.*	
41644754976	
296364998093824	

27436 . . .	6
2166 . .	36
76 .	216
164616 . . .	1296
12996 . .	
6498 . . .	
1296 .	
1512 . .	
1296	
172579056	④
1083 . .	
57 .	
6498 . .	
342 .	
171 . .	
216	
670536	③
2252680	

3719683107	9	7900 . . .	
378786	4	6420	
1515145	8	688	
19	2	8	
77	1	4	
385	7	3	
♂3721577522	4	6	⑤
30302917376 . .			5
23143776 . .			25
7856 .			125
151514586880 . . .			
115718880 . .			
46287552 . . .			
39280 .			
15712 .			
7856			
15157245614000.			④
11571888 . . .			
5892 . .			
57859440 . . .			
147300 . .			
125			
5787417125			③
4 28937085625			

[Table 6-8]

[Translator's Note: The rest of these tables, for the largest root above, and for the negative root considered next, are presented without explanation, as they follow the same scheme treated in great detail above for the first and second cases.]

4. If we add a single circle to 288 degrees; The sum will be 648, of which the fifth part 129: 36'. Thus the equation will be : 5③ -1⑤ - 5① = 117557050458 .

117557050458	(18096541
5	5①
1	1⑤ *added*
7	
5	5③ *taken*
217557 *Remain.*	
40	5①
†17899568	1⑤ *added*
2407125	
†26165	5③ *taken*
8874949542	
* 4389590645	5③ *added*
528340140	
45	5①
* 477139598708049	1⑤ *taken*
620054129	
4 29462098680	5③ *added*
3566263997	
30	5①
♂ 3214871446	1⑤ *taken*
51392551	

5 . . . 5①	3	1
53545 ⑤	1809	4832 . . . .
58545	224 . . .*Sq*	87918129
49087 5③	32481	5919918129 *Cube*
9558 *Div. Corr.*	2272481 *Sq.*	1 . . . . .
	1	1789568 ⑤
	94976 ④	
	211531895361	
	14214873631	

23328	9
1944	81
72 ..	729
209952	6561
1944	
15552	
1458 . .	
5103 . . .	
6561	
211531895361	④

[Table 6-9]

1	5	
1 . . .	10 ∣ 4 ∣	
1 . .	10 ∣ 6 ∣ 3	
1 .	5 ∣ 4 ∣ 3	
5	8	
10 . . .	64	
10 . .	512	
5 .	4096	
40	32768	
640 . . .		
5120 . .		
20480 .		
32768		
†1789568		(5)
4 . . .	8	
6 . .	64	
4 .	512	
32 . . .	4096	
384 . .		
2048 .		
4096		
94976		(4)
3 . .	8	
3 .	64	
24 . .	512	
192 .		
512		
4832		(3)
†24160		

104976	5	
5832	10 ∣ 4 ∣	
324	10 ∣ 6 ∣ 3	
18 . .	5 ∣ 4 ∣ 3	
524880	9	
58320	81	
3240	729	
90 . . .	6561	
4723920		
59320		
46656		
29160		
648		
2268		
590490 . .		
59049		
*477139598708049		(5)

	972	9	
	54 . .	81	
	8748	729	
	54 . .		
	432 . . .		
	729		
	87918129		(3)
	* 439590645		
	10709131895361	5	
	5919918129 . . .	10 ∣ 4 ∣	
	3272481 . .	10 ∣ 6 ∣ 3	
	1809 .	5 ∣ 4 ∣ 3	
535456594	76805		
59199	181290 . . .		
3	2724810 . .		
	9045		
3212739568	608	6	
355195	087	36	
1775975	438	216	
19	634	1296	
32	724	7776	
654	496		
	90		
♂ 3214871446	077		(5)
	23679672516 . . .	6	
	19634886 . .	36	
	7236 .	216	
	142078035096 . . .	1296	
	117809316 . .		
	58904658 . . .		
	43416 .		
	7236 . .		
	14472 . . .		
	1296		
	142148736316656		(4)
	9817443 . .	6	
	5427 .	36	
	58904658 . .	216	
	32562 .		
	16281 . .		
	216		
	5892419736		(3)
	4 29462098680		

[Table 6-10]