§21.1. ***Synopsis: Chapter Twenty One.***

In modern terms, the first part of the theorem demonstrated numerically amounts to: Given any triangle ABC, for any given point R within the triangle, to find a corresponding point S outside the triangle, (or vice versa) such that the ratios in continued proportion of the distances of R and S to the vertices are constant: i.e. the ratios RA:RB:RC and SA:SB:SC are equal to some fixed ratio. Briggs chooses the ratio to be 2 : 3 : 4 for his example: this being another instance of the use of logarithms to render a large amount of arithmetic into a do-able task. Prior to this , two lemmas are discussed: the first considers sums and differences of ratios, while the second is concerned with inverse points. The latter part of the theorem is concerned with finding the lengths of the sides of a right-angled triangle, given the ratio of their lengths, the area, and the length of the hypotenuse.

The whole chapter is an interesting *tour de force* of numerical calculations in two and three dimensions, in the years before the introduction of Cartesian co-ordinates.

§21.2. ***Twenty One.*** [p.57.]

For a given triangle to describe the arc, of which some point is to be distant from the vertices of the triangle, according to any possible given ratio. And given the base of the triangle, together with the area and the ratio of the legs; to find the legs[1].

So that this proposition is shown more conveniently, there are two Lemmas that I must explain first.

First Lemma

If any numbers are in continued proportion, then the sums and differences of the same numbers, either nearby, or equally far away are in continued proportion with the same ratio, as you see here[2].

					1 A						
				3 B		*2 C*					*4^{th} difference*
			9 D		*6 E*		*4 F*				*3^{rd} difference*
		27 G		*18 H*		*12 O*		*8 K*			*2^{nd} difference*
	81 L		*54 M*		*36 N*		*24 P*		*16 Q*		*1^{st} difference*
243 R		*162 S*		*108 T*		*72 V*		*48 X*		*32 Y*	

Continued proportions with the differences of the same [proportions]

Continued proportions with the sums of the same [proportions]

243 Y		*162 X*		*108 V*		*72 T*		*48 S*		*32 R*	
	405 Q		*270 P*		*180 N*		*120 M*		*80 L*		*1^{st} sum*
		675 K		*450 O*		*300 H*		*200 G*			*2^{nd} sum*
			1125 F		*750 E*		*500 D*				*3^{rd} sum*
				1875 C		*1250 B*					*4^{th} sum*
					3125 A						

[Table 21-1].

On both parts of this table, any numbers that are similarly placed, are in proportion. As B, F : D, O ; A, H : F, V; & which keep the same distance on the same line, are in continued proportion, as B, H, T ; D, O, Q.

This is all shown in Prop. 19, Book 5, Euclid, & by this axiom in general: If numbers in continued proportion are added or taken away from proportionals, the sums or differences are also in proportion.

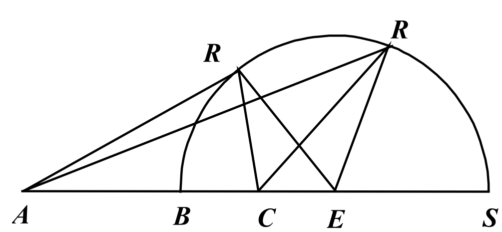
A Consequence.

Because of this, it is possible, with two given differences of third order proportionals, to find the proportionals themselves. For let H O K be three unknown proportionals, and let E, 6; F, 4 be the

[p.58.] given differences of the same [*i.e.* E = 6, etc]: taking C, 2 the given difference, they become C, F, K or C, E, H, in continued proportion, by Prop.11, Book 6, Euclid. They are the K or H noted in the table, & K + F, or H - E are the mean O. [The reader will note how to navigate the top half of the table by adding differences, and by subtracting the same in the bottom half.]

Second Lemma[3].

If three lines EC, EB, EA are in continued proportion, & with centre E and radius EB the semi-circle BRS is described, & from any point R on the periphery the straight lines RC, RA are drawn: [then] AB, BC : AR, RC are in proportion. For EA, ER, EC are proportionals from the Proposition and Definition 15, Book 1, Euclid. Therefore, the triangles AER, REC have equal angles by Prop. 6, Book 6, Euclid, or the same by Prop. 4, Book 6, Euclid. And the sides are in proportion: AE, AR : ER, RC, & alternatively, AE, ER : AR, RC. So they are [i.e. AR, RC], by the preceding



[Figure 21-1]

Lemma, as AE to ER or EB, thus for the difference of AB to BC; AB, BC : AR, RC are therefore proportionals. See Eutocius, Apollonius' problems in the commentary preface of the First Book: *Apollonius of Perga, On Conic Sections*.

