§28.1. *Synopsis: Chapter Twenty - Eight.*

The Lemma below is stated, relating the proportionality between the areas of inscribed and circumscribed regular figures, of a given order for a given circle, to the area of the inscribed figure with double the sides, and demonstrated with a number of examples:
(*i*) The inscribed regular hexagon has an area which is the mean proportional between the areas of the inscribed and circumscribed equilateral triangles; similarly, for the cases:
(*ii*) the regular inscribed/circumscribed hexagons with the regular inscribed dodecagon ;
(*iii*) the regular inscribed/circumscribed squares with the regular inscribed octagon;
(*iv*) the regular inscribed/circumscribed pentagons with the regular inscribed decagon.
[If a_n is the area of the inscribed n-gon, and A_n the area of the circumscribed n-gon, for a given circle, then $a_{2n} = \sqrt{(a_n.A_n)}$.]

A large list of calculated lengths of side for inscribed/circumscribed regular figures in a circle with unit radius is presented, together with their perimeters and areas. A number of scaling problems is then demonstrated, showing the great advantage of using logarithms.

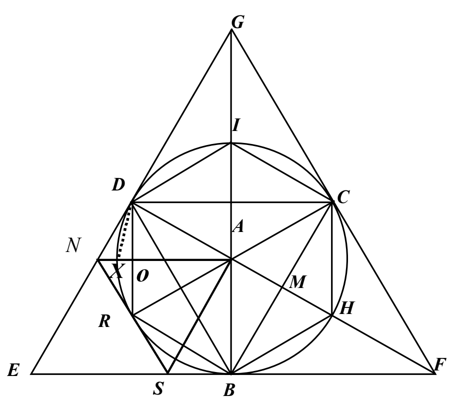
§28.2. *Chapter Twenty Eight.* [p.78.]

With the diameter of a circle given, to find the sides and areas of the regular Triangle, Square, Pentagon, Hexagon, Octagon, Decagon, Dodecagon, Hexadecagon, inscribed and circumscribed in the same circle.

Lemma

For regular figures ascribed in a circle: *Let two figures of the same kind be associated with the same circle, the one inscribed and the other circumscribed: if a third figure, of which the number of sides is equal to the sum of the remaining sides taken together, is to be inscribed in the same circle , then the area of this figure is the mean proportional between the remaining areas.*

[1. *Inscribed and circumscribed equilateral triangles with an inscribed regular hexagon*].



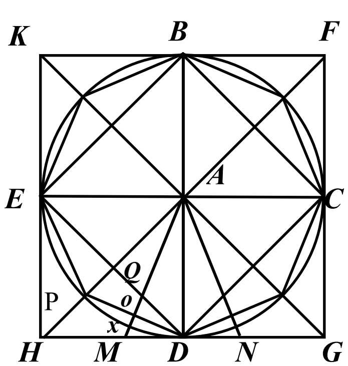
[Figure 28-1]

Let two triangles *DCB*, *GEF* be adscribed to the same triangle, to which the hexagon *DICHBR* is inscribed: I assert the area of the hexagon[1] to be the mean proportional between the areas of the adscribed triangles.

For the triangles *ACM, ACF* are similar: & *AM, AC* (or *AH*), *AF* are in continued proportion, see Figure 28-1. And therefore the triangles *ACM*, *ACH, ACF:* as they have the same altitude, their areas are proportional to their bases; that is, their areas are in continued proportion, as their bases; And the figures, [i.e. the areas of these triangles and the hexagon] are also in continued proportion. Let the radius of the circle *AD* be 1. *BC* is the side of the inscribed triangle ℓ.3, by Prop.12, Book13, Euclid. The perimeter of the triangle is ℓ.27. The semi-perimeter, ℓ.$6^{3}/_{4}$; *AM,* $^{1}/_{2}$. The area of the triangle BCD, ℓ. $^{27}/_{16}$; AO, ℓ.$^{3}/_{4}$; DR, 1. Triangle DAR, ℓ.$^{3}/_{16}$. The area of the hexagon *DICHBR* is ℓ.$^{27}/_{4}$; *EF,* ℓ.12; *AB*, 1; *EAF* ℓ.3; and the area *EGF* ℓ.27. [i.e. area ΔBCD × area ΔEGF = area squared hexagon DICHBR.]

[2. *Inscribed and circumscribed regular hexagons with an inscribed regular dodecagon*].

The line *DR* of the inscribed hexagon has length 1, [see Figure 28-1 again]. The circumscribed line NS, ℓ.$^{4}/_{3}$: for the lengths AO, ℓ.$^{3}/_{4}$; DR, 1; AR, 1; NS, ℓ.$^{4}/_{3}$ are in proportion. The area of the triangle *ANS* is ℓ.$^{1}/_{3}$, and of the circumscribed hexagon ℓ.12. Again, the area of the inscribed dodecagon is 3. For *XO* is 1 – ℓ.$^{3}/_{4}$, & the square of *XO,* $1^{3}/_{4}$ – ℓ.3; & the square of *DO,* $^{1}/_{4}$. Therefore the square of *DX* is 2 – ℓ.3; & the square of half the line *DX* is $^{1}/_{2}$ – ℓ.$^{3}/_{16}$. [From Pythagoras' Theorem] the square of the perpendicular from the point A to the line *DX* is $^{1}/_{2}$ + ℓ.$^{3}/_{16}$. The area of the triangle *ADX* is ℓ.$^{1}/_{16}$, or $^{1}/_{4}$, & the whole area of the inscribed dodecagon is 3. Which is the mean proportion between the areas of the inscribed hexagon ℓ.$^{27}/_{4}$ and the circumscribed hexagon ℓ.12.



[Figure 28-2]

[3. *Inscribed and circumscribed squares with an inscribed regular octagon*].

