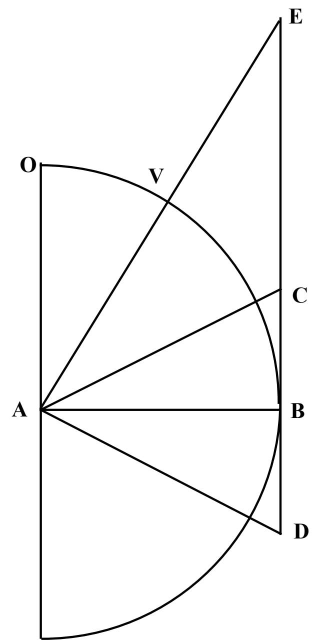
Chapter Seventeen.

Concerning Tangents and Secants and their Logarithms.

If the quadrant is cut into any number of equal parts, and tangents are prepared for half of the same of these from the beginning or the end of the quadrant: the tangents for the remaining angles can be found by addition or subtraction:
And the secants also of the other parts; as can be shown by the following Propositions

1. Half the sum of the tangents of any arc and its complement is equal to the secant of the difference of the same arcs.
2. Half the difference of the tangents of any arc and its complement is equal to the tangent of the difference of the same arcs.

Let CAB be 19:0', CAE or BAD 35:30'. BAE or CAD 45:30'. EAD 90:0'; CAE,CEA, and VAO are equal to the angle BAD 35:30' ;and BDA, BAE, CAD are equal 54:30' ; CA, CE therefore are equal; in the same manner CA, CD are equal by *Prop.* 5, *Book* 1, *Euclid*. And CA is equal to half the sum of the tangents EB 54:30' and BD, 35:30'. The secant of the angle CAB 19:0' is the difference of the angles EAB 54:30' and BAD 35:30'.[1]



[Figure 17-1]

Tangent BE 54$\underline{50}$	14019482945	
Tangent BD 33$\underline{50}$	7132930679	
Sum.	21152413624	
Difference	6886552266	
Half the sum	10976206812	AC *Secant* 19$\underline{00}$
Half the difference	3443276133	BC *Tangent* 19$\underline{00}$

[Table 17-1]

3. The sum of the secant and of the tangent of the same arc is equal to the tangent of the same arc increased by half the complement[2].
4. The difference of the secant and the tangent of the same arc is equal to the tangent of half the complement.

Let CAB be 33:0'. The complement CAO 57:0'. CASE, BAD 28:30'. EAB 61:30'.

Secant 33$\underline{00}$	AC	11923632928	
Tangent 33$\underline{00}$	CB	6494075932	
Sum	EB	18417708860	*Tangent* 61$\underline{50}$
Difference	BD	5429556996	*Tangent* 28$\underline{50}$

[Table 17-2]

5. The tangents of any arc you please and of half the complement is equal to the secant of the same arc. [See Figure 17-2].

Let CAB 47:0'. The complement CAO 43:0'. Half the complement 21:30' BAN.

Tangent $47\underline{00}$ CB	10723687100		
Tangent $21\underline{50}$ BN	3939104756		
Sum	14662791856	*Secant* $47\underline{00}$ CA or CN.	

[Table 17-3]

6. The tangent doubled of any arc together with the tangent of half the complement is equal to the tangent of the same arc increased by half the complement[3].
This is proved by *Props. 5 and 3*. For by 5, the tangents of the arc and of half the complement are equal to the secant of the same. To which if the tangent of the given arc is added, the sum multiplied by *3* is equal to the tangent of the same arc increases by half the complement.

Let BED be 39:0'. The complement BEO 51:0'. Half of the complement DEP 25:36'.

Tangent $39\underline{00}$	BD	8097840332	
	BG	8097840332	
Tangent $25\underline{50}$	[DP]	4769755327	
Sum	DF	20965435991	*Tangent* $64\underline{50}$

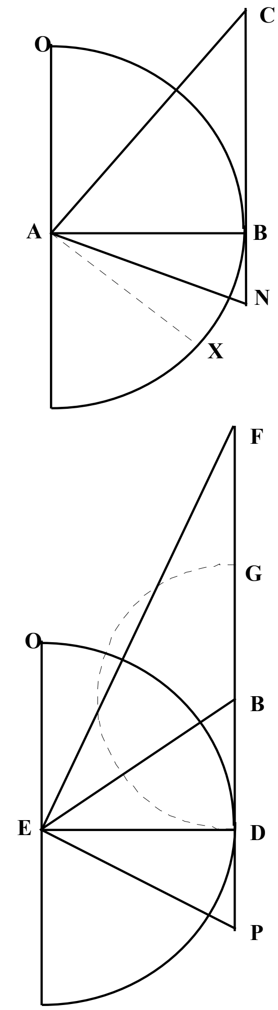
[Table 17-4]

7. The tangent of any arc you wish is taken from the tangent of the complement; there remains double of the tangent of the difference[4].

