AN ATTEMPT TO EXPLAIN THE CAUSES OF CELESTIAL MOTIONS.

From Actis Erud. Lips. ann. 1689.

It is agreed the ancients, especially those who had been pleased to follow after *Aristotle* and *Ptolomy*, at no time had acknowledged the grandeur of nature, which finally has become apparent to us and to the more recent age preceding ; according to which *Copernicus,* by recalling from oblivion the most noble hypothesis of the Pythagoreans, [*i.e.* that uniform motion in a circle is the natural thing for a body to do: which of course is true, if a constant central force is acting], which by itself perhaps may be more tainted with suspicion, that it should appear to be established correctly, has shown it to satisfy completely all the phenomena in a simple manner. But *Tycho* followed *Copernicus* in considering the whole of the system (with the exception of the sun and the earth being transposed) [*i.e.* Tycho took the earth as his observation platform, and treated the motion of the sun like that of any of the planets in his measurements of its latitude and longitude, to within 2 minutes of arc, by naked eye observations, using a giant quadrant: see e.g. C. Wilson, *Sci. Amer.* Mar. '72], according to the observations he added with a customary increased attention to accuracy and with the minimum apparatus, he had set out from the heavens the beauty of the orbits of the celestial bodies. But even if he had not acquired enough of the rewards from his Herculean labours, partially excluded by certain prejudices, and partially by death intervening, yet divine providence acted, so that the bulk of his observations came into the hand of an incomparable man, *Johann Kepler* whom the fates had protected from death, so that he might be the first man to publish the laws of heavenly matters and the true laws of the gods. This therefore he found, any of the primary planets describes an elliptic orbit, at one of the foci of which shall be the sun, with that law of motion, so that always areas will be swept out proportional to the times by the radius drawn from the sun to the planet. Likewise he grasped the rule, all the planets of the same system have periodic times in the three on two power of their mean distances from the sun, with the admirable selection of the laws becoming more triumphant, if he had known (which the famous *Cassini* noted) that the satellites of Jupiter and Saturn also were governed by the same laws, with respect of their primary planets, as with these acting towards the sun. But nevertheless he was unable to give the true cause of so great an agreement, either because of the understanding at the time, or feelings about the unexplained shining [of the sun] may have hindered his thinking, or because it was not yet the time for the inner [mechanism of the] geometry, and so the science of the motions from that, as now, might have made progress. Yet it has been uncovered and approached by investigative reasoning. [At this point Leibniz wanders away from the Newtonian model into the theories of Kepler and Descartes, involving vortices.] For the first evidence of the true cause of gravity, and of the nature of the law, on which gravity depends, because bodies by rotating are trying to recede from the centre along the tangent, and thus if straw or chaff may be present in water, with the vessel rotated the water acts as a whirlpool, denser than straw, and thus stronger than that, to be cast out from the middle, and the straw forced towards the centre ; just as distinctly in two or more places, as he set out in the *Epitome of Copernican Astronomy*; although at this stage the places are not entirely clear, and knowing not these forces themselves, nor knowing well enough how much they may follow [the customary laws of] physics, or especially of Astronomy. But from these then *Descartes* has made outstanding use, even if by his manner he might have concealed the originator. But I have marveled often, because *Descartes* indeed did not undertake the account of finding the laws of the heavens to *Kepler*, however much it be agreed ; either because it did not agree well enough with his principles, or because he might be happily ignorant of the discovery, or again because he might not have thought so hard by nature to have observed the discovery.

Again with minimal natural science it may be seen, neither indeed with machines worthy of God, can intelligent people assign the special directions of the journey made by the stars, as if by God the same reckonings required to be completed may be missing from the bodily laws; and truly the orbits of the solid bodies shall be sent out, but in a sympathetic manner, and by magnetism, and of that other kind of abstruse quality, either where they are not understood, or where they are understood, they may be considered the effect of apparent bodily forces ; certainly I consider nothing other to be present, than that the cause of the celestial motions shall be from the motion of the aether, or as it may be called in astronomical terms, indeed the forces may arise by being carried around in orbits, but by a fluid. This is a very old opinion, even if ignored : for *Leucippus Epicuros* in the first place expressed it thus : that in a system itself being formed vortices will have been used,, by the name $\delta\iota\nu\eta\varsigma$, and we have heard just as *Kepler* said, the motion of water acting in a whirlpool had an effect on a weight. And we learn from the travels of *Monconis*, now as well it was the sentiment of *Torricelli* (and as I suspect, even of *Galileo*, whose student he was) the whole aether with the planets to be turned around by the motion of the sun acting around its centre, just as still water whirled around its axis by a stick in the middle of the vessel ; and just as chaff or straw swimming in the water, thus stars, in an appropriate medium, go around more quickly. But these come to mind more generally without difficulty. Truly it has been proposed by us, and the matter will be taught to explain the laws of motion themselves more clearly, because it is by far too deep to be investigated. And since somehow in that manner a light shall shine forth for us, and the question may be seen to be successful both very conveniently and naturally, in that I have given hope of the true causes of the motions of the heavens to be approached by us.

[The following extra unpublished material has been added in the second paper from the Hanoverian Library, before the original individual labeled sections, to which also extra material has been added as noted:]

It is agreed from the outstanding considerations of Gilbert, as far as we know, magnetic properties can refer to bodies greater than the whole world, and besides directing the force, certain poles are to be considered, to have the strength of attracting known small bodies within its sphere of influence, which on earth we may call gravity, and by analogy we may transfer to the stars. [Leibniz does not seem to have made the connection between the weight of bodies on the earth, and considers gravity to be a magnetic force in the heavens, similar to the magnetism of the earth.] But it is not agreed well enough, what the true origin of this so widely apparent phenomenon shall be, and which shall be the same as in any magnet. But although it may never be possible to resolve this problem, yet we have such a wonderful agreement between these laws and they can be relied on to have a great similarity amongst themselves. Indeed it can be asserted, the attraction of weights to be a certain bodily radiation, for no solid substance must be used in explaining the phenomena of bodies being attracted. Therefore it is agreed, [taking for an example] within the body of an earthenware vessel, there could be unsuited matter trying to break loose either by being disturbed, or not sufficiently adapted according to the motion being exercised freely for the situated place, from which on being struck on all sided it may be drawn by other similar matter having a motion of the same kind, so that the motion of the attracted matter may become less disturbed within the vessel, as with the example of a flame, in which the expulsion of one flame and the attraction of another teach the senses about themselves [: here Leibniz may be thinking about the blacksmith's forge, where the bellows on the up-stroke draws air into cinders, and then forces it out as a flame on the down-stroke.]

But if anyone now should wish to consider the matter further, then [an example] may be thought to be a fluid globe, and that thus to have several inner motions within itself, accommodating themselves to the ambient motions, as in the example of oil droplets floating in water having become round, so that they may be disturbed minimally by the surroundings, and then each when it has become established a little, to have remained penetrable and the motion to be retained agreeing with the motions of the remaining fluid present. And indeed the nature of the fluid is to have several internal motions, by which when oil drops may be forced from the ambient motion, lest they fly away from the matter present, respond amongst themselves and thus go off in circles, and so that they try to form large circles, thus so that they retain the greatest effect in receding : as now they are thrust away from that body, from which now less of that fluid is enclosed or in which less fluid is trying to recede. [Thus Leibniz considers the centrifugal force as a mechanism helping to make a system more stable by moving the more excited elements to a greater distance from the centre.] And thus if the centrifugal force is being used, as found by Kepler, it must not only be deduced from the motion of the aether at the equator and from the parallel lines which are thrust away from the earth, but so that now I must remember to note in the future, also by great circles likewise concentric with the central globe, such as the meridians are, an example of this motion is apparent in the magnetism of the atmosphere, for although in the study of motion there shall be a variety of circles in all parts of any great circle, yet nothing is to be hindering certain poles, through the meridians of which there shall be a stronger flow of matter agreeing with the situation of the system. [An attempt is made to make the motions possible to be more general in three dimensions, by considering the magnetic field lines around the earth, which resemble the lines of longitude.] Thus various designated causes coincide together in explaining this reasoning and we have to consider likewise [several influences] agreeing with the centrifugal force: the radiation of spheres [*i.e.* sunlight, etc.], the attraction of spheres, an explosion from perturbations, the inner motion of fluids, and the circulation of the atmosphere.

But whatever shall be the cause of gravity, it can suffice for us that the rays of the globe of the attracting material be analogous to the rays of light to be propelled or impulses emitted along lines in any direction receding from the centre ; not as if it shall be necessary for the parts of the earth to extend as far as to a weight, but because the [fluid] matter by being impelled close to other matter is propagated as in light or sound, and in the motions of liquids. Although they may have erred [*i.e.* the natural philosophers], by which means the propagation was able to be persuaded by some other perceptible effect at some instant. But the rays, as thus I have said of a magnet, or by which the attraction is effected, stand together in the attempt [to make an object] recede by some imperceptible fluid, indeed the most finely divided, but the most dense, since which may be placed within porous bodies, such as earth-like bodies which are held within equal spaces, with not too much matter trying to recede from the centre and thus they are endowed with less levitation, and it is necessary with the fluid emitted prevailing, that earth-like planets be thrust towards the centre. [Leibniz thus considers that a planet, if not moved outwards by centrifugal force effected by some kind of invisible radiation or fluid, must move inwards; thus it must be endowed with another kind of more subtle fluid, to keep it in a steady orbit]. From which it must be apparent a fluid matter different from that fluid, because to make the weight and what we have said to be propelled from the centre, by far more subtle nor following its direction, but exercising its own motion, no less than that magnetic matter itself makes in air or water. For it is necessary that certain other matter shall pass through the pores of smaller terrestrial bodies but more frequently, excluding the former. And whence the greater part of the matter of terrestrial bodies is impervious to the first matter nor unless allowing that other but far more tenuous, by that kind of fluid the body can be considered to be heavier or more solid. And it can happen that all terrestrial bodies can be agreed to be made from homogeneous masses and thus by having equal dissemination from more subtle pores everywhere, moreover the specific gravity shall be greater or less from the smaller or larger size of its gaps and the porosity of these denser or with a heavier fluid of penetration arising copiously, in short so that from the same metal bodies of diverse specific weights are able to be formed, some of which from the kind of cavities sink in water, others float. [Thus Leibniz conceives that all bodies are made from the same basic material, but with different spaces between the particles designating different densities : of course, different substances have different chemical properties, so the original hypothesis would have to be extended; one can see what a hopeless task people faced at the time if the incorrect hypothesis is chosen initially.] But with everything else being equal, and with the same specific gravity present, yet the action tending towards the centre will be greater or less by an amount of radiation, which is required to be judged according to the example of light. Indeed just as before it has been shown by the learned men, bodies to be illuminated by a light in the inverse ratio of the square of the distances, thus it is said for some body to gravitate so much more of less by the square of the distance from the attracting body. Each is the same and the ratio is evident. Clearly about the centre of the light or the rays, R (fig. 19) concentric spherical surfaces can be described through A and through L or with the right lines drawn RAL, RCN of which parts of which may be cut with similar arcs and similarly by putting ABC, LMN, rotated about the axis or the radius of the arc bisecting RBM; again the light or the attracting force by some surface is spread out uniformly or equal parts of the same surface are illuminated equally or acted on equally; and thus the total light or force of the surface ABC is to the light or force of the surface LMN in a ratio composed of the intensity or illumination and of the extension or of the surfaces ; but there is just as great

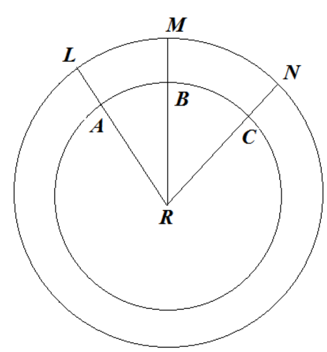


Fig.19

an amount of light or the force of attraction at the surface ABC as there shall be at the surface LMN, therefore the intensities are inversely as the extensions , that is, the illuminations or weights acting are inversely as the surfaces, or the surfaces ABC and LMN are as the squares of the diameters RA, RL. And thus the illuminations [*i.e.* light intensities] or weights acting are inversely as the squares of the distances from the radiating or attracting centres. But this was understood by us from the start, itself soon repeated by its own accord by a later analytical calculation from the common phenomena of the planets leading to a magnificent agreement arising between the ratios and the observations, and for a significant confirmation of the truth. For what follows, does not agree from hypothesis, but from phenomena concluded from the laws of motion and indeed may or may not be given from the attraction of the planets by the sun, it suffices for us to deduce from that the approach towards and departure from the sun, that is of the increments or decrements of the distance, that the planet will have if it may be attracted by a prescribed law. And whether it may actually travel round the sun in a circle, or not, thus it will suffice to change the position with respect to the sun so that it may move with the harmonic circulation, and hence the principles are required to be understood in an amazingly simple and fruitful manner, such I do not know or men at one time had dared to think or hope. But how much hence about the causes of these motions themselves can be concluded, we will leave the judgment of each to prudence, for perhaps from that the matter now has been led to some sort of conclusion, as the understanding poet [Lucan] dared to say in short to the astronomers :
Talia frustra
Quaerite quos agitat mundi labor, at mihi semper
Tu quaecunque paret tam crebros causa meatus
Ut superi voluere late.