If from any point H within the circumference BRS, or beyond; there are drawn two straight lines to the points A, C; HC is within the smaller arc, [if] *outside then* [within] *the greater arc for the given ratio.*[This has relevance for the second diagram, where H is either the point R, or the point S].

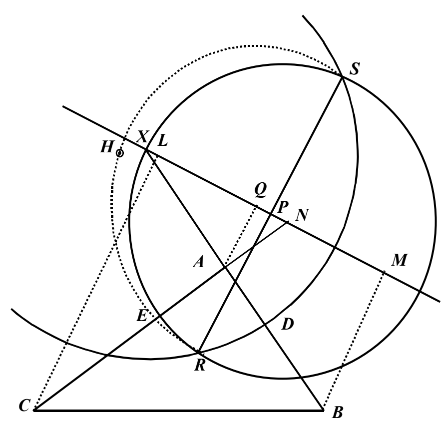
A Consequence.

With the diameter BS fixed [Fig. 21-1], the semi-circle BRS revolved, with this motion describing a spherical surface by Def. 14, Book 11, *Euclid*. There is no point on this surface [i.e. outside the plane of the figure], as none have been drawn from any point in the semi-circle BRS: & therefore if from any point M on this surface, two lines are drawn to the points A & C, AM, MC will be in proportion to AB, BC. [Just as AR, RC in the original plane are similarly proportional]

[*The main proposition is now demonstrated*].

With these lemmas shown, let there be given the triangle ABC of sides 6, 8, 10, & the two points R, S are sought so located in order that RA, RB, RC: and likewise SA, SB, SC are proportional to the numbers 2, 3, 4. The side AB is cut at any point D, according to the ratio of the neighbouring lines RA, RB [i.e. in the ratio 2:3]; & by the consequence of the First Lemma, making AX, DX, BX in continued proportion: & with centre X, radius XD, the arc RDS is drawn. [The distance AX can be calculated, and so AD found.] Then let AC similarly be cut in E (or BC), as AE, EC is proportional to the lines RA, RC, & the point N is found, & AN, EN, CN becoming continued proportionals. With centre N, radius NE the arc ERS is drawn, intersecting the first arc drawn in R and S. I assert: the lines RA, RB,

It is impossible for the arc ERS not to intersect the other arc, as these arcs preserve the ratio of the lines RA, RB, RC which must be satisfied. By departing from intercepting, the lines RA, RB, RC can have any proportions of numbers we wish, such as 1, 4, 2.



[Figure 21-2]

RC. Likewise the lines SA, SB, SC are proportional to the numbers 2, 3, 4. For RA, RC are proportional to AE, EC by the Second Lemma, that is, constructed from the numbers in the ratio 2:4. By the same reason, RA, RB are proportional to the lines AD, DB. That is, with the numbers in the ratio 2:3.

		$1^{1}/_{5}$					$2^{2}/_{3}$					$1^{1}/_{7}$		
	$3^{3}/_{5}$		$2^{2}/_{5}$			$5^{1}/_{3}$		$2^{2}/_{3}$			$5^{5}/_{7}$		$4^{2}/_{7}$	
$10^{4}/_{5}$	**DB**	$7^{1}/_{5}$	**AD**	$4^{4}/_{5}$	$10^{2}/_{3}$	**EC**	$5^{1}/_{3}$	**AE**	$2^{2}/_{3}$	$22^{6}/_{7}$	**GC**	$17^{1}/_{7}$	**BG**	$12^{6}/_{7}$
XB		**XD**		**XA**	**NC**		**NE**		**NA**	**CO**		**GO**		**BO**

[Table 21-2].

For if the line CB is cut in the point G, as BG to GC as 3 to 4, CB is continued to O, as OB $12^{6}/_{7}$,
[p.59.] OG $17^{1}/_{7}$, OC $22^{6}/_{7}$ are continued proportionals[4]. And with centre O, radius OG, the third arc is described, it also cuts the same points R and S. And these are the three circular arcs with centres X, N, O, on the same straight line.

