§5.   Synopsis of Propositions 28 - 36: A miscellany of theorems relating to the eye in optics, the intensity or power in a cone of rays, versions of Alhazen's problem, image viewing, etc.

   Prop. 28: The convergence or turning of the axes of the eyes is taken as a measure of the distance of an object from the viewer.
   Prop. 29: As the previous theorem, but for single eye vision. Note §5.29.2 views these theorems from a modern perspective.
   Prop. 30: It is impossible for the eye to form an image from converging rays.
   Prop. 31: For an object subtending an angle at some distance from the eye along the axis, the tangent of the half-angle varies inversely as the distance from the eye.
   Prop. 32: The previous theorem is extended to include lenses and mirrors. A corollary identifies the equality of the angle of vision with the angle subtended by the rays.
   Prop. 32: The radiant power within a cone of rays from a point source is in proportion to the square of the radius of the base of the intercepting circle.
   Prop. 33:   The radiating powers within cones of rays proceeding from a point source are as the squares of the chords of the semi-angles of the bases of their radiating cones. Rays are made to converge to equal areas for comparison without weakening using mirrors or lenses. The bases of the radiant cones are equidistant from the source. A first corollary extends the result to a radiating body, while a second corollary justifies the use of lenses or mirrors. We should note that this theorem is not the inverse square law, but merely a statement about the amount of radiation proceeding into solid angles. However, the theorem is used when Gregory comes to consider lenses and mirrors of differing diameters in telescopes, regarding their light-capturing abilities.
   Prop. 34: To find the point of reflection from a given regular smooth surface when a point is given on the incidence ray and also a point on the reflected ray. This is the famous problem of Alhazen: Gregory's geometrical solution seems almost trivial; however, one has to find the point L where the ellipse and the curve have a common tangent. One can only presume that this be done by trial and error in a graphical way, and so cannot be considered as a proper solution. The fact that Gregory was to spend more time on the problem when he became acquainted with analytical methods, as detailed on p. 437 of H. W. Turnbull's *James Gregory Tercentenary Memorial Volume*, would indicate that he was not convinced that this theorem actually solved the problem!
   Prop. 35: For the given surface of a transparent medium, the point of refraction is to be found where a point is given on the incident ray and another point is given on the refracted ray. This theorem is the refractive equivalent of the above mirror theorem, and deserves the same comment, for both theorems are of the form of the minimum transit time for the ray connecting the given points - examples of Fermat's Principle. Such theorems in general require calculus to effect a solution, which was not available to Gregory at this time.
   Prop. 35: Tracing the rays from a point to a reflecting or refracting surface to the eye, and locating the position of the image, if it exists.

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§5. Prop. 28.1. Prop. 28. Theorem.

If some point is observed by a person with normal eyesight, then the position of that point or its particular distance from the observer, is always estimated according to the turning of the axes of the eyes.

Let an observer's eyes be situated at the points M and N, and let A be any point in their field of view. If both the eyes are healthy and functioning properly, and there is nothing of interest to catch the observer's attention, then the axes of the eyes MO and NL are always parallel in their natural resting position. But if both eye look at some point A at a finite distance, then the ocular axes MO and NL are turned a little towards MA and NA,
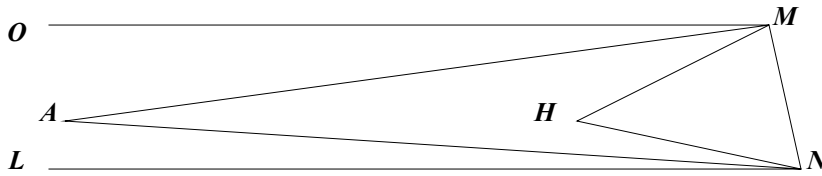
Prop. 28 - Figure 1.

