§26.1   *Chapter Twenty - Six: Synopsis.*

This is a long chapter concerned with surface and volume calculations of the ellipse and spheroid, and sections of the spheroid, according to the formulae of Archimedes; finally, the methods developed are applied to cask gauging, a popular mathematical pursuit at the time.

§1. For a given ellipse with major diameter AC, 8 [= $2a$], and minor diameter BD, 5 [= $2b$]: the area A is found initially from the relation: log A = log 8 + log 5 + Difference B. Note: this is equivalent to A = $\pi ab = 10\pi$ in modern terms, where Difference $B = \log(\pi/4)$ is a negative amount.

Or secondly, the area of the ellipse is the mean proportional between the areas of the circles with the major and minor diameters: 2 log A = (log 8^2 + Difference B) + (log 5^2 + Difference B); [equivalent to $\pi a^2/\pi ab = \pi ab/\pi b^2$].

Finally, the minor axis is to the major axis as the area of the ellipse is to the area of the minor circle: log A = (log 5^2 + Difference B) + log 8 - log 5; [equivalent to A = $\pi ab = \pi b^2 \times a/b$].

Note: in Table 26-3, Briggs prefers to add log 2 rather than subtract log 5 in the last method, thus using complementary arithmetic to avoid dealing with negative logarithms.

§2. Another method is presented for finding the area A of the ellipse. The difference in the areas of two concentric circles A1 and A2 (> A1), is equal to the area of an ellipse which has major and minor axes equal to the sum and difference of the diameters 8 and 5 of the circles, :
log A1 = (log 5^2 + Difference B) ; log A2 = (log 8^2 + Difference B); An ellipse is taken with axes 3 and 13, for 8^2 - 5^2 = 39, has area A = A2 - A1, while log A = log 3 + log 13 + Difference B.
[This is equivalent to $\pi(a^2 - b^2) = \pi(a - b)(a + b)$].

§3. The surface area and volume of a right cylinder with height D equal to its diameter are compared in turn with the surface area and volume of the circumscribed cube and inscribed sphere of the same width. The respective reduced ratios for both the areas and volumes of the cube, cylinder, and sphere in this order will be :-
diameter : (circumference of great circle)/4 : (circumference of great circle)/6.
[This is equivalent to 1 : $^\pi/_4$: $^\pi/_6$. The areas are then in the ratio $6D^2 \times$ (1 : $^\pi/_4$: $^\pi/_6$), while similarly the volumes are $D^3 \times$ (1 : $^\pi/_4$: $^\pi/_6$)].
The oblate spheroid is generated by rotating the semi-ellipse of the ellipse about the shorter axis, and the prolate spheroid similarly about the longer axis. See Fig.26-1. The volumes of the spheroids are mean proportionals between the volumes of the large and small spheres associated with the major and minor axes
[according to:- $^\pi/_6$.$(2a)^3$ > $^\pi/_6$.$(2a)^2$.$(2b)$ > $^\pi/_6$.$(2a)$.$(2b)^2$ > $^\pi/_6$.$(2b)^3$].

Briggs finds the logarithm of the volume of the spheroid by first evaluating the logarithm of the volume of the parallelepiped associated with the three axes; the logarithm of the volume of the circumscribed cylinder follows by taking Difference B [*i.e.* the logarithm of $^\pi/_4$.$(2a)^2$.$(2b)$ or $^\pi/_4$.$(2a)$.$(2b)^2$ is found]: the logarithm of the volume of the spheroid then follows by taking away the logarithm of $1^1/_2$ [i.e. corresponding to $^2/_3 \times$ $^\pi/_4$.$(2a)^2$.$(2b)$ = $^\pi/_6$.$(2a)^2$.$(2b)$, etc].

The volume of the spheroid V_{SD}, taken here as oblate, can also be found by proportion from the volume of the sphere V_S that shares a diameter a with the spheroid, according to $V_{SD}/V_S = 2b/2a$; or directly from the parallelepiped by taking Difference D [*i.e.* $^\pi/_6$.$(2a)$.$(2b)^2$
or $^\pi/_6$.$(2a)^2$.$(2b)$ directly].

§4. The volume of the section of a spheroid. According to Archimedes, for the prolate spheroid with the longer axis $2a$, if this axis is cut by a perpendicular vertical plane to give a short segment of length d, and a long segment of length f, so that $f + d = 2a$, (see Fig. 26-5 in the Notes), then:
(Volume of smaller spheroid segment)/$(f + a)$ = (Volume of smaller cone)/f; and similarly
(Volume of larger spheroid segment)/$(d + a)$ = (Volume of larger cone)/d; where the cones have the same base and height as their respective sections.
Briggs establishes the diameter of the prolate segment from proportions associated with the ellipse and its great circle, for given a, b, d and f. Subsequently, the area of the base circle and the volume of the cone are found; and from the above theorem, the volume of the segment of the spheroid found. The volume of the other segment of the spheroid is calculated in a similar way, and their sum compared with the volume of the spheroid, calculated separately. A similar set of calculations is performed for the oblate spheroid.

Briggs then considers the case of the spheroid with two sections, as an approximation to a wine cask. The volume of the prolate spheroid associated with the cask of given dimensions is found, and the volumes of the end segments calculated and removed. Another method for finding the cask volume involves averaging the area of cross-section: the area of the inner circle is taken with two thirds of the area of the ellipse with axes formed from the sum and difference of the max. and min. width of the cask; this area is multiplied by the height of the cask.

§26.2   ***Chapter Twenty Six.*** [p.69]

Concerning the Ellipse, Spheroid, and Cask.
1.

The ellipse is the common intersection of the surface of a cone & the surface of a plane,

cutting the whole cone. Furthermore, taking this figure for the ellipse itself; a section

of this kind may be considered to be an elongated circle or an oval shape; for in the works of

Archimedes spheroids are discussed which hold a likeness to the sphere, but with a shape

comprised of unequal distances from the centre. For the ellipse ABCD, either the periphery going

around is described, or the inside [i.e. the area] of the figure. The longest diameter is AC, the

shortest BD, the former intersecting at right angles in the centre E, [See Fig. 26-1].

   The ellipse is the mean proportional [area-wise] between the circles with the diameters *AC, BD*;

as shown in the work of *Archimedes*: *On Conoids and Spheroid,* Prop.5; for the oblong rectangle,

with the unequal sides of the squares described, is the mean

proportional between the squares: thus the area of the ellipse is the

mean proportional between the areas of the circles of the unequal

diameters. And the rectangle described by the diameters *AC, BD*,

is to the ellipse: as the square of the diameter is to the circle.
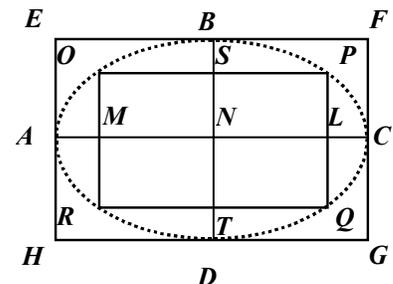
[Figure 26-1]

Therefore, given the diameters *AC, BD*; we can hence find the

area of the ellipse. From the logarithms of the diameters the difference ***B*** is taken [*i.e.* log(π/4)],

(which from the above chapter, taken from the logarithm of the square, leaves the logarithm of the

area of the circle) the remainder is the area of the ellipse. Let the diameters be AC, 8: BD, 5.