[Such frustration!
Ask who are disturbed by the labours of the world [in raising the billows],
But for me let whatever cause could be apparent of so very numerous wanderings,
Be such as the gods would wish to hide.]

(1) Therefore so that we may approach that matter itself, before everything can be shown, following the natural laws : *all bodies, which describe a curved line in a fluid, to be acted on by the motion of the fluid itself.* Indeed every curve requiring to be described tries to recede from that along the right line tangent (from the nature of the motion) therefore it is required that it shall be restrained. But there is nothing nearby, except the fluid (from the hypothesis) and nothing is trying to restrain it, except for the nearby moving fluid (from the nature of the body) : therefore by necessity to be itself in motion.

(2) Hence it follows, *the planets to be moved by their own aether,* or to have fluid orbits moving or being carried around. For everyone agrees curved lines are to be described, nor is it possible for the phenomena to be explained, by supposing they move with a rectilinear motion only. And thus (by the preceding) they are moved by the ambient fluid. Likewise it can be shown otherwise, because from that the motion of a planet is not uniform; or in equal times equal distances are being described. From which also it is necessary that the planet must be acted on by a nearby motion.

(3) I call the circulation harmonic, if the velocities of the circulation, which are at some body, shall be reciprocally proportional with the radii, or with the distances from the centre of rotation, or (which is the same) if these velocities circulating around the centre may decrease in proportion, at which the distance from the centre increase ; or most briefly, if the velocities of circulation increase in the proportion of these nearby. Thus indeed if the radii or the distances may increase uniformly, or arithmetically, the velocities will decrease in a harmonic progression. And thus not only in the arcs of a circle, but also can be found on some other curve describing the harmonic circulation at the place. We may put the moveable mass M to be carried in some curve $_3M_2M_1M$ (or $_1M_2M_3M$) and in equal elements of time to describe the elements (Fig. 18), of the curve $_3M_2M$, $_2M_1M$, a composite motion can be understood from a circular motion about some centre, such as $\odot$ (as $_3M_2T$, $_2M_1T$) and with a rectilinear motion such as $_2T_2M$, $_1T_1M$ (with $\odot_2T$ still equal to $\odot_3M$ and $\odot_1T$ equal to $\odot_2M$), such a motion can be understood also, while a ruler, or an indefinite right line $\odot N$, is moved around the centre $\odot$, and meanwhile the mobile mass M is moved along the right line $\odot N$. But there is no need to be concerned, that the motion shall be rectilinear only [i.e. it is to be considered as a second order effect], by which it may approach the centre, or recede from the same (which I call *paracentric motion*) : for it is carried by a circulatory motion of the mass M, so that $_3M_2T$ shall be to the other circulation of the same mass, $_2M_1T$, as $\odot_1M$ to $\odot_2M$, that is as if the circular motions made in equal elements of time shall be inversely as the radii. For since the arcs of the same elementary circular motions shall be composed in the ratio of the times and of the velocities, but the elementary times are assumed equal, the circular motions will be as the velocities, and thus also the velocities will be reciprocally as the radii, and thus the circulatory motion may be said to be harmonic.

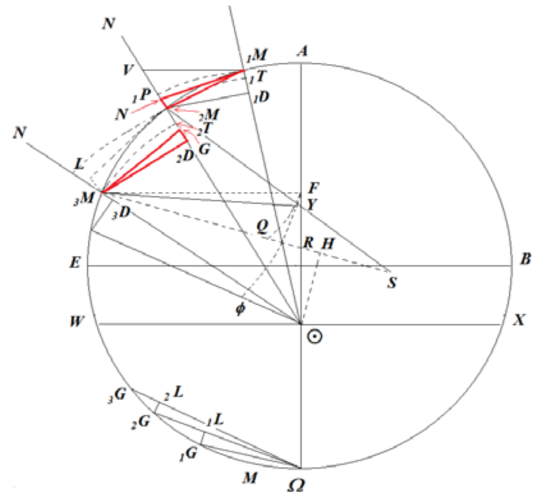


Fig. 18.

(4) *If a moving mass may be carried by a harmonic circulation* (whichever the paracentric motion shall be) *the areas cut by the radii drawn from the centre of the circulation to the mass will be taken proportional to the times, and vice versa.* For since the elementary circular arcs, such as $_1T_2M$, $_2T_3M$, shall be incomparably small, with respect of the radii $\odot_2M$, $\odot_3M$, the differences between the arc and the sine of the same right line (as between $_1T_2M$, and $_1D_2M$) by themselves incomparable with the differences, and hence (*by our analysis of the infinites*) these may be had as zero, and both the arcs and sines accordingly coinciding. Therefore $_1D_2M$, to $_2D_3M$, shall be as $\odot_2M$ to $\odot_1M$, or $\odot_1M$ by $_1D_2M$ equals $\odot_2M$ by $_2D_3M$, and therefore the half triangles of these are equal, surely $_1M_2M\odot$ and $_2M_3M\odot$, which since they shall be elements of the area $A\odot MA$, and

thus with the equality from the hypothesis of the elements of time taken, also the elements of the area are equal, and vice versa, and hence the areas A☉MA are proportional to the times, by which the arcs AM are traversed.

(5) I have inserted a demonstration between *incomparably small quantities*, for the sake of an example of the difference of two common quantities, with themselves being incomparable quantities. For thus indeed such, lest I be mistaken, can be set out most clearly. Therefore if anyone may be unwilling to use infinitely small quantities, it may be judged sufficient to consider them as being small, so that they shall be incomparable [*i.e.* different by an insignificant first order amount], and the error they produce between them of no consequence, indeed less with the given. Just as the earth may be taken as a point, or the diameter of the earth for an infinitely small line, with respect to the heavens, thus it can be shown, if the sides of the angle may have a base incomparably small to themselves, the angle contained to be incomparably smaller than a right angle, and the difference of the sides to be incomparable between themselves; likewise the difference of the whole sine, of the complement of the sine and of the secant, to be with incomparable [*i.e.* too small to be distinguished between] differences, likewise the difference of the chord, arc and tangent. For since these shall be infinitely small, the differences will be infinitely infinitely small, since these themselves shall be infinitely small, they will be the infinitely small differences of infinitely small quantities, and the versed sine also will be infinitely infinitely small, and thus incomparable with a right line. And the orders both of the infinitely large, as well as of the infinitely small, are infinite. And similar common triangles can be used for these unassignable ones, which have maximum use with tangents, maximas and minimas, and explaining the curvature of curves; likewise in nearly all of geometry applied to nature, for if the motion may be set out by a common line, that the moving mass may complete in a given time, the impetus, or the velocity may be set out by an infinitely small line, and the element itself of the velocity, such as it is for the action of the weight, or the centrifugal attempt, by an infinitely infinitely small line. And I have considered putting this *Lemma* in place, for our *Method of incomparable quantities, and the analysis of the infinites,* as the foundations of this new teaching.

[Leibniz is convinced about the planets being carried along by a vortex of circulating fluid ; however, to generate an elliptical orbit as required by Kepler's law, an extra second order force is required in addition to the fluid force, which he calls the paracentric force, acting towards or away from the sun : the nature of this force is not understood, it may be due to the weight of the body, or gravity; or it may be due to levitation, or some other cause; Kepler considered magnetism as a cause, as Gilbert's book *De Magnete* had recently been published. See Ch. 5 of Aiton's book : *The Vortex Theory of Planetary Motion*, for further details. Notwithstanding his lack of physical intuition, and the fact that clearly he had not read the *Principia* at this time: for Newton's idea of extending the force of gravity to the moon, and beyond, was the mental leap needed to bring about an understanding planetary motion - in this paper Leibniz provides an insight into his differential calculus, esp. the presence of higher order differences or derivatives and the occurance of a differential equation, as well as introducing the idea of *living* and *dead* forces, in which lay the germ of kinetic energy. The *vis viva* notion was to linger in

Dynamics for almost 200 years, before being appropriated at last into conservation laws; and of course it still has a meaning in Astronomy.]

(6) Now from these it is a consequence, the planets are to be moved in a harmonic circulation, the primary ones around the sun, and moons around their primary, as the centre. For with the radii drawn from the centre of the circulation, they describe areas proportional to the times (from observations.) Therefore with equal elements of the times put in place, the triangle ${}_1M_2M\odot$ is equal to the triangle ${}_2M_3M\odot$, and hence $\odot_1M$ to $\odot_2M$ is as ${}_2D_3M$ to ${}_1D_2M$, because it is to be a harmonic circulation.

(7) Also it is agreed, the aether, or *the fluid orbit of each planet to be moving in a harmonic circulation;* for it has been shown above, no body in the fluid is moving along a curve by its own accord, and therefore it will be the circulating aether, and that it is agreed to believe to account for the circulation of the planets, thus so that also the harmonic circulation of the aether of each planet, that is the fluid of the orbit of the planet, is known to be divided into innumerable concentric circular orbits of small thickness, any of which will have its own proper circulation with so much greater a velocity in proportion, as by how much each will be closer to the sun. But another more exact account of this motion in the aether will be rendered.

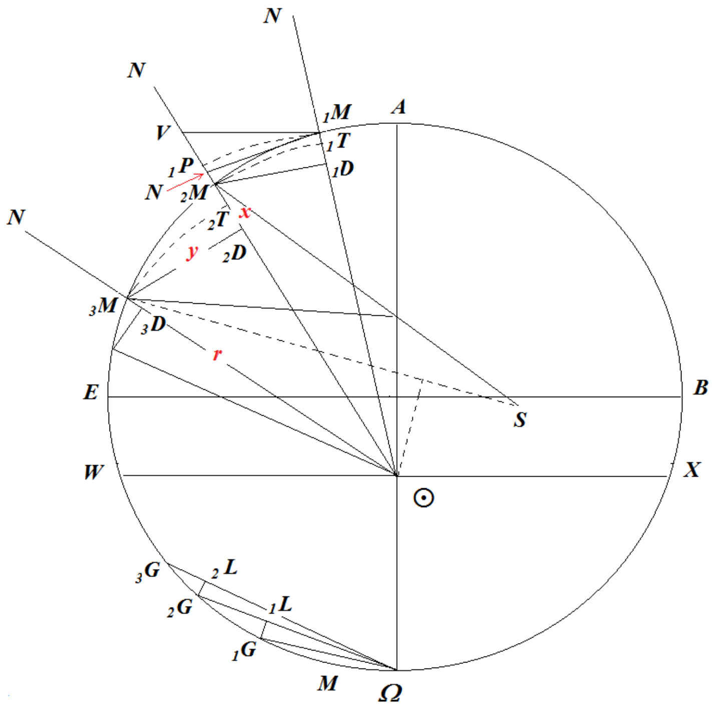
(8) And thus we may consider *a planet to be moved by a two-fold motion, or composed from the harmonic circulation of the orbit of its carrying fluid, and from a paracentric motion*, as if of a certain weight or an attraction, that is an impulse towards the sun, or the primary planet. But it is made by the circulation of the aether, so that a planet may travel in a circle harmonically, not as if by a proper motion, but as if by being carried around swimming quietly in the fluid, whose motion it follows, nor from which it retains any force of a more rapid circulation, which it would have in a lower or closer orbit, but by growing less, while it transfers into higher orbits (with a velocity greater than its resistance), it continually loses velocity, and approaches to that orbit to which it adapts itself insensibly. In turn, while it extends from higher to lower orbits, it takes the force of these. And thus there it shall be easier, because where once the motion of the planet is agreed on, with the present motion of the orbit, afterwards it will differ little from those nearby.

(9) With the harmonic circulation explained, *the paracentric motion of the planets is required to be explained, arising from the force of the circulation on a planet being made to change orbit, and composed from the attraction between itself and the sun*. Moreover, it may be permitted to call the attraction, although really it shall be an impulse, for certainly the sun can be considered just as a magnet ; moreover without doubt these magnetic forces are derived from the impulses of the fluid. From which also we will call the attraction of the weight, by considering the planet as a weight evidently drawn towards the centre of the sun. But the kinds of the obit depends on a specific law of attraction. Therefore we may consider what law of attraction may make the curve an ellipse, and so that we may follow this, it is necessary that we enter into the realms of geometry for a short while.

(10) Since every moving mass that will describe a curved line tries to recede along the tangent, it will be allowed hence to call this the attempt of '*shaking free*' [which we can understand to mean a perturbation or libration; a term that goes back to the original analogy of water rotating in a bucket with straw and chaff, which separated out on rotation] as in the motion of a sling, to which an equal force is required, by which the stone may be made to move, lest it may wander. *This attempt will be permitted to be measured to the perpendicular from a point following along the tangent of the preceding point by an unassignable distance.* And since the curve is circular, the celebrated *Huygens*, who first treated that geometrically, called this the *centrifugal* force. [*i.e.* part of the confusion which Newton clarified; the term is not mentions in his *Principia*, being an apparent force.] Moreover every attempt of 'shaking free' [or libration] is with respect to the velocity, or the force arising from a attempt repeated a very short time later is considered to be infinitely small, and just as by the action of gravity, which is of a homogeneous nature with that. From which and by the same cause each is confirmed. Nor hence is it a wonder, what Galileo wished, the repetition of blows to be infinite in explaining the force of gravity by comparison, or as I say of a simple attempt, the force of which I am accustomed to call a *dead force*, but which by acting continually, the force beginning from repeated blows, is to be rendered the *living force*.

