§15.1. *Synopsis: Chapter Fifteen.*

Briggs illustrates some of the more common uses of logarithms, such as finding the fourth proportion of three given proportions, and how to extend a set of given proportionalities in any desired way, by inserting additional terms, as in a geometric progression.

§15.2. *Chapter Fifteen.* [p.37.]

The Rule of Logarithmic proportion, or the quintessence of the Golden Rule.

The common rule of proportion shows how for three given numbers, the fourth proportional can be found through multiplication and division. This same rule is made better and easier through addition and subtraction [using logarithms].

If the logarithm of the first is taken from the sum of the second and third logarithms: the remainder is the logarithm of the fourth . For, let the given [numbers] be 3, 8, 18.

		Logarithms	
prop.	3	0,4771,21254	*A*
	8	0,9030,89987	*B*
	18	1,2552,72505	*C*
		2,1583,62492	*D*
	48	1,6812,41238	*E*

D is the sum of the means, from which *A* is taken, leaving E the logarithm of the fourth proportional, the number 48

[Table 15-1]

The truth of this proposition is demonstrated from the definition itself of Logarithms, and from the second Lemma of the first Chapter. Indeed the sum of the extremes is equal to the sum of the means, & therefore the difference of the first and second terms is the same as the difference of the third and fourth terms: and the ratio of the first to the second proportional is the same as the third to the fourth. We can find the logarithm of the same fourth proportional sought, if in place of the first logarithm the arithmetical complement of this is selected (about which, see chapter 11), and it is added to the same sum with the two given remaining proportionals. This sum is the fourth logarithm sought, if the one positioned in the first place towards the left is taken away. The reason for this being that the first logarithm ought to be taken away from the remaining logarithms: but not only have we taken nothing away, but we have added the complement of this logarithm to that remaining in addition: that is, the remainder of the logarithm of ten[1]. Thus, as the total is retained above the fourth proportional sought, and since the first logarithm and its complement is the whole number10: on that account, by taking away one from the first place which it always occupies (it is with ten being taken away), there remains the fourth logarithm sought, as you see here:

		Logarithms	
prop.	6	0,77815,125	
	15	1,17609,126	
	26	1,41497,335	
		2,59106,461	sum of means
	65	1,8129,1336	fourth proportional

prop.	6	9,22184,875	Complement.
	15	1,17609,126	
	26	1,41497,335	
	65	11[→]1,81291,336	Sum

[Table 15-2]

This shortcut was first suggested to me by my colleague and friend, the distinguished Mr. Edmund Gunter[1], Professor of Astronomy at Gresham College.

Because if two or more logarithms are taken away, and we can add the complement of any of these, taking away as many units from the total as there are complements.

But these are generally common proportions, if you remove the facility of working [with logarithms]: some proportions follow which [are evaluated] most easily through logarithms; they can hardly be performed without these. With three given numbers, if we consider any series of numbers in continued proportion which is placed between the given third and fourth proportional; and this series is made continuous on both sides: we can find any of the these of which the position has been given. I say: In a series of the continued proportions, if the ratio and the separation of two of the terms is given, together with any number you please; we can find a second term, whose separation above or below the given term has been noted.

The three given numbers are *A* 8, *B* 27, *C* 432, and the ratio of the first proportion to the fourth proportion is as *A* to *B*: and the proportional number sought is greater than *C*, between which and *C* there are 4 intermediary proportionals, or 5 intervals.

In the first place the logarithms of the numbers *A* and *B* are to be selected, and of these the difference, which is *F*. And as there are three intervals between the first [A] and the fourth [B], G taken as the third part of this difference. This fraction is the difference of the logarithms for one interval; or for whatever nearest numbers you like in the whole series.

Then the logarithm *D* of the given number *C* 432 is selected. And if we wish to have the nearest term above or below the given number, the difference G of one interval is added or taken away from the logarithm *D*. But if we seek another more removed term from the given; the difference G is to be multiplied by the ratio of the separation; then by addition to or subtraction from the logarithm of *D*: the sum or the remainder is found in the Chiliad of logarithms, and the number sought is shown in the margin. As you see here.