For the inscribed square *BCDE* the side has length ℓ.2, area 2 [see Figure 28-2]; the circumscribed square *FGHK* has side of length 2, and area 4.

For the inscribed octagon[2] the side DP has length ℓ.bin.2 - ℓ.2 . (For *AQ* is ℓ.$^{1}/_{2}$; *PQ* 1 - ℓ.$^{1}/_{2}$, and the square of

PQ $1\frac{1}{2} - \ell.2$. The square of *DQ*: $\frac{1}{2}$; therefore the square of *DP* is $2 - \ell.2$). The square *DO* ($\frac{1}{4}$ of the square *DP*) is $\frac{1}{2} - \ell.\frac{1}{8}$, which taken from the square of the radius *AD* 1, gives the square *AO* as $\frac{1}{2} + \ell.\frac{1}{8}$, and the product of AO with the line *OD*:

$\ell.\text{bin}.\frac{1}{2} + \ell.\frac{1}{8}$ times $\ell.\text{bin}.\frac{1}{2} - \ell.\frac{1}{8}$, which is $\ell.\frac{1}{8}$ the area of the triangle[3] *ADP*. The area of the inscribed octagon is $\ell.8$. Which is the mean proportional between the area of the inscribed square 2, and the circumscribed square 4.

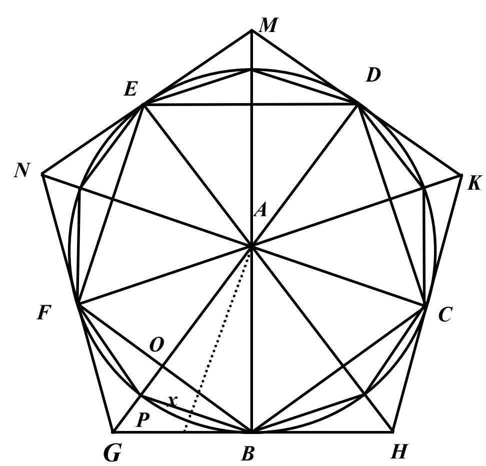
[4. *Inscribed and circumscribed octagons with an inscribed regular hexadecagon*].

The line MN of the circumscribed octagon has length $\ell.8 - 2$, [see Figure 28-2], (because HA, HN are equal, & therefore DN is $\ell.2 - 1$, & NM twice the same, is $\ell.8 - 2$. But also as HA, HN are equal, it is evident by Prop.5, Book 1, Euclid; that the angle HAN has the value $\frac{3}{4}$ of a right angle from the construction, & HNA is equal to the sum of the angles NGA, NAG by Prop.32, Book 1, *Euclid.)* The radius AD is 1; the product of the radius 1 by DN $\ell.2 - 1$ is $\ell.2 - 1$, is equal to the area of the triangle AMN, of which the corresponding 8-fold is $\ell.128 - 8$, equal to the area of the circumscribed octagon.[while the area of the inscribed octagon is $\ell.8$ from above].

With the inscribed hexadecagon [16-gon], the length of the side is $\ell.\text{bin } 32 - \ell.512$ [There is a typographical error here, but the correct value is given in Table 28-6].

[5. *Inscribed and circumscribed pentagons with the inscribed regular decagon*].

The length of the side of the inscribed pentagon is $\ell.\text{bin}.\frac{5}{2} - \ell.\frac{5}{4}$, the perpendicular to the centre is the length $\ell.\text{bin}.\frac{3}{8} + \ell.\frac{5}{64}$; the product of this perpendicular with the half of the side of the pentagon



[Figure 28-3]

$\ell.\text{bin}.{}^{5}/_{8} - \ell.{}^{5}/_{64}$ is the area of the triangle *ABC* $\ell.\text{bin}.{}^{5}/_{32} + \ell.{}^{5}/_{1024}$, and the total area of the inscribed pentagon[4] is $\ell.\text{bin}.{}^{125}/_{32} + \ell.{}^{3125}/_{1024}$. The side of the circumscribed pentagon is $\ell.\text{bin}.20 - \ell.320$. The product of the radius by the half of this side is $\ell.\text{bin}.5 - \ell.20$, the area of the triangle AGH, and the total area of the circum-scribed pentagon is $\ell.\text{bin}.125 - \ell.12500$.

The side of the inscribed decagon *BP* is $\ell.{}^{5}/_{4} - {}^{1}/_{2}$, of which the square is ${}^{3}/_{2} - \ell.{}^{5}/_{4}$. The square of the line *BX* is ${}^{3}/_{8} - \ell.{}^{5}/_{64}$ & the square of *AX* is ${}^{5}/_{8} + \ell.{}^{5}/_{64}$, the area of triangle *ABP* is $\ell.\text{bin}.{}^{5}/_{32} - \ell.{}^{5}/_{1024}$, and the total area of the decagon $\ell.\text{bin}.{}^{125}/_{8} - \ell.{}^{3125}/_{64}$, which is the mean proportional between the areas of the inscribed and the circumscribed pentagons. [The ratio of the inscribed pentagon to decagon is $\sqrt{(5 + 5\sqrt{5})/8}$, as also is the ratio of the inscribed decagon to the circumscribed pentagon].

[*End of Examples of Lemma*]

If we wish to construct some of these figures for a circle of which the diameter is given or found: in the first place the appropriate differences of the logarithms can be found for an individual figure; when I have shown these, then I can explain the rest briefly.

For the Circle and the Triangle.

