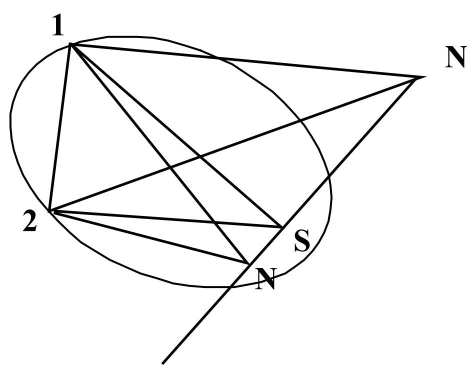
## Prop. 79. Problem.

*To find the central location of the nodes in the ecliptic, together with the distances of the planet from the sun for the same, from observations made on the earth.*

Let the first location of the earth be 1, and from that position the planet is observed in its node N without the latitude; and from the second location with the earth in location 2, the same planet is observed, at the same node N. Let the sun be at S, and the line SN shall be the common intersection of the plane of the ecliptic with the plane of the orbit of the planet. In the triangle 1S2, the sides 1S and 2S are given, and the angle 1S2; and hence the angles S12, S21 are given, and the side 12. And as N2S can be found from observation, it will give the angle N21; and in the same way N12 will be given. In the triangle N12, from the given 12, N12, N21, 2N will be found. Finally, in triangle N2S from the given N2, 2S, N2S, NS2 will be found for the position of the node N, and SN the nodal distance of the planet from the sun.

*Scholium.*

From such observations of the planets about the nodes, the most reliable method is picked of finding the mean motion of the planets, since they have the slowest motion at the nodes.




## Prop. 79. Problema.

*Loca centrica nodorum in ecliptica, una cum planetae, in iisdem, a sole distantiis, ex observationibus in terra habitis investigare.*

Sit primo terra in 1 ; & ex ea observetur planeta in nodo suo N carens latitudine: & secundo, ex terra in 2, observetur idem planera, in eodem nodo N; sitque; S sol; & recta SN, communis intersectio plani eclipticae, & plani orbitae planerae :

[**117**]

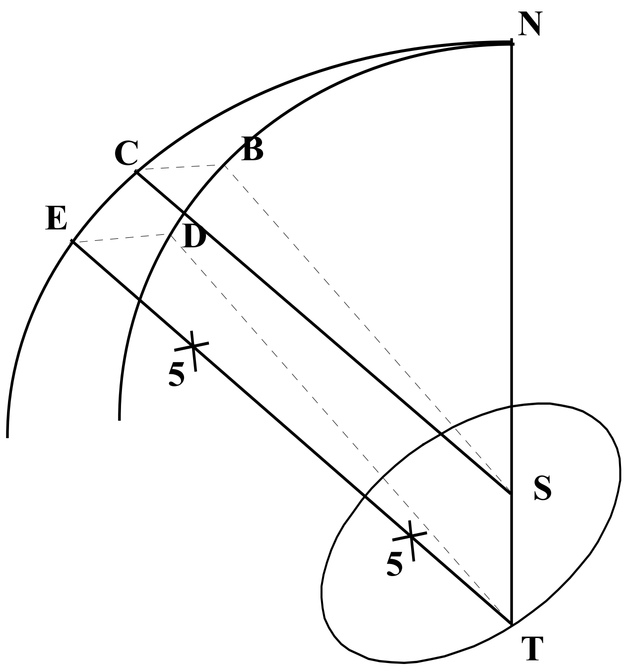
In triangulo 1S2, dantur latera 1S, S2, & angulus 1S2; & proinde dantur anguli S12, S21, & latus 12. Cumque detur N2S ex observatione, dabitur & angulus N21 ; eodemque modo dabitur N12. Et in triangulo N12, e datis 12, N12, N21, invenitur 2N. Denique in triangulo N2S e datis N2, 2S, N2S; inveniatur NS2 positio nodi N, & SN distantia planetae in nodo N a sole S.

*Scholium.*

Ex talibus observationibus planetarum in nodis, colligitur modus certissimus inveniendi motus medios planetarum ; quoniam nodi tardissimum habent motum.

## Prop. 80. Problem.

*To find the inclination of the orbit of a planet with the plane of the ecliptic.*




If the orbit of the planet is NCE, the intersection of which with the ecliptic, or with the nodal line NST from the earth in the line of the nodes arising from T, the planet 5 is observed at E, the latitude of which is ETD, the angle composed from the line T5E in the plane of the orbit of the planet, and the line TD, in the plane of the ecliptic. Another plane is drawn parallel to the plane of the latitude ETD, through the centre of the sun S ; and both these planes are normal to the plane of the ecliptic ; and the lines BS, CS are parallel to the lines DT, ET ; and therefore angle BSN = angle DTS, and angle ETD = angle CSB: but DTS and ETD are given from observation ; giving therefore BSN and CSB. Then in the spherical triangle CBN with the right angle at B, from the given sides CB, that is the angle CSB, and BN that is the angle BSN, the angle CNB can be found, the angle to the inclination sought.

## Prop. 80. Problema.

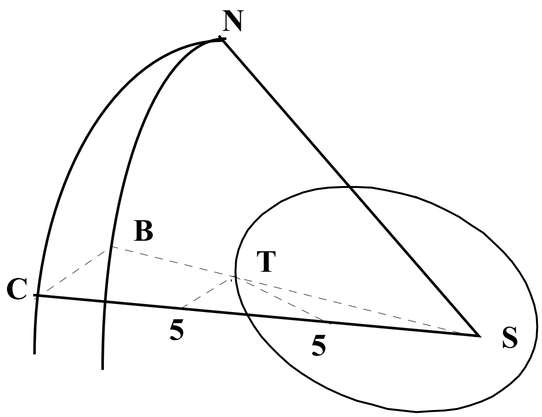
*Orbis planetarii cum ecliptica inclinationem invenire.*

Si orbita planetae NCE, cujus intersectio cum ecliptica, seu linea nodorum rectae NST a terra in linea nodorum existente T, obervetur planeta, in E, cujus latitudo ETD, angulus comprehensus a recta T5E in plano orbitae planetariae, & recta TD, in plano eclipticae. Plano latitudinis ETD parallelum, ducatur aliud planum, per centrum solis S ; eruntque ambo haec plana, plano eclipticae normalia ; & rectae BS, CS parallelae, rectis DT, ET ; & ideo BSN = DTS, ETD = CSB: dantur autem DTS, ETD ex observatione ; dantur igitur BSN,CSB. Deinde in triangulo sphaerico CBN rectangulo ad B, ex datis lateribus CB, hoc est angulo CSB, & BN hoc est angulo BSN ; inveniatur angulus CNB, inclinatio quaesita.

[**118**]

## Prop. 81. Problem.

*From the observation of the planet, in conjunction or apposition with the sun; setting aside the second inequality of this: to find the central latitude and the distance of the planet from the sun.*




The planet 5, of which the node is N, is observed from the earth T in conjunction or in apposition with the sun. The observed locus of the planet in its own orbit shall be C, in the ecliptic B. The angle BSN is given, indeed the separation of the node from the observed position of the planet in the ecliptic; therefore in the spherical triangle BNC with the right angle at B, from the given side BN, and from the angle BNC, the arc NC is found, for the distance of the planet in its orbit from the node N. Whenever the position of the node N is given, the position of the planet in its orbit is given; and hence the second inequality of this is set aside. Then in the same spherical triangle BCN the arc BC is found, the central latitude of the planet; and in the right angled triangle TS5, from the given angle TS5 for the central latitude of the planet, the observed latitude of the planet is TS, or the complement of this to the two lines at right angles, and from the side TS the distance of the earth to the sun, the length S5 is found, the distance of the planet from the sun.

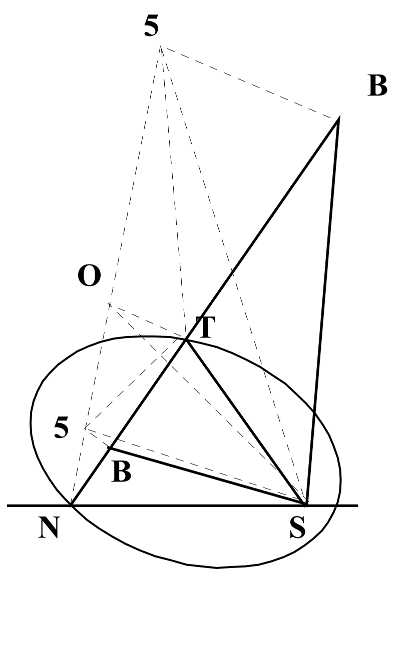
## Prop. 81. Problema.