	Logarithms		Numbers in continued proportion
A. 8.	0,90308,99869,9194		
B. 27.	1,43136,37641,5899		
Difference	0,52827,37771,6705. *F*		$3280\,{}^{1}/_{2}$ - 9
${}^{1}/_{3}$ Difference	0,17609,12590,5568. *G*		2187 - - 8
C. 432.	2,63548,37468,1491. *D*		1458 - - 7
	0,17609,12590,5568. 1*G*		972 - - - 6
648.	2,81157,50058.7059. *D* + 1*G*	27	648 - - - 5
972.	2,98766,62649,2627. *D* + 2*G*	18	432 Given 4
	0.88045,62952,7840. 5*G*	12	288 - - - 3
$3280\,{}^{1}/_{2}$	3,51594,00420,9331. *D* + 5*G*	8	192 - - - 2
C. 432.	2,63548,37468, 1491.		128 - - - 1
	0,52827,37771,6704. 3*G*		
128.	2,10720,99696,4787. *D* - 3*G*		

[Table 15-3]

$\left.\begin{array}{l} 8 \\ 27 \\ 432 \\ 1458 \end{array}\right\}$ pro-port.

And by this method we find 648 the next above *C*, by addition of the difference *G* ; & 972 the second, & $3280\,{}^{1}/_{2}$ the fifth from the same, from the same fraction G. In the same way, by subtraction of the difference *G* three times, we find 128 the third below *C*. The truth of all of this can be established from the strength of the first definitions: which is confirmed, these numbers are in proportion the logarithms of which are equi-different. And this truly is that golden rule of numbers in proportion, which presents for us, not only the third, or fourth, or the mean proportional; but any at all from continued proportion, between the third and the fourth, or beyond; without needing us to look beyond. For from this rule, just as from the most productive spring, most of what follows emanates.

§15.3. *Notes on Chapter Fifteen.*

[1] In modern terms, instead of dividing the numerator by the first number N, we multiply by $10^{10}/N$ initially, and

finally divide by 10^{10}.

[2] Edmund Gunter (1581 - 1626) was to provide the first tables of logarithms of sines, etc., and

a prototype slide rule. Briggs was very happy to be part of this enterprise.

§15.4.

Caput XV. [p.37.]

Regula proportionum Logarithmicae, vel Aureae regulae quinta essentia.

Regula proportionum vulgaris ostendit, quomodo datis tribus numeris inveniri possit quartus proportionalis, per multiplicationem & divisionem. Idem praestabit haec regula, sed multo facilius per additionem & subductionem.
Si e summa Logarithmorum secundi & tertii auferatur Logarithmus Primi: reliquus erit Logarithmus quarti. ut sunto dati 3.8.18.

		Logarithmi	
prop.	3	0,4771,21254	*A*
	8	0,9030,89987	*B*
	18	1,2552,72505	*C*
		2,1583,62492	*D*
	48	1,6812,41238	*E*

D est summa mediorum, e qua *A* ablatus, relinquit E Logarithmum numeri 4.8

Huius propositionis veritas demonstratur, ex ipsa definitione Logarithmorum, & secundo Lemmate Capitis primi. Est enim summa extremorum aequalis summae mediorum, & idcirco eadem est differentia primi & secundi, quae est tertii & quarti: & eadem ratio primi numeri ad secundum, quae est tertii ad quartum. Eundem quarti quaesiti Logarithmum inveniemus, si loco primi Logarithmi sumatur eius complementum Arithmeticum (de quo capite 11.) & conijciatur in eandem summam cum duobus reliquis datis. Summa haec si unitatis nota in primo versus sinistram loco sita auferatur, erit Logarithmus quarti quaesiti. cuius rei causa est, quod eum primus Logarithmus debuit e reliquis auferri: nos non modo nihil abstulimus, sed insuper reliquis adiecimus eius complementum, id est reliquum Denarii. adeo ut totus, supra quartum quaesitum, contineat, & primum Logarithmum, & eiusdem complementum, id est integrum Denarium. ablata idcirco unitatis nota, quae semper primam occupabit sedem (id est ablato Denario) restabit quartus Logarithmus. ut hic vides.

