VII.

Concerning Optical Curves and other matters.

Some time ago while travelling around on a quite distant journey, which I had undertaken on the command of my most Serene Prince, and searching the records here and there in the archives and libraries, certain monthly issues of the Leipzig *Actis* were presented to me by a friend, from which I might discern my lack of acquaintance of new books, which might have appeared in the Republic of Literature. Therefore in examining the June issue of the year 1688 a report appeared about the *Mathematical Principles of Natural Philosophy* by the most celebrated of men Isaac Newton, which although far removed from my present thoughts, I read eagerly with great delight.

[In 1688, Leibniz was travelling around visiting friends mainly, and eventually arrived in Munich, before moving on to Vienna, on his official business of researching the connection of the Braunischweig-Lünburg dynasty with the Erste family: see *Leibniz, An Intellectual Biography,* Maria Rosa Antognazza, CUP. The review mentioned appeared in the June '88 issue of the Acta, and it was due to Christoph Pfautz, professor of Mathematics at Leipzig, the detail Leibniz gives far exceeds what could be gleaned from the review article, and would seem to indicate that he was already familiar with Newton's work at this time ; Antognazza discusses this aspect on p. 295-6 : the present paper was a hasty response by Leibniz on a matter with which he was familiar already.]

For that man belongs to a small number of those, who have enlarged the boundaries of science, because they have been able to instruct even about these series, which Nicolas Mercator of Holstein had pursued by division, but Newton had adapted and put into effect equally by a much fuller discovery from the extraction of pure roots. As for myself, as I may say in passing, for advancing the method of series, besides the transformation of irrational curves into curves with rational symmetries (for I call curves rational, the ordinates of which always can be obtained from rational numbers) an account has been thought out for given transcending curves, where indeed there is no need for the extraction [of roots]. For I assume an arbitrary series, and by handling that by the rules of

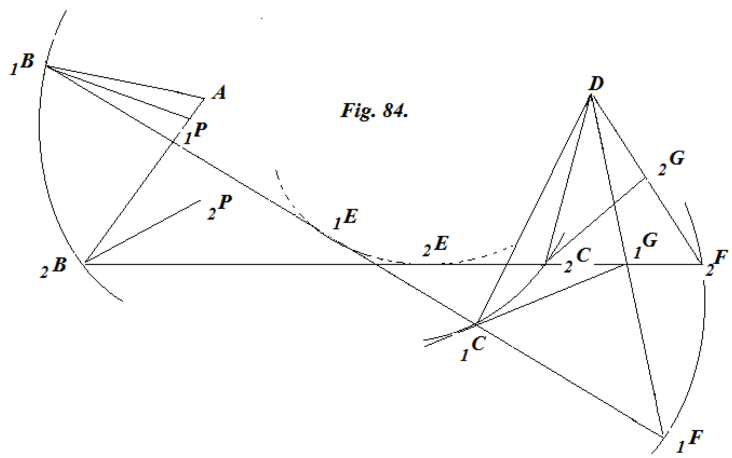


Fig. 84.

the problem, I obtain its coefficients. [Newton's Binomial Theorem applicable to square root expansions had been used in the *Principia*.] Again I hope for something outstanding from the present work of Newton, and from the account of the Acta I see in short, many new works evidently of great significance, while indeed some I have treated, but others have not been treated by me ; for besides the causes of celestial motion, as well as catoptric or dioptric curves [i.e. the shapes for mirrors and lenses], he has undertaken to explain the resistance of media. Descartes has investigated these optical curves, but concealed his results, nor were commentaries supplied; nor indeed has the matter been subjected to common analysis. I understand these after Huygens (but who has not yet published; [Huygens' *Optics* was not published until after his death]) and now these found by Newton. Even I have observed a different way to become known. And some time ago I had indeed a general method, however appropriately it gave the opportunity for the very elegant elicitations of our own Tschirnhaus to be published in the Acta, who considers whole lines as foci [*i.e.* the discovery of caustic curves : the curves followed by bundles of paraxial rays]. Which thence I have pursued, I will explain by an example, from which the rest may be understood. A shall be a point (fig. 84) and BB a given curve, reflecting the rays AB, the curve CC is sought, again reflecting the rays ABC into a common point D. This is the solution of the first action. From the given curve BB, with respect of its point A it is agreed to give a confocal curve or the curve EE [*i.e.* a curve on which all the rays lie]. Again from the two give confocal points, it is agreed possible to find another curve CC, from the one curve EE, and from the other point D, of which they are the foci which was sought. But better constructions arise, for

