§7.1. ***Synopsis: Chapter Seven.***

A region *L* in a series of continued means has been established in Chapter 6 in which there is finally proportionality between the fractional parts of the means and their known logarithms. Briggs shows how the logarithms of 2 and 3 can be found using a method based on this proportionality, upon forming a sequence of continued mean numbers based on the repeated square root extraction of a number derived from 2 or 3, that eventually lie in the above region. Initially, 2 is converted to $2^{10}/10^{3} = 1.024$ prior to forming the mean sequence: a process that reduces the number of means evaluated; in the region *L*, the fractional part of the 47^{th} continued mean of 1.024 is compared with a continued mean of 10, as we have indicated in the extended note in the last chapter, from which the logarithm of 2 is determined. The logarithm of 5, and multiples of 2, 5, and 10 can then be found. Similarly, in the case of 3, the logarithm of 6 is found from $6^{9}/10^{7}$ or 1.0077696, again reducing the number of operations slightly.

§7.2. *Chapter Seven*

With these continued means found between one and ten, together with their rational logarithms, by this method with the help of these the logarithms of all the other numbers can be found .

Between the given number (of which logarithm is sought) and one, the continued means shall be sought (as in the above chapter), until finally we find a number so very small that unity precedes fifteen zeros, and which may be followed by as many or more significant figures. Then (as we have shown at the end of the preceding chapter) these significant figures, by the rule of proportion, will give us the logarithm sought of that smallest mean number found. But in the last

2 - - - - - - - - - - -	
14142135623731	
11892071150027	
10905077326652	A number of the continued means between One and Two.
10442737824322	
10218971487053	
&c.	

[Table 7-1]

series of continued means for the logarithm of the first given number, we arrive at the logarithm of the smallest mean sought by bisection: Here, in the opposite direction, by doubling the logarithm found of the smallest mean, we will obtain the penultimate logarithm: And by the same method all of the remaining logarithms, until at last we shall have obtained the logarithm sought of the first number given.

And also this method itself has more than enough vexation: but we ourselves shall try to lessen this in part[1].

Since indeed previously we considered by lemma 1, chapter 1, from the logarithms of any two numbers in continued proportion, to be able to find the logarithm of any other number, the position of which from the given numbers will be known. And since the logarithm of the dividend, to be equal to the sum of the logarithms of the divisor and the quotient, by axiom 3, chapter 3, we shall be able, in the place of the given number, to substitute another more convenient number, of which the logarithm found, will also show the logarithm of the number given, by this method: The given number may be multiplied by itself and in products with itself, until the number *A* has been found, of which the first place to the left shall be one, as in the nearby position one or several ciphers follow. Then that number being divided by a number, which is to be described by one and as many ciphers as there will have been places in front of one in the number to be divided : and finally between this quotient and one the number of the continued means may be sought, until we have reached these limits, which we have assigned above, near to the number *L*. But those means have been found more easily, and from a smaller number, than those which were sought between the given number 2 and 1. Which all ought to be made clear by an example.

Let the logarithm of two be sought: a series in continued proportion shall be constructed by multiplication by 2 until 1024, ***A*,** is reached, that should be divided by 1000 *B*, the quotient will be $1\underline{024}$: when the logarithm of this number has been found, it is added to the logarithm of the divisor, and it will give the logarithm of the dividend *A,* 1024, by axiom 3, ch. 3. But the tenth part of this logarithm, by lemma 1, ch. 1, will be the logarithm of 2 sought.

Cont. Prop.	Indices
1	
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9
A1024	10

[Table 7-2]

Therefore , the root of the quotient found shall be sought, and the rest of the continued means, until we shall have come as far as to the forty-seventh mean, between the given quotient and one, which is 10000,00000,00000,016851605705394977, which as it shall be placed within the above established limits near *L*, the logarithm of that number is found by the golden rule: 0,00000,00000,00000,07318,55936,90623,9336.

