§20.1. ***Synopsis: Chapter 20.***

In which it is shown how to describe a triangle equal (in area) and isoperimetric to another given triangle, by making use of properties of the ellipse.

§20.2. ***Chapter Twenty.*** [p.55.]

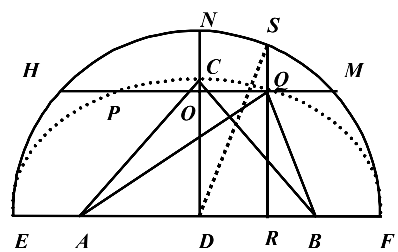
Above a given base, to describe a triangle equal (in area) and isoperimetric to another given triangle.

Let the given base, above which the triangle is to be described, be smaller than the largest side of the given triangle, but larger than the least.

Let the sides of the given triangle be 26, 25½, 12½. The base of which is 26, it has altitude 12, perimeter 64. For this other triangle is to be constructed with equal area and isoperimetric on the given base AB 24, of

			Logarithms
proportions	24	complement. arith.	861978876
	26		141497335
	12		107918125
	13		1|111394336

[Table 20-1]



[Figure 20-1].

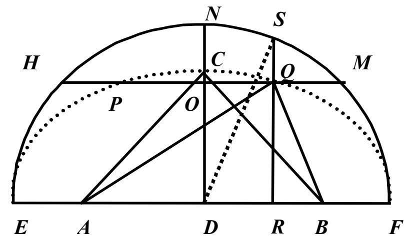
which the altitude is 13, because with triangles of equal area, the altitudes are in reciprocal proportion to the bases, by Prop. 15, Book 6, Euclid.

Then with the perpendicular ND drawn to the base AB, and DO to become equal to the altitude
[p.56.] found 13, & through the point O, HOM is drawn parallel with the base AB. Onto this parallel [line] is placed the vertex of the triangle sought. But since this triangle is isoperimetric with the given triangle: by taking away the base AB 24 from the given perimeter 64, there remains 40, the sum of the remaining lines. Let the isosceles triangle ABC be made, of which the leg on either side AC, BC is 20, and by bisecting the base in D, with centre D, radius DE 20, the arc of the circle ENF is described, & the elliptical arc ECF, intersecting the line HOM in the points P & Q. I assert for the lines AP, BP, or AQ, BQ: the triangle APB or AQB to be equal and isoperimetric with the given triangle. For with the ellipse, if AC, BC, DE, DF, are equal: for two lines from the same point on the periphery of the ellipse, taken together & drawn to A and B, are equal to the longer diameter EF, by Prop. 52, Book 3, Apollonius of Perga *On Conic Sections.*

But if we want to know the lengths of the lines AQ, BQ numerically; from the point Q the perpendicular QR is drawn, cutting the arc of the circle in the point S, & DC, DN : RQ, RS are in proportion; & by taking RS squared, from DS squared, there remains the line DR squared, by Prop. 47, Book 1, Euclid. And AR is the sum of the lines AD, DR; & BR is the difference of the same.

proportions		*Logarithms*
Square DC	256 compl. arith.	759176003
Square DN	400	260205999
Square RQ	169	222788670
Square RS	$264^1/_{16}$	1|242170672

[Table 20-2]



[Figure 20-2]

The sum of the squares RS & DR is equal to the square of DS, (or DN); the square of DR is therefore $135^{15}/_{16}$ [i.e. $DR^2 = DS^2 - RS^2$], & AR is 12 + ℓ.$135^{15}/_{16}{}^2$, and certainly BR is 12 – ℓ.$135^{11}/_{16}$. Which if multiplied by themselves, the squares are $279\underline{9375}$ + ℓ.78300, & $279\underline{9375}$ – ℓ.78300, to each of which if 169 is added, the square of the line RQ, the results are the square of the line AQ, $448\underline{9375}$ + ℓ.78300, & the square of the line BQ, $448\underline{9375}$ – ℓ.78300. If these square binomials [not in the modern sense] are reduced to absolute numbers, they are the square of AQ, $728\underline{7588715}$, and the square of BQ $169\underline{1161285}$. All of which particulars you see put together here.

The square root of the square 78300 is	$279\underline{8213715}$	
	$279\underline{9375}$	
Square of the right line RA	$559\underline{7588715}$ total	
Square of the right line RQ	169	
Square of the right line AQ	$728\underline{7588715}$	AQ. $26\underline{995534}$
The square root of the square 78300 is	$279\underline{8213715}$	
	$279\underline{9375}$	
Square of the right line BR	$\underline{1161285}$ remainder	
Square of the right line RQ	169	
Square of the right line BQ	$169\underline{1161285}$	BQ. $13\underline{004465}$
	AB.	24
	BQ.	$13\underline{004465}$
	AQ.	$26\underline{995534}$
The perimeter of the triangle ABQ will therefore be $63\underline{999999}$		

[Table 20-3]

& the area 156, is equal to the base AB 24 by half of the altitude RQ,6 . Which were to be shown.

§20.3. *Notes on Chapter 20.*

[1] A well-known property of the ellipse: for if DR is set equal to the ordinate x in standard form, and RQ the co-ordinate y, then $y^2 = b^2(1 - x^2/a^2)$; where a is the semi-major axis DF, and b the semi-minor axis DC. The co-ordinate y' from the circle, DN, is given by $y'^2 = a^2(1 - x^2/a^2)$; hence, $(y/y')^2 = b^2/a^2$.

