§1.1.   **Synopsis Proposition 1.**

In the first proposition the rationale behind the refraction method is explained. There is to be a correspondence set up between reflection and refraction by surfaces derived from conic sections. The former is already well known geometrically at this time, and Gregory intends showing that the latter can also be derives geometrically for surfaces of the same form. The opening premise is the refraction of a light ray towards or away from the normal in entering a more/less dense medium He borrows heavily from Kepler's *Optics* in considering limiting situations where the degree of refraction is extreme. Thus, a transparent sphere with an infinite density will refract a parallel beam in the air through the centre of the sphere: in a less extreme situation the parallel beam can be refracted to pass through a focus of an ellipsoid, which has an ellipse as cross-section. The far focus must be used in order that physically meaningful angles of refraction occur. In a similar manner, a parallel beam in the infinitely dense transparent medium can come to a focus in the air outside a plane bounding the dense medium: in a less extreme situation the parallel beam in the dense medium can be refracted by the surface of a hyperboloid through the focus of the other branch; in this case the cross-section is a hyperbola. The case of the paraboloid corresponds to no refraction. Hence, lenses with parabolic cross-sections will not occur in this work.

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**§1.2.**   **Proposition 1.**

The Proposition shall be to examine: which pray shall be the surface which measures refractions?[1]

It has been commonly observed by those involved in Mathematics, whatever the truth of what else they may say, that light rays passing through a less dense transparent medium and incident obliquely on another denser medium, are refracted at the surface of the second medium, and bend towards the perpendicular, excited by the points of incidence. Conversely, rays crossing the denser medium and incident obliquely on the rarer medium, are refracted by the surface of the second medium, and bend away from the before-mentioned perpendicular. It is not for us to set forth here the origins of this refraction : indeed *Alhazen, Vittellio, Kepler,* and many others have discussed these causes at length; moreover, since the measurements of these effects as set forth by these authors are revealed to be less than reliable, we shall attempt on that account to exert a little influence in the midst of all this confusion - which perhaps will be of some use to mathematicians. But concerning these things which are to be the subject of deeper considerations, it may be permitted to argue a little by analogy, before we approach the subject with geometrical rigor.

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It is clear enough from the elements of Optics that much of Reflection [Catoptrics] and Refraction [Dioptrics] have properties in common; therefore perhaps some common property will remain in the measurement of both reflection and refraction. But all the mystery of reflection that lies hidden in conic sections has been demonstrated - as will be shown in turn - and hence perhaps also a measure of refraction will be concealed therein. For the following cases considered, regular reflection cannot occur unless the reflecting surface is a conic section, and perhaps there may not be a rule for refraction either unless the refracting surface is a conic section also.
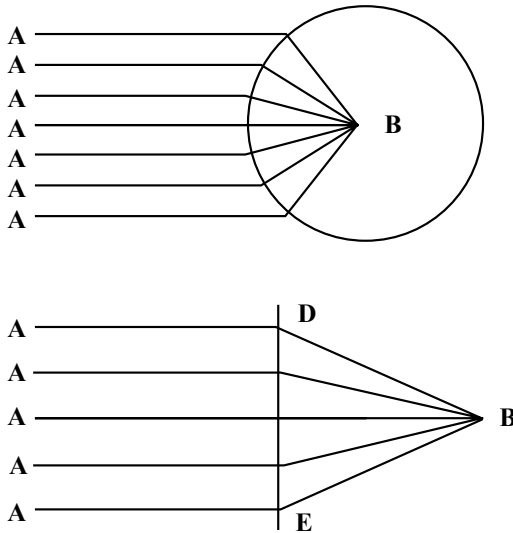
[Figures 1and 2.]