*Ex observatione planetae, in conjunctione, vel oppositione cum Sole; ejus inequalitatem secundam exuere: latitudinem centricam, & a sole dictantiam reperire.*

Observetur planeta, cujus nodus N, in conjunctione, vel opposition cum sole S, ex terra T. Sitque planetae, locus in ecliptica

[**119**]

observatus B, in sua orbita C. Datur angulus BSN, nempe distantia nodi a loco planetae in ecliptica observato; & ideo in triangulo sphaerico BNC rectangulo ad B, e datis latere BN, angulo BNC; reperitur arcus NC, distantia planetae in sua orbita, a node N; cumque detur locus nodi N, datur & locus planetae in sua orbita; & proinde exuitur ejus secunda inaequalitas. Deinde in eodem triangulo sphaerico BCN inveniatur arcus BC, latitudo planetae centrica ; & in triangulo rectilineo TS5, e datis angulis TS5 latitudine planetae centrica, TS latitudine planetae observata, vel ipsius complemento ad duos rectos, & latere TS distantia terrae a sole; inveniatur latus S5, distantia planetae a sole.




## Prop. 82. Problem.

*From any one observation of a planet, the second inequality of which is set aside, to find the central latitude and the distance from the sun.*

The planet 5 is observed from the earth T, the nodal line of which is NS, and with the apparent latitude 5TB. The plane of the apparent latitude is produced, until it cuts the plane of the orbit of the planet in the line 5N and the plane of the ecliptic in the line BTN. From the centre of the earth T, the perpendicular TO to the plane of the ecliptic may be raised all the way to the plane of the orbit of the planet at O; and the line SO is joined, which by necessity is in the plane of the orbit of the planet. In the triangle NST, the side ST is given from the theory of the earth [i.e. the earth: sun distance], the angle NST from the location of the known node, the angle NTS from observation; therefore the angle TNS is given, and the sides TN and NS. Then, for the given orbit of the planet with the inclination to the ecliptic, and the angle TSN, and the distance of the earth from the node; the angle OST will be given : and as TN to TS, thus the tangent of the angle OST, to the tangent of the angle ONT. Therefore the angle ONT is given, and the angle 5TB becomes known from observation. Therefore, in the triangle 5NT, from all the given angles, with the side NT, and the side 5N is given; and in the triangle 5NB with the right angle at B, from the given angle 5NB and the side 5N, the side NB becomes known. Then in the triangle BNS, from the given sides BN and NS, with the angle intercepted BNS, the side BS and the angle BSN are found for the central longitude of the planet in the ecliptic, computed from the line of the nodes. And in the spherical right angled triangle in the above figure [Prop. 81] CBN, from the given side BN, surely the angle BSN in this figure, and the angle BNC for the inclination of the orbit, the line BC is found. This is the angle BS5 [ in current diagram], for the central latitude, and the side CN [above] is the angle 5SN, the central longitude of the planet in its orbit, computed from the line of the nodes: and hence the second inequality of this is set aside. However in the triangle 5SB with the right angle at B, from BS and the angle BS5; the side S5 will not be disregarded, the distance of the planet from the sun.

## Prop. 82. Problema.

*Ex unica planetae observatione quacunque; ejus inequalitatem secundam exuere: latitudinem centricam, & a sole distantiam invenire.*

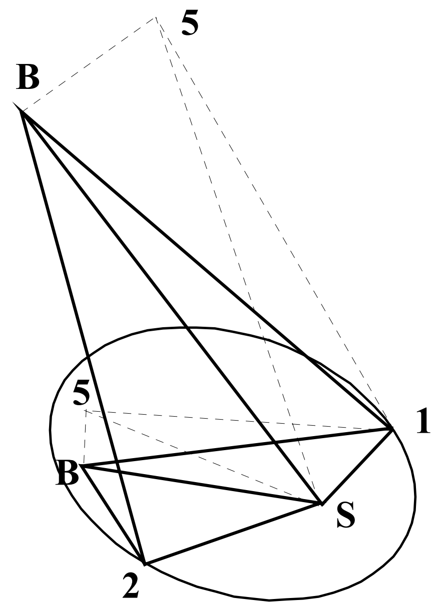
Observetur planeta 5, cujus linea nodorum NS, ex terra T, cum latitidine apparente TB. Producatur planum latitudinis apparentis, donec secuerit planum orbis planetarii in recta 5N, & planum eclipticae in recta BTN. E centro terrae T, erigatur plano

[**120**]

eclipticae perpendicularis TO, usq; ad planum orbis planetatii in O : & jungatur SO recta, quae necessario est in plano orbis planetarii. In triangulo NST, datur latus ST ex theoria terrae, angulus NST ex loco nodi cognito, angulus NTS ex observatione; dantur igitur angulus TNS, latus TN, latus NS. Deinde datis, orbis planetarii cum ecliptica inclinatione, & angulo TSN distantia terrae a nodo ; dabitur angulus OST : & ut TN ad TS, ita tangens anguli OST, ad tangentem anguli ONT ; datur igitur angulus ONT, & innotescit angulus 5TB observatione; ideoq; in triangulo 5NT, e datis omnibus angulis, cum latere NT, datur & latus 5N: & in triangulo 5NB rectangulo ad B, e datis angulo 5NB, & latere 5N, innotescit latus NB. Deinde in triangulo BNS, e datis lateribus BN, NS, cum angulo intercepto BNS, reperiuntur latus BS, & angulus B5N, longitudo centrica planetae in ecliptica, a linea nodorum computata; & in triangulo sphaerico rectangulo in superioribus figuris CBN, e datis latere BN, nempe angulo BSN in hac figura ; & angulo BNC orbium inclinatione, inveniatur BC latus, hoc est angulus BS5, latitudo centrica, & latus CN, hoc est angulus 5SN, longitudo planetae centrica in suo orbe, a linea nodorum computata :& proinde exuitur secunda ejus inaequalitas. Tandem in triangulo 5SB rectangulo ad B;e datis BS, BS5; non ignorabitur latus S5, distantia planetae a sole.

# Prop. 83. Problem.

*From two observations of the planet at the same point of the orbit, the distance of this planet from the sun, the central place in the ecliptic is found; and from which the longitude and the latitude of the planet are found; from the assumed theory of the earth.*




The earth shall be at 1 first, and the planet 5 is observed from this position, from which the perpendicular 5B may be dropped to the plane of the ecliptic; and the observed place of this in the ecliptic shall be B with latitude 51B; and after one or more whole revolutions of the planet, the planet 5 is again observed from the earth 2, and the sun shall be S. From the theory of the earth, all the sides and all the angles of the triangle 1S2 are given; also, the angles S1B and S2B are given from observation; therefore in triangle 21B, given the angles 21B and 12B ; and hence the angle 2B1 is given : and moreover given the side 21, from which the side B1 is found; & in the triangle 1B5, with the right angle at B, from the given side 1B, & from the angle B15, for the observed latitude; B5 can be found. Then in triangle B1S, from B1,1S, B1S given, BS is given, & 1SB, the central position of the planet in the plane of the ecliptic, from the earth in position 1 is computed; & in triangle BS5 with the right angle at B, from the given distances B5 and BS, the angle BS5 is found for the central latitude of the planet, and its distance S5 from the sun.

*Scholium.*

In this problem the position of the node may be given ; it is manifest that both the maximum inclination and the central position of the planet in its orbit are to be given, or ( if the maximum inclination shall be given) the position of the node and the central position of the planet in its orbit shall be given. In the right angled spherical triangle, the central latitude is one line, while the central longitude in the ecliptic is another line, computed from the line of the nodes; the central longitude of the planet in its orbit computed from the node is a third line, and the inclination of the orbit [to the ecliptic] is one angle. From which four, two are given and two are shown to be unknown. But from the two central places of the ecliptic and the two central latitudes, both the given maximum inclination and the position of the node in the ecliptic are given, by *Prop. 70* of this work. And from three positions of the planet in its orbit, both the position of the aphelion and the kind of the ellipse of the planet are given by *Coroll. 77* of this work. Therefore it is agreed, by applying the method many times, the inequalities of all the planets are to be found.

