Prop. 52. Problem.

With the distance and the angle of vision of any visible [object, or previously formed image] *given; to represent the image of that object in whatever given measure ; with the thickness of the lens given too.*

Let AVB be any visible object whatever, either radiating matter, or an image before or after the eye, provided it shall be plane; in which two points A, B are taken. Then the image of the radiant plane, AVB, is to be found, in which the images of the points A, B shall stand apart by the right distance R. In the line AB, any point whatever is taken, near the centre, namely V; from which is drawn VD, perpendicular to the plane AVB, equal to the distance of the given visible object, which is produced indefinitely. And DD taken in this, equal to the thickness of the lens; and it shall become, as AB to VD; thus, R to DI. And if the object is before the eye then the rays diverging from V ; or if past the eye converging to V; [in either case] diverging from the point I, with the help of the lens or mirror, with vertices DD {according to *Prop's.23, 24* etc of this work}. And through I a plane is drawn, parallel to the plane AVB. I say, the images of the points A, B shall be distant by the given right line R. Since indeed, for the object AVB,

[**78**]

supposed a plane, the apices of all the cones of rays of these points converge in the plane MIN. Hence in the plane MIN, as is stated clear enough from the scholium to *Prop.* 43 of this work. [Note: The text in the translator's copy of the O. P. is very unclear on this page, due to type from the previous page showing through.] The apices of the cones of rays of the points A, B therefore shall be the points M, N; and the points M, I, N shall be in a straight line. Since A, V, B are in one right line, indeed the rays AD, BD are in the same plane, for the rays AD, BD are in the same plane as the axis, and therefore the surface of these, either of reflection or refraction, is one and the same; therefore reflected or refracted in one plane with the axis, as the plane cuts the plane of the image in the right line MIN; and on account of the equality of the angles ADV, IDN; the triangles ADV, IDN are similar. And as:

$$DV: DI : : AV : IN;$$

In the same manner, as $\quad DV: DI : : VB : IM;$

but it was, as $\quad\quad\quad DV : DI : : AB : R;$ therefore R, & MN are equal. But the images of the points appear in the apices of the cones of their own rays, that is, in M and N, with the separation for R: which was to be done.

 Also by the same method, the images of visible objects at an infinite distance can be represented in terms of the given measure. *See the diagrams.* Let ADB be the angle of vision, of any visible object at infinite distance, that the right line AB subtends, and it shall be; as AB to R ; thus the perpendicular into the subtended, DV to DI ; & the rays, themselves parallel to DV, diverge from the point I by the lens or mirror, of which the vertices are D, D, by the same manner as previously stated.

Scholium.

But if the right line AB is small; so that it cannot be projected conveniently into a line R equal to itself: the image of AB itself may be projected into a line less than R, which image, by the same method, and if it might be visible itself, again projected into another image equal to R itself. Because if it still may not be possible to happen, the operation

will have to be repeated. Nevertheless, to be noted, if the right line R is too long with respect to the smallness of the lens or mirror ; the image MN is confused and indistinct towards the extremities M and N ; in the middle truly around the axis I of the lens or mirror produced, it will always be most distinct, since the image of the point V of the visible object is projected in I, to the geometrical figure.

   If indeed, the rays of the visible points shall gather in points of the plane of the image, and a white unpolished plane, the rays of the visible object, shining strongly, shall be fixed in the plane of the image, most clearly depicted in the white plane. For on account of the roughness of the plane, the rays of the individual points of the visible object, are concurring into images or points of the white plane. From the points of the same plane, rebounding from which, they strike the eyes of observers; & each visible point will appear depicted to be considered, in that point of the plane, from which all of these rays are reflected ; that is, in the apex of their own radiant cone ; or rather the pencils. But this image, projected by one mirror or lens, always appears in the inverted situation: as is apparent from *Prop. 46* of this work.

   But if the plane placed in the position of the image were polished ; being reflected, all the angles of incidence of the rays of the visible object are equal to the angles of reflection ; which therefore are nearly all reflected into the lens or mirror. And if the eye , from the side of the lens or mirror, may consider the aforementioned image, it will see a small part of that, in the surface of the polished plane. Which are all clear enough, and not in need of demonstration.

[**77**]
Prop. 52. Problema.

Cujuslibet visibilis, distantia, & angulo visorio, datis; illius imaginem in quavis data mensura representare: data quoque crassitie lentis.

Sit visibile AVB, quodlibet, sive materia radians, sive imago ante, vel post oculum, dummodo sit planum; in quo, sumantur duo puncta A, B. Sit igitur invenienda imago plani radiantis, AVB; in qua, punctorum A, B, imagines, distent recta data R. In recta AB, sumatur punctum quodlibet, prope ipsius medium, nimirum V ; e quo ducatur, plano AVB perpendicularis, VD, aequalis distantiae visibilis datae, quae producatur in infinitum: Et sumatur in ea, DD, aequalis lentis crassitiei ; fiatque, ut AB, ad VD ; ita R, ad DI. Et radii ab V divergentes, si visibile sit ante oculum ; vel ad V convergentes, si post oculum divergantur a puncto I, ope lentis, vel speculi, cujus vertices DD ; & per I, ducatur planum, plano visibilis AVB, parallelum. Dico, imagines punctorum A, B, distent recta data R. Quoniam enim, visibile AVB, supponatur planum , apices conarum radiosorum omnium, illorum puncti convergunt in plano MIN. Hunc in plano MIN,   ait e scholium ad *Prop.* 43 hujus satis paret. Sint igitur apices conorum radiosorum, punctorum A, B, puncti M, N; & erunt puncta M, I, N, in una linea recta; quoniam A, V, B, sunt in una recta, radii enim AD, BD, sunt in eodem plana, cum axe & ideo, superficies illorum, sive reflectionis, sive refractionis, una est & eadem; reflectuntur igitur, vel refringuntur, in una plano cum axe, quod planum secat planum imaginis, in recta MIN; & ob aequalitatem angulorum, ADV, IDN; triangula ADV, IDN, sunt similia,

et ut

$$DV: DI :: AV : IN;$$

Eodem modo, ut $\quad DV: DI :: VB : IM;$

erat autem, ut $\quad DV : DI :: AB : R;$ igitur R, & MN, sunt aequales. Imagines autem punctorum A, B apparent in apicibus, conorum suorum radiosorum, hoc est in M, & N. distantibus, recta R: quod faciendum erat.

Eodem etiam modo, possunt imagines visibilium, infinite distantium, in data mensura representari, *v. g.* Sit angulus ADB, angulus visorius, alicujus visibilis, infinite distantium, quem subtendat recta AB, fiatq; ut AB, ad R ; ita perpendicularis in subtendentem, DV, ad DI ; & radii, paralleli ipfi DV, divergantur a puncto I, lente vel speculo cujus vertices D, D , eodem modo, ut hactenus dictum.

Scholium.

Si autem recta AB fuerit parva ; ut non poterit commode projici, in rectam aequalem ipsi R : projiciat ipsius A, B imagino, in rectam, minorem ipso R, quae imago, eodem modo, ac si esset ipsum visibile, rursus projiciatur, in aliam imaginem, aequalem ipsi R: Quod si adhuc fieri non possit, reiteranda erit operatio. Notandum tamen si recta R, sit nimis longa respectu parvitatis lentis, vel speculi ; imaginem MN, fuerat confusam & indistinctam, versus extreminintes M & N; in medio vero I circiter axem; lentis vel speculis producantur; semper erat distinctissimo; quoniam imago puncti visibilis V, projictitur in I, ad figorem Geometricum.

[78]

Si vero; radii visibilis punctorum, congregentur in puncti plani imaginis, & planum album, & impolitum, radios visibilis, fortiter revibrans, figatur in plano imaginis; imago videbitur, clarissime depicta, in plano albo. Nam propter plani impolitiam, radii singulorum visibilis punctorum, in imaginis, vel plani albi puncta concurrentes ; ab iisdem plani punctis, undeque repercussi, oculos videntium feriunt; & unumquodque visibilis punctum , apparebit intuenti depictum, in illi plani puncto, a quo, omnes illius radii reflectuntur ; hoc est in apice sui coni radiosi ; vel potius penicilli: Apparet autem haec imago, unico speculo vel lente projecta, semper situ everso : ut patet per *Prop. 46* hujus.

Sed; si planum, in loco imaginis positum, fuerit politum ; reflectentur, omnes visibilis radii, ad angulos reflectionum, aequales angulis incidentibus ; qui propterea reflectentur pene omnes in lentem vel speculum. Et si oculus, a latere lentis vel speculi, praedictam imaginem intueatur ; videbit illius particulam, in plani politi fuperficie : quae omnia satis manifesta sunt, nec ulla egent demonstratione.