			Logarithms
Terms	Diameter of circle - - - - - - - - - - - - - -	2	0,30102,99956,6398
	Side of inscribed triangle - - - - - - - -	$\ell.3$	0,23856,06273,5983
	Side of circumscribed triangle - - -	$\ell.12$	0,53959,06230,2381
Difference of logs. for a triangle	inscribed		0,06246,93683,0415
	circumscribed		0,23856,06273,5983
			Logarithms
Terms	Circumference of circle	628318530718	0,79817,98683,5500
	Perimeter of triangle, inscribed	$\ell.27$	0,71568,18820,7949
	circumscribed	$\ell.108$	1,01671,18777,4347
Log. difference for the triangle	inscribed		0,08249,79862,7551
	circumscribed		0,21853,20093,8847
			Logarithms
Terms	Area of circle	314159265359	0,49714,98726,9102
	Area of triangle, inscribed	$\ell.{}^{27}/_{16}$	0,11362,18907,5153
	circumscribed	$\ell.27$	0,71568,18820,7949
Log. diff. for triangle	inscribed		0,38352,79819,3949
	circumscribed		0,21853,20093,8847

[Table 28-1]

If a circle with diameter 9 is given & the sides of the inscribed and circumscribed triangles are sought. [Note: Briggs favours an inverted ratio in the case where a negative logarithm results: as with diameter/length of inscribed side; subsequently, he subtracts this logarithm when scaling, as below].

		Logarithms	
	Log. of diameter 9 - - - - -	095424251	
Difference of the sides	inscribed	006246937	
	circumscribed	023856063	
Sides of the triangle	inscribed	089177314	7$\underline{794228}$
	circumscribed	119280314	15$\underline{588457}$

[Table 28-2]

The sides of the triangle are: inscribed 7$\underline{794228}$; circumscribed 15$\underline{588457}$.

If the side of length 6 of a triangle is given : & the diameters of the inscribed and circum-scribed circles are required.

		Logarithms	
	Log. of given side 6 - - - - -	077812125	
Difference for the triangle	inscribed	023856063	
	circumscribed	006246937	
Log. for the diameter	circumscribed	084062062	6$\underline{928406}$
	inscribed	053959062	3$\underline{464203}$

[Table 28-3]

The diameters of the circles is: inscribed 3$\underline{464203}$; circumscribed 6$\underline{928406}$.

If the area ℓ.243 of a triangle is given: & the areas of the inscribed and circumscribed circles are required.

		Logarithms	
	Log. of given area ℓ. 243 - - - - -	119280313	
Difference for the triangle	inscribed	038352798	
	circumscribed	021853201	
Log. of area of circle	circumscribed	157633111	37$\underline{699112}$
	inscribed	053959062	9$\underline{424778}$

[Table 28-4]

The areas of the circles is: inscribed 9$\underline{424778}$; circumscribed (four times that of the inscribed) 37$\underline{699112}$.

2. For the Circle & the Square.

			Logarithms
Terms	Diameter of circle	2	0,30102,99956,6398
	Side of inscribed square	ℓ.2	0,15051,49978,3199
	Side of circumscribed square	2	0,30102,99956,6398
Difference of logs. for sides of squares	inscribed		0,15051,49978,3199
	circumscribed		0,00000,00000,0000

				Logarithms
Terms	Circumference of circle		628318530718	0,79817,98683,5500
	Perimeter of square	inscribed ℓ.32		0,75257,4981,5995
		circumscribed 8		0,90308,99869,9194
Difference of logs. for perimeter of the square		inscribed		0,04560,48791,9505
		circumscribed		0,10491,01186,3694
				Logarithms
Terms	Area of circle		314159265359	0,49714,98726,9102
	Area of square	inscribed *2*		0,30102,99956,6398
		circumscribed 4		0,60205,99913,2796
Log. diff. for square		inscribed		0,19611,98770,2704
		circumscribed		0,10491,01186,3694

[Table 28-5]

For the circle and adscribed regular many-sided figures [Table 28-6].

				Logarithms
Circle		Diameter - - - - - - - - - - - - - - -	2	0,30102,9995
		Circumference	6283585307	0,79817,9868
		Area	3141592654	0,49714,9873
Pentagon	Side	inscribed ℓ. bin. $^5/_2$ – ℓ. $^5/_4$	1175570504	0,07024,8681
		circumscribed ℓ. bin. 20 – ℓ.320	1453085056	0,16229,1036
	Perimeter	inscribed ℓ. bin. $62^1/_2$ – ℓ. $781^1/_4$	5877852523	0,76921,8685
		circumscribed ℓ. bin.500 – ℓ.12500	726542528	0,86126,1040
	Area	inscribed ℓ. bin. $^{125}/_{32}$ + ℓ. $^{3125}/_{1024}$	237764129	0,37614,6310
		circumscribed ℓ. bin. 125 – ℓ.12500	363271264	0,56023,1045
Hexagon	Side	inscribed	1	0,00000,0000
		circumscribed ℓ $^4/_3$	1154700538	0,06246,9368
	Perimeter	inscribed	*6*	0,77815,1250
		circumscribed ℓ.48	6928202220	0,84062,0618
	Area	inscribed ℓ. $^{27}/_4$	2598076211	0,41465,1886
		dodecagon inscribed	3 - - - - - -	0,56023,1045
		circumscribed ℓ. 12	3464101651	0,53959,0623
Octagon	Side	inscribed ℓ.bin2 – ℓ.2	7653668647–	0,11613,0343
		circumscribed ℓ. 8 – 2	82842712 –	0,08174,5690
	Perimeter	inscribed ℓ.bin 128 – ℓ. 8192	6122934918	0,78695,9644
		circumscribed ℓ.512 – 16	662741696	0,82134,4298
	Area	inscribed ℓ. 8	282842712	0,45154,4993
		hexadecagon inscribed ℓ.bin 32 – ℓ.512	306146746	0,48592,9647
		circumscribed ℓ.128 – 8	332370850	0,52031,4302
Decagon	Side	inscribed ℓ.$^{27}/_4$ – $^1/_2$	6180339887–	0,20898,764
		circumscribed ℓ. bin 4 – ℓ.$^{64}/_5$	6498393925–	0,18719,397
	Perimeter	inscribed ℓ 125 – *5*	6180339887	0,79101,236
		circumscribed ℓ. bin 400 – ℓ.128000	6498393925	0,81280,603
	Area	inscribed ℓ. bin $^{125}/_8$ – ℓ. $^{3125}/_{64}$	2938926261	0,46818,869
		20-gon inscribed ℓ.ℓ..$^{4375}/_2$ – ℓ. 17578125	309016994	0,48998,236
		circumscribed ℓ . 100 – ℓ. 8000	324919696	0,51177,603
Dodecagon	Side	inscribed ℓ. bin. *2* – ℓ. 3	5176380902–	0,28597,3773
		circumscribed 4 – ℓ.12	5358983849–	0,27091,7552
	Perimeter	inscribed ℓ. bin 288 – ℓ.62208	6211657082	0,79320,7472
		circumscribed *48* – ℓ.1728	6430780618	0,80826,3694
	Area	inscribed - - - - - - - - - - - - - - - - - - -	3	0,47712,1255
		24-gon inscribed ℓ. bin 72 – ℓ. 3888	3105828541	0,49217,7477
		circumscribed ℓ. 100 – ℓ. 8000	3215399309	0,50723,3699