Let the arc PED be $27\underline{37}$. The complement DEF $62\underline{63}$. The difference DEB $35\underline{26}$.

Tangent PED $27\underline{37}$	5176866628	
Tangent DEF $62\underline{63}$	19316703943	
Difference	14139837315	
Half the Difference	7069918657	*Tangent* $35\underline{26}$ DB.

[Table 17-5]



[Figures 17- 2/3]

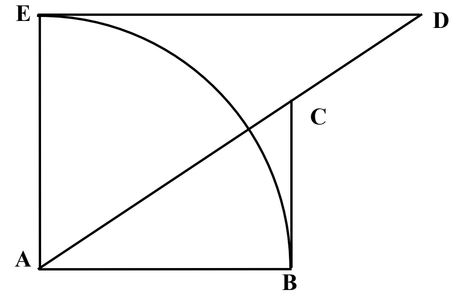
8. Double of the difference of the tangent of any arc you wish and of the complement is taken from the tangent of the larger arc, there remains the tangent of the smaller arc.

PED $39\underline{75}$, of which the complement DEF $50\underline{25}$, the difference DEB $10\underline{50}$.

Tangent of the Difference DEB $10\underline{50}$	1853390449	BD
Double	3706780898	GD
Tangent $50\underline{25}$	12023693107	FD
Tangent $39\underline{75}$	8316912209	FG, DP.

[Table 17-6]

9. The radius is the mean proportional between the tangents of any arc you please and of the complement.
CB is to BA in the same ratio as AE to ED. Because the triangles CBA and AED are equal angled. [Similar].

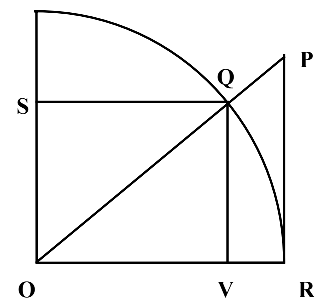


[Figure 17- 4]

10. Any Sine SQ, or OV, is to the sine of the complement QV: as the radius OR is to the tangent of the same complement RP.

11. The radius is the mean proportional between any sine and the secant of the complement.
SQ to QO; as OR to OP. Because the triangles SQO, ROP are equiangular.

12. The sine is to the tangent of its own arc; as the radius to the secant of the same.

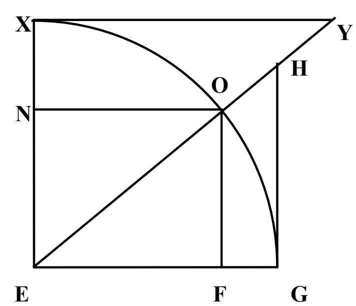


[Figure 17- 5]

Pro-port.	*Sine*	OF	*Pro-port.*	*Sine*	ON
	Tangent	HG		*Tangent*	YX
	Radius	EO		*Radius*	EO
	Secant	EH		*Secant*	EY

[Table 17-7]

For EOF, EHG, and in like manner, EON, EYX are similar triangles.



[Figure 17- 6]

13. The sine is to the tangent of its own arc, as the tangent of the complement to the secant of the same complement.

Pro-port.	*Sine*	OF
	Tangent	HG
	Tangent	XY
	Secant	EY

[Table 17-8]

Because the radius is the mean proportional between the second and the third, by *Prop*. 9, and between the first and the fourth by *Prop*. 11. And therefore the rectangle of the means is equal to the rectangle of the extremes. And by *Prop. 16, Book 6, Euclid*, the given lines are proportional.

14. The radius is to the secant of any arc, as the tangent of the complement to the secant of the same complement.

Pro-port.	*Radius*	EO
	Secant	EH
	Tangent	XY
	Secant	EY

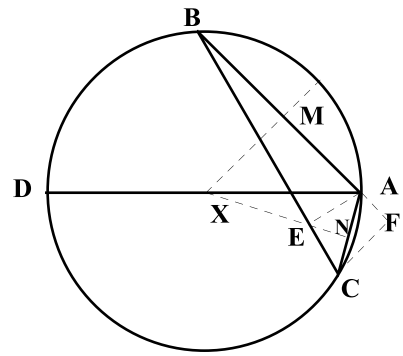
[Table 17-9]

For the first to the second; as OF to HG by *Prop.12*. And the third to the fourth as OF to HG by *Prop. 13.*

In Plane triangles.

15. The diameter of the circumscribed circle is to any side of inscribed triangle; as another side of the same triangle is to the perpendicular drawn from the angle comprised of the said sides, to the third side.