		Logarithms	
Diameters	8	090308999	
	5	06989700	
Sum		160205999	Area of Rectangle 40 formed from the
Difference ***B***		010491012	Diameters
Logarithm of the area		149714987	Area of the ellipse 31$_{41592692}$.

[Table 26-1].

The same area of which the logarithm is sought may be found, if in place of taking the difference B,

its complement is added. As we have shown in Chapter 15, thus

[p.70.]

		Logarithms
Diameters	8	090308999
	5	06989700
Sum		160205999
Complement of difference ***B***		989508988
		(1)149714987 Log. of the area

[Table 26-2]

The same area is produced, if the mean proportional is sought as per Ch.17, between the given

diameters of the circles, found as by Ch. 25.

		Logarithms	
Square of the diameter AC	64	180617997	
Difference ***B*** taken in both places		010491012	
Square of the diameter BD	25	139794001	
Remaining log. of area of major circle		170126985	Area of major circle 502654824
Remaining log. of area of minor circle		129302989	Area of minor circle 196349541
Sum of the logs		299429974	
Log. of mean proportional		149714987	Area of ellipse - - -3141592692

Proportions		*Logarithms*
or, as the minor diameter 5	Compl. Arith.	930102999
to the major diameter 8		090308999
thus the area of the minor circle		129302989
to the ellipse		1149714987

[Table 26-3]

2. *If two circles are concentric, the area lying between the peripheries of these is equal to the area*

of the ellipse, the diameters of which ellipse are the sum and difference of the given diameters of

the circles.

For as with squares, *the difference of the squares is equal to the rectangle described by the sum and*

difference of the sides: thus for circles, *the difference of the*

areas of the circles is equal to the area of the ellipse, of

which the diameters are equal to the sum and difference of

the given diameters. For the rectangle taken from the

diameters is the mean proportional between the squares of
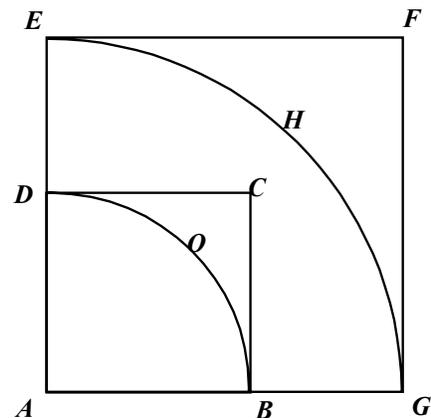
[Figure 26-2]

the diameters: & the area of the ellipse is the mean proportional between the areas of the circles of

these diameters.

   Therefore [area-wise], the square will be to the rectangle as the circle to the ellipse, and

conversely; as the square to the circle, so the rectangle to the ellipse: And the rectangle, which is

equal to the difference of the squares to the ellipse, which is equal to the difference of the area of

the squares to the circles.

<div align="center">30₆₃₀₅₂₈₄ ellipse. The difference.</div>

rectangle 39. The difference.
square 64   square 25   circle 50₂₆₅₄₈₂₄   circle 19₆₃₄₉₅₄₁

<div align="center">[Table 26-4]</div>

   Let the longer diameter be 8, the shorter 5. The squares are as 64: 25. The circles 50₂₆₅₄₈₂₄ :

19_{6349541.} The difference of the squares 39. The difference of the circles 30_{6305284.}

   Let the sum of the diameters of the ellipse be 13, and 3 the difference of the given diameters.

The rectangle is 39, of which the logarithm is 159106461. The difference **B** taken away, leaves

148615448, the logarithm of the area of the ellipse 30_{6305284.}

<div align="center">

Logarithms

</div>

		Logarithms	
Diameters	⌠13	111394335	
	⌡3	047712125	
Complement of difference **B**		989508988	
Log. of the diff. of the circles		1148615448	Area of the ellipse 30_{6305284.}

<div align="center">[Table 26-5].</div>

3. If for a cylinder, in which a sphere is inscribed, a cube is circumscribed: the quantities[1]

involved [in the order Cube : Cylinder : Sphere] are as the diameter of the sphere [D]; to the

quarter of the circumference of the great circle [$^1/_4\pi D$]; & as the sixth part of the same [$^1/_6\pi D$].

Thus the square of the diameter [D^2], to the area of the great circle [$^1/_4\pi D^2$], & as two thirds of the

area of the same circle [$^2/_3(\pi D^2/_4) = ^1/_6\pi D^2$]. And thus the surface of the cube [$6D^2$], to the surface

of the cylinder [$6 \times \pi D^2/4$], & the surface of the sphere [$6 \times \pi D^2/6$]. Thus also the volume of the

cube [D^3], to the volume of the cylinder [$^1/_4\pi D^3$] & the volume of the sphere [$^1/_6\pi D^3$].

Let the Diameter of the Sphere be 7 .
A Quarter of the Circumference is   5<u>4977871438</u>   The first three ratios with the
A Sixth of the Circumference   3<u>6651914292</u>   remaining ratios following in
                                                         proportion.

[p.71.]

Square of the Diameter	49
(Area of) Circle	38<u>484845100066</u>
$^2/_3$ of Circle	25<u>6563400044</u>
Surface of the Cube	294
Surface of the Cylinder	230<u>9070600396</u>
Surface of the Sphere	153<u>9380400264</u>
Volume of Cube	343
Volume of Cylinder	269<u>3915700462</u>
Volume of Sphere	179<u>5943800308</u>

[Table 26-6].

[Figure 26-3]

The spheroid is made by the rotation of half the ellipse by keeping other axis fixed. If the longer diameter AC remains, the spheroid is made long [*i.e.* prolate]; but if BD remains, the spheroid becomes broad [oblate], of which both are means in continued proportion, between spheres of unequal diameters: & each of these is in the ratio of two to three to the cylinder, of which the altitude is equal to the remaining diameter, the base is still circular, having been described by the movement of the semi-diameter turning in a circle[2].
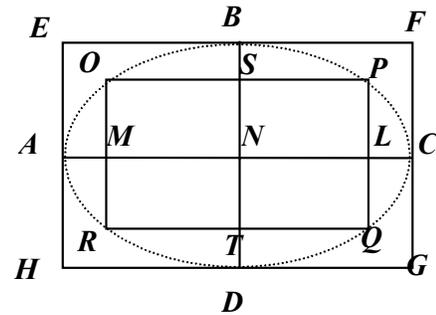
With the height and width of a spheroid given, the volume of which we can thus find: The logarithm of the height is added to the logarithm of the square of the thickness: the total is the logarithm of the parallelepiped of the same altitude and base of the square. Hence, the difference **B** of Chapter 25 is taken away. There remains the logarithm of the cylinder circumscribing the spheroid: from which if the difference of the ratio of one and a half [i.e. log 3/2] is taken away 017609125905568, there remains the logarithm of the spheroid. For, let the diameters of the ellipse be 8, 5. & by keeping the small diameter the spheroid becomes wide: the volume is sought[3].

		Terms	Logarithms
		3	047712125471966
of the ratio of one & half	{	2	030102999566398
		Diff. of logs.	017609125905568

	Logarithms
Wide diameter 8	090308999
Square 64	180617997
Height 5	069897000
Vol. of parallelepiped 320	250514997
Difference **B** being taken	010491012
Cylinder 251<u>32741</u>	240023985
Difference of the ratios by one & half	017609126
Vol. of spheroid 167<u>551605</u>	222414859

[Table 26-6]

Or, by Ch. 25, with the logarithms of the square & height, is added to the complement of the

difference B, & the difference of the ratio of one and a half: the total (taking away the first digit 2)

is the logarithm of the spheroid.

