§31.1. ***Synopsis: Chapter Thirty One.***

This chapter is the converse of the preceding one: in this case the regular shapes have the same areas, but the length of side is required.

§31.2. ***Chapter Thirty One.*** [p.85.]

Concerning Regular Figures of the same Area.

With different kinds of regular figures with the same area, it is not possible for any of the sides [of the different figures] to be ever equal. Nevertheless, when once the ratio of the sides of two equal areas is taken; the same kind of equality for the same [figures] is always kept. And conversely: if the ratio of the sides be kept the same, the figures themselves have equal areas. If the area situated inside any of the allowed regular Figures is 4, the side is:

	Side		*Log. of Side*
A Triangle	$\ell\ell$. $85^{1}/_{3}$ - - - - - - - -- - - - - - -or	$\frac{}{303934274}$	0,48277,9678
B Square	2	2	0,30102,9996
C Pentagon	$\ell\ell$. bin. $51^{1}/_{3} - \ell$. $2097^{19}/_{125}$ or	$1\frac{}{524774111}$	0,18320,5509
D Hexagon	ℓ. $^{64}/_{27}$ - - - - - - -- - - - - - - -or	$1\frac{}{240806479}$	0,09370,4052
E Heptagon	- - - - - - - - - - - - - - - - - - -- - - - -	$1\frac{}{049162516}$	0,02084,2766
F Octagon	ℓ. bin. 12 – ℓ.128 - - - - - - -or	$\frac{}{9101797211}$	– 0,04087,2845
G Nonagon	- - - - - - - - - - - - - - - - --	$\frac{}{8043992754}$	– 0,09452,8329
H Decagon	$\ell\ell$. bin. $2\frac{}{56} - 5\frac{}{24288}$	$\frac{}{7210211605}$	– 0,14205,1989
I Dodecagon	ℓ. bin $2^{2}/_{3} - \ell$. $5^{1}/_{3}$ - - - - - - or	$\frac{}{5977169814}$	– 0,22350,4405
J Circle	Diameter of the circle will be	$2\frac{}{2567583342}$	0,35348,5055

[Table 31-1]

We can make use of these logarithms: For given the side of any figure (of those placed above), to find [the length of] the side of another figure, of given equal area. For let the side of a hexagon be given of 5 parts; I want to find the side of the Dodecagon, with area equal to the Hexagon.

The difference of the Logarithms of the sides of the Hexagon and the Dodecagon added together, is taken from the Logarithm of five; the remainder is the Logarithm of the required side.

Or by Ch. 15:

proportionals	*Logarithms*
D. Side of the Hexagon	0,09370,4052
I. Side of the Dodecagon	– 0,22350,4405
Given side of Hexagon 5	0,69897,0004
Sum of means	0,47546,5599
Side of Decagon required 2408583614	0,38176,1547

[Table 31-2]

1. If we want to know the area of the Hexagon, of which the side of 5 parts is given; or of the Dodecagon, of which the side of 2408582914 parts is found : either Chapter 28 should be consulted: by taking the logarithm of double the side. Or for the Hexagon, per Ch. 28.

Proportions	*Logarithms*
The Square of the side of circumscribed Hexagon	0,12493,8736
Area of circumscribed hexagon	0,53959,0623
Square from side of given Hexagon 5	1,39794,0008
Sum of means	1,93753,0631
Area of the Hexagon of side 5 649519952	1,93753,0631

[Table 31-3]

By the same method for the Dodecagon.

		Logarithms
pro-portion-als	The square of the side of circumscribed Dodecagon	–0,57194,7546
	Area of circumscribed Dodecagon - - - - - - - - - - - -	0,47712,1255
	Square from side of found Dodecagon - - - - - - - - -	0,76352,3094
	Sum of means -	1,24064,4349
	Area of the Dudecagon of side 2408582614 - - - - - - - -	1,81259,1895

[Table 31-4]

The equality of these figures is taken from the equality of the logarithms.

2. Or the side of the square can be found from the logarithms of this chapter, equal in area to the same hexagon. For

Proportions	*Logarithms*
D. Side of Hexagon. Comp. Arith. - - - - - - - - - - -	9,90629,5948
B. Side of Square -	0,30102,9996
Side of given Hexagon 5 - - - - - - - - - - - - - - - - -	0,69897,0004
Required side of Square - - - - - - - - - - - - - - - - - -	10,90629,5948
Area of the Square itself - - - - - - - - - - - - - - - - -	1,81259,1896

[Table 31-5]

Therefore we have the area of the same three figures, the Hexagon, the Dodecagon, and the Square, all surely by this method equal to 649519053.

§15.1. # Caput XXXI. [p.85.]