[Thus here we have an introduction to Leibniz's idea of the *vis viva*, which came to be identified not as a force, but as the kinetic energy (without the factor of a half) acquired by a body from an infinite series of blows or a continuous force, rather than from a series of discrete impacts ; a *dead* force meant that nothing seemed to happen, or in the Newtonian sense, the forces were formed from action/reaction pairs, such as for a person standing on the ground experiencing the force of gravity downwards acting as it were from the centre of the earth, balanced by a reaction force upwards of the ground being compressed underfoot; the earth of course experiencing the equal and opposite reactions to these forces, and also balancing each other.]

(11) *The centrifugal attempt*, or the trial of breaking free from the circulation can be expressed by PN *the versed sine of the angle of the circulation* ${}_1M\odot N$ (or which may be returned likewise on account of the difference of the unassignable radii, by ${}_1D_1T$) for the versed sine is equal to the perpendicular drawn from one end point of the arc of the circle to the tangent of the other end, where we have expressed the breaking free attempt in the preceding (also the centrifugal attempt can be expressed by PV, the differential of the radius and of the secant of the same angle, of which difference the distinction from the versed sine is an infinity of infinities, infinitely small, and thus the smallest with respect to the radius.) Hence again since the versed sine shall be in the square ratio of the chord, or the unassignable arc, or of the velocity, it follows that the *centrifugal attempt of the moving object shall be equal to the motion to be described in equal circles in the square ratio of the velocities, in unequal circles to be described in a ratio composed from the squares of the velocities, and the reciprocals of the radii.*

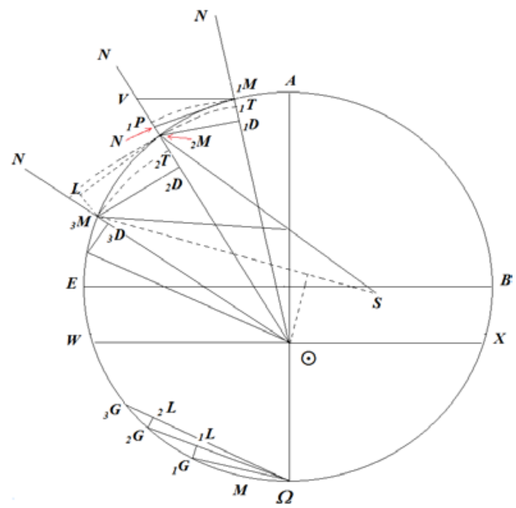


[Extra from the second version of this paper, from the Hanoverian Library, but unpublished at the time : For the analytical calculation there the beginning of the centrifugal force or the versed sine $_2D_2T$ shall be x, the velocity of the circulation or the right sine or the circulation $_2D_3M$ shall be y, radius $\odot_3M$ shall be r ; from common geometry there will be $x = yy : \overline{2r - x}$: or x to y shall be as y to $2r - x$ or x to y as y to $2r - x$. But x is incomparably less than r, therefore it vanishes in the quantity $2r - x$ and there shall be $x = yy : 2r$ or the centrifugal attempt $_2D_2T$ is to the circulation, $_2D_3M$ as the circulation is to the velocity, for which there shall be a need, for the whole diameter or for twice the distance from the centre, being traversed in the same element of time. [Clearly this result is incorrect, assuming the theory is correct, which of course it is not, as r is a constant, and ordinary differentiation gives $v = \frac{dx}{dt} = \frac{y}{r}\frac{dy}{dt}$].]

(12) *The centrifugal trials of the mobile are in the ratio of the reciprocals of the radii cubed with the harmonic circulation.* For by the preceding they are as the reciprocal of the radii and directly as the squares of the velocities, that is (because the velocities of the circulation are reciprocally as the radii) in the reciprocal square ratio of the radii; from the simple reciprocal with the square reciprocal there shall be the reciprocal cubed. For the calculation, θa shall be a constant plane area equal always to double the elementary triangle $_2M_3M\odot$ or to the rectangle formed from $_2D_3M$ by the radius $\odot_2M$ or r, therefore $_2D_3M$ will be $\theta a : r$ or θa divided by r, now [half] the centrifugal trial [or libration] $_2D_2T$ is equal to $_2D_3M$ squared divided by twice $\odot_3M$, therefore equal to $\theta\theta aa : 2r^3$.
[Since θ is the element of time *dt*, and if we denote the rate at which the angular motion of the radius is being performed by $\frac{d\varphi}{dt}$, following Aiton, p. 141, then the constant rate at which the area is swept put is given by

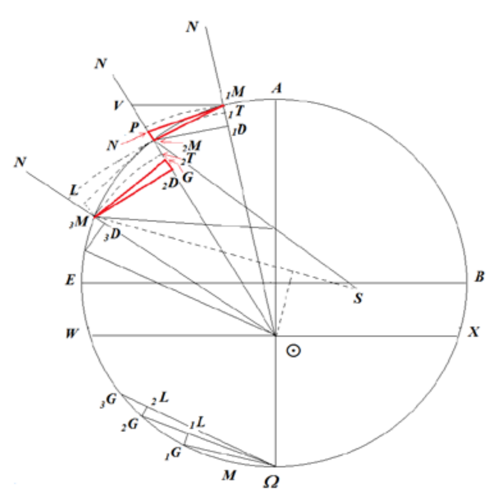
$\frac{1}{2}r^2\frac{d\varphi}{dt} = \frac{1}{2}rv = h$ or $r^2\frac{d\varphi}{dt} = rv =$ a above, and $r^2 d\varphi = \text{a}dt \to \text{a}dt$ as above; hence the centrifugal acceleration given by $\frac{v^2}{r} \to \frac{\text{aa}}{r^3}$ etc.]

(13) *If the paracentric motions* (receding or approaching the centre Ω itself) *shall be equal, and the circulation harmonic, the curve of the motion* ΩMG *will be a spiral beginning from the centre* Ω, of which this is a property, that the segments ΩGMΩ [*i.e.* the areas] shall be proportional to the radii, that is: with this position of the chord ΩG drawn from the centre, just as the areas, that is the segments, are (on account of the equal recessions) proportional to the radii with the times. There are many other remarkable properties of this spiral, nor the construction difficult. Indeed the general method is given in harmonic circulations, if from the radii the times may be given, either the velocities of the paracentric motion, or at least the elements of the impulses or of the actions of gravity, at any rate the curved lines can be constructed from the supposed quadratures.



(14) The *paracentric action, either of gravity or of levitation*, is expressed by the right line $_3$ML erected from the point $_3$M of the curve, for the point at an unassignable distance $_2$ML along the tangent from the preceding point $_2$M (produced to L), parallel to the preceding radius ⊙$_2$M (drawn from the centre ⊙ at the preceding point $_2$M).

(15) *In every harmonic circulation the element of the paracentric impetus* (that is, the increment or decrement of the velocity in falling towards the centre, or by ascending from the centre) *is the difference or sum of the paracentric actions* (that is the pushing together from gravity, or by levitation, or made by similar causes) *and of twice the centrifugal departure* (from that itself the harmonic circulation arose), indeed the sum of the levitations, if levitation shall be present ; the differences, if gravity shall be present : where with the action of gravity prevailing the descents will increase or the velocities of the ascents will decrease, as prevails with twice the centrifugal attempt, in the opposite direction. From $_1$M and $_3$M the normals at ⊙$_2$M shall be $_1$MN and $_3$M$_2$D ; therefore since the triangles $_1$M$_2$M⊙ and $_2$M$_3$M⊙ may be shown to be equal on account of the harmonic circulation, both the altitudes (on account of the common base ⊙$_2$M) $_I$MN and $_3$M$_2$D will be equal. Now with $_2$MG assumed to be equal to L$_3$M, $_3$MG may be joined parallel to $_2$ML itself ; therefore the triangles $_1$MN$_2$M and $_3$M$_2$DG will be congruent, and $_1$M$_2$M will be equal to



G_3M, and N_2M equal to G_2D. Again on the right line $\odot_2M$ (produced if there is a need, which is understood always) there may be taken $\odot P$ equal to $\odot_1M$, and $\odot_2T$ equal to $\odot_3M$, P_2M will be the difference between the radii $\odot_1M$ and $\odot_2M$; and ${}_2T_2M$ the difference between the radii $\odot_2M$ and $\odot_3M$. Now P_2M is equal to (N_2M or) $G_2D + NP$, and ${}_2T_2M$ is equal to ${}_2MG + G_2D - {}_2D_2T$, therefore $P_2M - {}_2T_2M$ (the difference of the differences) will be $NP + {}_2D_2T - {}_2MG$, that is (because NP and ${}_2D_2T$ are the versed sines of the two angle, and the incomparability of the difference of the radii coincide) twice ${}_2D_2T - {}_2MG$. Now the difference of the radii expresses the paracentric velocity, the difference of the differences expresses the element of the paracentric velocity. But ${}_2D_2T$ or NP is the centrifugal attempt of the circulation, certainly the versed sine (per 11) and ${}_2MG$ or ${}_3ML$ is the action of gravity (by the preceding). And thus the element of the paracentric velocity is equal to the difference between twice the centrifugal trial NP, or ${}_2D_2T$ and the simple action of gravity G_2M, or (because it may be concluded in the same manner) the sum from twice the centrifugal trial, and from the simple action of levitation.
[The presence of a force of levitation would appear to arise from a difference in the densities of the circulating aether fluid and the mobile masses within: yet such a levitation arises on earth from unequal pressures, and which eventually depend on gravitation……… Thus, Leibniz, as if in despair, has thrown all conceivable forces into his theory. The diagram incorporates both types of motion, either attractive inwards in the clockwise sense, and due to magnetism, weight , or levitation; or repulsive outwards in the anticlockwise sense, due to levitation or centrifugal, which Leibniz considered real. Whatever the origin of these forces, they are small, and account for the small departure of the planet from a purely circular motion to an elliptical one, in Leibniz's view.]

(16) From the given increments, the action of gravity or of levitation is given, or vice versa either to be ascending from the decrements of the velocity, or by descending from the increments. It is apparent from the preceding, for the centrifugal attempt is considered to be given always, since it shall be in the inverse triplicate ratio of the radii (by 12).

(17) With the equality of the elements of time the increments of the angles of the harmonic circulation, are in the inverse square ratio of the radii. For the circulations are in a ratio composed of the angles and of the radii, and the elementary circulations, since they shall be harmonic, are in the inverse ratio of the radii, therefore the elementary angles are in the ratio of the inverse squares of the radii. Such almost are the daily motions apparent seen from the sun (for here days are sufficiently small parts of time, for the more distant planets in the first place), which will be circular, in the inverse square ratio of the distances, thus so that with the distance doubled only a fourth part of the angle will be completed in the same element of time, with the distance tripled, only a ninth part.

(18) *If an ellipse may be described by the harmonic circulation a body moving around a focus as the centre of circulation, there will be between these three* [*motions*] : *the circulation* ${}_2T_3M$ or ${}_2D_3M$ (for these are similar and cannot be different); the *paracentric velocity* ${}_2D_2M$, *and the velocity of the moving body itself* (composed from these) *in that elliptic orbit itself, evidently* ${}_2M_3M$ *respectively, as these three others* : the transverse axis BE, the mean proportion between the difference and the sum of the distances of the foci between each other F$\odot$, and of the difference $\odot\phi$ of the distances of the point of the orbit ${}_3M$ from the foci, and finally twice the mean proportional between $\odot_3M$ and F_3M, the distances of the same point from the two foci. And these are true in the same manner for the hyperbola. In the parabola there with the quantities vanishing which are infinite, the quantities will become the circulation, the paracentric velocity,

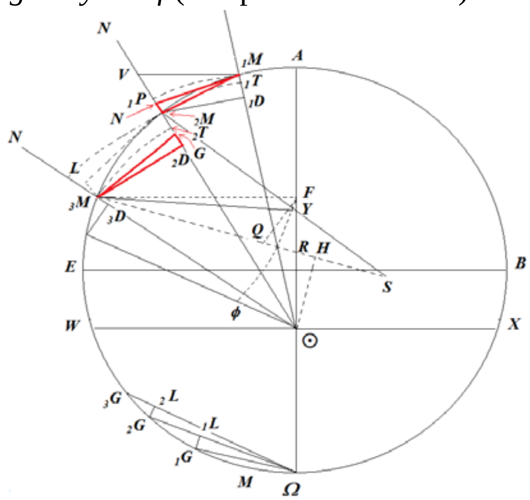


Fig. 18.

and the velocity composed from these, which is in the orbit itself respectively, as the latus rectum, the mean proportional between the latus rectum, and the excess of the radius over the smallest radius, (which is the fourth part of the latus rectum) and finally twice the mean proportional between the radius and the latus rectum. The truth of these can be derived from the common elements of conic sections, if the right line ${}_3MR$ may be perpendicular to the curve at MR (or of its tangent), to cross with the axis AΩ at R, and at that from the foci the normals to act FQ, $\odot$H; it is apparent for $\odot$H, H_3M, ${}_3M\odot$ themselves to be ${}_2M_2D$, ${}_2D\,{}_3M$, ${}_3M\,{}_2M$, that is proportional to the paracentric velocity, the circulation velocity, and to the [resultant] velocity in that orbit. Therefore is suffices to show the sides of the triangle ${}_3MH\odot$ to be between themselves, as we have said. Because it becomes easier, by considering the triangles ${}_3MQF$ and ${}_3MH\odot$ to be similar, and besides F_3M to be to $\odot_3M$, as FR to $\odot$R, thence the proposition can be concluded by common analysis. Hence it follows, with the foci interchanged, so that the one for the other becomes the centre of harmonic circulation, and with the attraction made the same as before, the account of the circulation remains the same, and of the paracentric velocity, at any point.