	Logarithms
Square 64	180617997
Height 5	069897000
Comp. Diff. **B**	989508988
Compl. Diff. ratio one & a half	982390874
Volume of spheroid	2222414859

[Table 26-7]

   We find the same volume of the spheroids by the rule of proportion. For the parallelepipeds of

which the bases are equal are in proportion with their own heights. In the same way as cylinders, so

with spheroids inscribed in cylinders: if they have the same width, they are in proportion with

heights. There are, therefore, a sphere to a spheroid of the same width; as the diameter or height of

[p.72.] the sphere to the height of the spheroid. Therefore the logarithm of the sphere for the given

diameter can be found, per Ch. 25, etc.

	Logarithms	
Diameter 8	090308999	1
Cube of the diameter	270926996	3
Difference **B** taken away	028100138	
Volume of sphere	242826858	

Proportions		*Logarithms*
Height of sphere 8	Compl. Arith.	909691001
Height of spheroid 5		069897000
Volume of sphere		242826858
Volume of spheroid		2222414859

[Table 26-8]

   Or because the parallelepiped is to the volume of the inscribed spheroid, as the cube to the

sphere: from the logarithm of the parallelepiped first found, the difference *D* is taken away. Ch. 25.

There is left the logarithm of the spheroid.

	Logarithms
Parallelepiped	250514997
Difference *D*	028100138
Volume of Spheroid	2222414859

[Table 26-9A]

And this is the flat spheroid. The oblong spheroid is found in the same way.

Let the flat diameter be 5, the height 8.

	Logarithms
Diameter 5	069897000
Cube of the diameter 125	209691001
Difference *D*	028100138
Volume of sphere	181590863

[Table 26-9B]

proportions		*Logarithms*
Height of sphere 5	Comp. Arith.	930102999
Height of spheroid 8		090308999
Sphere		181590864
Spheroid 104719755		1202002862

[Table 26-9C]

And in this way, we can find both the spheroids, which are in continued proportion between the

unequal diameters of the spheres 8 & 5.

Continued proportion.		*Logarithms*
Sphere with diameter 8	268082573	242826858
Spheroid width 8, height 5	167551608	222414860
Spheroid width 5, height 8	104719755	202002862
Sphere with diameter 5	65449847	181590863

[Table 26-9D]

[Figure 26-4]

4. If we want to know the volume of a segment of a spheroid[4], Propositions 31 & 32 are consulted,

[p.73.] from the book *On Conoids and Spheroids* by

Archimedes. Which I have tried to explain from

this source solely. *If a spheroid is cut by a plane*
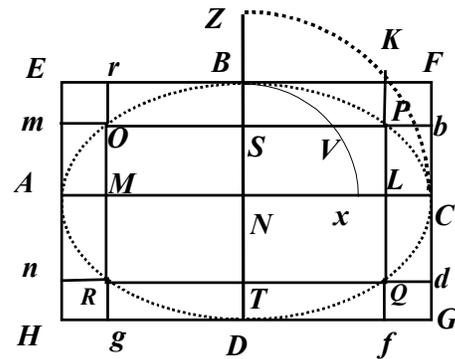
perpendicular to the axis: the segment of the spheroid is to the cone of equal height, having the

same base as the segment; as the sum of the half axis & the height of the remaining segment, is to

the height of the remaining segment. For let the [prolate] spheroid be *ABCD* cut by the plane

passing through PQ, & perpendicular to the axis AC: I assert the segment of the spheroid PCQ, to

be to the cone, of which the base is the circle of diameter PQ, with the height CL: as the sum of NA

& AL, to AL. Let AC, 20; BD, 12; CL, 2 ; PL is $3^3/_5$. For if two arcs from the ellipse ABCD are

described, the radii of which are equal to the lines NC, NB. & PL, PVS are drawn perpendicular to

the radii NC, NB; then NZ, NB : LK, LP are proportional; likewise NC, NX: SP, SV. But LK by

Prop. 13, Book 6, Euclid is the mean proportional between CL, 2 & LA, 18 [i.e. $LK^2 = AL.LC$];

therefore LK is 6, and LP 3̲6̲

[as LP = (NB/NZ).LK].

proport-ion	NZ	10
	NB	6
	LK	6
	LP	3̲6̲

proport-ion	AL	18
	LK	6
	LC	2

		Logarithms
Diameter PQ 7̲2̲		085733250
Square of PQ		171466500
Difference **B** taken away		010491012
Area of circle PQ	40̲7150408̲	160975488
Altitude CL 2	81̲4300816̲	030102999
Cylinder *PbdQ*		191078487
Log. of 3 taken away		047712125
Cone 1/3 of cylinder PCQ	27̲1433605̲	143366362

			Logarithms
pro-port--ion. {	*AL* 18	Compl. Arith.	874472749
	NA + AL 28		144715803
	Cone *PCQ*	27̲1433605̲	143366362
	Seg. of spheroid *PCQ*	42̲2230053̲	1162554914

		Logarithms
Circle *PL* 40̲7150408̲		160975488
Altitude *AL* 18		125527251
Cylinder *mPQn* 732̲8707343̲		286502739
log. of 3 taken away		047712125
Cone *APQ* $^1/_3$ of cylinder 244̲2902447̲		238790614

			Logarithms
pro-port--ion. {	*CL* 2	Compl. Arith.	969897001
	CN + CL 12		107918125
	Cone *PAQ*	244̲2902447̲	238790614
	Seg. of spheroid *PAQ*	1465̲7414684̲	1316605740

Segment PCQ	42̲2230053̲
Segment PAQ	1465̲7414684̲
Whole spheroid	1507̲9644737̲

		Logarithms
Diameter BD 12		107918125
Square of BD		215836249
Difference **B** taken away		010491012
Circle BD	113̲0973355̲	205345237
Axis AC 20		130102999
Cylinder	226̲1946710̲	335448236
Difference of half the ratio		017609126
Spheroid	1507̲9644737̲	317839110

[Table 26-10]

But if *BD* is the oblate axis of the spheroid, & crosses the plane cutting the line *OP*: *OP* are of

16 parts, & the circle of which the diameter *OP* is 201<u>0619298297472</u>; S*B*, 2<u>4</u>; *SD*, 9<u>6</u>. The cylinder

rOPK is 482<u>54863159</u>. The cone *OBP* is 160<u>84954386</u> .

[Figure 26-4]

		Logarithms
Diameter *OP* 16		120411998
Square of *OP* 256		240823996
Difference **B** taken away		<u>010491012</u>
Area of circle *OP*	201<u>06192983</u>	230332985
Altitude *BS* 2<u>4</u>	482<u>54863159</u>	<u>038021124</u>
Cylinder *rKPO*		268354109
Log. of 3 taken away		<u>047712125</u>
Cone *OBP*	160<u>84954386</u>	220641984

			Logarithms
pro-	*SD* 9<u>6</u>	compl. arith.	801772877
port-	*ND* + *SD* 15<u>6</u>		219312460
-ion.	Cone *OBP*	271<u>433605</u>	<u>220641984</u>
	Segment *OBP*	261<u>38030877</u>	1241727321
Circle *OP*		201<u>06192983</u>	230332985
Altitude *SD* 9<u>6</u>			<u>098227123</u>
Cylinder *OgfP*		1930<u>19452637</u>	328560108
log. of 3 taken away			047712125
Cone *ODP*		643<u>39817546</u>	280847983

			Logarithms
proportions.			
	BS 2<u>4</u>	Compl. Arith.	861978876
	BS + *BN* 84		192427929
	Cone *ODP*	643<u>39817546</u>	280847983
	Segment *OADCP*	2251<u>8936141</u>	1335254788
	Segment *OADCP*	2251<u>8936141</u>	
	Segment OBP	261<u>38030877</u>	
	Whole spheroid	2513<u>2741229</u>	

		Logarithms
Diameter *AC* 20		13010299956
Cube of *AC* 8000		39030899870
Difference **D** taken away		<u>02810013777</u>
Sphere	4188<u>7902047864</u>	36220886093

Proportions.			*Logarithms*
	AC 20	Compl. Arith.	86989700044
	BD 12		10791812460
	Sphere of diameter *AC*	4188<u>7902078644</u>	<u>36220886093</u>
	Spheroid *BADC*	2513<u>2741228718</u>	134002398597

[Table 26-11].