If in truth the reflecting surface is the concavity of a parabola, and the incident rays are parallel to the axis, then they are reflected into the focus. It can be asked therefore, for a given surface of refraction: is it the case that all the rays incident on this surface parallel to a certain specific line can be refracted into any one assigned point ? From the preceding it is probable - if such a surface can be made - that it shall be a conic section. But our search for such a surface making use of analogies may be undertaken by examining extreme situations. Thus, we may consider the medium in which the rays are incident to be the densest possible[2]: in which case the refracted rays will be perpendicular to the surface of the medium (see *Kepler. Ast. Opt. fo. 113*). The conic section is sought therefore for which all the perpendiculars are themselves concurrent in one point: the circle is such a section [Figure 1]. For the second case, the parallel rays are considered to be passing through the densest medium

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[*i.e.* to the left of DE]. The bundle of rays [Gregory calls this the form or shape] leaves along the same lines by which it enters; but the bundle enters with the lines perpendicular to the surface, as hitherto said, and so [tracing the rays backwards] the bundle of rays emerges from the densest medium turned away from the perpendicular lines[3]. Therefore the conic section is sought to which all the perpendiculars are parallel: but the straight line is such a section [Figure 2].

Some parallel rays A, A, A, etc. lying in one plane therefore are supposed to be refracted by the surface of the densest medium, where the refracting surface is a circle; in which case the refracted rays concur to the centre of the circle B. The single point [*i.e.* the focus] from an extreme point of view therefore is satisfactory. Also to be supposed, a number of rays arising from a single point B advancing in the rare medium, are to be refracted by the densest medium - the refracting surface of which is taken as the straight line DE - in which case all the refracted lines emerge parallel in the densest medium. Conversely, if these parallel rays lying in a single plane of the densest medium are considered to be refracted into the more rare medium by the surface DE, (because the

same bundle of rays leaves which enter) all the rays A, A, A, etc., are concurrent in the point B; thus the converse situation is satisfactory from this extreme point of view. If truly, everything is examined carefully, then it will be seem - on account of the aforementioned reasons - that all the rays, either parallel or non-parallel, which are incident on the circular surface of the densest medium for refraction, are concurrent in the centre of the circle. Now we ask: how does this come about? The answer is :- Well, however the line is drawn incident on the circle, (provided they are co-planar) an axis can be drawn parallel to it, and without doubt the circle can be considered as a kind of ellipse, so that any diameter can be called the axis, from which it appears that the special line sought is the axis of a conic section.

For the other case, from observation it will also be apparent that all the parallel rays in a single plane of the densest medium are not so much assigned to a single fixed point, but to any point you wish beyond the line DE, and the component parts concur in B. Also we ask : how does this come about? The reply is :- Well, (supposing the straight line to be the branch of a hyperbola) any point outside the densest medium can be accepted as the location of the focus, from which it can be seen that the focus is the required point of concurrence. But of the two foci of the hyperbola, either real or imaginary [depending on whether we have a real hyperbola or this degenerate straight line case], it will be the point of concurrence which stands furthest from the point of incidence of the rays, otherwise the angle of refraction would be greater than a right angle, which cannot happen [*i.e.* the focus of the far branch of the hyperbola is used].

From these pre-tests of the medium using extreme values, we may attempt to answer the following questions :- By considering rays passing either from the rare medium into the densest, or from the densest into the outermost rare medium,

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by necessity it follows that the rays from one medium incident on another of the same density, to be the mean between the two aforementioned extremes ; but in this case there shall be no refraction. For the parabola, therefore, (which is the mean between the circle and the straight line) all the lines parallel to the axis and co-planer with it coincident on it, ought to be concurrent in the focus by refraction. These are incident from points at the greatest distance, and so the focus shall be at an infinite distance from the vertex of the parabola. Therefore all the rays incident on the parabola, and drawn from the aforementioned imaginary focus, are parallel to the axis. If truly they are parallel to the axis both before and after incidence, then in general they are free from refraction, as is the proposition.

We may therefore conclude from the analogy that one is able to find a surface of refraction for all different kinds of transparent media, which shall be a conic section, in which coplanar parallel rays in one medium are refracted by another medium to concur at a point. Now when the rays are parallel in the denser medium and they concur in the rarer medium, then the surface of refraction approaches almost to the most obtuse of hyperbolas, *i.e.* a straight line. On the contrary, when the rays are parallel in the less dense medium and they concur in the denser medium then the surface of refraction approaches almost to the most obtuse of ellipses, *i.e.* a circle. Truly from these discarded analogous trifles we may come close to more reliable evidence for establishing the scientific origin of refraction.