[**80**]
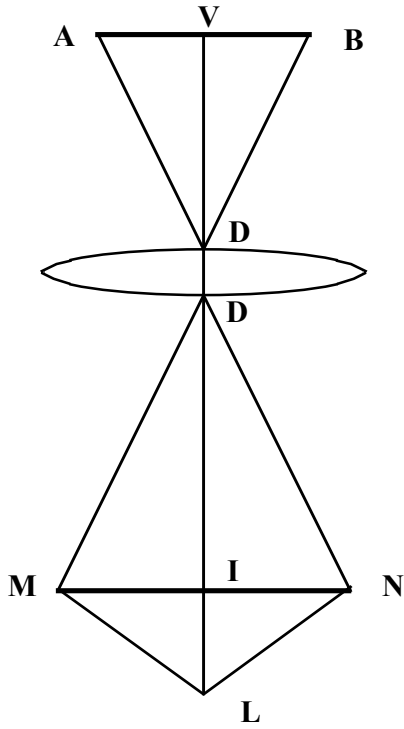
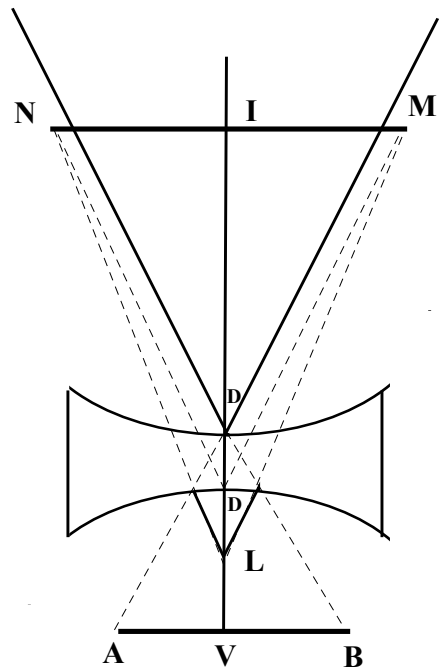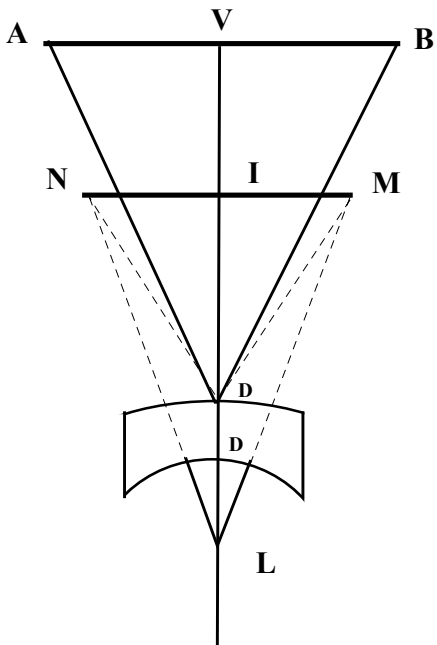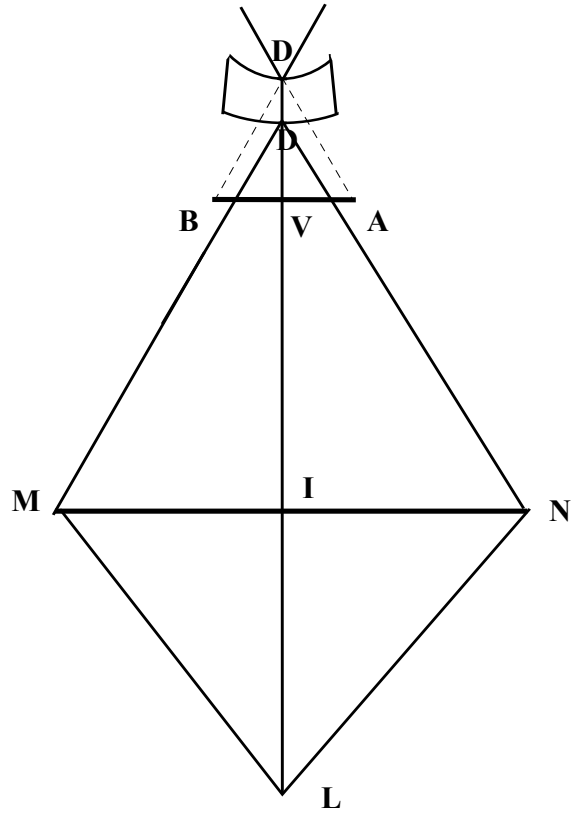**In Catoptrics: visibile ante oculum**   **Visibile post oculum**
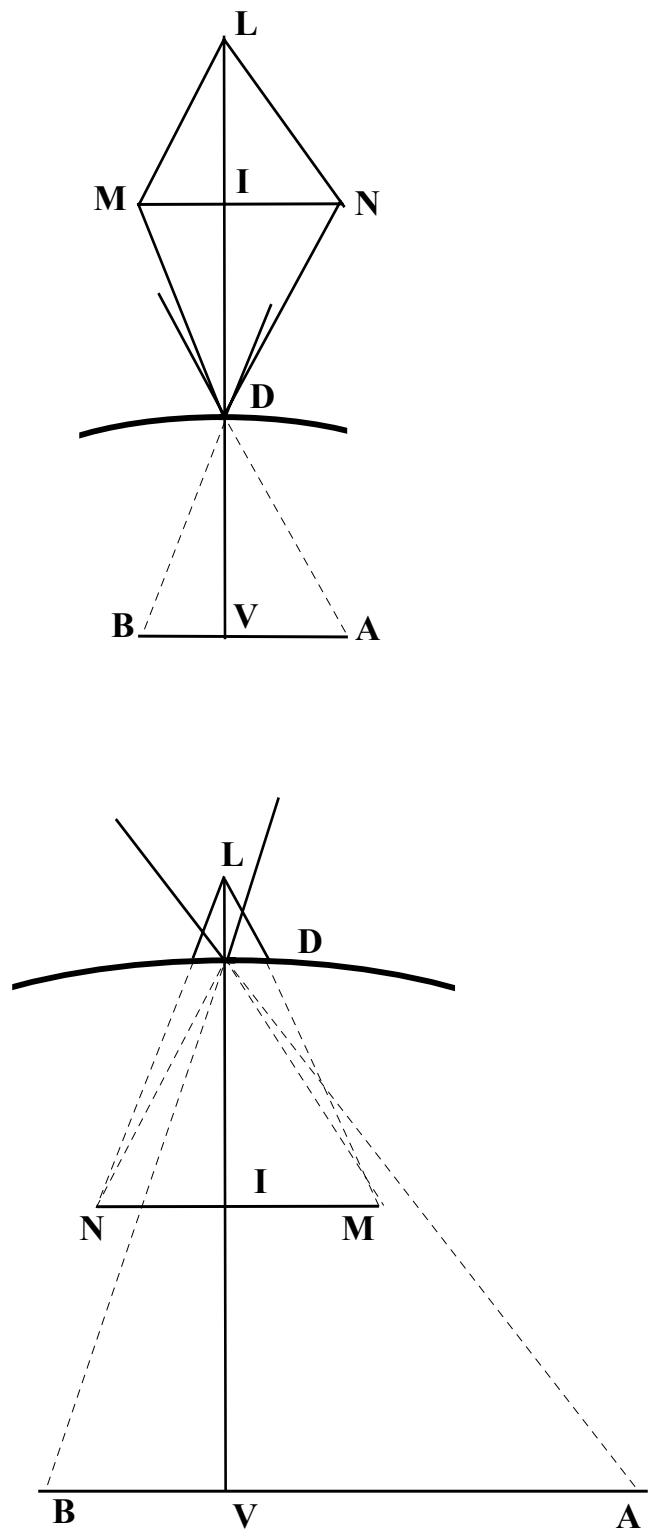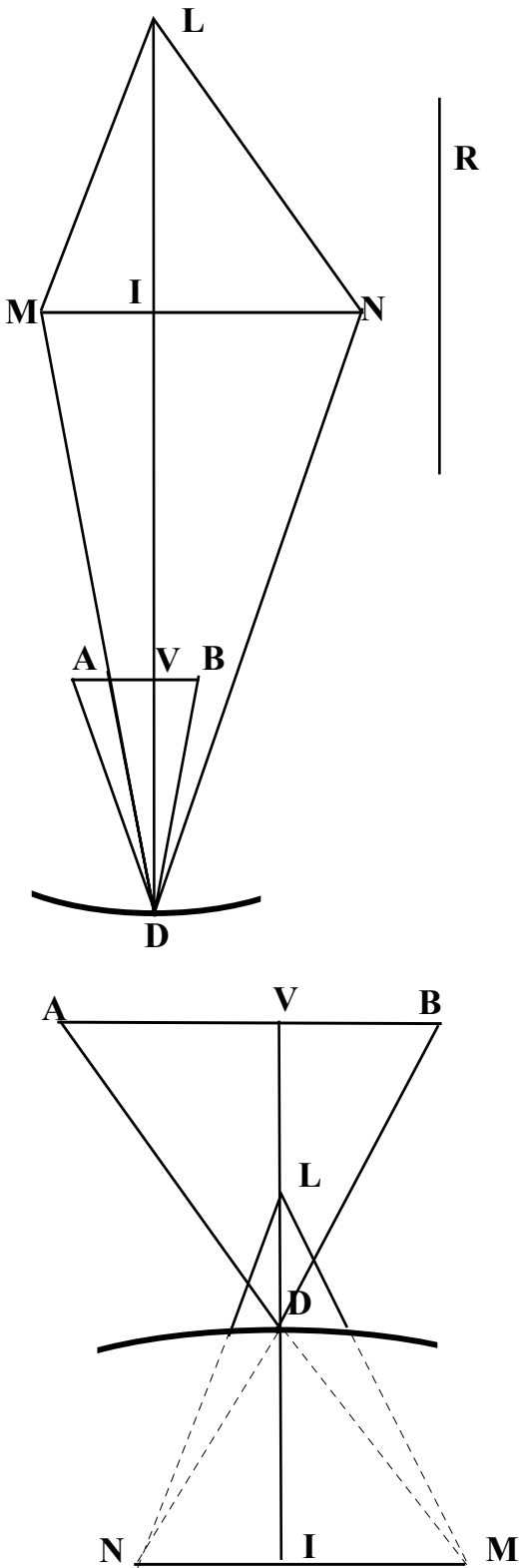
[**81**]

