Trigonometriae Britannicae

First Book

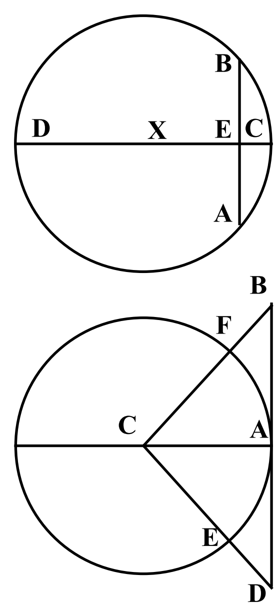
Chapter One

Since I am about to write about Trigonometry, I think it necessary that some things should be explained by me before I approach the science itself (among which there are a few things with which I seem to disagree with others[1].)

The foundation of trigonometry is based on the similarity of plane triangles for which only the equality of the angles or the ratio of the sides is required. Moreover, for these trigonometric calculations nothing is known with certainty except for the numbers expressed by the measurements of these quantities. Therefore, these calculations must be concerned not only with the circumferences of circles from which angles are measured, but also with straight lines: those especially which have been drawn inside the circle, marked off in some known amount and cut into some fraction, in order that we may more easily come to a decision on the lengths of other sections.

We divide the circumference of any circle whatever into 360 equal parts, that we call degrees [the Latin *Gradus,* means *step*], and any of these are divided as you please by the 60 – fold ratio into minutes & seconds, etc. Truly, I am persuaded by the authority of Viète's *Gregorian Calendar*, p. 29, and by the urging of others [to consider an alternative scheme]. There are 100 primary degrees that are divided by me in a 10 – fold ratio, and these can be divided in turn in the same ratio as often as you please, and these fractions give rise to a much easier and not less certain calculation.

So the radius or semi-diameter of the circle, that we take as one part, we will then divide into tenths of thousandths of hundredths of thousandths of thousandths of a thousandth or 1,000,000,000,000,000 equal small parts. [i.e. (${}^{1}/_{10}$ of ${}^{1}/_{1000}$) of (${}^{1}/_{100}$ of ${}^{1}/_{1000}$) of (${}^{1}/_{1000}$ of ${}^{1}/_{1000}$) or 14 places].



[Figure 1-1]

Also the remaining lines drawn to the circle are expressed by these same fractions. Some of these [lines] fall within the circle, being *subtended* or *inscribed*; which are terminated by the same points with which the arcs subtend these chords, thus AB is subtended by the arcs ACB and ADB.

Of these the halves are called the *Sines* of the half arcs, as AE is the Sine of the Arc AC & of the Arc AD.

The Sine is Half the Chord[2] of the two Arcs [i.e. applying to both the small arc with the acute angle and the large arc for the supplementary angle]. Or the Sine is the perpendicular from either arc of the circumference onto the Diameter crossing from the remaining arc. As AE is the perpendicular to the Diameter CED.

Other Perpendicular lines to the end of the Diameter we call *Tangents*, which are terminated by the lines drawn from the Centre which we call *Secants.* Thus the Tangents of the Arcs AF and AE are AB, AD; while the Secants of these are CB and CD.

And these Tangents and Secants are expressed in fractions of the given Radius, not less than of the Subtended Chords as Sines. But in the first place the Subtended Chords ought to be found, which here will have been noted, the finding of the Tangents and the Secants will be subservient.

The usual manner of finding Subtended Chords from Antiquity is expounded by *Ptolemy, Regiomontanus, Copernicus, Rheticus* & others; and before these from *Hipparchus* and *Menelaus*. Truly the time has come for another shorter and not less certain way from that [given].

Moreover, the usual method for these from Antiquity that I will consider briefly initially is taken together with these which *Ptolemy*[3] left us. Ptolemy divided the diameter of the circle into 120 equal parts, accommodating all the Subtended Chords for this Diameter, with whole number parts & Sexagesimal fractions as minutes[4]. But *Regiomontanus* & all the more recent except *Maur*: *Bressium* set up a Diameter of 200000 parts, with some number of zeros added, as it would appear.

Notes On Chapter One

[1] For this was a 'Golden Moment' in the evolution of Tables, in which to make a change from the sexagesimal division of the circle into 360 equal parts, to that of 100 parts only, with hundredths and thousandths of the new degree. This was a move favoured at least by some of the table makers, as suggested by Viète, in his *Calendarii Gregorianii*, page 29: but resisted by the table users, who wanted to stay with tradition. Briggs goes as far as to provide a basic table of 40 equally spaced angles of sines for the quadrant made up of 25 'degrees' for this new scheme in Chapter 14.

[2] The word *Sine* is of doubtful origin, according to the preamble to *Hutton's Mathematical Tables*, p.17. It is of some interest to note that the right - hand smaller arc of the first diagram in [Figure 1-1] can be thought of as representing a bow or *arc ACB*, while the string or cord is the *chord AB*; the length of the bolt or arrow is the *sagitta EC.* The tangent AB is the line in the lower diagram which touches the circle at A (*tangere*: to touch), while the secant BC is cut by the circle (*secare*: to cut) at F. Note in passing that what we now consider as *ratios* in elementary trigonometry or functions of the angle EXB or θ in analysis were originally considered as *lengths w.r.t.* a given radius, which was usually given by a large power of ten. There is no convenient explanation, however, for the use of the word *sine* for the length of the half chord EA. Hutton considers it to be of Latin origin, in which the word *sinus* has various related meanings, namely a fold (of the toga at the breast), a hollow, a bay or gulf, etc. This book also provides some useful information on some of the early tabulators mentioned by Briggs in this chapter.