And in this way if a spheroid is cut by a single plane, perpendicular to the axis, we can find the

volume of both segments.

There remains the segment that is described by the surface of the spheroid, & two planes

perpendicular to the axis and equidistant from the centre. It is our cask of whatever kind, the

capacity of which we can measure, following that which was said above from Archimedes. Thus

with Book 3, Ch. 10 of *Pantometria*, that learned book in the vernacular by T.D. [Thomas Digges], the most distinguished of men, is conscripted. Also Errardus *Barleduc*, Book 3, Ch. 10, and Clavius, *Geom. Practicae*, Book 5, Ch. 10.

Let the cask be *BPQDRO* of which the height is *ML* of 21 parts, while the width of the middle is *BD*, 14. The diameter of the base or the width of the end is *PQ* $9^1/_3$. To begin with the whole length of the spheroid *AC* is required, which we find thus: *PQ* $9^1/_3$ is taken from *BD* 14, and half the remainder $4^2/_3$ is *BS* $2^1/_3$. Again *SD* is $11^1/_3$, & *SV* the mean proportional between *BS* & *SD* [*i.e.*

again we use this useful theorem, to give $SV^2 = BS.ST$] , is $\ell.27^1/_9$, by Euclid, Book 6, Prop. 13, or 52174919477. But *SV, SP: NX* (or *NB*), *NC* are proportional [*i.e.* SV/SP = NX/NC]. NC therefore is ℓ.19845 or 1487228257 & LC 3587228357. The circle with diameter *PQ* has the area 6841690667819 . The cylinder *PbdQ* has the volume 24542706089, the cone *PCQ* has the volume 81809020297. With the two segments *PCQ, OAR* of volume 2573628361, taken from the spheroid, leaves the cask with volume *BPQDRO*. But if the circle of diameter *BD* 15393804400259, multiplied by the line *AC* 28174456514 makes 433712061456 for the volume of the cylinder *EFGH* , & the whole of the spheroid is 2891413743. From which the two segments 2573628361 taken will leave the volume of the cask 2643050907.

pro- port- ion.	AL	24587228257
	AL + AN	38674456514
	Cone	81809020297
	Segment	12868141805

[Table 26-12]

We come upon the volume of the same cask much more easily in the following way. The circle *DB* is sought with the mean width 14, & the base *PQ* $9^1/_3$. These circles are multiplied by the altitude *ML* 21: the products is the cylinders *rKfg* & *OPQR*. Taking then the difference of these cylinders, of which the difference $^1/_3$ is taken from the greater cylinder, or $^2/_3$ is added to the lesser cylinder gives the volume of the cask.

BD Diameter 14   *Logarithms*
The Square   196   229225607
Difference **B** taken away   010491012
Circle BD 153<u>9380400</u>   218734595
_____
PQ 9$^1/_3$
Square of PQ   19400735532
Difference **B** taken away   01049101186
Circle PQ 68<u>416906678</u>   18351634346
Large cylinder 3232<u>698840</u>
Small cylinder 1436<u>755040</u>
Difference 1795<u>943800</u>
_____
$^1/_3$ of difference 598<u>647933</u>
Cask 2634<u>050907</u>

[Table 26-13]

Or the area of the circle *PQ* is taken, & the ellipse of which the diameters are equal to the sum of the lines *BD*, *PQ* 23$^1/_3$, & two thirds of the difference of the same 3$^1/_9$: let this sum be multiplied by the given altitude ML 21: the product is equal to the volume of the cask.

Circle PQ 68<u>416906678</u>   *Logarithms*
Sum of BD, PQ 23$^1/_3$ 13679767853
$^2/_3$ Difference BD, PQ 3$^1/_9$ 04929155219
Circle BD 153<u>9380400</u>   218734595
Complement difference **B**   98950898814
Ellipse 570<u>140889</u>   19400735532
Circle + ellipse 1254<u>309956</u>   01049101186
Cask 2634<u>050907</u>

[Table 26-14]

The same volume as before has been found by this method[5]. Here is the reason for this procedure. The difference of the areas of the circles of the middle width BD, & of the ends *PQ*, is equal to the area of an ellipse, the diameters of which is equal to the sum & difference of the diameters *BD & PQ*, as I have shown in section 2 of this chapter. This is really the difference of the areas of the concentric Circles, that is, the base of the hollow cylinder, of which the third part is placed between the surfaces of the exterior cylinder and the Spheroid, the remaining two thirds are within the same spheroid. The area of the ellipse therefore which is added to the area of the small circle amounts to two thirds of the difference of the areas of the circles *BD, PQ* (for the whole difference of the diameters is not taken, but $^2/_3$ of the same 3$^1/_9$: because if I take the whole difference & as much as $^2/_3$ of the sum instead, the same ellipse comes out) which together with the

[p.76.] small circle is multiplied by the given height 21, makes the same volume which was first found for

the cask.

Let the cask be *PCQRAO*, of which the altitude *OR* 7$\underline{2}$ is the small width AC 20, & OP is 16.

The capacity of this cask is required. The sum of the diameters AC + OP is 36, their difference 4.

Square OP 256

Difference *B*

Circle OP 20$_{106192983}$

$^2/_3$ sum of the diameters 24

Difference of the diameters 4

Complement of difference *B*

Ellipse 75$_{3982235}$

Circle 201$_{0619298}$

base 276$_{4601533}$

Altitude   72

1990$_{513104}$

[Table 26-15]

2408239965

0104910119

2303329846

1380211242

0602059991

9895089881

11877361114
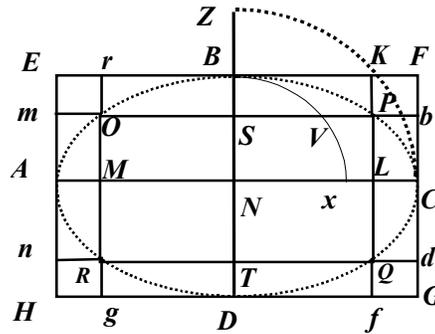
[Figure 26-4]

And by these means we can measure the sizes of these shapes, ellipses and spheroids.

If we wish to construct the equivalent circle of a given ellipse, the mean proportional between the

diameters of the ellipse is found: the circle of which the diameter is the mean proportional is equal

to the given ellipse. For it is the same ratio of the circle to the ellipse, of which the square is to the

oblong. If we want to describe the sphere equal to the spheroid; two continued mean proportionals

are found, between the height of the spheroid and the wide diameter. The sphere, of which the

diameter is the equal of that mean, which is nearer to the wide diameter; is equal to the given

spheroid. For thus the cube itself holds [the same ratio]to the parallelepiped with the square base, as

the sphere to the spheroid, for the parallelepiped with the same altitude. With the parallelepiped is

to the equal spheroid, as the square of the base to $^2/_3$ of the area of the circle.