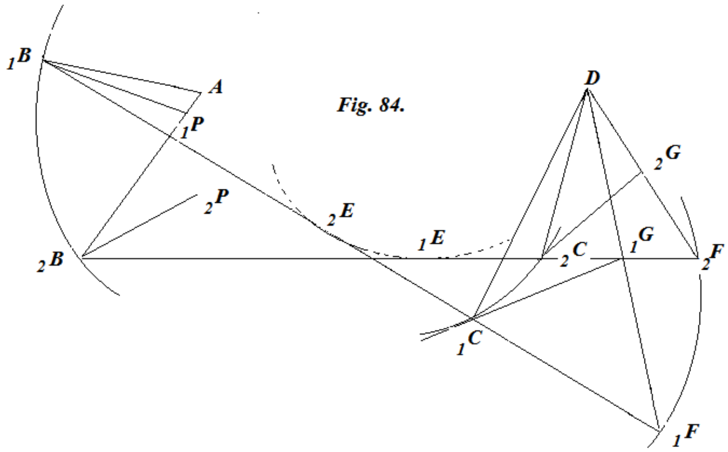
$$A_1B+{}_1B_1E+\text{arc.}{}_1E_2E = A_2B+{}_2B_2E \text{ and } D_2C+{}_2C_2E + \text{arc. } {}_2E_1E = D_1C+{}_1C_1E,$$

from which the whole $AB+BC+CD$ always is equal to a single constant right line. If a thread shall be wound around the curve EE and likewise it shall be tied to the point D, then from the evolute of the curve EE the point at the end of the string will describe the curve CC. But if the thread likewise shall be tied to the other end at the point A, the point ending the string will describe the curve BB. But from the hidden curve EE, the simplest construction of all will be produced: from the given constant right line (equal to $AB+BC+CD$) some given part AB may be removed AB, thus with the part drawn equal to the remainder BF, so that according to PB for the curve BB or its perpendicular to the tangent at B, the angle FBP may be made equal to ABP itself. The points D and F are joined, and from the mid-point G of DF, GC drawn out normally will cross BF itself at the point sought C, and indeed it is apparent, GC to be the tangent of the curve CC. Again, this figure may be rotated about the axis AD, and what we have said at the curves, also will have a place in the surfaces generated. And the same can be applied to dioptrics. I call the line EE *Acampte* [*i.e.* unbent, from the an. Greek : $\alpha\kappa\alpha\mu\pi\tau o\varsigma$: unbent or rigid], because it accepts the rays ABE without reflection or refraction. And these are given as *Aclastes* [$\alpha\kappa\lambda\alpha\sigma\tau o\varsigma$: unbroken], which same have not been refracted yet have been reflected, and they are these rays, which are described simply arising from EE, which first were considered by Huygens, although for another end. Such is the curve FF obtained, by putting $CF = CD$ (on producing to BC). If from the points A or D, or from one or the other, the line of the focus may be given, or from some point infinitely far away, where the radii become parallel, likewise they have their place in their own way. [Thus, another mirror surface may be used elsewhere, or the rays arise from infinity, in the case of the astronomical telescope.]

What I have embraced about the resistance of a medium I have pursued in a special paper, now from that time for the greater part of twelve years, and I have communicated several parts from that with Paris to the illustrious Royal Academy. Finally since also thoughts had come to me concerning the physical causes of celestial motion, I had a need to publish several completed parts from these. Indeed I had decided not to publish, until it were possible for me to bring together more carefully the laws of geometry with the latest phenomena of the astronomers; but (except that plainly I am adorned with other kinds of occupations, which scarcely allow any such hope) the work of Newton had excited me, so that I might allow any such kind to be put on record, so that more sparks of truth may be encouraged to leap out from the gathering together of the reasoning, and our shrewdness may have been of help to the cleverest of men.

VII.

DE LINEIS OPTICIS, ET ALIA.