[**121**]

## Prop. 83. Problema.

*Ex binis observationibus planetae, in eodem orbis puncto ; ejus distantiam a sole, locum centricum in ecliptica, quo ad longitudinem, & latitudinem invenire; supposita tantum terrae theoria.*

Sit primo terra in 1, & ex ea observetur planeta 5, e quo in eclipticae planum, demittatur perpendicularis 5B ;eritq; locus illius observatus in ecliptica B, cum latitudine 51B; & post unam, vel plures planetae revolutiones integras, rursus observetur planeta 5, ex terra 2, sitque sol S. Dantur ex theoria terrae, omnia latera, & omnes anguli, trianguli 1S2 ; dantur etiam anguli S1B, S2B, ex observatione; igitur in triangulo 21B, dantur anguli 21B, 12B; & proinde angulus 2B1 : datur autem & latus 21, e quibus inveniatur latus B1; & in triangulo 1B5, rectangulo ad B, e datis latere 1B, & angulo B15, latitudine observata; reperitur B5. Deinde in triangulo B1S, e datis B1,1S, B1S, datur BS, & 1SB, locus planetae centricus in ecliptica, a terra in 1 computatus; & in triangulo BS5 rectangulo ad B, e datis B5, BS, innotescit angulus BS5, latitudo planetae centrica; & latus S5, distantia illius a sole.

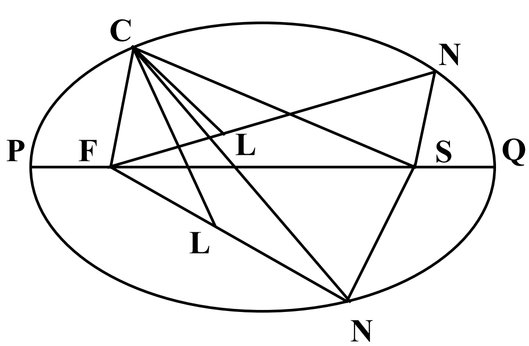
[**122**]

*Scholium.*

Manifestum est ( in hoc problemate daretur locus nodi) dari, & inclinationem maximam, & locum planetae centricum in suo orbe; vel ( si daretur inclinatio maxima ) dari locum nodi, & locum planetae centricum in suo orbe: In triangulo enim sphaerico rectangulo, latitudo centrica est latus unum, longitudo centrica in ecliptica, a linea nodorum computata, latus alterum ; longitudo planetae centrica in suo orbe, a nodo computata, latis tertium , & orbium inclinatio, unus angilus. E quibus quatuor, duo dati, duos ignotos semper manifestant. Ex duobus autem locis centricis in ecliptica, & duabus latitudinibus centricis; datur & inclinatio maxima, & nodi locus in ecliptica, per 70 hujus. Et ex tribus planetae locis centricis in suo orbe, datur & aphelii positio; & species ellipseos planetariae, per *Corol. 77* hujus. Constat igitur modus multiplex, inveniendi omnes planetarum inaequalitates.

## Prop. 84. Problem.

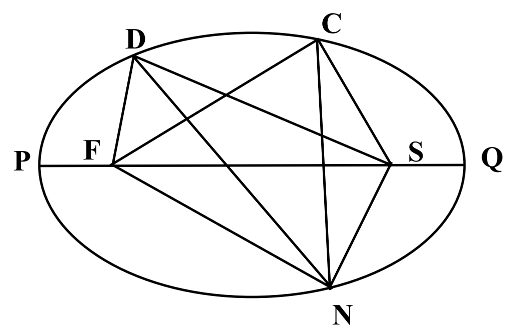
*From two given central positions of the planet in its orbit, together with the central motion of this planet, and the distances from the sun; to find the position of the aphelion and the kind of ellipse.*




Let PCQN be the ellipse of the planet, of which F is the focus of the central motion, or the sun S is the focus of the true motion, with PQ the transverse axis, and C and N shall be the positions of the planet. From the given sides SC and SN in the triangle CSN, to wit the distances of the planet from the sun ; and with the angle CSN taken from these and the known positions of the planet from the centre, the side CN and the angle NCS can be found. The lines FC and FN are drawn; and as NS + NF = CS + CF ; CS - SN will be equal to NF - CF and equal to LN: for FC = FL ; and in the isosceles triangle FLC, all the angles are given: truly LFC, from the central motion of the planet between the observations C and N and the angles FLC and

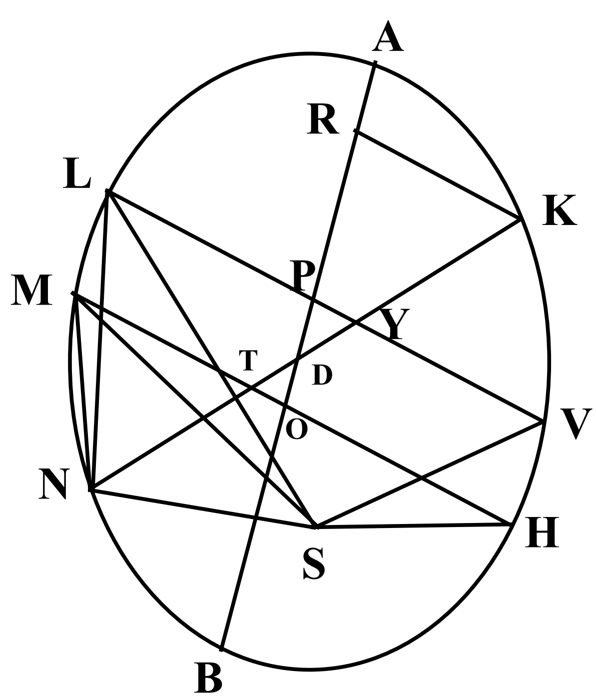
FCL on account of their equality ; therefore the angle CLN is given. Hence in triangle CLN, from the angle CLN, and the given sides NC and NL, the angle LCN is found ; hence the angle FCN = FCL + LCN is also given. In triangle FCN, from all the given angles, with the side CN given, the side CF will be given, and CF + CS = PQ. And then in triangle FCS, from FC and CS given, the angle FCS is known from the found angles FCN and NCS. [Hence] the distance FS between the foci, and the angle FSC for the location of the aphelion are found from the observation C.

*Scholium.*




But if anyone should want the resolution of this problem without the supposed central motion: by an observation from the earth D, and thus DN may found by the same method, by which CN was found in the [above] proposition. Let PQ 14 be the axis, and therefore FC = 14 - CS, & FN = 14 - NS will be given; therefore in the quadrilateral FCSN, all the sides are given, together with the diameter CN, from which the diameter FS may be found. In the quadrilateral FDSN too, all the sides are given from observation: SD, SN, FD = 14 - DS, and FN = 14 - NS; together with the diameter DN, from which again FS is sought. Therefore given the equation between FS first found, & FS found second, from the resolution of which everything sought shall be made clear.

But the most pretty of all the methods is the one set forth by *Dr. Ward on page 50 of Astronomiae Geometricae*, which neither supposes the central motion, nor the sun to be the focus of the ellipse; but however, the centre of the sun is fixed and the planets are moving in elliptical lines [curves]. From which the origin of the central motion, if such is given, and the position of the sun within the ellipse, being supposed [different] from the other methods, is inferred by a geometrical demonstration. But because of the excellence of this method, we ourselves will try to produce another example of the use of this in accordance with central motion. Let there be five points H, K, L, M, N on the ellipse, which shall nowhere determine parallel lines, and the lines NK, MH, are drawn cutting each other in T: while LV is drawn parallel to the line MH, cutting NK in Y. From observation, all the angles at [the sun] S are given by a line between S and the five points of the ellipse : hence the lines MN, NK, NL, and the angles MNS, KNS, MNT, NMS, HMS, NMT, NLY are



known. Then in the rectilinear triangle MNT, from all the given angles, with the side MN, the sides NT, MT are also given ; and in the rectilinear triangle NLY, from all the given angles, (for MTN = LYN) with the side LN; the sides NY, LY are given; and NT.TK : NY.YK : : MT.TH : LY.YV [From *Prop. 17*, *Appolonius, Book III;* see *e. g.* : T. Heath*, A History of Greek Mathematics,* p. 153; Dover] ; therefore YV is given, and hence the whole length LV. The lines LV [text has LY] and MH are bisected in P and O, and the line APDOB is drawn, surely the diameter of the ellipse; to which the ordinate lines PV and OH are joined ; and to the same diameter the ordinate line KR is joined. On account of the similarity of the triangles PDY and TDO: PY + TO : TY : : PY : DY ; and as the angle PYD is given, the lines PD, DO, DR, KR will be found. Therefore, let the lines be given by numbers: OH 10, PV 12, RK 8, PO 4, RP 2, with the angle KRB 60 [degrees]. Let BO be put as 1♃, and BP will be 1♃ + 4, but 100: 144 : : 1♃.OA : (1♃ + 4).PA [from $OH^2/PV^2$ = BO.OA/BP.PA]; therefore by the converse of *Prop. 99*, from the proportionalities of Gregory of St. Vincent, 100 is to 144 and is to the ratio 1♃ to 1♃ + 4 and to the ratio PA to OA = PA + 4; therefore PA is to PA + 4 as $\frac{144}{100}$♃ to 1♃ + 4, or as 144♃ to 100♃ + 400; and therefore the difference of these, surely

$$400 - 44\text{♃} : 144\text{♃} : : 4 : \frac{576\text{♃}}{400 - 44\text{♃}} = PA \quad ; \text{hence } PA + 4 = \frac{1600 + 400\text{♃}}{400 - 44\text{♃}} = AO,$$