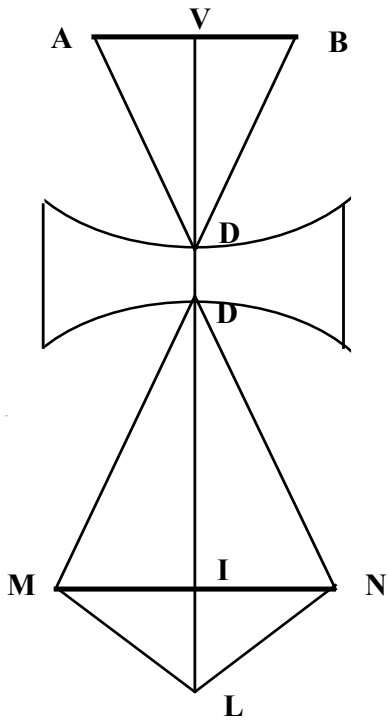**In Dioptricis intermediate diaphano densiore**

Visibilis ante oculum   **Visibilis post oculum**
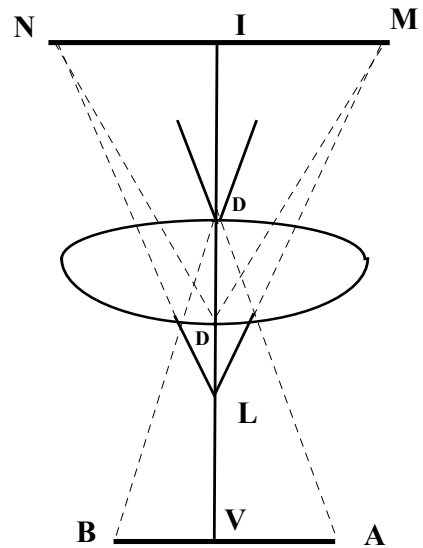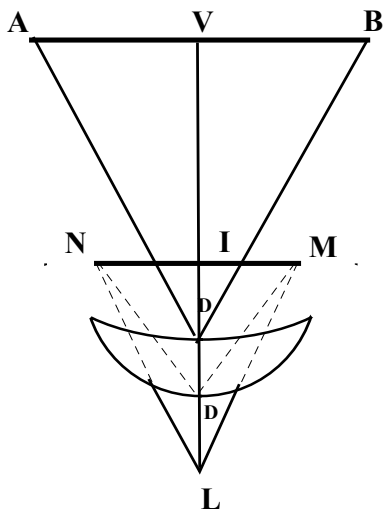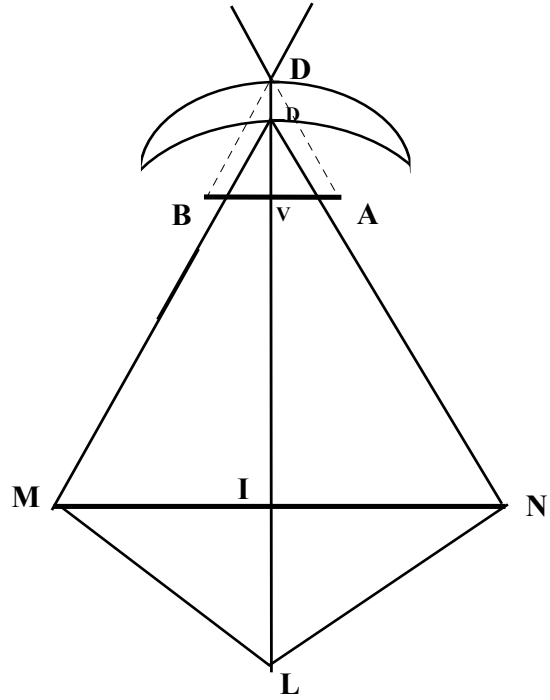
[**82**]

In Dioptricis intermediate diaphano rariore

Visibilis ante oculum   **Visibilis post oculum**

Prop. 53. Problem.

For a given visible image; projected with the help of some mirrors or lenses : and with the distances from each other in turn of the lenses, mirrors, and image given ; to find the angle of vision of the visible image from its own vertex of incidence.

With the same figures, the points M, N shall be the images of the two given points, projected with the help of some number of mirrors or lenses. However it is necessary for the right line M, N to cut the axis produced in I. And MI, IN, ID shall be given, and the distances from each other in turn of the lenses, mirrors, and the image. Therefore, with the sides NI & ID given in triangle NID; and the angle NDI given, or that equal to ADV. And it shall be as the distance of the image of AVB, from the vertex of its own emission; to the distance of the same AVB, from the vertex of its own incidence VD ; thus as the tangent of the angle ADV, to the tangent of the angle, by which AV is seen from the preceding angle of emergence. By the same method, the angle is found, by which VB appears from the same vertex of emergence; which added, will make the angle from which AB, or the second image, appears from its vertex of emergence ; which is equal to the angle by which the third angle appears, from its own vertex of incidence : from which given, gives the same between the pertinent parts to the second image and to the third ; which were given between the pertinent parts to the first and second parts ; therefore by the same reasoning, the angle will be found, by which the fourth image appears from its vertex of incidence , and at last the angle from which the visible object is seen from its angle of incidence.

Prop. 53. Problema.

Data imagino visibilie ; ope quotcunque speculorum vel lentium projecta : & datis lentium, vel speculum, & imaginum, a se invicem distantiis ; visibilis angulum visorium, ex vertice suae incidentiae invenire.

Sint in iisdem figures ; puncta M, N imagines duorum visibilis punctorum, ope quotcunque speculorum, vel lentium, projectae: Oportet tamen rectam M, N axem productum secare in I. Sintque; data, MI , IN, ID, & lentium vel speculorum, & imaginum, a se invicem distantiae. Datis igitur in triangulo rectangulo NID, lateribus, NI, & ID; datur & angulus NDI, seu illi aequalis ADV. Sitq; ut distantia imaginis AVB, a vertice suae emersionis; ad distantiam ejusdem AVB, a vertice suae incidentiae VD ; ita tangens anguli ADV, ad tangentem anguli, quo AV videtur ex praedicta vertice emersionis. Eodem modo, invenitur angulus, quo VB, apparet ex eadem vertice emersionis; qui additi, efficiunt angulum, quo AB, seu secunda imago, apparet ex vertice suae emersionis ; qui, aequalis est angulo, quo tertia imago apparet, ex vertice suae incidenriae: quo dato, dantur eadem, inter pertinentia ad imaginem secundam, & ad tertiam ; quae dabantur, inter pertinentia ad imaginem primam, & secundam ; eodem igitur ratiocinio, reperietur angulus, quo quarta imago, apparet ex vertice suae incidentiae, quo invenimus angulum tertiae, ex vertice ipsius incidentiae; & tandem angulus, quo visibile videtur ex vertice suae incidentiae.

[83] cont'd.

Prop. 54. Problem.

To illuminate the image of a visible object depicted in a plane in any given ratio.

Let AVB be the visible object, the image of which is depicted in the plane, by a lens or mirror, the diameter of which is DE; it shall be illuminated in the ratio R to S. Thus the square of the chord of the half angle DVE, to the square of the chord of the half angle FVG shall be as R to S. And the lens or mirror, of which the diameter is DE, is produced on both sides to G & F ; thus in order that FVG shall become the angle of the lens or mirror from the point V. Which is accomplished and displayed. For the lens or mirror will enlarge the illumination of the image in the ratio R to S: as this is apparent by Corollary 2 *Prop. 33* of this work.

Corollary.

Hence the method follows, the increased strengths of burning are devised in the given ratio ; for the greatest strengths of devising these, either shall be burning with the help of the sun or fire, always shall be present in the image of the sun or fire & the illumination to be increased in some ratio; and the burning increased in the same ratio: as by Prop. 33 the proof of this is obvious enough.
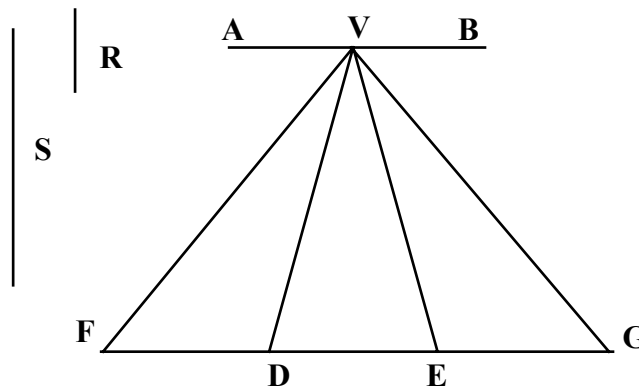
Prop. 54. Problema.

Imaginem visibilis, in plano depictam, in quacunque ratione data, illustrare.

Sit visibile AVB, cujus imago in plani depicta, lente vel speculo, cujus diameter DE; sit illustranda, in ratione R ad S. Sit ut R ad S, ita quadratum chordae, semissis anguli DVE, ad quadratum chordae semissis anguli FVG. Et producatur lens vel speculum, cujus diameter DE, ex omni parte in G, & F ; ita ut FVG, fiat angulus diametri, lentis, vel speculi , ex puncto V. Factumq; erit quod proponitur. Lens enim vel speculum, ampliabit imaginis illustrationem, in ratione R ad S: ut patet per Corollarium 2 *Prop. 33* hujus.

[84]
Corollarium.

Hinc sequitur modus, intendendi vires machinatum comburentium, in ratione data ; harum enim machinarum vires maximae, sive comburant ope solis, sive ignis, semper existunt in solis vel ignis imagine, & in quacunque ratione augmentatur illustratio ; in eadem ratione, augmentatur & ustio : ut per Prop. 33, & ejus consectarium [-*ia* in original text] satis patet.
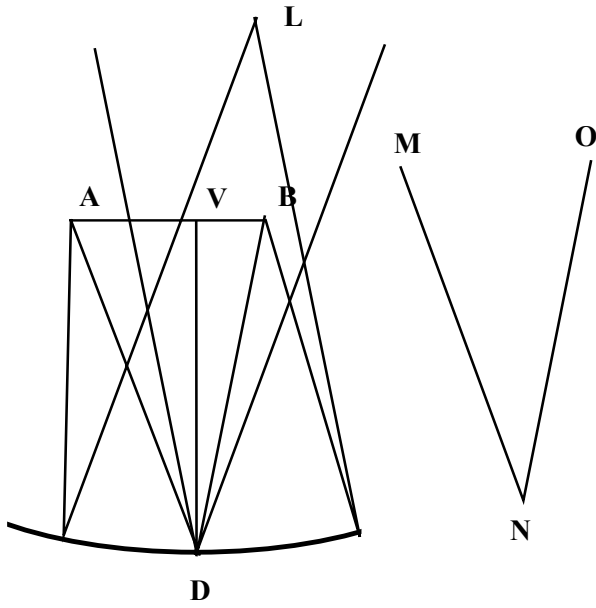
Prop. 55. Problem.