[2] As reported elsewhere, 'ℓ' is short for 'latus', meaning here 'square root', which would now be written √.

§20.4. **Caput XX.** [p.55.]

Super datam Basim triangulum describere aequale & isoperimetrum dato triangulo.

Sit data basis, super quam describendum est triangulum, minor, maximo latere dati trianguli, maior autem minimo.

Sunto latera dati trianguli 26, 25½, 12½.cuius basis sit 26, erit altitudo 12, perimeter 64. Huic aliud aequale & isoperimetrum constituendum est super datam basim AB 24, cuius altitudo debet esse 13. quia in triangulis aequalibus, altitudines sunt basibus reciproce proportionales, per 15.p.6.lib.Eucl.

Deinde ducatur ND perpendicularis basi AB, & fiat DO aequalis

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proport.	24	compl. arith.	861978876
		26	141497335
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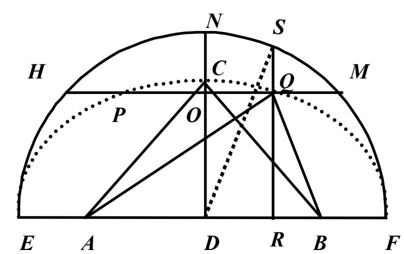
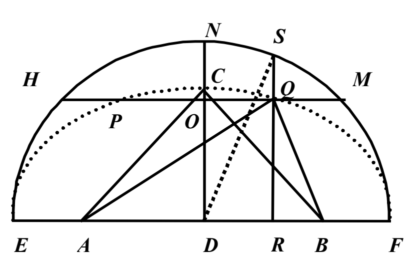
		13	1|111394336



inventae altitudini13, & per punctum O ducatur HOM parallela basi AB. in hac parallela [p.56.] situs erit vertex trianguli quaesiti. Cum autem hoc triangulum debeat esse isoperimetrum dato triangulo:ablata AB 24, e data perimetro 64, restabunt 40, summa reliquorum laterum. fiat ABC triangulum isosceles, cuius utrumque crus AC, BC sit 20, & biseca basi in D, centro D, radio DE 20, describatur peripheria circulis ENF, & peripheria Elliptica ECF, intersecans rectam HOM in punctis PQ. Aio ductis rectis AP, BP, vel AQ, BQ: triangula APB vel AQB esse aequalia & isoperimetra dato triangulo. Nam in Elleipsi, si AC, BC, DE, DF, aequentur: rectae duae ab oedem peripheriae Elleipticae puncto, ductae ad A & B, simul sumptae, aequantur diametro longiori EF, per 52.p.3.lib.Apollonij Pergaei de conicis sectionibus.

Quod si in numeris scire velimus longitudines rectarum AQ, BQ; a puncto Q ducatur perpendicularis QR, secans peripheriam circuli in puncto S, & DC, DN : RQ, RS fiant proportionales; & ablato quadrato RS, e quadrato DS, restabit quadratum, rectae DR , per 47.p.1.lib.Eucl. Eritque AR aggregatum rectarum AD, DR; & BR erit differentia earundem.



proportiones			*Logarithmi.*
Quadratum DC	256	compl. arith.	759176003
Quadratum DN	400		260205999
Quadratum RQ	169		222788670
Quadratum RS	$264^1/_{16}$		1|242170672

Quadrata RS & DR, aequantur quadrato DS, (vel DN); est igitur quadratum DR $135^{15}/_{16}$, & AR erit $12 + \ell.135^{15}/_{16}{}^{2}$, BR vero erit $12 - \ell.135^{11}/_{16}$. Quae si seipas multiplicent, earum quadrata erunt $279\underline{9375} + \ell.78300$, & $279\underline{9375} - \ell.78300$, quibus si addatur 169, quadratum rectae RQ, erit quadratum rectae AQ, $448\underline{9375} + \ell.78300$, & quadratum rectae BQ, erit $448\underline{9375} - \ell.78300$. & si haec quadrata Binomia reducantur ad numeros absolutos , erit quadratum AQ, $728\underline{7588715}$, & quadratum BQ $169\underline{1161285}$. quae omnia particularius hic adiecta vides

Latus quadrati 78300 est	$279\underline{8213715}$		
	$279\underline{9375}$		
Quadratum rectae RA	$559\underline{7588715}$	total	
Quadratum rectae RQ	169		
Quadratum rectae AQ	$728\underline{7588715}$		AQ. $26\underline{995534}$
Latus quadrati 78300 est	$279\underline{8213715}$		
	$279\underline{9375}$		
Quadratum rectae BR	$\underline{1161285}$	reliquum	
Quadratum rectae RQ	169		
Quadratum rectae BQ	$169\underline{1161285}$		BQ. $13\underline{004465}$
		AB.	24
		BQ.	$13\underline{004465}$
		AQ.	$26\underline{995534}$

Erit igitur perimeter trianguli ABQ $63\underline{999999}$

& aream 156, aequabitur facto a basi AB 24 in 6, semissem altitudinis RQ. Quae erant facienda.