(19) *If a moving body has a weight, or is drawn towards some point, such as we may put a planet with respect to the sun, it may be carried in an ellipse* (or in another conic section) *by a harmonic circulation, and at the focus of the ellipse both the centre of attraction as well as of the circulation, the attractions or of the gravity acting will be as the square of the circulation directly, or as the square of the radii, or of the distances of the foci inversely.* We have found this by an elegant example of our differential calculus, or analysis of the infinite quantities. AΩ shall be q; ⊙F e; BE, b (that is $\sqrt{\text{qq-ee}}$ [or in modern terms, $\sqrt{4a^2\text{-}4a^2e^2} = 2a\sqrt{1-e^2} = 2b$]), ⊙$_2$M the radius r; ⊙φ (or ⊙$_2$M – F$_2$M) $2\text{r} - \text{q}$, or by abbreviation p ; and the latus rectum WX shall be a, equal to bb : q.

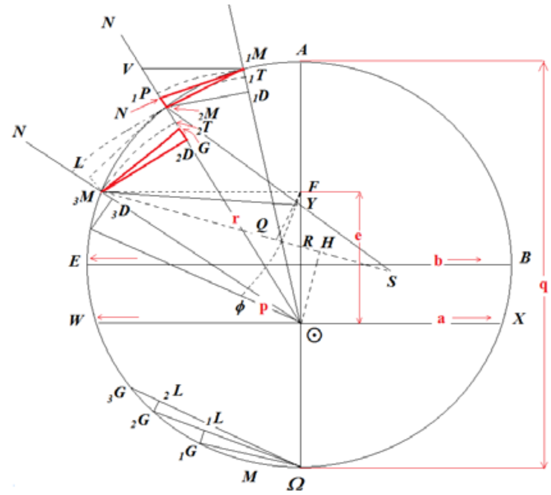


Fig. 18.

[In modern terms, $\text{bb} : \text{q} = 4a^2(1-e^2)/2a = 2a(1-e^2) = 2l$, where l is the semi-latus rectum. Note that in Leibniz's day, the eccentricity e was not yet known as a way of describing conic sections]. Double the element of the area of the triangle $_1$M$_2$M⊙ which is always the same, shall be θa, on putting a for the latus rectum, and with θ representing always the same element of time [dt]; and the circulation $_2$D$_3$M will be θa : r (see now 12 above) again the difference of the radii $_2$D$_2$M may be called dr, and the difference of the differences ddr. But by the preceding there is dr (or $_2$D$_2$M) to θa : r (or to $_2$D$_3$M) as $\sqrt{\text{ee-pp}}$ to b. Therefore $\text{brdr} = \theta\text{a}\sqrt{\text{ee-pp}}$, which is the equation of the differential. But the differential of the differential (*following the laws of the calculus from our other papers published in these Actis*) is : $\text{bdrdr} + \text{brddr} = -2\text{pa}\theta\text{dr} : \sqrt{\text{ee} - \text{pp}}$, of which with the aid of the two equations by removing dr, so that only ddr remains, it becomes $\text{ddr} = \text{bbaa}\theta\theta - 2\text{aaqr}\theta\theta,: \text{bbr}^3$, from which the proposition is obtained. For ddr, the element of the paracentric velocity, is the difference between $\text{bbaa}\theta\theta : \text{bbr}^3$ that is $\text{aa}\theta\theta : \text{r}^3$ which is twice the centrifugal trial (by 12 above) and between $2\text{aaqr}\theta\theta,: \text{bbr}^3$, that is (because $\text{bb} : \text{q} = \text{a}$) $2\text{a}\theta\theta : \text{rr}$: therefore it is required (by 15) so that $2\text{a}\theta\theta : \text{rr}$ shall be the action of gravity ; which multiplied by the constant a : 2 gives $\text{aa}\theta\theta : \text{rr}$, the square of the circulation. Therefore the actions of gravity are directly as the squares of the circulation, and therefore reciprocally as the squares of the radii. The same conclusion follows both for the *hyperbola* and the *parabola*, but especially for the *circle*, which is the simplest from these. But the account of the distinction between these conic sections, and when circles and ellipses may be generated before the others, will appear below.

[In modern terms, this differential equation is equivalent to :
$ddr = \frac{aa\theta\theta}{r^3} - \frac{2}{a} \cdot \frac{aa\theta\theta}{r^2}$, for the acceleration along the radius r; since θ is the element of time that we express now as *dt*, and $\frac{ddr}{dt^2} = \frac{aa}{r^3} - \frac{2}{a} \cdot \frac{aa}{r^2}$. This is the equation that may be derived starting from Newton's second law, the universal force of gravitation between the planet and the sun, and the conservation of angular momentum of the motion around the sun; or as we have just seen, by differentiation of the equation of an ellipse curve twice.]

(20) *The same planet is attracted in different ways, and indeed in the square ratio of the nearness to the sun* ; thus so that the same by being twice as near is attracted by four times stronger a force, three times nearer by a force stronger by nine times : by falling towards the sun a certain new force is acting always. It is apparent from the preceding, on considering a planet to describe an ellipse, and with a harmonious circulation, and in addition always to be drawn towards the sun. I see this proposition now while noted also by the most celebrated *Isaac Newton,* so that it is apparent from the relation of the forces, however thence I may not be able to judge, in what manner he arrived at that [hypothesis].

(21) *Also it is apparent the action of gravity on the planet to be to the centrifugal attempt of the planet* (or of the setting free from that harmonic circulation by snatching it in orbit, and thus ending the trial) so that the present distance from the sun to the quarter part of the latus rectum of the planet, or as r is to a : 4, and therefore the ratios of the gravitational force to the centrifugal trial are proportional to the distances of the planet from the sun.

[This follows from $\frac{ddr}{dt^2} = \frac{aa}{r^3} - \frac{2}{a} \cdot \frac{aa}{r^2} = 2 \times \text{centrifugal force-gravitational force}$, according to Leibniz, where a is the latus rectum. It is of some interest to note that on re-arranging this equation into the form : $\frac{d^2r}{dt^2} - \frac{aa}{r^3} = -\frac{2}{a} \cdot \frac{aa}{r^2}$, where $\frac{1}{2}a$ is the rate at which the planet sweeps out the area, we have a form similar to the modern form of the Newtonian equation for planetary motion, where the sun is assumed to have a mass much greater than the planet, and which we can write in a usual notation : $\frac{d^2r}{dt^2} - \frac{h^2}{r^3} = -\frac{GM}{r^2}$; note also that according to modern theory, the semi-latus rectum is given by $\frac{h^2}{GM}$, or $\frac{1}{2}a$. Thus, Leibniz's theory, when considered from different physical principles, as he knew nothing about the conservation of energy and angular momentum as such, comes up with the correct differential equation, the solution of which of course, is an ellipse in the case of bound orbits. One can see from (19) that he started from the equation of the ellipse, differentiated twice, and introduced forces and pseudo-forces to get a scheme that worked : it would appear that the aether vortex attended to the business of conserving the angular momentum. The interplay between attractive and repulsive forces introduced arose from a lack of understanding about the conservation of angular momentum, and of the sum of the potential and kinetic energies, all of which lay in the future. For at times the planet speeds up as it nears the sun, and at other time slows down as it moves away; Leibniz handled this behavior by introducing centrifugal and levitational forces of the correct sizes.]

(22) *The velocity of a planet is greater everywhere around the sun than the paracentric velocity, that is by approaching the sun, or receding from it.* For since the circulation shall be to the paracentric velocity as b to $\sqrt{\mathrm{ee}-\mathrm{pp}}$ (by 18, with the calculation added according to 19) that will be greater than this, if $\mathrm{bb}+\mathrm{pp}$ shall be greater than ee, because that certainly will be the case, when bb shall be greater than ee, or the transverse axis b, than the distance of the foci e. Truly that always happens for us with the planetary ellipses, which thus do not differ much from circles.

[In modern terms, as above, we have, using polar coordinates (r,θ) for the planet, centred on the sun of mass *M*, taken at rest :

$$\ddot{r}-r\dot{\theta}^2=-\frac{GM}{r^2};\quad \frac{1}{r}\frac{d\left(r^2\dot{\theta}\right)}{dt}=0\quad \therefore r^2\dot{\theta}=h=const.$$

As above, this reduces to $\frac{d^2r}{dt^2}-\frac{h^2}{r^3}=-\frac{GM}{r^2}$; on integrating with respect to *r*, this becomes :

$\frac{d^2r}{dt^2}-\frac{h^2}{r^3}=-\frac{GM}{r^2}$ giving $\dot{r}\frac{d\dot{r}}{dr}-\frac{h^2}{r^3}=-\frac{GM}{r^2}$ and $\dot{r}^2+\frac{h^2}{r^2}=GM\left(\frac{2}{r}-\frac{1}{a}\right)=v^2$. Here *v* is the total velocity, comprising the radial velocity v_r, and the tangential velocity v_θ, so that $v^2=v_r^2+v_\theta^2$. This is the conservation of energy equation : $\frac{1}{2}v_r^2+\frac{1}{2}v_\theta^2-\frac{GM}{r}=-\frac{GM}{2a}$, where *a* is the length of the semi-major axis. At the mean position, $r=a$, and $\frac{1}{2}v_r^2+\frac{1}{2}v_\theta^2=\frac{GM}{2a}$, while at the apses A and Ω, $r=a(1\pm e)$, $v_r=0$:
$\frac{1}{2}v_A^2=\frac{GM}{a(1-e)}-\frac{GM}{2a}$ and $\frac{1}{2}v_\Omega^2=\frac{GM}{a(1+e)}-\frac{GM}{2a}$. Again, at the ends of the latus rectum, where $r=l$ and the radial velocity has a turning point and a maximum, establishing $h=GMl$,
$\dot{r}^2_{\max}=GM\left(\frac{2}{l}-\frac{1}{a}\right)-\frac{h^2}{l^2}=GM\left(\frac{2}{l}-\frac{1}{a}\right)-\frac{GM}{l}=GM\left(\frac{1}{l}-\frac{1}{a}\right)$.]

(23) *At the aphelion A and the perihelion* **Ω** *there is only the circulation* [*i.e.* velocity] *without approaching or receding, at the perihelion a maximum, at the aphelion a minimum.* But at the mean distance of the planet from the sun (which is at the ends of the transverse axis itself, B and E, the velocity of accession or the velocity of recession is to the circulation in the ratio of the distance between the foci to the transverse axis, or as e to b. For there p vanishes.

(24) The velocity of the planet approaching the sun is a maximum or of receding from that is a maximum, when W☉ or X☉, the distance of the planet from the sun, is equal to half the latus rectum of the ellipse, for then (by 19 or 21) there shall be
$\mathrm{ddr}=0$ when $\mathrm{r}=\mathrm{a}:2$. And thus if from the sun as centre, with half the latus rectum as

the radius ⊙W, a circle may be described, it will cut the ellipse of the planet at the two points with the maximum paracentric velocity W and X, which in the one such as W it will be approaching, and in the other X receding. The minimum or zero is at the aphelion and perihelion, or at each vertex A and Ω of the ellipse.

(25) *Always in an ellipse, and therefore always at the planet, the attempt to recede from the sun, or trying to break free from the harmonic circulation, is less than the action of gravity or the attraction of the central sun.* For indeed (by 21) the attraction of the sun to the centrifugal trial is as the distance from the sun, or from the focus, to the fourth part of the latus rectum, but in an ellipse always at a distance from the focus greater than the fourth part of the latus rectum.

(26) *The forces which a planet may be considered to endure continually from the attraction of the sun during its journey, are as the angles of the circulation,* or which the radii contain, drawn from the sun to the first and last point of the journey ; or as if the motion were evident, or the journey were observed from the sun. Thus the force impressed duration the journey A_1M is to the force during the journey A_3M, as the angle A_1⊙M to the angle A⊙$_3$M. For the increments of the angles shall be as the forces of gravity (by 17 and 19) and therefore the sum shall be proportional to the sums, evidently of the angles completed in the circulation, with the sums of the forces, or from the forces thence considered. Hence at the point W where the ordinate from the sun crosses the ellipse normally, the force thence taken from the aphelion A, is half the part of the force taken from the aphelion to the perihelion; but there ⊙W is the distance from the sun, itself half the latus rectum. And the impetus from some journey considered is to that considered for a half revolution, as the angle of the circulation to two right angles. Moreover I understand the force from gravity, or from the attraction impressed by itself alone, without detractions nor with opposite impediments, computed from an expressed trial of libration.
[We consider now the equality of the areas traced out in equal times to be an expression of the conservation of angular momentum of the planet : there being no external torque acting to make this quantity change. Here the forces can only be an expression for the gravitational forces acting inwards at different angle throughout the motion; recall Newton's use of a polygon of forces to generate the centripetal force by a series of impulses.]