(for the clearest vision can only occur when the eyes are correctly orientated about their axes, which is in good agreement with experience, this indeed then is the most favoured explanation). Again if the person's eyes should be looking at some nearby point H closer than A, then the axes of the eyes MA and NA are turned more to give the directions MH and NH. It is hence possible to estimate the distance of an object from the viewer, a skill sharpened by the everyday experience of rotating the eyes about their axes.

Corollary.

Thus it follows that the point observed in the field of view always lies at the concurrence of the axes of the eyes.

[40]

§5. Prop. 28.2. Prop. 28. Theorema.

Si ambobus oculis sanis, punctum quodlibet aspiciatur ; distantia illius ab oculis singulis, hoc est ipsius locus, semper aestimatur, secundum contortionem axium oculorum.

Sit punctum quodlibet visibile A, oculorum centra MN. Si oculi M, N, nullo vitio laborent, & oculatus cogitabundus haereat nihil intuens; tunc oculorum axes MO, NL semper erunt parallel; nempe in situ suo naturali ; si vero ambo oculi intueantur punctum aliquod A e longinquo, paululum contorquentur axes oculares MO, NL in MA, NA, (quoniam in oculorum axibus solummodo sit visio distinctissima, ut experientia satis constat, ratione vero firmissime demonstratur:), & si ambo oculi intueantur punctum aliquod propinquius H, adhunc magis contorquentur axes oculorum MA, NA, in MH &

NH; atq; quotidiana experientia, majoris vel minoris contorsinis, natura est edocta, ut de distantia seu loco visibilis, conjecturam facere possit.

Corollarium.

Hinc sequitur, locum visibilis duobus oculis visu, semper esse in concursu axium ocularium.

[40]

§5. Prop. 29.1.   Prop. 29. Theorem.

If a point should be observed with only one eye, then the distance of the point from the eye, or the location of the point itself, is always estimated following the application of the eye to the field of view of that point.

A robust and healthy eye looking straight ahead and intercepting the parallel rays from a distant point is able to depict that object most clearly from the collected rays on the retina of the eye. If however a nearby point at some other angle is also shining and the single eye keeps its former position, then it is impossible for this point to be depicted clearly on the retina of the eye, as the ancient writers on optics show *ad nauseam*. The reason for this happening, which is innate to nature, according to these ancient writers, is that it is due to the fluids and moveable humors of the eye. These by some manner appear able to regulate nearby beams, in order that some may be depicted distinctly on the retina of the eye. From the change of these humors, large or small, the mind is able to discern (taught by day to day experience), whither a radiating point is at a small or large distance from the eye. Thus an estimate can be made about the position of the point. In the same manner that it can be said for short-sighted people, whose eyes can only view nearby objects.

Corollary 1.

Thus it follows in the first place, that the position of a point seen with single eye vision is always to be estimated from the point from which the rays emerge.

Corollary 2.

Thus it follows secondly, if many points are sending out diverging rays in the same manner as a single point but at different places, then no conclusion is drawn by the eye: regarding the location nor is there perfect vision.

Corollary 3.

Thus it follows thirdly, if the rays of a single point are sent parallel to the eye, the apparent location of that point is infinitely distant from the eye.

Corollary 4.

Thus it follows fourthly, if the rays from a single point are to converge to another point beyond the eye, it is not possible to assign the location of this point by looking at it (if anyone wanted to do so) , since the rays meet past the eye. An image of this kind generated by many points can conveniently be called an image behind the eye.

§5. Prop. 29.2.   Note.

Theorems 28 and 29 are a foray by Gregory into the worlds of binocular and monocular vision . The accommodation of the eye was not understood at the time of Gregory's account, and the humors within the eye were thought to be involved in some unknown way. Although Gregory was correct in assigning a role to the converging of the eyes using binocular vision, the twisting of the eyes is limited to objects less than some 200 metres away, the aim being to avoid double vision by superposing the images rather than to estimate distance. A number of contributing factors need to be considered in distance estimation, such as the degree of accommodation of the lens needed to form a sharp image slightly different views and other visual clues. Stereoscopic vision arises from the slightly different images presented by each eye for interpretation by the brain.