[ RA = AO - 6; BR = 6 +♃ ]

$$\& \ RA = \frac{664\text{♃} - 800}{400 - 44\text{♃}} \ ; \text{and } BO \times OA = \frac{1600\text{♃} + 400\,q}{400 - 44\text{♃}} : 100 : : BR \times RA = \frac{664q + 1384\text{♃} - 4800}{400 - 44\text{♃}} : 64;$$

$$\text{therefore } \frac{102400\text{♃} + 25600q}{400 - 44\text{♃}} = \frac{66400q + 318400\text{♃} - 480000}{400 - 44\text{♃}},$$

and from the reduced eqn. 17q = 200 - 15♃ ; as from $1\text{♃} = {}^{r}q\,\frac{21700 - 45}{17} = 3\underline{017}$ = BO is given.

[Note: The correct algebra gives $17x^2 + 15x - 200 = 0$, with the solution $x = (-15 + \sqrt{13825})/34 = 3.017$ as shown; the original text contains a number of errors - as shown below in the Latin text.]

And the conjugate diameters are easily given from these ; and from the conjugate diameters, and the angle of the conjugate axes of these by the example from *page 50 ast. Geometricae*, or with the aid of Prop. 72 and 73 for the ellipse from *Gregory of St. Vincent.* And finally from the above given, everything sought will be easily found, surely the centre of the mean motion, (if it shall be in the nature of things) the position of the sun within the ellipse, and the eccentricity, or the distance of the centre of the sun from the centre of the mean motion, and thus the kind of ellipse of the planet.

## Prop. 84. Problema.

*Ex datis duobus locis centricis planetae in suo orbe, una cum ipsius motu medio, & a sole distantiis : positionem aphelii, & ellipseos planetaria speciem, invenire.*

Sit PCQN ellipsis planetae,cujus focus medii motus F, focus veri motus, sive sol S, axis transversus PQ, loca planetae C, N: E datis in triangulo CSN lateribus SC, SN, distantiis nimirum planetae a sole ; & angulo ab illis comprehenso CSN,

[**123**]

e locis planetae centricis cognito ; inveniatur latus CN, & angulus NCS, Ducantur rectae FC, FN; & quoniam NS + NF = CS + CF ; erit CS - SN = NF - CF = LN: est enim FC = FL ; & in triangulo FLC isosceli, dantur omnes anguli, nempe LFC ; ex medio motu inter observationes planetae C, N, & anguli FLC, FCL propter eorum aequalitatem ; ergo datur angulus CLN, Deinde in triangulo CLN, e datis CLN, NC, NL, reperitur LCN ; datur ideo angulus FCN = FCL + LCN; & in triangulo FCN, e datis omnibus angulis , cum latere CN, datur latus CF: & CF + CS = PQ. Denique in triangulo FCS, e datis FC, CS [CF in text] & angulo FCS, ex angulis inventis FCN, NCS cognito ; inveniuntur FS distantia focurum; & FSC positio aphelii, ab observatione C.

*Scholium.*

Si autem quis desideret hujus Problematis resolutionem, motu medio non supposito: detur terria observatio D, & proinde eodem modo invenietur DN, quo in proportione invenitur CN. Sit axis PQ14 & ideo dabuntur FC = 14 - CS, & FN = 14 - NS ; in quadrilatero igitur FCSN, dantur omnia latera, una cum diametro CN, e quibus inveniatur duameter FS. In quadrilatero quoq; FDSN, dantur omnia latera SD, SN ex observatione, FD = 14 - DS, FN = 14 - NS una cum diametro DN, e quibus rursus inquiratur FS :datur igitur aequatio inter FS primo inventam, & FS secundo inventam, cujus resolutio manifestae omnia quaesita.

Methodus autem omnium pulcherrima, est illa quam tradit *D. Wardus ast. Geometriae, pag. 50,* quae nec supponit motum medium, nec solem esse ellipseos focum , sed tantum , solis centrum immobile, & planetas moveri, in lineis ellipticis; Unde centrum motus

[**124**]

medii, (si tale detur) & locum solis intra ellipsem, ab aliis supposita, demonstratione Geometrica concludit. Sed propter eximium hujus methodi, usum conabimur nos aliam illius praxem in medium afferre. Sint quinque puncta in ellipsi, H, K, L, M, N, quae nusquam terminent lineas parallelas, ducantur NK, MH, secantes sese in T: lineae autem MH ducatur parallela LV, secans NK in Y. Observationibus dantur omnes anguli ad S, & rectae inter S, & quinq; puncta ellipseos : & proinde cognoscuntur rectae MN, NK, NL, & anguli MNS, KNS, MNT, NMS, HMS, NMT, NLY. Deinde in triangulo rectilineo MNT, e datis omnibus angulis , cum latere MN, dantur etiam latera NT, MT; & in triangulo rectilineo NLY, e datis omnibus angulis, (est enim MTN = LYN) cum latere LN; dantur latera NY, LY ; at NT x TK : NY x YK :: MT x TH:LY x YV ; ergo datur YV, & proinde tota LV, Bisecentur rectae LY, MH, in P, & O, & ducatur recta APDOB, diameter nempe ellioseos, cui ordinatim applicantur PV, OH ; eidemq; diametro ordinatim applicetur recta KR : & ob similitudinem triangulorum PDY, TDO, erit PY + TL : TY : : PY : DY ; cumque detur angulus PYD, non latebunt PD,DO, DR, KR. Sint igitur, in numeris datae, rectae, OH 10, PV 12, RK 8, PO 4, RP2, cum angulo KRB 60. Ponatur BO 1♃, eritq; BP 1♃ + 4, sed 100: 144 : : 1♃ x OA : 1♃ + 4 x PA; igitur per conversum 99, de proportionalitatibus *Greg. a s. Vincentio*; 100 est ad 144 ; at radio 1♃ ad 1♃ + 4 est ad rationem PA ad OA

[**125**]

= PA + 4; igitur PA est ad PA + 4, ut $\frac{144}{100}$♃ ad 1, ♃ + 4, vel 144♃ ad 100♃ + 400; & ideo horum differentia, nempe

$400 - 44\text{♃} : 144\text{♃} :: 4 : \frac{576\text{♃}}{400 - 44\text{♃}} = PA$ ; proinde $PA + \text{♃} = \frac{1600 + 400\text{♃}}{400 - 44\text{♃}} = AO$,

& RA $= \frac{664\text{♃} - 800}{400 - 44\text{♃}}$ ; at BC x OA $= \frac{1600\text{♃} + 400\,q}{400 - 44\text{♃}} : 100 :: $ BR x RA $= \frac{664q + 3184\text{♃} - 4800}{400 - 44\text{♃}} : 64$;

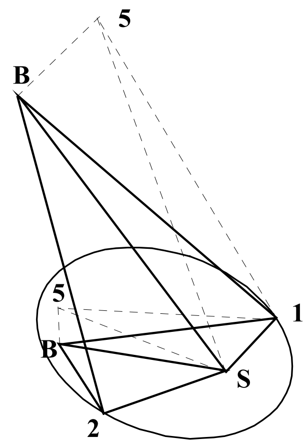
igitur $\frac{102400\text{♃} + 25600q}{400 - 44\text{♃}} = \frac{66400q + 318400\text{♃} - 480000}{400 - 44\text{♃}}$,

& aequatione reducta $17q = 200 - 90\text{♃}$ ; ut de datur $1\text{♃} = \frac{{}^{r}q\,5425 - 45}{17} = 1686 = BO$.