		Logarithmi.	
prop.	6	0,77815,125	
	15	1,17609,126	
	26	1,41497,335	
		2,59106,461	summa mediorus
	65	1,8129,1336	quartus.

prop.	6	9,22184,875	Complement.
	15	1,17609,126	
	26	1,41497,335	
	65	11[→]1,81291,336	Summa.

Quod si duo vel plures Logarithmi sint auferendi, poterimus uniuscuiusque complementum addere, auferentes e toto tot unitates, quot fuerint complementa.

Huius compendij primus me admonuit collega & amicus meus summus M. Edm. Gunter. Astronomiae Professor in Collegio Greshamensi

Sed haec fere vulgaria sunt, si operationis facilitatem excipias: sequuntur quaedam, quae per Logarithmos facillime; sine illis, vix praestari poterunt. Datis tribus numeris, si cogitemus seriem quotlibet numerorum continue proportionalium, sitam esse, inter tertium datum & quartum proportionalem; eamque seriem utrinque continuari: poterimus quemlibet eorundem, cuius situs datus fuerit invenire. Aio. In serie continue proportionalium, si datae fuerint ratio & distantia duorum terminorum, una cum numero quolibet; poterimus alterum, cuius distantia supra vel infra datum, nota fuerit, invenire.

[p.38.]

Sint dati tres numeri A 8. B 27. C 432.& sit ratio primi proportionalium ad quartum ut A ad B: quaeritur numerus supra C, inter quem & C sint quatuor intermedii, vel quinque interevalla.

Imprimis sumendi sunt Logarithmi numerorum A, B, & eorum differentia quae sit F. & cum tria sint intervalla inter primum & quartum, sumatur G pars tertia huius differentia. haec pars, erit differentia Logarithmorum pro unico intervallo; vel pro numeris quibuslibet in tota serie proximis.

Deinde sumatur dati numeri C 432, Logarithmus D. & si proximum supra datum vel infra habere velimus; addatur vel auferatur Logarithmo D, differentia G uinius intervalli. Sin quaeramus aliquem a dato remotiorem; differentia G multiplicanda est pro distantiae ratione; deinde addenda vel auferenda Logarithmo D: totus vel reliquis, inter Chiliadum Logarithmos repertus, ostendit in margine numerum quaesitum. ut hic vides.

	Logarithmi.		Numeri continued proportionales.
A. 8.	0,90308,99869,9194		
B. 27.	1,43136,37641,5899		
Differentia	0,52827,37771,6705. *F*		$3280\,{}^{1}/_{2}$ - 9
${}^{1}/_{3}$ Differentia	0,17609,12590,5568. *G*		2187 - - 8
C. 432.	2,63548,37468,1491. *D*		1458 - - 7
	0,17609,12590,5568. 1*G*		972 - - - 6
648.	2,81157,50058.7059. *D* + 1*G*	27	648 - - - 5
972.	2,98766,62649,2627. *D* + 2*G*	18	432 Datus 4
	0.88045,62952,7840. 5*G*	12	288 - - - 3
$3280\,{}^{1}/_{2}$	3,51594,00420,9331. *D* + 5*G*	8	192 - - - 2
C. 432.	2,63548,37468, 1491.		128 - - - 1
	0,52827,37771,6704. 3*G*		
128.	2,10720,99696,4787. *D* - 3*G*		

Atque ad hunc modum invenimus per additionem differentiae G, 648 proximum supra C: & 972 secundum, & $3280\,{}^{1}/_{2}$, quintum ab eodem, ex eadem parte. Eodem modo per subductionem differentiae G triplicatae, invenimus 128 tertium infra C. Horum omnium veritas constat ex primae definitionis vi: quae eos numeros proportionales esse confirmat, quorum Logarithmi sunt aequidistantes. Atque haec est illa numerorum proportionalium regula vere aurea, quae nobis exhibet, non tertium solum, aut quartum, aut medium proportionalem; sed omnino quemlibet e continue proportionalibus, intra teretium & quartum, vel extra; sive uberrimo fonte, plurima eorum quae sequuntur dimanarunt.

$$\left.\begin{matrix}8\\27\\432\\1458\end{matrix}\right\}\ \text{pro-port.}$$