Predatory animals have evolved binocular vision to give them a beneficial 3-D view of the world, while animals preyed upon have evolved eyes placed on the sides of the head that afford a much greater field of view for their safety, which gives them essentially monocular vision. Monocular vision can still use the cue of the degree of focusing required for a sharp image to judge distance, and also makes use of the other visual cues available for binocular vision : the relative motion induced in the image by moving the head around with respect to the far distance; the loss of definition or texture of distant objects and their increasing dimness and smallness, slowness of movement across the visual field, etc.

§5. Prop. 29.3.   Prop. 29. Theorema.

Si unico oculo, punctum quodlibet Asspiciatur; distantia puncti ab oculo, seu ipsius locus, semper aestimatur, secundum applicationem oculi, ad visionem illius puncti.

Oculus enim sanus, & bene valens, naturalem suum situm retinens, si puncti e longinquo radios intercipiat parallelos, his radiis collectis fortissime pingitur punctum visibile longinquum in oculi retina. Si vero punctum radians, appropinquet, oculo pristinum situm servante, impossibile est ut distincte pingitur radians in oculi retina, ut ad nauseam demonstrant Scriptores Optici. Qua de causa, naturae insitum est, ut humores oculi fluidos, & mobiles, aliquo modo disponat ad radiantia propinqua, ut distincte pingantur in oculi retina ; ex quorum humorum mutatione, magna vel parva, dignoscit natura (quotidianis experientiis edocta) num parvo, vel magno Intervallo, distet radians ab oculi ; atq; ita de illius loco conjecturam facit: eodem modo de Myopibus est dicendum, qui oculum habent, ad radiantia propinqua, a natura fabricatum.

Corollarium 1.

Hinc sequitur primo, locum puncti visibilis uno oculi visi, semper aestimari in puncto, ex quo proveniunt radii illius puncti visibilis.

Corollarium 2.

Hinc sequitur secundo, si unius puncti radii e diversis punctis divergantur, nullam dari, nec locum, nec visionem perfectam.

Corollarium 3.

Hinc sequitur tertio, si unius puncti radii in oculum paralleli mittantur, locum illius puncti apparentem, infinite distare ab oculo.

[41]

Corollarium 4.

Hinc sequitur quarto, si radii unius puncti, ad aliud punctum post oculum convergantur, nullum posse huic puncto locum assignari, nisi (si quis voluerit) post oculum in radiorum concursu; Unde imago ex talibus punctis conflata, Commode vocari potest, imago post oculum.

§5. Prop. 30.1.   Prop. 30. Theorem.

If the rays from an object point converge to an image point situated behind the eye, then it is impossible to form a distinct image.

Indeed the whole image is produced by the eyes, in order that it may view either remote distinct objects, which radiate almost parallel rays of light, or nearby objects which send out diverging rays. But the converging rays (which have been produced by a mirror or lens in a manner not often found in nature) cannot be depicted clearly on the retina of the eye; since the crystalline humor gathers these rays to a point within the vitreous humour, and sends the diverging rays to the retina. A blurred image arises from these disordered rays, as the works of Kepler have shown.

## §5. Prop. 30.1.   Prop. 30. Theorema.

Convergentibus unius puncti radiis, ad punctum situm post oculum, impossibile est fieri distinctam visionem.

Omnis enim oculis fabricatus est, ut aut remota distincta videat, quae radiant quasi parallele, aut propinqua, quae divergentes mittunt radios ; radiis autem convergentibus (qui ab arte, & non a natura ortum habent) nullius oculi retina distincte pingitur ; quoniam chrystallinus humor hos radios in punctum congregat, in humore vitreo, & disgregatos ad retinam mittit, ex qua disgregatione oritur confusa visio : ut videri est apud Keplerum.

§5. Prop. 31.1.   Prop. 31. Theorem.