Atque; ex his facile dantur diametri conjugatae ; & ex diametris conjugatis, & earum angulo, axes conjugati per praxem *pag. 50 ast. Geometricae*, vel ope 72, & 73 de ellipse *Grego. as. vincentio*. Et tandem ex praedictis datis, facile innotescunt omnia quasita, centrum nempe motus medii, (si sit in rerum natura) locus solis intra ellipsem, & excentricita, seu distantia centrij solaris a centro medii motus, & ellipseos planetariae species.

## Prop. 85. Problem.

*From the presumed theoretical positions of the earth and the planet, according to which the longitude, latitude, and the position of the planet from the earth are to be computed.*




The position is calculated from the given central motion of the earth itself, which shall be 1 in figure of Prop. 83 of this work [redrawn here]: and in short by the same method the position of the planet in its orbit, which shall be 5 ; and from the given distance from the node, and the inclination of the orbit, the angle BS5 may be found, the central latitude of the planet, and the position of the planet in the ecliptic B; and with the position of the earth in the ecliptic given 1, the angle 1SB is given. Given also from the theories of the earth and the planet SO, S5 and in right-angled triangle SB5, from 5S, BS5 given, BS is found. Then in triangle BS1 from BS and S1 given; and from the contained angle BS1, the side 1B is given, and the angle S1B, the position of the planet in the ecliptic from the earth 1. And finally, as 1B to SB, thus the tangent of the angle

[**126**]

BS5, to the tangent of the angle B15, the angle of the latitude sought.

## Prop. 85. Problema.

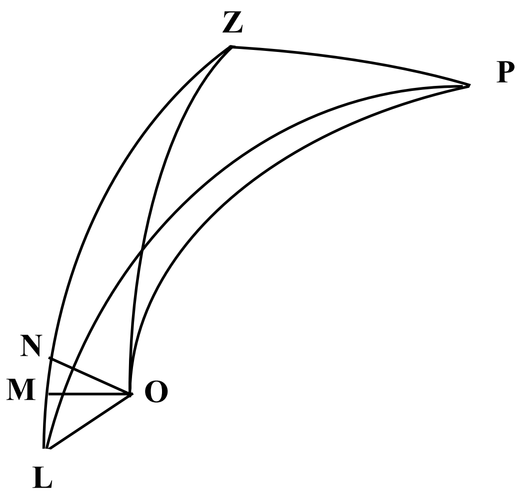
*Suppositis Theoriis Terrae, & Planetae ; locum Planetae e Terra, quo ad longitudenem, & latitudinem supputare.*

Ex terrae mediis motibus datis supputetur ipsius locus, qui in figura 83 hujus sit 1 {78 hujus} : & eodem prorsus modo supputetur locus planetae in suo orbe, qui sit 5; & data a nodo distantia & inclinatione orbium, inveniatur BS5, latitudo planetae centrica, & locus illius in ecliptica B; cumq; detur locus terrae in ecliptica 1, datur angulus 1SB. Dantur etiam ex theoriis terrae & planetae SO, S5 & in triangulo rectangulo SB5, e datis 5S, BS5 & in triangulo rectangulo BS5, e datis 5S, BS5 reperitur BS. Deinde in triangulo BS1 e datisBS, S1; & angulo comprehenso BS1, datur latus 1B, & angulus S1B, locus planetae in ecliptica, e terra 1. Et tandem, ut 1B ad SB, ita tangens

[**126**]
anguli BS, ad tangentem anguli B15, latitudinis quaesitae.

## Prop. 86. Problem.

*From one eclipse of the moon ; to find the lunar parallax and the diameter of the shadow for the moon in transit : with the supposition of the theory of the sun's motion only, and with the fixed positions of the stars.*




It may be observed in an eclipse of the moon how precisely the horns of the moon may become to the perpendicular, or arise in a single azimuth angle: (but only if it occurs), and at that point of time, the separation of the horns is observed, the height from one of the horns, and its declination and the right ascension according to the fixed stars; and hence the declination and the right ascension of the mid-point between the horns will be found. Let the distance of the mid-point between the horns from the [celestial] pole P be the arc PL, and let the complement of the apparent altitude, or the distance of the same apparent point from the vertical [zenith] Z be the arc ZL.

[**127**]

And in the spherical triangle ZPL, from the three given sides, the angle ZLP may be found. And the centre of the shadow, or the point opposite the sun, shall be O ; and since the declination and the right ascension of this shall be given in the spherical triangle LPO [the following quantities] shall be given: the side PO, the distance of the centre of the shadow from the pole; the angle OPL, the difference of the right ascensions; and the observed side LP; and therefore the side LO will be given ; and the angle OLP. Then O, from the centre of the shadow, may fall on the vertical circle ZL, and truly and more evidently, appearing to cross the horns of the moon in the perpendicular arc OM. And since there are two circles on the sphere [*i.e.* celestial], truly the circle of the moon, and the circle of the shadow, they mutually cut each other in the horns of the moon. The arc of one circle from the pole, the perpendicular joining the arc of their greatest circle of intersection, truly OM, divides the aforementioned arc in two in the point M ; M will be the true position of this point ; L is the apparent position of this join, surely of the mid-point between the horns [recall that this point is not seen 'head-on', but at an angle, hence the parallax] ; and the arc ML the points of the same parallax, which is found from the resolution of the spherical triangle LMO with the right-angle at M, from the given angle MLO = ZLP + PLO, and the side LO. Then N shall be the true position of one of the horns, and in the spherical triangle MNO with the right-angle at M [N in text], from the given MN, MO, NO is given, the semi-diameter of the shadow. But I must satisfy the two, from little geometrical tricks and our vision, that this parallax is the natural parallax of the centre of that circle in the body of the moon, which is drawn through the horns of the moon; for this given parallax, the geometrical parallax of the centre of the moon, can be found by Prop. 69 of this work, which in no circumstance differs sensibly from the parallax now found.

*Scholium.*

But because it is difficult to find the right ascension of the first fixed stars, and the parallax of the moon will be found much more exact from the common azimuth following Prop. 62, 63, and 64 of this work; rather it might be by the parallax of the moon thus found, and the converse of this problem, to find the right-ascension of the fixed stars. I was deciding to add certain things about the theory of the moon; but a stone sticks in the entrance, still unshaken by the astronomers, and entirely prohibiting a geometrical entrance: indeed the central lunar motions cannot be determined by a geometrical method up to this day. For these truly supposed, and from three positions of the moon in her orbit, from the second inequality of freedom, kinds of lunar ellipses can be found, and the position of the apogee in each one: from observation, by Corollary 77 of this work; if the motion of the apogee may be taken from both the central motion and the true motion ; and remaining may be put in the place of both the central motion and the true motion ; by preceding in the same way as is taught in that Corollary. In short I avoid the second inequality of the moon, since the astronomers themselves are still ignorant about this. However our mind is, also with some firm reason, that the moon has some annual inequality brought together with her first inequality, from which she might easily be set free, with the help of three lunar eclipses observed from one place of the ecliptic.