Let the inscribed triangle be ABC, the diameter of the circle AD, the perpendiculars are AE in BC, and CF in BA continued. I say that AD, AC: AB, AE are proportionals: For the triangles DAC, BAE are similar, because the angles at D and B are in the same section; and the angles ACD, AEB are right. Therefore DA, AC: BA, AE are proportionals. Also DAC, BCF are equal angled triangles: and therefore DA, AC: BC, CF are proportionals: and therefore DA, AC: BC, CF are proportionals[5].



[Figure 17- 7]

CONSEQUENCES.

The radius is to the sine of the angle, as the sine of the other angle is to half the perpendicular from the same parts drawn from the third angle to the opposite side. For these lines are the halves of these above which were in proportion. As XA the radius, AM the sine of the angle AXM or ACB equal to this , by *Prop. 20, Book 3, Euclid*. And AN the sine of the angle AXN or ABC.

16. If twice four lines shall be proportional; the Rectangles from similar terms being taken together are proportional.

For two ratios being given, the products from the homologous terms, have the ratio from the given ratios being composed. Therefore since ratios being composed from the same of these shall be the same [also], it is necessary [for] these rectangles (or the products expressing the sizes of the rectangles) to be in proportion. For let

4, 3: 8, 6 be proportionals. In the same way:
8, 7: 16, 14 are proportionals. The proportional products will be:
32, 21: 128, 84.

For Tangents

2. The Quadrant is cut in any number of equal parts; suppose, for example, eighteen. If the tangents are given for the first nine parts: the Tangents can be found for the remainder of the nine parts by addition only.

The given tangents are duplicated, and added to the tangents of half the complements: the sums will be (*by Proposition 6*) the tangents of the arcs composed from the given arcs and from half of the complements. For

Tangent 10:0' *doubled*	3526539614	*Tangent* 70:0' *doubled*	54949548390
Tangent 40:0' *complem.*	8390996312	*Tangent* 10:0' *complem.*	1763269807
Tangent 50:0' *addition*	11917535926	*Tangent* 80:0' *addition*	67712818197

[Table 17-10]

	Degrees	*Degrees*		*Degrees*
Given Tangents to be doubled	0	45	*Tangents found*	45
	10	40		50
	20	35		55
	30	30		60
	40	25		65
Found Tangents to be Doubled	50	20		70
	60	15		75
	70	10		80
	80	5		85

If however the tangents are given for the nine last parts of the quadrant; the remainder will be able to be found by subtraction.

If twice the tangent of the difference of the larger arc above the complement are taken away from the tangent of the larger arc: the tangent of the complement will remain, *by Proposition 6*.

Tangents given	*Tangents given to be doubled*	*Tangents found*
Degrees	*Degrees*	*Degrees*
85	80	5
80	70	10
75	60	15
70	50	20
	found Tangents to be doubled	
55	20	35
50	10	40
65	40	25
60	30	30
45	0	45

Tangent 85:0'	114300523028
Tangent 80:0' doubled	113425636392
Tangent 5:0'	874886635
Tangent 50:0'	11917535926
Tangent 10:0' doubled	352639614
Tangent 40:0'	8390996312

[Table 17-13]

3. With the same tangents of the semiquadrant; the secants can be found for the other parts. In the first place the tangents can be found for the separate parts of the whole quadrant: Then by *Prop. 5* the tangents of any arc you wish and half the complements are added ; the sum will give the secant of the same arc.

Tangents	*Tangents* $^1/_2$ *Comp.*	*Secants*
Degrees	*Degrees*	*Degrees*
40	25	40
30	30	30
20	35	20
10	40	10
0	45	0
50	20	50
60	15	60
70	10	70
80	5	80

[Table 17-14]

Tangent	80:0'	56712818196
Tangent	5:0'	874886635
Secant	80:0'	75787704831
Tangent	10:0'	1763269807
Tangent	40:0'	8390996312
Secant	10:0'	10154266119

[Table 17-15]

4. And by this method the tangents and secants can be prepared: the Logarithms of the same truly can be computed, either as the Sines by *Chapter 14, Arith. Logar.*; or preferably with the minimum bother and with none by the Logarithms of Sines first found, by *Propositions 10 and 11.*

Pro-port.	*Sine*	$6\underline{89}$	926462953.	996682805773424
	Sine	$22\underline{11}$.	376385967.	757563342320809
	Radius	$90\underline{00}$.	1000000000.	1000000000000000
	Tangent	$22\underline{11}$.	4062612173.	960880536547385

And by Prop. 9

Pro-port.	*Tangent*	$67\underline{89}$.	24614704959.	103911946345
	Radius	. . . .	10000000000.	100000000000
	Radius	. . . .	10000000000.	100000000000
	Tangent	$22\underline{11}$.	4062612173.	96088053655

[Table 17-16]

But with the logarithms for the tangents of the semiquadrant given; the logarithms are computed of the half left by subtraction alone from double the logarithm of the radius. And the logarithm of the tangent of this is the arithmetical complement of the complement of the tangent. And for this reason the logarithms from 45:0' as far as the end of the quadrant, can be omitted without much loss. However, it is better to add these, as the places that they occupy are left empty.