Consider the centre of the eye receding from or approaching in a straight line towards an object in the field of view. The ratio of the tangent of the semi-angle of the object as seen from the first eye-object distance to the tangent of the semi-angle from the second object-distance varies reciprocally as the first eye-object distance to the second eye-object distance.
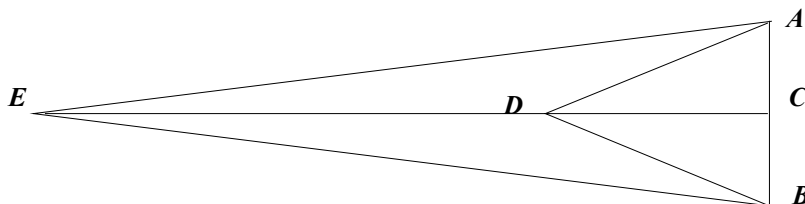
Let the line AB represent the visible object which is bisected

Prop. 31 - Figure 1.

by C, and from C the normal CE is erected to the line AB, on which the centre of the eye at the first position E is located ; and on the same line CE is located the position of the centre of the eye for the second position D. I say, therefore, that the ratio CD to CE thus varies reciprocally as the tangent of the angle CEB to the tangent of the angle CDB. Also (with the ray proceeding from the position CB) the ratio CD to CE varies directly as the tangent of the complement of angle CDB to the tangent of the complement of angle CEB.

[42]

For the tangents of arcs are in reciprocal proportion to the tangents of the complements of the same arcs. Hence, as the ratio CD to CE, so the tangent of the angle CEB to the tangent of the angle CDB. Qed.

§5. Prop. 31.2.   Prop. 31. Theorema.

Si centrum oculi, directea visibili recedat, vel ad illiud accedat ; erit ut tangens semianguli visorii unius stationis, ad tangentuem semianguli visorii alterius stationis ; ita reciproce, distantia centri ocularis, a centro visibilis unius stationis, ad distantiam centri ocularis, a centro visibilis alterius stationis.

Sit visibile recta AB , quae bisecetur in C, & a C erigatur ad rectam AB, normalis CE, in qua collocetur centrum oculare primae stationis E; & in eadem linea CE collocetur centrum oculare secundae stationis D: Dico igitur ut CD, ad CE, ita reciproce tangens anguli CEB, ad tangentem anguli CDB. Erit enim ut CD, ad CE, ita (posito CB radio) tangens complem. anguli CDB, ad tangentem complementi anguli CEB;
[42]
tangentes autem arcum sunt in reciproca proportione tangentium, complementum, eorumdem arcuum; erit igitur ut CD ad CE, ita tangens anguli CEB, ad tangentem anguli CDB ; quod demonstrare oportuit.

§5. Prop. 32.1.   Prop. 32. Theorem.

If any mirror or lens recedes from or proceeds towards an object point in a straight line, then the ratio of the tangent of the semi-angle of the lens for the first object distance, to the tangent of the same semi-angle of the second object distance thus varies reciprocally as the first object-lens distance to the second object-lens distance.

This theorem is demonstrated by the same method, from the preceding proposition.

Corollary.

From these propositions, if the eye, lens, or mirror, should recede from or advance towards the shining point source in a straight line, then the ratio will be as the the tangent of the visual semi-angle of the first station to the tangent of the visual semi-angle of the second position. This will be thus as the first tangent of the semi-angle of the rays to the second tangent of the semi-angle of the rays.

§5. Prop. 32.2. Prop. 32. Theorema.

Si speculum quodlibet, vel lens, directa a puncto radioso recedat, vel ad illud accedat; erit ut tangens semianguli lentis, vel speculi, e puncto radioso unius stationis, ad tangentem semianguli ejusdem alterius stationis ; ita reciproce distantia puncti a centro lentis unius stationis, ad distantiam puncti a centro lentis alterius stationis.

Hoc Theorema demonstratur eodem modo, quo Antecedens.

Corollarium.

