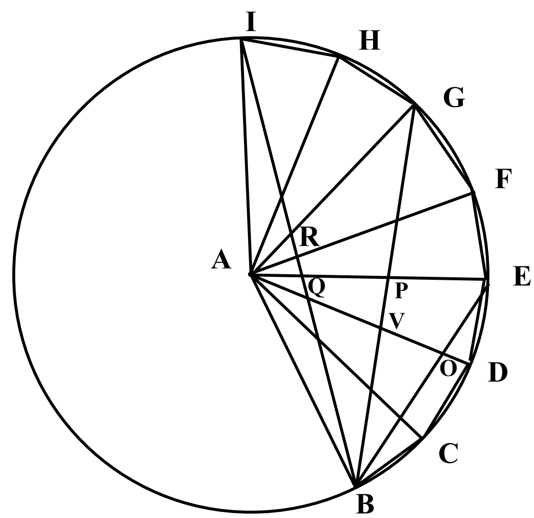
§7.1

Chapter Seven

Concerning Division by 7.

IF the arc BCDEFGHI is cut into these seven equal parts by the lines BE, BG, BI as drawn, OD will be 1 (2), and BO, 2 (1) — 1 (3). And the subtended chord of the seventh part, as here finally established, is equal to
7 (1) — 14 (3) + 7 (5) — 1 (7).



[Figure 7-1]

AB . . .	1
BC . . .	1 (1)
BO . . .	2 (1) — 1 (3)
VO . . .	2 (2) — 1 (4)
OD . . .	1 (2)
VD . . .	3 (2) — 1 (4)
AV . . .	1 — 3 (2) + 1 (4)
VP . . .	1 (1) — 3 (3) + 1 (5)
BV . . .	2 (1) — 1 (3)
BP . . .	3 (1) — 4 (3) + 1 (5)
BQ . . .	" " "
QP . . .	3 (2) — 4 (4) + 1 (6)
PE . . .	3 (2) — 1 (4)
VD . . .	" "
QE. . .	6 (2) — 5 (4) + 1 (6)
AQ . . .	1 — 6 (2) + 5 (4) + 1 (6)
QR . . .	1 (1) — 6 (3) + 5 (5) — 1 (7)
BQ. . .	3 (1) — 4 (3) + 1 (5)
IR . . .	3 (1) — 4 (3) + 1 (5)
IB . . .	7 (1) — 14 (3) + 7 (5) — 1 (7).

[Table 7-1]

Given therefore eight lines in continued proportion of which the first is the Radius, the second whatever subtended chord you wish: 7 times the subtended chord increased by 7 (5), is equal to 14 (3) + 1 (7) and corresponds to the subtended arc of seven times the angle.
And from this equation: for any given subtended chord, we can find the subtended chord for seven times the arc; or if on seven times the given arc there are one, two or three circles in place, we can find the subtending chord for the angle in excess of the angles of the whole circles.

However, it should be noted for all applications of this method involving section equations, if the number of the entire circles is even: the same signs of the equation are used, as is the case for the periphery altogether coming to less than 360 degrees; if on the other hand the number of whole circles is odd, then the opposite signs are put in placed. Let the given subtended chord angle be 140 degrees. Seven times 140 has the value 980. That is 260 beyond two whole circles.

For the Subtended 140 18793852415718167610822,
the equation 7 1 — 14 3 + 7 5 — 1 7.

7 1	1315569669100271737675754
7 5	16412589242269014888365139
Sum 7 1 + 7 5	17728158911369286626040893 *
14 3	{ 6638155724715450304324660
	{ 26552622289886180121729864
1 7	8281532008143860592945789
Sum 14 3 + 1 7	17574950022745491019000313
	17728158911369286626040893 *
Subtending 260 } 100 }	15328888623795607040580

See Chapter 5, Section 4. The number of continued proportions of which the first is the radius; the second is the subtended chord of 140:0':

[Table 7-2]

Let the subtended chord be 157:2':40": 196000039119128.
Seven times 157:2':40" has the value 1099:18':40" or 19: 18':40" beyond three whole circles.

7 1	137200273833896
7 5	2024802790966039
Sum 7 1 + 7 5	2162003064799935 †
14 3	752958108410275
	301183243364110
1 7	1111216207363663
Sum 14 3 + 1 7	2165357559138048
	2162003064799935 †
Subtending 19: 18':40":	03354494338113

See Chapter 5, Section 5. The number of continued proportions for the subtended chord of 157:2':40":

[Table 7-3]

Because the number of circles is more than a whole circle the equation will be
14 3 — 7 1 — 7 5 + 1 7.
Let the given subtended chord or 10 degrees be 17431148549. The chord of 70:0' is sought:

7 1	122018039843	
7 5	112649186	
Sum 7 1 + 7 5	122130689029 ††	
14 3	5296366280	
	2118546512	
1 7	488969	
Sum 14 3 + 1 7	7415401762	35
	122130689029	11††
	114715287266	76

See section 3, Ch. 5.

[Table 7-4]

2. We are able also by the same equation, for any given subtended arc, to find the subtended chord of the seventh part of the same arc, or the total composed from the given arc, and with one, two, or three circles.

The way of working will be scarcely different from that which was expounded above, for the subtended chords of the third and fifth parts. But the working for the multiplication of the terms is not without a little more labour.

And in this way the easiest equations are found and can be demonstrated: for any periphery you wish can be cut into equal parts however big, the smaller chord length is determined by the method which gives all the subtended chords of multiplied arcs, by the same preceding method, and the parts also, but not with the same facility. For where there are many arcs of equal segments, for these there are equations with more terms, and to any term is added a larger number: As with all operations the other method of dividing the arc will be the source of greater difficulty.

§7.2 Notes on Chapter Seven

1 This beautiful piece of elementary geometry is based on the set of similar isosceles that Briggs considered previously; see the Notes 1 of Chapters Three and Five. The constructions 'work' because the angle subtended at the centre is double the angle subtended at the circumference by the equal angles GBI, GBE, and EBC. As AH bisects the arc GI, etc, it follows that the angles at the centre such as HAI are equal to the angles GBI, etc. The rest follows from the set of similar isosceles discussed previously. Thus, for the first set of proportionalities, [Table 7-1] becomes:

AB	1
BC	p
BO	$2p - p^3$
VO	$2p^2 - p^3$
OD	P^2

[Table 5 - 1A]

VD	$3p^2 - p^3$
AV	$1 - 3p^2 + p^4$
VP	$P - 3p^3 + p^5$
BV	$2p - p^3$
BP	$3p - 4p^3 + p^5 = BQ$
PQ	$P^2 - 4p^4 + p^6$
PE	$3p^2 - p^4 = VD$
QE	$6p^2 - 5p^4 + p^6$
AQ	$1 - 6p^2 + 5p^4 - p^6$
QR	$p - 6p^3 + 5p^5 - p^7$
BQ	$3p - 4p^3 + p^5$
IR	$3p - 4p^3 + p^5$
IB	$7p - 14p^3 + 7p^5 - p^7$

[Table 5 - 2A]

Hence, the total length of the subtending chord IB = 2.BQ + QR = $7p - 14p^3 + 7p^5 - p^7$.