§26.3   *Notes On Chapter Twenty Six*

[1]   The ratios are taken in the order cube : cylinder : sphere. The original ratios given are :–

$1 : \pi/4 : \pi/6$, where we have succumbed to modern usage with π. For a diameter D, the area of the

cube is $6D^2$, the cylinder has a surface area, including the ends, of $3\pi D^2/2$; while the sphere has

surface area πD^2. These areas can also be put in the ratio $1 : \pi/4 : \pi/6$.

Similarly, the volumes are in the same ratio $1 : \pi/4 : \pi/6$. This is Briggs' rule of proportions.

[2]   For if m_1 and m_2 are the two means sought, starting from the larger, then we require:

$$\frac{(2a)^3}{m_1} = \frac{m_1}{m_2} = \frac{m_2}{(2b)^3}.$$ From which it follows that $m_1 = (2a)^2(2b)$ and $m_2 = (2a).(2b)^2$.

[3]   The volume of the ellipsoid with axis *2a, 2b, 2c* is $\pi(2a)(2b)(2c)/6$, where $a > b > c$. If two of

the axis are equal, then we have the volumes of the Spheroids: $V_1 = \pi(2a)^2(2c)/6$, if $b = c$;

and $V_2 = \pi(2a)(2b)^2/6$, if $b = c$.

In the first case, the circumscribing Parallelepiped has volume $(2a)(2b)^2$, while the circumscribed

cylinder has volume $\pi(2a)(2b)^2/4$. Thus, Briggs evaluates in order, the volumes $1.(2a)(2b)^2$,

$(\pi/4)(2a)(2b)^2$, and $(\pi/6)(2a)(2b)^2$. Briggs subsequently finds V_2 in a like manner.

[4]   Proposition 31,32 *On Conoids and Spheroids,*

Archimedes, may be stated, according to Figure 26-5,

in the form, where AC = *2a* : –

(Volume of smaller spheroid segment PCQ)/($f + a$)

= (Volume of cone PCQ)/f; and

(Volume of larger spheroid segment PAQ)/($d + a$)

= (Volume of cone PAQ)/d.
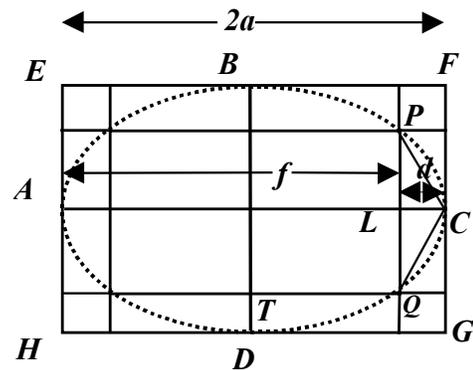
[Figure 26-5]

To check the results, we may write the volume of the small segment of the spheroid as

$V_R = (\pi/3)LP^2d(1 + a/f)$; and similarly

$V_L = (\pi/3)LP^2 f(1 + a/d)$ for the large segment. Subsequently, the total volume

$V = (\pi/3)LP^2[d(1 + a/f) + f(1 + a/d)] = (\pi/6)LP^2(2a)^3/fd$. But as in the text, $LP/LK = 2b/2a$, and

$LP^2 = (2b)^2/(2a)^2.fd$ hence $V = (\pi/6)(2b)^2(2a)$ as required.

Briggs leans on the work of Archimedes for this application of Logarithms. We may recall that

the treatise *The Method….*, in which Archimedes disclosed the method by which he had discovered

his wonderful results, lay undiscovered at this time. The Archimedes' Palimsest is again a centre of

active research on its re-emergence, after the detective work and genius of Heiberg in its discovery

and the production of a translation under difficult circumstances, and its subsequent disappearance.

[See, e.g. *Physics Today*, Volume 53, No. 6; June 2000, *'The Origins of Mathematical Physics:*

New Light on an Old Question', by Reviel Netz].

[5]   The area of the ellipse in question is $(\pi/4)(BD^2 - PQ^2) = (\pi/4)(2b - PQ)(2b + PQ)$. Now, $^2/_3$ of

the area of this ellipse is placed outside the circle with diameter PQ, giving a total area of:

$\pi. LP^2 + (^1/_6)\pi.(BD^2 - 4LP^2) = (^1/_6)\pi.(BD^2) + (^1/_3)\pi.(LP^2) = (^1/_6)\pi.(2b)^2 + (^1/_3)\pi.(2b/2a)^2.fd$, and

the volume of the cask $V_C = (^\pi/_6).(4b^2)[1 + fd/2a^2].2(a - d)$, as $PQ^2 = 4LP^2 = 4(2b/2a)^2.fd$; and

$BD = 2b$. We may subsequently write $V_C = (^\pi/_6).(2b)^2[2a - d^2/a - fd^2/a^2]$.

We have to reconcile this formula for V_C with that obtained above:–

The volume of the entire spheroid is $(^\pi/_6) ab^2$, Hence, the volume of the cask V_C is given by:

$V_C = (^\pi/_6)(2a)(2b)^2 - (\pi/3)(2b/2a)^2.fd.2d(1 + a/2) = (^\pi/_6)(2b)^2[2a - fd^2/a^2 - d^2/a]$, as above.

§26.4.   Caput XXVI. [p.69.]

De Elleipsi, Sphaeroide, & Dolio.

Elleipsis est communis intersectio superficiei conicae & superficiei planae, secantis conum ex omni parte. Sumitur etiam elleipsis pro ipsa figura, ab huiusmodi sectione comprehensa; quae dici poterit circulus oblongus, vel Cycloeides: ut apud Archimedem sphaeroeides dicitur, quae sphaerae similitudinem obtinens, inaequaliter distat a medio comprehensi spatij. ut ABCD Elleipsis, est vel peripheria ambiens, vel figura intus comprehensa. eius diameter longissima AC, brevissima BD, priorem ad rectos angulos intersecans in Centro E.
    Elleipsis est media proportionalis inter circulos Diametrorum *AC, BD*; *Archim. prop 5.lib.de Conoid.* nam ut Oblongo rectangulum, a lateribus inaequalium quadratorum comprehensum, est medium proportionale inter Quadrata: sic Elleipsis est media proportionalis inter Circulos diametrorum inaequalium. Et rectangulum a Diametris *AC, BD*

comprehensum, est ad Elleipsim: ut Quadratum Diametri ad Circulum. Datis idcirco Diametris *AC, BD*; area elleipsis sic inveniemus. A Logarithmis Diametrorum auferatur Differentia **B** , (quae superiori Capite ablata e Logarithmo Quadrati relinquebat Logarithmum circuli) reliquus erit Logarithmus Elleipsis. ut sunto Diametri AC, 8: BD, 5.

		Logarithmi.	
Diametri	8	090308999	
	5	06989700	
Summa		160205999	ectanguli 40 comprehensi a diametris.
Diff. **B**		010491012	
Logarithmus areae		149714987	area Elleipsis 31₄₁₅₉₂₆₉₂.