(27) *But it is worthwhile to explain more clearly the whole revolution of the planet from the assigned causes, and the steps towards and away from the sun.* Therefore with the planet at the maximum moving away point A, or certain smaller distance may be put in place and both a certain centrifugal trial of the circulation perturbation as well as the attraction of gravity is examined, as if the sun were closer. It is still at that distance, clearly at the more distant vertex from the sun the gravity is stronger by twice as much as the centrifugal trial (by 21) because the distance ⊙A of the aphelion, or of the more distant vertex from the sun, or from the focus is greater than half the latus rectum ⊙W. And thus the planet falls towards the sun along the path AMEWΩ, and the force increases continually by descending, as by the acceleration from gravity, as long as the new action of gravity remains stronger by being double the new centrifugal attempt ; for

as long as the force increases on approaching, above the force moving away, and thus the velocity of the approaching increases, while it may arrive at a place where the two new forces, that is at the place W, where the distance from the sun ⊙W is equal to half the latus rectum. Here therefore the velocity of approaching is a maximum, and the increase ceases (by 24). But after that the planet may approach towards the sun as far as to Ω, yet the velocity of the approach finally decreases, with the centrifugal trial prevailing over the force of gravity ; and that may be continued for some time, while the centrifugal forces are collected together into one force, from the beginning A, thus far, the forces of gravity, also from the start and thus far gathered together, are used up completely, or when the whole force of recession (considered from the individual centrifugal forces gathered together) finally may be equal to the whole force of approaching (from the forces of gravity continually repeated I consider), where the approach from everything may cease, and this is the place of the perihelion Ω, at which the planet is nearest to the sun. But afterwards by a continued motion, since until this point it has approached, now it begins to recede, and extends from Ω through X towards A. For double the centrifugal attempt which prevails over gravity has began, thence from W as far as to Ω at this stage it keeps on prevailing from Ω as far as to X, and hence since from Ω the planet may begin as it were to be moving anew, certainly with the former contrary forces mutually removed, also the recession thence from Ω prevails, and the velocity of recession continually increases until at X, but again its increment, or the new force decreases, until that new force for receding, or twice the centrifugal trial, of the new force for receding, or of gravity again shall be equal, evidently at X. And thus the maximum velocity of recession is at X. And from there gravity prevails, or the new force increasing, although hitherto the whole force of recession may prevail long enough, or the sum of all the receding forces acquired thence at Ω, above all the impediments of approaching, thence anew the force from Ω. But since here it may increase more than that, after X, at last for that shall be equal at A, where they mutually cancel out, and the recession ceases, that is it has returned to the aphelion A. And thus with all the former contrary forces used up in the compensation of equality, the situation returns to the first state ; and thus the whole performance may be repeated perpetually, while with the orbit completed perfectly for a long time, a notable change to the constitution of the orbit may be asserted.

(28) Therefore in the motion of a planet in an ellipse we have six especially noteworthy points : indeed four are along the way, A and Ω aphelion and the perihelion. Likewise E and B are the mean distances, (for ⊙B or ⊙E is half the major axis AΩ, and from that the arithmetical mean between ⊙A the maximum, and ⊙Ω the minimum digression) and with the two points added by us, W and X the ends of the latus rectum WX applied to the axis at the ordinate focus ⊙, which are the points of the maximum velocity, that one W receding, this one X approaching (by 21). Where also (by 26) the force from the continued action of gravity considered from A as far as W is exactly half of this, which is considered in the whole descent from A as far as Ω and similarly considered from Ω as far as to X, half of that which is considered from Ω as far as to A : and generally the forces considered from gravity through AW, WΩ, Ω X, XA are equal.

(29) Now is the time, that we may examine *the causes, which kind of ellipses the planets define. the focus of the ellipse may be given* ⊙, which is the place of the sun.

Now with the position given A where the sun begins to attract the planet, or so that the maximum distance of the planet, the vertex may be given more remote from that focus ellipse of the ellipses. Again with a given account of gravity or of the strength, by which the sun begins to pull on the planet, according to the centrifugal attempt, from which it may loosen the planet from a circular orbit, and it struggles to be repelled by the sun, and hence the ellipse is given with the principle latus rectum WX, or the applied ordinate at the focus ⊙. For with ⊙A given, it is to ⊙W the semilatus rectum, in the given ratio of the attraction of the sun to twice the centrifugal trial. Because if now the fourth part of the latus rectum may be taken away from the maximum digression given A⊙, the amount left will be to A⊙, as A⊙ to AΩ: therefore the major axis of the ellipse AΩ is given, or the transverse side. Therefore with the points ⊙, A, W or X given and Ω, and hence again C the centre of the ellipse, and the other focus F, and the transverse axis BE, and thus the ellipse. Nor less all may be given if for A initially Ω may be given.

(30) From these likewise it is apparent, *whatever kind of ellipse, or within which that circle may be contained, no other conic section may be described by the planet. And indeed a circle arises*, when the attraction of gravity, and twice the centrifugal force from the circulation arising from the initial attraction are equal ; for thus they will remain equal, with no cause emerging of approaching or of receding, but with the initial attraction (or in the state of the first contrary forces of approaching or of receding cancelling each other, which are equal at the beginning, that is at the aphelion or perihelion), and twice the centrifugal trial, are unequal, just as (by 25) the simple centrifugal attempt shall be less than the attraction, an *ellipse* will be described; and with the attraction prevailing the initial position is the aphelion, without there the double centrifuge arising may prevail, it is a perihelion. If the simple centrifugal trial shall be equal to the attraction, a *parabola* will arise; if greater, a hyperbola will arise, of which the focus shall be within the sun itself. But if the aforementioned shall not be gravitate but levitate, neither will it be drawn towards, but repelled from the sun, an opposite *hyperbola* will arise, whose focus shall be outside the sun itself.

Now there are two outstanding matters remaining in this argument, the one, as we have explained by which the motion of the aether may make the planet heavy or be pushed towards the sun, and indeed in the duplicate ratio of the more close ; then which shall be the cause of the comparative motions between diverse planets of the same system, thus so that the periodic times shall be in the three on two power of the mean distances, or because it returns with the same, the major axes of the ellipses: that is, the motion of the solar vortex or of the aether constituting each system must be explained more distinctly. But these with other matters requiring to be examined in more detail cannot be included in a paper of this brevity, and which may appear agreeable to us, it will be published separately more correctly.

TENTAMEN DE MOTUUM COELESTIUM CAUSIS.

Ex Actis Erud. Lips. ann. 1689.

Constat veteres, praesertim qui *Aristotelis*, & *Ptolomaei* placita secuti sunt, nondum agnovisse naturae majestatem, quae nostro demum, & praecedenti aevo praesentius illuxit ; ex quo *Copernicus* pulcherrimam Pythagoreorum Hypothesin, quam ipsi fortasse magis suppicione libasse, quam recte constituisse videntur, e tenebris revocatam, summa simplicitate phaenomenis satisfacere ostendit. *Tycho* autem *Copernicum* in summa systematis (excepta Solis, terraeque transpositione) secutus, ad observationes solito accuratiores animum adjecit, & orbium solidorum apparatum minime decorum ex coelo sustulit. Etsi autem ex *Herculeis* laboribus suis non satis fructus perceperit, partim praejudiciis quibusdam exclusus, partim morte praeventus, divina tamen providentia factum est, ut observationes ejus, & molimina venerint in manus Viri incomparabilis *Johannis Kepleri* cui fata servaverant, ut primus publicaret mortalibus.

Jura poli, rerumque fidem, legesque Deorum.

Hic ergo invenit, quemlibet planetam primarium orbitam describere ellipticam, in cujus altero focorum sit Sol, ea lege motus, ut radiis e Sole ad planetam ductis, areae semper abscindantur temporibus proportionales. Idem deprehendit, plures planetas ejusdem systematis habere tempora periodica in sesquiplicata ratione distandarum mediarum a Sole, mire prosecto triumphaturus, si scivisset (quod praeclare *Cassinus* notavit) etiam *Jovis,* & *Saturni* satellites easdem leges servare, respectu suorum planetarum primariorum, quas isti erga Solem. Sed tantarum tamque constantium veritatum causas dare nondum potuit, tum quod intelligentiis, aut sympathiarum radiationibus inexplicatis haberet praepeditam mentem, tum quod nondum illius tempore Geometria interior, & scientia motuum eo, quo nunc, profecissent. Aperuit tamen & rationibus indagandis aditum. Nam ipsi primum indicium debetur verae causae gravitatis, & hujus naturae legis, a qua gravitas pendet, quod corpora rotata conantur a centra recedere per tangemem, & ideo si in aqua festucae vel paleae innatent, rotato vase aqua in vorticem acta, festucis densior, atque ideo fortius quam ipsae, excussa a medio, festucas versus centrum compellit ; quemadmodum ipse diserte duobus, & amplius locis, in Epitome Astronomiae exposuit ; quanquam adhuc subdubitabundus, & suas ipse opes ignorans, nec satis conscius quanta inde sequerentur, tum in Physica, tum speciatim in Astronomia. Sed his deinde egregie usus est *Descartes*, etsi more suo autorem dissimulavit. Miratus autem saepe sum, quod *Descartes* legum coelestium a *Keplero* inventarum rationes reddere ne aggressus est quidem, quantum constat ; sive quod non satis concillare posset cum suis placitis, sive quod felicitatem inventi ignoraret, nec putaret tam studiose a natura observari.

Porro cum minime physicum videatur, imo nec admirandis Dei machinamentis dignum, *intelligentias* peculiares itineris directrices assignare sideribus, quasi Deo deessent rationes eadem corporeis legibus perficiendi ; & vero *orbes solidi* dudum sint explosi, *sympathicae* autem, & magnetismi, aliaeque id genus abstrusae qualitates, aut non intelligantur, aut ubi intelliguntur, corporearum impressionum effectus appariturae judicentur; nihil aliud ego quidem superesse judico, quam ut causa motuum coelestium a motibus aetheris, sive ut astronomice loquar, ab *orbibus deferentibus* quidem, sed *fluidis,* oriantur. Haec sententia vetustissima est, etsi neglectae Nam *Leucippus Epicuro* prior eam adeo expressit, ut in systemate formando ipsum adhibuerit *δινης* (*vorticis*) nomen, & audivimus quomodo *Keplerus* motu aquae in vorticem actae gravitatem adumbraverit. Et ex itinerario *Monconisii* discimus, jam tum *Torricelliii* fuisse sentenuam (& ut suppicor, etiam *Galilaei*, cujus iste discipulus erat) totum aetherem cum planetis motu Solis circa suum centrum acti circumagi, ut aqua a bacula in media vasis quiescentis circa suum axem rotato ; & ut paleas, seu festicus aquae innatantes, sic astra, media propiora, celerius circumire. Sed haec generaliora non difficulter in mentem veniunt. Nobis vero propositum est, ipsas motuum leges distinctius explicare, quod longe altioris indaginis esse, res docebit. Et cum aliqua in eo genere nobis lux affulserit, & inquisitio commode admodum, & naturaliter successisse videatur, in eam spem erectus sum, veris motuum caelestium causis a nobis appropinquatum esse.