Ex hisce, si oculus, lens vel speculum, directe a radioso recedat, vel ad illud accedat ; erit ut tangens semianguli visorii, unius stationis, ad tangentem semianguli visorii , alterius stationis ; ita tangens semianguli radiosi, ejusdem prioris stationis, ad tangentem semiangluli radiosi, alterius stationis.

§5. Prop. 33.1. Prop. 33. Theorem.

The radiating powers within cones of rays, either for illumination or burning, (where the rays are made to converge to equal areas for comparison, without being weakened), are as the squares of the chords of the semi-angles of the bases of their radiating cones.

Let the point A be radiating equally on every side to infinity, and let there be two cones of radiation DAE, BAC, of which the rays are gathered together in equal intervals GF and HI, without being weakened. I say that the strength of the illumination or conflagration[1] in GF, compared to the strength of the illumination or conflagration in HI, is in the duplicate, *i. e. square,* ratio of the chords of half the angle BAC to the chord of half the angle DAE. Indeed as often as the base area of the cone DAE is contained in the base area of the cone BAC, so an equal number of times is the radiant power of the rays of the cone DAE contained in the radiant power of the rays of the cone BAC. In the same way, whatever the proportion is between the bases, so it is the same with the radiant powers. That is, with the radiant cones intercepted by ideal lenses DE and BC radiating into equal areas GF and HI ; but the bases are equal to

Prop. 33 - Figure 1.

the circles of which the radii are chords of the semi-angles of the radiations, as shown by *Archimedes Book* 1, *Sphere & Cylinder,* Prop. 40. The base areas therefore are as the square ratio of these chords[2]. The radiant powers therefore, either for burning, or for illumination, are in the same ratio. Q.e.d. This theorem can be demonstrated in the same way, even if the areas HI and GF are reduced to bare points.
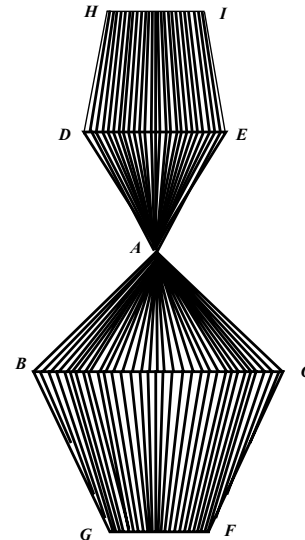
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Corollary 1.

It follows from this theorem, if an extended radiating body is located in the vicinity of A, and its rays converge in the equal areas HI and GF, then the strengths of all these rays for combustion or illumination are in the square ratio of the preceding chords. For the radiating body A can be divided up into its radiating points, and thus because these are proportional magnitudes. The square of the chord of the semi-angle DAE is to the square of the chord of the semi-angle BAC in the same ratio as the power of the body A radiating into the area HI, (for illumination or conflagration) is to the power of the body A radiating into the area GF, (for we suppose that the cones of all the radiating points converge in HI,

[44]

with the rays having equal radiating angles: and we suppose the same for the cones gathered together to GF). Thus the ratio of the powers for one of the first points to one of the second points is indeed as the square of the chord of the half-angle DAE to the square of the chord of the half-angle BAC. Thus, this shall be the ratio for the power of illumination or burning of the whole radiating body A, for all of the first points summed together radiating into the first area HI, to the power of all the second points summed similarly radiating into the second area GF.

Corollary 2.

Thus it follows secondly (if the radiating body should be at A, and with the help of mirrors or lenses DE and BC, the rays of single radiating points incident upon the lenses or mirrors converge to points of the segment HI, and likewise to points of the segment GF.) The ratio of the radiant powers for illumination or burning applied to the segments HI and GF shall be as the squares of the chords of the semi-angles DAE and BAC respectively. Indeed all these radiant magnitudes are in proportion to the squares of the chords of the semi-angles DAE and BAC, illuminating points in the one surface HI and points of the other surface GF. Whenever points of the first and second areas are present in equal amounts, the ratio of the power for a point of the first area to a point of the second area is as the square of the chords of half the angles DAE and BAC. The ratio for illumination or burning will be the same on adding the radiant powers for all the first points in HI, and similarly for all the second points in the segment GF.