And, if with the diameters staying fixed, these two circular arcs are turned in a circle [about the common line XNG]: generating spherical surfaces, of which by this motion, the common intersection is an arc, of which the diameter is RS: and this plane (containing the arc and RS) is perpendicular to the plane of the triangle ABC. And if from any point of this arc, three lines are drawn to the points A, B, C, these lines are proportional to the numbers 2, 3, 4, by the logical consequences to the Second Lemma. See the final problem in the appendix, at the end of François Viète's *Apollonius Galli*.

If anyone perhaps wants the theorem illustrated with numbers, beyond the geometrical demonstration : Given the lines AB, 6; AC, 8; CB, 10. and also given the ratios of the lines RA, RB, RC, 2, 3, 4, then AB is cut in the point D in the ratio of one and a half

If CAB had been obtuse angled, two perpendiculars might have been drawn from the points C, X to the sides AB, AC continued, which themselves will be proportional to AC, AX. With the segment of the line AC being found between the perpendicular and A, the distance of the point N to the perpendicular will have been noted, & the square of the line XN found[5].

[*i. e.* DB:AD :: 3 : 2], *as AD is* $2^{2}/_{5}$, *DB* $3^{3}/_{5}$. *Also AC is cut in the point E, in the ratio of two* [*i.e.* AE:EC :: 1: 2], *so AE is* $2^{2}/_{3}$, *EC* $5^{1}/_{3}$. *The segments are in continued proportion with the lengths XA, XD, XB: these proportionals are found by the consequences of the First Lemma XA,* $4^{4}/_{5}$; *XD,* $7^{1}/_{5}$; *XB,* $10^{4}/_{5}$. *In the same manner, NA,* $2^{2}/_{3}$; *NE,* $5^{1}/_{3}$; *NC,* $10^{2}/_{3}$. *In a nearby place the line XN is found; which, (as in this triangle the angles at A are right) is* $\ell.30^{34}/_{225}$, *by Prop. 47, Book 1, Euclid. That is 5491002742 approximately. Afterwards the perpendiculars AQ, BM, CL, RP are drawn, and these lengths are to be found, together with the segments NP, NQ, NL, NM. All of this you can see expressed in numbers*[5].

AQ	233108607
NQ	129504782
NP	61514772
PQ	67990010
RP	529773892
BM	524494365
CL	932434428
PM, *PL*	456504354

Sq. RA	92632933	RA	30435659
Sq. RB	208424099	RB	45653489
Sq. RC	370531732	RC	60871318
Sq. SA	586612350	SA	76590623
Sq. SB	1319877788	SB	114885934
Sq. SC	1346449400	SC	153181246

[Table 21-3].

And if with centre P, radius RP, the arc RHS is described: and we may imagine throwing up a right angle from the plane XCBM; as the point H projects perpendicularly above the point P: & you draw three lines to the point below H to the points A, B, and C: of these the squares will be (as before) proportional to the squares 4, 9,16.

Sq. AH	33962264	AH	58277152
Sq. BH	76415094	BH	87415728
Sq. CH	135849056	CH	116554304

[Table 21-4].

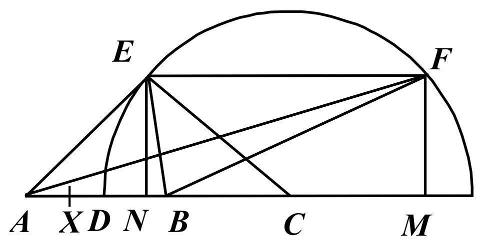
[*The last part of the Proposition: And given the base of the triangle, together with the area and the ratio of the legs; to find the legs*].

For, if with the given base AB 10 of the triangle, together with the area $\ell.1575$, & with the ratio of the sides 3 to 2, the sides can be found [Fig. 21-3]. The given base AB of 10 parts, is cut in the given ratio by the point D, as AD 6, is to DB 4. Of these segments, the difference DX is 2, thus itself has the same ratio to DB 4 the small segment, as the same BD 4 to BC 8, the smallest of three continued proportions, by the consequences of the First Lemma. Therefore CB, 8; CD, 12; CA, 18, are continued proportions, & with centre C, radius CD, the semicircle DEF is described. Then if the area of the given triangle ℓ.1575 is divided by 5, half of the given base, the quotient ℓ.63

[p.60.] is the altitude of the triangle. Which being found [proceed by] : drawing the perpendicular EN to the base and with the same equal altitude; and through the point E is drawn EF parallel to the base AB, intersecting the semicircle drawn before in the points E and F, and the points EA, EB joined, likewise FA, FB. I assert that the ratios: AE to EB, & AF to FB, as AD to DB, by the Second Lemma [see Figure 21-3]: that is from this construction, as 3 to 2. And the triangles AEB, AFB are equal in area between themselves, because of the equality of the same base. And the area of either is ℓ.1575. The truth of all of this is shown with numbers.

