§25.1. *Synopsis: Chapter Twenty -Five.*

In this chapter, Briggs demonstrates the power of logarithms in providing rapid solutions to some common arithmetical problems, such as: the area and circumference of a circle of given diameter; both the surface area and volume of a given sphere, according to Archimedes. The use of the symbol π had not yet been adopted, but the most recent evaluation by van Ceulen of the ratio of the circumference to diameter of the circle is used by Briggs to produce four differences, used in the formulae:
Difference *A* ($\equiv \log \pi$), to find the circumference from a given diameter D, or vice versa;
Difference *B* ($\equiv \log \pi/4$), to find the area of the circle in conjunction with D^2, being a negative logarithm;
Difference *C* ($\equiv \log \pi$), to find the area of the sphere with D^2, as difference *A*; and
Difference *D* ($\equiv \log \pi/6$), to find the area of the sphere with D^3, another negative logarithm.
A number of arithmetical problems are then addressed as examples.

§25.2. ***Chapter Twenty Five.*** [p.66.]

For a given diameter, to find the area and circumference of a circle; & for a sphere, the surface area and volume. And from whatever of these are given, to find the rest.

The ratio of the circumference to the diameter of a circle is, following Archimedes, less than three and one seventh; but more than three and ten over seventy one. Ptolemy, from Book Six of *Syntaxis Mathematica*, by taking the mean ratio [of Archimedes' upper and lower bounds], set the circumference to the diameter to be as 3:8:30 to 1 [Note: Sexagesimal notation is used here: 3:8:30 is equivalent to $3 + 8/60 + 30/60^2 = 3.14167$: See Heath, *A History of Greek Mathematics*, Vol. I, p.33. Dover]. On the other hand, the ratio has been established by Ludolph van Ceulen [1540 -1610: See Gullberg, *Mathematics....,* p.92. Norton], to be as 3<u>14159265358979</u> to 1. Which I have ascertained to accept as nearest the truth. And so that the reader can judge these ratios better, all of them are brought together to give a consensus, closely following each other to the same end. In this way.

Let the diameter be			<u>10000000000</u>
The circumference will be following	Archimedes	less than 22 to 7	31428571428
		more than 223 to 71	31408450704
	Ptolemy 3:8:30 to 1		31416666
	Ludolph van Ceulen		314159265359

[Table 25-1]

We have taken the very definition of logarithms from Ch.1: *If the differences of logarithms are equal, the numbers corresponding to the logarithms are in proportion.* The logarithms of the diameter & the circumference therefore are taken: likewise the squares of the diameter; both of the circle & of the spherical surface. Likewise the cube of the diameter for the sphere: together with their differences of logarithms. These differences added to the logarithm of the given number give the logarithm of any proportional number required[1].

		Logarithms	
Diameter	10	1000000000000000	
Circumference	314159265358979	149714987269102	
		049714987269102	Difference ***A***

[Table 25-2]

The area of the circle is equal to the rectangle taken from the half–diameter and the half circumference, by Prop.1 Archimedes, on the *Measurement of the Circle*; or from the diameter & a quarter of the circumference [neither of these is quite what Archimedes states: see T. Heath: *The Works of Archimedes*, p. 91.Dover]. Therefore if the logarithm of four is taken from the sum of the logarithms of the diameter and the circumference, there is left the logarithm of the area of the circle. For the product of the whole diameter, by the whole perimeter, is four times the area of the circle. And if the logarithm of the divisor is taken from the logarithm of the product or dividend the logarithm of the quotient is left, by Axiom 3, Ch.2.

[p.67.]