Constat et egregiis Gilberti cogitationibus, omne corpus mundanum majus , quoad nobis cognitum est, Magnetis referre naturam, et praeter vim directricem, polos quosdam respicientem, vim habere attrahendi cognata (minimum) corpora intra sphaeram suam, quam in terrestribus vocamus gravitatem, et analogia quadam ad sidera transferemus. Sed non satis constat, quae sit vera phaenomeni tam late patentis causa, et utrum eadem quae in Magnete. Quanquam autem problema demonstratione solvi nondum possit, habemus tamen quae mirifice consentiunt inter se magnaque verisimilitudine commendantur. Equidem asseri, Attractionem Gravium fieri radiatione quadam corporea, immaterialia enim explicandis corporeis phaenomenis adhiberi non debent. Deinde consentaneum est, esse in globi corpore conatum explosivum materiae inconvenientis sive perturbantis seu non satis apto ad motus liberrime exercendos loco positae, unde per circumpulsionem attrahatur alia consentiens seu motum ejusmodi habens, ut motum attrahentis in testinum minus perturbet, exemplo flammae, in qua expulsionem unius et attractionem alterius ipsi sensus docent. Quodsi quis jam altius ista repeti desideret, is cogitet Globum fluidum fuisse, atque adeo in se motum varium intestinum habuisse, motibus ambientium sese accommodantem, et instar guttae olei in aqua natantis fuisse rotundatum, ut ambientia quam minimum perturbarentur, et tunc quoque cum paulatim induruit, pervium mansisse et meatus convenientes motibus fluidi residui ingredientis retinuisse. Et quidem natura fluidi est motus intestinos habere varios, qui ubi ab ambientibus coercentur ne avolet materia, redeunt in se ipsos adeoque in circulares abeunt, et circulos magnos affectant, quia ita maximum retinent conatum recedendi. A quo jam detruduntur ea corpora, quibus minus hujus fluidi includitur seu in quibus minor conatus recedendi. Itaque si vis centrifuga adhibenda est ex Kepleri invento, non deduci debet ex motu aetheris in aequatore et parallelis qui ad axem telluris detruderet, sed ut jam olim annotare memini, circulis magnis quorum idem cum globo centrum, quales sunt meridiani, exemplo ejus motus qui in magnetis atmosphaera apparet, licet enim in fluido motus sit circularis varius in omnes partes in circulis magnis quibuscunque, nihil tamen prohibet esse quosdam polos, per quorum meridianos sit validior materiae cujusdam cursus ad systematis situm accommodatus. Ita variae causae assignatae coincidunt inter se hac explicandi ratione habemusque simul radiationem sphaericam, attractionem magnetis, explosionem perturbantis, fluidi motum intestinum, circulationem atmosphaerae conspirantes vim centrifugam. Sed quaecunque sit causa gratitatis, sufficere nobis potest Globum attrahentem radios materiales radiis lucis analogos propellere seu impulsuum lineas emittere in omnem plagam a centro recedentes ; non quasi partes a tellure ad grave usque pervenire necesse sit, sed quod materia materiam contiguam impellente impetus propagetur ut in lumine et sono, et liquoribus motis. Quanquam erraverint, quibus persuasum fuit propagationem alicujus effectus sensibilis fieri posse in instanti. Radii autem ut ita dicam Magnetici seu quibus attractio efficitur, consistunt in conatu recessivo fluidi cujusdam insensibilis, subtilissime quidem divisibilis, sed confertissimi, cui cum interponantur corpora porosa, qualia sunt terrestria quae non tantundem in pari spatio materiae a centro recedere conantis continent adeoque minora levitate praedita sunt, necesse est fluido emisso praevalente, terrestria versus centrum detrudi. Unde apparet esse debere aliam Materiam fluidam fluido illo, quod gravitatem facere et quod a centro propelli diximus, longe subtiliorem nec directionem ejus sequentem, sed proprios suos exercentem motus, non minus quam ipsa materia magnetica in aere aut aqua facit. Nam necesse est ut materia quaedam alia transeat per poros corporum terrestrium minores sed creberrimos, priorem excludentes. Et quo major pars corporis terrestris materiae priori impervia est nec nisi alteram illam longe tenuiorem admittens, eo gravius specie sit corpus seu solidius censeri potest. Et fieri posset ut omnia corpora terrestria constarent ex massa homogenea adeoque poros subtiliores ubique aequaliter disseminatos habente, gravitas autem specifica major minorve ex minore majoreve hiatuum ejus et porositatum illarum crassiorum seu gravisico fluido perviarum copia oriretur, prorsus ut ex eodem metallo corpora diversae gravitatis specificae formari possunt, quorum alia pro varietate cavitatum in aqua subsident, alia natabunt. Caeteris autem paribus, eademque existente gravitate specifica, solicitatio tamen ad centrum tendendi major minorve erit pro quantitate radiationis, quae aestimanda est ad exemplum lucis. Quemadmodum enim dudum demonstratum est a Viris doctis, corpora illuminari a lucido in ratione reciproca duplicata distantiarum, ita dicendum est corpora quoque attracta gravitare tanto minus quanto majus est quadratum distantiae ab attrahente. Utriusque eadem et manifesta ratio est. Nempe circa centrum lucidum vel radians R (fig. 19) describantur superficies sphaerieae concentricae per A et per L vel ductis rectis RAL, RCN abscindantur harum portiones quae sunt arcubus similibus et similiter positis ABC, LMN, rotatis circa axem seu radium arcus bisecantem RBM; porro lux vel vis attractiva per unamquamque superficiem uniformiter est diffusa seu partes aequales ejusdem superficiei aequaliter illuminantur sive solicitantur; itaque tota lux vel vis superficiei ABC est ad lucem vel vim superficiei LMN in ratione composita intensionis seu illuminationis et extensionis seu superficierum ; sed tantum est lucis seu vis attractivae in superficie ABC quantum in superficie LMN, ergo intensiones sunt reciproce ut extensiones , hoc est illuminationes vel gravitatis solicitationes sunt reciproce ut superficies, sed superficies ABC et LMN sunt ut quadrata diametrorum RA, RL. Itaque illuminationes vel gravitatis solicitationes sunt reciproce ut quadrata distantiarum a centro radiante vel attrahente. Hoc autem a priori nobis deprehensum, mox iterum sua sponte a posteriori per calculum analyticum ex phaenomenis planetarum communibus ductum nascetur mirifico consensu rationum et observationum, et insigni confirmatione veritatis. Quae enim sequuntur, non constant

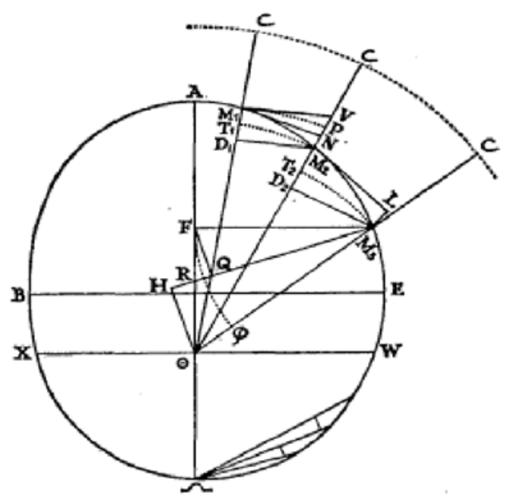
Hypothesibus, sed ex phaenomenis per leges motuum concluduntur sive enim detur sive non detur attractio planetarum ex sole, sufficit a nobis eum colligi accessum et recessum, hoc est distantiae incrementum vel decrementum , quem haberet si praescripta lege attraheretur. Et sive circuletur revera circa solem, sive non circuletur, sufficit ita situm mutare respectu solis ac si circulatione harmonica moveretur, et proinde Principia intelligendi mire simplicia et foecunda reperta esse, qualia nescio an olim homines vel sperare ausi fuissent. Quantum autem hinc de ipsis motuum causis sit concludendum prudentiae cujusque aestimandum relinquemus, fortasse enim eo res jam perducta est, ut Poeta intelligens non amplius dicere ausit Astronomis:

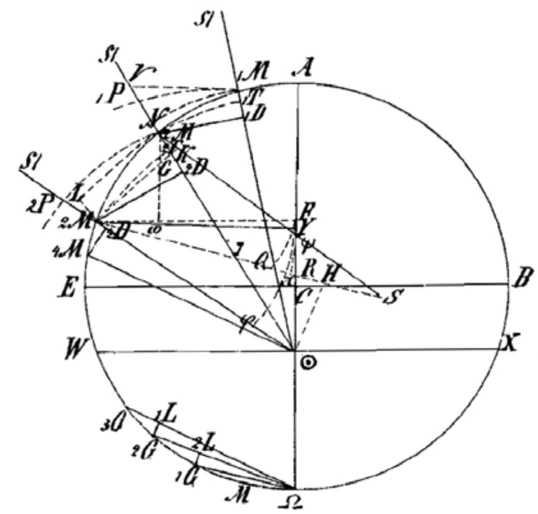
Talia frustra
Quaerite quos agitat mundi labor, at mihi semper
Tu quaecunque paret tam crebros causa meatus
Ut superi voluere late.

(1) Ut ergo rem ipsam aggrediamur, ante omnia demonstrari potest, secundum naturae leges *omnia corpora, quae in fluido lineam curvam describunt, ab ipsius fluidi motu agi.* Omnia enim curvam describentia ab ea recedere conantur per rectam tangentem (ex natura motus) oportet ergo esse quod coerceat. Nihil autem contiguum est, nisi fluidum (ex hypothesi) & nullus conatus coercetur, nisi a contiguo & moto (ex natura corporis) fluidum ergo ipsum in motu esse necesse est.

(2) Hinc sequitur, *planetas moveri a suo aethere*, seu habere orbes fluidos deferentes vel moventes. Omnium enim consensu lineas curvas describunt, nec possibile est phaenomena explicari, suppositis motibus rectilineis tantum. Itaque (per praecedentem) moventur a fluido ambiente. Idem aliter demonstrari potest, ex eo quod motus planetae non est aequabilis; seu aequalibus temporibus aequalia spatia describens. Unde etiam necesse est, ut a motu ambientis agatur.

(3) *Circulationem* voco *Harmonicam*, sive velocitates circulandi, quae sunt in aliquo corpore, sint radiis, seu distantiis a centra circulationis reciproce proportionales, vel (quod idem) si ea proportione decrescant velocitates circulandi circa centrum, in qua crescunt distantiae a centra ; vel brevissime, si crescant velocitates circulandi proportione viciniarum. Ita enim si radii seu distantiae crescant aequabiliter, seu





arithmetice, velocitates decrescent harmonica progressione. Itaque non tantum in arcubus circuli, sed & in curva alia quacunque describenda circulatio harmonica locum invenire potest. Ponamus mobile M ferri in curva quavis ${}_3M_2M_IM$ (vel ${}_1M_2M_3M$) & aequalibus temporis elementis describere (Fig. 18), elementa curvae, ${}_3M_2M$, ${}_2M_1M$, intelligi potest motus compositus ex circulari circa centrum aliquod, ut ⊙ (velut ${}_3M\,{}_2T$, ${}_2M_1T$) & rectilineo velut ${}_2T_2M$, ${}_1T_1M$ (sumtis ${\odot}_2T$ aequ. ${\odot}_3M$ et ${\odot}_1T$ aequ. ${\odot}_2M$), qualis motus intelligi etiam potest, dum regula, seu recta rigida indefinita ⊙n, movetur circa centrum ⊙, & interim mobile M movetur in recta ⊙n. Nihil autem refert, quis sit motus rectilineus, quo ad centrum acceditur, vel ab ipso receditur (quem voco *motum paracentricum*) modo circulatio ipsius mobilis M, ut ${}_3M_2T$ sit ad circulationem aliam ejusdem mobilis, ${}_2M_1T$, ut ${\odot}_1M$ ad ${\odot}_2M$, hoc est si circulationes aequalibus temporum elementis factae sint reciproce ut radii. Cum enim arcus ipsi elementarium circulationum sint in ratione composita temporum, & velocitatum, tempora autem elementaria assumantur aequalia, erunt circulationes ut velocitates, itaque & velocitates reciproce ut radii erunt, adeoque circulatio dicetur harmonica.

(4) *Si mobile feratur circulatione Harmonica* (quicumque sit motus Paracentricus) *erunt areae radiis ex centro circulationis ad mobile ductis abscissae temporibus insumtis proportionales, & vicissim.* Cum enim arcus Circulares Elementares, ut ${}_1T_2M$, ${}_2T_3M$, sint incomparabiliter parvi, respectu radiorum ${\odot}_2M$, ${\odot}_3M$, erunt differentiae inter arcus & sinus eorum rectos (ut inter ${}_1T_2M$, & ${}_1D_2M$) ipsismet differentibus incomparabiles, ac proinde (*per analysin nostram infinitiorum*) habentur ea pro nullis, & arcus, ac sinus pro coincidentibus. Ergo ${}_1D_2M$, ad ${}_2D_3M$, ut ${\odot}_2M$ ad ${\odot}_1M$, seu ${\odot}_1M$ in ${}_1D_2M$ aequ. ${\odot}_2M$ in ${}_2D_3M$, ergo & aequantur horum dimidia triangula nempe ${}_1M_2M\odot$ et ${}_2M_3M\odot$, quae cum sint elementa areae A⊙MA, itaque aequalibus ex hypothesi sumtis temporis elementis, etiam areae elementa sunt aequalia, & vicissim, ac proinde areae A⊙MA sunt temporibus, quibus percursi sunt arcus AM, proportionales.

(5) Assumsi inter demonstrandum *quantitates incomparabiliter parvas,* verbi gratia differentiam duarum quantitatum communium ipsis quantitatibus incomparabilem. Sic enim talia, ni fallor, lucidissime exponi possunt. Itaque si quis nolit adhibere *infinite parvas,* potest assumere tam parvas quam sufficere judicat, ut sint incomparabiles, & errorem nullius momenti, imo dato minorem, producant. Quemadmodum terra pro puncta, seu diameter terrae pro linea infinite parva habetur, respectu coeli, sic demonstrari potest, si anguli latera habeant basin ipsis incomparabiliter minorem, angulum comprehensum fore recto incomparabiliter minorem, & differentiam laterum fore ipsis differentibus incomparabilem; item differentiam sinus totius, sinus complememi & secantis, fore differentibus incomparabilem, item differentiam chordae, arcus, & tangentis. Unde cum hae sint ipsae infinite parvae, erunt differentiae *infinities infinite parvae,* & sinus versus etiam erit infinities infinite parvus, adeoque recto incomparabilis. Et infiniti sunt gradus tam infinitiorum, quam infinite parvorum. Et possunt adhiberi triangula communia *inassignabilibus* illis similla, quae in Tangentibus, Maximisque, & Minimis, & explicanda curvedine linearum usum habent maximum ; item in omni pene translatione Geometria ad naturam, nam si motus exponatur per lineam communem, quam data tempore mobile absolvit, impetus, seu velocitas exponetur per lineam infinite parvam, & ipsum elementum velocitatas, quale est gravitatis sollicitatio, vel conatus centrifugus, per lineam infinities infinite parvam. Atque haec *Lemmatum* loco annotanda duxi, pro *Methodo nostra quantitatum incomparabilium,* & *analysi infinitiorum,* tanquam Doctrinae hujus novae *Elementa.*

(6) Ex his jam consequens est, *planetas moveri circulatione Harmonicae* primarios circa Solem, satellites circa suum primarium, tanquam centrum. Radiis enim ex centra circulationis ductis areas describunt temporibus proportionales (per observationes.) Ergo temporum elementis positis aequalibus est triang. ${}_1M_2M\odot$ aequ. triang. ${}_2M_3M\odot$, & proinde $\odot_1M$ ad $\odot_2M$ est ut ${}_2D_3M$ ad ${}_1D_2M$, quod est circulationem harmonicam esse.