§5. Prop. 33.2.   Notes.

 Gregory compares the radiant powers using burning glasses of differing sizes, converging the rays to equal areas. The theory of heat was almost non-existent at the time, and so when Gregory talks about burning we assume he means the heating effect of radiation.

§5. Prop. 33.2.　　　　Prop. 33.　Theorema.

Vires Conorum radiosorum, in illustrando, vel comburendo (radiis nimirum in spatia aequalia absq; debilitatione congregatis) sunt in duplicata ratione Chordarum, suorum semiangulorum radiosorum.

Sit punctum, undiq; & aequaliter in infinitum radians A, sintq; coni duo radiosi DAE, BAC, quorumradii congregentur in spatia aequalis GF & HI, absq; debilitatione: Dico vim illustrationis, vel conflagrationis in GF, ad vim illustrationis, vel conflagrationis in HI, esse in duplicata ratione chordae, semissis anguli BAC, ad chordam semissis anguli DAE. Quoties enim continetur basis coni DAE, in base coni BAC, toties etiam continetur efficacia radiorum coni DAE, in efficacia radiorum coni BAC; & eodem modo, quae proportio est inter bases, eadem est & inter vires, conis scilicet in aequalia spatia congregatis ; Bases autem sunt aequales circulis, quorum radii snt chordae semiangulorum radiosorum, ut demonstrat *Archimedes Lib.* 1, *de sphae. & Cylind.* Prop. 40. Bases igitur sunt in duplicata ratione earum chordarum;vires ergo, sive in comburendo, sive in illustrando, sunt in eadem rarione; quod demonstrare oportuit. Eodem modo poterit hoc Theorema demonstrare, etiam spatia HI & GF sint mera puncta.

Corollarium 1.

Ex hocTheoremate sequitur, si corpus radiosum fuerit in A, & ejus radii congregentur in spatia aequalia HI, GF; eorum vires in comburendo, vel illustrando, esse in duplicata ratione praedictarum chordarum : Dividatur enim corpus radiosum A, in sua puncta radiantia; quoniam itaq; hae sunt magnitudines proportionales; nempe quadratum chordae semissis anguli DAE; quadratum chordae semissis anguli BAC; vis in illustrando; vel comburendo uniuscujusq; puncti corporis radiosi A, in spatia HI; & vis in illustrando, vel comburendo unius cujusq; puncti copporis radiosi A in spatio GF (supponimus enim omnium punctorum conos radiosos in HI,

[44]

 congregatos, aequales habere angulos radiosos; idem supponimus in conis ad GF congregatis) : erit ut una antecedentium, nempe quadratum chordae semissis anguli DAE; ad unam consequentium, nempe quadratum chordae semissis anguli BAC; ita omnes antecedentes, nempe vires in illustrando vel comburendo totius corporis radiosi A, in spatio HI; ad omnes consequentes, nempe vires in illustrando, vel comburendo, totius corporis radiosi A, in spatio GF.

Corollarium 2.

Hinc sequitur secundo (si corpus radiosum fuerit in A, & ope speculorum, vel lentium DE, BC, congregentur radii singulorum radiosi punctorum, in lentes, vel specula incidentes, in puncta spatii HI, & in puncta spatii GF) Vim in illustrando, vel comburendo spatii HI, esse ad vim in illustrando, vel comburendo spatii GF, ut quadratum chordae semissis anguli DAE, ad quadratum chordae semissis anguli BAC; sunt enim omnes hae magnitudines proportionales, quadratum chordae semissis anguli DAE, quadratum chordae semissis anguli BAC, illustratio unius-cujusq; puncti in spatio HI, & illustratio unius-cujusq puncti in spatio GF: Cumq; antecedentes, & consequentes sint numero aequales, propter aequalitatem spatiorum HI, GF; erit ut una antecedentium, nempe quadratum chordae semissis anguli DAE; ad unam consequentium, nempe

quadratum chordae semissis anguli BAC; ita omnes antecedentes, nempe tota illustratio, vel confligratio in spatio HI; ad omnes consequentes, nempe totam illustrationem, vel conflagrationem, in spatio GF.