Idem Areae quaesitae Logarithmus invenietur, si loco Differentiae B, auferendae, additur eius complementum. ut in Cap. 15 ostendimus, ut

		Logarithmi.
Diametri	8	090308999
	5	06989700
Sum		160205999
Compl. Diff.**B**		989508988
		(1)149714987 Logarith. areae
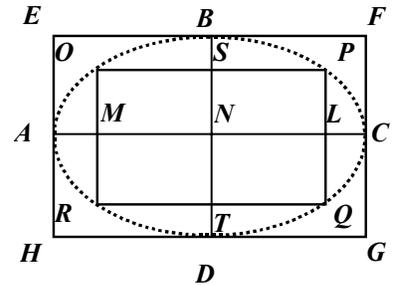
Eadem area prodibit, si quaeratur per cap.17 medius proportionalis inter datarum diametrorum Circulos, inventos per cap. 25.

[p.70.]

	Logarithmi.	
Quadratum diametri AC.64	180617997	
Differentia **B** auferenda ab utroque	010491012	
Quadratum diametri BD.25	139794001	
Reliquus Logar. circuli maioris	170126985	circlulus maior 50₂₆₅₄₈₂₄
Reliquus Logar. circuli minoris	129302989	circulus minor 19₆₃₄₉₅₄₁
Summae Logarithmorum	299429974	
Logarithmus medij proportionalis	149714987	Elleipsis - - -31₄₁₅₉₂₆₉₂

Proportionem

		Logarithmi.
vel, ut Diameter minor5	Compl. Arith.	930102999
ad Diametrum maiorem	8	090308999
sic circulus minor		129302989
ad Elleipsim		1149714987

2. *Si duo circuli sint concentrici, Area eorum peripherij interiecta aequatur Elleipsi, cuius Diametri sunt summae & differentia datarum Diametroram.*

Nam ut in Quadratis, *Differentia Quadratorum aequatur rectangulo comprehenso a summa & differentia Laterum:* sic in Circulis, *Differentia Circulorum aequatur Elleipsi, cuius Diametri aequantur sumae & differentia datarum diametris*. Est enim Rectangulum a Diametria comprehensum, medium proportionale inter Quadrata diametrorum : & Elleipsis est media proportionalis inter Circulos diametrorum.

Erit igitur ut Quadratum ad Rectangulum; sic Circulus ad Elleipsim, & alterne; ut Quadratum ad Circulum, sic rectangulum ad Elleipsim: & Rectangulum quod differentiae Quadratorum est aequale; ad Elleipsim, quae differentiae Circulorum aequabitur.
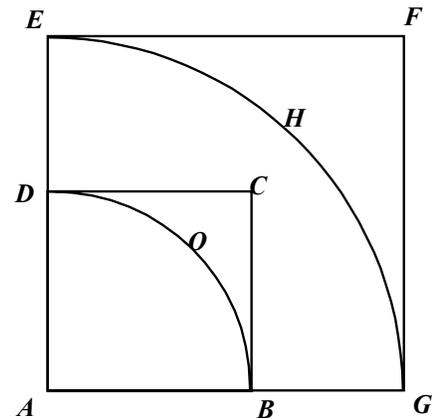
Rectang. 39. Differentia   306₃₀₅₂₈₄ Elleipsis Differentia.
Quad. 64   Quad. 25   Circulus 50₂₆₅₄₈₂₄   Circulus 19₆₃₄₉₅₄₁

Ut esto Diameter longior 8, minor 5. erunt Quadrata 64. 25. Circuli 502654824 : 196349541. Differentia Quadratorum 39. Differentia circuli 306305284.

Sunto Diametri Elleipsis,13 summa, 3 differentia diametrorum datarum. Rectangulum 39, eius Logarithmus 159106461. Differentia **B** ablata, relinquit 148615448. Logarithmum Elleipsis 306305284.

		Logarithmi
Diametri	13	111394335
	3	047712125
Compl. Diff. **B**		989508988
Logar. diff. circulorum		1148615448 Elleipsis 306305284.

3.   Si Cylindro, cui Sphaera inscribatur, circumscribatur Cubus: erunt ut Diameter Sphaerae, ad Quadrantem Peripheriae maximi Circuli, & ad Sextantem eiusdem: sic Quadratum Diametri, ad Circuli & ad duas tertias circuli. Et sic Superficies Cubi, ad Superficies Cylindri & Sphaerae : sic etiam Cubus, ad Cylindrum & Sphaeram.

Esto Diameter Sphaerae	7
Erit Quadrans Peripheriae	54977871438 tres primi tribus proximis,
Sextans Peripheriae	36651914292 reliquisq; sequentibus, sunt
	proportionales.
	[p.71.]
Quad. Diametro	49
Circulus	38484845100066
²/₃ Circuli	256563400044
Superficies Cubi	294
Superficies Cylindri	2309070600396
Superficies Sphaerae	1539380400264
Cubus	343
Cylindrus	2693915700462
Sphaerae	1795943800308

Sphaeroeides sit, conversione semielleipsis manente altera Diametro. Si maneat AC diameter longior, sit Sphaeroeides oblongata; sin maneat BD, Sphaeroeides lata. quae ambae sunt mediae continue proportionales, inter Sphaeras inaequalium diametrorum: & earum utraque, est in subsesquialtera ratione ad Cylindrum, cuius altitudo aequatur Diameter manenti, basis vero est circulus, motu semidiametri circumactae descriptus.

Datis altitudine & crassitudine Sphaeroeidis, eius soliditatem sic inveniemus. Addatur Logarithmo altitudinis, Logarithmo quadratae crassitudinis: totus erit Logarithmus parallelepipedi eiusdem altitudinis & basis quadratae. Hinc, auferenda est differentia **B** capitis 25. restabit Logarithmus Cylindri, Sphaeroeidi circumscripti : quo si auferatur Differentia rationis sesquialterae   017609125905568, restabit Logarithmus Sphaeroeidis. Ut sunto Diametri Elleipsis 8. 5. & manente diametro minore fiat Sphaeroeides lata: quaeritur soliditas.
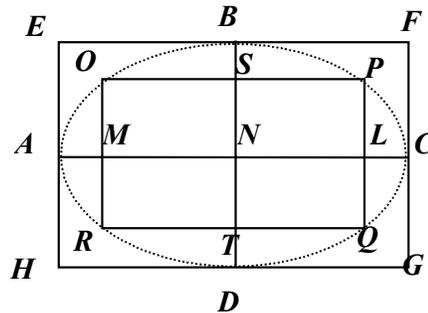
	Termini	Logarithmi.
	3	047712125471966
Rationis sesquialterae	2	030102999566398
	Diff. Logar.	017609125905568

	Logarithmi.
Diameter crassitudinis 8	090308999
Quadratum 64	180617997
Altitudo 5	069897000
Parallelepipedum 320	250514997
Differentia **B** auferenda	010491012
Cylindrus 25132741	240023985
Differentia rationis sesquialterae	017609126
Sphaeroeides 167551605	222414859

Vel, per cap. 25, Quadrati & altitudinis Logarithms, addantur complementa Differentia B, & Differentae rationis sesquialterae: Totus (dempta prima nota 2) erit Logarithmus sphaeroeidis.

	Logarithmi
Quadrati 64	180617997
Altitudinis 5	069897000
Compl. Differentiae **B**	989508988
Compl. Differentiae rationis sesquialterae	982390874
Sphaeroeides	2222414859

   Eandem soliditatem Sphaeroeidis, per proportionis regulam inveniemus. Sunt enim parallelepipeda quorum bases sunt aquales, ipsis altitudinibus proportionalia. eodem etiam modo tam Cylindri, quam Sphaeroeides cylindris inscriptae, si sint eiusdem crassitudinis, sunt altitudinibus proportionales. Erit igitur Sphaera, ad sphaeroeidem crassitudinis; ut Diameter vel altitudo

[p.72.]

   sphaerae, ad altitudinem Sphaeroeidis. Quaerendus est idcirco Logarithmus Sphaerae pro datis Diametro, per Cap. 25, &c.