To present a distinct image of any visible object for a long sighted person, with a given thickness of lens , for any visual angle you please, with the illumination at that angle.

Let AVB be anything visible whatever, either radiating matter, or an image before or after the eye, provided it shall be plane : the extreme points of which shall be A, B. For the distinct visible image AVB may be represented to the eye of the long sighted person thus in order that the outermost points A, B shall appear with a visual angle equal to the angle MNO, and the visible image AVB shall have the illumination of this angle. Any point V is taken in the line AB, near its middle ; from which the line VD is drawn, perpendicular to the plane of the visible object ; and from the chord AB is drawn the part of the circle containing the angle MNO, which shall cut the perpendicular in D: and DD shall become equal to the given thickness of the lens ; and the rays diverging from the point V, if the visible object AVB should be before the eye , or converging to the point V, if it should be past the eye, shall be reduced to being parallel [*Prop.* 19 & 21 of this work}, by the lens or mirror, the vertices of which are DD. And by the *Corollary* to *Prop.* 43 of this work, the rays of the individual points AB of the visible object, with the help of the same lens or mirror, also shall be reduced to being parallel. The eye then, receiving these rays, the visible image AB will always appear with the visual angle ADB, with the illumination of that angle and distinct for the long sighted person; which was to be established. {*Prop*. 44, Cor. 2; & *Prop*. 49.}

Scholium.

But if, on account of an inaccessible distance or smallness of the visible object AB, the angle ADB cannot be taken conveniently equal to the angle MNO ; the images of the given points A, B are projected into an equivalent line, which will be able to subtend the angle MNO conveniently enough ; and by the same way, with the preceding image, if it might be the given visible object.

In Catoptrics: visibile ante oculum

Visibile post oculum
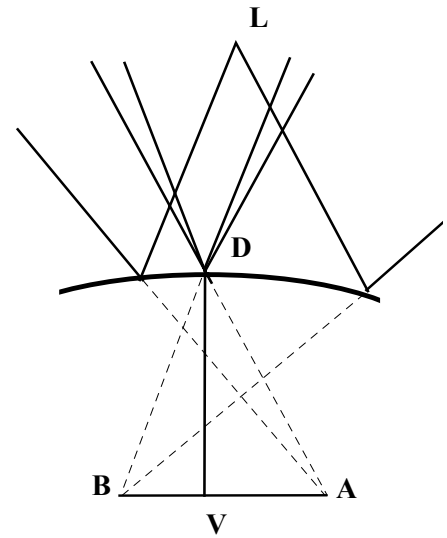
In Dioptricis intermediate diaphano densiore

Visibilis ante oculum

Visibilis post oculum

In Dioptricis intermediate diaphano rariore

Visibilis ante oculum   **Visibilis post oculum**

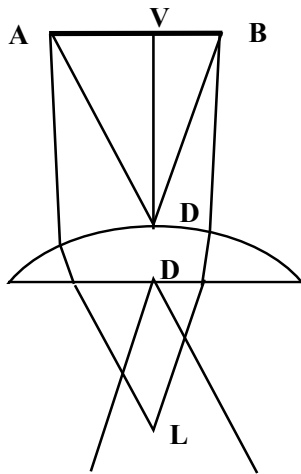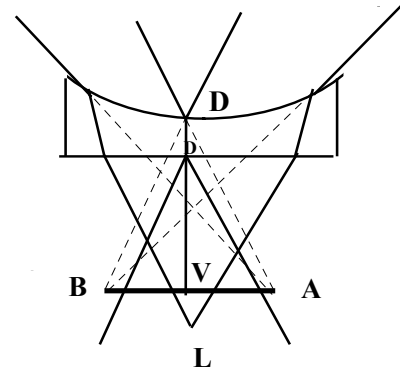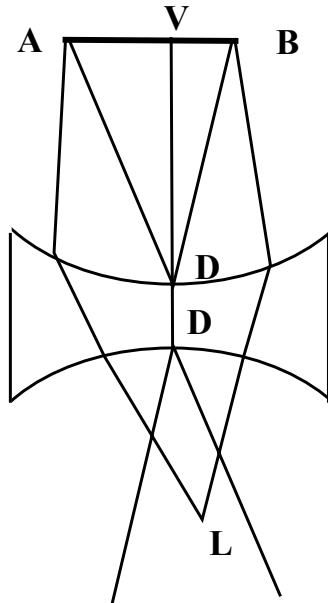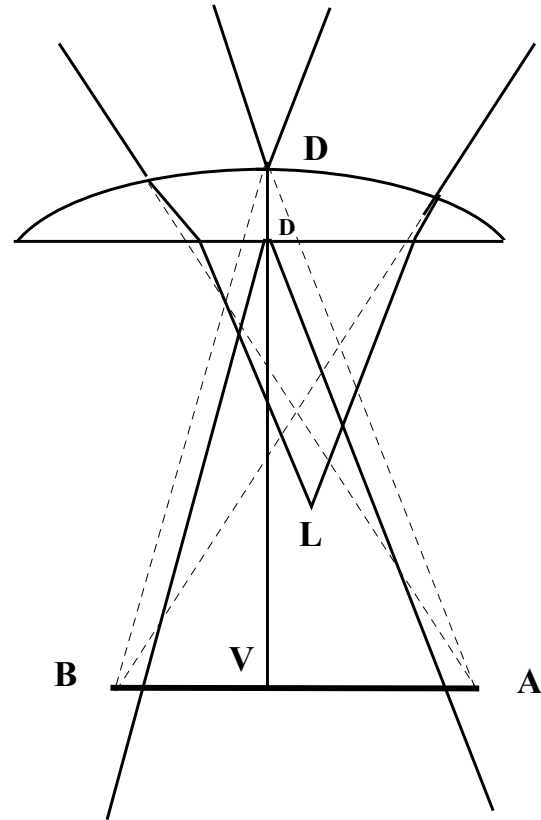## Prop. 55. Problema.

Cujuslibet visibilis imaginem, quovis angulo visorio, cum illius anguli illustratione, presbytis distinctam representare ; cum data lentis crassitie.

Sit AVB visibile quodlibet, sive materia radians, sive imago ante, sive post oculum, dummodo sit planum; cujus extrema puncta, sint A, B. Oculo Presbyti, repraesentetur imago visibilis AVB, distincta, ita ut puncta extrema A, B, appareant angulo visorio, aequali angulo MNO, & imago visibilis AVB habeat, illius anguli illustrationem. In linea AB, sumatur punctum quodlibet V, prope ipsius medium ; a quo, ducatur recta VD, plano visibilis perpendicularis ; & chorda AB, ducatur circuli portio continens angulum MNO, quae secet perpendicularem VD, in D: fiatq; D D aequalis datae lentis crassidei ; & radii, divergentes a puncto V, si visibile AVB, fuerit ante oculum ; vel ad punctum V, convergentes, si fuerit post oculum ; ad parallelismum reducantur {19 & 21 Hujus}, lente vel speculo, cujus vertices D, D: & per *Corollarium Prop. 43* hujus , radii, singulorum

visibilis AB punctorum, ope ejusdem lentis, vel speculi, etiam ad parallelismum reducentur; oculo igitur, radios hosce recipienti, semper apparebit imago visibilis AB, angulo visorio ADB, cum illius anguli illustratione: {44. Hujus *Cor*.2}& presbytis distincta; quod faciendum erat. {49. Hujus.}

[85]
Scholium.

Si autem, ob visibilis AB distantiam inaccessibilem, vel parvitatem, commode non poterit sumi angulus ADB, aequalis angulo MNO ; projiciantur imagines, punctorum A, B, in rectam, aequalem datae, quae satis commode, angulum MNO subtendere possit; & eodem modo, cum imagine procedendum; ac si esset visibile datum.

Prop. 56. Problem.
To represented the distinct image of any visible object, for any angle of vision, with the illumination of that angle; for the myopic eye, the distance of the vision of which should be known, with the thickness of the lens given too.

Let AVB be any visible object, or radiating matter, with the image either before or after the eye, provided it shall be plane; of which the most distant parts shall be A, B. For the myopic eye, of which the distance of [distinct] vision is S, the image of the visible object AVB may be shown distinctly ; thus in order that the extreme points of this A, B is shown with the angle of vision AOB given; and the image of the visible object AVB shall have the illumination of that angle. In the triangle AOB, OV shall be normal to the base AB; and OP shall be equal to the right line S ; then, RPT is drawn parallel to the base AB; and the images of the points A, B are projected into points separated by the distance RT. In the diagrams of Prop. 52 of this work, MIN shall be equal to RPT. Thus as NI shall be equal to RP; and MI equal to PT itself, and IL equal to the right line PO ; this is for the given right line S. I say, that the eye of the myopic person at L shall see the distinct image MIN, with the angle of vision AOB, and with the illumination of that angle. Since indeed, the triangles MLN and ROT are equal. & similarly; triangle MLN is similar to triangle AOB; & as

AB : MN : : VO : IL ;
but as
AB : MN : : VD : ID ;
then, as
VD : VO : : ID : IL;

and it shall be with, as the distance of the visible object from the vertex of incidence VD; to the separation of the visible object to the centre of the eye VO, thus the distance of the image from the vertex of emergence ID ; to the
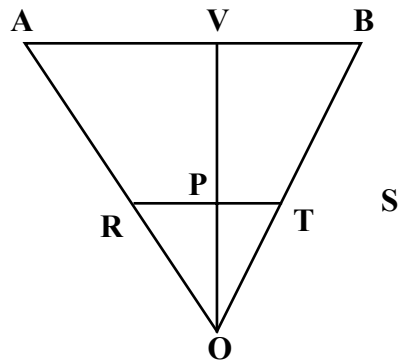
distance of the image from the centre of the eye. The image MN will appear at L with the same angle and illumination, with which the visible object appears to the eye at O : also

the image appears distinct, because IL is equal to the right line S, for the given distance of [clear] vision: Which were to be established.