	Logarithms
Diameter 10	100000000000000
Circumference 314159	149714987269102
The Log of 4 taken away	060205999132796
Area 7853981633974483	189508988136306

The square of the diameter forms a ratio with the area found :

	Logarithms	
Square of the Diameter 100	200000000000000	
Area of circle 7853981633974483	189508988136306	
	010491011863694	Difference ***B***

[Table 25-3]

Here it is noted [2], *the square of the diameter to the area of the circle as 14 to 11 gives rise to a larger ratio* [by Archimedes]. *For the ratio of 14 to 11 is as 100 to 7857142857. But the area of the* *circle is always less, namely, 785398. It happens for this reason: the ratio of the circumference to the diameter is less than 3 $^1/_7$.*

Then the ratio of the square of the diameter with the surface (area) of the sphere is considered. But the surface of the sphere is four times the area of the great circle, by Prop.30 Archimedes, *On the Sphere and Cylinder*, or equal to the rectangle described by the diameter and the circumference.

	Logarithms	
Square of the diameter 100	200000000000000	
Surface (area) of the sphere 314159265358979	249714987269102	
	049714987269102	Difference ***C***

[Table 25-4]

*The difference **C** is the same as the difference **A** first found. Because the square of the diameter is to the area of the sphere: as the same diameter is to the circumference of the great circle. With the square of the diameter and the* [area of the] *sphere produced by multiplication of the same diameter by itself, and by the circumference.* [*i.e.* the ratio corresponding to difference *A* is circumference: diameter, while the new ratio is diameter × circumference : diameter squared. Note that Briggs means *b/a*, when he says: 'The ratio of *a* to *b*'].

Finally, the ratio of the cube of the diameter is considered together with the volume of the sphere. The volume of the sphere is to the circumscribed cylinder, by the ratio two on three: Archimedes, following Prop. 31, Book 1, *On the Sphere and Cylinder*. Therefore, when the volume of this cylinder is found, by multiplication of the area of the circle by its own diameter; then with the same circular base area, the product with two thirds of the given diameter, gives the volume of the sphere.

	Logarithms	
Area of Circle - - - - 785398163397483	189508988136306	
$^2/_3$ of diameter - - - - - - - - - - - $6^2/_3$	082390874094432	
Sphere - - - - - - - - 523598775598322	2718998622307238	
Diameter of cube 1000 - - - - - - - - - - -	300000000000000	
	028100137759262	Difference ***D***

[Table 25-5]

As before, the ratio of the square of the diameter to the area of the circle is more than 14 to 11: thus the ratio of the cube to the sphere, by the same reason, is more than 21 to 11 [the ratio used by Archimedes].

Difference of logarithms	Diameter & circumference	049714987269102	***A***
	Square & circle	010491011863694	***B***
	Square and sphere	049714987269102	***C***
	Cube & sphere	028100137759262	***D***

[Table 25-6]

With these differences of logarithms found, *A, B, C, D*, we can, for any given diameter, find the circumference, the area of the circle, the surface of the sphere, & the volume of the sphere and vice-versa. When these differences are to be added, when they are to be taken away, the nature of the calculation itself will show, by what is given. All of which is illustrated with one example. Let the number 22 be given:

	Logarithms	
22 - - - - -	134242268	
Diff. ***A***	049714987 [log of π *(add/ subtracted)*]	
Sum	183957255	Circumference
Difference	084527281	Diameter

Let the 22 given be the diameter; the sum 183957255 is the logarithm of the circumference, 69<u>11503838</u>
But if 22 is the circumference; the remainder 084527281 is the logarithm of the diameter, 70<u>028175</u>.

[Table 25-7A]

[p.68.]

If the area of the circle is sought of which the diameter is 22, the area is 380<u>13272</u>.

[*Note that the Difference **B**, log(π/4), is a negative number*].

	Logarithms	
22 Diameter	134242268	
Square of the diameter	268484536	
Difference ***B***	010491012	
Final amount left	257993524	Area 380<u>13272</u>

[B]

If 22 is the area of the circle, the diameter is 5<u>2925676</u>.

	Logarithms	
22 Area of circle	134242268	
Difference ***B***	010491012	
Sum	144733280	Square of the diameter
	072366640	Diameter 5<u>2925676</u>.

[C]

If 22 is the square of the diameter, the area of the circle is 17<u>278760</u>

	Logarithms	
22 square of diameter	134242268	
Difference ***B***	010491012	
Difference	123751256	Area of circle 17<u>278760</u>

[D]

If 22 is the diameter of the circle, the surface area of the sphere is 1520<u>53083</u>.
[Difference ***C*** is log(π)].