With these logarithms found, if any whatever of these figures are proposed, given either their side, perimeter, or area, then we can find any of these other terms. Because we have shown the triangle in more detail, hence we will show only a single example from the rest.

Let the given side of a regular octagon be of 7 parts. The side is sought, perimeter, and area of the pentagon, with the same circle as the inscribed octagon.

The required side can be found thus:

With a circle of which the radius is unity, the side of the inscribed octagon is $\underline{7653668647}$, but the side of the pentagon is $1\underline{175579504}$. If the side of the octagon is taken as 7 parts, the side of the pentagon sought is the fourth proportion. Therefore the logarithms of the given terms are taken, so that the logarithm of the term required may be found (as in Ch. 15).

proportions	*Logarithms*
Side of the given octagon $\underline{7653668647}$	–0,11613034
Side of the octagon taken 7	0,84509804
Side of the given pentagon $1\underline{175579504}$	0,07024868
Side of the pentagon sought $10\underline{7516982}$	1,03147706

[Table 28-7]

Where it is observed that the logarithm of the first term (as this is less than unity) is negative, as we showed in Ch. 10; and because of this, it is not to be taken away from the sum of the means of the ratio, but rather added on; and the logarithm of the fourth proportional is 1,03147706, and the required side $10\underline{7516982}$.

This side, if we neglect to use the rule of proportion with logarithms, is the root of the four numbers[5] $122\,{}^1/_4 + \ell.\ 7503\,{}^1/_8 - \ell.\ 3001\,{}^1/_4 - \ell.1500\,{}^5/_8$. [For from the table, the length of the side by proportion is $7 \times \{\ \ell.\ \text{bin.}\ {}^5/_2 - \ \ell.\ {}^5/_4\} / \{\ \ell.\text{bin}2 - \ell.2\}$, which can be written as the square root of the number shown].

By the same method the perimeter of the pentagon is found, if for the third term the perimeter of the given pentagon is taken, and of this, the logarithm. As [Table 28-8]:

proportions	*Logarithms*
Side of the given octagon $\underline{7653668647}$	–0,11613034
Side of the octagon being taken 7	0,84509804
Perimeter of the given pentagon $5\underline{877852523}$	0,76921861
Perimeter of the pentagon sought $53\underline{758491}$	1,73044706

If we seek the area of the pentagon, with nothing given except the side of the octagon inscribed in the same circle: then we remember that the areas of similar plane figures are in the duplicate ratio with the logarithms of their corresponding sides. Therefore the side squared of the octagon can be found, in order to be able to establish area from the comparison. Thus, in this process, it is not necessary to find the length of the side of the pentagon, as we need only be concerned with the squares themselves, which will be easiest by the method of Ch. 16, as we have the logarithms of these. As you see here:

			Logarithms
pro-	Square of the side of the given octagon	$2 - \ell.2$	–0,0,232260686
port-	Square of the side taken	49 - - - - - - - - - - - -	1,690196080
ion-	Area of the given pentagon	$\ell.$ bin $^{125}/_{32}$ + $\ell.$ $^{3125}/_{1024}$	0,376146310
als	Area of the pentagon sought	$198\underline{8854794}$	12,298603076

[Table 28-9]

If the area of the given decagon is 6, and the side of the octagon circumscribing the same circle is sought, of which the decagon is inscribed; the calculation is:

			Logarithms
pro-	Area of the given decagon	$2\underline{93892626}$	0,46818869
port-	Area of decagon taken	6 - - - - - - - - - - - -	0,77815125
ion-	Square of the side of the given circumscribed octagon		$\underline{–0,16349138}$
als	Sum of the means		$\underline{0,61465987}$
	Square of the side of the sought circumscribed octagon		$\underline{0,14647118}$
	Side sought	$1\underline{18368622}$	0,07323559

[Table 28-10]

And thus in this way with a circle, and with these figures adscribed to the same circle, from a single term given, any other can be found by using logarithms.

§28.3. *Notes on Chapter Twenty Eight.*

[1] Briggs refers to figures according to the number of angles they contain; thus, 'sexangulum' for hexagon.