## Prop. 86. Problema.

*Ex unica eclipsi lunari ; parallaxem lunae, & diametrum umbrae in transitu lunae investigare : suppositis tantum solis theoria, & fixarum locis.*

Observetur quam exactissime in eclipsi lunae, cum cornua lunae fiant ad perpendiculum, sive existant in uno Azimuth: (si modo accidat) & in eo temporis articulo, observetur distantia cornuum, altitudo unius cornu, & illius declinatio, & ascensio recta, per stellas fixas; ac proinde non latebit, declinatio, & ascensio recta, puncti medii inter cornus. Sit distantia puncti medii inter cornua a polo P, arcus PL , sitq; complementum altitudinis apparentis, seu distantia apparens ejusdem puncti a vertice Z, arcus

[**127**]

ZL. Et in triangulo sphaerico ZPL, e datis tribus lateribus, inveniatur angulus ZLP. Sitq; centrum umbrae, seu oppositum solis O; & quoniam datur ipsius declinatio, & ascensio recta, dabuntur in triangulo sphaerico LPO, latus PO, distantia centri umbrae a polo, angulus OPL, differentia ascensionum rectarum , & latusLP observatum; dabitur ergo latus LO; & angulus OLP. Deinde a centro umbrae O cadat in circulum verticalem ZL, & vere, & apparenter, per cornua lunae transeuntem, arcus perpendicularis OM: Et quoniam duo circuli in sphaera, nempe circulus lunae, & circulus umbrae, se mutuo secant in lunae cornubus ; arcus a polo unius circuli, perpendicularis ad arcum circuli maximi eorum intersectiones jungentem, nempe OM, bifariam dividit praedictum arcum in puncti M ; erit igitur M locus verus illius ; juncti, cujus L est locus apparens, nimirum puncti medii inter cornu ; & arcus ML ejusdem puncti parallaxis, qui reperitur ex resolutione trianguli sphaerici LMO rectanguli ad M, e datis angulo MLO = ZLP + PLO, & latere LO. Deinde sit N verus locus unius cornu, & in triangulo sphaerico MNO rectangulo ad N, e datis MN, MO, datur NO, semidiameter umbrae. Ut autem geometricis ingeniis aliquantulum satisfaciam, duo hanc parallaxem esse genuinam parallaxem, centri illius circuli in corpore lunae, qui ducitur per illius cornua, & nostrum visum ;data autem hac parallaxi, poterit inveniri & geometrica parallaxis centri lunaris, per 69 hujus, quae nunquam sensibiliter differt a parallaxi nunc inventa.

*Scholium.*

Sed quoniam difficulter invenitur ascensio recta primae stellae fixae, & parallaxis lunae multo exactius inveniri poterit, per commune azimuth secundum 62, 64, 65 hujus ; satius esset per parallaxem lunae ita inventam, & hujus problematis conversum, ascensioinem rectam primae stellae fixae investigare. Statuebam de theoria lunae quaedam adjungere; sed in ipso vestibulo haeret saxum, ab astronomis adhuc inconcussum, & ingressum geometricum omnino prohibens: motus enim medii lunares in hunc usque diem, non determinantur methodo geometrica. Illis vero suppositis, & datis tribus locis lunae insua orbita, a secondis inaequalitatibus liberatis, poterit inveniri species ellipsess lunaris, & positio apogaei

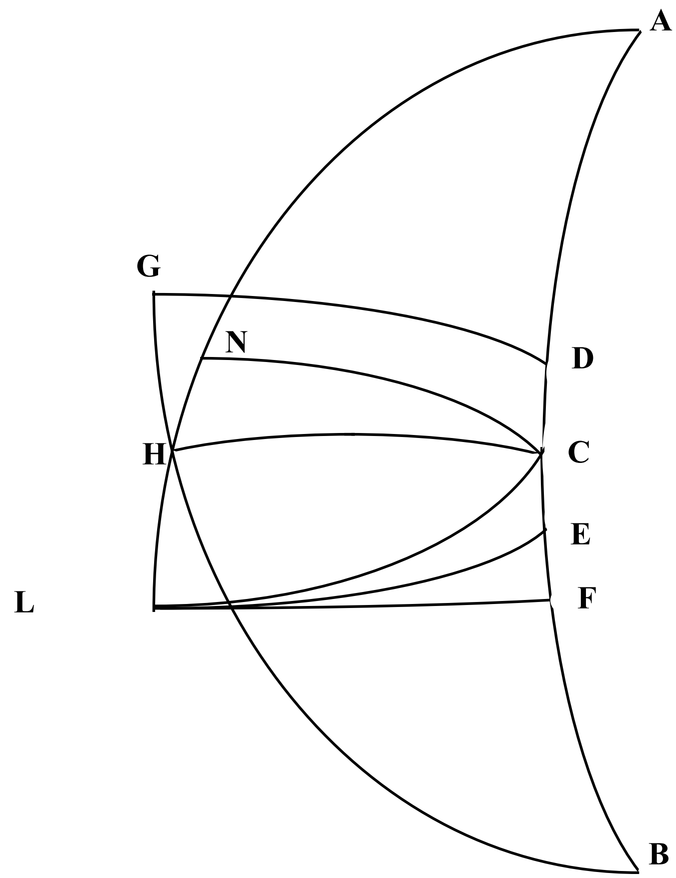
[**128**]

in unaquaq: observatione, per Corollarium 77 hujus; si auferatur motus apogaei, & a motu medio, & motu vero ; & residua ponantur loco motus medii, & motus veri ; eodem modo procedendo ut in illo Corollario docetur. A secundis lunae inaequalitatibus prorsus abstineo, quoniam adhuc ipsis astronomis sunt incognitae. Nostra tamen mens est, aliqua etiam ratione stabilita; lunam habere aliquam inaequalitatem annuam, cum prima sua inaequalitate commissam; a qua facile liberaretur, ope trium eclipsium lunarium, in uno eclipticae loco observatarum.

## Prop. 87. Problem.

*The parallaxes of two planets are to be investigated from the conjunction of each body.*

Let the true locations of the observations of the two planets be H and C, in the body conjunction: and the planet C is observed in the common azimuth of the two locations, the poles of which are A and B. Let the apparent positions of the planets C and H, from the position of which the pole is A be E and L ; and from the position of the pole B the apparent positions are D and G; and the arcs EL and DG of the angles CEL and GDC may be observed at the same time, according to Prop. 73 of this work. But the arcs AE and BD are given, to be observed finely enough to spread out the four parts; and hence the arcs AL and BG are given, and the angles HGD and HLE ; from which given the parallaxes CD, CE, HG, HL are to be found. But since it is impossible to find the arcs AE, AL; BD and BG with the same care that the rest of the arcs and angles are discovered; we therefore reject these from the calculation : yet they reveal with enough accuracy the ratio of the sines of the arcs DC, CE, and GH, HL, by Prop. 64 of this work. But the proportion of their distances from the centre of the earth is given by the theories of the planets C and H. Therefore by Prop. 69 of this work, given the proportion of the sine of the arc CE, to the sine of the arc HL; and the sine of the arc DC, to the sine of the arc HG. And therefore the mutual proportions are given between the sines of the parallaxes CE, CD, HG, HL. Hence from one given all become known; the perpendicular arc LF is dropped from the point L to the common azimuth ACB: and in the spherical triangle ELF, with the right-angle at F, from the given side LE, and from the angle LEF; the side FE may be found, and the side FL and the angle FLE: therefore the angle FLH will become known. The sine of the parallax CE may be put as 1ꝝ ; and from the given sines of the arcs FE and EC, the sine of the arc FC of the sum of these is formed: then in the spherical triangle LFC, with the right-angle at F, from the given sines of the sides FC, FL, the sine of the side LC and of the angle FLC may be found: and thus from the given sines of the angle HLF and the angle CLF, the sine of their difference may be found,




surely of the angle HLC. The perpendicular arc CN is dropped from the point C on the azimuth AHL: and thus in the spherical triangle LCN, with the right-angle at N ; from the given sine of the side LC and the sine of the angle CLN, the sines of the sides CN and LN may be found; but given the sine of the arc HN, without doubt the difference of the arcs LH and LN. And again in the spherical triangle HNC, with the right-angle at N ; from the given sines of the sides HN and NC, the sine of the side HC may be found. And thus by the same method the sine of the arc HC may be found from the given sines of the arcs DC and GH, in the fractional part of which the sine of the arc CE is 1♃ : therefore an equation will be given, between the sine of the arc HC found first, and the same sine found from the second calculation ; the resolution of which will give the value of the root, surely the sine of the parallax CE.

*Scholium.*

This prettiest of problems has a use, but perhaps a very laborious one, in the observations of Venus or Mercury obscuring a little part of the sun : indeed from such the parallax of the sun will be able to be investigated. Up to the present we have talked about parallaxes with respect to the globe of the earth; certain follow for parallaxes of the great orbit of the earth.