Scholium.

In the two previous propositions, we have been talking about the general method of putting together telescopes and microscopes, in which we have examined the distinct vision of the image, the enlargement and the illumination. But there remains another consideration, namely the angle by which a part of the image seen is viewed, but if the whole cannot be seen. First however, it is to be known, [regarding] the lenses and mirrors, not less than the spheres, the images of the visible object not arising at the foci, sensibly enough are to be projected; as the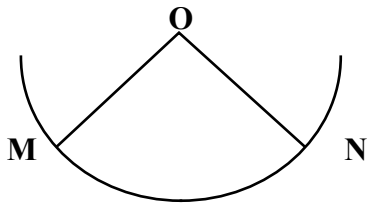 writers of optics explain in their own way with spheres. From which it happens, since the image of [those] related to the last image may be projected through the related images remaining, then incident at last on the eye. Yet knowing the visible images, and the images relating to the last image, to be made from the same rays; where indeed the rays emerging from one point of the visible object are concurrent in another point ; that point of concurrence is the image of the previous visible point, and also where the rays from one point of the visible from one related point to the final image are emerging, they are concurrent in the other ; that point of concurrence is the image of the first point pertaining to the final image; and in the same way so for all the points in the visible, as [for those] pertaining to the final image. With these things already touched on, we say the part of the image seen, to appear from that angle by which a part of the retina illuminated by the rays of the image related to the final image, shall appear from the centre of the eye. Let AB be the image related to the final image, the rays of which illuminate part of the retina MN, and let the centre of the eye be L, I say the part of the image seen to appear with the angle of vision MON. Since indeed the rays, which arrive from the related to the final image, also come from the visible object: therefore the same space MN is also illuminated by the rays from the visible object ; therefore the points MN are the furthest points of the visible object depicted in the retina of the eye, which appear with the angle of vision MON ; then the proposition is apparent. From this it follows (if the centre of the eye O is placed in the image related to the final image *e. g.* in the right line AB, which is always able to happen, when the second image is before the eye) to fill the whole part of the image seen belonging to the first image ; and since it belongs to the first image, it will be able to be made wider to please; also the angle of vision of the part of the image seen can be augmented to please : And thus. concerning the part of the image seen, to have said this little should be sufficient.

Prop. 56. Problema.

Cujuslibet visibilis imaginem, quovis angulo visorio, cum illius anguli illustratione; oculo myopis, cujus visus distantia sit cognita ; distinctam representare ; data quoque lentis crassitie.

Sit AVB visibile quodlibet, sive materia radians, sive imago ante, sive post oculum, dummodo sit planum; cujus extrema puncta, sint A, B. Oculo myopis, cujus visus distantia, est recta S, repraesentetur imago visibilis AVB, distincta ; ita ut ejus extrema puncta A, B, appareant angulo visorio, AOB, dato ; & visibilis AVB, imago habeat illius anguli illustrationem. Sit in triangulo AOB, OV normalis ad basem AB; sitq; OP, aequalis rectae S ; deinde, ducatur RPT, parallela basi AB, & imagines puncturom A, B, projiciantur in puncta, distantia per rectam RT. In figuris Prop. 52 hujus, sit MIN, aequalis RPT ; ita ut NI, sit aequalis RP; & MI, aequalis ipsi PT, & IL, aequalis rectae PO; hoc est rectae datae S. Dico, oculum myopis in L, videre imaginem MIN distinctam, cum angulo visorio AOB, & illius anguli illustratione. Quoniam enim, Triangula MLN, ROT, sunt aequalia,

[**87**]

& similia; triangulum MLN, est simile triangulo AOB; & ut

AB : MN :: VO : IL ;

ut autem

AB : MN : : VD : ID ;

igitur, ut

VD : VO : : ID : IL;

cumque sit, ut distantia visibilis a vertice incidentiae, VD ; ad distantiam visibilis, ab oculi centro, VO, ita distantia imaginis, a vertice emersionis, ID ; ad distantiam imaginis, ab oculi centro : eodem angulo, & eadem illustratione, apparebit imago MN, in L; quibus apparet visibile oculo in O : apparet etiam imago distincta ; quia IL, aequalis est rectae S, datae visus distantiae : Quae facienda erant.
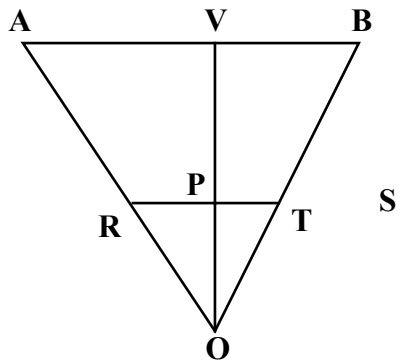
Scholium.

In prioribus duabus propositionibus, loquenti sumus de methode generali componendi telescopia, & microscopia; in quibus consideravimus imaginis distinctam visionem, augmentationem, & illustrationem : Restat autem aliud considerandum, nempe angulus, quo videtur pars imaginis visa; si modo tota non videatur. Primo tamen sciendum est lentes & specula conica, non minus quam sphaerica, visibilium in focis non existentium, imagines, satis sensibiliter projicere; ut in sphaericis suo modo demonstrant Opticae scriptores : unde evenit, quod imago pertinentis ad imaginem ultimam, projiciatur per pertinentia ad imagines reliquas, donec tandem in oculum incidat : Sciendum tamen

imagines visibilis, & imagines pertinentis ad imaginem ultimam, ex iisdem confici radiis; ubi enim radii ab uno puncto visibilis emergentes, in aliud punctum concurrunt ; illud punctum concursus, est imago prioris puncti visibilis; & ubi radii etiam visibilis ab uno puncto pertinentis

[**88**]

A ———————— B

O

M   N
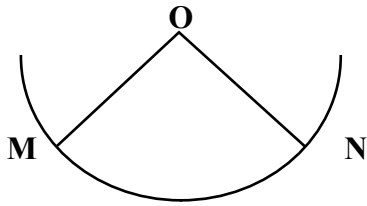
ad imaginem ultimam provenientes, in aliud concurrunt ; illud punctum concursus est imago prioris puncti pertinentis ad imaginem ultimam; & eodem modo de omnibus punctis tam in visibilis, quam in pertinente ad imaginem ultimam. Hisce praelibatis, dicimus partem imaginis visam, apparere eo angulo, quo pars retinae illustrata a radiis, imaginis, pertinentis ad imaginem ultimam, ex oculi centro apparet.   Sit imago pertinentis ad imaginem ultimam AB cujus radii illustrent partem retinae MN, sitque centrum oculi L. Dico portionem imaginis visam apparere angulo visorio MON. Quoniam enim radii, qui proveniunt a pertinente ad imaginem ultimam, etiam proveniunt a visibili: ideo idem spatium MN illustratur etiam a radiis visibilis ; igitur puncta MN sunt extrema visibilis puncta in oculi retina depicta, quae apparent angulo visorio MON ;

unde patet propositum. Ex hoc sequitur (si oculi centrum O ponatur in imagine pertinentis ad imaginem ultimam *e. g.* in recta AB, quod semper fieri potest, quando secunda imago est ante oculum) Partem imaginis visam totum pertinens ad imaginem primam implere ; cumque pertinens ad imaginem primam, poterit dilatari ad libitum ; potest quoque & angulus visorius partis imaginis visae augmentari ad libitum : De portione itaq ; imaginis visa haec pauca dixisse sufficiat.

Prop. 57. Problem.

For a given visual angle of the image, brought to the eye with the help of some mirrors or lenses; and with the distances by themselves of the lenses or mirrors, and of the eye, given in turn; to find the angle of vision of the visible object from its own vertex of incidence.

Let MLN be the visual angle in the figures of *Prop.* 52 & 53 of this work, where the image given is handled with the help of some lenses or mirrors; and the given angles shall be ILN, ILM, & LI shall be the given distance from the first image; therefore in the right angled triangle LIN, with the angle ILN & the side IL given, & the side IN given; and by the same method IM is found: with the image of the visible object MN given; projected with the help of some lenses or mirrors: & with the distances of the lenses or mirrors & of the image themselves given in turn ; the angle of vision of the visible object may be discovered from its own vertex of incidence, by *Prop.* 53 of this work : which was to be established.

Yet more straight forward ; as ID to IL, thus the tangent of the angle MLI, to the tangent of the angle MDI, or BDV; similarly too VDB is found; and thus with the distances of the lenses or mirrors & of the image, themselves given in turn ; & for the visual angle of the image, from the vertex of its own incidence, ADB; to be proceeding in the same manner as in the solution of *Prop*. 53 of this work.

Prop. 57. Problema.

Dato imaginis angulo visorio, ope quotcunque speculorum, vel lentium, in oculum allato; & datis, lentium vel speculorum, imaginum, & oculi, a se invicem distantiis; visibilis angulum visorium, ex vertice sua incidentiae invenire.