Proportions	12781,93493,20032,344161 16851,60570,53949,77 - -	The significant places after 15 zeros to be added to unity in the mean numbers.[2]
	55511,15123,12578,27021 73185,59369,06239,368 - -	The significant places after characteristic and 16 other zeros to be added to the Logarithms
	1 - - - - - - - - - - - - - - - - - - 16851,60570,53949,77	Places of the means as with prior proposition.
	43429,44819,03251,804 73185,59369,06239,368	Places of the Logarithms as with prior proposition.

A

B Divisor 1000)1024 (1024 Quotient

	Number of successive means	*Logarithms*
	1024 - - - - - - - - - - - - - - - - - - -	0,01029,99566,39811,95265,27744
47	10119,28851,25388,13862,397	0,00514,99783,19905,97632,63872
46	10059,46743,74634,83266,5424	0,00257,49891,,59952,98816,31936
45	10029,68064,49807,87373,6268	0,00128,74945,79976,49408,15968
44	1001483382,03790,41803,01838	0,00064,37472,89988,24704,07984
43	10007,41416,16998,35336,24906	0,00032,18736,44994,12352,03992
42	10003,70639,39821,00140,71761,5	0,00016,09368,22497,06176,01996
41	10001,85302,53059,10853,05827,77	0,00008,04684,11248,53088,00998
31	10000,00180,94275,48445,34363,95015,44	0,00000,00785,82432,85989,34375,977
21	10000,00000,17670,18930,57014,19482,62	0,00000,00000,76740,65708,97396,85154
11	10000,00000,00017,25604,42423,25943,477	0,00000,00000,00074,94204,79391,98911,283
1	10000,00000,00000,01685,16057,05394,977	0,00000,00000,00000,07318,55936,90623,9368

[Table 7-3]

1	
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9
1024	10
1048576	20
1073741824	30
1099511627776	40
14073488355328	47

[Table 7-4]

Here [Table 7-3] we have some of the continued means between 1 and 1024. The total number of these is 47, of which the first shall be the smallest and closest to 1: the logarithm of this is found by that method which is explained at the end of the sixth chapter. The rest we shall find by doubling this one found, and by successive doubling, until the logarithm of the quotient 1024 itself will have been reached. Or, if it were considered, to seek the logarithm by omitting some more removed intermediate means. A series of numbers from unity in continued proportion with the ratio of 2 shall be made, with which the common indices may be associated. And the number may be taken, situated across from these indices, which is equal to the number of the interval between the given and sought logarithms, and that number shall multiply the given logarithm, and the product will be the logarithm sought. As, for the sake of an example: if the nearest logarithm is being sought (because it is a single interval) it is multiplied by two; if the second from that given, by four; if the tenth from the given, multiply by 1024, because the index of this number is 10 : if the forty seventh from the given, the product will be 140737488355328. The logarithm of the first mean given may be multiplied by that forty seventh number, the product will be the logarithm of the given quotient; between which and the first there are 47 intervals. The reason for this is as follows. These logarithms are continued proportions with a ratio of 2; that is, they are themselves proportional to these numbers. Therefore, as the first of these numbers, to the eleventh or the forty eighth: thus, the first logarithm, to the eleventh or the forty eighth logarithm.

Finally, having found the logarithm of the quotient, the logarithm of the dividend is to be sought, which is found by axiom 3, chapter 2.

		Logarithms
pro-port.	- - - - - - - 1	0,0000
	Divisor 1000	3,00000,00000,00000,00
	Quotient1024	0,01029,99566,39811,95
	Dividend 1024	3,01029,99566,39811,95

[Table 7-5]

It is indeed the given logarithm of the divisor by chapter 4, and by axiom 2, chapter 2, which added to the logarithm found of the quotient, gives the logarithm of the dividend.

But since the dividend 1024, and the given 2, of which the logarithm is sought, shall be placed in the same sequence of continued proportionals from one, and of these the indices 10 and 1 shall be given: it is clear by axiom 1, chapter 2. The logarithm to be divided tenfold to be the logarithm sought, which ought to be assigned to 2 . Therefore the logarithm of 2 is: 0,30102,99956,63981,195.