	Logarithms	
22 diameter	134242268	
Square of diameter	268484536	
Difference ***C***	049714987	
Total	318199523	Area of sphere 152053083 .

[E]

The area of the sphere is four times the area of the great circle, which was found previously to be 38013272.

If the diameter is 22, the volume of the sphere is 55752799.
[*Note that the difference D, the log(π/6), is also a negative number*].

	Logarithms	
Cube of diameter	402726804	
Difference ***D***	0281001378	
Total	374626666	Vol. of sphere 55752799 .

[F]

If 22 is the volume of the sphere, the diameter is 347649296.

	Logarithms	
Vol. of sphere 22	134242268	
Difference ***D***	028100138	
	162342406	cube of diameter
Total	054114135	diameter 347649296

[G]

Also, the same can be found otherwise. For, *if from the logarithm of the area of the circle & doubled the diameter, the logarithm of 3 is taken away, the remainder is the logarithm of the volume of the sphere.*

	Logarithms	
Double the diameter 44	164345268	
Area of circle 38013272	257993524	
Total	422338792	double volume of cylinder
Three	047712126	
Remaining	374626666	Volume of sphere 55752797

[H]

These things are true because: The product of the area of the circle by the diameter doubled is double the volume of that cylinder, to which the sphere can be inscribed. Therefore the sphere is a third part of the cylinder itself duplicated. & therefore by taking away the logarithm of three, the remainder is the logarithm of a third of the volume of the sphere.

If the logarithm doubled of the half – diameter is added to the Difference ***A***; the total is the logarithm of the area of the circle: [Table 25-6I]

	Logarithms	
Semi-diameter 11	1041392685	
Log. of Semi-diameter doubled	2082785370	Square of the radius 121
Difference ***A***	0497149872	
	2579935242	Circle (of area) 38013272

The square of the radius is to the area of the circle, as the radius to the semi-circumference.

Therefore the difference ***A*** (and the same when it is used, from the definition of logarithms, if the ratio is the same, which is of the diameter to the circumference) added to the logarithm of the square of the radius gives the logarithm of the area of the circle[3].

§25.3. *Notes on Chapter Twenty Five.*

1 Note here the absence of π from the computations: the formal identification of the ratio of the circumference to the diameter by π was due, according to Gullberg, to William Jones in his *Synopsis palmariorum matheseos*, in 1706. Briggs developes a ratio in the form of a difference of logarithms for each circumstance which he investigates.

2 For choosing $^{22}/_7$ as an approximation for π gives $D^2/\pi r^2$ =14/11, as proposed by Archimedes as an approximation.

3 What every schoolboy and schoolgirl now knows as πr^2.

§25.4. **Caput XXV.** [p.66.]

Data Diametro, invenire Circuli Peripheriam & Aream; & Sphaerae, Surficiem & Soliditatem. Et data harum quaelibet, invenire quamlibet reliquarum.

Ratio Pheripheriae ad Diametrum est secundem Archimedem minor quam tripla sesquiseptima; maior autem quam tripla superpartiens decem septuagesimas primas. Ptolemeus mediam secutus rationem, sexto libro Mathematicae syntaxis, ponit Pheripheriam ad Diametrum esse, ut 3:8:30 ad 1. At Ludolphus van Culen , ut 3<u>14159265358979</u> ad 1. quem ego proxime accedere ad veritatem comperi. Atque ut Lector melius possit de hisce rationibus iudicare, eas omnes redigendas censui ad eundem terminum consequentem. ad hunc modum.