Sit in figuris *Prop*. 52 & 53 hujus ; MLN, angulus visorius, quo comprehenditur imago ope quotcunque lentium vel speculorum, datus ; sintq; dati anguli ILN, ILM, & LI, distantia oculi ab imagine prima ; in triangulo igitur rectangulo LIN, datis, angulo ILN, & latere IL, datur & latus IN ; eodemq; modo reperitur IM: data igitur visibilis imagine, MN; ope quotcunque lentium, vel speculorum, projecta : & datis, lentium, vel speculorum, & imaginum, a se invicem, distantiis ; inveniatur, visibilis angulus visorius, ex vertice suae incidentiae per *Prop*. 53 hujus : quod erat faciendum.

Expeditius tamen ; ut ID ad IL, ita tangens anguli MLI, ad tangentem anguli MDI, seu BDV; similiter quoque reperitur VDB; datis itaq; lentium, vel speculorum, & imaginum a se invicem distantiis, & angulo visorio imaginis, ex vertice suae incidentiae, ADB; eodem modo procedendum ut in solutione *Prop*. 53 hujus.

Prop. 58. Problem.

With one focus and the position of the axis given for a kind of ellipse ; and with the position given of a line cutting the axis : to find the ellipse of which the given line shall be perpendicular to the circumference.

Let AB be the straight line produced in either direction, cutting the axis with the given position FA, in the point A: and the ellipse shall be found, of which the focus is F, and the axis [lies] in the line FA. The ratio of the separation of the foci to the axis of the ellipse shall be as R to S ; thus, as the right line BA, incident on the ellipse at the point E, shall be perpendicular to the line of the tangent EM of the ellipse. As R to S thus FA is to FE ; and the angle AEL is equal to FEA ; the line EL is joined. I say that the points F, L are the focal points; and FE, EL together are equal to the axis of the ellipse sought. For with the ellipse described to such an extent, the line BAE, dividing the angle LEF in two equal parts, will be perpendicular to the tangent, surely ME, in the point E. Since the angles LEA, AEF are equal ; thus as FE will be to FA, so EL to LA; thus as FE to FA,

that is S to R, hence FE, EL likewise, surely the axis of the ellipse, to FA, AD likewise, surely the separation of the foci: therefore the ellipse with foci F, L has been described, passing through the point E, the ellipse sought; which was to be shown.

Prop. 58. Problema.

Datis; specie ellipseos, uno foco, & positione axeos ; dataq; linea, axem, positione datum, secante: ellipsim invenire, cujus circumferentiae, perpendicularis sit linea data.
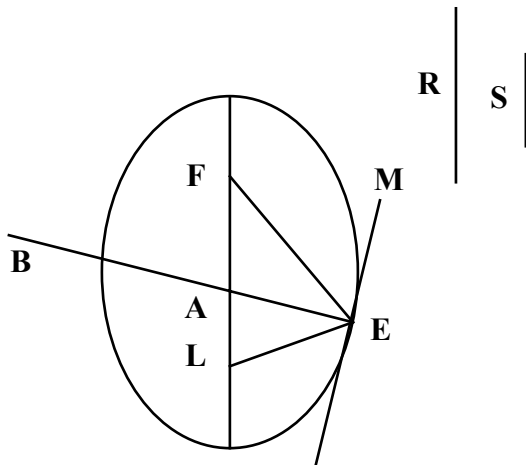
Sit recta AB, utrinq; producta ; secans axem, positione datum FA, in puncto A: sitque invenienda ellipsis, cujus focus F, & axis in linea FA ; ratio distantiae focorum, ad axem ellipseos, ut R ad S ; ita, ut BA recta, incidens in circumferentiam ellipseos, in puncto E, sit rectae, EM ellipsim tangenti, in puncto E, perpendicularis. Sit ut R ad S, ita FA, ad FE ; sitq ; angulus AEL, aequalis angulo FEA ; & jungatur recta EL. Dico F, L, puncta, esse focos, & FE, EL, simul, esse aequales axi ellipseos quaesitae. Descripta enim tali ellipsi, recta BAE dividens angulum LEF bifariam, erit perpendicularis ad contingentem in puncto E, nempe ME ; & quoniam anguli LEA, AEF sunt aequales; erit ut FE, ad FA, ita EL, ad LA; igitur ut FE ad FA, hoc est S ad R, ita FE, EL simul,

[**90**]

nempe axis ellipseos, ad FA, AL simul, nempe FL distantiam focorum : est igitur ellipsis, focis F, L, descripta, & transiens per punctum E, ellipsis quaesita ; quod ostendendum erat.

Prop. 59. Problem.

To build a telescope from a single lens, with the aid of which far-sighted eyes may perceive the angle of vision of a distant visible object a great deal enlarged, with distinct vision.

Let AEB be the angle of vision of a distant visible object; and let a telescope be constructed from a single lens, with the help of which a relaxed eye can see the image of a distant object distinctly, with an angle of vision AOB : AEB, AOB shall be two triangles above the same base AB; and the right line EOL shall be drawn perpendicular to the base AB, and cutting this at L in two equal parts. Then with the focus L, and with the axial position LE, a dense ellipse may be described of that denser medium, from which

the lens is to be made, and [another] from air, in which the eye exists; thus as the right line BE produced shall be the perpendicular of this circumference in F; & for the same ellipse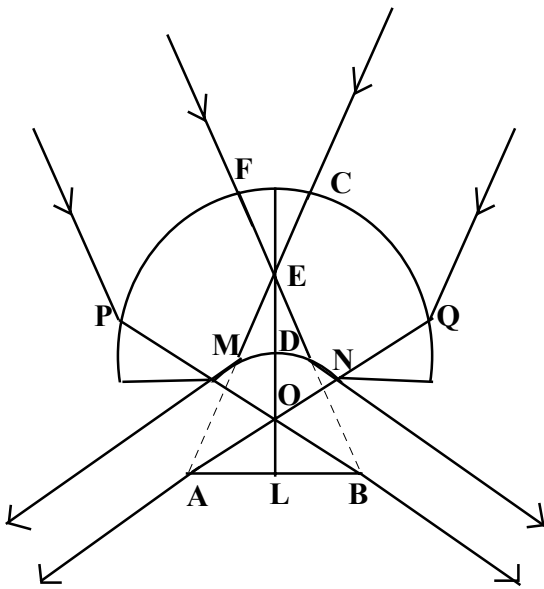, AE produced will be the perpendicular to the circumference in C; with the same focus L too, and with the same axial position LE, an ellipse of the first density [i.e. air] is also described ; thus as the right line BO produced will be the perpendicular of this circumference in M; & for the same ellipse, AO produced will be the perpendicular to the circumference in N; & from the revolution of these ellipsis about the common axis EL, a certain lens shall be made, from the before mentioned denser medium, with a convex part FC, and a concave part MN. I say that a long-sighted eye [*i. e.* relaxed eye] in the concavity MN, sees a distinct distant visible object, of which the angle of vision is AEB, having the centre in the axis of the lens produced, through the [increased] angle of vision MON or AOB. For indeed the extreme points of visibility are in a single plane with the axis of the lens by hypothesis; because the centre of visibility is on the axis of the lens produced; & therefore the surface of common refraction is the same figure, from the rotation of which the lens has been generated, surely FC, MN; & EC, EF produced deal exactly with the visible object, since the angle FEC is equal to this angle of vision; & therefore the rays of the extreme points of the visible object are incident normally on the common surface of refraction, that is the axes of the pencils of the extreme points of vision, are the rays CE, FE, & therefore the apices of the pencils, of the extreme points of vision, are B, A , surely the common sections of the axis of the pencils of the extreme points of the visible object with the plane drawn through the focus L, to which the perpendicular is the axis of the lens. Then if AO, BO are produced within the common surface of refraction to the surface of the lens at P & Q ; & the rays are drawn of the extreme visible points incident at P & Q ; & these rays may be refracted with regard to the apices of their pencils A & B, and as they shall be straight lines to the common surface of refraction at M & N; but all the rays falling on the surface of refraction NDM itself, & extending to the points of the line ALB, shall be reduced to being parallel with these rays directly incident on the surface, and entering without refraction; as is easily deduced from *Prop.* 17 & 37 of this work : therefore all the rays of the extreme points of the visible object are reduced to being parallel with the lines OB, OA ; & therefore the visible object is seen with an angle of vision equal to AOB itself ; but it appears, & to the far sighted eye distinct, since the rays of the individual visible points are parallel after leaving the lens ; which was to be shown.
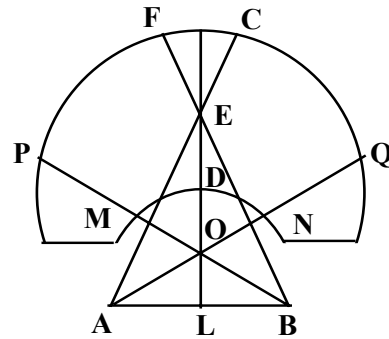
Scholium.

This problem could also be resolved with the help of a rarer medium between the visible object and the eye ; but since it is of no more usefulness, we omit the demonstration of this, but it is brought about with the aid of a dense hyperbola, this being brought about with the help of an ellipse. Also the same illustration is able to demonstrate this, for

which in the preceding, indeed with the angle of vision always proportional. But I relinquish the demonstration, for the sake of exercising the talent of the studious.

Note on Prop. 59. Problem: We are dealing with paraxial rays incident on an ellipsoid. In modern terms, the refractive index is chosen as the inverse of the eccentricity: thus, rays parallel to the optical axis converge on the more distant focal point L; while paraxial rays converge to a point in this focal plane. As Gregory shows, a ray through the near focus E and perpendicular to the surface can be used to locate this point, A or B. Hence the position of the first image AB is located. The same reasoning is applied to the second ellipse, which is considered as an ellipse of air in a glass medium: just as a parallel beam of paraxial rays converge for a dense ellipse, so a beam converging to the far focus of a less dense ellipse are rendered parallel to the ray through the near focus.