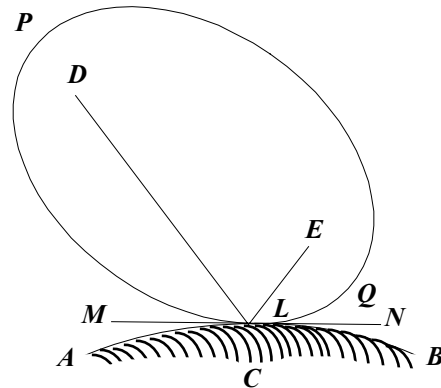## §5. Prop. 34.1.   Prop. 34. Problem.

For a given regular smooth surface, to find the point of reflection from the surface when an incident ray passes through a given point, and the reflected ray passes through another given point.

Let ACB be the given smooth regular surface; and let the points D, E be given; from the foci D, E a spheroid PLQ is described, touching the surface ACB in the point L: I say that L is the point sought. Through L is drawn the plane MLN, and touching the other surface; & through the points DEL

[45]

the other plane is drawn, cutting the same spheroid through the axis; and with a common section making the ellipse LPQ, from which the spheroid is generated, cutting the plane MLN normally in the line LMN too, which touches the ellipse PLQ in the point L. Therefore the ellipse PLQ and the line LMN are in the sought plane of reflection, and the lines drawn from the foci of the ellipse make the angles DLM, ELN equal: evidently the angles of incidence and reflection. L is therefore the point of incidence Q.e.d.

Prop. 34 - Figure 1.

§5. Prop. 34.2.   Prop. 34. Problema.

Data regulari politi superficie, & datis, uno puncto in linea incidentiae, & altero in linea reflectionis ; punctum reflectionis invenire.

Sit data regularis politi superficies ACB ; sintq; data puncta D, E; focis D, E describatur sphaerois PLQ, tangens superficiem ACB in puncto L: Dico L esse punctum quaesitum. Per L ducatur planum MLN, utramq; superficiem tangens; & per puncta DEL
**[45]**
ducatur aliud planum, secans sphaeroidem per axem; & communi sectione faciens ellipsen LPQ, ex qua genera est sphaerois, secans quoq; planum MLN normaliter, in linea LMN, quae ellipsen PLQ tangit in puncto L. Sunt igitur ellipsis PLQ, & linea LMN, in Plano reflectionis quaesito; & lineae ex ellipseos focis ductae, faciunt angulos DLM, ELN, nimirum incidentiae, & reflectionis, aequales; est igitur L punctum incidentiae, quod ostendendum erat.

§5. Prop. 35.1.   Prop. 35. Problem.

For the given surface of a transparent medium, the point of refraction is to be found where a point is given on the incident ray and another point is given on the refracted ray.
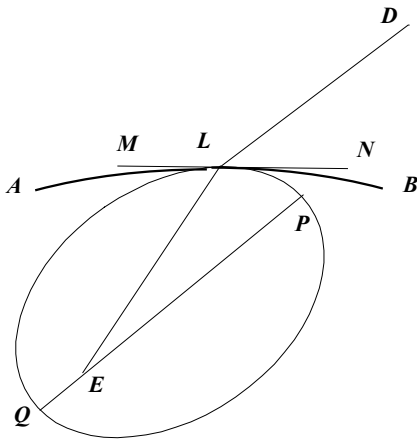
Prop. 35 - Figure 1.