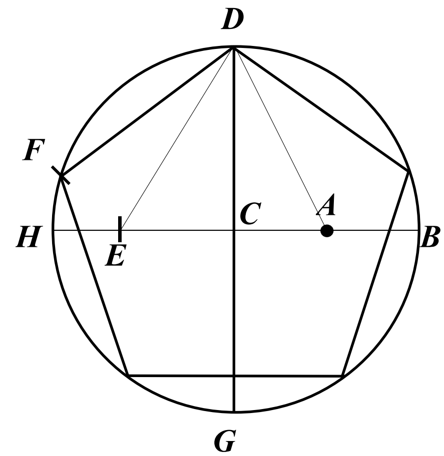
[2] $\ell.$ means 'latus' or side, being one of the abbreviations for the square root sign at this time, while bin. is short for 'bin-us, -a, -um', meaning 'a pair or two', and indicates in this case that the square root of both terms is included The expression thus means $\sqrt{(2 - \sqrt{2})}$.

[3] Here we have a good example of the nature of algebraic manipulations at the time. In modern terms, we have

$DO^2 = {}^1/_4 DP^2 = {}^1/_2 - \sqrt{{}^1/_8}$. Then $AO^2 = {}^1/_2 + \sqrt{{}^1/_8}$, and the product

$AO.OD = \sqrt{({}^1/_2 + \sqrt{{}^1/_8})}\, \sqrt{({}^1/_2 - \sqrt{{}^1/_8})} = \sqrt{{}^1/_8}$, the area of ΔADP.

4 The construction of the regular pentagon is contained in Proposition 11, Book 1V, Euclid. As this development may not be so obvious as the previous constructions have been, we give a construction of the pentagon, attributed to Ptolemy, as shown by Henry E. Dudeney in his *Amusements in Mathematics*, Nelson, 1917, p. 38. [Heath, Volume 2 of *The Elements* , p.104 of the Dover edition, gives another method, due to H. M. Taylor]. In this construction, the radius of the (circumscribed) circle is taken as unity. Two perpendicular diameters are constructed. The mid-point A of the line BC is found, and the length AD used to mark off with compasses the equal length AE: of size $\sqrt{5}/2$ by Pythagoras. The length EC is $(\sqrt{5} - 1)/2$, and hence



[Figure 28-4]

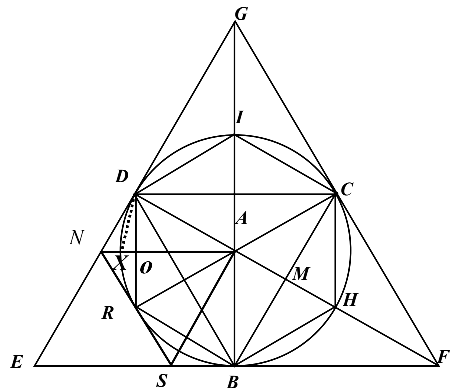
$ED = \sqrt{(5/2 - \sqrt{5}/2)}$, and the equal length FD marked off with the compasses as a side of the pentagon. The other sides are produced by drawing equal arcs around the circle. It suffices to show that this is indeed the length of side of the inscribed pentagon: for if the radius of the circumscribed circle is 1, then the length of the side a is given by $a = 2\sin(\pi/5)$, where $\sin(\pi/5) = \sqrt{\{(5 - \sqrt{5})/8\}}$. Thus, the radius of the in-circle to the pentagon is $\sqrt{\{(3+\sqrt{5})/8\}}$, while the area of ΔABC is $\sqrt{\{(5 + \sqrt{5})/32\}}$, etc. An up to date presentation can be found 'on the web' currently at www.cut-the-knot.org

§28.4. **Caput XXVIII.** [p.78.]

Data Diametro Circli, invenire Latera & Areas ordinatorum Trianguli, Quadrati, Quinquanguli, Sexanguli, Octanguli, Decagon, Decanguli, Dodecanguli, Sedecanguli, eidem circulo inscripti, & circumscriptorum.
Lemma.

In figuris circulo ordinatis ascriptis. *Si duae figurae homogeneae circulo adscribantur, una intus, reliqua extra: tertia, cuius latera sunt numero aequalia lateribus reliquorum simul sumptis, eidem circulo inscripta, erit media proportionalis inter reliquas.*

Sunto duo Triangula *DCB*, *GEF* eidem circulo adscripta, cui inscribatur Sexangulum *DICHBR* :aio Sexangulum esse medium proportionale inter Triangula adscripta.



Sunt enim Triangula *ACM, ACF* similia: & *AM, AC* (vel *AH*), *AF* continue proportionalia. & idcirco Triangula *ACM*, *ACH, ACF* cum sint aequealta, sunt ut bases: id est sunt continue proportionalia; ut ipsae bases; & Figurae, horum Triangulorum Sextae, sunt etiam continue proportionales. Esto *AD* radius circuli 1. erit *BC* Latus trianguli inscripti $\ell.3$, per 12.p.13.1.Eucl. Perimeter Trianguli $\ell.27$. semiperimeter $\ell.6^3/_4$. *AM,* $^1/_2$. Area Trianguli $\ell.\,^{27}/_{16}$. AO, $\ell.^3/_4$. DR 1. Triangulum DAR, $\ell.^3/_{16}$. Sexangulum *DICHBR* $\ell.^{27}/_4$;

EF, $\ell.12$; *AB*, 1; *EAF* $\ell.3$; *EGF* $\ell.27$.

Latus Sexanguli inscripti *DR,*1. Circumscripti latus NS, $\ell.^4/_3$: Sunt enim AO, $\ell.^3/_4$; DR, 1; AR, 1; NS, $\ell.^4/_3$ proportionales. & Triangulum *ANS* $\ell.^1/_3$, & Sexangulum circumscriptum $\ell.12$. Dodecangulum autem inscriptum est 3. est enim *XO* is 1 – $\ell.^3/_4$, &Qu. *XO,* $1^3/_4$– $\ell.3$; & Qu. *DO,* $^1/_4$. est igitur Qu. *DX* 2 – $\ell.3$; & Qu. semissis rectae *DX* $^1/_2$ – $\ell.^3/_{16}$. & Quad. perpendicularis a puncto *A* in rectam *DX* est $^1/_2$ + $\ell.^3/_{16}$. Triangulum igitur *ADX* $\ell.^1/_{16}$, vel $^1/_4$, & totum Dodecangulum inscriptum est 3. Quod est medium

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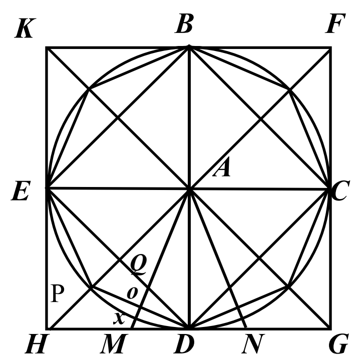
proportionale inter Sexangulum inscripti $\ell.^{27}/_4$ & Sexangulum circumscriptum $\ell.12$.