	Logarithmi.	
Diameter 8	090308999	1
Cubus Diametri	270926996	3
Differentia **B** auferenda	028100138	
Sphaere	242826858	

Proportiones		Logarithmi.
⌠Altitudo Sphaerae 8	Compl. Arith.	909691001
⎰Altitudo Sphaeroeidis 5		069897000
Sphaera		242826858
⌡Sphaeroeides		2222414859

   Vel quia parallelepipedum est ad Sphaeroeidem, ut Cubus ad Sphaeram: e Logarithmo parallelepipedi prius invento, auferatur Difference *D*. cap. 25. reliquus erit Logarithmus Sphaeroeidis.

	Logarithmi.
Parallelepipedum	250514997
Differentia *D*	028100138
Sphaeroeides	2222414859

   Atque haec est Sphaeroeides Lata. Sphaeroeides Oblonga eodem modo invenietur.
   Esto Diameter crassitudinis 5, Altitudo 8.

	Logarithmi.
Diameter 5	069897000
Cubus Diametri 125	209691001
Differentia *D*	028100138
Sphaera	181590863

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	proportionem		*Logarithmi*
⌠	Altitudo Sphaerae 5	Comp. Arith.	930102999
⎰	Altitudo Sphaeroeides 8		090308999
	Sphaera		181590864
⌡	Sphaeroeides 104719755		1202002862

Atque ad hunc modum, utrasque Sphaeroeides invenimus, quae sunt continue proportionales inter Sphaeras Diamerrorum inaequalium 8 & 5.

Contin. propor.		Logarithmi.
⌠ Sphaera cuius Diameter 8	268082573	242826858
⎰ Sphaeroeides crassa 8, alta 5	167551608	222414860
⎰ Sphaeroeides crassa 5, alta 8	104719755	202002862
⌡ Sphaera cuius Diameter 5	65449847	181590863

4. Si segmenti Sphaeroeidis soliditatem scire velimus, consulendem sunt Archimedis prop.31 & 33.lib. de Conoid. Quas unica hac exprimere conatus sum. *Si Sphaeroeides plano secetur perpendiculari ad axem: segmentum* Sphaeroeidis *est ad Conum aequealium, habentem eandem cum segmento basim; ut composita ex axe dimidiato & altitudine reliqui segmenti, est ad altitudinem reliqui segmenti.*

   Ut esto Sphaerois *ABCD*, secta plano transeunte per PQ, & perpendiculari axi AC: aio segmentum PCQ, esse Conum, cuius basis est Circulus Diametri PQ, altitudo vero CLm

<div style="text-align:center">[p.73.]</div>

: ut composita ex NA & AL, ad AL. Esto AC, 20; BD, 12; CL, 2 ; PL erit $3^3/_5$. Nam si in Elleipsi ABCD describantur duae peripherea quarum radij aequantur rectis NC, NB. & ducantur PL, PVS perpendiculares radij NC, NB; erunt NZ, NB : LK, LP ; item NC, NX: SP, SV proportionales. Est autem LK per 13.prop.6.lib.Eucl. media proportionalis inter CL, 2 & LA, 18, erit igitur LK 6, & LP 3<u>6</u>
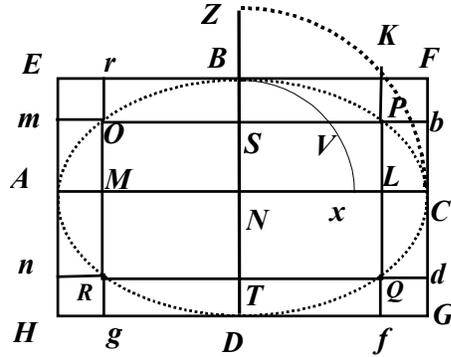
prop-		NZ	10	prop.		AL	18
port.	{	NB	6		{	LK	6
		LK	6			LC	2
		LP	3<u>6</u>				

		Logarithmi.
PQ Diameter 7<u>2</u>		085733250
Quadrantum PQ		171466500
Differentia **B** auferenda		<u>010491012</u>
Circulus PQ	40<u>7150408</u>	160975488
Altitudo CL 2	81<u>4300816</u>	<u>030102999</u>
Cylindrus *PbdQ*		191078487
3.Logar auferendus		<u>047712125</u>
Conus 1/3 cylindri PCQ	27<u>1433605</u>	143366362

			Logarithmi.
pro-		*AL* 18	Compl. Arith.	874472749
port.	{	*NA + AL* 28		144715803
		Conus *PCQ*	27<u>1433605</u>	<u>143366362</u>
		Segm. Sphaeidis *PCQ*	42<u>2230053</u>	1162554914

		Logarithms
Circulus *PL* 40<u>7150408</u>		160975488
Altitudo *AL* 18		<u>125527251</u>
Cylindrus *mPQn* 732<u>8707343</u>		286502739
3. Logarithmus auferendus		<u>047712125</u>
Conus *APQ* $^1/_3$ Cylindri 244<u>2902447</u>		238790614

			Logarithms
pro-		*CL* 2	Compl. Arith.	969897001
port.	{	*CN + CL* 12		107918125
		Cone *PAQ*	244<u>2902447</u>	<u>238790614</u>
		Segm. of sphaer. *PAQ*	1465<u>7414684</u>	1316605740

Segmentum PCQ	42<u>2230053</u>
Segmentum PAQ	1465<u>7414684</u>
Tota Sphaerois	1507<u>9644737</u>

		Logarithmi
Diameter BD 12		107918125
Quadratum BD		215836249
Differentia **B** auferenda		<u>010491012</u>
Circulus BD	1130<u>973355</u>	205345237
Axis AC 20		<u>130102999</u>
Cylindrus	2261<u>946710</u>	335448236
Differentia rationis sesquialtera		<u>017609126</u>
Sphaeroeides	1507<u>9644737</u>	317839110

Sin fuerit *BD* axis sphaeroeidis Lata, & transeat planum secans per rectam *OP*: erit *OP* partium16 , &Circulus cuius Diameter *OP* 2010619298297472; S*B*, 24; *SD*, 96. Cylindrus *rOPK* 48254863159. Conus *OBP* 16084954386 .

[p.74.]

		Logarithmi.
Diametri *OP* 16		120411998
Quadratum *OP* 256		240823996
Differentia **B** auferenda		010491012
Circulus *OP*	2010619_2983_	230332985
Altitudo *BS* 24	48254_863159_	038021124
Cylindrus *rKPO*		268354109
3.Logarithmus auferendus		047712125
Conus *OBP*	16084954386	220641984

			Logarithmi.
pro- port.	SD 96	compl. arith.	801772877
	ND + SD 156		219312460
	Conus *OBP*	271_433605_	220641984
	Segmentum *OBP*	2613_8030877_	1241727321

Circulo *OP*	2010_6192983_	230332985
Altitudo *SD* 96		098227123
Cylindrus *OgfP*	1930_19452637_	328560108
3.Logarithmus auferendus		047712125
Conus *ODP*	6433_9817546_	280847983

			Logarithmi.
proportiones.			
	BS 24	Compl. Arith.	861978876
	BS + BN 84		192427929
	Conus *ODP*	6433_9817546_	280847983
	Segmentum *OADCP*	2251_8936141_	1335254788

Segmentum *OADCP*	2251_8936141_
Segmentum OBP	2613_8030877_
Tota Sphaerois	2513_2741229_

		Logarithmi
Diameter *AC* 20		13010299956
Cubus Diametri 8000		39030899870
Differentia **D** auferenda		02810013777
Sphaera	41887_902047864_	36220886093

			Logarithmi
Proportiones.			
	AC 20	Compl. Arith.	86989700044
	BD 12		10791812460
	Sphaera Diametri *AC*	41887_902078644_	36220886093
	Sphaera *BADC*	25132_741228718_	134002398597

Atque ad hunc modum si Sphaeroeides unico secatur plano, perpendiculari ad axem, poterimus utriusque segmenti soliditatem invenire.