Let the surface of regular density be AB, and let the given point on the line of incidence be E, and on the line of refraction : from the more distant focus E, a spheroid QLP of the dense medium is described, touching the dense surface in the point L, thus as the line DL is parallel to the axis of the spheroid: I say that L is the sought point of refraction. Through L is drawn the plane MLN tangent to the other surface; and through the points D, L, and P another plane is drawn cutting the axis of the spheroid and making a common elliptic section with the dense medium normal to the plane MLN, which therefore is the surface of refraction. Therefore the ray DL is parallel to the axis QP, incident in that less dense mediun and refracted through E in the denser medium. In the opposite direction, the ray EL passing through the point L is refracted through D. Therefore the point L is the point of refraction sought, Q.e.d.

§5. Prop. 35.2.   Prop. 35. Problema.

Datis regulari densi superficie, & dats, uno puncto in linea incidentiae, & altero in linea refractionis ; punctum refractionis invenire.

Sit regularis densi superficies AB, sitq; datum punctum in linea incidentiae E, & in linea refractionis D : foco remotiore E, describatur sphaerois densitatis QLP, tangens superficiem densi AB in puncto L, ita ut recta DL sit axi sphaeroidis QP parallela: Dico L esse punctum refractionis quaesitum. Per L ducatur planum MLN utramq; superficiem tangens ; perq; puncta D, L, P ducatur aliud planum, quod transibit per axem sphaeroidis,faciens communem sectionem ellipsem densitatis, ad Planum MLN rectam, quae igitur est superficies refractionis ; & ideo radius DL axi QP parallelos, in eam incidens, refringetur in E; & e contra EL ex puncto L refrangitur in D. Punctum igitur L est punctum refractionis quaesitum, quod ostendendum erat.
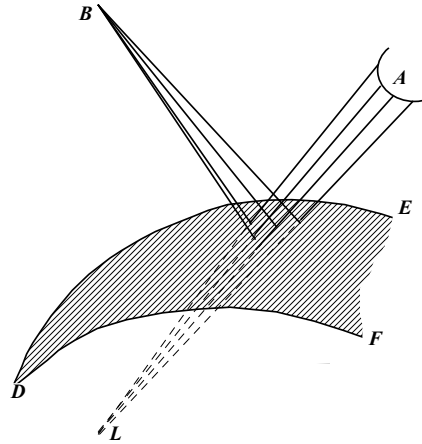
§5. Prop. 36.1.   Prop. 36. Problem.

For a given regular smooth reflecting surface, or the smooth surface of a refracting medium, and with a given visible point as the object and the eye ; it is required to find the location of the visible image of the point formed either by reflection or refraction (provided the image point is distinct).

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Let the regular polished surface be EDF, A shall be the eye, and B the visible point object. Thus with the point B positioned in the line of an incidence ray, and with individual points of the pupil of the eye A present separately in the lines of reflection, the image point of the reflected rays can be found. All the lines of reflection are drawn from the points of the pupil through their points of reflection, which concurr in L, and - provided this happens - the location of the image point B appears to the eye. If however the rays do not concurr in one point, then nothing distinct will be seen, and no point that determines the position of the image of the visible point B is given.

Prop. 36 - Figure 1.

[47]

 All of which are apparent from the corollaries to Prop. 29 of this work: in the same way, the locus of the image can be found for refraction.

§5. Prop. 36.2. Prop. 36. Problema.

Data regulari, densi, aut politi superficie ; & datis, puncto visibili, & oculo ; locum imaginis puncti visibilis reflexum, vel refractum (si modo detur distinctus) invenire.

Sit regularis politi superficies EDF, sit oculus A, & punctum visibile B: positis itaq; puncto B in linea incidentiae, & singulis punctis pupillae oculi A seorsum in lineis reflectionum, inveniantur puncta reflectionum : & a punctis pupilae, per sua reflectionum puncta, ducantur omnes lineae reflectionum, in quarum concursu nempe L, (si modo concurrant) erit locus apparens imaginis puncti B.
[47]
Si vero in uno puncto non concurrant, nullus dabitur, distinctus, & determinatus locus imaginis, puncti visibilis B. Quae omnia patent ex corollariis ad 29 Prop. hujus : eodem quoque modo, inveniendus est locus imaginis in Dioptricis.