Quadrati *BCDE* inscripti Latus $\ell.2$, Area 2 , Circumscripti *FGHK* Latus 2, Area 4.



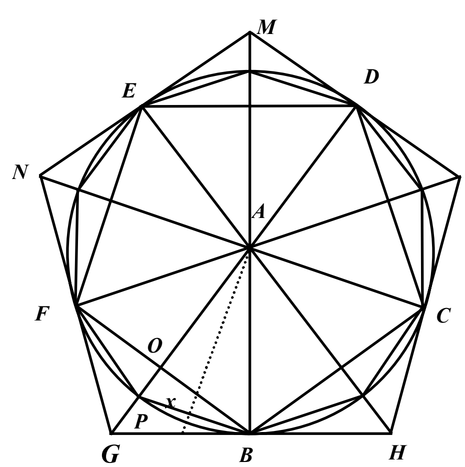
Octanguli inscripti Latus, DP

ℓ.bin.2 - $\ell.2$. (est enim *AQ* $\ell.^1/_2$; *PQ* 1 - $\ell.^1/_2$, & Qu.

PQ $1^1/_2$ - $\ell.2$. Qu. *DQ*: $^1/_2$; idcirco Qu. *DP* 2 - $\ell.2$). Qu. *DO* ($^1/_4$ Qu. *DP*) erit $^1/_2$ - $\ell.^1/_8$. quo ablato e quadrato Radiij *AD* 1, restabit Qu. *AO* $^1/_2$ + $\ell.^1/_8$,& factus a recta AO ℓ.bin.$^1/_2$ + $\ell.^1/_8$ in *OD*: ℓ.bin.$^1/_2$ - $\ell.^1/_8$, erit $\ell.^1/_8$ Area trianguli *ADP*. & Octangulum inscriptum erit $\ell.8$. Quod medium est proportionale inter Quadratum inscriptum 2, & Quadratum circumscriptum 4.

Octanguli circumscripti Latus MN, $\ell.8$ - 2, (quia HA, HN aequantur, &idcirco DN est $\ell.2$ - 1, & NM dupla eiusdem, $\ell.8$ - 2. Quod autem HA, HN aequantur, patet per 5.pro.1.lib.Eucl.; quia HAN valet $^3/_4$ recti ex fabrica, & HNA aequatur angulis NGA, NAG per 32.p.lib.1. Eucl.) *AD* radius 1. factus a radio 1 in DN $\ell.2$ - 1,. est is $\ell.2$ - 1, aequalis Areae Trianguli AMN, cuius octuplum $\ell.128$ - 8, aequatur Octangulo circumscripto.

Sedecanguli inscripti Latus est ℓ.bin 32 – $\ell.512$. Latus inscripti Quinquanguli ℓ.bin.$^5/_2$ - $\ell.^5/_4$, perpendicularis a centro in Latus ℓ.bin.$^3/_8$ + $\ell.^5/_{64}$. factus ab hac perpendiculari in semissem lateris Quinquanguli ℓ.bin.$^5/_8$ - $\ell.^5/_{64}$, erit Triangulum ABC ℓ.bin.$^5/_{32}$ + $\ell.^5/_{1024}$. & Area totius inscripti Quinquanguli ℓ.bin.$^{125}/_{32}$ + $\ell.^{3125}/_{1024}$. Latus Quinquanguli circumscripti, erit ℓ.bin.20 - $\ell.320$. Factus a Radio in semissem huius lateris erit ℓ.bin.5 - $\ell.20$, Triangulum AGH, & totum Quinquangulum circumscriptum ℓ.bin.125 - $\ell.12500$.



Latus Decanguli inscripti *BP* $\ell.^5/_4$ - $^1/_2$, cuius quadratum $^3/_2$ - $\ell.^5/_4$. Quadratum rectae *BX*

$^3/_8$ - $\ell.^5/_{64}$ & Qu. *AX* $^5/_8$ + $\ell.^5/_{64}$, area Trianguli ABP

ℓ.bin.$^5/_{32}$ - $\ell.^5/_{1024}$, & totum Decangulum ℓ.bin.$^{125}/_8$ - $\ell.^{3125}/_{64}$, quod proportione medium est inter Quinquangula inscriptum & circumscriptum.

Si circulo cuius diameter data vel quaesita fuerit, harum figurarum aliquam adscribere velimus: inprimis: quaerendae sunt Logarithmorum unicuique figurae convenientium Differentiae. quas ubi exhibuero, reliqua qua potero brevitate expediam. Pro Circulo & Triangulo.