About the Logarithms of Secants

4. As by Prop. 11 the radius is the mean proportional between any sine and the secant of the complement: If the logarithm of the sine of any of these is taken from the logarithm of the radius doubled, then the remainder will be the logarithm of the complement of the Secant. As

				Logarithms
Pro-	*Sine*	1367.	2363294117	9373517774
port.	*Radius*	. . . .	10000000000.	10000000000
	Radius	. . . .	10000000000.	10000000000
	Secant	7633	42313819208	10626482226

[Table 17-17]

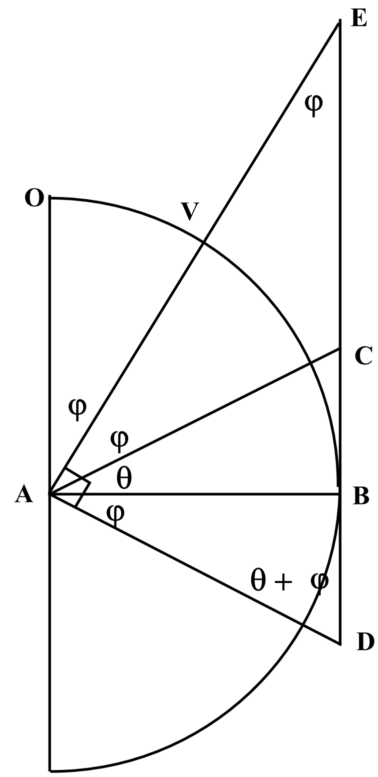
Because of this, the logarithms of the secants are found so much easier from the logarithms of the sines, and I have considered these should be omitted altogether from my considerations. For I do not want to stand in the way of diligent people, [who might wish to complete these tables], by offering an excess of ratios.

To be noted, that if these numbers were found beforehand, then the remainder can be added on most conveniently by quinquisection, as was shown for sines previously[5].

END OF THE FIRST BOOK.

Notes on Chapter Seventeen

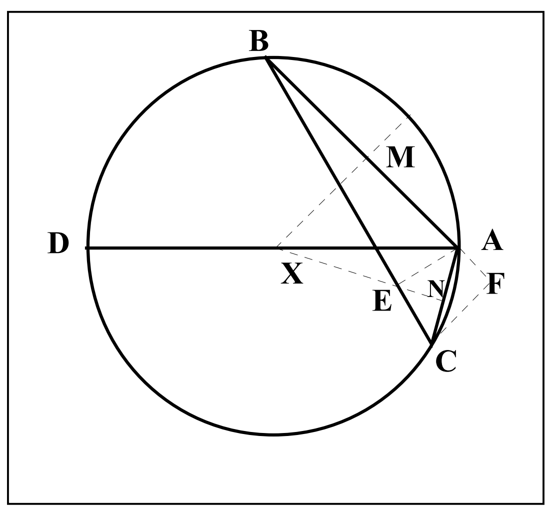
[1] These results are similar to those considered in Chapter 15, where the same construction has been employed. Thus, two tangents are known, corresponding to BE and BD. Half their sum is the secant AC of θ, while half their difference is the tangent BC of θ.



[2] For AC + CB = BE. For the complement of θ is 2φ, and half the complement is φ: BE is the tangent for the arc $\theta + \varphi$. Similarly, AC - CB = BD , which is the tangent of φ, half the complement.

[3] For 2BC + BD = CD + BC = BE.

[4] For BF - DP = BP - DP = BD, and DF = BF + DB ; hence, DF - DP = BD + (BF - DP) = 2BD.



[5] In modern terms, we set AD = $2R$; AC = b; BC = a, and CF = $a \sin B$. Then: Prop. 15 becomes $2R/b = a/(a \sin B)$, or $\sin B/b = 2R$. As similar results follow for the other sides, the Sine Rule is established. This result is apparent directly from proposition 15.

[6] Thus, in this rather abrupt manner, the last words are given by Briggs in this work. One may presume his health was failing during this time: as we see in the preface by Gellibrand, no attempt had been made to start the Book II. Here too the translation comes to an end, for Gellibrand's contribution has long since been rendered into English, and of course it lacks the wonderful *ab initio* constructs, both geometrical and numerical, used by Briggs to produce this work, his last masterpiece.