§10.1. ***Synopsis: Chapter Ten.***

In this chapter Briggs points out initially that the idea of a fraction can be extended to include whole numbers and improper fractions, which he refers to as multitudinous numbers, which can all be represented and added like conventional fractions less than one, with a numerator and denominator for a given ratio. The logarithms of all fractional numbers are then presented in a straight-forward way that needs no elaboration.

§10.2. *Chapter Ten.* [p.19.]

By this method, we can find the logarithms of all whole numbers, whether they be prime or composite. There remain fractions, which some call the minutia, which also have their logarithms. Now fractions are thus properly said to be numbers less than one, and which numbers Euclid, Def.2, Book 7, accordingly would seem to exclude from the natural ones. However, usually we can answer questions well enough to do with numbers involving fractions, which whenever added, they are to equal or there remains, a number of the multitudinous kind; and since they have a certain and obvious ratio, it is evident that these homogeneous fractions of the multitudinous kind represent a number, and therefore there can be a tallying up between the numbers. Fractions, therefore, in this context, are taken with a little more generality: for any number put in the form of fractions is described by a numerator and a denominator. Whether it shall be a whole number such as $\frac{7}{1}\ \frac{12}{4}$; or mixed, when whole number parts are placed together, as $\frac{5}{4}\ \frac{13}{5}$; or thus properly called fractions, evidently less than one, such as $\frac{3}{4}\ \frac{7}{10}$. For all these, the ratio of numerator to denominator is the same, to which one is to the given fraction, and for that reason, by Prop.20.Book.7 Euclid, they are in proportion.

	4		4		5	Denominator
pro-	3	pro-	5	pro-	13	Numerator
port.	1	port.	1	port.	1	Unity
	$\frac{3}{4}$		$\frac{5}{4}$		$\frac{13}{5}$	Fraction

[Table 10-1]

To find the Logarithm of a given fraction.

If the logarithm of the denominator is taken from the logarithm of the numerator, there is left the logarithm of the fraction.

For with four numbers in proportion, the sum of the logarithms of the means is equal to the sum of the logarithms of the extremes[1]. One mean, unity, has the logarithm zero, and it is the logarithm of the remaining mean, the numerator only, to be equal to the logarithms of the extremes: that is of the denominator, which is the first of the four proportions, and the fraction, which is the fourth proportion, as we see in this example :

		Logarithms			*Logarithms*
pro- port.	5	06989700043	pro- port.	4	06020599913
	13	11139433523		3	04471212547
	1	00000000000		1	00000000000
	$\frac{13}{5}$	04149733480		$\frac{3}{4}$	- 01249787366

[Table 10-2]

From which the improper fractions $\frac{13}{5}$ or $2\frac{3}{5}$ have an abundant logarithm[2]; but proper fractions thus called, such as $\frac{3}{4}$, since they are smaller than one, have a deficient logarithm: that is less than zero. Or the remainder of a small part from which a large part has been taken away: which is usual in the works of the Arithmeticians. As if from three is taken away five, there remains[3] —2.

All of these things are obvious enough, with these numbers which are in continued proportion with the decuplet ratio. In which, where the greater a number will be, there the logarithm has increased the more above 0; and where the lesser the fractions will be, there the logarithm will be most reduced below 0. As we examine here: where the three numbers $12\frac{5}{10}, 1\frac{25}{100}, \frac{125}{1000}$ are continued proportion, and of which the logarithms keep the same differences. For the logarithm of one is 0, which term we set down by definition, from which the positive logarithms are to increase, and those negative so begin to decrease. But the interval between *C* and *D* is much less than the interval is between *D* and *E* ; nevertheless, if the interval of the logarithm of *B* lies beyond the constituted term 0; is added to the same interval of *D* above : the total interval between *B* and *D* is equal to the interval between *D* and *E*, or between *A* and *B*. So with the negative logarithms of fractions, the characteristic changes in the same way as with positive numbers. For from one to ten, the characteristic is 0; from ten to one hundred, 1; from a hundred to a thousand, 2; and so successively, as we showed in Ch.4. In the same way with negative logarithms: from unity to a tenth part[4], 0; from a tenth to a hundredth part, 1; from a hundredth to a thousandth part, 2; etc, as we see here.