(7) Consentaneum etiam est, Aetherem, seu *Orbem fluidum cujusque planetae moveri circulatione harmonica;* nam supra ostensum est, nullum corpus in fluido sponte moveri linea curva, erit ergo & in aethere circulatio, eamque rationis est credere consentientem circulationi planetae, ita ut sit etiam circulatio Aetheris cujusque planetae harmonica, hoc est orbis planetae fluidus, in innumeros orbes circulares concentricos exiguae crassitudinis cogitatione dividatur, quilibet suam habebit propriam circulationem tanto velociorem proportione, quanta quisque erit propior Soli. Sed hujus motus aethere alias exactius reddetur ratio.

(8) Itaque ponemus planetam *moveri motu duplici, seu composito ex circulatione harmonica, orbis sui fluidi deferentis, & motu paracentrico,* quasi cujusdam gravitatis, seu attractionis, hoc est impulsus *versus Solem, seu planetam primarium.* Facit autem circulatio aetheris, ut planeta circuletur harmonice, non velut motu proprio, sed quasi tranquilla natatione in fluido deferente, cujus motum sequitur, unde nec impetum circulandi velociorem retinet, quem habuerat in orbe inferiore, seu propiore, sed cum elanguescentem, dum superiores (majori velocitati quam suae resistentes) trajicit, continue deponit, & sese orbi quem accedit, insensibiliter accommodat. Vicissim dum a

superioribus ad inferiores tendit, impetum eorum accipit. Idque eo facilius sit, quia ubi semel consensit planetae motus, cum praesentis orbis motu, postea a proximis parum differt.

(9) Explicata circulatione harmonica, veniendum est ad *Motum paracentricum planetarium, ortum ex impressione excussoria circulationis, & attractione solari* inter se compositis. Liceat autem appellare attractionem, licet revera sit impulsus, utique enim Sol quadam ratione tanquam magnes concipi potest ; ipsae autem actiones magneticae a fluidorum impulsibus haud dubie derivantur. Unde etiam vocabimus *Sollicitationem Gravitatis,* concipiendo planetam tanquam grave tendens ad centrum, nempe Solem. Pendet autem species orbitae a speciali lege attractionis. Videamus igitur quae lex attrahendi lineam ellipticam faciat, idque ut consequamur, in Geometria adyta parumper ingrediamur necesse est.

(10) Cum omne mobile a linea curva quam describit recedere conetur per Tangentem, licebit *conatum* hunc vocare *excussorium,* ut in motu fundae, cui aequalis aequiritur vis, quae mobile coercet, ne evagetur. *Hunc conatum metiri licebit perpendiculari ex puncto sequenti in tangentem puncti praecedentis inassignabiliter distantis.* Et cum linea est circularis, hanc *vim* celeberrimus *Hugenius,* qui primus eam Geometrice tractavit, appellavit *centrifugam.* Omnis autem conatus excussorius est respectu velocitatis, seu impetus ex conatu repetito aliquandiutino concepti infinite parvus, quemadmodum & solicitatio gravitatis, quae homogeneae cum ipso est naturae. Unde et eadem causa utriusque confirmatur. Nec proinde mirum est, quod voluit *Galileus,* percussionem esse infinitam comparatione gravitatis nudae, seu, ut ego loquor, simplicis conatus, cujus *vim ego mortuam* vocare soleo, quae agenda demum concipiens impetum repetitis impressionibus, *viva* redditur.

(11) *Conatus centrifugus,* seu conatus excussorius circulationis *exprimi* potest *per* PN *sinum versum anguli circulationis* ${}_1M\odot N$ (vel quod ob differentiam radiorum inassignabilem eadem redit, per ${}_1D_1T$) nam sinus versus aequatur perpendiculari ex uno extremo arcus circuli puncto, in tangentem alterius ductae, qua conatom excussorium expressimus in praecedenti (potest etiam exprimi conatus centrifugus per PV, differentiam radii & secantis ejusdem anguli, cujus differentiae discrimen a sinu verso est *infinitesies infinities, infinite parvum,* adeoque nullisimum, respectu radii.) Hinc porro cum sinus versus sit in duplicata ratione chordae, seu arcus inassignabilis, sive velocitatis, sequitur *conatus centrifugos mobilium aequabili motu aequales circulos describentium esse in duplicata ratione velocitatum, inaequales describentium esse in ratione composita ex quadrata velocitatum, & reciproca radiorum.*

(12) *Conatus centrifugi mobilis harmonice circulantis sunt in ratione radiorum reciproca triplicata.* Sunt enim (per praecedentem) in reciproca radiorum, & directa duplicata velocitatum, id est (quia velocitates circulationis harmonicae sunt reciproce ut radii) duplicata reciproca radiorum; ex simplice autem reciproca, & duplicata reciproca sit reciproca triplicata. Pro calculo sit θa planum constans aequale semper duplo triangulo

elementari $_2M_3M\odot$ seu rectangulo $_2D_3M$ in $\odot_2M$ radium seu r, ergo $_2D_3M$ erit θa : r seu θa divis. per r, jam $_2D_2T$ conatus centrifugus aequ. $_2D_3M$ quadr. divis. per bis $\odot_3M$, ergo aequ. $\theta\theta$ aae $2r^3$.

(13) Si *motus paracentricus* (recessus a centro Ω vel ad ipsum accessus) *sit aequabilis,* & circulatio harmonica, *linea motus* ΩMG *erit spiralis ex centra* Ω*, incipiens, cujus* ea est proprietas, ut segmenta ΩGMΩ, sint *proportionalia radiis,* id est hoc loco chordis ΩG ex centra eductis, sunt enim tam areae, hoc est, segmenta, quam (ob aequabilem recessum) radii temporibus proportionales. MuItae sunt aliae notabiles hujus spiralis proprietases, nec difficilis constructio. Imo *generalis datur methodus* in circulatione Harmonica, si ex radiis dentur tempora, aut velocitates paracentrici motus, aut saItem elementa impetuum seu sollicitationes gravitatis, construendi lineas saItem suppositis quadraturis.

(14) *Sollicitatio paracentrica, seu gravitatis, vel levitatis,* exprimitur recta $_3ML$ ex puncto curvae $_3M$ in puncti praecedentis inassignabiliter distantis $_2M$ tangentem $_2ML$ (productam in L) acta, radio praecedenti $\odot_2M$ (ex centra $\odot$ in punctum praecedens $_2M$ ducto) parallela.

(15) *In omni circulatione harmonica elementum impetus paracentrici* (hoc est incrementum, aut decrementum velocitatis descendendi versus centrum, vel ascendendi a centra) *est differentia, vel summa sollicitationis paracentricae* (hoc est impressionis a gravitate, vel levitate, aut causa simili factae) *et dupli conatus centrifugi* (ab ipsa circulatione harmonica orti), summa quidem levitas, si levitas adsit ; differentia, si gravitas : ubi praevalente gravitatis sollicitatione crescit descendendi, vel decrescit ascendendi velocitas, ut praevalente duplo conatu centrifugo, contra. Ex $_1M$ & $_3M$ normales in $\odot_2M$ sint $_1M$ N & $_3M_2D$; cum ergo triangula $_1M_2M\odot$ et $_2M_3M\odot$ sint aequalia ostensa ob circulationem harmonicam, erunt (ob basin communem $\odot_2M$) et altitudines $_1MN$, et $_3M_2D$ aequales. Jam sumta $_2MG$ aequali L_3M, jungatur $_3MG$ parallela ipsi $_2ML$; igitur congrua erunt triangula $_1MN_2M$, et $_3M_2DG$, et erit $_1M_2M$ aequ. G_3M, & N_2M aequ. G_2D. Porro in recta $\odot_2M$ (si opus producta, quod semper subintelligo) sumatur $\odot P$ aequ. $\odot_1M$, & $\odot_2T$ aequ. $\odot_3M$, erit P_2M differentia inter radios $\odot_1M$, & $\odot_2M$; & $_2T_2M$, differentia inter radios $\odot_2M$ et $\odot_3M$. Jam P_2M aequ. (N_2M seu) $G_2D + NP$, et $_2T_2M$ aequ. $_2MG + G_2D - {}_2D_2T$, ergo $P_2M - {}_2T_2M$ (differentia differentiarum) erit $NP + {}_2D_2T - {}_2MG$, hoc est (quia NP et $_2D_2T$ sinus versi duorum angulorum, et radiorum incomparabiliter differentium coincidunt) bis ${}_2D_2T - {}_2MG$. Jam differentia radiorum exprimit velocitatem paracentricam, differentia differentiarum exprimit elementum velocitatis paracentricae. Est autem $_2D_2T$ vel NP conatus centrifugus circulationis, quippe sinus versus (per 11) & $_2MG$ seu $_3ML$ est solicitatio gravitatis (per praecedentem). Itaque elementum velocitatis paracentricae aequatur differentiae inter duplum conatum centrifugum NP, seu $_2D_2T$, & simplicem solicitationem gravitatis G_2M, aut (quod eadem modo concluditur) summae ex duplo conatu centrifugo, & simplici solicitatione levitatis.

(16) Datis incrementis, aut decrementis velocitatis ascendendi, aut descendendi, datur solicitatio gravitatis levitatisve, aut vice versa. Patet ex praecedenti, nam conatus centrifugus semper dari censetur, cum sit in ratione triplicata reciproca radiorum (per 12).

(17) Aequalibus temporum elementis incrementa angulorum circulationis harmonicae, sunt in ratione duplicata reciproca radiorum. Nam circulationes sunt in ratione composita angulorum, & radiorum, & circulationes elementares, cum sint harmonicae, sunt in ratione reciproca radiorum, ergo anguli elementares sunt in ratione radiorum reciproca duplicata. Tales sunt fere motus apparentes diurni ex Sole spectati (dies enim hic sufficienter exiguae funt partes temporis, inprimis pro planetis remotioribus), qui erunt circiter, in ratione reciproca quadratorum distantiae, ita ut in distantia dupla tantum quarta pars anguli eadem temporis elemento absolvatur, in tripla tantum nona.

(18) *Si ellipsis describatur circulatione mobilis harmonica circa focum tanquam ctrculationis centrum, erunt inter se haec tria circulatio* ${}_2T_3M$ vel ${}_2D_3M$ (haec enim comparabiliter non differunt), *velocitas paracentrica* ${}_2D_2M$, *& velocitas ipsius mobilis* (ex ipsis composita) *in ipso orbita elliptica, nempe* ${}_2M_3M$ *respective ut haec alia tria* ; axis transversus BE; media proportionalis inter defferentiam, & summam distantiae focorum inter se F⊙, & differentiae $\odot\varphi$ distantiarum puncti orbitae ${}_3M$ a focis, ac denique dupla media proportionalis inter $\odot_3M$ et F_3M, distantias ejusdem puncti a duobus focis. Eadem haec suo modo et in hyperbola vera sunt. In parabola quantitatibus quae ibi infinitae sunt evanescentibus, fient circulatio, velocitas paracentrica, et velocitas ex ipsis composita, quae est in ipsa orbita respective, ut latus rectum ; media proportionalis inter latus rectum, & excessum radii super radium omnium minimum, (qui est quarta pars lateris recti) & denique dupla media proportionalis inter radium, & latus rectum. Horum veritas ex communibus conicorum elementis derivari potest, si ponatur rectam ${}_3MR$ curvae (vel ejus tangenti) perpendicularem in MR Axi $A\Omega$ occurrere in R, & in eam ex focis normales agi FQ, ⊙H; patet ⊙H, H_3M, ${}_3M\odot$; esse ipsis ${}_2M\,{}_2D$, ${}_2D\,{}_3M$, ${}_3M\,{}_2M$, hoc est velocitati paracentricae, circulationi, et velocitati in ipsa orbita, proportionales. Sufficit igitur ostendi latera trianguli ${}_3MH\odot$ esse inter se, ut enuntiavimus. Quod facilius fiet, considerando triangula ${}_3MQF$, & ${}_3MH\odot$ esse similla, et praeterea esse F_3M ad $\odot_3M$, ut FR ad ⊙R, unde per analysin communem propositum concludetur. Sequitur hinc, permutatis licet focis, ut alter pro altero centrum circulationis harmonicae, attractionisque fiat, eamdem quae ante, manere rationem circulationis, et velocitatis paracentricae, in quovis puncto.