For the half chord EA or EB, Briggs uses the word '*Subtensus, -a, -um*' as an adjectival passive past participle, meaning, '(being) stretched or (being) held under' the corresponding arc, where the use of the passive 'being' is optional in English, and does not change the meaning. For convenience, we will always call this the 'subtended chord', or just 'chord' though the word 'chord' is not in the original text of Briggs. Thus, if the

Latin text states '*Subtensae AC*', we translate this as' of the [Subtending] Chord AC', or 'to/for the Chord AC'.

[3] Anyone wishing to know more about the origins of the table of sines can do little better than to spend some time reading *Book I* of Ptolemy's *Almagest.* This is readily accessible, e.g. in Volume 16 of *The Great Books of the Western World, (Ency. Brit.),* where the famous table of Chords is set out on pp.21 - 24, and an explanation given for their construction.

For example, the entry for 90 degrees is the chord 84:51:10. The radius R has 60 parts, and the sought chord has length 2R sin45 = 84.852814 parts. According to Ptolemy, this is $84 + {}^{51}/_{60} + {}^{10}/_{3600} = 84 + 0.85 + 0.0027777 = 84.85277$.

A good general source is *A History of Greek Mathematics*, Vol. I and II, by Sir Thomas Heath (Dover).

[1]

LIBER PRIMUS.
CAPUT PRIMUM.

De Trigononometra scripturus, quaedam (inter quae pauca sunt in quibus ab aliis discrepare videor) mihi necessario explicanda censeo ante quam ipsam doctrinam aggrediar.

Trigonometriae fundamentum positum est in similitudine Triangulorum planorum ad quam requiruntur tantummodo aequalitas Angulorum vel proportio Crurum; harum autem nulla potest esse certa cognito nisi numeris exprimantur earum mensurae. Debent igitur non solum Peripheriae Circulares, quibus Angulos metimur, sed etiam lineae, illae praesertim quae Circulis sunt adscriptae, in notas aliquot & certas partes secari, ut de utrarum que magnitudine rectius & facilius possimus iudicare.

Peripheriam igitur quamlibet in 360 partes secamus aequales, quas appellamus *Gradus*; & horum quaelibet Sexagecupla ratione in *Minuta, & Secunda,* &c. Ego vero adductas authoritate Vietae pag. 29.Calendarii Gregoriani, & aliorum hortatu, Gradus partior decupla ratione in partes primarias 100, & harum quamlibet in partes 10 quarum quaelibet secatur eadem ratione. Atque hae partes culculum reddunt multo facilorem, & non minus certum. Radium autem vel Semi-Diametrum Circuli, statuimus esse unius partis, quam dividimus deinceps in particulas aequales *Decies Centies Millies Mille*; vel 1,000,000,000,000,000 .

Reliquae etiam Rectae Circulo adscriptae, exprimuntur in iisdem partibus. Earum quaedam intra Circulum cadunt, ut *Subtensae* vel *inscriptae*; quae iisdem punctis terminantur quibus Peripheriae quas subtendunt, ut AB Subtensa Peripheriarum ACB, & ADB.

Harum Semisses appellentur *Sinus* Arcuum Dimidiatorum, ut AE Sinus Arcus AC & Arcus AD.

Sinus est Demissis Subtensae Dupli Arcus. Vel Sinus est Perpendicularis ab altero Termino Peripheriae, in Diametrum per reliquum Terminum transeuntem. Ut AE Perpendicularis Diametro CED.

Alias lineas extremae Diametro Perpendiculares appellamus Tangentes, quae terminantur a rectis ductis a Centro, quas appellamus Secantes. Ut Peripheriarum AF, AE, Tangentes sunt AB, AD. Secantes autem earundem sunt CB, CD.

Atque hae Tangentes & Secantes exprimuntur in partibus Radii datis, non minus quam Subtensae ut Sinus, Imprimis autem inveniri debent Subtensae, quae ubi notae fuerint, Tangentium & Secantium inventioni subserviunt.

[2]

Modus inveniendi Subtensas ab Antiquis usitatus traditur a *Ptolemaeo, Regiomontano, Copernico, Rhetico* & aliia: & ante hos ab Hipparcho & Meneao: Ista vero aetas alium modum invenit magis compendiarium, & non minus certam.

Modus autem ab iis Antiquis usitatus, primo in loco quam potero brevissime dicatur, sumpto unitio ab iis quae Ptolemaeus nobis reliquit. Qui diametrum Circuli possuit partium aequalium 120, Subtensas omnes huic Diametro accommodans, in integris partibus Sexagesimis utpote Minutis. At Regiomontanus & recentiore omnes praeter Maur: Bressium statuunt Diametrum partium 200000, adiectis si visum sit quolibet Ciiphris. .