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§1.3.   **Notes on Proposition 1:**

[1]   The author is unaware of the now customary form of Snell's Law of refraction : he intends to relate the 'optical density' to the focal property of a conic section, which we show below in modern terms for a medium of refractive index *n*. Initially he argues by analogy, assuming that an unknown law of refraction can be applied to conic sections just as the law of reflection can be applied, which certainly is the case as reflecting surfaces can bring parallel rays to a focus. In this matter, he follows the lead of Kepler in considering extreme cases, a ploy still used in understanding new physical phenomena.

[2]   That is, consider an infinite refractive index. This at least has the effect of changing a parallel beam into a focused beam; and conversely, by making use of a circular cross-section and a plane surface respectively. See the *James Gregory Tercentenary Volume*. Page 454 onwards. See also, p. 127 of Kepler's *Optics*, translated by W. Donahue, Green Lion Press, 2000.

[3]   Thus showing the principle of reversibility of a light ray.

<div align="center">[5]</div>

§1.4.   **Propositio 1.**

Propositum sit inquirere, quaenam sit superficies quae metitur refractiones.

Omnibus in Mathesi vel leviter veratis, vulgo notum est, radios luminosos per diaphanum rarius transeuntes, & in aliud diaphamum densius obliquè incidentes, refringi in superficie secundi diaphani, & ad perpendiculares vergere ab incidentiae punctis excitatas ; & è contrario radios per diaphanam densius transeuntes, & in aliud diaphanum tenuius obliq ; incidentes, refringi in superficie secundi diaphani, & a praedictis perpendicularibus divergere. Cujus refractionis causus & elementa non nostrum est hic explicare, abunde enim de his disputarunt *Alkazanus, Vitellio, Keplerus, & mulit alii*: Quoniamve: ob quae de earum mensurâ ab authoribus profaeruntur minus solida videntur, paucula quaedam de hac re (Mathematicis forsan non inutilia) in medium adducere conabimur. De his autem quae altioris sunt considerationis, liceat paululum analogicè disputare, priusquam ad $\alpha\chi\rho\iota\beta\epsilon\iota\alpha\nu$ geometricam accedamus.

<div align="center">[6]</div>

Satis patet ex Opticis elementis, multa Catoptricae, & Dioptricae esse communia ; forsan igitur; & in reflectionum, & in refractionum mensuris, aliquid commune haerebit : Totum autem reflectionem mysterium, in sectionibus conicus latere compertum est; (ut deinceps patebit) forte igitur & refractionum mensura illic latebit. Secundo non fit regularis reflectio, nisi superficies reflectionis sit sectio conica ; fortassis ergo nec regularis refractio, nisi refractionis superficies sit sectio etiam conica.

Si vero superficies reflectionis sit concavitas parabolae, & radii incidentes axi paralleli, omnes reflectuntur in focum : Quaeritur ergo num possit dati superficies refractionis, ita ut omni radii in eam incidentes, speciali cuidam lineae paralleli, refringantur in unum aliquod punctum determinatum? ex praedictis probabile est (si talis detur) hanc superficiem esse conicam sectionem : ut autem talem superficiem analogicè inquiramur, ab extremis incipiatur ; & concipiamus medium in quod incidunt radii esse densissimum ; radii refracti, ad superficiem medii perpendiculares erunt. (*Keplerus Ast. Opt. fo. 113*) Quaeritur igitur sectio Conica, cuius omnes perpendiculares ad sui lineam, in unum punctum concurrant ? talis autem est circulus. Secundo concipiatur illud medium densissimum, per quod transeunt