(19) *Si mobile quod gravitatem habet,* vel ad centrum aliquod trahitur, qualem planetam respectu Solis ponimus, *feratur in ellipsi* (aut alia sectioe coni) *circulatione harmonica, sitque in foco ellipseos centrum tam attractionis; quam circulationis, erunt attractiones, seu gravitatis solicitationes, ut quadrata circulationum directe, seu ut quadrata radiorum, sive distantiarum a foco reciproce.* Hoc ita invenimus, non ineleganti specimine nostri calculi differentialis, vel analyseos infinitiorum. $A\Omega$ sit q; ⊙F e; BE, b (hoc est $\sqrt{qq\text{-}ee}$) $\odot_2M$ radius r; ⊙φ (seu $\odot_2M - F_2M$) $2r - q$, seu per compendium p ; & latus rectum WX sit a, aequ. bb : q. Duplum elementum areae seu duplum triangulum

${}_1M_2M\odot$ quod semper aequale est, sit θa, posito a latere recto, et θ repraesentante elementum temporis semper aequale ; & ${}_2D_3M$ circulatio erit $\theta a : r$ (vid. jam supra 12) porro differentia radiorum ${}_2D_2M$ vocetur dr, & differentia differentiarum ddr. Per praecedentem autem est dr (seu ${}_2D_2M$) ad $\theta a : r$ (seu ad ${}_2D_3M$) ut $\sqrt{ee\text{-}pp}$ ad b. Ergo $brdr = \theta a\sqrt{ee\text{-}pp}$, quae est *aequatio differentialis.* Hujus autem *aequatio differentio-differentialis (secundum Leges calculi a nobis alias in Actis istis explicati)* est $bdrdr + brddr = -2pa\theta dr : \sqrt{ee-pp}$, quarum duarum aequationum ope tollendo dr, ut restet tantum ddr, fiet $ddr = bbaa\theta\theta - 2aaqr\theta\theta,: bbr^3$, unde habetur propositum. Nam ddr, velocitatis paracentricae elementum, est differentia inter $bbaa\theta\theta : bbr^3$ hoc est $aa\theta\theta : r^3$ qui est duplus conatus centrifugus (per 12 supra) & inter $2aaqr\theta\theta,: bbr^3$, hoc est (quia $bb : q = a$) $2a\theta\theta : rr$: oportet ergo (per 15) ut $2a\theta\theta : rr$ sit solicitatio gravitatis ; quae ducta in constantem a : 2 dat $aa\theta\theta : rr$, quadratum circulationis. Sunt ergo solicitationes gravitatis ut quadrata circulationis directae, & proinde ut quadrata radiorum reciproce. Eadem conclusio, et in *hyperbola* et *parabola* succedit, maxime autem in *circulo,* qui est simplicissima est ipsis. Ratio autem discriminis inter has conicas sectioes, & quando circuli & ellipses prae aliis generentur, infra apparebit.

(20) *Planeta idem attrahitur a Sole diversimode, & quidem in duplicata ratione viciniarum* ; ita ut idem duplo vicinior quadruplo fortius, triplo vicinior noncuplo fortius ad descendendum versus Solem nova quadam impressione perpetuo solicitetur. Patet ex praecedenti, posito Planetam ellipsin describere, ac circulari harmonice, ac praeterea continuo impelli versus Solem. Video hanc propositionem jam tum innotuisse etiam viro celeberrimo *Isaacc Newtono,* ut ex relatione Actorum apparet, licet inde non possim judicare, quomodo ad eam pervenerit.

(21) Patet etiam *solicitationem gravitatis in Planetam esse ad conatum Planetae centrifugum* (seu excussorium ab ipsa circulatione harmonica eum rapiente in orbem, atque adeo excutere conante prosectum) ut *distantia praesens a Sole ad quartam partem lateris recti ellipseos planetariae,* seu ut r ad a : 4, ac proinde rationes ipsae gravitatis ad conatum centrifugum sunt planetae distantiis a Sole proportionales.

(22) *Velocitas planetae circa Solem ubique major est velocitate paracentrica, hoc est accedendi ad Solem, vel ab eo recedendi.* Cum enim sit circulatio ad paracentricam ut b ad $\sqrt{ee-pp}$ (per 18, adde calculum ad 19) erit major illa quam haec, ssi $bb + pp$ major quam ee, quod utique sit, cum bb major quam ee, seu b axis transversus, quam e distantia focorum. Id vero in ellipsibus planetariis nobis notis semper contingit, quae non usque adeo a circulis differunt.

(23) *In Aphelio A et Perihelia Ω sola est circulatio sine accessu et recessu, in Perihelio maxima, in Aphelia minima.* In media autem planetae *distantia* a Sole (quae est in ipsis extremis axis transversi, B & E velocitas accessus, recessusve est ad

circularionem in ratione distantiae inter focos, ad axem transversum, seu e ad b. Ibi enim p evanescit.

(24) *Maxima* est *planetae velocitas accedendi ad Solem, vel ab eo recedendi,* cum W⊙ vel X⊙, distantia planetae a Sole, est aequalis dimidio ellipseos lateri recto, tunc enim (per 19 vel 21) sit $ddr = 0$ cum $r = a : 2$. Itaque si ex Sole tanquam centro, dimidio latere recto ⊙W, tanquam radio, describatur circulus, is ellipsin planetae in duobus punctis maxima paracentricae velocitatis W & X secabit, quae in uno ut W erit accedendi, in altero X recedendi. Minima sive nulla est in Aphelio & Perihelio, sive in ellipsis utroque vertice A et Ω.

(25) *Semper in ellipsi, adeoque et semper in planeta conatus centrifugus* recedendi a Sole, seu conatus excussorius *circulationis* harmonicae, *minor est* solicitatione gravitatis, seu *attractione* centrali *Solis.* Est enim (per 21) attractio ad conatum centrifugum ut distantia a Sole, seu foco, ad quartam partem lateris recti, semper autem in ellipsi distantia a foco major quarta lateris recti parte.

(26) *Impetus quos planeta attractione Solis continuata, durante itinere concepit, sunt ut anguli circulationis,* seu quos radii ex Sole ad primum et postremum itineris punctum ducti comprehendunt ; sive ut motus apparens, seu iter spectatum ex Sole. Sic impetus impressus durante itinere A_1M est ad impetum impressum durante itinere A_3M, ut angulus A_1⊙M ad angulum A$⊙_3$M. Sunt enim angulorum incrementa ut impressiones gravitatis (per 17 et 19) ergo et summae summis proportionales, nempe anguli circulatione absoluti, summis impressionum, seu impetibus inde conceptis. Hinc in puncto W ubi normalis ordinata ex Sole ellipsi occurrit, impetus inde ab Aphelia A conceptus, est dimidia
pars impetus concepti ab Aphelio ad Perihelium; est autem ibi ⊙W distantia a Sole, ipsum dimidium latus rectum. Et impetus itinere quovis conceptus est ad conceptum semirevolutione, ut angulus circulationis ad duos rectos. Intelligo autem impetus a gravitate, vel attractione impressos per se ac solos, non detractis, nec computatis impetibus contrariis ab excussorio conatu impressis.

(27) Sed operae pretium est distinctius ex causis assignatis *explicare totam Planetae revolutionem, gradusque accessus et recessus erga Solem.* Planeta igitur in maxima digressione A, seu Aphelio positus minorem quidem et conatum centrifugum circulationis excutientis, et attractorium gravitatis solicitantis experitur, quam si Soli propior esset. Est tamen in ea distantia, nempe in vertice remotiore a Sole fortior gravitas, quam duplus conatus centrifugus (per 21) quia ⊙A distantia Aphelii, seu verticis remotioris a Sole, seu foco major est dimidio latere recto ⊙W. Descendit itaque planeta versus Solem itinere AMEWΩ, & continue crescit descendendi impetus, ut in gravibus acceleratis, quamdiu manet nova gravitatis solicitatio fortior duplo novo conatu centrifugo ; tamdiu enim crescit impressio accedenti, super impressionem recedendi, adeoque absolute crescit accedendi velocitas, donec in locum perveniatur, ubi aequantur duae illae novae contrariae impressiones, id est in locum W, ubi distantia a Sole ⊙W aequatur dimidio lateri recto. Ibi ergo velocitas accedendi est maxima, & crescere desinit

(per 24). Exinde autem etsi pergat planeta accedere ad Solem, usque ad Ω, velocitas tamen accedeni rursus decrescit, praevalente conatu duplo centrifugo super gravitatis impressionem ; idque tamdiu continuatur, donec impressiones centrifugae in unum collectae, ab initio A, hucusque, impressiones gravitatis, etiam ab initio hucusque collectas, praecise consumunt, seu quando totus impetus recedendi (conceptus ex singulis impressionibus centrifugis collectis) toti impetui accedendi (ex gravitatis impressionibus continue repetitis concepto) tandem aequatur, ubi cessat omnis accessio, atque is locus ipsum est Perihelium Ω, in quo Planeta est Soli maxime vicinus. Postea autem continuato motu, cum hactenus accesserit, nunc recedere incipit, tenditque ab Ω per X versus A. Nam duplus conatus centrifugus qui praevalere coeperat super gravitatem, inde a W usque ad Ω adhuc pergit praevalere ab Ω usque ad X, ac proinde cum ab Ω incipiat planeta quasi de novo moveri, quippe prioribus impetibus contrariis mutuo sublatis, praevalet etiam recessus inde ab Ω, & recedendi velocitas continue crescit usque ad X, sed incrementum tamen ejus, seu nova impressio decrescit, donec ista nova impressio ad recedendum, seu duplus conatus centrifugus, novae impressioni ad recedendum, seu gravitati iterum sit aequalis, nempe in X. Itaque in X est maxima recedendi velocitas. Et ex eo praevalet gravitas, seu nova impressio accedendi, licet adhuc satis diu praevaleat totus recedendi impetus, seu summa omnium impressionum recedendi inde Ω acquisitarum, super totum impetum accedendi, inde ab Ω denuo impressum. Sed cum tamen hic magis crescat quam ille, post X, tandem ei sit aequalis in A, ubi mutua destruuntur, & recessus cessat, id est reditur ad Aphelium A. Atque ita omnibus impressionibus pristinis contrariarum aequalium compensatione consumtis, res redit ad statum primum ; atque omnia de integro perpetuis lusibus repetuntur, donec longa dies perfecto temporis orbe, rerum constitutioni mutationem notabilem asserat.

(28) Habemus ergo *in motu Planetae elliptico sex puncta inprimis notabilia* : quatuor quidem obvia, A et Ω Aphelii, & Perihelii. Itemque E & B mediae distantiae, (nam ⊙B vel ⊙E est dimidius axis major AΩ, adeoque medium arithmeticum inter ⊙A maximam, & ⊙Ω minimam digressionem) & duo a nobis addita, W et X extrema lateris recti WX ad axem in foco ⊙ ordinatim applicati, quae sunt puncta maximae velocitatis, illud W recedendi, hoc X accedendi (per 21). Ubi etiam (per 26) impetus a continua gravitatis impressione conceptus ab A usque ad W praecise est dimidius ejus, qui toto descensu ab A usque ad Ω concipitur similiter conceptus ab Ω usque ad X, dimidius ejus qui concipitur ab Ω usque ad A : et omnino impetus a gravitate concepti per AW, WΩ, Ω X, XA sunt aequales.

(29) Tempus jam est, ut tradamus *causas, quae speciem ellipseos Planetariae definiunt.* Datur focus ellipseos ⊙, qui est locus Solis. Dato jam loco A ubi Planetam Sol trahere incipit, vel ut maxima planetae distantia, datur remotior ab hoc foco ellipseos vertex. Data porro ratione gravitatis seu virtutis, qua Sol planetam trahere incipit, ad conatum centrifugum, qua ibi circulatio planetam excutere, et a Sole repellere nititur, hinc datur & latus rectum ellipseos principale WX, seu ordinatim applicata in foco ⊙. Nam ⊙A data, est ad ⊙W semilatus rectum in ratione data attractionis. Solaris ad duplum conatum centrifugum. Quod si jam quarta pars lateris recti detrahatur a maxima digressione data A⊙, erit residuum ad A⊙, ut A⊙ ad AΩ: datur ergo AΩ major axis

ellipseos, seu latus transversum. Datis ergo punctis ⊙, A, W vel X datur et Ω, atque hinc porro & C centrum ellipseos, & alter focus F, & axis transversus BE, adeoque ellipsis. Nec minus dantur omnia si pro A initio daretur Ω.

(30) Ex his simul patet, *quomodo ellipsis, vel qui sub ea continetur circulus, non alia conica sectio, a planetis discribatur*. *Et circulus* quidem oritur, cum attractio gravitatis, et dupla vis centrifuga a circulatione orta ab initio attractionis sunt aequales; ita enim aequales manebunt, nulla existente causa accessus, aut recessus, sed cum initio (vel in statu destructorum priorum impetuum contrariorum accedendi, recedendive, qui initio aequivalent, hoc est in Aphelio vel Perihelio) attractio, & duplus conatus centrifugus, sunt inaequales, modo (per 25) conatus centrifugus simplex sit minor attractione, describitur *ellipsis*; & praevalente attractione initium est Aphelium, sine praevaleat duplus eo natus centrifugus, est Perihelium. Si conatus centrifugus simplex attractioni sit aequalis, *parabola* ; si major, hyperbola orietur, cujus focus intra ipsam sit Sol. Quod si Planeta non gravitate, sed levitate esset praeditus, nec traheretur, sed repelleretur a Sole, *hyperbole opposita* orietur, cujus nempe focus extra ipsam Sol esset.

Duo jam in hoc argumento potissimum praestanda supersunt, unum, ut explicemus quis motus aetheris planetas graves faciat, seu versus Solem pellat, et quidem in duplicata ratione viciniarum ; deinde quae sit causa comparationis motuum inter diversos planetas systematis ejusdem, ita ut tempora periodica sint in sesquiplicata ratione mediarum distantiarum, seu quod eodem redit, axium majorum ellipticorum : id est, distinctius explicar debet motus vorticis Solaris, seu aetheris, Systema unumquodque constituentis. Sed haec cum altius repetenda sint brevitati hujus Schediasmatis includi non possunt, et quid nobis consentaneum visum sit, rectius separatim exponetur.