		2 XD		
	6 AD		4 DB	
18 CA		12 CD		8 CB

proportions			*Logarithms*
XD	2	Comp. Arith.	9,69897,001
DB	4		0,60205,999
DB	4		0,60205,999
BC	8	- - - - - - - - - - -	(1)0,90308,999



[Figure 21-3]

EN, FM: altitude ℓ. 63

Sq. EN	63				
Sq. EC	144	CM.9			
Sq. NC or MC	81	NC.9	Sq. EN.63	AB.10	Sq.AN.81
		CB.8	Sq.NB.1	NB.1	Sq.NE.63
		NB.1	Sq.EB.64 EB.8	AN.9	Sq.AF.144 AE.12

CM.9	Sq. BM.289	AC.18	Sq.AM.729
CB.8	Sq.FM.63	CM 9	Sq. FM. 63
BM.17	Sq.BF.352	AM.27	Sq. AF.792

proport- -ions		proport- -ions		proport- -ions	
	2		BD.4		Sq. BD. 16
	3		AD.6		Sq. AD. 36
	BD.4		EB.8		Sq. FB. 352
	AD.6		EA.12		Sq. FA. 792
			FB.ℓ.352		
			FA.ℓ.792		

[Table 21-5].

And if the altitude EN ℓ.63, is multiplied by ℓ.25, half the base AB, the product ℓ.1575 is the area.

§21.3. *Notes on Chapter 21.*

1 The work of Apollonius has been a continued source of ideas for geometers over the centuries. A brief account of this theorem and its extensions can be found in the *CRC Handbook of Mathematics*, p. 57, by Eric Weisstein, Chapman & Hall/CRC, 1999.

See also, perhaps, '*Excursion in Geometry*', by Stanley G. Ogilvey, Dover, pp 14 - 23. Briggs' motivation is, of course, to show how logarithms can be used to validate the geometrical reasoning for a particular triangle. As Briggs mentions, he has drawn his inspiration from Viète's work: *Appolonius Gallus.* This can be accessed in the original by the modern reader in the reprint of Schooten's collection of Viète's works: *Opera Mathematica*, p.345, problem V, (Georg Olms Verlag, 1970).

2 Briggs' example amounts to the finite sequence, for $1 \leq n \leq 6$:

$a_{n+1} = \frac{3}{2} a_n$; where $a_1 = 32,\ a_2 = 48,\ a_3 = 72,$ etc. In general, by induction, we have:

$a_n = a_1(p/q)^{n-1}$, where $n, p,$ and q are positive integers, and $a_{n+1}/a_n = p/q$, the proportionality ratio. (In the present case, $p/q = {}^3/_2$.) In general, the differences of succeeding orders are derived sequences related to the original according to:

1st order : $b_{n+1} = a_{n+1} - a_n = \left(\frac{p}{q} - 1\right) a_n$;

2nd order : $c_{n+2} = b_{n+2} - b_{n+1} = \left(\frac{p}{q} - 1\right)^2 a_n \ (= \frac{1}{4} a_n)$;

3rd order : $d_{n+3} = c_{n+3} - c_{n+2} = \left(\frac{p}{q} - 1\right)^3 a_n \ (= \frac{1}{8} a_n)$;

similarly 4th order : $e_{n+4} = \left(\frac{p}{q} - 1\right)^4 a_n \ (= \frac{1}{16} a_n)$,

and 5th order : $f_{n+5} = \left(\frac{p}{q} - 1\right)^5 a_n \ (= \frac{1}{32} a_n = 1$, when $n = 1)$.

We may write terms in the equivalent form: $b_{n+1} = (1 - q/p)a_{n+1}$, etc. The diagonal relations follow from these two forms, relating differing orders. These difference sequences of course, have the same proportionality between their terms as the original sequence: the sums of succeeding orders follow a similar pattern.