			Logarithmi.
Termini	Diameter circuli - - - - - - - - - - - - -	2	0,30102,99956,6398
	Latus Trianguli inscripti - - - - -	ℓ.3	0,23856,06273,5983
	Latus Trianguli circumscripti - - -	ℓ.12	0,53959,06230,2381
Differentia Logarith. pro Triangulo	inscribed		0,06246,93683,0415
	circumscripti		0,23856,06273,5983

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				Logarithmi.
Termini	Peripheria Circuli		$6\underline{28318530718}$	0,79817,98683,5500
	Perimeter trianguli	inscripti	ℓ.27	0,71568,18820,7949
		circumscripti	ℓ.108	1,01671,18777,4347
Differentia Logarithm.pro Triangulo		inscripti		0,08249,79862,7551
		circumscripti		0,21853,20093,8847

				Logarithmi.
Termini	Area Circuli		$3\underline{14159265359}$	0,49714,98726,9102
	Area Trianguli	inscribed	ℓ.$^{27}/_{16}$	0,11362,18907,5153
		circumscribed	ℓ.27	$\underline{0,71568,18820,7949}$
Differentia Logarithm. pro Triangle		inscripti		0,38352,79819,3949
		circumscripti		0,21853,20093,8847

Si Data sit Circuli Diameter 9 & quaeruntur latera Triangulorum inscripti & circumscripti.

		Logarithms	
Logarith. Diametri 9 - - - - -		095424251	
Differentia pro lateribus	inscripti	006246937	
	circumscripti	023856063	
Latera Triangulorum	inscribed	089177314	$7\underline{794228}$
	circumscribed	119280314	$15\underline{588457}$

Erunt latera Triangulorum: inscripti $7\underline{794228}$; circumscripti $15\underline{588457}$.

If datum sit Trianguli Latus 6 : & quaeruntur Diametri circulorum inscripti & circumscripti.

		Logarithmi	
Logarithmus. Lateris dati 6 - - - - -		077812125	
Differentia pro Triangulo	inscripti	023856063	
	circumscripti	006246937	
Logarith. pro Diametro	circumscripti	084062062	$6\underline{928406}$
	inscripti	053959062	$3\underline{464203}$

Erunt diametri circulorum: inscripti $3\underline{464203}$; circumscripti $6\underline{928406}$.

Si data sit Area trianguli ℓ.243 : & quaerantur Areae circulorum inscripti & circumscripti.

		Logarithmi	
Logsrithmus Areae datae ℓ. 243		119280313	
Differentiae pro triangulo	inscripti	038352798	
	circumscripti	021853201	
Logar. pro Areis Areis circulorum	circumscripti	157633111	$37\underline{699112}$
	inscripti	053959062	$9\underline{424778}$

Circuli inscripti Area erit $9\underline{424778}$; circumscripti (quadrupla inscripti) $37\underline{699112}$.

2. Pro Circulo & Quadrato.

			Logarithmi.
Termini	Diameter circuli 2		0,30102,99956,6398
	Latera Quadrati	inscripti *ℓ*.2	0,15051,49978,3199
		circumscripti 2	0,30102,99956,6398
Differentia logarith. pro Latera Quadrati		inscripti	0,15051,49978,3199
		circumscripti	0,00000,00000,0000

			Logarithmi.
Termini	Peripheria circuli	$6\underline{28318530718}$	0,79817,98683,5500
	Perimeter Quadrati	inscripti *ℓ*.32	0,75257,4981,5995
		circumscripti 8	0,90308,99869,9194
Differentia Logarith. pro Perimetro Quadrati		inscripti	0,04560,48791,9505
		circumscripti	0,10491,01186,3694

			Logarithmi.
Termini	Area Circuli	$3\underline{14159265359}$	0,49714,98726,9102
	Area Quadrati	inscripti *2*	0,30102,99956,6398
		circumscripti 4	0,60205,99913,2796
Differentia Logar. pro Area Quadrati		inscripti	0,19611,98770,2704
		circumscripti	0,10491,01186,3694

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Pro Circulo, et adscriptis Multangulis Ordinatis.

				Logarithmi
Circuli		Diameter - - -- - - - - - - - - - - - -	2	0,30102,9995
		Peripheria	$6\underline{283585307}$	0,79817,9868
		Area	$3\underline{141592654}$	0,49714,9873
Quinquanguli	Latus	inscripti *ℓ*. bin. $^5/_2$ – *ℓ*. $^5/_4$	$1\underline{175570504}$	0,07024,8681
		circumscripti *ℓ*. bin. 20 – *ℓ*.320	$1\underline{453085056}$	0,16229,1036
	Perimeter	inscripti *ℓ*. bin. $62^1/_2$ – *ℓ*. $781^1/_4$	$5\underline{877852523}$	0,76921,8685
		circumscripti *ℓ*. bin.500 – *ℓ*.12500	$7\underline{26542528}$	0,86126,1040
	Area	inscripti *ℓ*. bin. $^{125}/_{32}$ + *ℓ*. $^{3125}/_{1024}$	$2\underline{37764129}$	0,37614,6310
		circumscripti ℓ. bin. 125 – *ℓ*.12500	$3\underline{63271264}$	0,56023,1045
Hexagon	Latus	inscripti 1		0,00000,0000
		circumscripti ℓ $^4/_3$	$1\underline{154700538}$	0,06246,9368
	Perimeter	inscripti *6*		0,77815,1250
		circumscripti ℓ.48	$6\underline{928202220}$	0,84062,0618
	Area	inscribed *ℓ*. $^{27}/_4$	$2\underline{598076211}$	0,41465,1886
		dodecagon inscripti	3 - - - - - -	0,56023,1045
		circumscripti ℓ. 12	$3\underline{464101651}$	0,53959,0623
Octanguli	Latus	inscripti ℓ.bin2 – ℓ.2	$\underline{7653668647}$ –	0,11613,0343
		circumscripti *ℓ*. 8 – 2	$\underline{82842712}$ –	0,08174,5690
	Perimeter	inscripti ℓ.bin 128 – ℓ. 8192	$6\underline{122934918}$	0,78695,9644
		circumscripti *ℓ*.512 – 16	$6\underline{62741696}$	0,82134,4298
	Area	inscripti ℓ. 8	$2\underline{82842712}$	0,45154,4993
		hexadecagon inscripti ℓ.bin 32 – ℓ.512	$3\underline{06146746}$	0,48592,9647
		circumscripti ℓ.128 – 8	$3\underline{32370850}$	0,52031,4302
Decagon	Latus	inscripti ℓ.$^{27}/_4$ – $^1/_2$	$\underline{6180339887}$ –	0,20898,764
		circumscripti ℓ. bin 4 – ℓ.$^{64}/_5$	$\underline{6498393925}$ –	0,18719,397
	Perimeter	inscripti *ℓ* 125 – *5*	$6\underline{180339887}$	0,79101,236
		circumscripti *ℓ*. bin 400 – ℓ.128000	$6\underline{498393925}$	0,81280,603
	Area	inscripti ℓ. bin $^{125}/_8$ – ℓ. $^{3125}/_{64}$	$2\underline{938926261}$	0,46818,869
		20anguli inscripti ℓℓ$^{4375}/_2$ – ℓ $\underline{17578125}$	$3\underline{09016994}$	0,48998,236
		Circumscripti ℓ . 100 – *ℓ*. 8000	$3\underline{24919696}$	0,51177,603
Dodecagon	Latus	inscripti *ℓ*. bin. *2* – *ℓ*. 3	$\underline{5176380902}$ –	0,28597,3773
		circumscripti 4 – *ℓ*.12	$\underline{5358983849}$ –	0,27091,7552
	Perimeter	inscripti *ℓ*. bin 288 – *ℓ*.62208	$6\underline{211657082}$	0,79320,7472
		circumscripti *48* – *ℓ*.1728	$6\underline{430780618}$	0,80826,3694
	Area	inscripti - - - - - - - - - - - - - - - - - -	3	0,47712,1255
		24 anguli inscripti *ℓ*. bin 72 – *ℓ*. 3888	$3\underline{105828541}$	0,49217,7477
		Circumscripti *ℓ*. 100 – *ℓ*. 8000	$3\underline{215399309}$	0,50723,3699