## Prop. 87. Problem.

*Ex duorum Planetarum conjunctione corporali; utriusq; planeta Parallaxes investigari.*

Sint loca vera duorum Planetarum, in conjunctione corporali observatorum H, C: & observetur planeta C in communi azimuth duorum locorum, quorum vertices A, B: sintq; planetarum C, H, e loco cujus vertex A, loca apparentia E, L; & e loco cujus vertex B, loca apparentia D, G; & eodem instante observentur per 73 hujus arcus EL, DG, anguli CEL, GDC: Dantur autem arcus AE, BD, quadrantibus ad intentum satis subtiliter observari; & proinde dabuntur arcus AL, BG, & anguli HGD, HLF ; e quibus datis, inveniendae sint parallexes CD, CE, HG, HL : Quoniam autem impossibile est arcus AE, AL, BD, BG, eadem diligentia invenire, qua reperiuntur arcus, & anguli reliqui; ideo eos e calculo rejicimus: Manifestant tamen satis accurate, rationem sinuum , arcuum DC, CE, & GH, HL, per 64 hujus. Datur autem ex planetarum C, H, theoriis, proportio suarum distantiarum a centro terrae, & igitur per 69 hujus, datur proportio sinus arcus CE, ad sinum arcus HL; & sinus arcus DC, ad sinum arcus HG: & ideo dantur proportiones mutuae inter sinus parallaxium CE, CD, HG, HL : & proinde uno dato innotescunt omnes, ex puncto L in azimuth commune ACB, demittatur arcus perpendicularis LF: & in triangulo sphaerico ELF, rectangulo ad F, e [E in original] datis latere LE, & angulo LEF; inveniantur latus FE, latus FL, & angulus FLE: innotescit igitur angulus FLH. Ponatur sinus parallaxi [parallaxeos in original] CE 1♃ ; & e datis sinubus arcuum FE, EC, inveniatur sinus arcus FC eorum summae: deinde in triangulo sphaerico LFC, rectangulo ad F, e datis sinubus laterum FC, FL, inveniatur sinus lateris LC, & anguli FLC: e datis itaq; sinu anguli HLF, & sinu anguli CLF, inveniatur sinus eorum differentiae, nempe anguli HLC:

[**130**]

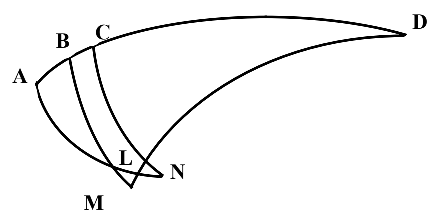
& demittatur a puncto C in azimuth AHL, arcus perpendicularis CN : in triangulo itaq; sphaerico LCN, rectangulo ad N ; e datis, sinu lateris LC, sinu anguli CLN; inveniatur sinus laterum CN, LN; datur autem sinus arcus HN, differentiae nimirum arcuum LH, LN. Et tandem in triangulo sphaerico HNC, rectangulo ad N ; e datis sinubus laterum HN, NC, inveniatur sinus lateris HC. Et eodem prorsus modo inveniatur sinus arcus HC, e datis sinubus arcuum DC, GH, in partibus, quarum sinus arcus CE 1♃ : dabitur ergo aequatio, inter sinum arcus HC primo inventum , & sinum eundem secundo inventum ; cujus resolutio dabit valorem radicis, nempe sinum parallexeos CE.

*Scholium.*

Hoc Problema pulcherrimum habet usum, sed forsan laboriosum, in observationibus Veneris, vel Mercurii particulam Solis obscurantis : ex talibus enim solis parallexis investigari poterit. Hactenus loquuti sumus de parallaxibus respectu globi terrestris: sequuntur quaedam de parallaxibus magni orbis terrae.

## Prop. 88. Problem.

*To investigate the parallax of fixed stars* (*by a sensing method*).




Let ABCD be an arc of the ecliptic, in which A may be assigned to a point in apposition to the sun; and let the true position of the star be L, N appearing as the apparent location. The declination is found from the meridian altitude of the star; and the right-ascension from any equal motion, or by *Scholium* 86 of this work, and hence the latitude itself will be found, which shall be NC ; and the position in the ecliptic C. Then, in the spherical right-angled triangle ANC ; from the given sides AC and CN ; the side AN is found, and the angle CAN. Again, with the apposition of the sun holding, an observer at point D shall observe the latitude MB, and the place in the ecliptic B of the same star (Of which the true position is L, but now appearing at M) and by the same way as before, the arc MD and the angle MDB of the same star are found. Finally in the spherical triangle ADL, from the given angles ADL and DAL ; and from the side AD,

[**131**]

from the known motion of the sun, the side AL will be given, which taken from the arc AN, leaves the parallax LN, of the observation N ; and the side DL, which taken from the arc DM, leaves LM, the parallax of the observation M.

## Prop. 88. Problema.

*Parallaxem stellae fixae* (*modo sensibilem*) *investigare.*

Sit arcus eclipticae ABCD, in quo assignetur punctum soli oppositum A ; sitq; verus locus stellae L, apparens N; cujus loci apparentis, declinatio invenitur per altitudinem stellae meridianam; & ascensio recta, per motum aliquem aequabilem ; vel *Scholium* 86 hujus ; & proinde non latebit ipsius latitudo, quo sit NC ; & locus in ecliptica C. Deinde, in triangulo sphaerico rectangulo, ANC ; e datis lateribus AC, CN ; invenitur latus AN, & angulus CAN. Rursus opposito solis punctum D tenente, observetur ejusdem stellae (cujus verus locus L, nunc autem apparens locus M) latitudo MB, & locus in ecliptica B; & eodem modo quo ante, invenitur arcus MD, & angulus MDB. Tandem in triangulo sphaerico ADL, e datis angulis ADL, DAL ; & latere AD,

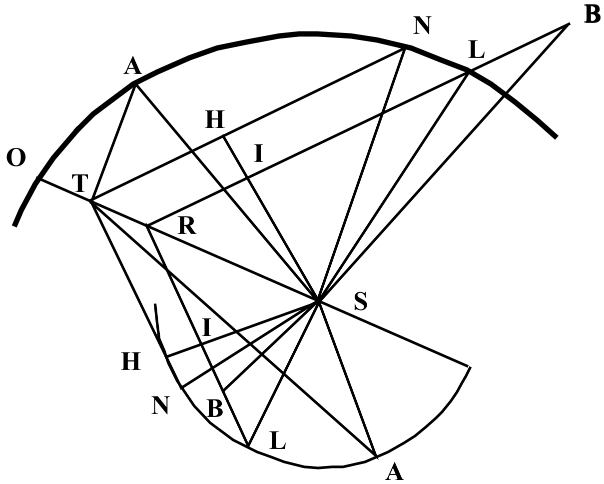
[**131**]

per motum solis cognito, dabitur latus AL, quod ab arcu AN ablatum, relinquit LN parallaxem, observationis N ; & latus DL, quod ab arcu DM ablatum ; relinquit LM, parallexem observationis M.

## Prop. 89. Theorem.

*The ratio, that can be observed of the sine of the parallax of a phenomenon to the sine derived from another parallax of the same phenomenon at another time, is composed from the direct proportion of the sines of the apparent distances from the sun; from the direct proportion of the distances of the earth from the sun ; and the reciprocal proportion of the distances of the phenomenon from the sun.*

Let the sun be S, with the earth T for the first time of observation, and the phenomenon for the same time A; the earth R for the time of the second observation, the phenomenon B, with O opposite the sun. With Centre S and radius SA the circle ANL is described, and the right lines SRTO, TA, SA, RB, SB are drawn. I say that the ratio: the sine of the parallax TAS to the sine of the parallax RBS, is to be composed from the ratio of the sine of the angle STA to the sine of the angle SRB; with the ratio TS to RS ; and from the ratio BS to AS. For the right line TN is drawn parallel to RB: and to the intersections of the lines RB and TN with the circle ANL, surely L and N, the lines SL and SN are drawn; and to the lines TN and RB, the perpendicular SH is drawn. And (by Prop. 64 of this work) the sine of the angle TAS to the sine of the angle TNS shall thus be as the sine of the angle STA to the sine of the angle STN: and as the sine of the angle TNS, surely ( putting NS for the radius) SH; to the sine of the angle RLS, surely (for the radius has put LS = NS) SI; thus TS to RS: and as the sine of the angle RLS the sine of the angle RBS; thus BS to LS = SA. If therefore there were as many as, surely, the sine of the angle TAS, the sine of the angle TNS, the sine of the angle RLS, the sine of the angle RBS; the ratio of the first to the last, surely the sine of the parallax TAS, to the sine RBS, composed from the mean



ratio, which have been shown with the same ratio; the sine of the angle STA, to the sine of the angle SRL; TS to RS ; BS to AS: therefore the proposition is shown.

Comment on Prop. 89.