3 Algebraically, *if* $a : b = c : d$, then $(a - c):(b - d) = a : b$, where $c < a$ and $d < b$.

See Heath, Vol. 2 , *Euclid, The Thirteen Books of The Elements*, Dover (1954), p. 174, for a discussion of geometric proportionality. We note that the second lemma is related to finding the inverse point of a given point with respect to a circle with a given radius. (Note that ΔECR is similar to ΔERB, as EC/ER = ER/EA, and the angle at E is common). A related discussion is to be found on pages 198 - 200, *ibid*.

The two positions of R shown by Briggs illustrate that the result is true for acute or obtuse angles at this stage: in the next section two extended sides of the triangle ABC and two circular arcs localise the points R and S.

It may be of interest to some readers to recall the extreme usefulness of this lemma when applied to physics, as a means of calculating in a simple manner from pure geometry, the gravitational force acting on a small mass outside a uniform shell of matter, as introduced by William Thompson (see, e.g. *Treatise on Natural Philosophy*, *Part II*, Kelvin & Tait. Cambridge, (1890), page 13 - 14) in his method of images. This is in sharp contrast to the original method used by Newton in the *Principia*, Prop. LXXI. The three dimensional aspect of the problem is thus highly suggestive of things to come: this writer has often wondered why Newton did not go down this road in the development of his gravitational theory, rather than using the horrendous method he adopted. As far as one can tell, Newton seems to have ignored Briggs' work, to his own detriment, and also robbing Briggs of a more prominent place in the evolution of mathematics at this time.

4 Recall that BC = 10, and note that neither O, G, nor the arc are shown on Figure 21-2, presumably to avoid confusion. To find these lengths: let BO = x, then we solve $(x+30/7)^2 = x(x+10)$, according to Lemma 2, to give x = BO = $12\,^6/_7$, etc.

5 The extra perpendicular lines enable the lengths (RA, RB, RC), and (SA, SB, SC) to be evaluated from Pythagoras' Theorem. E.g. $RA^2 = PQ^2 + (RP - AQ)^2$. To find these, use similar triangles: e.g. AQ = AN.AX/XN.

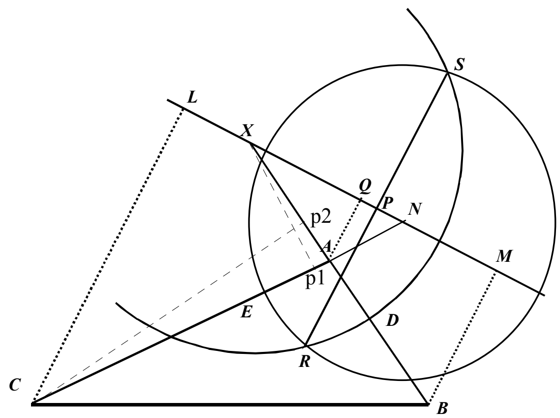


Figure 21-3.

6 In the case where the angle CAB is obtuse, further construction lines are necessary. Thus, if the foot of the perpendicular from X to AC is p1, and similarly C to AB p2, then the distance AP1 can be found from trigonometry, as the angle BAC is known, and similarly Xp1: hence Xp1N is a known right angled triangle, and hence XN can be found, etc.

§15.1. **Caput XXI.** [p.56.]

Dato triangulo peripheriam describere, cuius quodlibet punctum distet ab angulis dati trianguli, secundum datas quascunque possibiles rationes. Et data basi trianguli, una cum Area & ratione crurum; invenire crura.

Ut haec commodius ostendantur, sunt mihi Lemmata duo prius demonstranda.

Lemma primum.