<div align="center">[7]</div>

 radii paralleli ; & quoniam eisdem lineis egreditur forma quibus ingreditur ; ingreditur autem forma, lineis superficiei perpendicularibus, ut hactenus dictum ; ergo egrediuntur radii, sive forma, e medio densissimo lineis perpendicularibus : Quaeritur igitur sectio conica cujus omnes perpendiculares ad sui lineam sint

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paralleli ? talis autem est linea recta. Supponendo igitur, radios parallelos quotcunq; A, A, A, &c. in uno plano, refringi in superficie medii densissimi, & superficiem refractionis esse circulum ; radii refracti concurrent in centrum circuli B: uni igitur ex extremis est satisfactum. Supponendo etiam radios quotcunq,; ex puncto B existente in medio raro prodeuntes, refringi in medio densissimo cujus superficies refractionis comprehendetur linea recta DE, omnes lineae refractae evadent parallelae in medio densissimo. Si vero hi radii paralleli in uno plano medii densissimi, concipiantur in medii rarioris superficie DE refringi, (quoniam iisdem lineis egreditur forma quibus ingreditur) concurrent omnes radii A, A, A, &c. in punctum B; atq; ita altera ex extremis est satisfactum. Si vero, rem diligenter quis intueatur, videbit (propter praedictas rationes) omnes radios, sive parallelos, sive non parallelos, in superficiem refractionis circularem medii densissimi incidentes, in circuli centrum concurrere. Quaeritur autem unde hoc proveniat ? Respondetur ; videtur hoc provenire ex eo, quod quomodocunq ducatur linea in circulum incidens, (dummodo cum circulo in eodem sit plano) axis ipsi parallelus duci possit ; concipiendo nimirum, circulum esse ellipseps speciem, quaevis illius diameter potest dici axis ; unde videtur ; axem sectionum conicarum, esse lineam specialem quaesitam. Animadverdenti quoque patebit, omnes radios, in uno plano medii densissimi parallelos, non in unum tantum, sed in quodlibet punctum assignatum, extra lineam DE, ad partes B concurrere : Quaeritur etiam unde hoc proveniat ? Respondetur; hoc vedetur provenire ex eo quod, (supponens lineam rectam esse hyperbolam) quodlibet punctum extra ipsam possit sumi loco foci ; unde videtur focum esse punctum concursus quaesitum. E duobus autem focis, vel realibus, vel imaginariis, is erit punctum concursus, qui longissime a radiorum incidentia distat, alioquin angulus refractionis esset major recto, quod fieri non potest. Hisce de extremis praelibatis medium tentemus : Si autem radios provenientes e diaphano raro in densissimum, & e diaphano

densissimo in rarum extrema concipiamus ; necessario sequitur radios ex uno diaphano, in aliud ejusdem densitatis incidentes, esse medium inter praedicta duo extrema ; in hoc autem casu nulla sit refractio ; In parabola ergo (quae media est inter circulum & lineam rectam) omnes lineae axi parallelae, & in eodem cum parabolâ plano, in ipsam incidentes, debent per refractionem concurrere in focum, a punctis incidentiae maximè remotum : at focus iste a vertice parabolae infinitè distat ; omnes igitur radii in parabolam incidentes, & ad praedictum focum imaginatium ducti, sunt axi paralleli : si vero & ante, & post incidentiam, sint axi paralleli, a refractione omnino sunt liberi, quod est propositum. Concludimus igitur analogicè pro omni diaphanorum diversitate, inveniri posse superficiem refractionis (quae sit sectio conica) in quâ lineae parallelae in plano unius diaphani, in altero refractae concurrant in punctum : quo autem densius fuerit diaphanum in quo radii sunt paralleli, & quo rarius diaphanum in quo concurrunt ; eo propius accedit superficies refractionis ad hyperbolarum obtusissimum, id est lineam rectam : & e contra; quo rarius fuerit diaphanum, in quo radii sunt paralleli, & quo densius fuerit diaphanum, in quo concurrunt, eo propius accedit superficies refractionis ad ellipsium obtussimam, id est circulum. Verùm relictis hisce analogiae nugis, ad experientiae scientiarum originis certiora testimonia accedamus.