	Logarithms
1000	3,00000,00
100	2,00000,00
10	1,00000,00
1	0
$\frac{1}{10}$	–1,00000
$\frac{1}{100}$	–2,00000
$\frac{1}{1000}$	–3,00000

		Logarithms
$\frac{10000}{8}$	1250	3,09691,00
$\frac{1000}{8}$	125	2,09691,00
$\frac{100}{8}$	$12\frac{5}{10}$	1,09691,00 ***E***
$\frac{10}{8}$	$1\frac{25}{100}$	0,09691,00 ***D***
	Unity	0,00000,00 ***C***
$\frac{1}{8}$	$\frac{125}{100}$	– 0,90308,99 ***B***
$\frac{1}{80}$	$\frac{125}{1000}$	– 1,90308,99 ***A***
$\frac{1}{800}$	$\frac{125}{10000}$	–2,90308,99
$\frac{1}{8000}$	$\frac{125}{100000}$	–3,90308,99
of the same	value.	

	Logarithms
2570	3409933
257	2409933
$25\frac{7}{10}$	1409933
$2\frac{57}{100}$	0409933
$\frac{257}{1000}$	–0590067
$\frac{257}{10000}$	–1590067
$\frac{257}{100000}$	–2590067
	Logarithms
3	0,44712,1254
24	1,38021,1241
$\frac{3}{24}$	*–0,90308,9987*+
125	2,09691,00
1000	3,00000,00
$\frac{125}{1000}$	*–0,90308,9987*
1	*0,00000,0000*
8	*0,90308,9987*
$\frac{1}{8}$	*–0,90308,9987*

[Table 10-3]

But since the value of a fraction is always the same, if the ratio of the numerator to the denominator is the same, we can write the same fraction in many ways, of these however the logarithm is always the same, the differences of course of the numerator and the denominator. Which with all proportions is the same, from the definition of logarithms itself. Thus $\frac{3}{24}, \frac{1}{8}, \frac{125}{1000}$ express for us the same fraction: of which the logarithm is the same – 0,90308,9987.+

§10.3. *Notes On Chapter Ten.*

[1] Briggs writes ratios in the form *b : a :: 1* : (*b/a*); or, in modern terms, $\frac{b}{a} = \frac{1}{a/b}$; *b* and *b/a* are the extremes of the ratio, while *a* and 1 are the means.

[2] Positive numbers are called 'abundant' by Briggs, while negative numbers are termed 'deficient'.

[3] Briggs uses a very long stroke for the minus sign. This may have been to make it more obvious in his working with logarithms. As a rule, Briggs endeavours to avoid working with negative numbers.

[4] The usual '–' sign is implicit as we are considering fractions with negative logs.

§10.4. **Caput X.** [p.19.]

Ad hunc modum invenire poterimus, omnium integrorum numerorum Logarithmos, sive sint primi sive compositi. Supersunt Partes, quas aliqui minutias appellant, quae suos etiam habent Logarithmos. Sunt autem Partes proprie sic dictae numeri minores Unitate, quas licet Eucl.2.def.lib.7 excludere videatur a natura numeri; tamen, cum per Partes saepissime satis apposite respondemus quaestioni de numero; quae additae, quandoque adaequant, quandoque superant numerum multitudinis; ad eumque certam habeant & manifestam rationem: manifestum est, eas esse numero multitudinis homogeneas, & idcirco inter numeros censendas. Partes igitur, hoc in loco, paulo generalius sumantur, pro numero quolibet, qui partium formam induens, per numeratorem & denominatorem describitur: sive sit integer ut $\frac{7}{1}\,\frac{12}{4}$ sive mixtus, cum integris partes adhaeserint, ut $\frac{5}{4}\,\frac{13}{5}$ sive partes proprie sic dicte, minores scilicet Unitate, ut $\frac{3}{4}\,\frac{7}{10}$. In his omnibus, eadem est ratio Denominatoris ad Numeratorem, quae est Unitate ad datas partes. est enim factus a Numeratore in Unitate aequalis facto a Denominatore in ipsas partes. & idcirco per 20.p.7.Eucl. sunt proportionales.

pro -port.	4	pro- port.	4	pro- port.	5	Denominator.
	3		5		13	Numerator.
	1		1		1	Unitas.
	$\frac{3}{4}$		$\frac{5}{4}$		$\frac{13}{5}$	Partes.

Datarum partium Logarithmum invenire.