Si quotlibet numeri sint continue proportiones: tam summae quam differentia eorundem, proximae, vel aeque remotae: sunt continue proportionales in eadem ratione, ut hic vides

					1 A						
				3 B		*2 C*					*differentiae quartae*
			9 D		*6 E*		*4 F*				*differentiae tertiae*
		27 G		*18 H*		*12 O*		*8 K*			*differentiae secundae*
	81 L		*54 M*		*36 N*		*24 P*		*16 Q*		*differentiae primae*
243 R		*162 S*		*108 T*		*72 V*		*48 X*		*32 Y*	

Continue proportionales cum differenijs eorundem

Continue proportionales cum aggregatis eorundem

243 Y		*162 X*		*108 V*		*72 T*		*48 S*		*32 R*	
	405 Q		*270 P*		*180 N*		*120 M*		*80 L*		*summae primae*
		675 K		*450 O*		*300 H*		*200 G*			*summae secundae*
			1125 F		*750 E*		*500 D*				*summae tertiae*
				1875 C		*1250 B*					*summae quartae*
					3125 A						

In his utrisque, Quicunque sunt similiter siti, sunt proportionales.
Ut B, F : D, O ; A, H : F, V; & qui in eadem recta eandem servant distantium, sunt continue proportionales, ut B, H, T ; D, O, Q.

Haec omnia demonstrantur per 19.p.5.lib.Eucl., & per generale illud axioma. Si proportionalia proportionalibus addantur vel auferantur; tota vel reliqua sunt proportionalia.

Consectarium.

Hinc licebat, Datis duabus differentijs, trium continue proportionalium; ipsos proportionales invenire, ut sunto tres continue proportionales ignoti, H O K & sint datae E, 6; F, 4 differentiae eorundem: sumatur C, 2 differentia datarum,

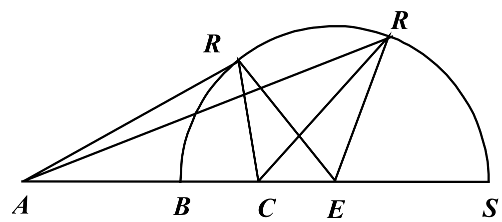
[p.58.]

& fiant C, F, K vel C, E, H, continue proportionales, per 11.p.6.lib.Eucl. erunt K vel H noti, & K + F, or H - E erit O medius.

2 *Secunda Lemma.*

Si tres rectae EC, EB, EA sint continue proportionales, & centro E radio EB describatur peripheria BRS, & a puncto quolibet R in peripheria ducantur rectae RC, RA : erunt AB, BC : AR, RC proportionales. Sunt enim EA, ER, EC proportionales ex thesi & 15.d.1.lib.Eucl. & idcirco, triangula AER, REC aequiangula per 6.p.6.lib.Eucl. & similia per 4.p.6.lib.Eucl. eruntque latera AE, AR : ER, RC, proportionalia: & alterne, AE, ER : AR, RC. Sunt autem per praecedens Lemma, ut AE ad ER vel EB, sic differentia AB ad BC. Sunt igitur AB, BC : AR, RC proportionales. Vide Eutocium in commentario in praefationem libri primi Apol. Pergae de conicis sectionibus.

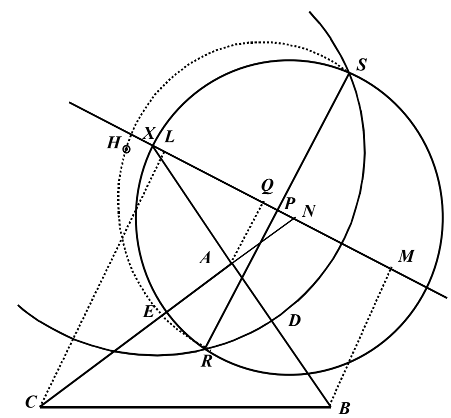
Si ab puncto H intra peripheriam BRS, vel extra;ducantur dua rectae ad puncta A, C; erit HC intra peripheriam minor, extra vero maior quam pro data ratione.



Consectarium.

Si diametro BS manente, circumagatur semipheripheria BRS, motu hoc describetur superficies sphaerica per 14.d.11.lib.Eucl. In hac superficie nullum est punctum, quod non fuit descriptum ab aliquo puncto in semiperipheria BRS: & idcirco si ab aliquo puncto M in hac superficie sphaerica, ducantur duae rectae ad puncto AC, erunt AM, MC proportionales rectis AB, BC.