The logarithm of two found, the logarithm of five itself will be shown. Indeed 2, 1, 10, 5 are in proportional, and by lemma 2, chapter 1, with the logarithms of three proportionals given, the logarithm of the fourth will be disclosed.

Proportions	*Logarithms*
2	0,30102,99956,63981,195
1	0
10	1,00000,00000,00000,000
5	0,69897,00043,36018,805

[Table 7-6]

Besides the logarithm of five, the logarithms of all will be known, which came about in any way whatever from multiplication, or the division, by these three 2,5,10 among themselves. From the multiplication of two alone, by itself and into its products gives 4, 8, 16, 32, 64, etc; likewise of five into itself and its products gives 25, 125, 625, 3125, etc. ; products of two and five give 250, 1250, 6250, 3125, etc. ; of two with ten 20,200, 2000, 40, 400, 80, 800, etc.; by division, $2\frac{1}{2}, 1\frac{1}{4}, 12\frac{1}{2}, 6\frac{1}{4}, 1\frac{3}{5}, 3\frac{1}{5}, 6\frac{2}{5}$, etc., which shall all become clear by axioms 2 and 3 of chapter 2.

The next after the place of two and five comes three, of which the logarithm will be most easily found from the logarithm of six, which I seek by that method described above; therefore, let a series in continued proportion be made in which 6 will be nearest to 1, and 10077696 may be taken, which being divided by 10000000 gives the quotient 1$\underline{0077696}$, between this quotient and one I seek 46 continued means, from which the nearest to one is 10000,00000,00000,01099,85934,58815,57186,6, of which the logarithm by the golden rule is 0,00000,00000,00000,04776,62844,78608,0304, and this logarithm being doubled 46 times, will give the logarithm 0,00336,12534,52792,69 of the quotient 1$\underline{0077696}$. To which the logarithm of the divisor 7,00000,00000,0000 added will give 7,00336,12534,52792,69, the logarithm of the dividend 10077696; of which the ninth part (because the index of the divisor is 9) will be 0,77815,12503,83643,63, the logarithm sought of six , from which if the logarithm of two is taken, away, the logarithm of three will remain, all of which are in agreement with what was explained above by this chapter itself.

1	
6	1
36	2
216	3
1296	4
7776	5
46656	6
279936	7
1699616	8
10077696	9

Proportions		*Logarithms*
Unity	1 - - - - - - -	0,00000
Divisor	10000000	7,00336,12534,52792,69
Quotient	1<u>0077696</u>	0,00336,12534,5279259
Dividend	10077696	7,00336,12534,5279269

Indices of the numbers	10077696 - - -	9 - - - - - - - -	proportionals
	6 - - - - - - - - -	1 - - - - - - - -	
Logarithms of the numbers	10077696 - - -	7,00336,12534,5279269	
	6 - - - - - - - - -	0,77815,12503,83643,63	

proportionals	6.	0,77815,12503,83643,63	Logarithms of 6, 2, 3.
	2.	0,30102,99956,63981,19	
	3.	0,47712,12547,19662,44	
	1.	00	

[Table 7-7]

§7.3. ***Notes On Chapter Seven.***

1 The ammended method proposed has fewer root extractions; in addition, Briggs has a method of extracting the square roots of numbers close to one, involving finite differences, that is far less inconvenent than the 'completing the square' routine that he presumably used initially, the use of the new method he explains in the next chapter. One gets the feeling that he went through a rather exasperating time until he had found a method that was not too trying even on his considerable computational skills. We tend to forget that all the working was done by hand, and simple additions/subtractions, division by two, etc., at all times were highly preferable to long multiplications, division, and the tedious root extraction procedure. One might estimate a time of several hours being needed to perform *one* root extraction to the 30 plus places Briggs worked to: the labour being more intense at the start. Hence, we are looking at something in the order of 100 to 200 hours work for the complete extraction down to the 50 th plus root. Given other commitments, we are looking perhaps at some 6 weeks work or more. Given the added annoyance of mistakes made along the way, which meant returning to an earlier stage again, one can only marvel at the dedication shown by Briggs to his task.

