**§30.1.** ***Synopsis: Chapter Thirty.***

The areas of a series of regular shapes starting with the equilateral triangle and ending with the dodecagon, each with the same perimeter, are evaluated, and compared with the area of the corresponding circle. The areas of isoperimetric regular shapes can be computed readily via logarithms.

**§30.2.** ***Chapter Thirty.*** [p.84.]

*Concerning Regular Figures of the same Perimeter.*

In figures of this kind, the area is greater where there are more sides: which is always equal to the rectangle described by the semi-perimeter of the polygon, and the radius of the circle inscribed in the same.

Let the perimeters of the subsequent regular isoperimetric figures be 4; they are:

| | *Area* | | *Log. of the Area* |
|---|---|---|---|
| A Triangle | $\ell.\ ^{48}/_{81}$ - - - - - - -- - - - - - or | 7698003589 | – 0,11362,1890 |
| B Square | 1 | | 0,00000,0000 |
| C Pentagon | $\ell.$ bin. $^{16}/_{25} + l.\ ^{1024}/_{3125}$ or | 11011055364 | 0,04182,8946 |
| D Hexagon | $\ell.\ ^{4}/_{5}$ - - - - - - -- - - - - - or | 11547005384 | 0,06246,9368 |
| E Heptagon | - - - - - - - - - - - - - - - - -- | 11865836552 | 0,07429,8362 |
| F Octagon | $\ell.$ bin. $^{3}/_{4} + \ell.\ ^{1}/_{3125}$ - - - or | 12071067812 | 0,08174,5690 |
| G Nonagon | - - - - - - - - - - - - - - - - -- | 12211010752 | 0,08675,1614 |
| H Decagon | $\ell.$ bin. $^{4}/_{5} + \ell.\ ^{64}/_{125}$ | 12310734148 | 0,09028,3953 |
| I Dodecagon | $^{2}/_{3} + \ell.\ ^{1}/_{3}$ - - - - - - - - - or | 12440169358 | 0,09482,6293 |
| K Circle | - - - - - - - - - - - - - - - - -- | 12732395447 | 0,10491,0119 |

[Table 30-1]

If from two figures of this kind, the area of one is given, the area of the remaining one also is known through these logarithms, in this way: Let the square be given, of which the area is 49: I want to know the area of the decagon isoperimetric with the square. The difference of the Logarithms is taken of the square and the decagon placed here in the table, this is added to the Logarithm of the given square, the total is the logarithm of the required decagon. So:

| | *Logarithms* |
|---|---|
| Logarithm of the given Square 49: | 1,69019,6080 |
| Difference of the logs. of the Square and the Decagon - - - - | 0,09028,3953 |
| Area of the decagon of which the perimeter is 28 is 603225973 | 1,78048,0033 |

[Table 30-2]

If the area of the given triangle is 49 parts; and the area of the decagon is required; for the difference of the logarithms, the sum is taken, ( because the logarithm of the triangle is negative) 0,20390,5843: which added to the given logarithm of 49, gives the logarithm of the required decagon, 1,89410,1923.

Or, by Chapter 15:

| proportions. | | *Logarithms* |
|---|---|---|
| | A. Triangle | – 0,11361,1890 |
| | H. Decagon | 0,09028,3953 |
| | Triangle given 49 | 1,69019,6080 |
| | Sum of means | 1,78048,0033 |
| | Decagon required 783613525 | 1,89410,2023 |

[Table 30-3]

After the area of the polygon is found or given, if you want to know the side or perimeter of the same figure, you should consult Ch. 28.

Finally, if the area is divided by the semi-perimeter, the quotient is the radius of the inscribed circle.

## §15.3. No Notes.

## §15.4. De Figuris Ordinatis Isoperimetris. [p.84.]

In huiusmodi figuris, quo plura sunt latera, eo maior est Area: quae semper aequatur Rectangulo comprehenso semiperimetro multanguli, et Radio circuli eidem inscripti.

Esto Perimeters figurarum ordinatarum isoperimetrarum subsequentium 4; erunt:

| | *Areae* | | *Arearum Logarithmi.* |
|---|---|---|---|
| A Trianguli | $\ell.\ ^{48}/_{81}$ - - - - - - --- - - - vel | 7698003589 | – 0,11362,1890 |
| B Quadrati | 1 | | 0,00000,0000 |
| C Quinquanguli | $\ell.$ bin. $^{16}/_{25} + l.\ ^{1024}/_{3125}$ vel | 11011055364 | 0,04182,8946 |
| D Sexanguli | $\ell.\ ^{4}/_{5}$ - - - - - - -- - - - - vel | 11547005384 | 0,06246,9368 |
| E Septanguli | - - - - - - - - - - - - - - - - -- | 11865836552 | 0,07429,8362 |
| F Octanguli | $\ell.$ bin. $^{3}/_{4} + \ell.\ ^{1}/_{3125}$ - - - vel | 12071067812 | 0,08174,5690 |
| G Nonanguli | - - - - - - - - - - - - - - - - -- | 12211010752 | 0,08675,1614 |
| H Decanguli | $\ell.$ bin. $^{4}/_{5} + \ell.\ ^{64}/_{125}$ | 12310734148 | 0,09028,3953 |
| I Dodecanguli | $^{2}/_{3} + \ell.\ ^{1}/_{3}$ - - - - - - - - vel | 12440169358 | 0,09482,6293 |
| K Circuli | - - - - - - - - - - - - - - - - -- | 12732395447 | 0,10491,0119 |

Si e duabus figuris huiusmodi, alterius area data fuerit: reliquae etiam nota erit, per hos Logarithmos, ad hunc modum. Esto Datum Quadratum, cuius area sit 49: cupio scire Area Decanguli isoperimetri quadrato. Sumatur differentia Logarithmorum Quadrati et decanguli hic positorum, ea addatur Logarithmo dati quadrati, totus erit Logarithmus quaesiti Decanguli. ut:

| | *Logarithms* |
|---|---|
| Logarithmus dati Quadrati 49: | 1,69019,6080 |
| Differentia Logarithmorum Quadrati et Decanguli - - - - | 0,09028,3953 |
| Area decanguli cuius perimeter est 28 60$\frac{3225973}{}$ | 1,78048,0033 |

[p.85.]

Si Trianguli area data sit partium 49; et quaeratur area Decanguli; pro differentia Logarithmorum, sumi debet summa, (quia trianguli Logarithmus est defectiuus) 0,20390,5843: quae adiecta Logarithmo 49 dato, dat Logarithmum quaesiti Decanguli 1,89410,1923.

Vel, per cap.15:

| | | *Logarithmi.* |
|---|---|---|
| pro-port. | A. Trianguli | – 0,11361,1890 |
| | H. Decanguli | 0,09028,3953 |
| | Trianguli dati 49 | 1,69019,6080 |
| | aggregatum mediorum | 1,78048,0033 |
| | Decanguli quaesiti 78$\frac{3613525}{}$ | 1,89410,2023 |

Postquam data vel inventa fuerit area polygoni: si latus vel perimetrum eiusdem scire desideres, consulere debes Cap. 28.

Et, si semiperimeter aream diviserit : quotus erit radius circuli inscripti.