His Logarithmis inventis, si proposita harum figurarum qualibet, detur eius latus vel perimeter vel area; poterimus alterius cuiusvis harum, quemlibet terminum invenire. Quod in triangulo ostendimus fusius, idem unico aut altero examplo in reliquis deinceps ostendimus.

Esto datum latus Octaguli ordinati partium 7. quaeruntur Latus, Perimeter et Area quinquanguli, eidem circulo cum Octagulo inscripti.

Latus quaesitum sic invenietur:

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In circulo cuius Radius est unitas, latus inscripti Octanguli est 7653668647, Latus autem Quinquanguli est 11755579504. et si latus Octanguli sumatur partium 7, erit latus Quinquanguli quaesitum, quartum proportione. Sunt igitur semendi Logarithmi datorum terminorum, ut (per cap. 15) inveniatur Logarithmus termini quaesiti.

proportiones	*Logarithmi.*
Latus Octanguli datum 7653668647	–0,11613034
Latus Octanguli sumptum7	0,84509804
Latus Quinquanguli datum 1175579504	0,07024868
Latus Quinquanguli quaesitum 107516982	1,03147706

Ubi animaduertendum, Logarithmum primi termini (cum is sit minor unitate) esse defectum, ut cap. 10 ostendimus; atque ea de causa, non esse auferendum e summa mediorum, sed addendum potius; eritque Logarithmus quartus 1,03147706, et latus quaesitum 107516982.

Hoc latus, si neglectis Logarithmis per proportionis regulum quaeritur, erit latus Quadrinomij $122\,^{1}/_{4} + \ell.\ 7503\,^{1}/_{8} - \ell.\ 3001\,^{1}/_{4} - \ell.1500\,^{5}/_{8}$.

Eodem modo inveniri poterit perimeter Quinquanguli, si pro tertio termino sumatur perimeter dati Quinquanguli, eiusque, logarithmus. ut :

proportiones	*Logarithms*
Latus octanguli datum 7653668647	–0,11613034
Latus octanguli sumptum 7	0,84509804
Perimeter Quinquanguli data 5877852523	0,76921861
Perimeter Quinquanguli quaesita 53758491	1,73044706

Si quaerimus Aream Quinquanguli,cum nihil aliud sit datum praeter latus Octanguli eidem circulo inscripti : meminisse debemus, figuras similes planas, esse in duplicata ratione homologorum laterum: et idcirco laterum Octanguli quadrata esse sumenda, ut rite institui possit comparatio. Veruntamen, in hoc negotio, non opus erit, ut de ipsis quadratis simus solliciti, modo eorum Logarithmos, quod per cap. 16 facillimum erit, habuerimus. ut hic vides.

			Logarithmi.
pro-	Quadratum lateris Octanguli data	$2 - \ell.2$	–0,0,232260686
port.	Quadratum lateris sumpti	49 - - - - - - - - - - - -	1,690196080
	Area Quinquanguli dati	$\ell.$ bin $^{125}/_{32} + \ell.\ ^{3125}/_{1024}$	0,376146310
	Area Quinquanguli quaesita	1988854794	12,298603076

Si area dati Decanguli sit 6, et quaeratur latus Octanguli circumscripti eidem circulo, cui Decangulum inscribitur; erunt:

			Logarithms
pro-	Area decanguli dati	293892626	0,46818869
port.	Area Decanguli sumpta	6 - - - - - - - - - - - -	0,77815125
	Quadratum lateris Octanguli circumscripti dati		–0,16349138
	aggregatum mediorum		0,61465987
	Quadratum lateris Octanguli circumscripti quaesiti		0,14647118
	Latus quaesitum 118368622		0,07323559

Atque ad hunc modum in circulo, et in his figuris eidem circulo adscripti, ex unico termino, poterit alius quilibet per Logarithmos inveniri.