Sit Diameter -			<u>10000000000</u>
Pheripheria erit secundum	Archimedem	minor quam 22 ad 7 - - - -	31428571428
		maior quam 223 ad 71 - - - -	31408450704
	Ptolemaeum 3:8:30 to 1	- - - - - - - - - - - -	31416666
	Ludolphum van Culen	- - - - - - - - - - - - - -	314159265359

Accepimus ex ipsa Logarithmorum definitione cap.1. *Si Logarithmorum differentiae sint aequales: numeri Logarithmis respondentes sunt proportiones.* Summantur igitur *L*ogarithmi Diametri & Pheripheriae: item quadrati e diametro; & Circuli & Superficiei Sphaericae. Cubi item & Sphaerae: una cum eorum differentijs. Hae differentiae, additae Logarithmo dati numeri, dabunt Logarithmum numeri cuiuscunque proportionalis quaesiti.

		Logarithmi.	
Diameter	10	1000000000000000	
Pheripheria	31<u>4159265358979</u>	<u>149714987269102</u>	
		049714987269102	Differentia ***A***

Area circuli, aequatur rectangulo comprehenso a semidiametro & semipheripheria per 1.pr.Archim. de dimensione circuli. vel e diametro & quadrante Pheripheriae. idcirco si Logarithmus quaternarij auferatur e Logarithmis Diametri & Pheripheriae, restabit Logarithmus Areae circularis. Est enim factus a tota Diametro, in totam Pheripheriam, quadruplus Areae circularis. Et si auferatur Logarithmus divisoris, e Logarithmo facti vel dividendi, restabit Logarithmus quoti. per 3.ax.2.cap.

[p.67.]

	Logarithmi
Diameter 10	100000000000000
Pheripheriae $31\underline{4159}$	149714987269102
4 Logarithmus auferendus	$\underline{060205999132796}$
Areae $78\underline{53981633974483}$	189508988136306

Cum Area inventa conferatur Quadratum Diametri:

	Logarithmi.	
Quadratum Diametri 100	200000000000000	
Area circuli $78\underline{53981633974483}$	$\underline{189508988136306}$	
	010491011863694	Differentia ***B***

Ubi notandum, Quadratum Diametri maiorem obtinere rationem ad Aream circuli, quam 14 ad 11. nam ratio 14 ad 11 est uy 100 ad $78\underline{57142857}$. *At circulus est aliquanto minor. nempe* $78\underline{5398}$. *Quod ideo contingit, quia ratio Pheripheriae ad diametrum, minor est quam tripla sesquiseptima.*

Deinde conferatur Quadratum diametri cum superficie Sphaeriea. Est autem superficies sphaerica quadrupla maximi circuli, per 30.pr. Archimedis, de Sphaera & Cylindro. vel aequalis rectangulo comprehenso a *D*iametro & Pheripheria.

	Logarithmi.	
Quadratum diametri 100	200000000000000	
Superficies sphaerica $314\underline{159265358979}$	$\underline{249714987269102}$	
	049714987269102	Differentia ***C***

*Differentia **C** est eadem cum differentia **A** prius inventa. Quia quadratum Diametri est ad superficiem Sphaericam: ut ipsa Diameter Pheripheriam. cum Quadratum & Sphaericum, fiant ex multiplicatione eiusdem Diametri, in seipsam & in Pheripheriam,*

Postremo conferatur Cubus Diametri cum Sphaera.

Sphaera est ad Cylindrum circumscriptum, in ratione subsequialtera. Archim. consect.pr.31.lib.1 de Sphaera & Cyl. Idcirco, cum fiat hic cylindrus, ex multiplicatione circuli in suam Diametrum; eadem basis circularis, ducta in duas tertias Diametri, dabit soliditatem Sphaerae.

		Logarithmi.	
Circulus	- - - - $78\underline{5398163397483}$	189508988136306	
$^2/_3$ Diametri	$6^2/_3$	$\underline{082390874094432}$	
Sphaere	- - - - - - $52\underline{3598775598322}$	2718998622307238	
Cubus Diametri	1000 - - - - - --- - --	$\underline{300000000000000}$	
		028100137759262	Differenctia ***D***

Ut antea, ratio Quadrati ad Circulum erat maior quam 14 ad 11: sic ratio Cubi ad Sphaeram, eandem ob causam, maior est quam 21 ad 11.