Si Logarithmus Denominatoris auferatur e Logarithmo Numeratoris, restabit Logarithmus Partium. Cum enim in quatuor proportionalibus, Logarithmi mediorum aequentur Logarithmis extremorum; & Unitas, mediorum alter nullum habeat Logarithmum;

[p.20.]

reliqui medii solius, id est Numeratoris solius, Logarithmus aequatur Logarithmis extremorum: id est Denominatoris, qui primus est e quatuor proportionalibus, & Partium, qui numerus est quartus. ut in his exemplis videmus.

		Logarithmi.			*Logarithmi.*
pro- port.	5	06989700043	pro- port.	4	06020599913
	13	11139433523		3	04471212547
	1	00000000000		1	00000000000
	$\frac{13}{5}$	04149733480		$\frac{3}{4}$	— 01249787366

In quibus partes impropriae $\frac{13}{5}$ vel $2\frac{3}{5}$ habent Logarithmum abundantem: at partes proprie sic dictae $\frac{3}{4}$ cum sint minores Unitate, Logarithmum habent defectivum : id est minorem 0. vel reliquam minoris a quo major sublatus fuit. quod apud Arithmeticos est usitatum. ut si e tribus auferantur quinque, restabunt -2.
Haec omnia satis erunt manifesta, in his numeris, qui sunt continue proportionales in decupla ratio. in quibus quo major erit numerus, eo altius ascendit eius Logarithmus supra 0: & quo minores erunt partes, eo magis deprimitur earum Logarithmus infra 0. ut hic cernimus, ubi tres numeri $12\frac{5}{10}, 1\frac{25}{100}, \frac{125}{1000}$ sunt continue proportionales, eorumque Logarithmi easdem servant differentias. Est enim Logarithmus Unitatis 0: quem terminum constituimus, a quo Logarithmi Abundantes crescere, Defectivi autem decrescere incipiunt. Distantia autem inter C & D multo minor est quam inter D

& E; veruntamen si distantia Logarithmi B, infra constitutum terminum 0; addatur distantiae D, supra eundem,. tota distantia inter B & D, aequibitur distantiae inter D & E, vel inter B & A. Isti autem Partium Logarithmi Defectivi, Characteristicam mutant eodem modo quo Abundantes. nam ab Unitate ad Decem, Characteristica est, 0. a Decem ad Centum, 1. a Centum ad Mille, 2.& sic deinceps, ut cap.4 ostendimus. eodem modo in Defectivis: ab Unitate ad partem Decimam, 0. a parte Decima ad Centesimam, 1. a Centesima ad Millesimam, 2. &c. ut hic videmus.

	Logarithmi.			*Logarithmi.*
1000	3,00000,00	$\frac{10000}{8}$	1250	3,09691,00
100	2,00000,00	$\frac{1000}{8}$	125	2,09691,00
10	1,00000,00	$\frac{100}{8}$	$12\frac{5}{10}$	1,09691,00 ***E***
1	0	$\frac{10}{8}$	$1\frac{25}{100}$	0,09691,00 ***D***
			Unitas	0,00000,00 ***C***
$\frac{1}{10}$	−1,00000	$\frac{1}{8}$	$\frac{125}{100}$	− 0,90308,99 ***B***
$\frac{1}{100}$	−2,00000	$\frac{1}{80}$	$\frac{125}{1000}$	− 1,90308,99 ***A***
$\frac{1}{1000}$	−3,00000	$\frac{1}{800}$	$\frac{125}{10000}$	−2,90308,99
		$\frac{1}{8000}$	$\frac{125}{100000}$	−3,90308,99
		eiusdem	valoris.	

	Logarithmi.
2570	3409933
257	2409933
$25\frac{7}{10}$	1409933
$2\frac{57}{100}$	0409933
$\frac{257}{1000}$	−0590067
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125	2,09691,00
1000	3,00000,00
$\frac{125}{1000}$	*−0,90308,9987*
1	*0,00000,0000*
8	*0,90308,9987*
$\frac{1}{8}$	*−0,90308,9987*

Cum autem Partium idem semper sit valor, si eadem fuerit ratio numeratoris ad denominatorem. poterimus easdem partes scribere pluribus modis, earum tamen Logarithmus semper erit idem: differentia scilicet Logarithmorum numeratoris & denominatoris. quae in proportionalibus omnibus est eadem,

[p.21.]

ex ipsa Logarithmorum definitione. ut $\frac{3}{24}, \frac{1}{8}, \frac{125}{1000}$ easdem partes nobis exprimunt: quarum idem est Logarithmus − 0,90308,9987. +