De Figuris Ordinatis aequalibus.

In Figuris heterogeneis Ordinatis aequalibus, nulla unquam esse potest laterum aequalis. Veruneame, quae semel deprehensa fuerit, in duabus aequalibus, laterum ratio; eadem in aequalibus eorundem generum, semper servabitur. Et contra: si eadem servetur laterum ratio, ipsae figurae erunt aequales.

Si area Ordinatae cuiuslibet figurae infra positae fit 4: erit latus,

	Latera		*Logarithmi Laterum.*
A Trianguli,	$\ell\ell$. $85^1/_3$ - - - - - - - - - - - - - - vel	303934274	0,48277,9678
B Quadrati,	2	2	0,30102,9996
C Quinquanguli	$\ell\ell$. bin. $51^1/_3$ – ℓ. $2097^{19}/_{125}$ vel	1524774111	0,18320,5509
D Sexanguli	ℓ. $^{64}/_{27}$ - - - - - - - - - - - - - - vel	1240806479	0,09370,4052
E Septanguli	- - - - - - - - - - - - - - - - - - - -	1049162516	0,02084,2766
F Octanguli	ℓ. bin. 12 – ℓ.128 - - - - - - vel	9101797211	– 0,04087,2845
G Nonanguli	- - - - - - - - - - - - - - - - -	8043992754	– 0,09452,8329
H Decanguli	$\ell\ell$. bin. 256 – 524288	7210211605	– 0,14205,1989
I Dodecanguli	ℓ. bin $2^2/_3$ – ℓ. $5^1/_3$ - - - - - vel	5977169814	– 0,22350,4405
K Diameter Circuli erit - - - - - - - - - - -		22567583342	0,35348,5055

Horum Logarithmorum adiumento poterimus : Dato cuiuscunque (superius positae) figurae latere; alterius, datae prorsus aequalis, latus invenire. Ut esto latus Sexanguli datum partium 5; cupio invenire latus Dodecanguli, Sexangulo aequalis.

Differentia Logarithmorum lateribus Sexanguli et Dudecanguli convenientium, auferatur e Logarithmo quinarij five;reliquus erit Logarithmus lateris quaesiti.

Vel per Cap.15:

		Logarithmi.
propport.	D. Lateris Sexanguli - - - - - -	0,09370,4052
	I. Lateris Dodecanguli - - - - -	– 0,22350,4405
	Lateris Sexangulidati 5 - - - - -	0,69897,0004
	aggregatum mediorum - - - - -	0,47546,5599
	Lateris Decanguli quaesiti 2408583614	0,38176,1547

1. Si aream Sexanguli, cuius latus datur partium 5 ; vel Dodecanguli, cuius latus invenitur partium 2408582914 scire velimus: Vel consulendum est cap.28: sumpto lateris Logarithmo duplicato.

		Logarithmi.
proport.	Quadrati e latere circumscripti Sexanguli - - - - -	0,12493,8736
	Areae circumscripti Sexanguli - - - - - - - - - -	0,53959,0623
	Quadrati e latere Sexanguli dato 5- - - - - - - - -	1,39794,0008
	aggregatum mediorum - - - - - - - - - - - - - - -	1,93753,0631
	Areae Sexanguli cuius latus 5 649519952- - - - - -	1,93753,0631

Eodem modo pro Dodecangulo.

		Logarithmi
proport	Quadrati e latere circumscripti Dodecanguli	–0,57194,7546
	Areae circumscripti Dodecanguli - - - - - - - - - - -	0,47712,1255
	Quadrati e latere Dodecanguli invento - - - - - - - -	0,76352,3094
	aggregatum mediorum - - - - - - - - - - - - - - - - - - -	1,24064,4349
	Areae Dodecanguli cuius latus 2408582614 - - - - - - - -	1,81259,1895

Aequalitas harum figurarum deprehenditur ex aequalitate Logarithmorum

Vel per Logarithmos huius capitis quaerendum rest latus Quadrati, eidem Sexangulo aequalis. Vt

2.

		Logarithmi.
pro-	D. Lateris Sexanguli Comp. Arith. - - - - - - - - - - -	9,90629,5948
port.	B. Lateris quadrati - - - - - - - - - - - - - - - - - - -	0,30102,9996
	Lateris Sexanguli dati 5 - - - - - - - - - - - - - - - - -	0,69897,0004
	Lateris quadrati quaesiti - - - - - - - - - - - - - - - - - -	10,90629,5948
	Ipsius quadrati - - - - - - - - - - - - - - - - - -	1,81259,1896

Habemus igitur areas harum trium figurarum Sexanguli, Dodecanguli, et Quadrati, omni modo prorsus aequales 649519053.