Versanti mihi dudum in longinquo satis itinere, quod Serenissimi Principis mei jussu sus cepi, et passim monumenta in Archiviis et Bibliothecis excutienti, oblati sunt ab amico quodam Actorum Lipsiensium menses. unde jamdiu novorum librorum expers discerem, quid in Republica Literaria ageretur. Inspicienti igitur Junium anni 1688 occurit relatio de Principiis Naturae Mathematicis Viri Clarissimi lsaaci Newtoni, quam licet a praesentibus meis cogitationibus longe remotam avide et magna cum delectatione legi. Est enim vir ille ex paucorum illorum numero, qui scientiarum pomoeria protulere, quod vel solae illae series docere possunt, quas Nicolaus Mercator Holsatus per divisionem erat assecutus, sed Newtonus longe ampliore invento extractionibus radicum purarum pariter et affectarum accommodavit. A me, ut obiter hic dicam, methodo serierum promovendae, praeter transformationem irrationalium linearum in rationales symmetras (voco autem rationales, quarum ordinatae semper ex abscissis haberi possunt in numeris rationalibus) excogitata est ratio pro curvis transcendenter datis, ubi ne extractio quidem locum habet. Assumo enim seriem arbitrariam, eamque ex legibus problematis tractando, obtineo ejus coefficientes. Porro a praesenti opere Newtoniano praeclara quaeque expecto, et ex relatione Actorum video, cum multa prorsus nova magni sane momenti, tum quaedam ibi tradi, a me nonnihil tractata ; nam praeter motuum coelestium causas, etiam lineas catoptricas vel dioptricas et resistentiam medii explicare aggressus est. Lineas illas Opticas Cartesius habuit, sed celavit, nec suppleverunt commentatores; neque enim res communi analysi subest. Eas postea ab Hugenio (sed qui nondum edidit) et nunc a Newtono inventas intelligo. Etiam mihi, sed per diversam, ut arbitror, viam innotuere. Et habebam quidem methodos generales dudum, sed proprias perelegantes eruendi occasionem dedit egregium inventum Dn. Tschirnhausii nostri in Actis publicatum, qui integras lineas tanquam focos considerat. Quid inde consecutus sum, exemplo explicabo, unde reliqua intelligantur. Sit punctum A (fig. 84) et linea data BB, reflectens radios AB, quaeritur linea CC, radios ABC iterum reflectens in unum commune punctum D. Solutio primae aggressionis haec est. Data linea BB, ejus respectu constat dari puncti A confocum linearem seu lineam EE. Rursus datis duobus confocis, uno lineari EE, altero puncto D, constat inveniri posse aliquam lineam CC, cujus sunt foci, quae erit quaesita. Meliores autem constructiones prodeunt, nam

$A_1B + {}_1B_1E + arc.{}_1E_2E_2 = A_2B + {}_2B_2E$ et $D_2C + {}_2C_2E + arc.\ {}_2E_1E = D_1C + {}_1C_1E,$

unde tota AB+BC+CD
semper est aequalis uni constanti rectae. Si filum circumligatum sit lineae EE simulque alligatum puncto D, tunc evolutione curvae EE stylus filum intendens describet lineam CC. Sin filum idem altero extremo alligatum sit puncto A, stylus filum intendens describet lineam BB. Sed dissimulate curva EE, prodit constructio simplicissima talis: a data recta constante (aequal. $AB + BC + CD$) detrahatur quaevis data AB, residuae sumatur aequalis BF ita ducta, ut ad PB curvae BB vel ejus tangenti perpendicularem in B faciat angulum FBP ipsi ABP aequalem. Jungatur DF, et ex puncto ipsius DF medio G normaliter educta GC occurret ipsi BF in quaesito puncto C, et quidem patet, GC esse lineae CC tangentem. Rotetur porro figura haec circa axem AD, et quae in lineis diximus, etiam in superficiebus genitis locum habebunt. Eadem et dioptricis applicari possunt. Lineam EE voco Acampton , quae radios ABE sine reflexione et refractione accipit. Dantur et Aclastae, quae eosdem non refringunt et tamen reflectunt, et sunt illae, quae ipsius EE evolutione simplici describuntur, quod primus licet alio fine consideravit Hugenius. Talis est FF, posita CF (in producta BC) $= CD$. Si pro A aut D punctis, aut alterutro, foci lineares darentur, aut punctum infinite abesset, ubi radii fierent paralleli, eadem suo modo locum haberent.

Quae de resistentia medii peculiari scheda complexus sum, jam pro magna parte Parisiis duodecim abhinc annis eram assecutus, et illustri Academiae Regiae nonnulla ex illis communicavi. Denique cum mihi quoque meditationes inciderint de causa physica motuum coelestium, operae pretium duxi peculiari schediasmate nonnullus ex illis in publicum proferre. Decreveram quidem premere, donec mihi liceret leges Geometricas diligentius conferre cum phaenomenis novissimis Astronomorum; sed (praeterquam quod alterius plane generis occupationibus distinguor, quae vix quicquam tale sperare patiuntur) excitavit me Newtonianum opus, ut haec qualiacunque extare paterer, quo magis collatione rationum excussae emicent scintillae veritatis, et ingeniosissimi viri acumine adjuvemur.