Superest Segment quod superficie Sphaeroeidis, & duobus planis perpendicularibus Axi & aequidistantibus a centro comprehenditur. cuiusmodi est Dolium nostrum, cuius capacitatem metiri poterimus, secundum ea quae superius dicta sunt ex Archimede. sic Pantometria lib.3.cap.10. quem librum vir clarissimus T.D. lingua vernacula eruditissime conscripsit. sic Errardus Barleduc, lib.3.cap.10. sic Clavius Geom. Practicae lib.5.cap.10.

Ut esto Dolium *BPQDRO* cuius altitudo sit *ML* partium 21, crassitudo autem media sit *BD*, 14. Diameter basis vel crassitudo extrema sit *PQ* 9$^1/_3$. imprimis quaerenda est longitudo integrae sphaeroid *AC* , quam sit inveniemus: *PQ* 9$^1/_3$ auferatur *BD* 14, semissis reliqui 4$^2/_3$ erit *BS* 2$^1/_3$. vero erit *SD* 11$^1/_3$, & *SV* media proportionalis inter *BS* & *SD* , erit ℓ.27$^1/_9$, per13.p.6.lib. Eucl. vel

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52174919477. sunt autem *SV, SP: NX* (vel *NB*), *NC* proportionales. erit igitur NC ℓ.19845 vel 1487228257 & LC 3587228357. Circulus Diametri *PQ* 6841690667819 . Cylindrus *PbdQ* 24542706089, Conus *PCQ* 81809020297. Duo segmenta *PCQ*, *OAR* 25736283611, quae ablata e Sphaeroeid, relinquunt Dolium *BPQDRO*. Est autem Circulus Diametri *BD* 15393804400259, qui ductus in rectam *AC* 28174456514 facit 433712061456, cylindrum *EFGH* & totam Sphaeroeidem 2891413743.e qua si demantur duo segmenta 2573628361 restabit Dolium 2643050907.

pro- ⎧ AL   24_587228257_
port. ⎨ AL + AN   38_674456514_
      ⎩ Conus   81_809020297_
        Segm.   12_868141805_

    Eandem Dolij capacitatem multo facilius assequemur, ad hunc modum. Quaerantur circuli DB crassitudinis mediae14, & basis *PQ* 9$^{1}/_{3}$. hos circulos multiplicet altitudo *ML* 21: facti erunt Cylinderi *rKfg* & *OPQR*. sumatur deinde horum Cylindrorum differentia, huius differentiae $^{1}/_{3}$ ablata e Cylindro maiore, vel $^{2}/_{3}$ adiectae Cylindro minori dabunt soliditatem Dolij.

BD Diameter 14	Logarithmi.
Quadratum 196	229225607
Differentia **B** auferenda	010491012
Circulus BD 1539_380400_	218734595
PQ 9$^{1}/_{3}$	
Quadratum PQ	19400735532
Differentia **B** auferenda	01049101186
Circulus PQ 68_416906678_	18351634346
Cylindrus ma. 3232_698840_	
Cylindrus mi. 1436_755040_	
Differentia 1795_943800_	
$^{1}/_{3}$ Differentiae 598_647933_	
Dolium 2634_050907_	

    Vel sumatur Circulus *PQ* , & Elleipsis cuius diametri aequantur summae rectarum *BD*, *PQ* 23$^{1}/_{3}$, & duabus tertijs differentiae earundem 3$^{1}/_{9}$:horum summam multiplicet altitudo data ML 21: factus aequabitur Dolio.

Circulus PQ 68_416906678_	*Logarithmi.*
Summa BD, PQ 23$^{1}/_{3}$	13679767853
$^{2}/_{3}$ Differentiae BD, PQ 3$^{1}/_{9}$	04929155219
[Circulus BD 1539_380400_	218734595]
Complem. Differentiae **B**	98950898814
Elleipsis 570_140889_	19400735532
Circ. + Elleipsi 1254_309956_	01049101186
Dolium 2634_050907_	

    Hoc modo inventa est eadem soliditas quae antea. Cuius rei ratio haec est. Differentia Circulorum mediae crassitudinis BD, & extremae *PQ*, aequatur Elleipssi, cuius Diametri aequantur summae & differentiae Diametorum *BD* & *PQ*, ut ostendi, 2.sect.huius capitis. Ista vero Circulorum concentricorum differentia,est basis Cylindri Concavi, cuius pars tertia sita est inter superficies Cylindri exterioris & Sphaeroeidis, reliquae duae tertiae sunt intra Sphaeroeidem. Elleipssis idcirco quae circulo minori adijcitur valet tantum duas tertias illius differentiae Circulorum *BD, PQ* (sumebatur enim non integra Diametrorum differentia, sed tantum $^{2}/_{3}$ eiusdem 3$^{1}/_{9}$: quod si totam differentiam & tantum $^{2}/_{3}$ summae sumpsissem, eadem Elleipsis evenisset) quae una cum Circulo

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minore multiplicata per datam altitudinem 21, facit eandem quae prius inventa est soliditatem Dolij.
    Esto Dolium *PCQRAO*, cuius altitudo *OR* 7_2_ sit minor crassitudine AC 20, & sit OP 16. Quaeritur huius Dolij capacitas. Summa Diametrorum AC + OP 36, differentia 4.

Quadratum OP 256	2408239965	
Differentia **B**	0104910119	
Circulus OP 201_06192983_	2303329846	
$^{2}/_{3}$ summae Diametrorum 24	1380211242	
Differentia Diametrorum 4	0602059991	
Compl. Difference **B**	9895089881	
Elleipsis 753_982235_	11877361114	
Circulus 201_0619298_		
basis 2764_601533_		
Altitudo 7_2_		

Dolium 1990<u>513104</u>

Atque his modis metiri poterimus magnitudines harum figurarum, Elleipsis & Sphaeroeidis.

5   Si datae Elleipsi Circulum aequalem construere velimus, quaerenda est media proportionalis inter Diametros Elleipsis: Circulos cuius Diameter aequatur mediae, erit aequalis datae Elleipsi. Est enim eadem ratio Circuli ad Elleipsim, quae est Quadratum ad Oblongum. Si Sphaeroeidi Sphaeram describere velimus; inveniendae sunt duae mediae continue proportionales, inter altitudinem Sphaeroeidis & Diametrum crassitudinis. Sphaera, cuius diameter aequabitur illi mediae, quae est crassitudinis diametro propior; erit aequalis datae Sphaeroeidi. nam ita se habet Cubus ad Parallelepipedum quadratae basis, ut Sphaera ad Sphaeroeidem eiusdem cui Parallelepipedo altitudinis. cum Parallelepipedum sit ad Sphaeroeidem aequealtam, ut Quadrata basis ad $^2/_3$ Circuli.