Prop. 59. Problema.

Ex unica lente, telescopium fabricare, cujus ope presbyti comprehendant angulum visorium visibilis longinqui quantumlibet auctum, cum distincta visione.

Sit angulus visorius visibilis longinqui AEB sitq; telesciopium ex unica lente construendum, cujus ope presbytus videat, visibilis longinqui imaginem distinctam, cum angulo visorio AOB: sint duo triangula isoscelia AEB, AOB super eadem basi AB; ducaturq; recta EOL, in basem AB perpendicularis, & eam bifariam secans in L. Deinde foco L , axeos positione LE, describant ellipsis densitatis illius diaphani densioris, e quo fabriqanda est lens, & aeris, in quo existat [existas in ms.] oculus ; ita ut recta BE producta, illius circumferentiae sit perpendicularis in F; & in eadem ellipsi, AE producta erit perpendicularis circumferentiae in C; eodem quoq; foco L, & eadem axeos positione LE, describatur ellipsis prioris etiam densitatis, ita ut recta BO producta, illius circumferentiae sit perpendicularis in M; & in eadem ellipsi, AO producta, erit perpendicularis circumferentiae in N ; & ex revolutione harum ellipsium circa

communem axem EL, fiat lens quaedam, ex diaphano densiore praedicto, convexa ad partes FC, & concavitate ad partes MN. Dico oculum presbyti in concavitate MN, videre visibile longinquum, cujus angulus visorius AEB, habens centrum in axe lentis producto, distinctum, per angulum visorium MON seu AOB. Sunt enim extrema visibilis puncta in uno plano cum axe lentis ex hypothesi ; quoniam centrum visibilis est in axe lentis producto ; & igitur communis earum superficies refractionis, est eadem figura ex cujus revolutione genita est lens, nempe FC, MN, & EC, EF productae exacte comprehendunt visibile, quoniam angulus FEC est aequalis ejus angulo visorio; & igitur radii extremorum visibilis punctorum in superficiem refractionis communem normaliter incidentes, id est axes penicillorum, extremorum visibilis punctorum, sunt radii CE, FE, & igitur apeces penicillorum, extremorum visibilis punctorum, sunt B,A, communes nempe sectiones axium penicillorum extremorum visibilis punctorum cum plano per foco L ducto, cui perpendicularis est axis lentis. Deinde si AO, BO in communi refractionis superficie producantur ad lentis fuperficiem in P, & Q, & ducantur radii extremorum visibilis punctorum incidentes in P & Q ; refringentur hi radii, in apices suorum penicillorum A & B; cumq; sint recti ad superficiem communem refractionis in M & N; irrefracte penetrabunt ad A, & B ; superficies autem refractionis NDM omnes radios in se incidentes, & ad puncta rectae ALB tendentes, reducit ad parallelismum illis radiis directe in superficiem MDN incidentibus, & irrefracte penetrantibus ; ut facile deducitur ex Prop. 17, & 37 hujus : omnes igitur radii extremorum visibilis punctorum reducuntur ad parallelismum lineis OB, OA ; & igitur visibile videtur angulo visorio aequali ipsi AOB ; apparet autem, & presbytis distinctum, quoniam singulorum visibilis punctorum radii, post egressum e lente sunt paralleli ; quod ostendendum erat.

Scholium.

Poterat etiam hoc Problema resolvi, ope diaphani rarioris inter visibile & oculum ; sed quoniam nullius est utilitatis, demonstrationem ejus praetermittimus, perficitur autem ope hyperbolae densitatis, sicuti haec perficitur ope ellipseos. Eadem etiam illustratio demonstrari poterat hic, quae in prioribus, nempe angulo visorio semper proportionata. Sed studiosis exercendi ingenii gratia, demonstrationem relinquo.
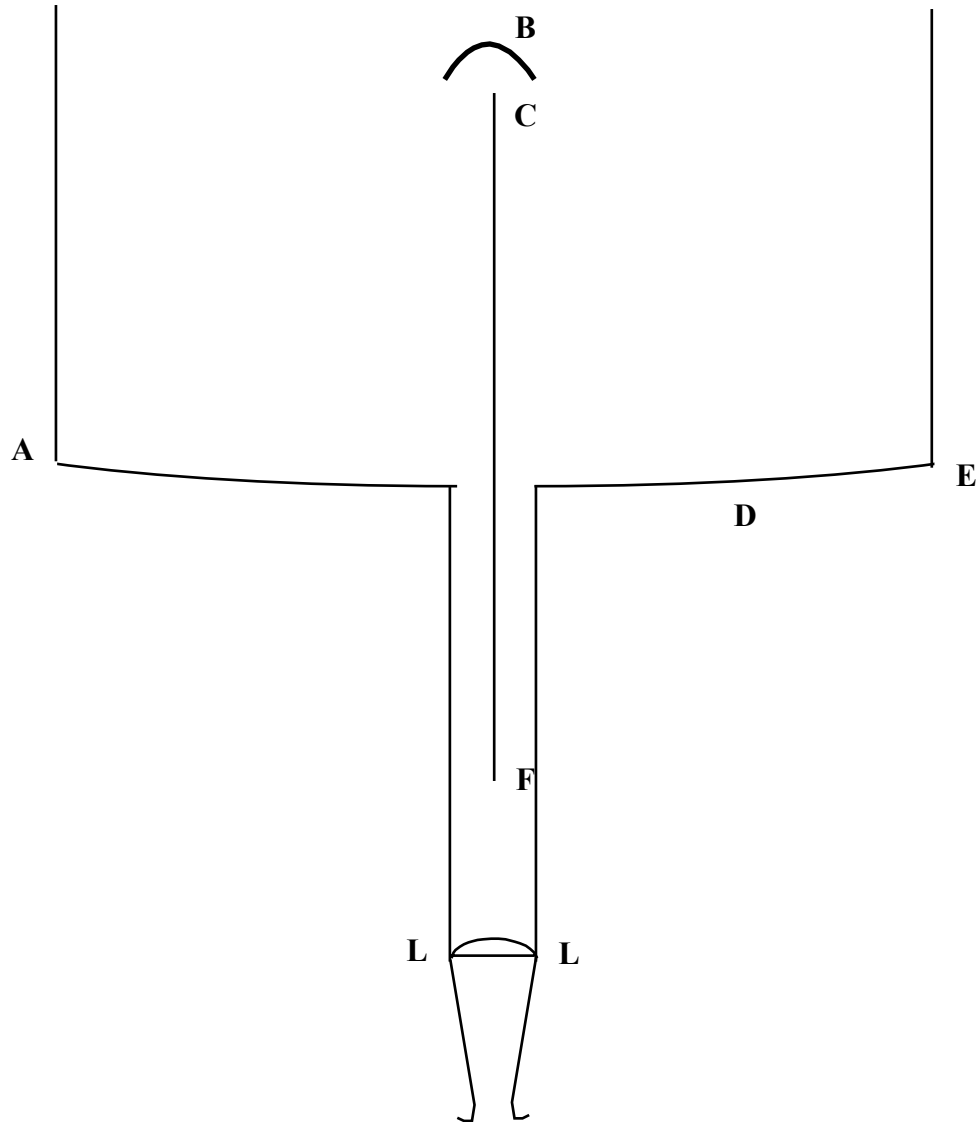

Epilogue.

From these few propositions explained here generally, there depends not only the whole principle of the telescope, but also everything about catoptrics and dioptrics. Indeed it is shown when the image shall be made distinct, when confused; in which place the image of the visible object shall arise, and how great a part of that may be seen by the eye; in what ratio the image shall be enlarged or diminished, made bright or dim; how the image is to be made, how it is to be calculated. Now it remains for us to say a little about certain kinds of telescope. These two particular optical instruments are so much in daily use: the telescope for distant viewing, and the microscope for the near; to which we may add a third, namely the iconoscope, for projecting images of
visible objects, shown in *Prop.* 52 of this work. These instruments are of three kinds: without doubt from pure dioptrics, being recognised so much until now; or from pure catoptrics; or from a mixture, part from catoptrics and part dioptrics. Then in each one, to be able to generate countless kinds, indeed from 2, 3, 4, etc lenses in the first, mirrors in the second, and from mirrors and lenses in the third. Within each of these kinds there are still two subdivisions, the one shows the image situated erect, the other inverted; from