2 Rows one and three are the numbers in row P of the large table in the previous chapter, representing the non-zero digits after 15 ciphers; the first for the 54 square root of 10, and similarly the third for the logarithm. The 47th root of 1.024, or mean proportional between 1 and 1.024, is in row two. The number in row one is adjusted to 1 for ease in future work, and the corresponding log in row three adjusted in proportion. The log of the number in row two is found by Briggs' 'Golden Rule' and placed in row four. Note that in modern terms, in the over all process, Briggs has evaluated the natural logarithm of 1.024, to the accuracy required, from the relation

$(1.024^{1/2^{47}} - 1).2^{47} \sim \ln(1.024)$, and converts to base 10 logarithms, on dividing by ln(10).

§7.4. **Caput VII. [p.12.]**

Inventis hisce continue mediis inter Unitatem & Denarium, una cum eorum Logarithmis rationalibus; aliorum omnium numerorum Logarithmi, horum ope inveniri poterunt, ad hunc modum.

Inter datum numerum (cuius Logarithmus quaeritur) & unitatem, quaerantur continue medii, (ut in superiore capite) donec tandem inveniamus numerum adeo exiguum, ut Unitas praecedat quindecim cyphras, quas totidem vel plures notae significativae sequantur. Deinde (ut ad finem praecedentis capitis ostendimus) hae notae significativae, per proportionis regulum dabunt nobis quaesitum Logarithmum, illius numeri medii ultimo inventi.

In priore autem illa continue mediorum serie, a primi numeri Logarithmo dato, bisecando devenimus ad ultimi medii Logarithmum quaesitum: hic contra, duplicando ultimi medii Logarithmum inventum, penultimi Logarithmum assequemur: eodemque modo reliquorum omnium; donec tandem dati primi numeri quaesitum Logarithmum nacti simus.

2 - - - - - - - - - - -	
14142135623731	Numeri continue medij inter Unitatem & Binarium
11892071150027	
10905077326652	
10442737824322	
10218971487053	
&c.	

Atque hic modus etiam in se habet molestiae plus satis. nos autem eam conabimur aliqua ex parte minuere.

Cum enim antea accepimus, in continue proportionalibus, datis duorum quorumlibet Logarithmis, posse alterius cuiusvis, cuius situs a datis notus erit, Logarithmum inveniri: per Lemma 1. cap. 1 & Logarithmum divisi, aequari Logarithmis divisoris & quoti, per 3. ax. cap. 3. poterimus, loco dati numeri, alium substituere commodiorum, cuius Logarithmus inventus, dati etiam numeri Logarithmum ostendet. ad hoc modum. Multiplicetur datus numerus in seipsum, & in factos a seipso, donec inveniatur numerus *A*, cuius prima nota versus sinistram sit Unitas, quam proximo in loco sequatur cyphra una aut plures: deinde ille numerus dividatur, per numerum qui describitur per unitatis notam & tot cyphras quot fuerint notae in dividendo praeter Unitatem : tandemque inter quotum hunc & Unitatem quaerantur numeri continue medii, donec ad illos terminos pervenerimus, quos supra assignavimus, iuxta numerum L. Isti autem medii sunt & numero pauciores, & inventu faciliores, quam illi qui quaerebantur inter datum numerum 2 & Unitatem. Quae omnia sunt exemplo illustranda. Quaerendus esto Logarithmum Binarii: fiat per multiplicationem [p.13.] Binarii, series continue proportionalium donec inveniatur 1024. A. is dividatur per 1000.B. quotus erit 1024, cuius Logarithmus ubi inventus fuerit, additus Logarithmo divisoris, dabit Logarithmum numeri A divisi 1024. per 3.ax. cap.3. huius autem Logarithmi pars decima per Lemma 1. cap. 1. erit Logarithus Binarii quaesitus.

Quaerantur idcirco latus quoti inventi, reliquique; continue medii, usque dum pervererimus ad quadragesimum septimum medium, inter datum quotum &Unitatem. qui est 10000,00000,00000,016851605705394977. qui cum intra limites superius constitutos iuxta L situs sit, eius Logarithmus per auream regulam invenitur 0,00000,00000,00000,07318,55936,90623,9336.