From Theorems 63 and 64, we have the parallax of the observation, namely the difference of the angles subtended between the vertical and the phenomenon, for an observer located on the surface of the earth and the angle measured from the centre of the earth; from the theorems it is easily shown that the ratio of the sines of the parallaxes is the same as the ratio of the sines of the angles to the vertical, for two observers at different locations at the same time. In the present theorem, the sine rule applied to triangles TAS and TNS, noting SA = SN, gives sin TAS / sin TNS = sin STA / sin STN. Also, from the right-angled triangles and the parallel bases, sin TNS = HS/SN, and sin RLS = SI / SL, giving sin TNS / sin RLS = HS / SI = TS / RS.
Also, sin RLS / sin RBS = BS / LS. Hence, if we consider sin TAS / sin TNS, and sin RLS / sin RBS:
sin TAS / sin RBS = sin TNS / sin RLS × BS / LS × sin STA / sin SRL =
TS /RS × BS / AS × sin STA / sin SRL.

The situation in the theorem might occur if the phenomenon were a comet, and the positions of the comet A and B were observed at time intervals of 12 hours from the same location on earth, and TR is related to the diameter of the earth.

## Prop. 89. Theorema.

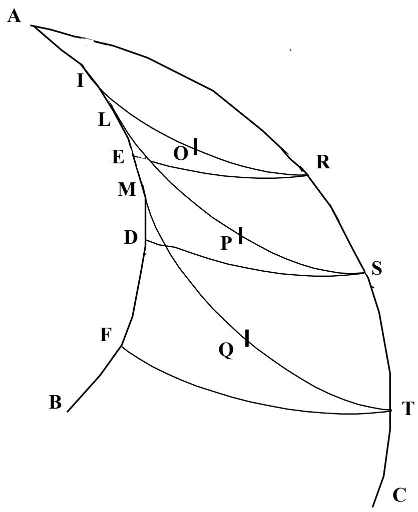
*Ratio, sinus parallaxeos Phaenomeni, ad sinum, alterino parallaxeos ejusdem Phaenomeni, alio tempore observati; est composita ex directecta proportione sinuum, distantiarum apparentium a sole ; directa proportione distantiarum terrae a sole; & reciproca proportione distantiarum Phaenomeni a sole.*

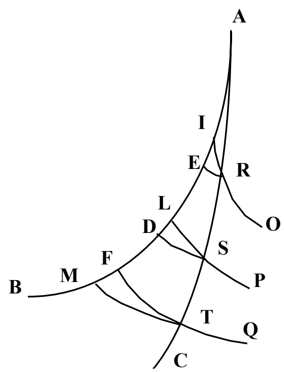
Sit sol S, terra tempore primae observationis T, phaenomenon eodem tempore A; terra tempore secundae observationis R, Phaenomenon B, solis oppositio O. Centro S & radio SA describatur circulus ANL, & ducantur rectae SRTO, TA, SA, RB, SB. Dico rationem, sinus parallaxeos TAS, ad sinum parallaxeos RBS, esse compositam ; ex ratione sinus anguli STA, ad sinum anguli SRB; ratione TS ad RS ; & ratione BS, ad AS. Rectae RB, parallela ducatur TN: & ad intersectiones rectarum RB, TN, cum circulo ANL, nempe L, N, ducantur rectae SL, SN; & ad rectas TN, RB, perpendicularis ducatur SH. Eritque (per hujus 64) ut sinus anguli TAS, ad sinum anguli TNS ; ita sinus anguli STA, ad sinum anguli STN: & ut sinus anguli TNS, nempe ( positio NS ratio) SH; ad sinum anguli RLS. nempe (posito ratio LS = NS) SI; ita TS, ad RS: & ut sinus anguli RLS ad sinum anguli RBS; ita BS, ad LS = SA. Si igitur fuerint quotcunque quantitates, nempe sinus anguli TAS, sinus anguli TNS, sinus anguli RLS, sinus anguli RBS; ratio primae ad ultimam, nempe sinus parallaxeos TAS, ad sinum RBS, componentur ex rationibus mediarum , quae eadem demonstratae sunt cum rationibus ; sinus anguli STA, ad sinum anguli SRL; TS ad TS ; BS ad AS: patet ergo propositum.

## Prop. 90. Problem.

*From three given longitudes and latitudes of any phenomenon moving around the sun, on setting aside the second inequality of this and the inclination of the orbit to the ecliptic; for each observation of the phenomenon to find the distance from the sun with the supposition however of the theory of the earth, and with the ratios of the distances of the phenomenon from the sun.*

Let AB be an arc of the ecliptic, and AC shall be the orbit of the phenomenon: three locations R, S, T of the phenomenon are observed from the earth in its orbit [with reference points] O, P, Q. Because the positions of the phenomenon observed from the earth are given ; the same apparent distances will also be given, either from the direction of the sun or from the direction opposite the sun, surely IO, LP, and MQ; and the arcs IL and LM ; and the angles OIA, PLA, and QMA; and by the preceding proposition the ratios of the sines of the parallaxes RO, SP, and TQ become known. The sine of the arc RO is put as 1♃ ; therefore the sines of the arcs SP, TQ will be given in terms of unknown numbers : but the sines of the arcs IO, LP and MQ are given; therefore the sines of the arcs IR, LS, MT are found in terms of unknown numbers. And from the given sines of the arcs IR, LS, MT, with the sines of the angles RIF, SLD, TME ; the sines of the perpendicular arcs RE, SD, and TF can be found and the sines of the arcs ED and DF. Then from the given sines of the arcs ER, DS and ED; the sine of the angle CAB is sought: by Prop. 70 of this work: and in the same way from the given sines of the arcs DS, FT and DF, the sine of the same angle CAB is found: and it will give an equation between the sine of the angle CAB first found, and the second found, the solution of which equation will reveal all sought.




*Scholium.*

But this work and labour are to find the proportions of the distances of the phenomenon from the sun ; however they are acquired with enough probable conjecture, through the change of the diameter [of the orbit that appears], the slowing down or speeding up of the central motion; yet everything most certainly has to be done from a combination of many observations: but our task has not been to set up non-geometrical methods to provide more detailed explanations.

Comment on Prop. 90 : The problem of determining the orbit of a comet had to await the coming of analytical methods as set out by Isaac Newton in the third book of the *Principia,* some 20 years later. It is interesting to note that Gregory seems to be hinting at such a development in the final sentence.

The End.

## Prop. 90. Problema.

*E datis tribus longitudinibus, & latitudinibus, Phaenomeni cujuslibet, circa solem agitati ; ipsius inaequalitatem secundam exuere ; orbis inclinationem cum ecliptica, & in unaquaq; observatione, Phaenomeni a sole distantiam investigare : suppositis tantum , telluris theoria, & rationibus distantiarum Phaenomeni a Sole.*

Sit arcus eclipticae AB, orbis Phaenomeni AC, tria loca Phaenomeni e terra observata O, P, Q, in suo orbe R, S, T. Quoniam loca phaenomeni e terra observata dantur ; dabuntur etiam ejusdem, vel a sole, vel ab oppositionibus solis, distantiae apparentes, nempe IO, LP, MQ; & arcus IL, LM ; & anguli OIA, PLA, QMA, & perProp. praecedentem innotescunt rationes sinuum, parallaxium RO, SP, TQ. Ponatur sinus arcus RO 1♃ ; dabantur igitur sinus arcuum SP, TQ in numeris cofficis :dantur autem sinus arcuum IO, LP, MQ ; deprehenduntur ergo sinus arcuum IR, LS, MT, in numeris cofficis. Et ex datis sinubus arcuum IR, LS, MT, cum sinuus angulorum RIF, SLD, TME ; inveniantur sinus arcuum perpendicularium RE, SD, TF, & sinus arcuum ED, DF. Deinde ex datis sinubus arcuum ER, DS, ED; quaeratur sinus anguli CAB: per 70 hujus:& eodem modo e datis sinubus arcuum DS, FT, DF, inquiratur sinus ejusdem anguli CAB: dabiturque aequatio inter sinum anguli CAB primo inventum, & eundem secundo inventum, cujus aequationis resolutio manifestabit omnia quaesita.

*Scholium.*

Proportiones autem distantiarum Phaenomeni a sole invenire, hic labor, hoc opus ; nihilominus acquiruntur probabili satis conjectura, per diametri apparentis mutationem, motus centrici tarditatem vel velocitatem; omnium tamen certissime ex multarum observationum collatione : sed nostri non est instituti methodos ageometricas fusius explicare.

FINIS.