Differentia Logarithmorum	Diametri & Pheripheriae	049714987269102	***A***
	Quadrati & Circuli	010491011863694	***B***
	Quadrati & Sphaerici	049714987269102	***C***
	Cubi & Sphaerae	028100137759262	***D***

Inventis his Logarithmorum differentijs *A, B, C, D*, poterimus pro data quacunque Diametro invenire Pheripheriam Aream circuli, superficiem sphaericam, & Sphaeram. & contra. Quando hae differentiae sunt addendae, quando auferendae, rei ipsius datae natura ostendet. Quae omnia unicum exemplum illustrabit. Esto datus numerus 22 :

	Logarithmi.	
22 - - - - -	134242268	
Diff. ***A***	049714987	
Totus	183957255	Pheripheriae
Reliquum	084527281	Diametri

Si datus 22 sit Diameter; erit Totus 183957255 Logarithmus Pheripheriae, 6911503838
Sin 22 sit peripheria; Reliquus 084527281 erit Logarithmus Diametri, 70028175 .

[p.68.]

Si quaeratur Area Circuli cuius Diameter est 22, erit Area 38013272.

	Logarithmi.	
22 Diameter	134242268	
Quadratum Diametri	268484536	
Differentia ***B***	010491012	
Reliquus	257993524	Areae 38013272

Si 22 sit Area circuli, erit Diameter 52925676.

	Logarithmi.	
22 Area of circuli	134242268	
Differentia ***B***	010491012	
totus	144733280	Quadrati e Diametro
	072366640	Diametri 52925676.

Si 22 sit Quadratum Diametri, erit Area circuli 17278760

	Logarithmi.	
22 Quadratum diameter	134242268	
Differentia ***B***	010491012	
Reliquus	123751256	Area circuli 17278760

Si 22 sit Diameter circuli erit sphaericum 152053083.

	Logarithmi.	
22 Diameter	134242268	
Quadratus Diam.	268484536	
Differentia ***C***	049714987	
Totus	318199523	Sphaerici 152053083 .

[E]

Sphaericum est quadruplum maximi circuli, qui antea inventus fuit 38013272.
22 Diametererit Sphaera 55752799.

	Logarithmi.	
Cubus Diametri	402726804	
Differentia ***D***	0281001378	
Reliquus	374626666	Sphaerae 55752799 .

[F]

Si 22sit Sphaere erit Diameter 347649296.

	Logarithmi.	
Sphaera 22	134242268	
Differentia ***D***	028100138	
	162342406	cubi e Diametro
Totus	054114135	diametri 347649296

poterunt etiam eadem inveniri aliter. ut *si a Logarithmis circuli & Diametri duplicatae, auferatur Logarithmus Ternarij, reliquus erit Logarithmus Sphaera.*

	Logarithms	
Diameter duplicata 44	164345268	
Circulus 380$\underline{13272}$	257993524	
Totus	422338792	Cylindri duplicati
Ternarius	047712126	
Reliquus	374626666	Sphaerae 5575$\underline{2797}$

Huius rei haec est causa. Factus a Circulo in Diametrum duplicatam, est duplus Cylindri illius, cui Sphaere inscribi poterit. Est igitur Sphaera pars tertia Cylindri istius duplicati. & idcirco ablato Logarithmo ternarij, restabit Logarithmus trientis: id est, Sphaerae.

Si Logarithmo semidiametri duplicato, addatur Differentia ***A***; totus erit Logarithmus Circuli.

[p. 69.]

	Logarithmi.	
Semidiameter 11	1041392685	
Logar.Semidiametri duplicatus	2082785370	Quadrati e radio 121
Differentia ***A***	0497149872	
	2579935242	Circuli 380$\underline{13272}$

Quadratum Radij est ad Circulum, ut Radius ad semipheripheriam. Idcirco differentia ***A*** (quae eadem ubique servatur, ex Logarithmorum definitione, si ratio sit eadem, quae est Diametri ad Pheripheriam) adiecta Logarithmo Quadrati e Radio, dabit Logarithmum Circuli.