which that is always to be equally preferred with the others which represents the image situated true and erect. But of the first kind, indeed from pure dioptrics, this is the only property, as many lenses as you please can be continued thus in order that the image may be enlarged as much as you please; but this has disadvantages. In the first place, the imagines themselves generally are not able to grow in separation in a manageable enough manner. Secondly, pertaining to the final image, it is not able to be enlarged enough with regard to the illuminating images, except by weakening the rays of the visible object by the great width. Thirdly, the thickness of the individual lenses weaken the rays slightly. These indeed are the properties of the second kind [of telescope]: first because, pertaining to the final image, it can be dilated as you please, without any weakening of the rays; secondly, because the images themselves need not grow in separation; but with this they have not a small disadvantage, as it will be hardly possible to continue beyond two mirrors. But the third golden kind has no disadvantage, and it is able to have all the properties of the former kinds, if lenses and mirrors may be duely disposed; this is if mirrors shall be applicable to the ultimate and penultimate images, and lenses applicable to the remainder of the images. And thus, for the sake of an example, we will describe one telescope of this most perfect kind . Let AFCE be a concave parabolic mirror most carefully polished, in the focus C of which is placed a small concave elliptical mirror having a common focus, and a common axes with the concave parabolic mirror, and may be fastened in this position; but it is necessary that the preceding focus of this elliptic mirror shall approach as close as possible to its vertex, and the other focus shall be the greatest distance from this vertex; the other focus of this elliptic mirror shall be F, with a common axis produced beyond the parabolic mirror, and in the vertex of the of the parabolic mirror a round hole MN is hollowed out, in which hole a tube is placed having the same axis as the mirrors, big enough to be receiving the reflected rays of the visible object from the concave elliptic mirror, and the tube is produced to L, which is close to F itself, and a crystalline lens is attached at L, of which F is the outer focus, but planar in the direction of the eye, having a common axis with the mirrors and tube. And this will be best resolved telescope made for far vision, indeed remote visible objects will appear to the eye through the tube most distinctly enlarged, which is almost in the ratio of the distances of the vertices from the common foci; and made clear in the same way by which vision may be may be explained for such a visual angle; only if the diameter related to the final image shall permit the pupil of the eye to be filled with rays; but how this may be known we have explained in the scholium to *Prop*. 51 of this work. But concerning the magnification of all telescopes, microscopes, and iconoscopes, this rule is to be observed : (indeed it is geometrical, if the centre of vision shall be so, as the eye, on the axis of the instrument, the image shall never depart sensibly from the truth) as the distance of the first image from the vertex of its emergence, to the distance of the same from the centre of the eye, thus the tangent of the semi - angle of the first image seen from the centre of the eye, to the tangent of the semi - angle of the same first image, from the vertex of its emergence. Again, as the distance of the second image from the vertex of its emergence, to the distance of the same from the vertex of incidence, thus the tangent of the semi - angle of the second image seen from the vertex of its emergence, to the tangent of the semi - angle of the second image from the vertex of its emergence; or the tangent of the semi - angle of the third image seen from the vertex of its incidence. Thus it may proceed to the tangent of the semi - angle of the ultimate image seen from the

vertex of its emergence or of the visible object from its own vertex of incidence, which semi - angle doubled gives the angle of the object seen from its own vertex of incidence. One may look at the demonstration in *Prop.* 53 & 57 of this work.

   Concerning the mechanical construction of these mirrors and lenses, from others attempted in vain; I say nothing, as I am less versatile with mechanics. Yet to be asserted boldly, the perfection of optics in lenses and mirrors is sought in vain. If however it should be pleasing to someone, he will be able to apply the particular propositions of this little tract, although indeed imperfect. Indeed the portion of a sphere (although it may not concentrate the parallel rays in one point) presents the position of the image in one spherical surface, to have the same centre as the the portion, which surface is not always able to concur with other spherical surfaces, yet concurrence is required as it should appear satisfactory, at least to the perception. Also, other imperfections of the spherical lenses are, arising from the stated; just as with telescopes with much enlarging, concerning the final image, it will hardly be able to be widened beyond two or three steps; as will be evident enough from working it out, then horrible obscurity appears. But against hyperbolic lenses, it is only objected that nothing will be able to be most clearly seen, except a visible point arising on the axis of the instrument. But this weakness (if thus it is allowed to be called) is sufficiently revealed in the eye itself, yet not to be imputing nature, for which nothing is in vain, as all will be most suitably carried through to the end. Nevertheless, with conical lenses and mirrors not permitted, it will be rather with spherical parts used in the place of spheroids and paraboloids in catopotrics; as with hyperboloids in dioptrics, with which, parts of spheres are less appropriate.

With these we go to the stars.

Epilogus.

Ex hisce paucis Propositionibus generaliter hic demonstratis, pendent non solum tota telescopiorum doctrina, sed etiam universa catoptrica , & dioptrica ; ostenditur enim, quando fiat visio distincta, quando confusa, in quo loco existat visibilis imago, & quanta illius pars ab oculo videatur, in qua ratione fiat imaginis augmentatio, vel diminutio, illustratio, vel obscuratio, quomodo facienda, quomodo calculanda: nunc restat, ut paucula quaedam de telescopiorum generibus dicamus. In hunc usq; diem, duae sunt tantum machinae opticae praecipuae; nempe telescopia ad remota aspicienda, & microscopium ad propinqua; quibus nos tertiam adjicimus; nempe Icoscopium, ad projiciendas visibilium imagines, in *Prop*. 52 hujus demonstratum: atq; hae machinae sunt trium generum; nimirum vel ex puris dioptricis, hactenus tantum cognitis; vel ex puris catoptricis; vel ex mixis, partim catoptricis, partim dioptrics : Deinde in unoquoq; genere infinitae possunt esse species, nempe, ex 2, 3, 4, &c. lentibus in primo; speculis in secundo; lentibus, & fpeculis in tertio : & in singulis illis speciebus sunt adhuc duae subdivisiones, una repraesentat imaginem in situ erecti, altera in situ everso, e quibus illa

semper est preferenda caeteris paribus, quae repraesentat imaginem in situ vero, & erecto. Primi autem generis, nempe e puris dioptricis, haec est sola proprietas; quod posit lentibus quotvis continuari, ita ut visibile quantum libet amplificetur; haec autem habet incommoda; primo ipsius species plerunq; in longitudinem non satis tractabilem excrescunt; secundo pertinens ad imaginem ultimam non potest satis dilatari ad illustrandas imagines,

[**93**]

absque magna crassitie radios visibilis debilitante ; tertio singularum lentium crassities radios visibilis aliquantulum debilitant. Secundi vero generis hae sunt proprietates, primo quod pertinens ad imaginem ultimam possit ad libitum dilatari, absque ulla radiorum debilitatione ; secundo quod ipsius species non excrescant in longitudinem; hoc autem habent incommodum non exiguum, quod vix possint earum specula continuari ultra duo. Tertium autem genus aureum nulla habet incommoda, & omnes priorum generum proprietates habere potest; si lentes & specula rite disponantur; hoc est si pertinentia ad imaginem ultimam, & penultimam sint specula, & pertinentia ad imagines reliquas sint lentes. Nos itaq; exempli gratia unum hujus perfectissimi generis telescopium describemus: Sit ADCE speculum parabolicum concavum exquisitissime politum, in cujus foco C, ponatur parvum speculum ellipticum concavum habens communem focum, & communem axem cum speculo parabolico concavo, & in hoc situ figatur ; oportet autem, ut hujus speculi elliptici focus praedictus quam proxime accedat ad ipsius verticem, & alter quam longissime ab ea distet, sit focus ipsius alter F, in communi axe producto extra speculum parabolicum, & in parabolici speculi vertice excavetur foramen rotundum MN, in quo foramine ponatur tubus eundem habens axem cum speculis, satis amplus ad recipiendos radios visibilis a speculo concavo elliptico reflexos, & producatur in L quam proxime ipsi F ; & figatur in L lens chrystallinae, cujus focus exterior F, plana autem ad partes oculi, habens commumem axem cum speculis, & tubo ; eritq; haec fabrica telescopium optimum presbytis destinatum : visibilia enim longinqua, oculo apparebunt, per tubum distinctissime ampliata, quam proxime in ratione distantiarum verticum a focis communibus; & illustrata eodem modo, quo illustraretur visibile tali angulo visum; si modo diameter pertinentis ad imaginem ultimam permittat uveam oculi radiis impleri; quomodo autem hoc sciatur docuimus in Scholio *Prop*. 51 hujus. Sed de omni Telescopiorum, Microscopiorum, & Icoscopiorum amplificatione, sit haec regula observanda:(geometrica enim est, si tam centrum visibilis, quam centrum oculi, sint in axe machinae, nunquam sensibiliter a veritate aberrat) ut distantia imaginis primae, a vertice suae emersionis, ad distantiam

[**95**]

ejusdem ab oculi centro, ita tangens semianguli visorii imaginis primae ex oculi centro, ad tangentum semianguli visorii ejusdem imaginis primae, ex vertice suae emersionis, vel semianguli visorii imaginis secundae, ex vertice suae incidentiae: & rursus ut distantia imaginis secundae a vertice suae emersionis, ad distantiam ejusdem a vertex incidentiae, ita tangentem semianguli visorii imaginis secundae e vertice suae emersionis, vel semianguli visorii imaginis tertiae ex vertice suae incidentiae; & ita progrediatur ad semiangulum visorium imaginis ultimae ex vertice suae emersionis seu visibilis ex vertice suae incidentiae, qui semiangulus duplicatus dat angulum visibilis quaesitum ex vertice suae incidentiae. Demonstrationem videre licet in Prop. 53 & 57 hujus.

De Mechanica horum speculorum, & lentium, ab aliis frustra tentata; ego in mechanicis minus versatus nihil dico: audacter tamen asseri, opticae perfectionem in lentibus & speculis sphaericis frustra quaeri. Si vero cui placeat poterit praecipuas hujus tractatuli propositiones sphaericis applicare; etsi non adeo perfecte: Portio enim sphaerae (praeterquam quod radios parallelos in unum punctum non congreget) locum imaginis praebet in una superficie sphaerica, habere idem centrum cum portione;quae superficies non potest omnimodo concurrere cum alia superficie sphaerica : [tasis?] tamen concursus requiritur, saltem ad sensum, ut ex praedictis satis apparet. Aliae etiam sunt lentium sphaericorum imperfectiones, a dictis emergentes; veluti, quod in telescopiis multum amplificantibus, pertinens ad imaginem ultimam, vix poterit ultra duos vel tres gradus dilatari; ut computanti fatis patebit : unde provenit horrenda obscuritas. At contra, lentes hyperbolicas solum objicitur, quod nihil possit distinctissime videm, praeter punctum visibilis, in axe machinae existens :Sed haec infirmitas (si ita appellare liceat) in ipso oculo est satis manifesta; non tamen naturae imputanda ; quae nihil frustra, sed omnia quam commodissime peragit. Nihilominus, lentibus, & speculis conicis non concessis; satius erit portionibus sphaericis uti loco sphaerideon, & conoideon parabolicatum ; in catoptrica; quam hyperbolicarum in dioptrica; cum quibus portiones sphaericae minus conveniunt.

His itur ad astra.