His demonstratis, esto datum triangulum ABC laterum 6.8.10, & quaerantur duo puncta R, S eo situ posita, ut RA, RB, RC: item SA, SB, SC sint proportionales numeris 2, 3, 4. Secatur latus quodlibet AB in puncto D, secundum rationem conterminarium rectarium RA, RB , & per Consectarium primi Lemmatis, fiant AX, DX, BX continue proportiones: & centro X, radio XD, describatur peripheria RDS. Deinde secatur AC (vel BC) similiter in E, ut AE, EC sint proportionales rectis RA, RC, & inveniatur punctum N, & AN, EN, CN continue proportionales. & centro N radio NE describatur peripheria ERS, intersecans peripheriam prius descriptam in punctis R S. aio: rectas RA, RB, RC; item SA, SB, SC esse proportionales numeris 2, 3, 4. Sunt enim RA, RC per secundum Lemma proportionales rectis AE, EC , id est ex fabrica numeris 2:4. eadem de causa, RA, RB erunt proportionales rectis AD, DB. Id est numeris 2:3.



Si peripheria ERS non intersecet reliquum, impossibile est ut eae serventur rationes rectarum RA, RB, RC qui imperantur. quod evenire deprehendemus, si rectae RA, RB, RC habere velimus proportionales numeris 1, 4, 2.

		$1\frac{1}{5}$		
	$3\frac{3}{5}$		$2\frac{2}{5}$	
$10\frac{4}{5}$	**DB**	$7\frac{1}{5}$	**AD**	$4\frac{4}{5}$
XB		**XD**		**XA**

		$2\frac{2}{3}$		
	$5\frac{1}{3}$		$2\frac{2}{3}$	
$10\frac{2}{3}$	**EC**	$5\frac{1}{3}$	**AE**	$2\frac{2}{3}$
NC		**NE**		**NA**

		$1\frac{1}{7}$		
	$5\frac{5}{7}$		$4\frac{2}{7}$	
$22\frac{6}{7}$	**GC**	$17\frac{1}{7}$	**BG**	$12\frac{6}{7}$
CO		**GO**		**BO**

Quod si recta CB recta in puncto G, ut BG sit ad GC, ut 3 ad 4, continuanda erit CB usque O, ut OB $12\frac{6}{7}$, OG $17\frac{1}{7}$, OC $22\frac{6}{7}$ sint continue proportionales. BC = 10, Et centro O, radio OG,

[p.59.]

descripta tertia peripheria, transibit teiam per puncta eadem RS. Suntque harum trium peripheririum centra X, N, O, in eadem recta.

Et, si manentibus diametris, harum semiperipheriarum duae circumagantur : superficierum sphaericarum eo motu descriptarum communis intersectio erit peripheria, cuius diameter erit recta RS: eiusque planum erit perpendiculare plano trianguli ABC. Et si ab huius peripheriae puncto quolibet, ducantur tres rectae ad puncta A B C, erunt hae rectae proportionales numeris 2.3.4, per secundi Lemmatis consectarium. Vide ultimum problema, in appendicula Francisci Vietae ad finem Apollonij Galli.

Si quis forte praeter demonstrationem Geometricam, harum rerum illustrationem per numeros desideret. dantur AB, 6; AC, 8; CB, 10.dantur etiam rationes rectarium RA, RB, RC, 2.3.4 deinde secatur AB in puncto D in ratio sesquialtera, ut AD sit $2^2/_5$, DB $3^3/_5$. secatur etiam AC in puncto E, in ratione dupla,ut AE sit $2^2/_3$, EC $5^1/_3$. quae segmenta cum sint differentiae trium rectarum XA, XD, XB continue proportionalium : ipsae proportionales per consectarium primi Lemmatis inveniuntur XA, $4^4/_5$; XD, $7^1/_5$; XB, $10^4/_5$. Eodem modo, NA, $2^2/_3$; NE, $5^1/_3$; NC, $10^2/_3$. proximo in loco quaerenda est recta XN; quae, (cum in hoc triangulo, anguli ad A sint recti) erit $\ell.30^{34}/_{225}$, per 47.p.1.lib.Eucl. id est 5$\underline{491002742}$ proxime. postea ducenda sunt perpendiculares AQ, BM, CL, RP & earum longitudines investigandae, una cum segmentis NP, NQ, NL, NM. Quae omnia hic in numeris expressa vides.

Si CAB fuisses obliquus, duae perpendiculares ducende fuissent a punctis C, X in latero AB, AC continuata, quae erunt proportionales ipsis AC, AX. & inventa segmento rectae AC inter perpendicularem & A,nota erit distantia puncti N a perpendiculari, & invenietur quadratum rectae XN .