Cont. Proport.	Indices.
1	
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9
A1024	10

Pro-port.	12781,93493,20032,344161	Notae significativae post quindecim cyphras unitati adijciendae in numeris medijs
	16851,60570,53949,77 - -	
	55511,15123,12578,27021	Notae significativae post characteristicam & alias sexdecim cyphras Logarithmis adijciendae
	73185,59369,06239,368 - -	
Pro-port.	1 - - - - - - - - - - - - - - - - - -	Notae Mediorum ut prius.
	16851,60570,53949,77	
	43429,44819,03251,804	Logarithmorum notae ut prius.
	73185,59369,06239,368	

A

B Divisor 1000)1024 (1024 Quotus

	Numeri continue medij	*Logarithmi.*
	1024 - - - - - - - - - - - - - - - - - - -	0,01029,99566,39811,95265,27744
47	10119,28851,25388,13862,397	0,00514,99783,19905,97632,63872
46	10059,46743,74634,83266,5424	0,00257,49891,,59952,98816,31936
45	10029,68064,49807,87373,6268	0,00128,74945,79976,49408,15968
44	1001483382,03790,41803,01838	0,00064,37472,89988,24704,07984
43	10007,41416,16998,35336,24906	0,00032,18736,44994,12352,03992
42	10003,70639,39821,00140,71761,5	0,00016,09368,22497,06176,01996
41	10001,85302,53059,10853,05827,77	0,00008,04684,11248,53088,00998
31	10000,00180,94275,48445,34363,95015,44	0,00000,00785,82432,85989,34375,977
21	10000,00000,17670,18930,57014,19482,62	0,00000,00000,76740,65708,97396,85154
11	10000,00000,00017,25604,42423,25943,477	0,00000,00000,00074,94204,79391,98911,283
1	10000,00000,00000,01685,16057,05394,977	0,00000,00000,00000,07318,55936,90623,9368

Hic habemus aliquot e continue mediis inter Unitatem & 1024. totus eorum numerus est 47, quorum minimus & unitati proximis sit primus. huius Logarithmus invenitur eo modo qui traditur ad finem sexti Capitis. reliquos vero inveniemus duplicando hunc inventum, reliquosque per continuam duplicationem inventos, donec ad ipsius quoti 1024. Logarithmum perventum fuerit. vel, si visum fuerit, omissis aliquot intermediis, remotioris alicuius Logarithmum quaerere; fiat series numerorum ab unitate continue proportionalium in ratione dupla, quibus adiungantur Indices vulgares. & sumatur numerus, situs e regione Indicis illius, qui aequatur numero intervallorum inter datum Logarithmum & quesitum, is numerus multiplicet datum Logarithmum; factus erit Logarithmus quaesitas. ut si proximus quaeratur Logarithmus (quia unicum est intervallum) binarius multiplicet; si secundas a dato, quaternarius; si decimus a dato, multiplicet 1024. quia huius numeri Index est 10: si quadragesimus septimus a dato, factor erit 140737488355328. exempli causa; primi medii datus Logarithmus multiplicetur per illum quadragesimum septmum numerum, factus erit Logarithmus dati Quoti; inter quem & primum sunt 47 intervalla. Huius rei causa haec est. Logarithmi hi sunt [p.14.] continue proportionales in dupla ratione; id est, sunt his ipsis numeris proportionales. & idcirco, ut horum numerorum primus, ad undecimum vel quadragesimum octavum: sic primus Logarithmus, ad undecimum vel quadragesimum octavum Logarithmum.

Invento tandem Quoti Logarithmo, quaerendus est Logarithmus Divisi. qui habetur per 3. ax. cap. 2.

1	
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9
1024	10
1048576	20
1073741824	30
1099511627776	40
14073488355328	47

		Logarithmi.
pro- port.	- - - - - - - 1	0,0000
	Divisor 1000	3,00000,00000,00000,00
	Quotient1024	0,01029,99566,39811,95
	Dividend 1024	3,01029,99566,39811,95

Est enim Logarithmus Divisoris datus, per cap. 4. & per 2. ax. cap2. qui additus quoti Logarithmo invento, dat Logarithmum divisi.