AQ	2$\underline{33108607}$
NQ	1$\underline{29504782}$
NP	$\underline{61514772}$
PQ	$\underline{67990010}$
RP	5$\underline{29773892}$
BM	5$\underline{24494365}$
CL	9$\underline{32434428}$
PM, *PL* }	4$\underline{56504354}$

Qu. RA	9$\underline{2632933}$	RA	3$\underline{0435659}$
Qu. RB	20$\underline{8424099}$	RB	4$\underline{5653489}$
Qu. RC	37$\underline{0531732}$	RC	6$\underline{0871318}$
Qu. SA	58$\underline{6612350}$	SA	7$\underline{6590623}$
Qu. SB	131$\underline{9877788}$	SB	11$\underline{4885934}$
Qu. SC	134$\underline{6449400}$	SC	15$\underline{3181246}$

Et si centro P, radio RP, describatur peripheria RHS: & cogitemus eius planum ad rectos angulos erectum subiecto plano XCBM;ut punctum H perpendiculariter immineat puncto P: & rectas tres ductas a puncto sublimi Had puncta A B C: earum quadrata erunt (ut priora) proportionalia quadratis 4.9.16.

Qu. AH	33$\underline{962264}$	AH	5$\underline{8277152}$
Qu. BH	76$\underline{415094}$	BH	8$\underline{7415728}$
Qu. CH	135$\underline{849056}$	CH	11$\underline{6554304}$

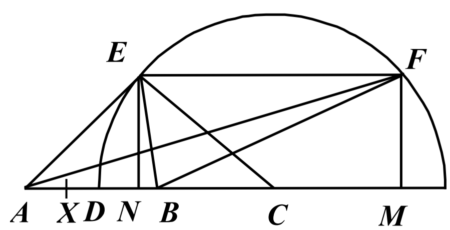
Quod, si data Basi trianguli AB 10, una cum area ℓ.1575, & ratione laterum 3 ad 2, quaerantur latera. Data basis AB partium10, secetur data ratione in puncto D, ut sit AD 6, DB 4. horum segmentorum, differentia DX 2, ita se habet ad DB 4 minus segmentum, ut idem BD 4 ad BC 8 minimum e tribus continue proportionalibus: per Consectarium primi Lemmatis. Erunt igitur CB, 8; CD, 12; CA, 18, continue proportionales, & centro C, radio CD, describenda est peripheria DEF. Deinde si area trianguli data ℓ.1575 dividatur per 5, semissem datae basis, quotus ℓ.63; erit altitudo trianguli: qua inventa, ducatur EN perpendicularis basi & eidem altitudini aequalis; & per punctum E ducatur EF, parallela basi AB, intersecans peripheriam prius descriptam in punctis E F,

[p.60]

& connectantur puncta EA, EB , item FA, FB. aio AE esse ad EB, & AF ad FB, ut AD ad DB, per Secundum Lemma: id est ex fabrica, ut 3 ad 2. eruntque triangula AEB, AFB aequalia inter se, quia aequealta in eadem basi. estque area alterutrius ℓ.1575. Horum omnium veritas erit in numeris manifesta.

	2 XD	
6 AD		4 DB
18 CA	12 CD	8 CB

proportiones			*Logarithmi*
XD	2	Comp. Arith.	9,69897,001
DB	4		0,60205,999
DB	4		0,60205,999
BC	8	- - - - - - - - - - -	(1)0,90308,999



EN, FM altitudo ℓ. 63

Qu. EN	63				
Qu.. EC	144	CM.9			
Qu.. NC	81	NC.9	Qu.. EN.63	AB.10	Qu..AN.81
vel MC		CB.8	Qu..NB.1	NB.1	Qu..NE.63
		NB.1	Qu..EB.64 EB.8	AN.9	Qu..AF.144 AE.12

CM.9	Qu.. BM.289	AC.18	Qu..AM.729
CB.8	Qu..FM.63	CM 9	Qu.. FM. 63
BM.17	Qu..BF.352	AM.27	Qu.. AF.792

proport- -ions	2 3 BD.4 AD.6	proport- -ions	BD.4 AD.6 EB.8 EA.12 FB.ℓ.352 FA.ℓ.792	proport- -ions	Sq. BD. 16 Sq. AD. 36 Sq. FB. 352 Sq. FA. 792

Et si EN altitudo ℓ.63, multiplicet ℓ.25, semissem basis AB, factus ℓ.1575 erit area.