Cum autem Divisus 1024, & datus 2, cuius Logarithmus quaeritur, siti sint in eadem serie continue proportionalium ab unitate, eorumque Indices 10 & 1 sint dati: manifestum est per 1. ax. cap. 2. Divisi Logarithmum decuplum esse Logarithmi quaesiti, qui Binario tribui debet. Est igitur Logarithmus Binarii. 0,30102,99956,63981,195.

Binarii Logarithmus inventus, Quinarii Logarithmum secum exhibebit. Sunt enim 2.1.10.5 proportionales. & per 2. Lem. cap.1 datis trium proportionalium Logarithmis, quartis proportionalis Logarithmus manifestus erit.

		Logarithmi.
Pro- port.	2	0,30102,99956,63981,195
	1	0
	10	1,00000,00000,00000,000
	5	0,69897,00043,36018,805

Praeter quinarii Logarithmum, noti erunt Logarithmi omnium, qui quovis modo proveniunt ex multiplicatione, vel divisione horum trium 2.5.10. inter se.

Ex multiplicatione Binarii solius in seipsum & in suos factos, 4, 8, 16, 32, 64, etc. item Quinarii in se & suos factos, 25, 125, 625, 3125, etc. Binarii in factos a Quinario, 250, 1250, 6250, 3125, etc. Binarii in Denarium 20,200, 2000, 40, 400, 80, 800, etc.

Ex divisione $2\frac{1}{2}, 1\frac{1}{4}, 12\frac{1}{2}, 6\frac{1}{4}, 1\frac{3}{5}, 3\frac{1}{5}, 6\frac{2}{5}$, etc., quae omnia fiunt manifesta per 2 & 3.ax. cap. 2.

Proximos post Binarium & Quinarium loco venit Ternarius, cuius Logarithmus commodissime inveniri per Logarithmum Senarii. quem quaero ad eum qui superius traditur modum, fiat igitur series continue proportionalium, in quibus 6 sit proximus unitati, & sumatur 10077696, qui divisus per 10000000 dat quotum $1\underline{0077696}$, inter hunc quotum & unitatem, quaerto continue medios quadraginta sex, e quibus unitati proximus est 10000,00000,00000,01099,85934,58815,57186,6, cuius Logarithmus per proportionis regulum 0,00000,00000,00000,04776,62844,78608,0304, atque hic Logarithmus quadragies sexies duplicatus, dabit 0,00336,12534,52792,69 Logarithmum Quoti $1\underline{0077696}$. Cui Logarithmus divisoris 7,00000,00000,0000 adiectus dabit 7,00336,12534,52792,69, Logarithmum divisi 10077696; cuius pars nona (quia divisi Index est 9) erit 0,77815,12503,83643,63 Logarithmus Senarii) quaesitus, a quo si ablatus fuerit [p.15.] Logarithmus Binarii, restabit Logarithmus Ternarii. quae omnia sunt iis consentanea quae hoc ipso Capite traduntur superius.

1	
6	1
36	2
216	3
1296	4
7776	5
46656	6
279936	7
1699616	8
10077696	9

Proportions		*Logarithmi.*
Unitas.	1 - - - - - - -	0,00000
Divisor.	10000000	7,00336,12534,52792,69
Quotus.	$1\underline{0077696}$	0,00336,12534,5279259
Divisus	10077696	7,00336,12534,5279269

Indices numerorum	10077696 - - -	9 - - - - - - - -	proportionales
	6 - - - - - - - - -	1 - - - - - - - -	
Logarithmi numerorum	10077696 - - -	7,00336,12534,5279269	
	6 - - - - - - - - -	0,77815,12503,83643,63	

pro-port.	6. 0,77815,12503,83643,63	Logarith. 6, 2, 3.
	2. 0,30102,99956,63981,19	
	3. 0,47712,12547,19662,44	
	1. 00	